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Abstract

This paper investigates long-run effects of inflation and deflation in a monetary life-cycle model that incorporates both capital stock and elastic labor supply as production factors. The model also introduces the zero lower bound on the nominal interest rate. The findings of this paper are twofold. First, in contrast to a result obtained from most neoclassical monetary models with an infinitely lived representative agent, the Friedman rule is not optimal and mild inflation can be desirable in this model. The Tobin effect on capital stock is encouraged by redistribution among households and therefore dominates distortionary effects of the inflation tax on labor supply and consumption. Importantly, the optimal rate of inflation depends on how inflation tax revenues are rebated to households. Second, there is a remarkable asymmetry in terms of welfare costs between inflation and deflation. For a lower rate of inflation than the rate that makes the nominal interest rate just zero, the Tobin effect works strongly in a deflationary direction because households are willing to hold more money, thus depressing aggregate output and social welfare significantly. This result reinforces the validity of pursuing mild inflation to evade the risk of hitting the zero lower bound.

Keywords: Friedman rule; Zero lower bound; Tobin effect; Inflation tax; Redistribution
JEL classification: E31, E58, O42

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1 Introduction

Since the 2000s most advanced countries have set a price stability target of around 2 percent per year. According to Hammond (2012), about half of the G20 countries had already adopted a full inflation targeting regime at the beginning of 2012. Now the Federal Reserve Bank, European Central Bank, and Bank of Japan also virtually have an inflation target rate of 2 percent. There are a few practical reasons why central banks place great importance on pursuing a price stability target of around 2 percent. One of these is a statistically technical issue involved in measurement errors (upward bias) of the consumer price index. Another reason is to ensure a preemptive margin against the risk of hitting the zero lower bound on the nominal interest rate in the presence of negative shocks, as proposed by Summers (1991) and others.[1]

From the perspective of monetary economics, however, it is not necessarily straightforward to ask how high a price stability target is desirable for economic welfare in the long run. Some classical monetary models propose the notion of the super-neutrality of money (for example, Sidrauski [1967]). Roughly speaking, this notion implies that the inflation rate does not affect real economic activity in the long run. In what is commonly known as the Friedman rule, Friedman (1969) argues that optimal monetary policy means setting the opportunity cost of holding money (the nominal interest rate) to zero. This argument holds robustly in many monetary models with modifications including the neoclassical growth model and shopping time model (for example, Chari, Christiano, and Kehoe [1996]). However, it is not acceptable to policy practitioners in general, because it implies that mild deflation is desirable as long as the real interest rate is moderately positive in the long run. On the other hand, in the standard New Keynesian economy with price stickiness, zero percent inflation becomes optimal because it does dissolve the distortion of resource allocations caused by variations in relative prices (for example, Rotemberg and Woodford [1998]). Based on recent research developments in monetary economics, Schmitt-Grohé and Uribe (2010) conclude that leading theories do not provide strong support for a significantly positive level of the inflation rate. These facts are very well known as showing a large gap between theory and practice in regard to the optimal rate of inflation. The objective of this paper is to provide a possible explanation to fill this gap.

In a neoclassical growth model into which money is introduced, there are two typical channels through which inflation affects households' economic decisions and hence real aggregates in the long

[1] In addition, some economists may emphasize consistency with a situation where people perceive general prices stable in the long run, building on the fact that the expected long-term inflation rate stayed stably around 2 percent in Europe and the United States before the occurrence of the financial crisis.

run: the Tobin effects (Tobin [1965]) and distortionary effects of the inflation tax (for example, Lucas and Stokey [1983] and Cooley and Hansen [1989]). The former is a channel affecting substitution between money and capital stock in the asset portfolios of households. Higher inflation reduces a real return on money holdings, thus shifting asset demand from money to capital stock. The latter is a channel affecting substitution between consumption and leisure, or between consumption that requires money at hand and consumption otherwise. Higher inflation shifts demand from consumption requiring cash to leisure not requiring cash. Consequently, the former (latter) is likely to have positive (negative) effects on aggregate output. An important question here is which channel is dominant quantitatively.[2] The literature mentioned above shows that the latter is dominant over the former and the Friedman rule is optimal for most neoclassical growth models with an infinitely lived representative agent. On the other hand, not many studies address these issues by taking account of agents' heterogeneity. Moreover, in a monetary growth model with a representative agent, it is impossible to differentiate effects on aggregate output and social welfare among steady states for various rates of deflation that drive the nominal interest rate to the zero lower bound.[3]

Against these issues, a strand of the literature (Ireland [2005], Bhattacharya, Haslag, and Russell [2005], Bhattacharya *et al.* [2008], and others) has rethought the Friedman rule using a monetary growth model with heterogeneous agents.[4] For whether the Friedman rule is optimal or suboptimal, these studies stress the importance of distributional effects as an additional channel through which inflation affects real economic activity. When the government redistributes inflation tax revenues equally among households in a lump-sum fashion, inflation induces resource transfers away from households with much money towards households with little money. As a result, this can lead to

[2] As empirical research on the long-run relationship between economic growth and inflation, for example, Barro (1996) finds a negative correlation between economic growth and inflation, at least for moderate and high inflation. On the contrary, Nikitin and Russell (2006) provide supportive evidence for the Tobin effect, namely, a positive relationship between economic growth and inflation for negative or positive but low rates of inflation.

[3] This comes from the fact that at a steady state the real interest rate is pinned down by the subjective discount factor from the Euler equation with respect to inter-temporal consumption. Therefore, the inflation rate is uniquely determined from the Fisher equation with the nominal interest rate zero, while real balance of money is not uniquely determined. Krugman (1998) and Svensson (1999) describe such a state of liquidity trap by employing monetary growth models with an infinitely lived representative agent. They warn that central banks cease to be able to stimulate the economy and to control price levels when the nominal interest rate is zero. However, they do not necessarily point out that deflation can be costly in terms of social welfare. In fact, the Friedman rule remains optimal in their models. See Ireland (2005) as for a detailed discussion.

[4] Ireland (2005) introduces money into á la Blanchard-Yaari model that can take account of differences in the birth date of households and the population growth rate. He shows that the model can identify welfare costs of different rates of deflation in the presence of the zero lower bound on the nominal interest rate. Bhattacharya, Haslag, and Russell (2005) incorporate money into an overlapping generations model that can deal with finite lifespan as well. They discuss differences from a monetary model with an infinitely lived representative agent. Bhattacharya *et al.* (2008) extend the standard pure exchange model by including two types of agents that differ in their preference for holding money.

improving considerably the utility of poor households with little money under positive inflation, thus breaking the optimality of the Friedman rule. While these preceding studies take account of households' heterogeneity, they make simplification assumptions on production technology to derive intuitive insights analytically: in particular, in that there exists no capital stock and/or in that labor supply is inelastic.[5] Therefore, they do not capture sufficiently the above-mentioned trade-off between the Tobin effect and distortionary effects of the inflation tax.

In contrast to these preceding studies, this paper constructs a monetary life-cycle model that incorporates both capital stock and elastic labor supply to take into consideration not only the Tobin effect but also distortionary effects of the inflation tax on labor supply. In evaluating the effects of deflation as well, this model also has the zero lower bound imposed on the nominal interest rate. Furthermore, it is worthwhile to note that the impact of redistribution among households induced by inflation can vary with realistic age-specific profiles of money holdings, labor supply, and consumption. In this regard, the model is calibrated to match both key macroeconomic variables and age-specific profiles in the Japanese economy. In other words, this paper's contribution is to make quantitative assessments of effects of inflation and deflation on aggregate output and social welfare (the relative importance of the Tobin effect and the inflation tax given redistribution among households) using an empirically realistic model in terms of life-cycle profiles. The computational results show that redistribution among households and the associated Tobin effect play important roles in evaluating aggregate output and welfare costs for various rates of inflation and deflation. Moreover, if redistribution among households is an important determinant for the optimal inflation rate as highlighted by the preceding studies (and this paper), the optimal rate is likely to vary according to how the government adjusts variations in fiscal space produced by the inflation tax. Therefore, this paper computes aggregates and welfare costs at steady states for different levels of the inflation rate under different fiscal schemes that replace the inflation tax by another tax. In this sense, this paper also adds quantitative experiments on the relationship between optimal inflation and government tax policy to theoretical analyses conducted by the preceding studies.

The remainder of this paper is organized as follows. Section 2 describes the setup of our model.

[5] To the best of my knowledge, there have already been a few studies that address the optimality of long-run inflation using a two-period overlapping generations model that incorporates capital stock as well as money *under the assumption that labor is inelastically supplied.* Palivos (2005) introduces money demand in the form of money-in-the-utility, and focuses on the role of distributional effects of inflation by allowing for a difference in the preference for altruism. He suggests that a small but positive rate of inflation can be desirable even if the Tobin effect does not work. Bhattacharya, Haslag, and Martin (2009) formulate the role of money in the specification of random relocation with informational and spatial constraints, and highlight the importance of neoclassical production technology with knowledge externality for the determination of the optimal inflation rate. They show that the Tobin effect is always operative with neoclassical production technology and shifts up the optimal inflation rate in a positive direction. These preceding studies are most closely related to this paper.

Section 3 explains calibration methodology and data sources used in this paper. Section 4 provides simulation results on long-run effects of inflation and deflation in the Japanese economy. Section 5 concludes.

2 Model

This section presents the model and defines the competitive equilibrium. The model is a neoclassical growth model with overlapping generations that consists of three sectors: a firm, households, and government.[6] There exist competitive factor markets for production: labor and capital inputs. The features of the model are (1) to incorporate money through the cash-in-advance (CIA) constraint, (2) to impose the zero lower bound on the nominal interest rate, and (3) to be calibrated so that it can replicate households' life-cycle profiles of money holdings, labor supply, and consumption.

2.1 Production Technology

There is a representative firm that produces goods with a Cobb-Douglas constant returns to scale technology given the income share of capital, α:

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}, \tag{1}$$

where Y_t, L_t, and K_t are aggregate output, labor and capital inputs at time t. Z_t is deterministic total factor productivity at time t, which grows at the rate $g_t^{1-\alpha}$. The firm rents capital at the rental rate R_t and hires labor at the wage rate w_t from households in competitive markets. The firm's profit maximization yields the first-order conditions:

$$\max_{L_t,\, K_t} Y_t - w_t L_t - R_t K_t;$$

$$w_t = (1-\alpha) Z_t \left(\frac{K_t}{L_t}\right)^{\alpha} \quad \text{and} \quad R_t = \alpha Z_t \left(\frac{K_t}{L_t}\right)^{\alpha-1}. \tag{2}$$

The rate of return on capital r_t is defined as the marginal product of capital net of depreciation of capital: $r_t = R_t - \delta$, where δ is the depreciation rate of capital. Here the real interest rate is defined as the after-tax rate of return on capital: $x_t = \left(1 - \tau_t^k\right) r_t$, where τ_t^k is the tax rate on capital income levied by the government. With aggregate investment I_t at time t, aggregate capital

[6] An advantage in using an overlapping generations model is that it is easier to replicate realistic hump-shaped profiles of consumption, labor supply, and asset holdings. This model is also computationally tractable with the zero lower bound occasionally binding.

stock evolves as follows:

$$K_{t+1} = I_t + (1-\delta) K_t. \tag{3}$$

2.2 Government

The government raises revenues by injecting money into the economy according to the rule at the growth rate σ_t:

$$M_{t+1} = (1+\sigma_t) M_t, \tag{4}$$

where M_t is aggregate quantity of money at the beginning of time t. The government sets the growth rate of money supply to control the inflation rate, $\pi_t = P_t/P_{t-1} - 1$, where P_t is the price level at time t. The government also raises revenues by issuing one-period government bonds and by levying taxes on households' capital income, labor income, and consumption at the flat rates, τ_t^k, τ_t^l, and τ_t^c, to finance its expenditure that is the sum of government purchases and social transfer. The budget constraint is expressed as follows:

$$P_t G_t + M_t + (1+i_t) B_t + P_t SS_t = M_{t+1} + B_{t+1} + P_t \tau_t^c C_t + P_t \tau_t^l w_t L_t + P_t \tau_t^k r_t K_t + P_t \tau_t, \tag{5}$$

where G_t, B_t, C_t, and τ_t are government purchases, government bonds, and aggregate consumption, lump-sum tax/transfer at time t. SS_t is the total amount of social security transfer to retirees. The nominal interest rate at time t is defined as the Fischer equation:

$$1 + i_t = (1+\pi_t)(1+x_t) \geq 1. \tag{6}$$

There is the zero lower bound on the nominal interest rate. I impose the present value condition on the real returns on capital and bonds so that agents are indifferent to the choice between these two assets. Government bonds and government purchases are set constant exogenously in this model. In the subsequent simulations, I also assume that the government adjusts one of the tax instruments (the lump-sum tax/transfer and the tax rates on consumption, labor income, and capital income, $\{\tau_t, \tau_t^c, \tau_t^l, \tau_t^k\}$) to satisfy the budget constraint with the other instruments kept exogenously constant.

2.3 Households

The household sector comprises J cohorts of households. At every time t, a new generation of households comes into the economy, while the other existing generations of households become older by one age and the oldest generation of households of age J exits from the economy. Each household can be economically active for J periods (from age 1 to age J), but is exposed to mortality risk. Namely, households of age $j-1$ at time $t-1$ are faced with the conditional probability, $\psi_{j,t}$, that they can survive to age j at time t, where $\psi_{J+1,t}=0$ by assumption. The population of age j households and the growth rate of the new generation at time t are given by

$$n_{j,t}=\psi_{j,t}n_{j-1,t-1} \quad \text{and} \quad n_{1,t}=\left(1+\rho_{1,t}\right)n_{1,t-1}. \tag{7}$$

The total population of households at time t and the population share of households of age j at time t are given by

$$N_t=\sum_{j=1}^{J}n_{j,t} \quad \text{and} \quad \mu_{j,t}=\frac{n_{j,t}}{N_t}. \tag{8}$$

The population growth rate at time t is given by $1+\rho_t=N_{t+1}/N_t$.

Until mandatory retirement, households of age j at time t supply labor $h_{j,t}$ for production and earn wage income according to their age-specific labor efficiency ε_j. Households retire at age T and thereafter receive social security benefits ss_t from the government, which is assumed to cover a certain proportion θ of the average labor income at the contemporaneous time: $ss_t=\theta w_t L_t$. Then the total amount in (5) is expressed by

$$SS_t=\sum_{j=T}^{J}n_{j,t}ss_t=\theta w_t L_t\sum_{j=T}^{J}n_{j,t}. \tag{9}$$

Households own capital to rent it for production, hold government bonds to earn the interest rate, and hold money to buy some goods. $k_{j,t}$, $b_{j,t}$ and $m_{j,t}$ are capital, bond, and money holdings of age j households at the beginning of time t. Following the definition of Lucas and Stokey (1983) described later, I assume that households consume "cash goods" and "credit goods" at time t, denoted by $c^s_{j,t}$ and $c^d_{j,t}$, respectively. Here I define by $c_{j,t}$ and $a_{j,t}$ consumption and real asset holdings of age j households at time t: $c_{j,t}=c^s_{j,t}+c^d_{j,t}$ and $a_{j,t}=k_{j,t}+m_{j,t}/P_{t-1}+b_{j,t}/P_{t-1}$. I assume that households have a bequest motive and obtain a direct utility from leaving their assets at age J for their descendants who are born into the economy in the next period. With this assumption, the new born households can consume cash goods in the initial period. In addition, all households alive at time t receive equally accidental bequests from households that die at time $t-1$. The total amount of accidental bequests at time t is given by

$$\varsigma_t=\sum_{j=2}^{J}\left(1-\psi_{j,t}\right)n_{j-1,t-1}a_{j,t}. \tag{10}$$

Households born at time s choose sequences of consumption $\left\{c_{j,t}^s, c_{j,t}^d\right\}$, labor inputs $h_{j,t}$, capital, bond, and money holdings $\{k_{j,t}, m_{j,t}, b_{j,t}\}$, and intended bequests $a_{J+1,s+J}$, where $t = s-1+j$ holds for $j = 1, ..., J$, in order to maximize their expected lifetime utility discounted by the subjective discount factor β:

$$E\left[\sum_{j=1}^{J}\beta^{j-1}\left(\prod_{i=1}^{j}\psi_{i,s-1+i}\right)u\left(c_{j,t}^s, c_{j,t}^d, 1-h_{j,t}\right)+\beta^J\left(\prod_{i=1}^{J}\psi_{i,s-1+i}\right)U\left(a_{J+1,s+J}\right)\right]. \quad (11)$$

$u(\cdot)$ and $U(\cdot)$ are instantaneous utility functions from consumption and leisure and from intended bequests. Then the budget constraints over their lifetime are given by

$$\begin{aligned} & P_t\left(1+\tau_t^c\right)\left(c_{j,t}^s + c_{j,t}^d\right) + P_t k_{j+1,t+1} + m_{j+1,t+1} + b_{j+1,t+1} \qquad \text{for} \quad j < T \\ = \;& P_t(1-\tau_t^l)w_t\varepsilon_j h_{j,t} - P_t\tau_t + P_t\left(1+x_t\right)k_{j,t} + m_{j,t} + \left(1+i_t\right)b_{j,t} + P_t\varsigma_t; \qquad (12) \\ & P_t\left(1+\tau_t^c\right)\left(c_{j,t}^s + c_{j,t}^d\right) + P_t k_{j+1,t+1} + m_{j+1,t+1} + b_{j+1,t+1} \qquad \text{for} \quad j \geq T \\ = \;& P_t ss_t - P_t\tau_t + P_t\left(1+x_t\right)k_{j,t} + m_{j,t} + \left(1+i_t\right)b_{j,t} + P_t\varsigma_t. \qquad (13) \end{aligned}$$

In addition, households are required to hold some money at the beginning of the period to consume cash goods. In other words, households are subject to the cash-in-advance (CIA) constraint as proposed by Lucas and Stokey (1983):[7]

$$P_t\left(1+\tau_t^c\right)c_{j,t}^s \leq m_{j,t}. \quad (14)$$

When the nominal interest rate is positive ($i_t > 0$), households economize on holding money, implying that this constraint holds with equality. Then higher inflation causes substitution from cash goods to credit goods and leisure that are not subject to the CIA constraint. When the nominal interest rate is zero ($i_t = 0$), households are willing to hold as much money as is supplied by the government, implying that it holds with inequality.

2.4 Competitive Equilibrium

An equilibrium consists of sequences of prices $\{r_t \text{ (or } x_t),\, w_t,\, \pi_t \text{ (or } i_t)\}$, households' decisions $\left\{c_{j,t}^s,\, c_{j,t}^d,\, h_{j,t},\, k_{j+1,t+1},\, m_{j+1,t+1},\, b_{j+1,t+1}\right\}$, government policy $\left\{G_t,\, M_t,\, B_t,\, \tau_t,\, \tau_t^l,\, \tau_t^c,\, \tau_t^k,\, ss_t\right\}$, and aggregate factor inputs $\{K_t,\, L_t\}$ such that, at every time t, (i) households maximize their lifetime utility (11) subject to their budget constraints (12) and (13) and CIA constraints (14); (ii) factor

[7] As discussed in Section 3, the cash-credit model enables us to reproduce the actual money-output ratio by calibrating the value for the weight on cash goods in the utility function.

prices are determined by (2) from the firm's profit maximization; (*iii*) the government satisfies the money supply rule (4) and the budget constraint (5); (*iv*) the market clearing conditions hold:

$$P_t K_{t+1} + M_{t+1} + B_{t+1} = P_t \sum_{j=1}^{J} n_{j,t} a_{j,t+1} \quad \text{for the asset market;} \tag{15}$$

$$L_t = \sum_{j=1}^{T-1} n_{j,t} \varepsilon_j h_{j,t} \quad \text{for the labor market;} \tag{16}$$

$$M_t = \sum_{j=1}^{J} n_{j,t} m_{j,t} \quad \text{for the money market;} \tag{17}$$

$$Y_t = C_t + I_t + G_t \quad \text{for the final good market, where } C_t = \sum_{j=1}^{J} n_{j,t} c_{j,t}. \tag{18}$$

3 Calibration

One period of the model corresponds to a decade. The household sector is classified into eight age groups ($J = 8$): Under 30, 30s, 40s, 50s, 60s, 70s, 80s, and 90s.[8] Mandatory retirement is applied to the 80s ($T = 7$).[9] Table 1 summarizes the values for main calibrated parameters and exogenous variables. All the values are expressed on the annual basis. Most of these values are calculated from data for either the average of the 2000s or the year 2009. Almost all of the values for technology parameters and policy parameters are calculated from the Japanese National Accounts,[10] based on the methodology presented by Hayashi and Prescott (2002). The baseline value of the ratio of lump-sum tax to output is set to satisfy the government budget constraint (5) with the baseline values of the others and zero inflation rate.

The population growth rate ρ_t and the surviving probability by 10-year age group $\{\psi_j\}_{j=1}^{8}$ are calculated from the Annual Report on Current Population Estimate (ARCPE), published by the Statistics Bureau of the Ministry of International Affairs and Communication. The age-specific profile of labor efficiency $\{\varepsilon_j\}_{j=1}^{6}$ is obtained from Braun, Ikeda, and Joines (2009). This is based on the Basic Survey and Wage Structure (BSWS), administered by the Ministry of Health, Labour and Welfare.

[8] Data on age-specific profiles are provided by the five-year or 10-year age group in the Japanese statistics used in this paper. The 70s, 80s, and 90s in the model are categorized as Over 70 in the Japanese statistics.

[9] Although mandatory retirement for many Japanese firms is usually set between age 60 and 65, Japanese labor statistics show that labor force participation above age 65 (even for the 70s) is significantly above zero in Japan.

[10] I use the 1993 System of National Accounts on a year 2000 basis.

I assume the following functional forms of instantaneous utility from consumption and leisure and from intended bequests:

$$u\left(c_{j,t}^{s}, c_{j,t}^{d}, 1-h_{j,t}\right) = \frac{\left[\left(c_{j,t}^{s}/\kappa_j\right)^{\gamma_j}\left(c_{j,t}^{d}/\kappa_j\right)^{1-\gamma_j}\left(1-h_{j,t}\right)^{\lambda_j}\right]^{1-\eta}}{1-\eta}, \quad (19)$$

$$U\left(a_{J+1,t}\right) = \xi\frac{\left(a_{J+1,t}\right)^{1-\eta}}{1-\eta}, \quad (20)$$

where with $\lambda_j = 0$ for $j = 7$ and 8 by construction. The family scale factor $\{\kappa_j\}_{j=1}^{8}$ is based on the number of dependent children at the parents' age j.[11] This number is obtained from the National Survey of Family Income and Expenditure (NSFIE), administered by the Ministry of Health, Labour and Welfare. The intertemporal elasticity of substitution η is set at a standard value of 2.0. The subjective discount factor β is set to match the capital-GNP ratio, 2.86 in 2009. The weight on intended bequests ξ is set at such a value that can replicate the wealth ratio of the oldest three cohorts (over 70: 70s, 80s, and 90s in the model) to the youngest (Under 30). This wealth ratio is estimated from the NSFIE. The weights on cash good consumption and on leisure, $\left\{\gamma_j\right\}_{j=1}^{8}$ and $\{\lambda_j\}_{j=1}^{6}$, are chosen so that the model-generated profiles can mimic well the actual profiles of money holdings and labor supply, respectively.[12,13] The former is estimated from the NSFIE and its level is rescaled so that the model-generated money-output ratio matches the average value of the M1-GNP ratio, 0.26 during the period before 1995.[14] The latter is calculated from the BSWS and the Labor Force Survey (LFS), and its level is rescaled so that the model-generated hours worked are about 35 percent of the available time endowment.[15]

The profiles of these preference weights are shown in Figure 1 together with the actual profiles of consumption, labor supply, money and total asset holdings. As this figure shows, the model

[11] Following a few studies (for example, Nishiyama and Smetters [2005]), I set $\kappa_j = \left(1+\frac{\nu_j}{2}\right)^{\frac{1}{4}}$ where ν_j is the number of dependent children at the parents' age j.

[12] Unfortunately, there are no data on the age-specific profile of cash holdings available from the NSFIE. Therefore, as an alternative, I use data on age-specific profiles of deposits as a proxy for an age-specific profile of money holdings.

[13] Here recall that households' money demand is formulated by the CIA constraint as transaction motive to purchase cash goods in the model. The weight on cash goods is supposed to capture other motives to hold money (for example, demand for its liquidity and safety characteristic observed in reality) as well when this parameter is calibrated to match the model-generated profile of money holdings with the actual one. In addition, this parameter is assumed to differ with age. As shown in Figure 1, this parameter is higher for older households. This might imply that older households with shorter life expectancy put a higher value on the liquidity and safety characteristic of money.

[14] Preceding studies such as Cooley and Hansen (1989) define money in the CIA model as M1, based on the fact that the ratio of M1 to output (or consumption of non-durables and services) moved stably in the past. I follow this convention, although the period over which money is to be held is longer in this paper than in preceding studies. As we observe movements in the nominal interest rate on ordinary deposits in Japan, it turns out that the zero interest rate started virtually from 1995. After 1995, the M1-GNP ratio increased sharply due to low interest rates and deflation, while it was relatively stable before that year. In this paper, I refer to the average value of the M1-GNP ratio until 1994 as the standard value that represents Japanese households' preference for holding money.

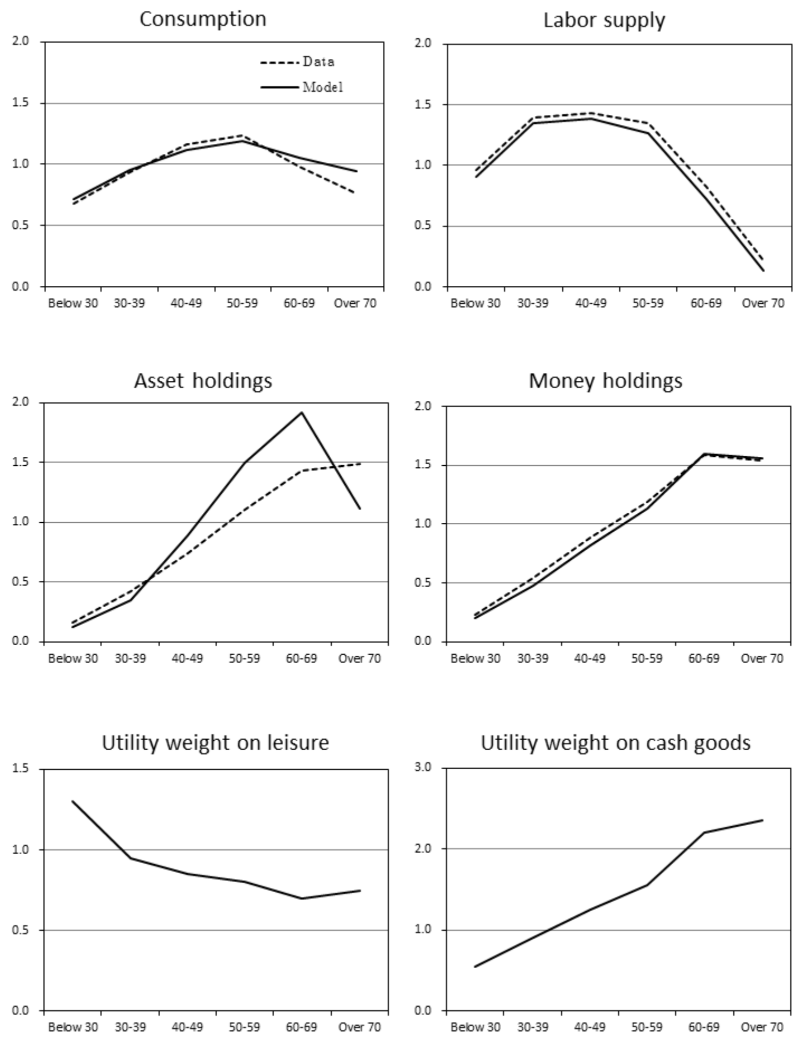
[15] Here the labor supply of households is defined as hours worked (from the BSWS) times the employment rate (from the LFS).

can replicate reasonably general patterns of life-cycle profiles. Consumption takes a hump-shape with a peak at the 50s. Labor supply is relatively small at the youngest group (the Under 30) and declines largely at the 60s and 70s. Money holdings and assets holdings (net worth) keep increasing until the 60s. The young (the Under 30 and 30s) have a smaller amount of assets (money) than half of the average, while the old (the 60s and Over 70) have a larger amount of assets (money). Therefore, inflation induces redistribution from the 60s and Over 70 to the Under 30 and 30s when the government returns inflation tax revenues to households equally in a lump-sum way.

Table 1: Calibrated Parameters and Baseline Exogenous Variables

Parameter		Value	Target
Demographics, etc.			
Population growth rate	ρ	0.00	Data (ARCPE)
Surviving probability	$\{\psi_j\}_{j=1}^{8}$	—	Data (ARCPE)
Family scale	$\{\kappa_j\}_{j=1}^{8}$	—	Data (NSFIE)
Labor efficiency	$\{\varepsilon_j\}_{j=1}^{6}$	—	Data (BSWS) and Braun, Ikeda, and Joines (2009)
Preference			
Discount factor	β	1.013	$K/Y = 2.86$
Risk aversion	η	2.0	Assumed
Weight on cash goods	$\{\gamma_j\}_{j=1}^{8}$	Fig. 1	Age profile of money holdings (NSFIE) and $M/Y = 0.26$
Weight on leisure	$\{\lambda_j\}_{j=1}^{6}$	Fig. 1	Age profile of hours worked (BSWS) and average hours worked = 0.35
Weight on bequests	ξ	0.034	$a_{\text{over70}}/a_{\text{under30}} = 9.2$
Technology			
Capital share	α	0.396	Hayashi and Prescott (2002) updated
Depreciation rate	δ	0.078	Hayashi and Prescott (2002) updated
Growth rate	g	0.01	Hayashi and Prescott (2002) updated
Government policy			
Replacement ratio	θ	0.40	Oshio and Yashiro (1997) and others
Labor income tax rate	τ^l	0.089	Hayashi and Prescott (2002) updated
Consumption tax rate	τ^c	0.080	Hayashi and Prescott (2002) updated
Capital income tax rate	τ^k	0.362	Hayashi and Prescott (2002) updated
Lump–sum tax-output ratio	τ/Y	0.074	Government budget constraint (5)
Purchases-output ratio	G/Y	0.194	Hayashi and Prescott (2002) updated
Debt-output ratio	B/Y	1.17	Data (Flow of Funds, etc.)

Figure 1: Age-Specific Profiles



Note: Each of the first four panels is normalized by the average that is equal to unity.

4 Results

In this section, I show the computational results of aggregates and welfare costs at steady states for different levels of the inflation rate.[16] The optimal inflation rate here is the rate that minimizes the welfare cost. To begin with, I show the importance of redistribution among households induced by inflation for the optimal inflation rate. To this end, following a few preceding studies,

[16] Following the convention in steady-state welfare analyses conducted by an overlapping generations model, I define social welfare as households' expected lifetime utility (11) in this paper. The welfare cost is measured by the so-called consumption equivalent variation χ, which is defined as the percentage increment in consumption that makes households as well off at the inflation rate as at zero percent inflation rate:

$$W_0 = \sum_{j=1}^{J} \beta^{j-1} \varphi_j u \left((1+\chi) c_j^s, (1+\chi) c_j^d, 1-h_j \right) + \beta^J \varphi_J U \left((1+\chi) a_{J+1} \right),$$

where W_0 is social welfare at the steady state of zero percent inflation rate and $\varphi_j = \Pi_{i=1}^{j} \psi_i$ is the unconditional probability of surviving from birth to age j. Here time subscripts are omitted.

I compare results under a fiscal scheme——(a) a complementary tax/transfer——which neutralizes distributional effects of inflation, with those under another fiscal scheme——(b) a lump-sum tax/transfer——which is commonly assumed in many monetary models.

Next, I turn to three fiscal schemes that rely on a distortionary tax as a realistic source of government revenues: (c) labor income tax, (d) consumption tax, and (e) capital income tax. The government may be able to utilize only distortionary taxes (the government may not be able to utilize a lump-sum tax/transfer) to finance its expenditure in reality. Then the government should compensate revenue losses due to lower inflation by increasing the distortionary taxes. Phelps (1973) is known to have pointed out the possibility that the Friedman rule may become suboptimal from this perspective of public finance. However, this conclusion has not widely supported by theoretical analyses based on many monetary models with an infinitely lived representative agent (for example, Chari, Christiano, and Kehoe [1996]). In contrast, in the presence of redistribution among heterogeneous households, I show that the Friedman rule is not optimal when the inflation tax is replaced by other distortionary taxes. I also show that redistribution among households (and hence the optimal rate of inflation) depends on which distortionary tax the government adjusts in response to variations in the inflation tax. These findings have something in common with results in the optimal tax literature.

4.1 Complementary Tax/Transfer

In scheme (a), the government is assumed to rebate the same amount of resource as is levied on households by inflation to the households. In other words, each household is fully compensated for the inflation tax by the government, because it can receive as many resources as it has lost due to inflation. Here distributional effects (redistribution) of the inflation tax are offset by the additional fiscal transfer $\widetilde{\tau}_j^m$ that differ with money holdings by age, as long as the nominal interest rate is positive:

$$\widetilde{\tau}_j^m = \frac{\pi}{1+\pi}\widetilde{m}_j \quad \text{for} \quad i > 0; \qquad \widetilde{\tau}_j^m = \frac{\pi}{1+\pi}\widetilde{M} \quad \text{for} \quad i = 0.$$

Bhattacharya, Haslag, and Russell (2005) and Ireland (2005) apply such an experimental scheme and show that the Friedman rule becomes optimal once again. The same thing holds in the model that incorporates both capital stock and elastic labor supply as neoclassical production factors.

Table 2 reports aggregates and welfare costs for different rates of inflation under scheme (a). The welfare cost is minimized around the inflation rate of -4 percent that makes the nominal interest rate just zero. As the inflation rate rises above the optimal rate, labor supply, output, and consumption falls. This obstructs capital accumulation simultaneously. In other words, distortionary effects of the inflation tax are dominant over the Tobin effect, as Cooley and Hansen (1989) have shown using the CIA model with an infinitely lived representative agent.

On the other hand, once the inflation rate falls below the optimal rate, then the real balance of money increases sharply and in return capital stock declines sharply. This means that the Tobin effect is strongly operative in a deflationary direction. Once the nominal interest rate reaches zero, money and capital become perfect substitutes. Then households are willing to hold more money as deflation raises the real return on money. On the reverse side, capital stock is crowded out by money so that the real return on capital is equalized to that on money. As a result of the sharp decline in capital stock, output and social welfare decrease dramatically. Labor supply also decreases slightly because of both the wealth effect on older working households with much money and the substitution effect on young working households that suffer from the decline in the real wage rate. Even with an inflation rate that is only 0.4 percentage point lower than the optimal rate, the welfare cost worsens by more than 4 percentage points. The welfare loss at −4.5 percent deflation (3.8 percent) is much larger than that at 5 percent inflation (1.4 percent). There is a striking asymmetry in terms of the welfare cost between inflation and deflation in the presence of the zero lower bound on the nominal interest rate.

Table 2: Effects of Steady-State Inflation (1)

π (%)	i (%)	w	χ (%)	Y	L	C	K/Y	M/Y
10.0	14.8	0.998	3.103	0.982	0.984	0.986	2.835	0.265
5.0	9.6	0.998	1.440	0.988	0.990	0.990	2.834	0.262
2.0	6.5	0.999	0.537	0.994	0.995	0.995	2.838	0.260
1.0	5.4	0.999	0.261	0.997	0.997	0.997	2.841	0.259
0.0	4.4	1.000	0.000	1.000	1.000	1.000	2.844	0.258
−1.0	3.3	1.001	−0.242	1.004	1.003	1.003	2.848	0.257
−4.1	0.0	1.005	−0.840	1.019	1.014	1.015	2.866	0.252
−4.5	0.0	0.963	3.849	0.970	1.007	0.984	2.686	0.612

All the values are expressed on an annual basis.

$w, Y, L,$ and C are normalized by the value for zero percent inflation.

4.2 Lump-Sum Tax/Transfer

In scheme (b), the government is assumed to rebate inflation tax revenues equally among all the living households in a lump sum way, $\widetilde{\tau}^m$:

$$\widetilde{\tau}^m = \frac{\pi}{1+\pi}\widetilde{M} \quad \text{for all households.}$$

Old households that hold more money than the average lose part of their resources because the inflation tax levied on their money holdings exceeds this additional lump-sum transfer:

$$\frac{\pi}{1+\pi}\widetilde{m}_j > \frac{\pi}{1+\pi}\widetilde{M} = \widetilde{\tau}^m \quad \text{for cohort } j \text{ such that } \widetilde{m}_j > \widetilde{M}.$$

On the contrary, young households benefit from this scheme. There occurs redistribution from old households with more money to young households with less money.

Table 3 reports aggregates and welfare costs for different rates of inflation under scheme (b). The welfare cost is minimized around the inflation rate of zero percent, implying that the optimality of the Friedman rule does not hold any longer. As the inflation rate increases above the optimal rate, labor supply decreases, but capital stock and output increase. In other words, the Tobin effect dominates distortionary effects of the inflation tax on labor supply. The reason for this observation is that young households have a higher propensity to save, while old households have a higher propensity to consume in the life-cycle model. Consequently, the redistribution from the old to the young encourages capital accumulation in the whole economy. Although output expands with higher inflation, consumption declines, because old households cut their consumption due to the negative redistribution and because all the households refrain from cash good consumption. Similarly to the result observed in scheme (a), the Tobin effect works strongly in a deflationary direction when the inflation rate falls below the rate that makes the nominal interest rate zero. In the following subsections, I consider three types of distortionary tax that the government adjusts to return inflation tax revenues to households.

Table 3: Effects of Steady-State Inflation (2)

π (%)	i (%)	w	χ (%)	Y	L	C	K/Y	M/Y
10.0	14.6	1.016	1.398	1.005	0.989	0.998	2.918	0.266
5.0	9.5	1.009	0.384	1.002	0.993	0.998	2.888	0.263
2.0	6.4	1.004	0.052	1.000	0.997	0.999	2.866	0.260
1.0	5.4	1.002	0.006	1.000	0.998	0.999	2.858	0.259
0.0	4.3	1.000	0.000	1.000	1.000	1.000	2.850	0.258
−1.0	3.3	0.998	0.039	1.000	1.002	1.001	2.841	0.257
−4.3	0.0	0.985	1.198	0.994	1.009	1.000	2.784	0.323
−4.5	0.0	0.962	3.989	0.968	1.006	0.983	2.686	0.612

All the values are expressed on an annual basis.

w, Y, L, and C are normalized by the value for zero percent inflation.

4.3 Labor Income Tax

In scheme (c), the government is assumed to rebate inflation tax revenues to working households by reducing labor income tax.[17] Table 4 reports aggregates and welfare costs for different rates of inflation under scheme (c). The welfare cost is minimized around the inflation rate of 2 percent, implying that the optimality of the Friedman rule does not hold any longer.

The point is that distortion on labor supply is mitigated by reduction in the labor income tax. Lower labor income tax encourages labor supply. Here note that this scheme intensifies redistribution from old households to young households, because only working households receive a sort of additional fiscal transfer financed by the inflation tax through reduction in the labor income tax. This slightly strengthen the Tobin effect. Consequently, compared with the results observed in scheme (b), output expands and the welfare cost decreases in inflationary cases because both capital accumulation and labor supply increase with higher inflation. On the other hand, relatively to the results in scheme (b), the welfare cost increases in deflationary cases because deflation leads to a higher distortionary tax on labor supply, thus discouraging labor supply and dampening output further.

Table 4: Effects of Steady-State Inflation (3)

π (%)	i (%)	w	χ (%)	Y	L	C	K/Y	M/Y
10.0	14.6	1.018	0.846	1.027	1.009	1.019	2.926	0.266
5.0	9.5	1.010	0.047	1.016	1.005	1.011	2.892	0.263
2.0	6.4	1.004	−0.103	1.007	1.002	1.005	2.868	0.260
1.0	5.4	1.002	−0.075	1.004	1.001	1.002	2.859	0.259
0.0	4.4	1.000	0.000	1.000	1.000	1.000	2.849	0.258
−1.0	3.3	0.998	0.128	0.996	0.999	0.997	2.838	0.257
−4.3	0.0	0.985	1.485	0.975	0.990	0.981	2.784	0.295
−4.5	0.0	0.962	4.714	0.930	0.966	0.945	2.686	0.567

All the values are expressed on an annual basis.

w, Y, L, and C are normalized by the value for zero percent inflation.

4.4 Consumption Tax

In scheme (d), the government is assumed to rebate inflation tax revenues to households by reducing consumption tax.[18] Table 5 reports aggregates and welfare costs for different rates of inflation under scheme (d). The welfare cost is minimized around the inflation rate of −1 percent, implying that

[17] The tax rate on labor income τ^l adjusts so that the government budget constraint (5) should be satisfied.

[18] The tax rate on consumption τ^c adjusts so that the government budget constraint (5) should be satisfied.

the optimality of the Friedman rule does not hold any longer.

The point is that capital accumulation is not much encouraged although distortion on consumption and labor supply is mitigated by reduction in the consumption tax. Lower consumption tax shifts households' demand away from leisure toward consumption. Consequently, compared with the results observed in scheme (b), consumption and labor supply increase, but output decreases slightly in inflationary cases because capital accumulation declines. Here note that scheme (d) does not intensify redistribution from old households to young households, because all the households receive a kind of additional fiscal transfer according to their consumption. Although inflation levies more tax amount on old households, old working households (in particular, the 50s and 60s) can receive more an additional fiscal transfer because they consume more. In other words, old households are somewhat compensated for higher inflation tax by lower consumption tax, relative to the results in Scheme (b). Consequently, the Tobin effect weakens and the welfare cost increases more in inflationary cases under scheme (d) than under scheme (b). On the other hand, relative to the results in scheme (b), the welfare cost increases in the cases of deflation with zero nominal interest rate because deflation leads to a higher distortionary tax on consumption.

Table 5: Effects of Steady-State Inflation (4)

π (%)	i (%)	w	χ (%)	Y	L	C	K/Y	M/Y
10.0	14.7	1.009	1.827	1.013	1.003	1.008	2.894	0.260
5.0	9.5	1.005	0.654	1.007	1.002	1.004	2.875	0.259
2.0	6.4	1.002	0.178	1.003	1.001	1.002	2.863	0.259
1.0	5.4	1.001	0.073	1.001	1.000	1.001	2.858	0.259
0.0	4.3	1.000	0.000	1.000	1.000	1.000	2.854	0.259
−1.0	3.3	0.999	−0.036	0.999	1.000	0.999	2.850	0.259
−4.3	0.0	0.984	1.747	0.970	0.986	0.976	2.784	0.486
−4.5	0.0	0.961	5.113	0.922	0.960	0.937	2.686	0.946

All the values are expressed on an annual basis.

w, Y, L, and C are normalized by the value for zero percent inflation.

4.5 Capital Income Tax

In scheme (e), the government is assumed to rebate inflation tax revenues to households by reducing capital income tax.[19] Table 6 reports aggregates and welfare costs for different rates of inflation under scheme (e). The welfare cost is minimized around the inflation rate of 2 percent, implying

[19] The tax rate on capital income τ^k adjusts so that the government budget constraint (5) should be satisfied.

that the optimality of the Friedman rule does not hold any longer.

The point is that distortion on capital accumulation is mitigated by reduction in the capital income tax. Higher inflation raises the (after-tax) real return on capital and thus encourages capital accumulation, while still discouraging labor supply. Output increases with higher inflation because the former (Tobin effect) dominates the latter (inflation tax). Compared with the results observed in scheme (b), output increases and the welfare cost decreases in inflationary cases. Similar to scheme (d), scheme (e) weakens redistribution from old households to young households because all the households receive a kind of additional fiscal transfer according to their asset holdings. Old households can receive more additional fiscal transfer because they own more assets than young households. In other words, old households are well compensated for higher inflation tax by lower capital income tax. Nonetheless, capital accumulation is encouraged because of reduction in the capital income tax. On the other hand, relatively to the results in the other schemes, the welfare cost expands greatly in deflationary cases because deflation leads to a higher distortionary tax on capital accumulation.

Table 6: Effects of Steady-State Inflation (5)

π (%)	i (%)	w	χ (%)	Y	L	C	K/Y	M/Y
10.0	14.8	1.034	1.017	1.029	0.996	1.014	2.985	0.262
5.0	9.5	1.020	0.132	1.017	0.997	1.008	2.925	0.260
2.0	6.4	1.009	−0.071	1.007	0.998	1.003	2.877	0.259
1.0	5.4	1.005	−0.060	1.004	0.999	1.002	2.859	0.259
0.0	4.4	1.000	0.000	1.000	1.000	1.000	2.838	0.259
−1.0	3.3	0.995	0.116	0.996	1.001	0.998	2.816	0.258
−4.3	0.0	0.878	9.499	0.863	0.984	0.906	2.327	0.963
−4.5	0.0	0.739	26.356	0.706	0.956	0.778	1.788	1.891

All the values are expressed on an annual basis.

w, Y, L, and C are normalized by the value for zero percent inflation.

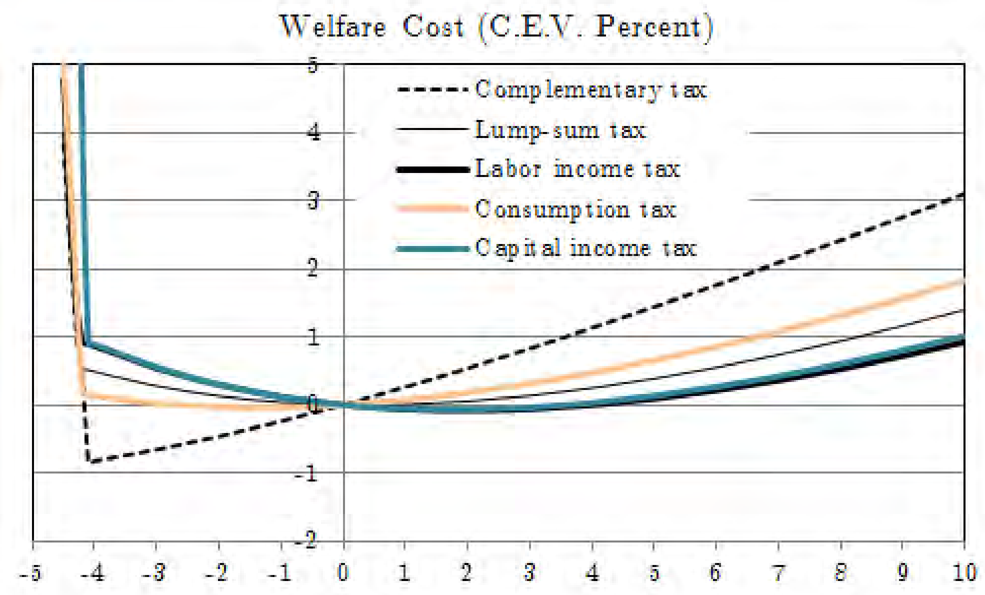
4.6 Summary of the Results

Figure 2 depicts steady-state welfare costs for various rates of inflation under each of the five types of fiscal scheme. Table 7 summarizes the welfare costs and the effects on output at the optimal inflation rates under the five types of fiscal scheme. As I have discussed thus far, the optimality of the Friedman rule breaks except for experimental scheme (a). Importantly, the level of the optimal inflation rate varies depending on the fiscal scheme, namely, what kind of tax the government adjusts to return inflation tax revenues to households. Mild inflation is likely to be desirable for social welfare when the government reduces the tax rate on either labor income or capital income instead. However, it is worthwhile to note that there are not large differences in welfare costs around zero percent of the inflation rate. Even so, it is possible that the impact on output is significant when inflation tax revenues are rebated to households through reduction in the tax rate on either labor income or capital income. In contrast, as is clear in Figure 2, there is a remarkable asymmetry between inflation and deflation in the sense that welfare costs of high deflation are quite large in the presence of the zero lower bound on the nominal interest rate while those of high inflation are modest. This result is robust for the fiscal schemes. Combined with the fact that the welfare cost of high inflation is not so large, this result suggests that it should be rational for central banks to pursue an expansionary monetary policy when the economy is faced with deflationary pressure.

Table 7: Optimal Inflation, Welfare Cost, and Impact on Output

	Optimal inflation (%)	Welfare cost (%)	Impact on output
(a) Complementary transfer/tax	−4.1	−0.8	+1.9
(b) Lump-sum transfer/tax	+0.4	−0.0	+0.0
(c) Labor income tax	+2.1	−0.1	+0.7
(d) Consumption tax	−1.4	−0.0	−0.2
(e) Capital income tax	+1.7	−0.1	+0.7

The impact on output for the optimal inflation is expressed in percentage point increment from the value for zero percent inflation.



5 Conclusion

In this paper I have explored the optimal rate of inflation using a neoclassical monetary model with overlapping generations, where inflation induces redistribution from old cohorts to young cohorts. Unlike almost all of the monetary models with overlapping generations used in the literature, the model incorporates not only capital stock to capture the Tobin effect, but also elastic labor supply to capture distortionary effects of the inflation tax on labor supply. The model also has the zero lower bound on the nominal interest rate imposed for the assessment of effects of deflation. I have calibrated the model so that it can replicate the age-specific profiles of consumption, labor supply, and money holdings as well as key macroeconomic indicators in Japan. From the computational results of steady states for different rates of inflation, I have shown that redistribution among cohorts and the associated Tobin effect play critical roles in breaking the optimality of the Friedman rule. These are also important for assessing large welfare costs of deflation that drives the nominal interest rate to the zero lower bound. Furthermore, it has turned out that the optimal inflation rate depends on how the government rebates inflation tax revenues to households.

Although such a monetary life-cycle model as used in this paper seems to provide us with useful insights into effects of the long-run inflation and deflation, we should note that this kind of model has some important limitations. For instance, a monetary model within a neoclassical framework builds fully on the money supply rule when it comes to the determinant of the inflation rate. Deflation is caused by a continuous contraction in the money supply and a central bank can raise the inflation rate anytime only if it expands the money supply. Consequently, it is difficult for this model to produce an empirically plausible situation where deflation continues chronically even though a central bank increases the money supply to a vast degree: Japan's experience over the past two decades.[20] As another caveat, we should note that this paper does not take account of heterogeneity within a cohort, but merely heterogeneity among cohorts. Naturally, inflation induces redistribution within every cohort as well as among all cohorts. Rather, wealth inequality tends to expand with age, reflecting differences in historically accumulated income. In this setting, households that receive resource transfers are not necessarily the young who have a higher propensity to save. This fact may prevent redistribution induced by inflation from improving aggregate output through the Tobin effect. As a result, inflation may become difficult to support when we extend the model by incorporating intra-cohort heterogeneity. I leave this extension for the future, although there are some restrictions on data availability in Japan.

[20] Needless to say, this model abstracts from some important factors for explaining inflation dynamics, such as the inflation expectation and uncertain disturbance to an economy.

A Computation of the Equilibrium

Here I describe briefly the computational procedure of the equilibrium. I begin by defining real variables normalized by total population N_t and efficiency unit $Z_t^{1/(1-\alpha)}$:

$$
\begin{aligned}
\widetilde{X}_t &= \frac{X_t}{N_t} \quad \text{for} \quad X_t \in \{L_t, h_{j,t}\}\,; \quad \widetilde{X}_t = \frac{X_t}{N_t Z_t^{1/(1-\alpha)} P_{t-1}} \quad \text{for} \quad X_t \in \{M_t, B_t, m_{j,t}, b_{j,t}\}\,; \\
\widetilde{X}_t &= \frac{X_t}{N_t Z_t^{1/(1-\alpha)}} \quad \text{for} \quad X_t \in \{\text{others}\}\,.
\end{aligned}
$$

Then the money supply rule (4) can be rewritten as follows and used to pin down either the inflation rate or aggregate real balance of money:

$$
(1+\pi_t)(1+z_t)\widetilde{M}_{t+1} = (1+\sigma_t)\widetilde{M}_t, \quad \text{where} \quad (1+z_t) = (1+\rho_t)(1+g_t)\,. \tag{21}
$$

One of the tax rates is adjusted to satisfy the government budget constraint (5):

$$
\widetilde{G}_t + \frac{\widetilde{M}_t}{1+\pi_t} + (1+x_t)\widetilde{B}_t + \widetilde{SS}_t = (1+z_t)\widetilde{M}_{t+1} + (1+z_t)\widetilde{B}_{t+1} + \tau_t^c \widetilde{C}_t + \tau_t^l w_t \widetilde{L}_t + \tau_t^k r_t \widetilde{K}_t + \widetilde{\tau}_t. \tag{22}
$$

Similarly, the households' budget constraints ((12) and (13)) are rewritten in real terms as follows:

$$
\begin{aligned}
& (1+\tau_t^c)\left(\widetilde{c}_{j,t}^s + \widetilde{c}_{j,t}^d\right) + (1+z_t)\left(\widetilde{k}_{j+1,t+1} + \widetilde{m}_{j+1,t+1} + \widetilde{b}_{j+1,t+1}\right) \\
= \;& \widetilde{\Omega}_t - \widetilde{\tau}_t + (1+x_t)\widetilde{k}_{j,t} + \frac{\widetilde{m}_{j,t}}{1+\pi_t} + (1+x_t)\widetilde{b}_{j,t} + \widetilde{\varsigma}_t,
\end{aligned} \tag{23}
$$

where $\widetilde{\Omega}_t$ is $(1-\tau_t^l) w_t \varepsilon_j h_{j,t}$ for $j < T$ and ss_t for $j \geq T$. The CIA constraint (14) can be expressed as

$$
i_t \left[\frac{\widetilde{m}_{j,t}}{1+\pi_t} - (1+\tau_t^c)\widetilde{c}_{j,t}^s\right] = 0 \quad \text{with} \quad i_t \geq 0 \quad \text{and} \quad \frac{\widetilde{m}_{j,t}}{1+\pi_t} \geq (1+\tau_t^c)\widetilde{c}_{j,t}^s. \tag{24}
$$

Combining these constraints yields an alternative formula for determining the real asset holdings of individual households, $\widetilde{a}_{j,t} = \widetilde{k}_{j,t} + \widetilde{m}_{j,t} + \widetilde{b}_{j,t}$:

$$
(1+\tau_t^c)\left[(1+i_t)\widetilde{c}_{j,t}^s + \widetilde{c}_{j,t}^d\right] + (1+z_t)\widetilde{a}_{j+1,t+1} = \widetilde{\Omega}_t - \widetilde{\tau}_t + (1+x_t)\widetilde{a}_{j,t} + \widetilde{\varsigma}_t. \tag{25}
$$

After solving the households' utility maximization, obtain aggregate real asset holdings by summing them up over j: $\widetilde{A}_t = \sum_{j=1}^{J} \mu_{j,t} \widetilde{a}_{j,t}$. When $i_t > 0$, real balances of money holdings $\widetilde{m}_{j,t}$ are also uniquely determined by the CIA constraint, implying that aggregate money balance $\widetilde{M}_t$ is computed by summing them up over j. With aggregate government bonds $\widetilde{B}_t$ exogenously given in the model, derive aggregate capital stock from the definition of real asset holdings: $\widetilde{K}_t = \widetilde{A}_t - \widetilde{M}_t - \widetilde{B}_t$. On the other hand, when $i_t = 0$, it is not possible any longer to pin down money holdings uniquely from the CIA constraint. Instead, however, it is possible to determine the inflation rate, real interest rate, aggregate capital stock and aggregate real balance of money, $\left\{\pi_t, x_t, \widetilde{K}_t, \widetilde{M}_t\right\}$, by solving the simultaneous equation system that consists of the Fisher equation, the firm's first order condition for the rate of return on capital, and the definition of real asset holdings as well as the money supply rule (21):

$$1 + x_t = \frac{1}{1+\pi_t}, \quad x_t = \left(1 - \tau_t^k\right)\left[\alpha\left(\frac{\widetilde{K}_t}{\widetilde{L}_t}\right)^{\alpha-1} - \delta\right], \text{ and } \widetilde{M}_t = \widetilde{A}_t - \widetilde{K}_t - \widetilde{B}_t.$$

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