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Sohei Kaihatsu, Koichiro Kamada, and Mitsuru Katagiri

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Abstract

This paper presents theoretical foundations for quantitative easing (QE). Since the late 2000s, with no room for lowering policy interest rates, central banks in the major advanced economies have adopted various unconventional monetary policies. QE is one of those unconventional policies and has so far achieved visible results in practice. However, our theoretical understanding of how QE achieves these results remains incomplete. The purpose of this paper is to introduce an inflation-sensitive money provision rule and show theoretically how QE helps an economy escape from a liquidity trap.

Keywords: Liquidity trap; Quantitative easing; Monetary policy rule
JEL classification: E31, E52, E58

*****Director and Senior Economist, Monetary Affairs Department, Bank of Japan (E-mail: souhei.kaihatsu@boj.or.jp)
******Associate Director-General and Senior Economist, Institute for Monetary and Economic Studies, Bank of Japan (E-mail: kouichirou.kamada@boj.or.jp)
*******Deputy Director and Economist, Research and Statistics Department, Bank of Japan (E-mail: mitsuru.katagiri@boj.or.jp)

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1. INTRODUCTION

In the late 2000s, faced with global financial turmoil and economic stagnation, central banks in the major advanced economies seemed to have exhausted the tools in their conventional monetary policy arsenal. Even their most drastic policies failed to stop the slowdown of economic growth and the decline in inflation rates. As a result, short-term nominal interest rates fell to zero percent or even below. With no room for lowering short-term nominal interest rates, the central banks began to implement various unconventional monetary policies. Quantitative easing (QE) is one of those policies; it was adopted by the Federal Reserve from 2008 to 2014, the Bank of Japan (BoJ) from 2013, and the European Central Bank from 2015. QE had previously been implemented by the BoJ from 2001 to 2006. Central banks in other economies were thought to be learning from Japan's experience, though few would have imagined they would be calling for QE so soon.

QE has so far achieved visible, though not necessarily sufficient, results. In the US, three versions of QE were implemented from 2008 and the economy recovered so strongly that the Fed exited from QE in 2014, and in 2015 finally raised its policy interest rate. In Japan, quantitative and qualitative easing (QQE)—a combination of QE and the lengthening of bond maturities—seems to have had positive effects on labor markets, although the inflation rate has not yet reached the BoJ's target. Meanwhile, in Europe, the inflation rate has continued declining steadily. Our experience, though quite short, shows that it takes years for QE to have visible effects. Thus, the central banks that introduced QE early in the global financial crisis seem to have gained benefits from the policy, while those that delayed adopting QE have reaped little return as yet.

However, neither academics nor practitioners have a clear theoretical understanding of how QE affects price developments. As former Federal Reserve chairman Ben Bernanke has remarked, "The problem with QE is it works in practice but it doesn't work in theory" (Brookings Institution, 2014). In fact, standard theoretical models tend to deny the effectiveness of QE altogether or to deem it at most quite limited. For instance, Eggertsson and Woodford (2003) propose the *irrelevance* *proposition*, which states that an increase in base money has no effects on the equilibrium of the economy when the policy interest rate is stuck at the zero lower bound. Note, however, that the proposition holds only under specific assumptions, as they argue carefully.[1]

The purpose of this paper is to offer some theoretical underpinnings for QE within the framework of a standard modern macroeconomic model. There are various economic arguments in the literature, some taking the shape of a formal model and others providing only verbal reasoning, that seek to show how price developments can be influenced in a low interest rate environment. We pick up some influential works, including Krugman (1998) and Benhabib et al. (2002), and discuss how useful their proposals are when considering the generation of inflationary pressure. We then introduce an *inflation-sensitive money provision rule* to provide the theoretical backbone for QE and show how it helps an economy escape from a liquidity trap, with an emphasis on differences from other policy measures including Leeper's (1991) fiscal theory of the price level.

The remainder of this paper is organized as follows. Section 2 shows how a liquidity trap emerges when the Taylor rule combines with the zero lower bound on nominal interest rates. Section 3 reviews a range of policy measures that have been proposed for escaping from a liquidity trap. Section 4 introduces an inflation-sensitive money provision rule and examines its properties. Section 5 concludes.

2. TWIN STEADY STATES

There are three types of economic agent in this paper: households, a fiscal authority, and a central bank. We treat the central bank and the fiscal authority separately, and do not consider a consolidated government. To begin with, we present the intertemporal budget constraints with which these agents are faced. We exploit these budget

[1] See also Woodford (2012) which offers an intensive discussion on monetary policy measures implementable in a low interest rate environment, especially forward guidance and balance-sheet policies.

constraints to analyze various policy measures proposed as means of saving an economy from a liquidity trap in later sections. Then, we show how the Taylor rule and the zero-lower bound for nominal interest rates combine to produce *twin steady states*, an inflationary steady state and a deflationary steady state.

2.1. Intertemporal budget constraints

A. Households

Let $u(\cdot)$ be a utility function and $\beta \in (0,1)$ be a subjective discount rate, which is assumed constant. Households maximize their lifetime utility $\sum_{k=0}^{\infty} \beta^k u(C_{t+k}/P_{t+k})$, subject to the following budget constraint,

$$C_t + B_t^h + M_t = R_{t-1}\left(B_{t-1}^h + M_{t-1}\right) + Y_t - T_t, \tag{1}$$

where C_t and P_t denote consumption and prices, respectively; B_t^h and M_t denote government bonds and base money held by households, respectively.[2] R_{t-1} denotes the (gross) nominal interest rate on government bonds. We assume that interest is paid on base money or reserves.[3,4] Since the rate is assumed to be the same as applied to government bonds, money and government bonds are indifferent for households. This reflects recent central banking practice in the major advanced economies. Y_t denotes households' income before tax. T_t denotes lump sum taxes on households (or subsidies if negative). In this paper, nominal variables are indicated in capital letters, real ones in lower case letters.

We divide equation (1), the nominal budget constraint, by P_t to express it in real

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[2] We assume that there is no private money creation and that base money is never drawn from but remains at the central bank. Thus, the money stock is always equivalent to base money.

[3] There are central banks, including the BoJ, that pay interest only on excess reserves. For simplicity, however, we assume that interest is paid on all reserve balances.

[4] An alternative assumption to generate the money demand for households is the money-in-utility (MIU) assumption (see, e.g., Woodford, 2003). Though not so well established as the MIU assumption, we focus on the interest-on-reserves assumption and investigate its implications in this paper.

terms.

$$c_t + b_t^h + m_t = r_{t-1}\left(b_{t-1}^h + m_{t-1}\right) + y_t - \tau_t. \tag{2}$$

Define the (gross) inflation rate as $\pi_t \equiv P_t/P_{t-1}$. Then, the real interest rate is given by $r_{t-1} \equiv R_{t-1}/\pi_t$.

After repeated substitution, we obtain the intertemporal budget constraint for households.

$$\begin{aligned} & r_{t-1}\left(b_{t-1}^h + m_{t-1}\right) + \left[y_t + \sum_{k=1}^{\infty} \lambda_{t+k-1|t} y_{t+k}\right] \\ &= \left[c_t + \sum_{k=1}^{\infty} \lambda_{t+k-1|t}\, c_{t+k}\right] + \left[\tau_t + \sum_{k=1}^{\infty} \lambda_{t+k-1|t} \tau_{t+k}\right] \\ &\quad + \lim_{k\to\infty} \lambda_{t+k-1|t}\left(b_{t+k}^h + m_{t+k}\right), \end{aligned} \tag{3}$$

where $\lambda_{j|i}$ is the cumulative discount rate, defined as follows.

$$\lambda_{j|i} = \left(\prod_{k=i}^{j} r_k\right)^{-1}. \tag{4}$$

Below we assume

$$\lim_{j\to\infty} \lambda_{j|i} = 0 \quad \text{for any } i < \infty. \tag{5}$$

This condition is satisfied quite easily. For instance, it is satisfied if the real interest rate is positive in a steady state.

The left-hand side of equation (3) indicates households' total wealth, including current and future income. On the right-hand side, the first term indicates the sum of current and future consumption, the second term the sum of current and future taxes, and the third term households' disposable assets left unused at the end of their life. We assume additionally that households are faced with the borrowing constraint.

$$\lim_{k\to\infty} \lambda_{t+k-1|t}\left(b_{t+k}^{h} + m_{t+k}\right) \geq 0. \tag{6}$$

This is called the no-Ponzi game constraint in the literature and should be satisfied in equilibrium. If this inequality is violated, households borrow money infinitely to increase consumption. This contradicts the definition of equilibrium.

B. Fiscal authority

The budget constraint for the fiscal authority is given by

$$T_t + B_t = R_{t-1}B_{t-1}, \tag{7}$$

where $B_t \equiv B_t^h + B_t^c$ and B_t^c indicates government bonds held by the central bank. For simplicity, we assume away government spending here.

We divide equation (7), the nominal budget constraint, by P_t to obtain the real budget constraint for the fiscal authority.

$$\tau_t + b_t = r_{t-1}b_{t-1}. \tag{8}$$

Then the intertemporal budget constraint for the fiscal authority is given by

$$r_{t-1}b_{t-1} = \left[\tau_t + \sum_{k=1}^{\infty} \lambda_{t+k-1|t}\tau_{t+k}\right] + \lim_{k\to\infty} \lambda_{t+k-1|t}b_{t+k}. \tag{9}$$

That is, government bonds issued up to the previous period and interest on them must be funded by taxes or held forever by households or the central bank.

C. Central bank

The central bank is faced with the following budget constraint.

$$M_t = R_{t-1}M_{t-1} + Q_t, \tag{10}$$

where Q_t denotes the central bank's new provision of base money. We assume that when providing (absorbing) base money, the central bank buys (sells) the equivalent value of government bonds. This implies[5]

$$B_t^c = M_t. \tag{11}$$

Recall that the central bank is assumed to pay interest on base money or reserves held by households and that the interest rate on reserves is the same as that paid on government bonds. In this case, all interest paid by the fiscal authority to the central bank is paid by the central bank to households. Therefore, the central bank makes no gains from seigniorage.

We divide equations (10) and (11), the nominal budget constraints, by P_t to obtain the real budget constraints for the central bank.

$$m_t = r_{t-1}m_{t-1} + q_t; \tag{12}$$

$$b_t^c = m_t. \tag{13}$$

Repeated substitution in equation (12) gives us the central bank's intertemporal budget constraint as follows.

$$r_{t-1}m_{t-1} = -\left[q_t + \sum_{k=1}^{\infty} \lambda_{t+k-1|t}\, q_{t+k}\right] + \lim_{k\to\infty} \lambda_{t+k-1|t} m_{t+k}\,. \tag{14}$$

That is, base money provided up to the previous period and interest on it must be absorbed by selling operations or left forever at the central bank.

2.2. Taylor rule, zero lower bound, and twin steady states

As shown by Benhabib et al. (2001), two steady states exist when the Taylor rule becomes non-linear due to the zero lower bound on nominal interest rates: an inflationary steady state, and a deflationary steady state.[6] Below we reformulate their

[5] Compositional changes in the central bank's balance sheet may have an impact on price developments, as mentioned by Svensson (2003), Bernanke and Reinhart (2004), and others, but these are not discussed here as being beyond the scope of the current paper.

[6] Bullard (2010) summarizes the controversy about whether the US economy was trapped in a deflationary steady state in the 2000s. Recently, Aruoba et al. (2013) have reported that the Japanese economy shifted to a deflationary steady state in the late 1990s; and that the

argument, using the intertemporal budget constraints constructed just above.

In the standard macroeconomic model, households are assumed to use up their assets by the end of their life. That is,

$$\lim_{k\to\infty} \lambda_{t+k-1|t}\left(b_{t+k}^{h} + m_{t+k}\right) = 0. \tag{15}$$

This is called the transversality condition in the literature. If households maximize their utility, this condition must be satisfied in equilibrium. Otherwise, households could increase their utility by spending their remaining assets, which would contradict the definition of equilibrium.

Substituting $b_t^c = m_t$ into equation (15), we have

$$\lim_{k\to\infty} \lambda_{t+k-1|t} b_{t+k} = 0, \tag{16}$$

which means that the fiscal authority refunds all government bonds eventually. It is also assumed that households use up their money by the end of their life. This implies that

$$\lim_{k\to\infty} \lambda_{t+k-1|t} m_{t+k} = 0. \tag{17}$$

No specific forms are assumed for fiscal and monetary rules here. Any actions may be taken, so long as they satisfy conditions (16) and (17).[7]

Due to the interest on reserves, the central bank has at its disposal two policy instruments which it can target independently: the nominal interest rate, R_t, and the provision of base money, q_t.[8] We assume that the central bank follows the Taylor rule

U.S. economy remained in an inflationary steady state even after the policy interest rate fell nearly to zero percent in 2009.

[7] Given equation (16), equation (17) can be derived from more general premise that the fiscal policy is conducted to keep the real value of government bonds held by households as a finite value, i.e., $\left|b_{t+k}^{h}\right| < \infty$.

[8] Separation of interest rate policy from balance sheet policy is an advantage of introducing the interest-on-reserves policy, as pointed out by Goodfriend (2002) and Keister et al.

to determine the nominal interest rate.[9] That is,

$$R_t = \bar{R}\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi}. \tag{18}$$

Denote the real interest rate in the inflationary steady state by $\bar{r} > 1$.[10] Let $\bar{\pi} > 1$ be the inflation rate in the inflationary steady state, which is equal to the central bank's target rate of inflation. Using the Fisher equation, the nominal interest rate achieved in the inflationary steady state is given by $\bar{R} \equiv \bar{r}\bar{\pi} > 1$. We also assume that the central bank follows the Taylor principle, $\phi > 1$.

In a low interest rate environment, however, the nominal interest rate is often subject to the zero lower bound. Equation (18) ceases to be applicable as it is. Instead, we rewrite it as follows.

$$R_t = max\left\{1, \bar{R}\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi}\right\}. \tag{19}$$

Clearly, the zero lower bound on the nominal interest rate introduces non-linearity in the Taylor rule. This gives rise to twin steady states.

In modern macroeconomics, households are assumed to make an optimal choice among consumption, financial assets, etc., subject to their budget constraint, taking fiscal and monetary rules as given. Then the market clearing conditions are imposed to solve for an equilibrium path. We do not specify a particular model here, since our argument below is more or less applicable to a broad range of the economic models used in the literature. Suppose that an economy reaches the inflationary steady state. Then, the inflation rate is given by $\pi_t = \bar{\pi}$; the nominal interest rate by $R_t = \bar{R}$; and the real interest rate by $r_t = \bar{r}$. This is what the central bank expects to happen eventually, when it uses the Taylor rule.

(2008).

[9] Here, we assume away the output gap term from the Taylor rule, as in Benhabib et al. (2001).

[10] The natural rate of interest is treated as positive in this paper, but after the recent global financial crisis is likely to be negative in practice. See empirical works by Laubach and Williams (2015) for the U.S. economy and Imakubo et al. (2015) for the Japanese economy.

In addition to the inflationary steady state, we define the other steady state, i.e., the deflationary steady state as follows. Let $\pi_t = 1/\bar{r}$ and $r_t = \bar{r}$. Then we have

$$\bar{R}\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi} = \bar{R}^{1-\phi} < 1. \tag{20}$$

The nominal interest rate hits the zero lower bound. Thus, we have $R_t = 1$ from equation (19). Since $\bar{r} > 1$ by definition, we have $\pi_t < 1$, which means deflation. This situation is named the deflationary steady state here and is also called a liquidity trap in the literature. We use "liquidity trap" and "deflationary steady state" interchangeably below. Note that the real interest rate in the deflationary steady state is the same as that observed in the inflationary steady state. As is clear in equation (3), the real interest rate determines relative prices between current and future consumption goods. As long as the real interest rate is unchanged, households do not change their actions, and there are therefore no differences in real terms between the two steady states.[11]

Nonetheless, the deflationary steady state is not equally preferable to the inflationary steady state (see, e.g., Bullard, 2010). If we could ignore external shocks and nominal frictions, it would not matter which steady state an economy were in. Households would enjoy the same amount of consumption. However, it does matter since we cannot ignore external shocks and nominal frictions. Suppose that the economy is in the inflationary steady state initially. If the economy is hit by an external shock, the central bank adjusts the nominal interest rate immediately and the economy is returned to the initial state as promptly as possible. The welfare loss will be relatively small in this case even with nominal frictions. Alternatively, suppose that the economy is in the deflationary steady state initially. As the zero lower bound is binding, the central bank cannot lower the nominal interest rate in response to adverse shocks and therefore fails to push the economy back to the initial state. The welfare loss caused by nominal frictions will consequently be larger.

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[11] In Benhabib et al. (2001), the inflationary and deflationary steady states are also referred to as the *active* and *passive* steady states, or as the *intended* and *unintended* steady states, respectively.

3. ESCAPING FROM DEFLATION: A LITERATURE REVIEW

In this section, we review some of the policies most frequently discussed in the literature as potential means of influencing price levels or inflation rates in a low interest rate environment: Eggertsson and Woodford's (2003) commitment to a future path of nominal interest rates; Krugman's (1998) commitment to a future path of the money stock; Leeper's (1991) fiscal theory of the price level; and Benhabib et al.'s (2002) inflation-sensitive taxation rule. We examine each of these policy measurers and discuss its effectiveness in saving an economy from the deflationary steady state or a liquidity trap. In Subsections 3.1 and 3.2, we continue to assume that households' transversality condition, i.e., equation (15), is satisfied around the deflationary steady state. The cases where the condition is violated are discussed in Subsection 3.3.

3.1. Managing expectations through commitment

Suppose that someone succeeds in lowing future real interest rates, r's, in some way. This lifts up future cumulative discount rates, λ's, in equation (3), making future goods expensive relative to current goods. Households accordingly rearrange their consumption schedule so as to maximize their lifetime utility, increasing current consumption and decreasing future consumption. This creates excess demand for current goods, which generates inflation in the current period.

A question arises, however: Who lowers future real interest rates and how? Eggertsson and Woodford (2003) propose that the central bank should commit to continuing low interest rate policy into the future. For instance, the central bank can promise to continue with a zero interest rate policy even if the Taylor rule demands a shift to positive nominal interest rates. Nessén and Vestin (2005) propose that the central bank should use a modified Taylor rule, which responds to a moving average of past inflation rates instead of the current inflation rate. This is called a history-dependent policy in the literature. Price level targeting is another history-dependent policy mentioned frequently.

Krugman (1998) proposes that the central bank should commit to keeping the money stock at a high level in the future.[12] Suppose that nominal interest rates become positive sometime in the future. If no interest is paid on money, the relationship between the money stock and prices will be restored at that time. For instance, money holdings will be minimized to what is required for cash payment, so that the quantity theory of money $Mv = Py$ holds, where v is the velocity of money (presumed stable). If the money stock is large, price levels will be high. Expectations of these future high prices raise current prices and generate positive inflation in the current period.

This begs the question of how the central bank can credibly commit to a given future money stock. Krugman (1998) insists that it is enough to "credibly promise to be irresponsible." It is, however, clear that this policy suffers from the time inconsistency problem. That is, the central bank might absorb money as soon as inflation actually takes hold. If households expect this, the policy has no effects on prices in the current period. Similarly, the proposal by Eggertsson and Woodford (2003) is not immune to the time inconsistency problem. If households expect the central bank to raise interest rate once inflation rates actually increases, the policy has no impact on the current prices.

3.2. Fiscal theory of the price level

The above arguments, both Eggertsson and Woodford (2003) and Krugman (1998), hinge on a crucial assumption: it is expected that nominal interest rates will be positive or, at least, that upward pressure will be exerted on nominal interest rates in the future. However, when an economy is trapped in the deflationary steady state, nominal interest rates are expected to be zero percent permanently. Thus, their proposals simply never get off the ground in practice.

It is worth touching on the fiscal theory of the price level (FTPL) here. This theory makes no assumptions regarding future nominal interest rates and thus works even in the deflationary steady state. Suppose that the fiscal authority announces a permanent

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[12] Auerbach and Obstfeld (2005) generalize Krugman's (1998) two-period model into an infinite-horizon model and demonstrate the validity of his model in a general setting.

tax reduction. That is, in equation (9), current τ is reduced, while future τ's are unchanged. We maintain the transversality condition here. Thus, the last term in equation (9) is zero. Consequently, $r_{t-1}(= R_{t-1}/\pi_t)$ on the left-hand side must decline for the equality to hold. This necessitates an unanticipated rise in P_t, and concomitant increase in $\pi_t(= P_t/P_{t-1})$.[13] In other words, a fiscal policy based on FTPL has inflationary effects.

This FTPL-based fiscal policy, however, does not eliminate the deflationary steady state, and thus cannot stop an economy from going back to it again. To see this, consider the following example. Initially, the economy is in the deflationary steady state with $b_{t-1} = \bar{b}$. At time t, the fiscal authority announces that the tax rate is reduced by $\Delta > 0$ at time t, and that it will go back to the steady state tax rate $\bar{\tau} \equiv (\bar{r} - 1)\bar{b}$ from time $t + 1$ onward.[14] That is, $\tau_t = \bar{\tau} - \Delta$ and $\tau_s = \bar{\tau}$ for $s = t + 1, t + 2, \cdots$. In addition, suppose $r_s = \bar{r}$ for $s = t, t + 1, \cdots$, as was in the deflationary steady state. Then, using equation (8), the law of motion of government bonds is given by

$$b_t - \bar{b} = (r_{t-1} - \bar{r})\bar{b} + \Delta; \tag{21}$$

$$b_{s+1} - \bar{b} = \bar{r}(b_s - \bar{b}) \text{ for } s = t, t + 1, \cdots. \tag{22}$$

Repeated substitution then gives us

$$b_{t+k} - \bar{b} = \bar{r}^k(b_t - \bar{b}). \tag{23}$$

[13] π_{t+1} and beyond do not change at all, since the rise in P_t is already fully reflected in P_{t+1} and beyond.

[14] $\bar{b}$ is not necessarily zero, and can be strictly positive. For instance, government bonds are used as collateral in financial transactions. Furthermore, a regulation requires life insurance companies and other financial institutions to hold government bonds as a safe asset.

Using equation (23) with $\lambda_{t+k-1|t} = \bar{r}^{-k}$, we have

$$\lim_{k\to\infty} \lambda_{t+k-1|t} b_{t+k} = \lim_{k\to\infty} \bar{r}^{-k}\, \bar{b} + \lim_{k\to\infty} \left(b_t - \bar{b}\right)$$

$$= b_t - \bar{b}. \tag{24}$$

Therefore, the transversality condition, i.e., equation (15) or (16), is satisfied if and only if $b_t - \bar{b} = 0$. Clearly, this is achieved by lowering r_{t-1} in equation (21), i.e., raising π_t or P_t by $\Delta/\bar{b}$. Thus, the FTPL policy causes an inflation rate hike at time t. However, the inflation rate decreases from time $t+1$ onward, and the deflationary steady state emerges again eventually.

3.3. Inflation-sensitive taxation rule

The above discussion of FTPL is suggestive, since it shows that fiscal policy has the potential to generate inflation even in a low interest rate environment. However, as long as households' deflationary expectations remain entrenched, the economy will return to the deflationary steady state or liquidity trap as soon as the fiscal authority's willingness to countenance further permanent tax reductions is exhausted. In this regard, Benhabib et al.'s (2002) inflation-sensitive taxation rule is interesting, since it is proposed as a measure for saving an economy from a liquidity trap. In this subsection, we reformulate their proposal to conform to our framework.

We start with the fiscal authority's reaction function. A taxation rule is given as a function of government bonds, as follows.

$$\tau_t = \bar{\tau} - \alpha_t^b r_{t-1}\left(b_{t-1} - \bar{b}\right) + (r_{t-1} - \bar{r})\bar{b}. \tag{25}$$

Substituting this into equation (8) gives

$$b_t - \bar{b} = r_{t-1}\left(1 + \alpha_t^b\right)\left(b_{t-1} - \bar{b}\right). \tag{26}$$

Thus, we have

$$b_{t+k} - \bar{b} = \left(\lambda_{t+k-1|t}\right)^{-1} \mu_{t+k|t+1}^{b} \left(b_t - \bar{b}\right), \tag{27}$$

where

$$\mu_{j|i}^{b} = \prod_{k=i}^{j} \left(1 + \alpha_k^b\right). \tag{28}$$

Using the relationship $m_t = b_t^c$ and equation (27), we have

$$\lim_{k\to\infty} \lambda_{t+k-1|t} \left(b_{t+k}^{h} + m_{t+k}\right) = \lim_{k\to\infty} \lambda_{t+k-1|t}\, b_{t+k}$$

$$= \lim_{k\to\infty} \lambda_{t+k-1|t}\, \bar{b} + \lim_{k\to\infty} \mu_{t+k|t+1}^{b} \left(b_t - \bar{b}\right). \tag{29}$$

Now choose any $\tilde{\pi} \in [1/\bar{r}, \bar{\pi}]$. Then, the following strategy enables the fiscal authority to exclude an equilibrium with π_t stuck permanently below $\tilde{\pi}$ (i.e., $\pi_t \leq \tilde{\pi}$ permanently), in particular the deflationary steady state where $\pi_t = 1/\bar{r}$. First, the fiscal authority sets $b_t > \bar{b}$, if necessary, by adjusting τ_t, and then uses the taxation rule (25), changing the value of parameter α_t^b in the following way.[15]

$$\alpha_t^b = \begin{cases} \alpha^{b+} \geq 0 & \text{for} \quad \pi_t \leq \tilde{\pi}; \\ \alpha^{b-} < 0 & \text{for} \quad \pi_t > \tilde{\pi}. \end{cases} \tag{30}$$

Focus on the last line of equation (29). Suppose that an economy is in equilibrium with $\pi_t \leq \tilde{\pi}$ permanently. The first term is zero by equation (5). The second term is positive, as demanded by the upper case in equation (30). Consequently, the last line of equation (29) is positive as a whole, violating households' transversality condition in equation (15). Households find themselves looking at disposable assets left unused at the end of their life and thus increase consumption to use up those assets, which raises the inflation rate. This contradicts the assumption of the economy being in equilibrium.

[15] Equation (25) includes Benhabib et al.'s (2002) rule as a special case. If we assume $\bar{b} = 0$ and $r_{t-1} = \bar{r}$, the equation is simplified to $\tau_t = -\alpha_t^b \bar{r} b_{t-1}$, which is almost the same as Benhabib et al.'s (2002) rule, given the condition for α_t^b specified in equation (30).

Suppose alternatively that an economy is in equilibrium with $\pi_t > \tilde{\pi}$. The first term is zero by equation (5). The second term is also zero, as this time it is the bottom case of equation (30) that is applied. Consequently, if b_t is a finite value, the last line of equation (29) becomes zero as a whole, satisfying households' transversality condition.

The inflation-sensitive taxation rule differs from the FTPL policy exemplified in Subsection 3.2. To see this, consider the following example. Initially, the economy is in the deflationary steady state with $b_{t-1} = \bar{b}$. At time t, the fiscal authority announces that it reduces the tax rate by $\Delta > 0$ and then follows the inflation-sensitive taxation rule in equation (25) with $\alpha^{b+} = 0$. That is, $\tau_t = \bar{\tau} + (r_{t-1} - \bar{r})\bar{b} - \Delta$ and $\tau_s = \bar{\tau} + (r_{s-1} - \bar{r})\bar{b}$ for $s = t+1, t+2, \cdots$. In addition, suppose that the economy would be back to the deflationary steady state, where $r_s = \bar{r}$ for $s = t, t+1, \cdots$, from time $t+1$ onward. Then, the taxation rule is simplified to $\tau_s = \bar{\tau}$ for $s = t+1, t+2, \cdots$. Using equation (8), the law of motion of government bonds is given by

$$b_t - \bar{b} = \Delta, \tag{31}$$

and equation (22) for $s = t+1, t+2, \cdots$. Thus, the transversality condition is satisfied if and only if $b_t - \bar{b} = 0$, as in Subsection 3.2. From equation (31), however, this is by no means achieved here. Thus, the economy could not go back to the deflationary steady state in the present case and would converge to the inflationary steady sate.

A key difference between the FTPL policy and the inflation-sensitive taxation rule is in the last term of equation (25). According to the FTPL, tax reduction breaks down the transversality condition temporarily, but at the same time, an increase in assets, i.e., government bonds, creates inflationary pressure, which lowers r_{t-1} ex post and reestablishes the transversality condition. This mechanism does not work under the inflation-sensitive taxation rule, however. Due to the last term of equation (25), $(r_{t-1} - \bar{r})\bar{b}$, τ_t is reduced further in response to the decline in r_{t-1} and offsets the decrease in fiscal burden induced by the decline in r_{t-1}. This generates additional wealth effects and violates the transversality condition again. This means that the economy could not go back to the deflationary steady state.

4. A THEORETICAL BACKBONE FOR QUANTITATIVE EASING

In this section, we present some theoretical underpinnings for QE. In the model of Benhabib et al. (2002), it is the fiscal authority that deploys the policy instruments of taxation and bond issuance to control price developments. This set-up chooses to ignore the fact that in today's major advanced economies price stability is part of the mandate of the central bank. The challenge, therefore, is to replace the fiscal authority with the central bank. Below we take up this challenge, presenting a formal model that provides a theoretical backbone for QE.

4.1. Inflation-sensitive money provision rule

To focus on the behavior of the central bank, we simplify that of the fiscal authority: it acts in passive accordance with a rule to keep constant the amount of government bonds held by households in real terms.

$$b_t^h = \bar{b}^h, \tag{32}$$

which means that the government follows a Ricardian fiscal policy in terms of government bonds held by households.[16] The central bank announces the following money provision rule.

$$q_t = \bar{q} + \alpha_t^m r_{t-1}(m_{t-1} - \bar{m}) - (r_{t-1} - \bar{r})\bar{m}, \tag{33}$$

where $\bar{q} = (1 - \bar{r})\bar{m}$.[17] Substituting this into equation (12), we have

$$m_t - \bar{m} = r_{t-1}(1 + \alpha_t^m)(m_{t-1} - \bar{m}). \tag{34}$$

Repeated substitution then gives us

$$m_{t+k} - \bar{m} = \left(\lambda_{t+k-1|t}\right)^{-1} \mu_{t+k|t+1}^m (m_t - \bar{m}), \tag{35}$$

[16] This condition can be replaced by a weaker condition, $|b_t^h| < \infty$, as before.

[17] $\bar{m}$ is not necessarily zero, and can be strictly positive. For instance, commercial banks are required to make deposits in their central bank accounts when they create deposit money.

where

$$\mu_{j|i}^{m} = \prod_{k=i}^{j}(1+\alpha_k^m)\,. \tag{36}$$

Using equations (32) and (35), we have

$$\lim_{k\to\infty}\lambda_{t+k-1|t}\left(b_{t+k}^h + m_{t+k}\right) = \lim_{k\to\infty}\lambda_{t+k-1|t}\,\bar{b}^h + \lim_{k\to\infty}\lambda_{t+k-1|t}\overline{m}$$

$$+\lim\nolimits_{k\to\infty}\mu_{t+k|t+1}^{m}(m_t-\overline{m}). \tag{37}$$

Choose any $\tilde{\pi} \in [1/\bar{r}, \bar{\pi}]$. Then, the following strategy enables the central bank to exclude an equilibrium with π_t stuck permanently below $\tilde{\pi}$ (i.e., $\pi_t \le \tilde{\pi}$ permanently), in particular the deflationary steady state, where $\pi_t = 1/\bar{r}$. First, the central bank sets $m_t > \overline{m}$, if necessary, by adjusting q_t, and then provides money following equation (33), changing the value of parameter α_t^m in the following way.

$$\alpha_t^m = \begin{cases} \alpha^{m+} \ge 0 & \text{for } \pi_t \le \tilde{\pi}; \\ \alpha^{m-} < 0 & \text{for } \pi_t > \tilde{\pi}. \end{cases} \tag{38}$$

Focus on the right-hand side of equation (37). The first and second terms are zero by equation (5). Suppose that an economy is in equilibrium with $\pi_t \le \tilde{\pi}$ permanently. The last term of equation (37) is positive, as demanded by the upper case in equation (38). As a result, the right-hand side of equation (37) is positive as a whole, violating households' transversality condition. This entails positive wealth effects: households find themselves with disposable assets left unused at the end of their life and thus expand demand for goods, which generates inflationary pressure.[18] This contradicts the assumption of the economy being in equilibrium with $\pi_t \le \tilde{\pi}$. Suppose alternatively that an economy is in equilibrium with $\pi_t > \tilde{\pi}$. The third term of equation (37) is zero, since this time the bottom case of equation (38) applies. Thus, the right-hand side is zero as a whole, satisfying households' transversality condition, and

[18] Ueda (2013) suggests that a central bank may generate positive wealth effects by purchasing various types of assets even in a low interest rate environment.

no inflation pressure is generated.

A comment is appropriate here on the similarity of taxation and money provision. Substituting $b_t^h = \bar{b}^h$ in equation (8), we have

$$\tau_t + \bar{b}^h + b_t^c = r_{t-1}\left(\bar{b}^h + b_{t-1}^c\right). \tag{39}$$

Substitute $m_t = b_t^c$ in equation (12). Then we have

$$b_t^c = r_{t-1} b_{t-1}^c + q_t. \tag{40}$$

Subtracting equation (40) from (39) gives

$$\tau_t + q_t = (r_{t-1} - 1)\bar{b}^h. \tag{41}$$

The equation indicates that taxation and money provision are interchangeable in the sense that interest payments on government bonds held by households can be funded either through taxation by the fiscal authority or money provision by the central bank. It is worthwhile recalling Ricardian equivalence theorem here. According to the theorem, tax reduction by issuing bonds has no effects on the economy if households expect a future tax hike. A similar theorem holds for money provision, as is pointed out by Bullard (2010). QE is effective if the money provision is permanent, i.e., if the money will not be absorbed as long as deflation prevails.[19]

The above interchangeability between the actions of the central bank and fiscal authority has implications for Cochrane's (2014) discussion of *fiscal backing*.[20] The central bank is prohibited by law from buying bonds directly from the fiscal authority. Thus, the central bank buys government bonds from households and provides money

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[19] Buiter (2014) explains the effectiveness of QE differently, starting from the assumption of fiat money being irredeemable.

[20] Fiscal backing should be distinguished from *overt monetary finance of fiscal deficits* by Turner (2013) and *money-financed fiscal stimulus* discussed by Galì (2014). The difference appears quite subtle, but has great importance from the viewpoint of central bank independence. As argued by Michael Woodford in Reichlin et al. (2013), QE is preferable since it preserves the separation between monetary and fiscal policy, while it has the same inflationary effects as a money-financed fiscal transfer.

to them; the fiscal authority issues new government bonds to households and raises money from them; and the money is used to fund tax reduction. In this process, first, the central bank's bond purchases decrease b_t^h and increase b_t^c; then, the following bond issuance by the fiscal authority increases b_t^h up to the initial level of $\bar{b}^h$. As a consequence, b_t increases overall, which in turn generates inflationary pressure through the wealth effects. Suppose that the fiscal authority did not issue new bonds. Then b_t would not change overall, and thus no inflationary pressure would be generated. What this means is that, for QE to be effective, it is essential for the fiscal authority to follow a Ricardian fiscal policy in terms of government bonds held by households and as a result behave in accordance with the actions of the central bank, whether intentionally or unintentionally.

Many central bank officials have dwelt on the potential for QE to help an economy escape from a liquidity trap. Among them, Bernanke (2003) has argued that a policy package of tax reduction, bond purchases by the central bank, and price level targeting would be effective in stopping Japan's deflation. We have already addressed the effects of tax reduction and central bank bond purchases. Price level targeting is not indispensable in the current model, but would have the following two merits. First, the policy would play a role as forward guidance, indicating clearly the central bank's intentions. Second, the policy would strengthen the effects of our inflation-sensitive money provision rule, due to its property of historical dependency.[21]

4.2. Relationship with one-time permanent money provision

It is worth examining the relationship between the inflation-sensitive money provision rule and one-time permanent money provision. Let us consider the following situation, which is comparable to the one presented in Subsection 3.2 in the discussion of the effects of the FTPL-based fiscal policy. Initially, the economy is in the deflationary steady state with $m_{t-1} = \bar{m}$. At time t, the central bank announces that money

[21] The central bank can adopt average inflation targeting discussed by Nessén and Vestine (2005) as an alternative, depending on the relative importance of the forward-looking factor in the Phillips curve.

provision is increased by $\Delta > 0$ at time t, and that it will go back to the steady state level from time $t+1$ onward. That is, $q_t = \bar{q} + \Delta$ and $q_s = \bar{q}$ for $s = t+1, t+2, \cdots$. In addition, suppose $r_s = \bar{r}$ for $s = t, t+1, \cdots$, as was in the deflationary steady state. Then, using equation (12), the law of motion of base money is given by

$$m_t - \bar{m} = (r_{t-1} - \bar{r})\bar{m} + \Delta; \tag{42}$$

$$m_{s+1} - \bar{m} = \bar{r}(m_s - \bar{m}) \text{ for } s = t, t+1, \cdots. \tag{43}$$

Repeated substitution then gives us

$$m_{t+k} - \bar{m} = \bar{r}^k (m_t - \bar{m}). \tag{44}$$

Using equation (44) with $\lambda_{t+k-1|t} = \bar{r}^{-k}$, we have

$$\begin{aligned} \lim_{k\to\infty} \lambda_{t+k-1|t}\left(b_{t+k}^h + m_{t+k}\right) &= \lim_{k\to\infty} \bar{r}^{-k}\,\bar{b}^h + \lim_{k\to\infty} \bar{r}^{-k}\,\bar{m} \\ &\quad + \lim\nolimits_{k\to\infty}(m_t - \bar{m}) \\ &= m_t - \bar{m}. \end{aligned} \tag{45}$$

Thus, the transversality condition, i.e., equation (15), is satisfied if and only if $m_t - \bar{m} = 0$. Clearly, this is achieved by lowering r_{t-1} in equation (42), which requires an increase in π_t, an inflation rate hike at time t. However, the inflation rate decreases from time $t+1$ onward, and the deflationary steady state comes back eventually.

Our inflation-sensitive money provision rule generates different consequences from the above one-time money provision. To see this, consider the following example. At time t, the central bank announces that it increases money provision by $\Delta > 0$ and then follows the inflation-sensitive money provision rule with $\alpha^{m+} = 0$. That is, $q_t = \bar{q} + (r_{t-1} - \bar{r})\bar{m} + \Delta$ and $q_s = \bar{q} + (r_{s-1} - \bar{r})\bar{m}$ for $s = t+1, t+2, \cdots$. In addition, suppose that from time $t+1$ onward, the economy would be back to the deflationary steady state, where $r_s = \bar{r}$ for $s = t, t+1, \cdots$. Then, the money provision rule is simplified to $q_s = \bar{q}$ for $s = t+1, t+2, \cdots$. Using equation (12), the law of motion of money is given by

$$m_t - \bar{m} = \Delta, \tag{46}$$

and equation (43) for $s = t+1, t+2, \cdots$. Thus, the transversality condition is satisfied if and only if $m_t - \bar{m} = 0$, as above. From equation (46), however, this is by no means achieved here. Two things happen off equilibrium, when money provision is increased under the inflation-sensitive money provision rule. In the first step, an increase in money provision breaks down the transversality condition temporarily, but at the same time, an increase in assets, i.e., base money, generates inflationary pressure, i.e., an increase in P_t and π_t, which lowers r_{t-1} *ex post* and reestablishes the transversality condition. In the second step, due to the last term of equation (25), $-(r_{t-1} - \bar{r})\bar{m}$, q_t is increased further in response to the decline in r_{t-1}. Intuitively, the central bank provides additional base money in order to complement the decrease in the real value of base money caused by the increase in P_t in the first step. This additional money provision violates the transversality condition again, which implies that the economy could not go back to the deflationary steady state.

4.3. Dynamics of inflation and money

Here we investigate the dynamic properties of our inflation-sensitive money provision rule. We assume perfect foresight and flexible prices for simplicity, following Benhabib et al. (2002). Note, however, that our rule is designed so as to function in a more realistic model setting as well.

We start with the following Euler equation.

$$u'(c_t) = \beta u'(c_{t+1}) r_t. \tag{47}$$

We deal with a simple endowment economy where households are given fixed income of $y_t = \bar{y}$ every period. There is no investment, government expenditure, etc. Goods and services are consumed exclusively by households. This means

$$c_t = \bar{y}. \tag{48}$$

Combining equations (47) and (48), we have

$$r_t = \frac{1}{\beta}. \tag{49}$$

This implies $\bar{r} = 1/\beta$.

Define π^0 as the inflation rate at which equation (18) dictates a nominal interest rate of zero percent. That is,

$$1 \equiv \bar{R}\left(\frac{\pi^0}{\bar{\pi}}\right)^{\phi}. \tag{50}$$

We assume $\tilde{\pi} = \pi^0$ throughout this subsection, just for explanatory purposes. Suppose $\pi_t > \pi^0$. Combining the Taylor rule in equation (18) and the Fisher equation, $R_t = r_t\pi_{t+1}$, we have

$$\frac{\pi_{t+1}}{\pi_t} = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi-1}. \tag{51}$$

Taking logs of equation (51) then gives us

$$ln\, \pi_{t+1} - ln\, \pi_t = (\phi - 1)(ln\, \pi_t - ln\bar{\pi}\,). \tag{52}$$

According to this equation, once π_t deviates from $\bar{\pi}$ it will never return. We assume $(1 + \alpha^{m-})/\beta < 1$. Then, according to equation (34), even if m_t deviates from $\bar{m}$ it will eventually converge back to it.[22]

Suppose alternatively that $\pi_t \leq \pi^0$. Since $R_t = 1$ and $r_t = 1/\beta$ in this case, the Fisher equation implies

$$\pi_{t+1} = \beta\,. \tag{53}$$

According to this equation, even when π_t deviates from β, it immediately returns next period. Conversely, since $(1 + \alpha^{m+})/\beta > 1$, according to equation (34), deviations of m_t from $\bar{m}$ result in divergence: m_t never returns to $\bar{m}$.

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[22] If $(1 + \alpha^{m-})/\beta \geq 1$, m does not converge to $\bar{m}$. However, since the transversality condition is still satisfied, we can still find an equilibrium path to the inflationary steady state.

Figure 1 visualizes the above analysis of the dynamics of π_t and m_t, which summarized by equations (34), (38), (51) and (53). In the figure, E^+ denotes the inflationary steady state, while E^- denotes the deflationary steady state. Under our base money provision rule, dynamic paths converging to E^- are not equilibrium paths, and thus, the equilibrium path converging to E^+ is left as the only remaining equilibrium. Suppose that the economy is at E^- initially. Announcing our inflation-sensitive money provision rule will cause π_t to jump from β to $\bar{\pi}$ immediately.

For comparison, we consider the case where α_t^m always takes a negative value. In this case, the dynamics of π_t and m_t are changed as in Figure 2. Note that both E^+ and E^- are steady states in this case. Therefore, the economy cannot escape from the deflationary steady state once it is trapped there.

4.4. Notes on α^m

Here we consider necessary conditions to be satisfied by α_t^m more closely. So far we have assumed that α_t^m switches between two values as in equation (38). This is, however, one of many possible relationships between α_t^m and π_t. Let us denote those relationships as a function, $\alpha_t^m = f(\pi_t)$, in general. We assume that function $f(\cdot)$ does not vary over time.

Function f should satisfy the two conditions.

$$f(\pi_t) \geq 0 \quad \text{for } \pi_t \leq 1/\bar{r}; \tag{54}$$

$$f(\pi_t) < 0 \quad \text{for } \pi_t > \bar{\pi}. \tag{55}$$

Graphically, function $f(\cdot)$ should avoid the two shaded areas located in the upper right and lower left corners in Figure 3, where the actual inflation rate is on the horizontal axis, while a value of α_t^m is on the vertical axis. In the lower left corner, the deflationary steady state is not eliminated, and thus the economy is caught in a liquidity trap (i.e., indeterminacy). In the upper right corner, the inflationary steady state is eliminated, and thus the economy suffers hyper-inflation (i.e., no stable solution).

Note that equation (38), drawn as a step function in Figure 3, satisfies conditions (54) and (55). Alternatively, we can draw function $f(\cdot)$ as a continuous function, as shown in the same figure. In the latter case, we redefine $\tilde{\pi} \in [1/\bar{r}, \bar{\pi}]$ implicitly by $f(\tilde{\pi}) \equiv 0$. Then we have

$$\alpha_t^m = \begin{cases} f(\pi_t) \geq 0 & \text{for} \quad \pi_t \leq \tilde{\pi}; \\ f(\pi_t) < 0 & \text{for} \quad \pi_t > \tilde{\pi}. \end{cases} \tag{56}$$

Clearly, equation (56) is a continuous version of equation (38). Though simple, equation (38) is enough to save an economy from a liquidity trap in theory. Nonetheless, equation (56) is a meaningful generalization to reflect actual practice. In fact, the Fed lowered the pace of its bond purchases and money provision gradually (eight times) before exiting from QE III in 2014.

There are several things to note here. First, the relative timing of the interest rate rise, π^0, and the cessation of money provision, $\tilde{\pi}$, is a matter of choice. All three plans, (a) $\tilde{\pi} < \pi^0$, (b) $\tilde{\pi} > \pi^0$, and (c) $\tilde{\pi} = \pi^0$, are acceptable in theory for the purpose of eliminating the deflationary steady state and producing inflationary pressure.[23] Second, the pace of base money provision and absorption, measured by $|f(\pi_t)|$ or $|\alpha_t^m|$, need not be large in theory. In practice, however, large-scale asset purchase is thought to be useful until our inflation-sensitive money provision rule gains sufficient credibility among households. Third, exit from QE does not necessarily mean reducing base money in nominal terms. The target nominal money supply, $\bar{M} \equiv \bar{m}P$, increases at the speed of $\bar{\pi}$ in the inflationary steady state. Actual nominal money must converge to this target, but need not necessarily decrease, and may even increase so long as it does so sufficiently slowly.

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[23] See Yamaoka and Syed (2010) for Japan's exit from the QE implemented from 2001 to 2006 and Board of Governors of the Federal Reserve System (2011, 2014) for the recent QE in the U.S.

5. CONCLUSION

Motivated by the low inflation rates or outright deflation observed recently in the major advanced economies, this paper focuses on the deflationary steady state, otherwise known as a liquidity trap, and considers potential policy measures for enabling an economy to escape from it. The paper's major results are summarized as follows.

The deflationary steady state emerges when the Taylor rule combines with the zero lower bound on the nominal interest rate. This is a natural consequence of how monetary policy implementation is practiced today and thus a real risk that central banks are likely to confront. It is indeed quite probable that the Japanese economy has been trapped in such a deflationary steady state since the 2000s, and central banks in other economies can no longer ignore this risk.

This paper picks up economic policies mentioned frequently in the literature as measures implementable even in a low interest rate environment. Among them, policies committing to a future path of interest rates or the money stock are likely to be ineffective in a situation where nominal interest rates are expected to remain stuck at zero percent permanently, as in the case of the deflationary steady state.

On the other hand, a fiscal rule which dictates continuous tax reductions and bond issuance helps an economy escape from the deflationary steady state theoretically. Crucially, this rule generates wealth effects and thus inflationary pressure. This resets households' deflationary expectations, turning them positive.

Our inflation-sensitive money provision rule works via a similar mechanism, substituting money for tax. As long as deflation prevails, the central bank continues providing money and the fiscal authority keeps issuing bonds. This produces wealth effects and thus generates inflationary pressure. One of the merits of our inflation-sensitive money provision rule is that it grants the initiative for controlling price developments to the central bank, which is the authority entrusted in practice with the primary responsibility for maintaining price stability.

In this regard, note that our inflation-sensitive money provision rule is different from the money finance that is prohibited by the fiscal law, but rather close to fiscal backing *à la* Cochrane (2014), i.e., an endogenous response of the Ricardian fiscal authority. The central bank acquires bonds actively to achieve its mandate of price stability, while the fiscal authority issues new government bonds in passive accordance with a rule to keep constant the amount held by the public. There is no intention of money finance here.

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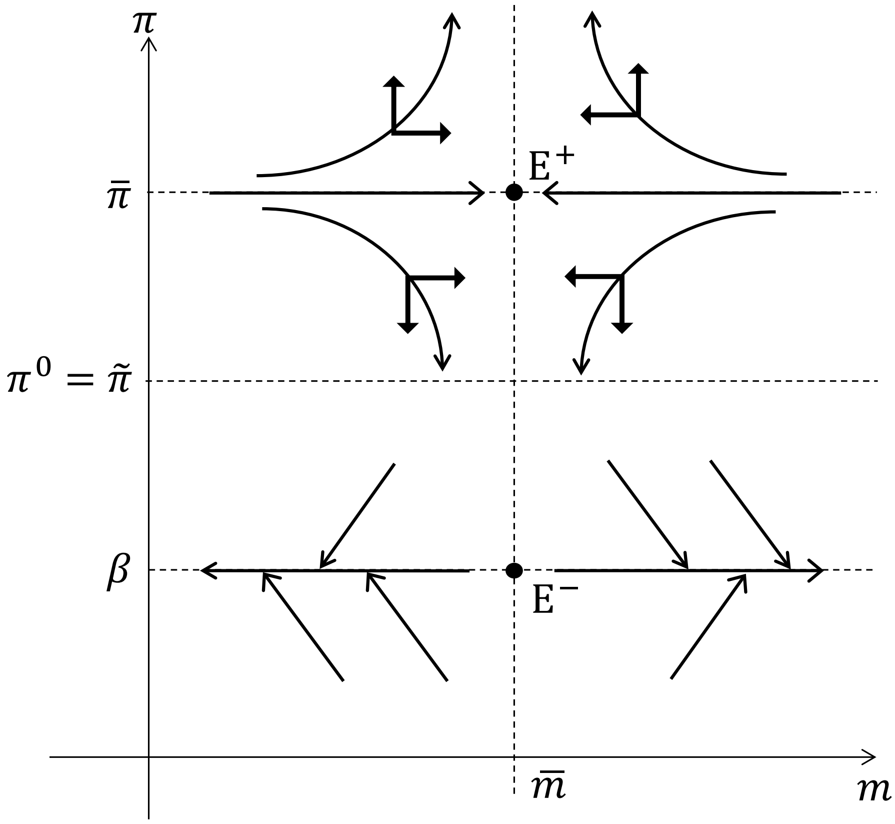

Figure 1: Inflation-sensitive money provision rule and economic dynamics

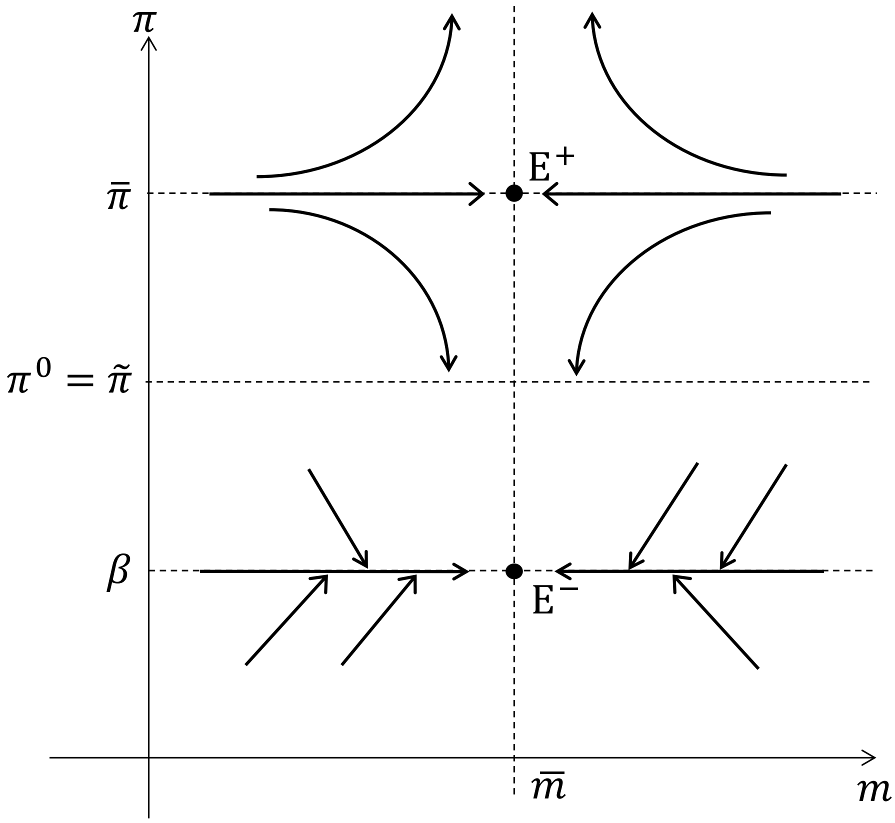

Figure 2: Inflation-insensitive money provision rule and economic dynamics

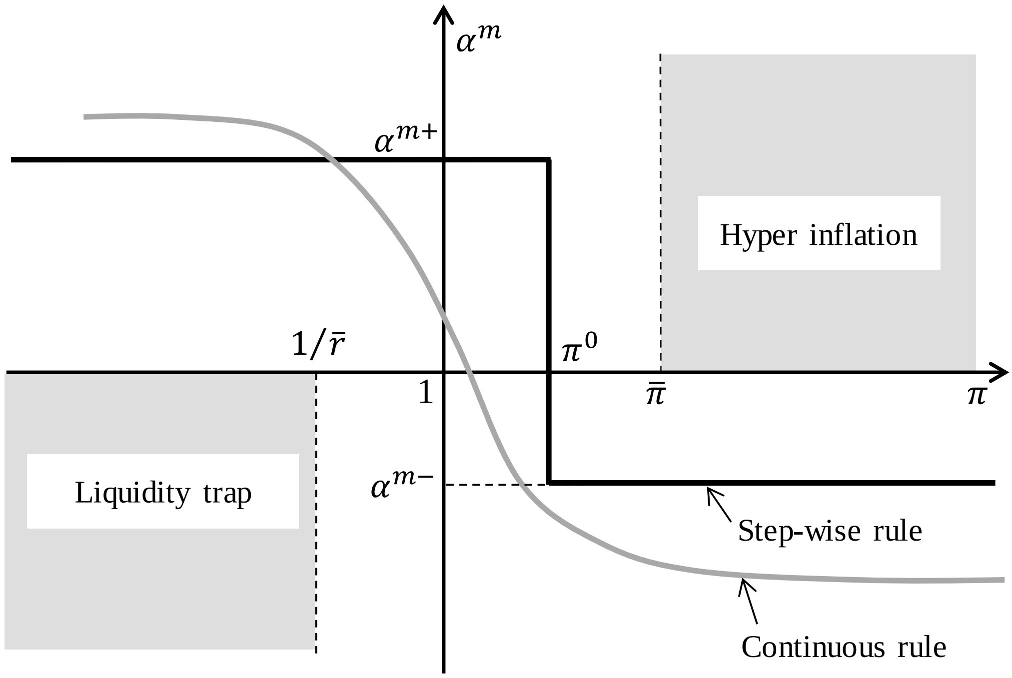

Figure 3: Step-wise and continuous money provision rules