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Risk Aggregation with Copula for Banking Industry

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Abstract

This paper surveys several applications of parametric copulas to market portfolios, credit portfolios, and enterprise risk management in the banking industry, focusing on how to capture stressed conditions. First, we show two simple applications for market portfolios: correlation structures for returns on three stock indices and a risk aggregation for a stock and bond portfolio. Second, we show two simple applications for credit portfolios: credit portfolio risk measurement in the banking industry and the application of copulas to CDO valuation, emphasizing the similarity to their application to market portfolios. In this way, we demonstrate the importance of capturing stressed conditions. Finally, we introduce practical applications to enterprise risk management for advanced banks and certain problems that remain open at this time.

Keywords: copula; multivariate distribution; tail dependence; risk aggregation; economic capital

JEL classification: G17, G21, G32

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1. Introduction

Risk managers in the banking industry evaluate the risks associated with financial asset and credit portfolios based on the assumption that underlying risk factors follow multivariate probability distributions that consist of marginal distributions and correlation structures. Most financial asset returns that determine marginal distributions have heavier tails than Gaussian distributions. In addition, each pair of returns is mutually dependent, particularly in the tail part of the distribution. For example, stock prices will plunge simultaneously in the event of financial turmoil like the Lehman shock of September 2008, while prices correlate moderately under more typical conditions. This suggests that the dependence between stock prices in the lower tail is stronger than in other parts of the multivariate distribution.

Parametric copulas are widely used in financial risk management to capture the various correlation structures that hold between risk factors. A copula is a function that joins, or couples, univariate distribution functions to a multivariate distribution function, as denoted by C in the equation

$$\Pr(X_1 \leq x_1, \dots, X_d \leq x_d) = C(\Pr(X_1 \leq x_1), \dots, \Pr(X_d \leq x_d)), \quad (1)$$

for the multivariate random vector $(X_1, \dots, X_d)$ (for examples, see Joe [1997] or Nelsen [2006]). A copula extracts the dependence structure from the joint distribution, independent of marginal distributions. We can construct the joint distribution of financial assets returns by specifying the marginal distributions and the parametric copula. Due to their tractability, copulas are used in evaluations of certain portfolio risks involving several assets and in evaluations of collateralized debt obligations (CDOs) comprising several bonds associated with default risks.

A Gaussian copula is the most popular copula for representing correlation structures for asset returns. Although the copula is easy to estimate, it cannot capture lower tail dependencies. One solution is to assume copulas capable of representing correlation structures under stressed conditions.

This paper overviews several applications of copulas in the banking industry to market portfolios, credit portfolios, and enterprise risk management, referring to work by Tozaka and Yoshiba (2005), Shintani, Yamada, and Yoshiba (2010), and Yoshiba (2013). Using parametric copulas to capture stressed conditions is a particular focus.

This paper is organized as follows. Section 2 overviews risk aggregation for market portfolios for which copulas are used. Section 3 discusses the use of copulas to assess credit portfolio risks. Section 4 describes how advanced banks apply copulas to enterprise risk management. Section 5 gives concluding remarks.

2. Risk aggregation for market portfolios

With regard to risk aggregation for market portfolios, risk managers determine correlation structures for certain risk factors non-parametrically or parametrically. Popular non-parametric methods include historical simulation methods, which capture only patterns of risk factors for both correlation structures and marginal distributions. Popular parametric methods assume a Gaussian copula for the correlation structure of risk factors and a Gaussian distribution for marginal distributions. (See, for example, Jorion [2006]).

Most banks adopt an unconditional approach to measure relevant market risks for one day, ten days, and other short-term periods. Unconditional approaches assume that the relevant risk factors for the period in question follow the same distribution and do not incorporate time series characteristics, such as serial correlations. Following the standard practice of risk measurement in banks, we measure market portfolio risk unconditionally for a one-day period based on daily log returns for risk factors.[1]

2.1. Correlation structures for three major stock indices

Shintani, Yamada, and Yoshiba (2010) analyzed correlation structures for three major stock indices daily returns in the US (the S&P500), the Eurozone (Euro Stoxx 50), and for Japan (the Nikkei 225). The observation period was from January 2001 to September 2009.

They began by analyzing pairwise lower tail dependence with 5% and 1% thresholds: $\lambda_L(0.05)$ and $\lambda_L(0.01)$. The lower tail dependence with a certain threshold u is defined as

[1] For differences in unconditional and conditional approaches, see McNeil, Frey, and Embrechts (2005) and Isogai (2014).

$$\lambda_L(u) = \Pr[F_2(X_2) < u | F_1(X_1) < u] = \frac{C(u,u)}{u}, \quad (2)$$

where $F_1(X_1)$ and $F_2(X_2)$ are marginal distribution functions of the first and the second variable. We adopt an empirical distribution for the marginal distributions $F_1(\cdot)$ and $F_2(\cdot)$. We estimate each parametric copula parameter by equating the sample correlation and theoretic correlation after calculating Kendall's sample rank correlation $\hat{\tau}_K$.

Table 1 summarizes the expressions and the rank correlations for some of the bivariate copulas used in this subsection. Although the Gumbel copula is upper-tail dependent, it can be applied to lower-tail dependent data with the rotated-Gumbel copula. The rotated copula $C(u_1, u_2)$ for a copula $\hat{C}(u_1, u_2)$ is obtained by $(1 - u_1, 1 - u_2) \sim \hat{C}$ and is defined as $C(u_1, u_2) = u_1 + u_2 - 1 + \hat{C}(1 - u_1, 1 - u_2)$. The rank correlation of the rotated copula is equal to that of the original copula.

Table 1. Bivariate parametric copulas and their rank correlation

Copula	Parameter	Expression $C(u_1, u_2)$	Kendall's tau τ_K
Gaussian	ρ	$\boldsymbol{\Phi}_\rho(\Phi^{-1}(u_1), \Phi^{-1}(u_2))$	$(2/\pi)\arcsin\rho$
t	ρ, ν	$\boldsymbol{T}_{\nu,\rho}(T_\nu^{-1}(u_1), T_\nu^{-1}(u_2))$	$(2/\pi)\arcsin\rho$
Clayton	α	$(u_1^{-\alpha} + u_2^{-\alpha} - 1)^{-1/\alpha}$	$\alpha/(\alpha+2)$
Gumbel	γ	$\exp\{-((-\ln u_1)^\gamma + (-\ln u_2)^\gamma)^{1/\gamma}\}$	$1 - 1/\gamma$

Notes: $\boldsymbol{\Phi}_\rho(\cdot,\cdot)$ denotes a bivariate standard Gaussian distribution function with correlation ρ. $\boldsymbol{T}_{\nu,\rho}(\cdot,\cdot)$ denotes a bivariate *t* distribution function with degree of freedom ν and correlation ρ. $\Phi^{-1}(\cdot)$ denotes the inverse function of the univariate standard Gaussian distribution function. $T_\nu^{-1}(\cdot)$ denotes the inverse function of univariate *t* distribution function.

Table 2 shows the sample lower tail dependence $\hat{\lambda}_L(u)$ and the theoretical value by each parametric copula. The parameter for each parametric copula is obtained by equating it to Kendall's tau τ_K. We see that the sample lower tail dependence is much greater than the theoretical value given by the Gaussian copula. The sample lower tail dependence with a 5% threshold approaches the theoretical value of the *t* copula with degree of freedom parameter $\nu = 3$. The lower tail dependence in this period is much stronger than that implied by the Gaussian copula.

Table 2. Sample lower tail dependence and theoretical value given by each parametric copula between each pair of daily returns for three major stock indices

	Pair	$\hat{\lambda}_L(u)$	Gaussian	t(6)	t(3)	Rotated-Gumbel	Clayton
$\lambda_L(0.05)$	US/Euro	0.39	0.25	0.32	0.37	0.47	0.52
	US/JPN	0.35	0.20	0.30	0.36	0.40	0.43
	Euro/JPN	0.24	0.13	0.20	0.25	0.30	0.28
$\lambda_L(0.01)$	US/Euro	0.27	0.13	0.24	0.33	0.43	0.51
	US/JPN	0.29	0.09	0.23	0.32	0.36	0.41
	Euro/JPN	0.18	0.05	0.14	0.22	0.24	0.23

Notes: The data used for estimations for the US/JPN pair is one day lagged data for JPN. The parameter ν for t copula is fixed to 6 or 3 in this table.

Next, we estimate parameters for each parametric copula by maximizing the likelihood for the same data with empirical cumulative probabilities for each index return. Table 3 shows the maximum likelihood estimates and Schwarz's Bayesian information criterion (BIC) for each parametric copula. We adopt BIC to select the optimal parametric copula. The *t* copula with low degree of freedom parameters from 3 to 5 is selected by BIC. A Gaussian copula is worse than the *t* or rotated-Gumbel copula in terms of BIC. The result is consistent with the results of Tsafack (2009), who selected a copula based on Akaike information criterion (AIC) and BIC for weekly stock return data for United States and Canada up to 2004.[2] We see that a Gaussian copula with weak lower tail dependence does not adequately capture the overall correlation structure in terms of likelihood.

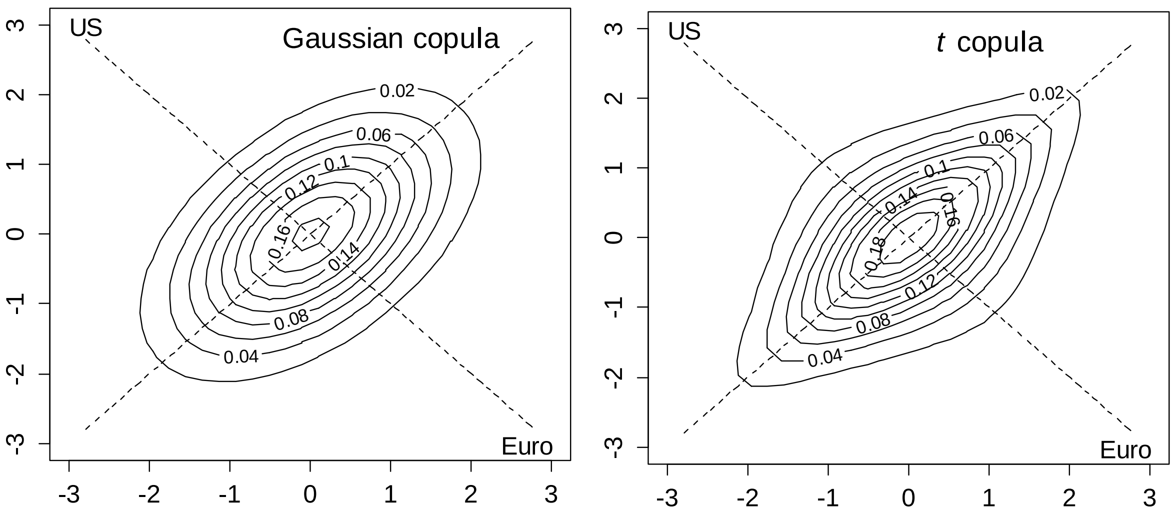
[2] Some information criteria, such as BIC and AIC, are used to select the optimal copula. Both criteria are calculated based on log-likelihood, with certain penalties applied by number of parameters. We adopt BIC for the criteria, which imposes more penalties in number of parameters than AIC. BIC is calculated by $-2l(\xi) + p \ln N$, where $l(\xi)$ is the maximum log-likelihood, p the number of parameters, and N the sample size. The model with the lowest BIC is selected.

Table 3. Maximum likelihood estimates and BIC for each parametric copula for each pair of daily returns for three major stock indices

Pair	Gaussian	t	Rotated-Gumbel	Clayton	Gumbel
	ρ	ρ ν	γ	α	γ
US/Euro	0.519 (–785)	0.520 3 (–1,055)	1.539 (–868)	0.831 (–695)	1.533 (–887)
US/JPN	0.443 (–544)	0.427 3 (–637)	1.389 (–583)	0.640 (–488)	1.366 (–542)
Euro/JPN	0.270 (–183)	0.273 5 (–253)	1.220 (–248)	0.387 (–217)	1.180 (–165)

Notes: The upper values in each cell are maximum likelihood estimates. The lower values in the parenthesis are the BICs. The parameter ν for the t copula is estimated with the restriction that ν is an integer value greater than 2.

Figure 1 depicts a joint density contour of the estimated copula with standard Gaussian margins for daily stock returns for the US and the Eurozone. Figure 1(a) depicts a Gaussian copula with standard Gaussian margins. Figure 1(b) depicts Student's t copula with standard Gaussian margins, adopted by BIC as the best-fit copula among several alternatives. We see that the lower tail dependence (the bottom-left corner) for Student's t copula is stronger than that for the Gaussian copula.



(a) Gaussian copula **(b) Student's *t* copula**

Figure 1. Contour plot of copula density for two stock returns with standard Gaussian margins for (a) Gaussian copula and (b) Student's *t* copula

2.2. Correlation structures for stock prices and interest rates

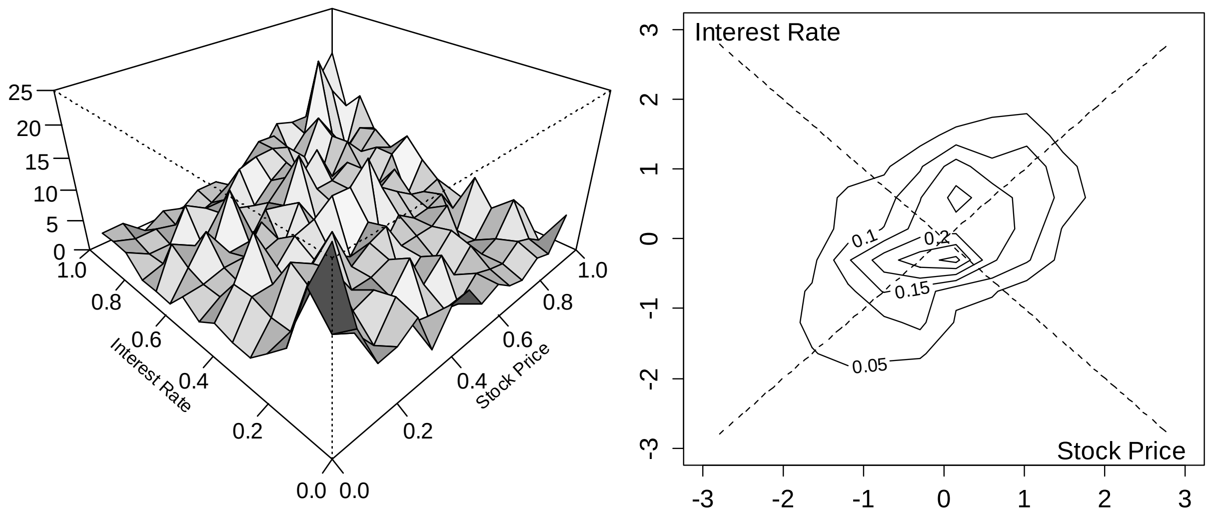
Japanese daily market data from 2007 to 2012 indicate a positive linear correlation between stock prices and interest rates, suggesting an increase in bond prices paired with a fall in stock prices. The measured risk of value-at-risk (VaR) or expected shortfall (ES) for the aggregated bond and stock portfolio becomes much smaller than the sum of the risk measures for those sub-portfolios. The reduction in risk measures for an aggregated portfolio is known as the diversification effect. Widely used aggregation methods that analyze recent Japanese data sometimes show up to a diversification effect of up to 60%.

Yoshiba (2013) has proposed using copulas for risk factors, focusing on stressed conditions. First, a bivariate copula with both positive and negative linear correlations is applied to the copula. Second, a copula estimated from stressed data such as Eurozone crisis data or post-Bubble data in Japan is applied to the copula. This paper gives an overview of the first method.

We select two risk factors; daily log returns for the Nikkei 225 index and daily changes in 5-year government interest rates.[3] The observation period is from October 1, 2007 to October 1, 2012. Marginal distributions are estimated by the skew-*t* distribution proposal by Azzalini and Capitanio (2003). Using the estimated marginal distribution functions $\widehat{F_1}(x)$ and $\widehat{F_2}(x)$ estimated, respectively, from stock returns and interest rate movement data, the pseudo sample $\{(u_{11}, u_{21}) \dots, (u_{1N}, u_{2N})\}$ is obtained by $u_{ij} = \widehat{F_i}(x_{ij})$ for $i = 1,2; j = 1, \dots, N$. Figure 2 (a) depicts a joint histogram of the pseudo sample $\{(u_{11}, u_{21}) \dots, (u_{1N}, u_{2N})\}$ during this period. The front side with zero for both axes indicates the largest drop in stock prices and interest rates. The figure indicates a relatively high frequency, suggesting that bond values will rise when stock prices fall, mitigating portfolio losses. Figure 2 (b) plots the joint density contour after converting each u_{ij} to the quantiles of the standard Gaussian distribution, $\Phi^{-1}(u_{ij})$.[4] This contour rises diagonally up to the right with an elliptical shape, suggesting that this pseudo sample exhibits a positive linear correlation.

[3] We use generic interest rates calculated by Bloomberg for 5-year interest rates of government bonds.

[4] Joint density contours with standard Gaussian margins are visual representations of the various dependencies in the center and the tail area (see Joe [1997]). If the copula is Gaussian, the contour is elliptical (see Figure 1[a]).



(a) Histogram **(b) Joint density contour**

Figure 2. Joint histogram and contour plot for the pseudo sample

Table 4 is the result of the maximum likelihood estimation. Here, we consider parametric copulas in Table 1 and the rotated ones for Gumbel and Clayton. We also consider a mixed-Gaussian copula implied in a mixed-Gaussian distribution. When we consider two state in the bivariate case, the mixed-Gaussian distribution is mixed with a negatively correlated Gaussian distribution and a positively correlated Gaussian distribution in the ratio of $\theta : (1-\theta)$. The copula is expressed as follows:

$$C(u_1, u_2) = \theta \mathbf{\Phi}_{\rho_1}\left(\Phi^{-1}(u_1), \Phi^{-1}(u_2)\right) + (1-\theta)\mathbf{\Phi}_{\rho_2}\left(\Phi^{-1}(u_1), \Phi^{-1}(u_2)\right). \quad (3)$$

The mixed Gaussian copula extracts a negatively correlated Gaussian component from positively correlated data. The mixed-Gaussian copula is selected by BIC.[5]

[5] The mixed-Gaussian copula is not always selected by BIC. For example, Yoshiba (2013) shows *t* copula is selected by BIC for data related to the Euro crisis and the post-bubble period in Japan. As for more complicated copulas, we can construct, as examples, mixed-*t* or mixed-Gaussian-*t* copulas. An examination of these copulas is left for the future.

Table 4. MLE for the pseudo sample

	Parameter	Est. Value	Std. Err.	BIC
Gumbel	γ	1.385	0.031	–239.8
Rotated-Gumbel	γ	1.416	0.031	–282.0
Clayton	α	0.662	0.050	–236.8
Rotated-Clayton	α	0.567	0.047	–184.5
Gaussian	ρ	0.436	0.021	–251.9
t	ρ	0.466	0.024	–307.2
	ν	5.481	0.918	
Mixed-Gaussian	ρ_1	–0.458	0.124	–313.0
	ρ_2	0.616	0.026	
	θ	0.145	0.036	

Notes: The parameter ν for *t* copula is estimated without restrictions on integer value.

We consider a sample portfolio consisting of 50 billion yen in stocks and 700 billion yen in 5-year discount bonds, which is representative of the average portfolio held by Japanese regional banks. We adopt a daily 99% VaR and 97.5% expected shortfall (ES) estimates for risk measures.[6] For each category of stocks and bonds, we estimate the return distribution by applying a skew-*t* distribution. Table 5 shows the VaR and ES for each category, along with the simple sum of those risk measures.

Table 5. VaR and ES for stocks and bonds, and their simple sum

	stocks	bonds	simple sum
VaR (99%)	2.61	2.47	5.08
ES (97.5%)	2.82	2.77	5.59

(billion yen)

Table 6 summarizes VaR and ES for the sample portfolio while accounting for the diversification effect. The diversification effect is given by the reduction rate of the aggregated VaR or ES from the simple sum of VaR or ES as follows:

$$\text{diversification effect} = \frac{\text{simple sum VaR/ES} - \text{aggregate VaR/ES}}{\text{simple sum VaR/ES}}. \qquad (4)$$

The joint distribution for risk factors is constructed by each estimated copula with estimated marginal distributions. For the nonparametric copula, the pseudo sample is converted into a set of risk factors by taking quantiles for marginal skew-*t* distributions.

[6] 99% VaR is the 99th percentile for a portfolio loss distribution. A 97.5% ES is the average of the losses in the 2.5% tail of the loss distribution. If the portfolio profit–loss distribution is Gaussian, a 97.5% ES nearly equals 99% VaR.

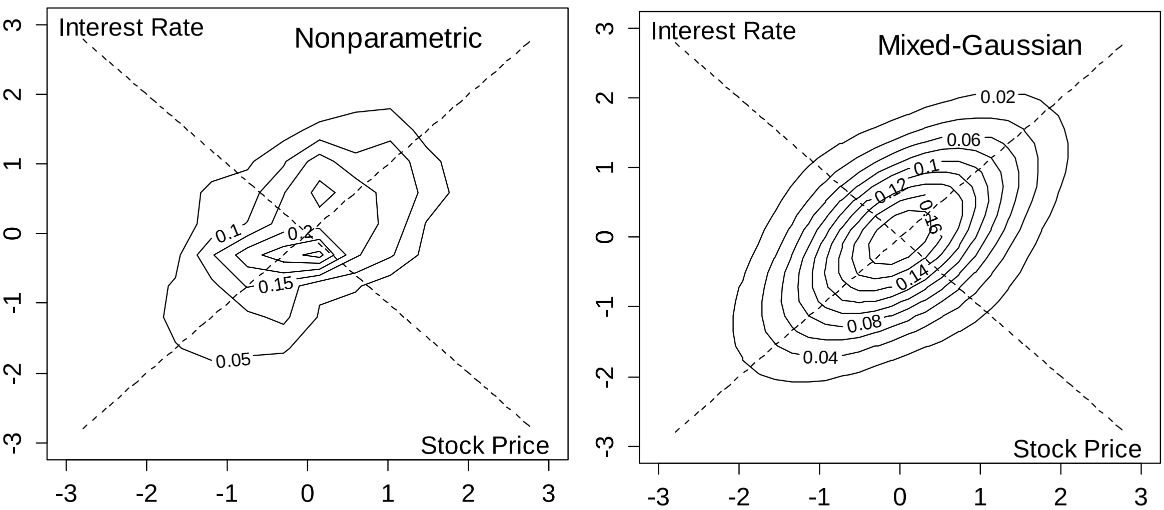
For each parametric copula, we generate 100,000 random bivariate vectors and calculate VaR and ES with marginal skew-*t* distributions. Iterating the procedure 100 times, we obtain averages and standard deviations for VaR and ES.

Table 6. VaR and ES using the estimated copula

Copula	VaR(99%)	Std. dev.	diversification effect	ES(97.5%)	Std. dev.	diversification effect
Nonparametric	3.01	––	41%	3.24	––	42%
Gumbel	2.66	0.03	48%	2.90	0.04	48%
Rotated-Gumbel	2.58	0.03	49%	2.84	0.04	49%
Clayton	2.68	0.03	47%	2.96	0.04	47%
Rotated-Clayton	2.81	0.03	45%	3.05	0.04	45%
Gaussian	2.65	0.03	48%	2.95	0.04	47%
t	2.60	0.03	49%	2.85	0.04	49%
Mixed-Gaussian	4.21	0.04	17%	4.55	0.05	19%

Notes: VaR and ES are given in units of billions of yen.

The diversification effect for a mixed-Gaussian copula (VaR:17%; ES:19%) selected by BIC is the smallest and is much smaller than that for other copulas. Unlike other parametric copulas, a mixed-Gaussian copula can capture both positive and negative linear correlations, allowing a better fit to the pseudo sample. The results indicate a negative linear correlation can be captured at a frequency of θ = 14.5% (see Figure 3). This correlation structure increases estimates for portfolio VaR and ES and reduces diversification effects.



(a) Nonparametric copula **(b) Mixed Gaussian copula**

Figure 3. Contour plot of joint density with Gaussian margins

3. Risk aggregation for credit portfolios

Correlation structures for firm asset values are a key element for credit portfolio risk aggregation. First, we will survey risk measurements for a banking industry credit portfolio by addressing prudent correlation structures. Second, we will provide an overview of applications to CDO valuation.

3.1. Credit portfolio risk measurement in the banking industry

In the banking industry, the credit portfolio risk of lending exposure is assessed by VaR with confidence levels of up to 99.9% in accordance with the Basel Accords (Basel Committee on Banking Supervision 2011). In models for credit VaR, the asset log values for the lending firms follow a multivariate Gaussian distribution with a correlation matrix based on the structural model developed by Merton (1974). If the firm's asset value at maturity T is less than the firm's liability, the firm defaults. The maturity T is usually fixed to one year, which is the time interval required to cover the potential loss of the bank by economic capital. Bank losses are given by the loss given default (LGD) multiplied by the exposure at default (EAD). In most cases, EAD is capped at the lending amount. LGD is also fixed exogenously.

Banks estimate the default probability of each firm until maturity by applying a statistical model based on financial indicators, not by the above structural model. Even in the latter case, banks capture the same correlation structures for firm asset values as a multivariate Gaussian distribution, according to Merton (1974). The correlation structure is a Gaussian copula. Many credit VaR models, including J.P. Morgan's CreditMetrics, assume a Gaussian copula between firm asset values.

Frey, McNeil, and Nyfeler (2001) have calculated the number of defaulting firms using a Gaussian copula and t copula with several degree of freedom parameters for a lending portfolio involving loans to N homogenous firms with 0.5% default probability and 0.038 asset correlation. Following Frey, McNeil, and Nyfeler (2001), Tozaka and Yoshiba (2005) have applied Gaussian, t with degree of freedom parameter of 10, Clayton, and rotated-Gumbel copulas to asset correlation structures and calculated the number of defaulting firms for N = 10,000. Table 7 gives the distribution of the numbers obtained by a Monte Carlo simulation involving 100,000 iterations.

Table 7. Number of defaulting firms given by each copula for specific confidence levels

Copula	50%	90%	95%	99%	99.9%
Gaussian	43	90	109	155	227
t(10)	9	133	240	586	1,305
Rotated-Gumbel	42	56	66	156	1,176
Clayton	26	122	179	343	643

Table 7 indicates that a tail dependent copula like *t*(10) and the rotated-Gumbel gives an extremely large number of defaulting firms with high confidence level (99.9%).

This is a simple example of applying a copula to capture credit portfolio risk. In practice, the correlation structure may vary from industry to industry. A vine copula[7] or hierarchical Archimedean copula (HAC)[8] may be applied to capture flexible asset correlation structures. See Kawaguchi, Yamanaka, and Tashiro (2014) for a study of flexible asset correlation structures using vine copulas.

3.2. Applying copulas to CDO valuation

Li (2000) has applied copulas to the valuation of CDOs, which consist of many CDS (credit default swap) to firms with 5-year maturity. To capture the number of defaulting firms at maturity, Li (2000) uses a copula to represent the correlation structure for the underlying firm asset values at maturity. Credit ratings agencies, where Gaussian copulas are widely used for CDO valuations, have been criticized for the inability of their models to capture tail dependencies when the price of CDOs plunged around the time of the Lehman shock. The explanation for this shortcoming is that Gaussian copulas have no asymptotic tail dependence.

Burtschell, Gregory, and Laurent (2009) have investigated various copulas with few parameters to fit iTraxx market data in August 2005. On the other hand, Shintani, Yamada, and Yoshiba (2010) comparatively investigated the effects on credit spread by applying various copulas determined by historical asset value data.

Following standard settings for CDOs, we set a CDO with 100 homogenous underlying assets, 5-year maturity, and 40% recovery rate for each asset. The default

[7] For vine copulas, see the handbook by Kurowicka and Joe (2010).
[8] For HAC, see Savu and Trede (2010).

probability for each asset was 5% per 5 years, somewhat higher than usual. The asset correlation ρ for the Gaussian copula is 0.15, a standard setting used by ratings agencies. In this case, the Kendall's tau τ_K of the Gaussian copula is 0.096. Equating this value to the theoretical value of each copula, we obtain the parameter for each copula as *t*'s ρ: 0.15, Clayton's α: 0.21, and the rotated-Gumbel's γ: 1.11. We can divide the CDO portfolio into tranches of [0%, 6%] for equity, [6%, 18%] for mezzanine, [18%, 36%] for senior, and [36%, 100%] for super-senior, thereby maintaining ratings above AAA for super-senior, AA–AAA for senior, BBB–A for mezzanine by Gaussian copula's valuation.[9] Table 8 shows the credit spread for each tranche derived from expected loss rates in this case.[10] The rows of *t*(20), *t*(6), and *t*(3) are resulting from a *t* copula with fixed degree of freedom parameters ν =20, 6, and 3, respectively. The Gaussian copula evaluates the lowest credit spread for the upper tranches (senior, super-senior), while the rotated-Gumbel copula evaluates the highest credit spread for equity. For example, the credit spread for the senior tranche is 0.65bp by the Gaussian copula, which is much lower than 21.81bp by *t*(3) copula or 19.04bp by rotated-Gumbel copula. We see that the recognition of tail dependencies yields large differences in risk sensitivity for the upper tranches.

Table 8. Credit spread for each tranche (bp)

Copula	Equity	Mezzanine	Senior	Super-senior
Gaussian	1147.43	63.38	0.65	0.000
t(20)	1061.07	86.94	2.33	0.002
t(6)	899.52	127.82	9.11	0.043
t(3)	735.55	165.40	21.81	0.196
Rotated-Gumbel	1018.34	59.01	19.04	2.685
Clayton	860.61	135.77	12.65	0.099

4. Enterprise risk management

Banks measure risks within market, credit, and operational risk categories. Many major banks aggregate firm-wide risks for each risk category to calculate the economic capital needed out to a one-year horizon. The economic capital is allocated to each

[9] The tranche with [*a*%, *b*%] covers the portfolio loss, while the loss rate is in [*a*%, *b*%].
[10] Credit spread s is calculated as $s = -\ln(1-EL)/T$, where *EL* is the expected loss for the tranche and *T* is 5-year maturity.

business unit and used as performance measures (see, for example, Klaassen and van Eeghen [2009]). Rosenberg and Schuermann (2006) have explored various risk aggregation methods empirically, and Brockmann and Kalkbrener (2010) have introduced the Deutsche Bank model.

Since banks apply different methods to each risk category, the first problem in aggregating firm-wide risk is to determine which level of the correlation structure we should notice (see Figure 4). Some advanced banks apply a tail dependent *t* copula or a high correlation of Gaussian copula to profit–loss for each risk category. This is called top level aggregation. Profit–loss is sometimes proxied by a time series of some index. Some other advanced banks apply a *t* copula to the joint distribution of risk factors (RFs) in each risk category. This is called base level or bottom up aggregation.

With top level aggregation, banks sometimes calibrate Gaussian or *t* copula parameters (including ν for *t* copula) by using each time series for market and credit profit–loss. They determine correlations with other risk categories *a priori,* without statistical estimates. In this context, intuitively identifiable correlation structures are desirable for actual practice.

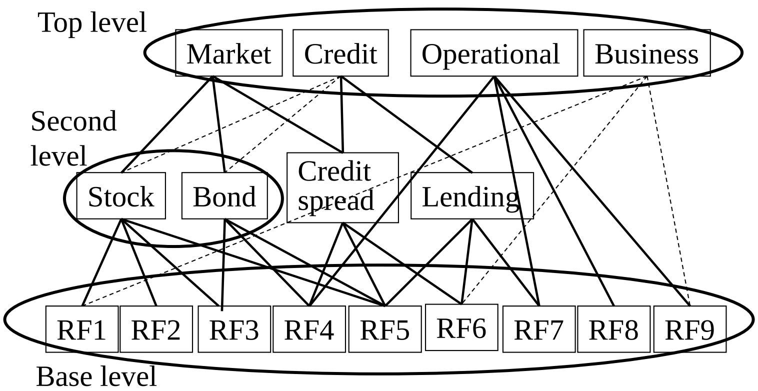


Figure 4. Aggregation level

Differences in risk evaluation periods for each risk category represent a practical issue for enterprise risk aggregation. Although the risk horizon for calculating economic capital is one year, banks assume some positions may close in a short-term period of high market liquidity. One and ten days are standard risk evaluation periods for liquid market risks for trading positions. Three months and six months are typical evaluation periods for non-trading positions. Selecting the optimal time intervals for given correlation structures and methods for selecting ideal serial correlation structures when estimating economic capital remain outstanding issues.

5. Concluding remarks

This paper surveys the application of copulas in the banking industry, focusing in particular on ways to incorporate stressed situations into risk measurements.

The application of copulas in the banking industry can differ from academic convention, incorporating drastic assumptions in certain cases. Research on applications that account for the needs of the banking industry is likely to prove beneficial.

References

Azzalini, Adelchi and Antonella Capitanio, "Distributions generated by perturbation of symmetry with emphasis on a multivariate skew *t*-distribution," *Journal of the Royal Statistical Society Series B*, **65**(2), 2003, pp.367–389.

Basel Committee on Banking Supervision, Joint Forum "Developments in modelling risk aggregation," 21 October, 2010.

Basel Committee on Banking Supervision, "Basel III: A global regulatory framework for more resilient banks and banking systems - revised version June 2011," 2011.

Brockmann, Michael and Michael Kalkbrener, "On the aggregation of risk," *Journal of Risk*, **12**(3), 2010, pp. 45–68.

Burtschell, Xavior, Jon Gregory, and Jean-Paul Laurent, "A comparative analysis of CDO pricing models under the factor copula framework," *Journal of Derivatives*, **16**(4), 2009, pp.9–37.

Frey, Rüdiger, Alexander McNeil, and Mark Nyfeler, "Copulas and credit models," *Risk*, **14**(10), 2001, pp.111–114.

Isogai, Takashi, "Benchmarking unconditional VaR and ES calculation methods: a numerical simulation with fitted truncated stable distribution," Bank of Japan Working Paper Series No.14-E-1, 2014.

Joe, Harry, *Multivariate Models and Dependence Concepts*, Chapman & Hall, 1997.

Jorion, Philippe, *Value at Risk: The New Benchmark for Managing Financial Risk*, 3rd edition, McGraw-Hill, 2006.

Kawaguchi, Muneki, Suguru Yamanaka, and Yusuke Tashiro, Improvement of Credit

Concentration Risk Evaluation Using Copulas, *Securities Analysts Journal*, **52**(3), 2014, pp.43–51.

Klaassen, Pieter and Idzard van Eeghen, *Economic Capital: How It Works and What Every Manager Needs to Know*, Elsevier Science, 2009.

Kurowicka, Dorota and Harry Joe, *Dependence Modeling: Vine Copula Handbook*, World Scientific Publishing, 2010.

Li, David X., "On default correlation: A copula function approach," *Journal of Fixed Income*, **9**(4), 2000, pp.43–54.

McNeil, Alexander J., Rüdiger Frey, and Paul Embrechts, *Quantitative Risk Management: Concepts, Techniques, and Tools,* Princeton University Press, 2005.

Merton, Robert C., "On the pricing of corporate debt: the risk structure of interest rates," *Journal of Finance,* **29**(2), 1974, pp.449–470.

Nelsen, Roger B., *An Introduction to Copulas*, 2nd edition, Springer, 2006.

Rosenberg, Joshua V. and Til Schuermann, "A general approach to integrated risk management with skewed, fat-tailed risks," *Journal of Financial Economics*, **79**(3), 2006, pp.569–614.

Savu, Cornelia and Mark Trede, "Hierarchies of Archimedean copulas," *Quantitative Finance*, **10**(3), 2010, pp.295–304.

Shintani, Kohei, Tetsuya Yamada, and Toshinao Yoshiba, "Correlation structures between assets' returns in financial crisis: Evaluation using copulas," *Kin'yu-kenkyu*, **29**(3), 2010, pp.89–122 (in Japanese).

Tsafack, Georges, "Asymmetric dependence implications for extreme risk management," *Journal of Derivatives*, **17**(1), 2009, pp.7–20.

Tozaka, Hironobu and Toshinao Yoshiba, "Commentary on concrete applications of copula in financial practices," *Kin'yu-kenkyu*, **24**(S-2), 2005, pp.115–162 (in Japanese).

Yoshiba, Toshinao, "Risk aggregation by a copula with a stressed condition," Bank of Japan Working Paper Series No.13-E-12, 2013.