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The Zero Lower Bound and Parameter Bias in an Estimated DSGE Model

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Abstract

This paper examines how and to what extent parameter estimates can be biased in a dynamic stochastic general equilibrium (DSGE) model that omits the zero lower bound (ZLB) constraint on the nominal interest rate. Our Monte Carlo experiments using a standard sticky-price DSGE model show that no significant bias is detected in parameter estimates and that the estimated impulse response functions are quite similar to the true ones. However, as the probability of hitting the ZLB increases, the parameter bias becomes larger and therefore leads to substantial differences between the estimated and true impulse responses. It is also demonstrated that the model missing the ZLB causes biased estimates of structural shocks even with the virtually unbiased parameters.

Keywords: Zero lower bound; DSGE model; Parameter bias; Bayesian estimation
JEL classification: C32, E30, E52

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1 Introduction

Dynamic stochastic general equilibrium (DSGE) models have become a prominent tool for policy analysis. In particular, following the development of Bayesian estimation and evaluation techniques, estimated DSGE models have been extensively used by a range of policy institutions, including central banks. At the same time, the zero lower bound (ZLB) constraint on the nominal interest rates has been a primary concern for policymakers. Much work has been devoted to understand how the economy works and how policy should be conducted in the presence of this constraint from a theoretical perspective.[1] However, empirical studies that estimate DSGE models including the ZLB are still scarce because of computational difficulties in the treatment of nonlinearity arising from the bound,[2] and hence most practitioners continue to estimate linearized DSGE models without explicitly considering the ZLB.

This paper examines how and to what extent the estimates of structural parameters can be biased in an estimated DSGE model where the existence of the ZLB is omitted in the estimation process. Suppose that there is a ZLB constraint in the economy and that an econometrician fits a model without taking into account this constraint. We would then expect the parameter estimates in the model to be biased. If significant biases were detected, it would cast some doubt on the common practice in which the ZLB constraint is not explicitly taken into account and would motivate researchers toward the use of an estimation procedure that deals with the constraint. Conversely, if the biases involved are negligible, it would at least assure practitioners that their common practice leads to reliable estimates in its own way.

More specifically, we construct artificial time series simulated from a standard sticky-price DSGE model that incorporates an occasionally binding constraint on the nominal interest rate. The parameters calibrated in this data generating process (DGP) are regarded as true values.

[1] See, for instance, Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005), Adam and Billi (2006, 2007), Christiano, Eichenbaum, and Rebelo (2011), Aruoba and Schorfheide (2013), Bodenstein, Guerrieri, and Gust (2013), Braun, Körber, and Waki (2012), Erceg and Lindé (2014), Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2012), Gavin, Keen, Richter, and Throckmorton (2013), Nakata (2013a, 2013b), and Ngo (2014).

[2] A remarkable exception is Gust, López-Salido, and Smith (2012) who estimate a nonlinear DSGE model in which the ZLB is occasionally binding. Based on the estimated model, they quantify the effect of the ZLB constraint on the recent economic slump in the US.

The solution algorithm follows from Erceg and Lindé (2014) and Bodenstein, Guerrieri, and Gust (2013).[3] In their algorithm, if the model-implied nominal interest rate falls below zero, a sequence of contractionary monetary policy shocks is added in both the current and anticipated periods so that the contemporaneous and expected interest rates at the lower bound are zero. Then, using the simulated data, a Monte Carlo experiment is conducted, in which the model is estimated without imposing the ZLB. In the estimation, we employ Bayesian methods, which are now extensively used to estimate DSGE models. We set the prior means to the true parameter values and assess the parameter biases from neglecting the ZLB by comparing the posterior means and Bayesian credible intervals with the true values.

One could argue that the DGP in our analysis should be replaced with a fully nonlinear model as in Braun, Körber, and Waki (2012), Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2012), Gust, López-Salido, and Smith (2012), Gavin, Keen, Richter, and Throckmorton (2013), Nakata (2013a, 2013b), and Ngo (2014). These authors argue that the solution algorithm in which nonlinearity is considered only in monetary policy rules but the remaining equilibrium conditions are linearized may lead to an inaccurate assessment of the ZLB.[4] However, if the DGP is characterized by a fully nonlinear model, we cannot identify the source of parameter bias: whether the exclusion of the ZLB or the linearization of the model. In this regard, the solution method employed in this paper is compatible with the standard solution algorithms for linear rational expectations models, such as Blanchard and Kahn (1980) and Sims (2002). Therefore, our analysis points to parameter bias only resulting from omitting the ZLB constraint.

Our main results are summarized as follows. In the baseline economy, where the true parameter values are calibrated such that the probability of hitting the ZLB is about six percent,[5] we find that

[3]These studies apply the method in Laséen and Svensson (2011), who propose a convenient algorithm to construct policy projections conditional on alternative anticipated policy rate paths in linearized DSGE models.

[4]In particular, Nakata (2013a) and Ngo (2014) stress that the difference between such a quasi-linear method as used in our analysis and the fully nonlinear method becomes larger when monetary policy deviates from the optimal commitment policy. Because a Taylor-type monetary policy rule is assumed in our model, our approach is subject to the issue raised by them.

[5]This probability is in line with the simulation result in Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2012), who simulate a similar model to ours calibrated for the US economy in a fully nonlinear

no substantial bias is detected while the estimates of the monetary policy parameters are slightly biased. We demonstrate that these biases in parameter estimates do not amount to substantial biases in the estimates of the impulse response functions.[6] However, if the true parameter values in the DGP are altered such that the probability of binding at the constraint increases, we show that the biases in the parameter estimates become larger and that the estimated impulse responses can then substantially differ from the true responses. This finding suggests that researchers in a very low interest rate environment should make use of an estimation procedure that explicitly takes into account the ZLB.

Let us emphasize that parameter bias is not the only thing that matters in the estimation of DSGE models. As Aoki and Ueno (2012) have argued, missing the ZLB can cause biased estimates of structural shocks. Concerning this issue, we demonstrate that substantial differences between true shocks and the corresponding smoothed estimates are found particularly in monetary policy shocks even though the estimated parameters are virtually unbiased.

The remainder of the paper proceeds as follows. Section 2 describes the stylized DSGE model used for our analysis and the solution algorithm that incorporates the occasionally binding constraint on the nominal interest rate into the linear rational expectations system. Section 3 explains our Monte Carlo experiments and presents our results. Section 4 conducts a robustness analysis with a larger-scale DSGE model. Section 5 is the conclusion.

2 The Model and Solution Method

This section describes the DGP used in our analysis. The DGP consists of a small-scale DSGE model with sticky prices and a monetary policy rule. In order to incorporate the ZLB constraint on the nominal interest rate, the model is solved using the method employed in Erceg and Lindé (2014) and Bodenstein, Guerrieri, and Gust (2013).

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setting.

[6]Our robustness analysis shows that these results hold even if we use a larger-scale DSGE model à la Smets and Wouters (2003, 2007).

2.1 The Model

In the model economy, there are households, perfectly competitive final-good firms, monopolistically competitive intermediate-good firms that face price stickiness, and a monetary authority. For empirical validity, the model features habit persistence in consumption preferences, price indexation to recent past and steady-state inflation, and monetary policy smoothing. A similar model is used in Dennis (2004, 2009) and Milani and Treadwell (2012), among others.

The log-linearized equilibrium conditions are summarized as follows:[7]

$$\tilde{Y}_t = \frac{1}{1+\gamma} E_t \tilde{Y}_{t+1} + \frac{\gamma}{1+\gamma} \tilde{Y}_{t-1} - \frac{1-\gamma}{\sigma(1+\gamma)} \left(\tilde{r}_t^n - E_t \tilde{\pi}_{t+1} \right) + z_t^d, \tag{1}$$

$$\tilde{\pi}_t = \frac{\beta}{1+\beta\iota} E_t \tilde{\pi}_{t+1} + \frac{\iota}{1+\beta\iota} \tilde{\pi}_{t-1} + \frac{(1-\xi)(1-\xi\beta)}{\xi(1+\beta\iota)} \left[\left(\chi + \frac{\sigma}{1-\gamma} \right) \tilde{Y}_t - \frac{\sigma\gamma}{1-\gamma} \tilde{Y}_{t-1} \right] + z_t^p, \tag{2}$$

$$\tilde{r}_t^n = \max[\phi_r \tilde{r}_{t-1}^n + (1-\phi_r) \left(\phi_\pi \tilde{\pi}_t + \phi_y \tilde{Y}_t \right) + z_t^r, \ -(\bar{r}+\bar{\pi})]. \tag{3}$$

Eq. (1) is the spending Euler equation, where $\tilde{Y}_t$, $\tilde{r}_t^n$ and $\tilde{\pi}_t$ are output, the nominal interest rate, and inflation in terms of the percentage deviations from their steady-state values, z_t^d is a shock to households' preferences, interpreted as a demand shock, $\gamma \in [0,1]$ is the degree of habit persistence, and $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution.

Eq. (2) is the New Keynesian Phillips curve, where $\beta \in (0,1)$ is the subjective discount factor determined by the steady-state relationship $\beta = 1/r$, where r is the steady-state gross real interest rate, $\iota \in [0,1]$ is the weight of price indexation to recent past inflation π_{t-1} relative to steady-state inflation π, $\xi \in (0,1)$ is the so called Calvo parameter, which measures the degree of price stickiness, and $\chi > 0$ is the inverse of the elasticity of labor supply. z_t^p is a cost-push shock.

Eq. (3) is a Taylor (1993) type monetary policy rule with the ZLB on the nominal interest rate, where $\phi_r \in [0,1)$ is the degree of interest rate smoothing, $\phi_\pi > 1$ and $\phi_y > 0$ represent the degrees of interest rate policy responses to inflation and output gap respectively, z_t^r is a monetary policy shock, and $\bar{\pi} = 100(\pi - 1)$ and $\bar{r} = 100(r-1)$ are the steady-state values for the real interest rate and inflation, respectively. As $\tilde{r}_t^n$ is expressed as the deviation from the steady-state nominal interest rate $\bar{r} + \bar{\pi}$, the max function constrains the level of the nominal interest rate to be greater than or equal to zero.

[7] See the Appendix for the full description of the model.

Each shock z_t^x, $x \in \{d, p, r\}$ is governed by a stationary first-order autoregressive process:

$$z_t^x = \rho_x z_{t-1}^x + \varepsilon_t^x, \tag{4}$$

where $\rho_x \in [0, 1)$ is an autoregressive coefficient and ε_t^x is a normally distributed innovation with mean zero and standard deviation σ_x.

In the DGP, the percentage deviation of output from the steady state $100 \log(Y_t/Y)$, the inflation rate $100 \log \pi_t$, and the nominal interest rate $100 \log r_t^n$ are assumed to be observable. Then, these observables are related to the model-implied variables by the following observation equations:

$$\begin{bmatrix} 100 \log(Y_t/Y) \\ 100 \log \pi_t \\ 100 \log r_t^n \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{\pi} \\ \bar{r} + \bar{\pi} \end{bmatrix} + \begin{bmatrix} \tilde{Y}_t \\ \tilde{\pi}_t \\ \tilde{r}_t^n \end{bmatrix}. \tag{5}$$

2.2 Parameter Setting

The model is parameterized according to standard choices in the literature in order to make our test economy as representative as possible. Table 1 summarizes our baseline parameter setting.

The inverse of intertemporal elasticity of substitution σ is set to be unity, so that the households' preferences are characterized by the log-utility function. We assign the inverse of the elasticity of labor supply $\chi = 2$. The habit persistence and price indexation parameters, γ and ι, are both set at 0.5. As is often the case in the New Keynesian literature, we set the Calvo parameter $\xi = 0.75$, which implies that the average duration of prices is four quarters. The monetary policy parameters ($\phi_\pi = 1.5$, $\phi_y = 0.5$) that represent the degrees of interest rate responses to inflation and output gap follow from the coefficients in the original Taylor (1993) rule. The policy-smoothing parameter ϕ_r is set to be 0.5. The steady-state inflation rate $\bar{\pi}$ is set at 0.5, implying that the central bank's target inflation rate is two percent annually. The steady-state value of the real interest rate ($\bar{r} = 0.5$) is almost the same as the average of the ex-post real interest rate calculated from the three-month Treasury bill rate and changes in the GDP implicit deflator in the post-1980s US sample.

For each shock, moderate persistency is assumed: $\rho_d = 0.5$, $\rho_p = 0.5$, $\rho_r = 0.5$. The standard deviation of each shock ($\sigma_d = 0.25$, $\sigma_p = 0.25$, $\sigma_r = 0.25$) is calibrated in line with the prior mean often used in the literature that estimate similar DSGE models using Bayesian methods.

2.3 Solution Method

The solution algorithm follows from Erceg and Lindé (2014) and Bodenstein, Guerrieri, and Gust (2013). They apply the method in Laséen and Svensson (2011), who propose a convenient algorithm to construct policy projections conditional on alternative anticipated policy rate paths in linearized DSGE models. This solution method is compatible with the standard solution algorithms for linear rational expectations models, such as Blanchard and Kahn (1980) and Sims (2002), and hence yields a straightforward interpretation about the implications of the ZLB constraint. For a detailed description of the solution method, see the Appendix A to Bodenstein, Guerrieri, and Gust (2013).

To be specific to our analysis, Eq. (3) that includes the max operator is replaced with:

$$\tilde{r}_t^n = \phi_r \tilde{r}_{t-1}^n + (1-\phi_r)\left(\phi_\pi \tilde{\pi}_t + \phi_y \tilde{Y}_t\right) + z_t^r + m_t, \tag{6}$$

where m_t is the contractionary monetary policy shock that enforces the ZLB constraint. If the unconstrained nominal interest rate $\bar{r} + \bar{\pi} + \phi_r \tilde{r}_{t-1}^n + (1-\phi_r)\left(\phi_\pi \tilde{\pi}_t + \phi_y \tilde{Y}_t\right) + z_t^r$ (in terms of percentage level) falls below zero, a positive m_t is endogenously determined so that $\bar{r} + \bar{\pi} + \phi_r \tilde{r}_{t-1}^n + (1-\phi_r)\left(\phi_\pi \tilde{\pi}_t + \phi_y \tilde{Y}_t\right) + z_t^r + m_t = 0$. We call this m_t a binding shock.

m_t should be treated not only as an unanticipated shock but also as an anticipated shock given agents facing large negative shocks in the economy expect that the nominal interest rate should be at the ZLB for some periods in the future. To take such expectation channel into account, m_t is extended as follows:

$$\begin{aligned}
m_t &= m_{t-1}^1 + \nu_t^0 \\
m_t^1 &= m_{t-1}^2 + \nu_t^1 \\
m_t^2 &= m_{t-1}^3 + \nu_t^2 \\
&\vdots \\
m_t^{K-1} &= m_{t-1}^K + \nu_t^{K-1} \\
m_t^K &= \nu_t^K,
\end{aligned}$$

where K is the maximum number of future periods in which the unconstrained monetary policy rule implies the negative nominal interest rate. Based on this specification of the binding shock, each ν_t^k, $k = 0, 1, \ldots, K$ has an effect on $E_t \tilde{r}_{t+k}^n$ since $E_t m_{t+k} = \nu_t^k$. Therefore, if $\bar{r} + \bar{\pi} + E_t \tilde{r}_{t+k}^n < 0$

without the binding shocks, $\bar{r} + \bar{\pi} + E_t \tilde{r}^n_{t+k} = 0$ can be enforced by adjusting ν^k_t. As these binding shocks are chosen depending on the state of the economy, the expected duration of ZLB periods is endogenously determined. In practice, ν^k_t affects $E_t \tilde{r}^n_{t+k'}$ for $k' \neq k$ because of the dynamic structure of the model, but we can exactly find a set of ν^k_t for $k = 0, 1, \ldots, K$ that ensures the ZLB since there are as many binding shocks as there are periods for the zero nominal interest rate.

3 Monte Carlo Experiments

In this section, we conduct Monte Carlo experiments to examine how the parameter estimates can be biased if the existence of the ZLB is excluded in the estimation process. Then, we analyze the effects of these parameter biases on the model properties by comparing the estimated impulse responses with those based on the true parameter values. Moreover, we investigate whether the model omitting the ZLB causes biased estimates of structural shocks.

3.1 Design

Our Monte Carlo experiments proceed as follows. First, we generate an artificial time series of output (deviation from the steady state), inflation, and the nominal interest rate from the DGP as described in the previous section, given the parameter values presented in Table 1. Thus, the simulated series can be regarded as those that reflect the existence of the ZLB constraint on the nominal interest rate. The simulated sample size is 200 observations,[8] which corresponds to quarterly observations over a period of 50 years. This sample size is chosen because it is comparable to the size with which many researchers estimate DSGE models in practice.

Next, using the simulated data, we estimate the DSGE model that consists of Eq. (1)–(4) together with the observation equations (5) without imposing the ZLB constraint. In the estimation, we employ Bayesian methods. The prior distributions of the model parameters are given in Table 2, where each prior mean is set to the corresponding true parameter value used in generating the data. To obtain the posterior distributions, we generate 200,000 draws using the random-walk Metropolis–Hastings algorithm and discard the first 25 percent of these draws.[9]

[8]The model is simulated for 300 periods, and the first 100 observations are eliminated.

[9]The scale factor for the jumping distribution in the Metropolis–Hastings algorithm is adjusted so that the accep-

These steps are replicated for 500 times, and the posterior means and Bayesian credible intervals for the parameters are averaged over the replications. Then, we can evaluate the parameter bias arising from missing the ZLB by examining how the resulting posterior means and credible intervals differ from the true parameter values.

According to the simulated sample of 100,000 periods (500 Monte Carlo replications with a sample size of 200 observations) using the baseline parameter setting presented in Table 1, our model economy is at the ZLB for 6.4 percent of quarters. The distribution of the duration of the ZLB spells is shown in Figure 1, where the average duration at the ZLB is 1.8 quarters.[10] These statistics are quite comparable to the simulation results in Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2012), who simulate a similar model to ours calibrated for the US economy in a fully nonlinear setting and show that the economy spends 5.5 percent of quarters at the ZLB and that the average duration of a spell at the ZLB is 2.1 quarters.

3.2 Results for Baseline Experiment

The second to fourth columns in Table 3 compare the true parameter values with the averages of the posterior means and the 90 percent credible intervals for the estimated parameters in the baseline experiment. We can see that most of the estimated parameters are not biased as the posterior mean estimates are very close to the true parameter values and the 90 percent credible intervals contain the true values. This may be good news for many researchers in that the structural parameters relating to preferences and nominal rigidities, the so-called deep parameters, are not much affected by ignoring the ZLB. We detect some biases in the estimates of parameters that characterize the monetary policy rule, ϕ_π and σ_r, in the sense that the mean estimates for these parameters differ from the corresponding true values compared with the other parameters. However, these biases are not substantial because the credible intervals for these parameters still include the true values.

tance rate of candidate draws is approximately 25 percent.

[10] We acknowledge that the average duration of the ZLB spells is short, compared with the recent experience that major central banks have faced the ZLB for a prolonged period. We tried to parameterize the DGP so that the duration was longer, but then our solution method frequently failed in finding the equilibrium paths because the contractionary effects of the binding shocks became too large. This issue made it difficult to simulate samples in our Monte Carlo experiments.

To assure that the small biases detected in the estimates of ϕ_π and σ_r are because of omitting the ZLB rather than because of the identification problems as argued by Canova and Sala (2009) and Iskrev (2010a, 2010b), we conduct an additional Monte Carlo experiment where the ZLB constraint is not imposed in the DGP. The results are presented in the last two columns in Table 3 (No ZLB in DGP). Because all the mean estimates are almost the same as the true values, we confirm that the parameter estimates are unbiased if the exclusion of the ZLB is not an issue in the estimation.

An intuitive explanation for these biased estimates is the following. In the DGP, the monetary policy reaction function has a kink where the ZLB becomes binding, that is, the reaction function has positive slopes with respect to inflation, output, and the lagged nominal interest rate if the unconstrained nominal interest rate is positive, but the slopes become flat if it is negative. However, when such a kink is omitted in the estimation process, as is the case in our experiment, the estimated slopes are approximated to lie between the positive and flat slopes in the DGP, and thus the parameters in the monetary policy rule can be underestimated. In our experiment, the biases emerge as a change in the estimate of ϕ_π, although the estimate of ϕ_y could also be potentially affected. The reason for the upward bias in the estimate of σ_r is also straightforward. In the simulated data, the nominal interest rate never falls below zero because the DGP takes account of the ZLB constraint. In the estimation process, however, the model-implied nominal interest rate can be negative, and such a discrepancy must be captured by the monetary policy shock ε_t^r, which results in the increase in σ_r.

These causes of the parameter biases could have some influence on the estimates of the other structural parameters that characterize the Euler equation and the Phillips curve as we apply the system-based estimation approach. However, our experiment has revealed that such influence is quite limited.

An important issue is whether the parameter biases detected in the present experiment can lead to a sizeable difference in the dynamic properties of the model. To investigate this issue, we compare the impulse response functions estimated without considering the ZLB with those computed using the true parameter values. Figure 2 depicts the impulse responses of output, inflation, and the nominal interest rate to one standard deviation shocks in demand, cost-push, and monetary policy. The responses are expressed in terms of the percentage deviation from the steady state in order to exclude the effects of possible biases in the steady-state parameters and to focus on the changes in the transmission of shocks. In each panel, the solid thick line represents the true response, and the solid thin line and dashed lines are respectively the posterior mean and 90 percent credible interval for the estimated response. Throughout the figure, although small differences are found between the mean estimates and the true response, all the credible intervals include the true responses. Therefore, the parameter biases in the baseline experiment do not amount to substantial biases in the impulse response functions.

The results presented so far suggest that no substantial biases are detected in the parameter estimates even if the ZLB constraint is omitted in the estimation. However, parameter bias may not be the only thing that matters. As Aoki and Ueno (2012) emphasize, ignoring the ZLB can cause biased estimates of the structural shocks. To investigate this issue, we conduct the following experiment. First, we generate a single sample of 200 observations using the baseline DGP. A set of simulated data are shown in Figure 3. Next, using the simulated data, we estimate the model without imposing the ZLB and compute the Kalman smoothed estimates of the shocks. Then, we assess whether the estimated shocks are biased by comparing them with the true shocks used to generate the data.[11] Figure 4 compares the true shocks (solid lines) and the mean estimates of the smoothed shocks (dashed lines) regarding demand, cost-push, and monetary policy. To make the difference obvious, the smoothed estimates for the shock variables z_t^x, $x \in \{d, p, r\}$ that follow the first-order autoregressive processes are presented instead of those for the i.i.d. disturbances ε_t^x, $x \in \{d, p, r\}$. While the differences are small in the demand and cost-push shocks, substantial differences are found in the monetary policy shock. In particular, the huge differences coincide with the periods when the nominal interest rate is bounded at zero in Figure 3. Thus, the model ignoring the ZLB can lead to biased estimates of the shocks even though parameter bias is marginal.

[11] One could argue that the smoothed estimates of the shocks might be biased even if the ZLB was not an issue. To examine this possibility, we generated a sample without imposing the ZLB and conducted the same experiment. We found that the true and estimated shocks were almost the same.

3.3 Results for Alternative Experiments

According to the simulated time series in our baseline experiment, the nominal interest rate is bounded at zero for 6.4 percent of quarters. As the probability of hitting the ZLB increases in the DGP, it could be possible that parameter biases from excluding the ZLB in the estimation become large. To analyze such a possibility, we change the parameters assigned to the DGP so that the model economy is more frequently constrained by the ZLB. We then conduct the same Monte Carlo exercises as the baseline experiment using the data simulated by the DGP with the alternative parameter settings. Specifically, we consider two cases. One is where the true steady-state real interest rate $\bar{r}$ falls by 0.3 (1.2 percent annually) so that $\bar{r} = 0.2$. In this case, the probability of being stuck at the ZLB is 12.9 percent of the simulated sample. The other is where the standard deviation of the demand shock σ_d increases from 0.25 to 0.55. In the latter case, the number of zero interest rate periods increases because large negative shocks are more likely to depress the economy and to lower the nominal interest rate, the model economy is at the ZLB for 14.3 percent of quarters. The distribution of the duration of the ZLB spells in each case is presented in Figure 5. In both cases, the average duration at the ZLB is 1.9 quarters, which is almost the same as in the baseline experiment.

The middle two columns in Table 4 (Case of low $\bar{r}$) present the averages of the posterior means and the 90 percent credible intervals for the parameters in the experiment where the true parameter value for the real interest rate decreases to $\bar{r} = 0.2$. Compared with the baseline experiment, the differences between the mean estimates and true values are obvious for most of the parameters. In particular, the estimates of the monetary policy parameter ϕ_π, the autoregressive coefficient ρ_r and the standard deviation σ_r of the monetary policy shock exhibit substantial biases in the sense that the credible intervals for these parameters do not include the true values. Regarding the biases in the estimates of ϕ_π and σ_r, the same intuition as in the baseline experiment applies. The upward bias in ρ_r is considered as a result of capturing persistent dynamics of the simulated nominal interest rate during the zero interest rate spells.

The last two columns in Table 4 (Case of large σ_d) show the parameter estimates in the experiment where the true standard deviation of the demand shock increases to $\sigma_d = 0.55$. In addition to the biased parameters mentioned in the previous case, significant biases are found in the estimates of the habit persistence and price indexation parameters, γ and ι, the policy-smoothing parameter ϕ_r, and the steady-state real interest rate $\bar{r}$. The downward biases in the estimates of γ, ι, and ϕ_r, all of which determine the internal persistency of the model economy, may contribute to lower the model-implied volatilities of the observed variables increased by the upward bias in ρ_r and σ_r. The reason for the upward bias in $\bar{r}$ is as follows. The presence of the ZLB forces the nominal interest rate to be equal or greater than zero. However, if the model estimation fails to consider the ZLB, the nominal interest rate can be negative, and the mean of the model-implied nominal interest rate declines. Then, the estimate of the steady-state nominal interest rate must rise to adjust the difference between the mean of the model-implied nominal interest rate and that of the corresponding series simulated from the DGP with the ZLB, given the model is log-linearized around the steady state. In this experiment, such an adjustment has mostly emerged as a change in the estimate of $\bar{r}$,[12] rather than $\bar{\pi}$.

To gain more insight about the parameter biases detected in the two experiments, we compare the sample properties implied by the model without the ZLB to those implied by the model with the ZLB. Table 5 shows the covariances of output, inflation, and the nominal interest rate implied by the DGP (ZLB with true parameters), the model omitting the ZLB with the true parameters (No ZLB with true parameters), and the model omitting the ZLB with the biased parameters (No ZLB with biased parameters) in (1) the case of low $\bar{r}$ and (2) the case of large σ_d. In the model with the biased parameters, we replace the true parameters with the corresponding mean estimates in Table 4 only for the parameters that exhibit substantial bias mentioned in the preceding paragraphs; that is, in the case of low $\bar{r}$, the true parameters of ϕ_π, ρ_r, and σ_r are replaced with the biased mean estimates, and, in the case of large σ_d, the true parameters of γ, ι, ϕ_π, ρ_r, $\bar{r}$, and σ_r are replaced. In both cases, the covariances implied by the model omitting the ZLB with the true parameters considerably change from those implied by the DGP. However, the model omitting the ZLB with the biased parameters helps to make most of the components in the covariances close to those of the DGP; the biased parameters contribute to matching all the components except for the covariance of inflation and the interest rate in the former case and to matching the variances of output, inflation,

[12] A change in $\bar{r}$ affects the subjective discount factor β through the steady-state relationship $\beta = 1/r$ and the definition $r = \bar{r}/100 + 1$, but the resulting change in β is quite marginal in its magnitude.

and the interest rate and the covariance of output and inflation in the latter case.

Figure 6 and 7 present the estimated and true impulse responses in order to examine the effects of the parameter biases on the model properties in the alternative experiments. Compared with the responses in the baseline experiment (Figure 2), the differences between the mean estimates and the true responses become large in most of the panels. In the case of low $\bar{r}$, particularly, the credible intervals for the output responses to the cost-push and monetary policy shocks and the inflation response to the monetary policy shock do not contain the true responses in the first few periods after the shocks. In the case of large σ_d, the true responses are outside the credible intervals regarding all the responses to the demand and monetary policy shocks. Therefore, the dynamic properties of the estimated model are no longer the same as the true ones.

4 Robustness Analysis

While we have employed a small-scale DSGE model in our analysis, an increased number of policy institutions are actively developing and estimating larger-scale DSGE models à la Smets and Wouters (2003, 2007). Thus, as a robustness analysis, we conduct a similar exercise as in the baseline experiment using a version of the Smets and Wouters (2007) model.

4.1 Smets–Wouters Type Model

The model employed in this section is a slightly modified version of the Smets and Wouters (2007) model. This version differs from their original model in the following two aspects. First, for the purpose of comparison with our baseline experiment, the monetary policy rule is replaced with the one specified in our baseline model whereas Smets and Wouters employ a generalized Taylor rule in which the policy rate is adjusted in response to the level and changes in the theoretical output gap (i.e., the gap between real output and output that would be obtained in the absence of nominal rigidities) in addition to inflation. Second, as in the baseline experiment, the binding shocks m_t are incorporated into the monetary policy rule in order to take account of the ZLB constraint on the nominal interest rate in the DGP.

The model consists of the following log-linearized equations.[13] In what follows, all the variables are expressed in terms of their percentage deviation from the steady-state balanced growth path.

$$
\begin{aligned}
\tilde{c}_t &= \frac{\lambda/\gamma}{1+\lambda/\gamma}\tilde{c}_{t-1} + \frac{1}{1+\lambda/\gamma}E_t\tilde{c}_{t+1} + \frac{(\sigma_c-1)w^h l/c}{\sigma_c(1+\lambda/\gamma)}\left(\tilde{l}_t - E_t\tilde{l}_{t+1}\right) \\
&\quad - \frac{1-\lambda/\gamma}{\sigma_c(1+\lambda/\gamma)}\left(\tilde{r}_t - E_t\tilde{\pi}_{t+1} + \varepsilon_t^b\right), && (7) \\
\tilde{\imath}_t &= \frac{1}{1+\beta\gamma^{1-\sigma_c}}\tilde{\imath}_{t-1} + \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}}E_t\tilde{\imath}_{t+1} + \frac{1}{\gamma^2\varphi(1+\beta\gamma^{1-\sigma_c})}\tilde{q}_t + \varepsilon_t^i, && (8) \\
\tilde{q}_t &= \frac{1-\delta}{r^k+1-\delta}E_t\tilde{q}_{t+1} + \frac{r^k}{r^k+1-\delta}E_t\tilde{r}_{t+1}^k - \left(\tilde{r}_t - E_t\tilde{\pi}_{t+1} + \varepsilon_t^b\right), && (9) \\
\tilde{y}_t &= \phi_p\left[\alpha\tilde{k}_t^s + (1-\alpha)\tilde{l}_t + \varepsilon_t^a\right], && (10) \\
\tilde{k}_t^s &= \tilde{z}_t + \tilde{k}_{t-1}, && (11) \\
\tilde{z}_t &= \frac{1-\psi}{\psi}\tilde{r}_t^k, && (12) \\
\tilde{k}_t &= \frac{1-\delta}{\gamma}\tilde{k}_{t-1} + \left(1-\frac{1-\delta}{\gamma}\right)\left[\tilde{\imath}_t + \gamma^2\varphi(1+\beta\gamma^{1-\sigma_c})\varepsilon_t^i\right], && (13) \\
\tilde{y}_t &= c_y\tilde{c}_t + i_y\tilde{\imath}_t + r^k k_y\tilde{z}_t + \varepsilon_t^g, && (14) \\
\tilde{\mu}_t^p &= \alpha\left(\tilde{k}_t^s - \tilde{l}_t\right) - \tilde{w}_t, && (15) \\
\tilde{\pi}_t &= \frac{\iota_p}{1+\beta\gamma^{1-\sigma_c}\iota_p}\tilde{\pi}_{t-1} + \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}\iota_p}E_t\tilde{\pi}_{t+1} - \frac{(1-\xi_p)(1-\beta\gamma^{1-\sigma_c}\xi_p)}{\xi_p(1+\beta\gamma^{1-\sigma_c}\iota_p)[(\phi_p-1)\varepsilon_p+1]}\tilde{\mu}_t^p + \varepsilon_t^p, && (16) \\
\tilde{r}_t^k &= -\left(\tilde{k}_t^s - \tilde{l}_t\right) + \tilde{w}_t, && (17) \\
\tilde{\mu}_t^w &= \tilde{w}_t - \left\{\sigma_l\tilde{l}_t + \frac{1}{1-\lambda/\gamma}\left[\tilde{c}_t - \frac{\lambda}{\gamma}\tilde{c}_{t-1}\right]\right\}, && (18) \\
\tilde{w}_t &= \frac{1}{1+\beta\gamma^{1-\sigma_c}}\tilde{w}_{t-1} + \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}}\left(E_t\tilde{w}_{t+1} + E_t\tilde{\pi}_{t+1}\right) - \frac{1+\beta\gamma^{1-\sigma_c}\iota_w}{1+\beta\gamma^{1-\sigma_c}}\tilde{\pi}_t \\
&\quad + \frac{\iota_w}{1+\beta\gamma^{1-\sigma_c}}\tilde{\pi}_{t-1} - \frac{(1-\xi_w)(1-\beta\gamma^{1-\sigma_c}\xi_w)}{\xi_w(1+\beta\gamma^{1-\sigma_c})[(\phi_w-1)\varepsilon_w+1]}\tilde{\mu}_t^w + \varepsilon_t^w, && (19) \\
\tilde{r}_t &= r_r\tilde{r}_{t-1} + (1-r_r)\left(r_\pi\tilde{\pi}_t + r_y\tilde{y}_t\right) + \varepsilon_t^r + m_t. && (20)
\end{aligned}
$$

Eq. (7) is the consumption Euler equation, where $\tilde{c}_t$ denotes consumption, $\tilde{l}_t$ is labor input, $\tilde{r}_t$ is the short-term nominal interest rate, $\tilde{\pi}_t$ is the inflation rate, ε_t^b captures the risk premium shock in the return on assets held by households, λ is the degree of external habit persistence in consumption preferences, γ is the steady-state growth rate, σ_c is the degree of relative risk aversion, and $w^h l/c$ is the steady-state value of labor relative to consumption. Eq. (8) is the investment adjustment

[13] See the appendix to Smets and Wouters (2007) for a full description of the model.

equation, where $\tilde{\imath}_t$ represents investment, $\tilde{q}_t$ denotes the real value of the existing capital stock, ε_t^i represents the shock to investment efficiency, β is the subjective discount factor, and φ is the steady-state elasticity of investment adjustment costs. Eq. (9) is the no-arbitrage condition for the value of capital, where $\tilde{r}_t^k$ is the real rental rate of capital, δ is the depreciation rate of capital, and r^k is the steady-state real rental rate of capital. Eq. (10) is the Cobb–Douglas production function with fixed costs where $\tilde{y}_t$ denotes output, $\tilde{k}_t^s$ is effective capital services, ε_t^a represents the TFP shock, ϕ_p is one plus the share of fixed costs in output, α is the share of capital in production. Eq. (11) gives the effective capital services used in production, where $\tilde{z}_t$ and $\tilde{k}_{t-1}$ denote the capital utilization rate and capital installed in the previous period. Eq. (12) is the condition for the capital utilization rate, where ψ is determined by a function of the steady-state elasticity of the rate adjustment costs. Eq. (13) is the capital accumulation equation. Eq. (14) is the aggregate resource constraint, where ε_t^g represents the exogenous spending shock, c_y, i_y, k_y are the steady-state output ratios of consumption, investment, and capital. Eq. (15) is the equation for the price markup $\tilde{\mu}_t^p$, where $\tilde{w}_t$ is the real wage. Eq. (16) is the New Keynesian Phillips curve, where ε_t^p represents the price markup shock, ξ_p and ι_p are the degrees of price stickiness and price indexation to past inflation, $(\phi_p - 1)$ is the steady-state goods market markup, and ε_p is the curvature of the Kimball (1995) goods market aggregator. Eq. (17) is the condition for capital and labor inputs in production. Eq. (18) is the equation for the wage markup $\tilde{\mu}_t^w$, where σ_l is the inverse elasticity of labor supply. Eq. (19) is the wage equation, where ε_t^w represents the wage markup shock, ξ_w and ι_w are the degrees of wage stickiness and wage indexation to past inflation, $(\phi_w - 1)$ is the steady-state labor market markup, and ε_w is the curvature of the Kimball (1995) labor market aggregator. Eq. (20) is the monetary policy rule, where r_r is the degree of interest rate smoothing, r_π and r_y represent the degrees of interest rate policy responses to inflation and output gap, respectively, ε_t^r is the monetary policy shock, and m_t is the binding shock.

There are seven structural shocks in the model. While the shocks regarding the TFP, the risk premium, external demand, investment, and monetary policy follow first-order autoregressive processes

$$\varepsilon_t^x = \rho_x \varepsilon_{t-1}^x + \eta_t^x, \ x \in \{a, b, g, i, r, \},$$

the shocks to the price markup and the wage markup follow ARMA(1,1) processes

$$\varepsilon_t^x = \rho_x \varepsilon_{t-1}^x + \eta_t^x - \mu_x \eta_{t-1}^x, \ x \in \{p, w\},$$

where ρ_x and μ_x are autoregressive and moving average coefficients, and η_t^x is a normally distributed innovation with mean zero and standard deviation σ_x.

The observation equations are

$$\begin{bmatrix} 100\Delta \log Y_t \\ 100\Delta \log C_t \\ 100\Delta \log I_t \\ 100\Delta \log W_t \\ 100 \log l_t \\ 100\Delta \log P_t \\ 100 \log r_t \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{l} \\ \bar{\pi} \\ \bar{r} \end{bmatrix} + \begin{bmatrix} \tilde{y}_t - \tilde{y}_{t-1} + \varepsilon_t^a \\ \tilde{c}_t - \tilde{c}_{t-1} + \varepsilon_t^a \\ \tilde{\imath}_t - \tilde{\imath}_{t-1} + \varepsilon_t^a \\ \tilde{w}_t - \tilde{w}_{t-1} + \varepsilon_t^a \\ \tilde{l}_t \\ \tilde{\pi}_t \\ \tilde{r}_t \end{bmatrix},$$

where $\bar{\gamma} = 100(\gamma - 1)$, $\bar{l}$ is the steady-state hours worked, $\bar{\pi} = 100(\pi - 1)$, and $\bar{r} = 100(\beta^{-1}\gamma^{\sigma_c}\pi - 1)$.

4.2 Design for the Monte Carlo Experiment

As shown in the second column in Table 6, the true parameter values that characterize the DGP basically follow from the prior means used in Smets and Wouters (2007). However, the standard deviations of the structural shocks σ_x, $x \in \{a, b, g, i, r, p, w\}$ are all changed from 0.1 to 0.45 so that the resulting probability of hitting the ZLB is the same (6.4 percent of quarters) as that in our baseline experiment.[14]

Then, the model without the ZLB constraint is estimated using the artificial time series generated from the DGP presented above. In the same way as the previous section, each prior mean is set to the corresponding true parameter value. For the standard deviations of the priors, we use the same values as set in Smets and Wouters (2007). We generate 125,000 posterior draws and discard the first 20 percent of the draws.

We conduct 200 Monte Carlo replications using a sample size of 200 observations. As in the preceding experiments, the posterior means and Bayesian credible intervals for the parameters are

[14] If these changes were not made, i.e., $\sigma_x = 0.1$, $x \in \{a, b, g, i, r, p, w\}$, the probability of hitting the ZLB was zero.

averaged over the replications.

The distribution of the duration of the ZLB spells is shown in Figure 8, based on the simulated sample of 40,000 periods (200 Monte Carlo replications with a sample size of 200 observations). In this experiment, the average duration at the ZLB is 1.7 quarters, which is almost the same as in the baseline experiment.

4.3 Results for the Robustness Analysis

The third to fourth columns in Table 6 show the posterior means and 90 percent credible intervals for the estimated parameters in the experiment using the Smets–Wouters type model. Most of the posterior mean estimates are almost the same as the true parameter values, and all the 90 percent credible intervals contain the true values. Thus, no substantial bias is detected in this experiment. The estimates of the monetary policy responses to inflation and output, r_π and r_y, and the subjective discount factor $100(\beta^{-1} - 1)$—related to the steady-state real interest rate—seems to be slightly biased, depending on one's point of view. For these biases, the same explanation as provided in the previous section applies.

As in the baseline experiment, we confirm from the last two columns in Table 6 (No ZLB in DGP) that the parameter estimates are unbiased when the ZLB constraint is not imposed in the DGP.

Figure 9 compares the estimated and true impulse responses of output, inflation, and the nominal interest rate to one standard deviation shocks to the TFP, the risk premium, external demand, investment, monetary policy, the price markup, and the wage markup. As with the preceding experiments, in each panel, the solid thick line represents the true response, the solid thin and dashed lines are respectively the posterior mean and 90 percent credible interval for the estimated response, and the responses are shown in terms of the percentage deviation from the steady state. Because the parameter estimates are virtually unbiased, the mean estimates are very close to the true responses in all of the panels.

To investigate the possibility that ignoring the ZLB can cause biased estimates of the structural shocks, we simulate a single sample of 200 observations, as shown in Figure 10, using the present DGP. Then, using the simulated sample, we estimate the model without imposing the ZLB and compute the Kalman smoothed estimates of the shocks. In Figure 11, the mean estimates of the smoothed shocks (dashed lines) about the TFP ε_t^a, the risk premium ε_t^b, external demand ε_t^g, investment ε_t^i, monetary policy ε_t^r, the price markup ε_t^p, and the wage markup ε_t^w are compared with the corresponding true shocks (solid lines). Non-negligible differences between the smoothed estimates and true shocks are detected in the monetary policy shocks during the periods when the nominal interest rate is stuck at zero in Figure 10. Therefore, we reach the same conclusion as in the baseline experiment that the model omitting the ZLB leads to biased estimates of the shocks even with the unbiased parameters.

5 Conclusion

This paper has investigated parameter bias in an estimated DSGE model that excludes the ZLB constraint on the nominal interest rate. To this end, we have conducted Monte Carlo experiments using a standard sticky-price DSGE model. According to the results in our baseline experiment, no significant bias is detected in the estimated parameters whereas the parameter estimates associated with the monetary policy rule are slightly biased. With the increased probability of hitting the ZLB, however, the parameter biases become large and substantially affect the dynamic properties of the model. Moreover, we have demonstrated that the estimation of the model omitting the ZLB leads to biased estimates of the structural shocks even though the estimated parameters are unbiased. These findings caution researchers against the common practice of estimating linearized DSGE models without considering the ZLB.

However, as Braun, Körber, and Waki (2012), Gust, López-Salido, and Smith (2012), Nakata (2013a), and Ngo (2014) among others suggest, the quasi-linear solution method used in this paper may be inaccurate in the assessment of the ZLB. In this regard, it is an important research agenda to examine how parameter estimates in linearized DSGE models can be affected by ignoring the true economic structures characterized by fully nonlinear models.

Appendix

This appendix presents the full description of the model. In the model economy, there are a continuum of households, a representative final-good firm, a continuum of intermediate-good firms, and a monetary authority.

Each household $h \in [0,1]$ consumes final goods $C_{h,t}$, supplies labor $l_{h,t}$, and purchases one-period riskless bonds $B_{h,t}$ so as to maximize the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_{h,t} - \gamma C_{t-1})^{1-\sigma}}{1-\sigma} - \frac{l_{h,t}^{1+\chi}}{1+\chi} \right]$$

subject to the budget constraint

$$P_t C_{h,t} + B_{h,t} = P_t W_t l_{h,t} + r_{t-1}^n B_{h,t-1} + T_{h,t},$$

where E_t is the expectation operator conditional on information available in period t, $\beta \in (0,1)$ is the subjective discount factor, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution, $\gamma \in [0,1]$ is the degree of external habit persistence in consumption preferences, $\chi > 0$ is the inverse of the elasticity of labor supply, P_t is the price of final goods, W_t is the real wage, r_t^n is the gross nominal interest rate, and $T_{h,t}$ is the sum of a lump-sum public transfer and profits received from firms. The first-order conditions for optimal decisions on consumption, labor supply, and bond-holding are identical among households and therefore become

$$\Lambda_t = (C_t - \gamma C_{t-1})^{-\sigma}, \tag{A.1}$$

$$W_t = \frac{l_t^{\chi}}{\Lambda_t}, \tag{A.2}$$

$$1 = E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{r_t^n}{\pi_{t+1}}, \tag{A.3}$$

where Λ_t is the marginal utility of consumption and $\pi_t = P_t / P_{t-1}$ denotes gross inflation.

The representative final-good firm produces output Y_t under perfect competition by choosing a combination of intermediate inputs $\{Y_{f,t}\}$ so as to maximize profit $P_t Y_t - \int_0^1 P_{f,t} Y_{f,t} df$ subject to a CES production technology $Y_t = \left(\int_0^1 Y_{f,t}^{1/(1+\lambda^p)} df \right)^{1+\lambda^p}$, where $P_{f,t}$ is the price of intermediate good f and $\lambda^p \geq 0$ denotes the intermediate-good price markup. The first-order condition for profit maximization yields the final-good firm's demand for intermediate good f, $Y_{f,t} = Y_t \left(P_{f,t} / P_t \right)^{-(1+\lambda^p)/\lambda^p}$, while perfect competition in the final-good market leads to $P_t = \left(\int_0^1 P_{f,t}^{-1/\lambda^p} df \right)^{-\lambda^p}$.

Each intermediate-good firm f produces one kind of differentiated goods $Y_{f,t}$ under monopolistic competition by choosing a cost-minimizing labor input l_t given the real wage W_t subject to the production function

$$Y_{f,t} = A_t l_{f,t},$$

where A_t represents the exogenous technology level. The first-order condition for cost minimization shows that real marginal cost is identical among intermediate-good firms and is given by

$$mc_t = \frac{W_t}{A_t}. \tag{A.4}$$

In the face of the final-good firm's demand and marginal cost, the intermediate-good firms set prices of their products on a staggered basis as in Calvo (1983). In each period, a fraction $1-\xi \in (0,1)$ of intermediate-good firms re-optimize their prices while the remaining fraction ξ indexes prices to a weighted average of past inflation π_{t-1} and steady-state inflation π. The firms that re-optimize their prices in the current period then maximize expected profit

$$E_t \sum_{j=0}^{\infty} \xi^j \frac{\beta^j \Lambda_{t+j}}{\Lambda_t} \left[\frac{P_{f,t}}{P_{t+j}} \prod_{k=1}^{j} \left(\pi_{t+k-1}^{\iota} \pi^{1-\iota} \right) - mc_{t+j} \right] Y_{f,t+j}$$

subject to the final-good firm's demand

$$Y_{f,t+j} = Y_{t+j} \left[\frac{P_{f,t}}{P_{t+j}} \prod_{k=1}^{j} \left(\pi_{t+k-1}^{\iota} \pi^{1-\iota} \right) \right]^{-\frac{1+\lambda^p}{\lambda^p}},$$

where $\iota \in (0,1)$ denotes the weight of price indexation to past inflation relative to steady-state inflation. The first-order condition for the reoptimized price P_t^o is given by

$$E_t \sum_{j=0}^{\infty} \left\{ \begin{array}{l} (\beta\xi)^j \dfrac{\Lambda_{t+j}}{\Lambda_t} Y_{t+j} \left[\dfrac{P_t^o}{P_t} \displaystyle\prod_{k=1}^{j} \left(\dfrac{\pi_{t+k-1}}{\pi} \right)^{\iota} \dfrac{\pi}{\pi_{t+k}} \right]^{-\frac{1+\lambda^p}{\lambda^p}} \\ \times \left[\dfrac{P_t^o}{P_t} \displaystyle\prod_{k=1}^{j} \left(\dfrac{\pi_{t+k-1}}{\pi} \right)^{\iota} \dfrac{\pi}{\pi_{t+k}} - (1+\lambda^p)\, mc_{t+j} \right] \end{array} \right\} = 0. \tag{A.5}$$

Moreover, the final-good's price $P_t = \left(\int_0^1 P_{f,t}^{-1/\lambda^p} df \right)^{-\lambda^p}$ can be rewritten as

$$1 = (1-\xi) \left(\frac{P_t^o}{P_t} \right)^{-\frac{1}{\lambda^p}} + \xi \left[\left(\frac{\pi_{t-1}}{\pi} \right)^{\iota} \frac{\pi}{\pi_t} \right]^{-\frac{1}{\lambda^p}}. \tag{A.6}$$

The final-good market clearing condition is

$$Y_t = C_t, \tag{A.7}$$

while the labor market clearing condition leads to

$$\frac{Y_t d_t}{A_t} = \int_0^1 l_{f,t} df = l_t, \tag{A.8}$$

where $d_t = \int_0^1 (P_{f,t}/P_t)^{-(1+\lambda^p)/\lambda^p} df$ represents price dispersion across the intermediate-good firms. Note that this dispersion is of second-order under the staggered price setting and that its steady-state value is unity.

A monetary policy rule is specified as

$$\log r_t^n = \max\left[\phi_r \log r_{t-1}^n + (1-\phi_r)\left(\log r^n + \phi_\pi \log \frac{\pi_t}{\pi} + \phi_y \log \frac{Y_t}{Y}\right) + z_t^r,\ 0\right]. \tag{A.9}$$

In the absence of the ZLB on the nominal interest rate, the monetary authority adjusts the interest rate following a Taylor (1993) type monetary policy rule where $\phi_r \in [0,1)$ is the degree of interest rate smoothing, r^n is the steady-state gross nominal interest rate, and $\phi_\pi, \phi_y \geq 0$ are the degrees of interest rate policy responses to inflation and output. z_t^r is a monetary policy shock, which captures unsystematic components of monetary policy.

The equilibrium conditions are (A.1)–(A.9). Log-linearizing the equilibrium conditions around the steady state and rearranging the resulting equations yields Eq. (1)–(3). The demand shock z_t^d and cost shock z_t^p have been introduced in a reduced form manner, and interpreted as a shock to consumption preferences and time-varying price markup of intermediate-goods respectively.

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Table 1: Parameter setting

Parameter	Economic interpretation	Assigned value
σ	Inverse of intertemporal elasticity of substitution	1.000
χ	Inverse of elasticity of labor supply	2.000
γ	Habit persistence	0.500
ξ	Price stickiness	0.750
ι	Price indexation	0.500
ϕ_π	Policy response to inflation	1.500
ϕ_y	Policy response to output	0.500
ϕ_r	Interest rate smoothing	0.500
$\bar{\pi}$	Steady-state inflation rate	0.500
$\bar{r}$	Steady-state real interest rate	0.500
ρ_d	Persistence of demand shock	0.500
ρ_p	Persistence of cost shock	0.500
ρ_r	Persistence of policy shock	0.500
σ_d	Standard deviation of demand shock	0.250
σ_p	Standard deviation of cost shock	0.250
σ_r	Standard deviation of policy shock	0.250

Table 2: Prior distributions

Parameter	Distribution	Mean	Standard deviation
σ	Gamma	1.000	0.150
χ	Gamma	2.000	0.150
γ	Beta	0.500	0.150
ξ	Beta	0.750	0.150
ι	Beta	0.500	0.150
ϕ_π	Gamma	1.500	0.150
ϕ_y	Gamma	0.500	0.150
ϕ_r	Beta	0.500	0.150
$\bar{\pi}$	Gamma	0.500	0.150
$\bar{r}$	Gamma	0.500	0.150
ρ_d	Beta	0.500	0.150
ρ_p	Beta	0.500	0.150
ρ_r	Beta	0.500	0.150
σ_d	Inverse Gamma	0.250	2.000
σ_p	Inverse Gamma	0.250	2.000
σ_r	Inverse Gamma	0.250	2.000

Table 3: Posterior distributions of parameters in baseline experiment

		Baseline		No ZLB in DGP	
Parameter	True value	Mean	90% interval	Mean	90% interval
σ	1.000	1.000	[0.774, 1.222]	1.012	[0.784, 1.237]
χ	2.000	2.000	[1.754, 2.244]	2.003	[1.756, 2.247]
γ	0.500	0.476	[0.350, 0.602]	0.510	[0.384, 0.637]
ξ	0.750	0.765	[0.708, 0.822]	0.763	[0.702, 0.823]
ι	0.500	0.475	[0.297, 0.651]	0.504	[0.317, 0.689]
ϕ_π	1.500	1.363	[1.178, 1.545]	1.490	[1.318, 1.659]
ϕ_y	0.500	0.523	[0.351, 0.691]	0.497	[0.353, 0.638]
ϕ_r	0.500	0.487	[0.429, 0.545]	0.500	[0.448, 0.552]
$\bar{\pi}$	0.500	0.476	[0.346, 0.603]	0.497	[0.379, 0.614]
$\bar{r}$	0.500	0.537	[0.449, 0.626]	0.495	[0.412, 0.578]
ρ_d	0.500	0.498	[0.373, 0.623]	0.491	[0.359, 0.624]
ρ_p	0.500	0.499	[0.366, 0.633]	0.492	[0.356, 0.630]
ρ_r	0.500	0.526	[0.425, 0.628]	0.488	[0.383, 0.593]
σ_d	0.250	0.253	[0.213, 0.293]	0.248	[0.210, 0.286]
σ_p	0.250	0.239	[0.191, 0.286]	0.246	[0.197, 0.294]
σ_r	0.250	0.281	[0.245, 0.316]	0.248	[0.221, 0.274]

Notes: Means and 90% intervals are the averages of posterior means and 90% credible intervals based on 500 Monte Carlo replications using a sample size of 200 observations. In each of the 500 replications, 200,000 MCMC draws are generated and the first 25 percent of the draws are discarded.

Table 4: Posterior distributions of parameters in alternative experiments

		Case of low $\bar{r}$		Case of large σ_d	
Parameter	True value	Mean	90% interval	Mean	90% interval
σ	1.000	1.012	[0.790, 1.231]	0.994	[0.763, 1.221]
χ	2.000	1.998	[1.751, 2.242]	1.994	[1.748, 2.238]
γ	0.500	0.422	[0.300, 0.543]	0.348	[0.215, 0.480]
ξ	0.750	0.760	[0.711, 0.809]	0.768	[0.740, 0.796]
ι	0.500	0.407	[0.253, 0.559]	0.295	[0.184, 0.403]
ϕ_π	1.500	1.293	[1.082, 1.492]	1.295	[1.081, 1.499]
ϕ_y	0.500	0.571	[0.347, 0.790]	0.420	[0.242, 0.594]
ϕ_r	0.500	0.477	[0.412, 0.543]	0.404	[0.331, 0.477]
$\bar{\pi}$	0.500	0.467	[0.320, 0.611]	0.497	[0.331, 0.659]
$\bar{r}$	0.200	0.294	[0.190, 0.398]	-	-
$\bar{r}$	0.500	-	-	0.688	[0.531, 0.845]
ρ_d	0.500	0.497	[0.384, 0.611]	0.529	[0.424, 0.636]
ρ_p	0.500	0.522	[0.401, 0.644]	0.580	[0.474, 0.685]
ρ_r	0.500	0.605	[0.525, 0.686]	0.682	[0.615, 0.751]
σ_d	0.550	-	-	0.570	[0.477, 0.662]
σ_d	0.250	0.268	[0.226, 0.310]	-	-
σ_p	0.250	0.235	[0.189, 0.281]	0.215	[0.173, 0.256]
σ_r	0.250	0.366	[0.308, 0.422]	0.589	[0.478, 0.698]

Notes: In the case of low $\bar{r}$, the true values are set at $\bar{r} = 0.2$ and $\sigma_d = 0.25$. In the case of large σ_d, the true values are set at $\bar{r} = 0.5$ and $\sigma_d = 0.55$. Means and 90% intervals are the averages of posterior means and 90% credible intervals based on 500 Monte Carlo replications using a sample size of 200 observations. In each replication, 200,000 MCMC draws are generated and the first 25 percent of the draws are discarded.

Table 5: Covariances of observed variables in alternative experiments

	ZLB with true parameters			No ZLB with true parameters			No ZLB with biased parameters		
	Output	Inflation	Interest rate	Output	Inflation	Interest rate	Output	Inflation	Interest rate
(1) Case of low $\bar{r}$									
Output	0.683			0.565			0.741		
Inflation	0.159	0.694		-0.087	0.444		0.145	0.717	
Interest rate	0.020	0.359	0.335	-0.093	0.364	0.459	-0.037	0.411	0.457
(2) Case of large σ_d									
Output	2.410			1.376			1.645		
Inflation	1.220	1.383		0.332	0.659		0.920	1.181	
Interest rate	0.737	0.768	0.825	0.583	0.738	1.221	0.557	0.664	0.795

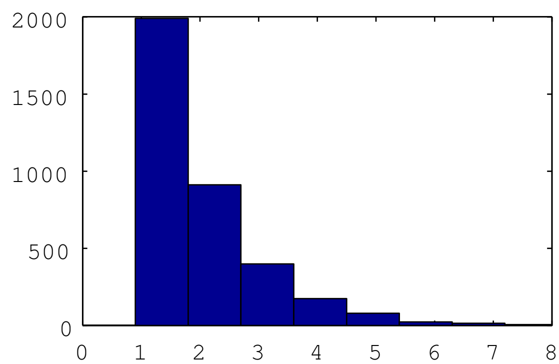
Notes: In the case of the models including the ZLB with true parameters, the empirical covariances are calculated by the averages of the sample covariances based on 500 Monte Carlo replications with a sample size of 200 observations. In the case of the models omitting the ZLB with true and biased parameters, the theoretical covariances are computed by solving the discrete Lyapunov equations.

Table 6: Posterior distributions of parameters in the Smets–Wouters model

		Smets–Wouters model		No ZLB in DGP	
Parameter	True value	Mean	90% interval	Mean	90% interval
φ	4.000	4.044	[3.357, 4.714]	4.121	[3.418, 4.808]
σ_c	1.500	1.557	[1.345, 1.765]	1.534	[1.330, 1.735]
λ	0.700	0.693	[0.644, 0.742]	0.698	[0.650, 0.746]
ξ_w	0.500	0.498	[0.446, 0.549]	0.506	[0.456, 0.556]
σ_l	2.000	2.039	[1.253, 2.808]	2.128	[1.339, 2.901]
ξ_p	0.500	0.494	[0.456, 0.533]	0.499	[0.461, 0.536]
ι_w	0.500	0.490	[0.365, 0.615]	0.496	[0.372, 0.619]
ι_p	0.500	0.493	[0.400, 0.587]	0.502	[0.410, 0.594]
ψ	0.500	0.507	[0.435, 0.578]	0.507	[0.435, 0.579]
ϕ_p	1.250	1.260	[1.208, 1.312]	1.257	[1.205, 1.308]
r_π	1.500	1.423	[1.159, 1.678]	1.535	[1.244, 1.818]
ρ	0.750	0.741	[0.693, 0.791]	0.755	[0.710, 0.801]
r_y	0.125	0.100	[0.036, 0.164]	0.121	[0.055, 0.187]
$\bar{\pi}$	0.625	0.617	[0.480, 0.751]	0.618	[0.483, 0.753]
$100(\beta^{-1}-1)$	0.250	0.272	[0.130, 0.410]	0.244	[0.111, 0.372]
$\bar{l}$	0.000	0.014	[-0.380, 0.407]	-0.001	[-0.391, 0.387]
$\bar{\gamma}$	0.400	0.400	[0.397, 0.404]	0.400	[0.396, 0.404]
α	0.300	0.304	[0.275, 0.332]	0.302	[0.274, 0.330]
ρ_a	0.500	0.495	[0.407, 0.584]	0.496	[0.407, 0.585]
ρ_b	0.500	0.482	[0.401, 0.564]	0.500	[0.423, 0.577]
ρ_g	0.500	0.493	[0.402, 0.585]	0.494	[0.401, 0.586]
ρ_I	0.500	0.499	[0.399, 0.599]	0.498	[0.397, 0.599]
ρ_r	0.500	0.477	[0.379, 0.576]	0.478	[0.380, 0.577]
ρ_p	0.500	0.473	[0.222, 0.723]	0.470	[0.217, 0.722]
ρ_w	0.500	0.474	[0.222, 0.724]	0.467	[0.213, 0.719]
μ_p	0.500	0.456	[0.194, 0.715]	0.459	[0.199, 0.716]
μ_w	0.500	0.456	[0.196, 0.715]	0.461	[0.202, 0.716]
ρ_{ga}	0.500	0.502	[0.387, 0.618]	0.502	[0.387, 0.617]
σ_a	0.450	0.447	[0.407, 0.487]	0.447	[0.407, 0.487]
σ_b	0.450	0.449	[0.379, 0.518]	0.446	[0.378, 0.514]
σ_g	0.450	0.452	[0.412, 0.491]	0.452	[0.412, 0.491]
σ_I	0.450	0.450	[0.388, 0.510]	0.449	[0.388, 0.509]
σ_r	0.450	0.444	[0.402, 0.485]	0.446	[0.404, 0.487]
σ_p	0.450	0.446	[0.396, 0.496]	0.444	[0.394, 0.493]
σ_w	0.450	0.444	[0.394, 0.494]	0.446	[0.396, 0.495]

Notes: Means and 90% intervals are the averages of posterior means and 90% credible intervals based on 200 Monte Carlo replications using a sample size of 200 observations. In each replication, 125,000 MCMC draws are generated and the first 20 percent of the draws are discarded.

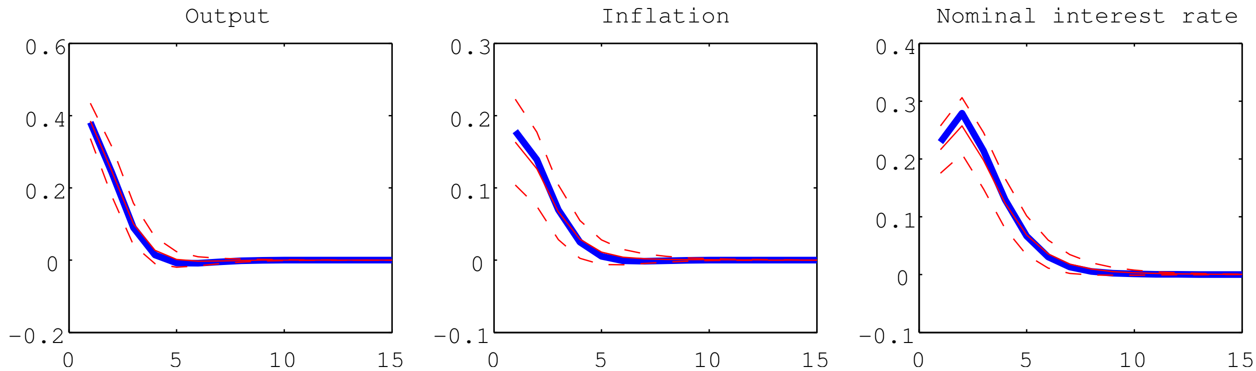
Figure 1: Distribution of duration of ZLB spells in baseline experiment



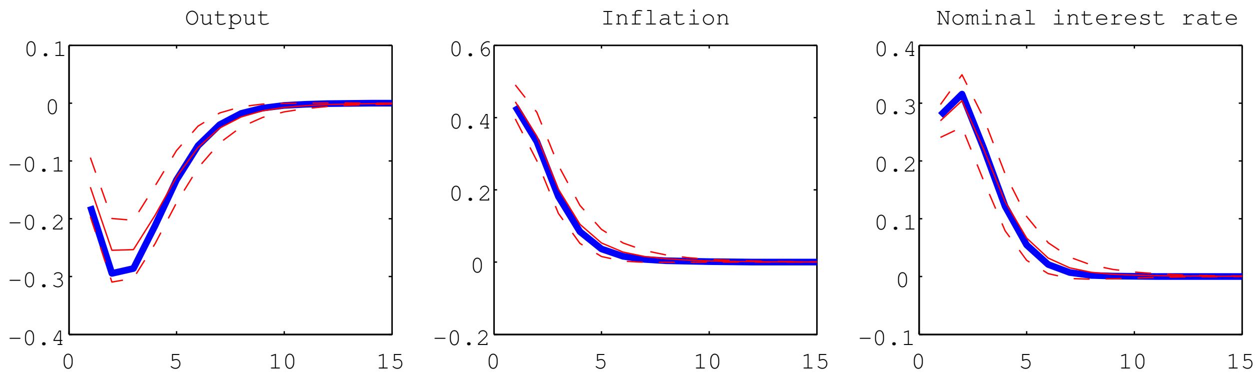
Note: The figure shows the histogram of the duration at the ZLB based on the simulated sample of 100,000 periods (500 Monte Carlo replications $\times$ 200 quarters).

Figure 2: Impulse responses in baseline experiment

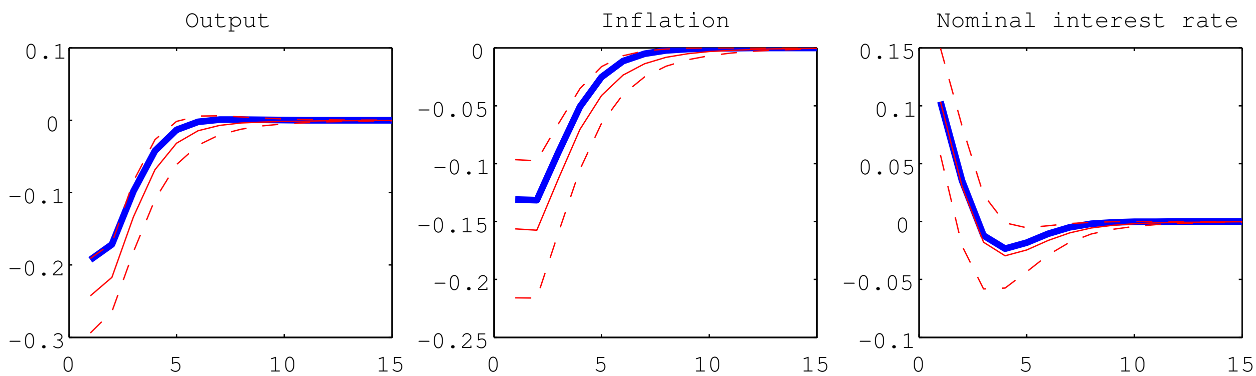
(1) Demand shock



(2) Cost-push shock

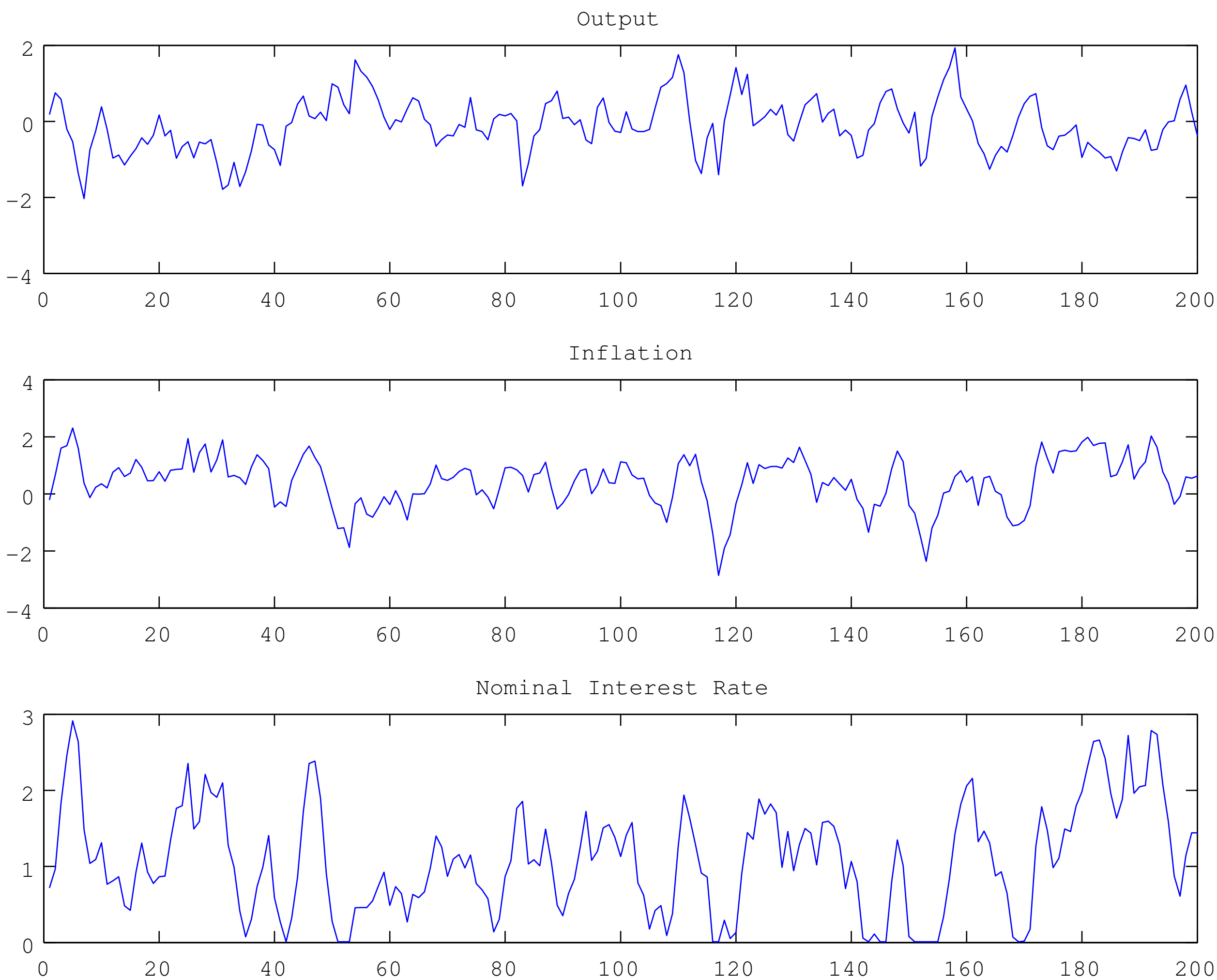


(3) Monetary policy shock



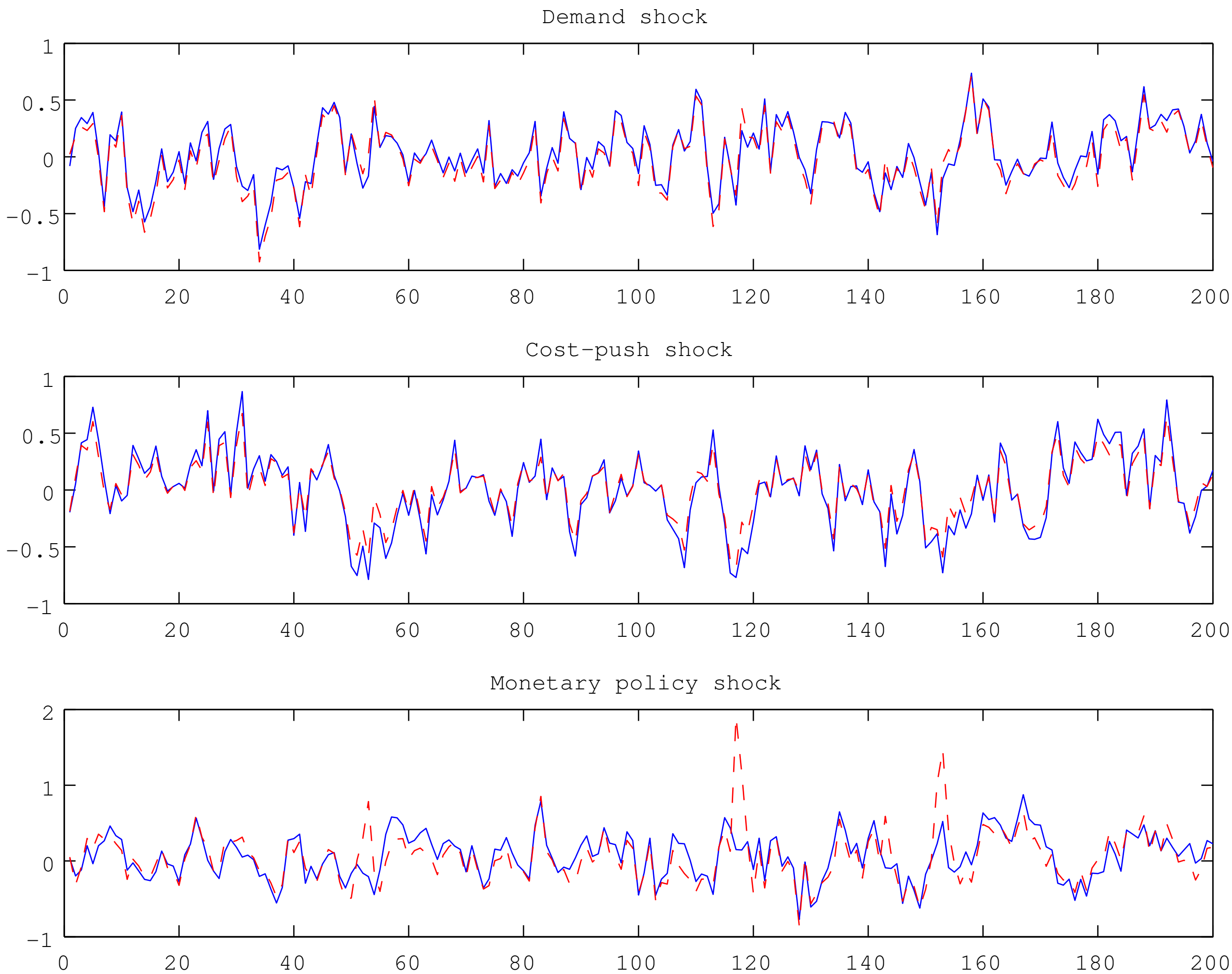
Notes: The figure shows the impulse responses of output, inflation, and the nominal interest rate (in terms of the percentage deviation from the steady state) to one standard deviation shocks in demand, cost-push, and monetary policy. The solid thick lines represent the true responses, and the solid thin lines and dashed lines are respectively the averages of posterior means and 90% credible intervals for the estimated responses, where the averages are taken over 500 Monte Carlo replications.

Figure 3: Simulated sample in baseline experiment



Note: The figure shows a sample of output, inflation, and the nominal interest rate simulated from the baseline DGP.

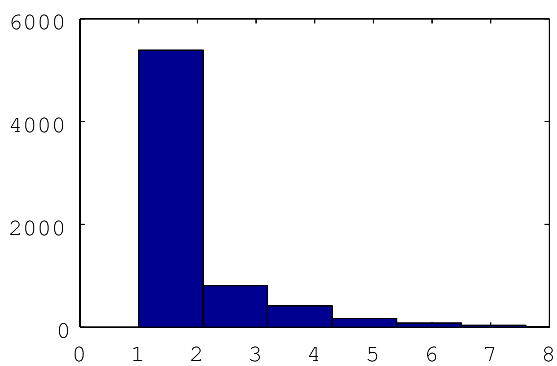
Figure 4: Smoothed estimates of shocks in baseline experiment



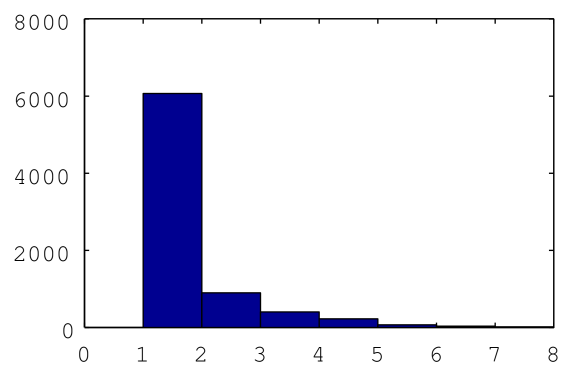
Note: The figure compares the true shocks (solid lines) in demand, cost-push, and monetary policy with the mean estimates of the Kalman smoothed shocks (dashed lines) based on the sample presented in Figure 3.

Figure 5: Distribution of duration of ZLB spells in alternative cases

(1) $\bar{r} = 0.20$



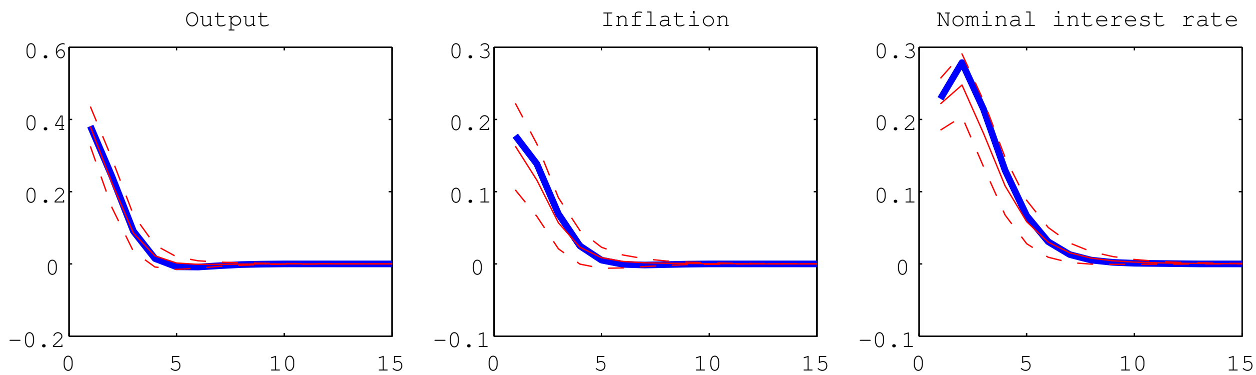
(2) $\sigma_d = 0.55$



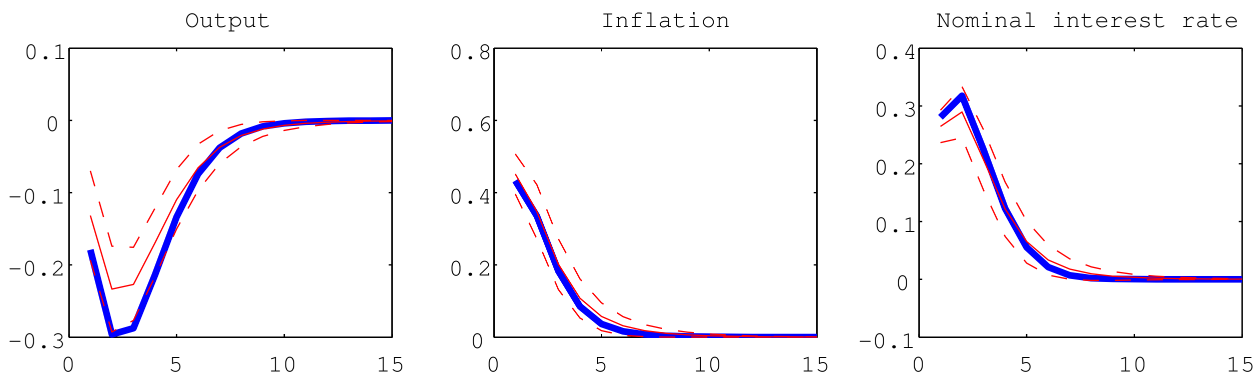
Note: The figure shows the histograms of the duration at the ZLB based on the simulated samples of 100,000 periods (500 Monte Carlo replications × 200 quarters).

Figure 6: Impulse responses in alternative case I ($\bar{r} = 0.2$)

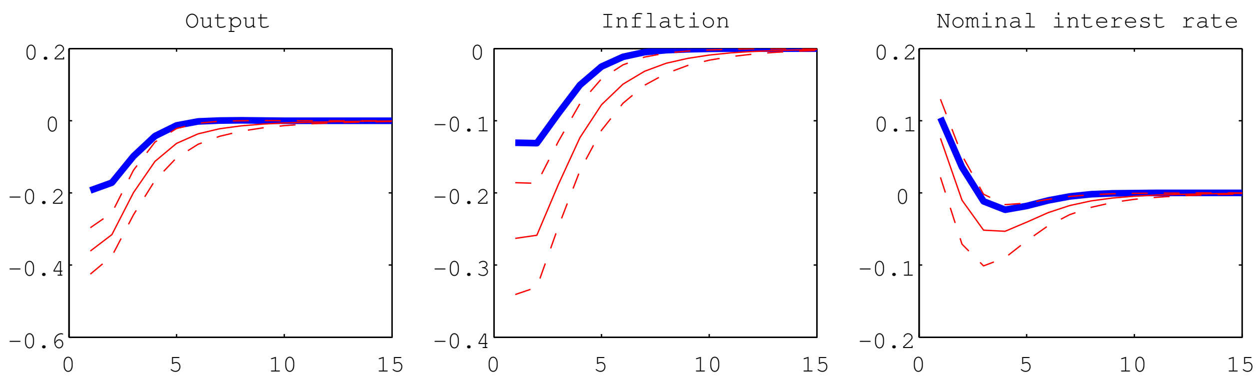
(1) Demand shock



(2) Cost-push shock



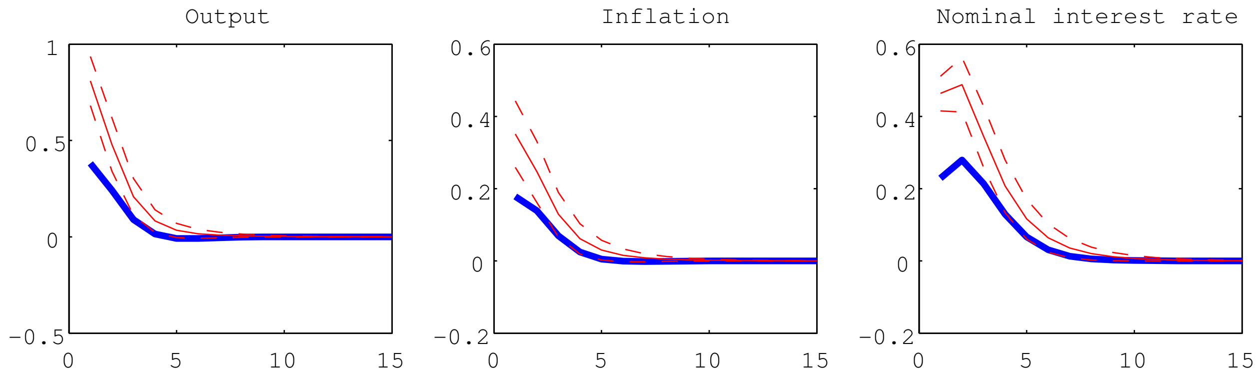
(3) Monetary policy shock



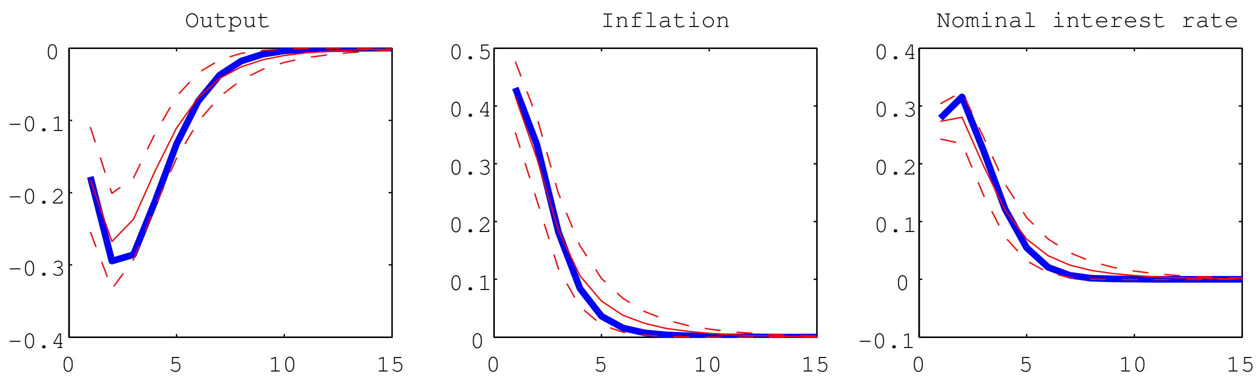
Notes: The figure shows the impulse responses of output, inflation, and the nominal interest rate (in terms of the percentage deviation from the steady state) to one standard deviation shocks in demand, cost-push, and monetary policy. The solid thick lines represent the true responses, and the solid thin lines and dashed lines are respectively the posterior means and 90 percent credible intervals for the estimated responses. The means and the intervals are the averages of posterior means and 90% credible intervals based on 500 Monte Carlo replications.

Figure 7: Impulse responses in alternative case II ($\sigma_d = 0.55$)

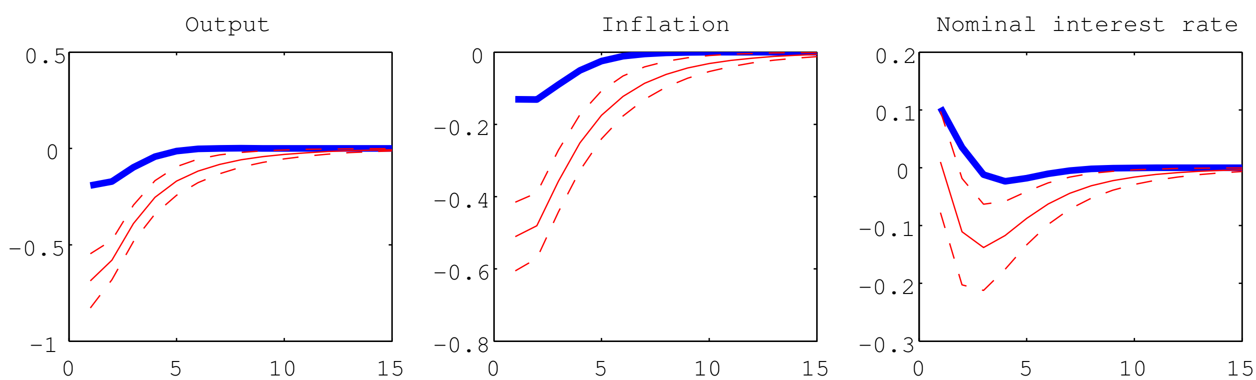
(1) Demand shock



(2) Cost-push shock

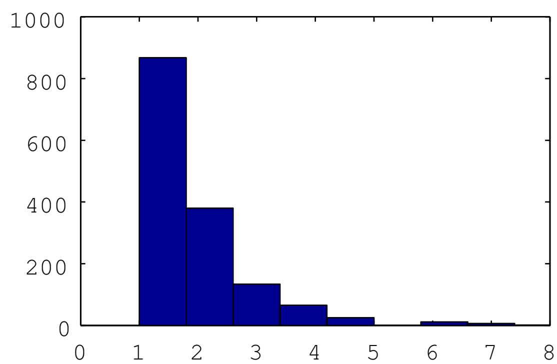


(3) Monetary policy shock



Notes: The figure shows the impulse responses of output, inflation, and the nominal interest rate (in terms of the percentage deviation from the steady state) to one standard deviation shocks in demand, cost-push, and monetary policy. The solid thick lines represent the true responses, and the solid thin lines and dashed lines are respectively the averages of posterior means and 90% credible intervals for the estimated responses, where the averages are over 500 Monte Carlo replications.

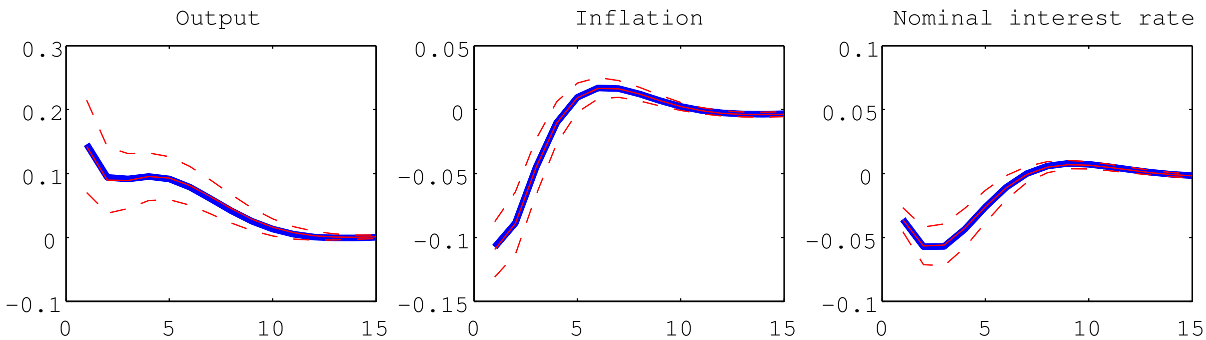
Figure 8: Distribution of duration of ZLB spells in robustness analysis



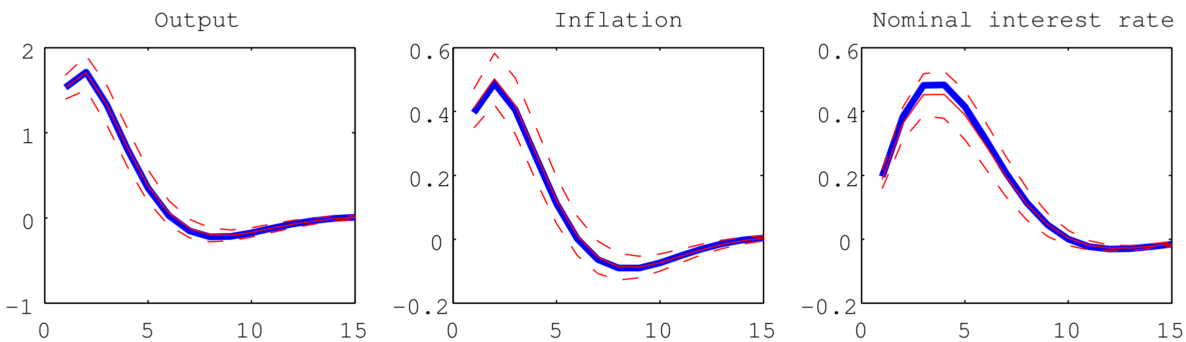
Note: The figure shows the histogram of the duration at the ZLB based on the simulated sample of 40,000 periods (200 Monte Carlo replications $\times$ 200 quarters).

Figure 9: Impulse responses in robustness analysis

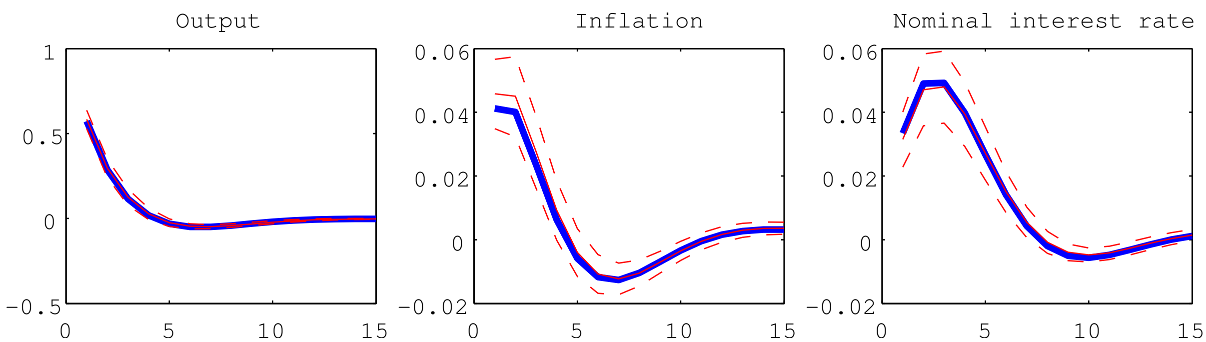
(1) TFP shock



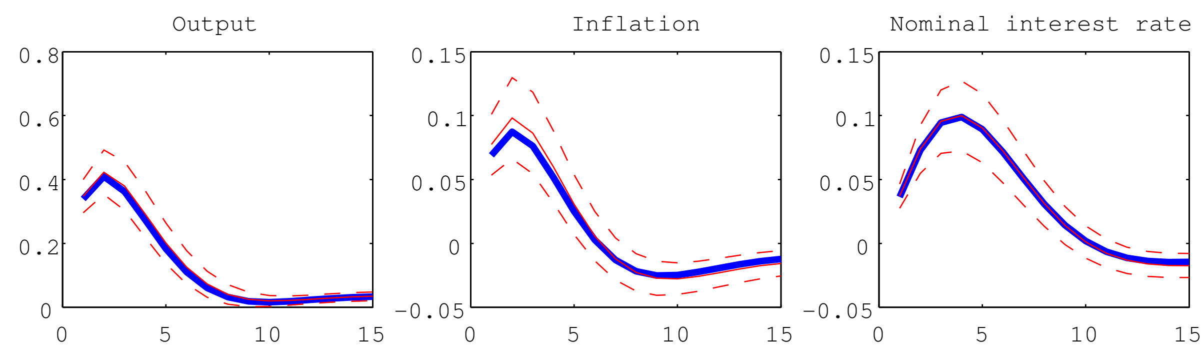
(2) Risk premium shock



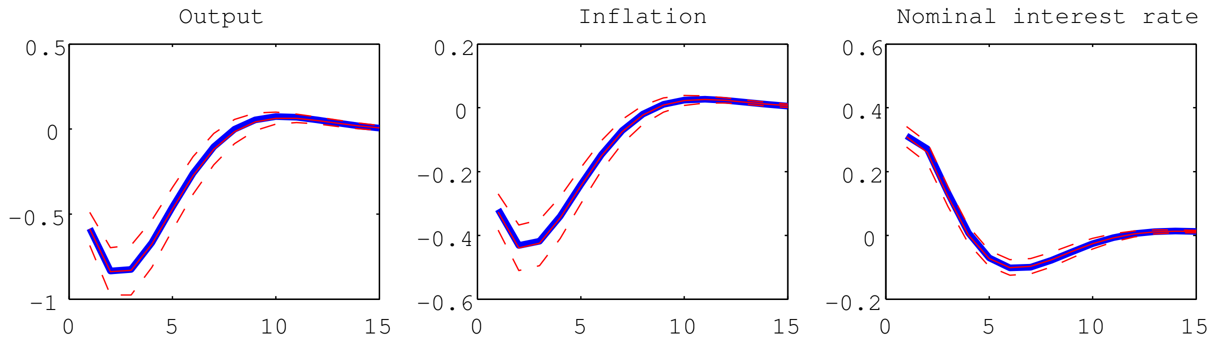
(3) External demand shock



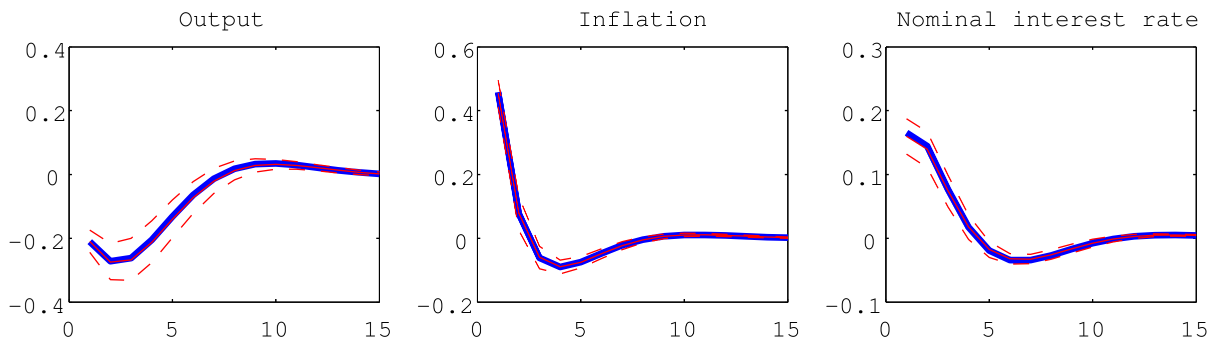
(4) Investment shock



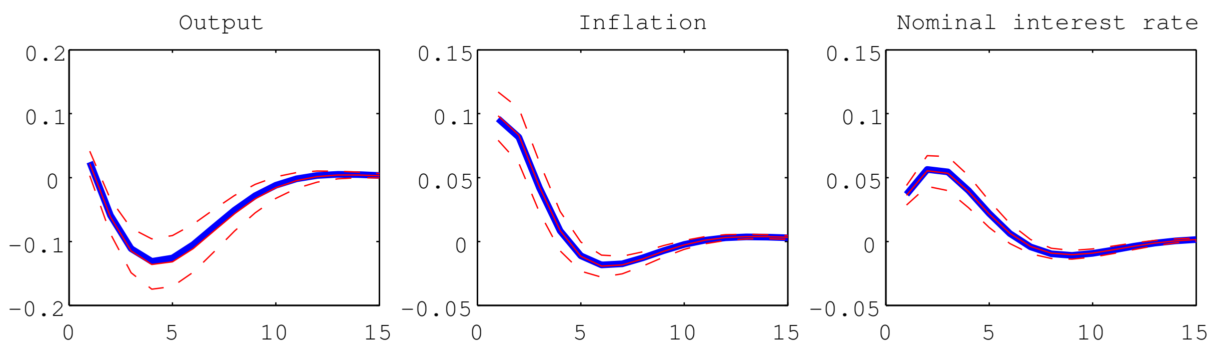
(5) Monetary policy shock



(6) Price markup shock

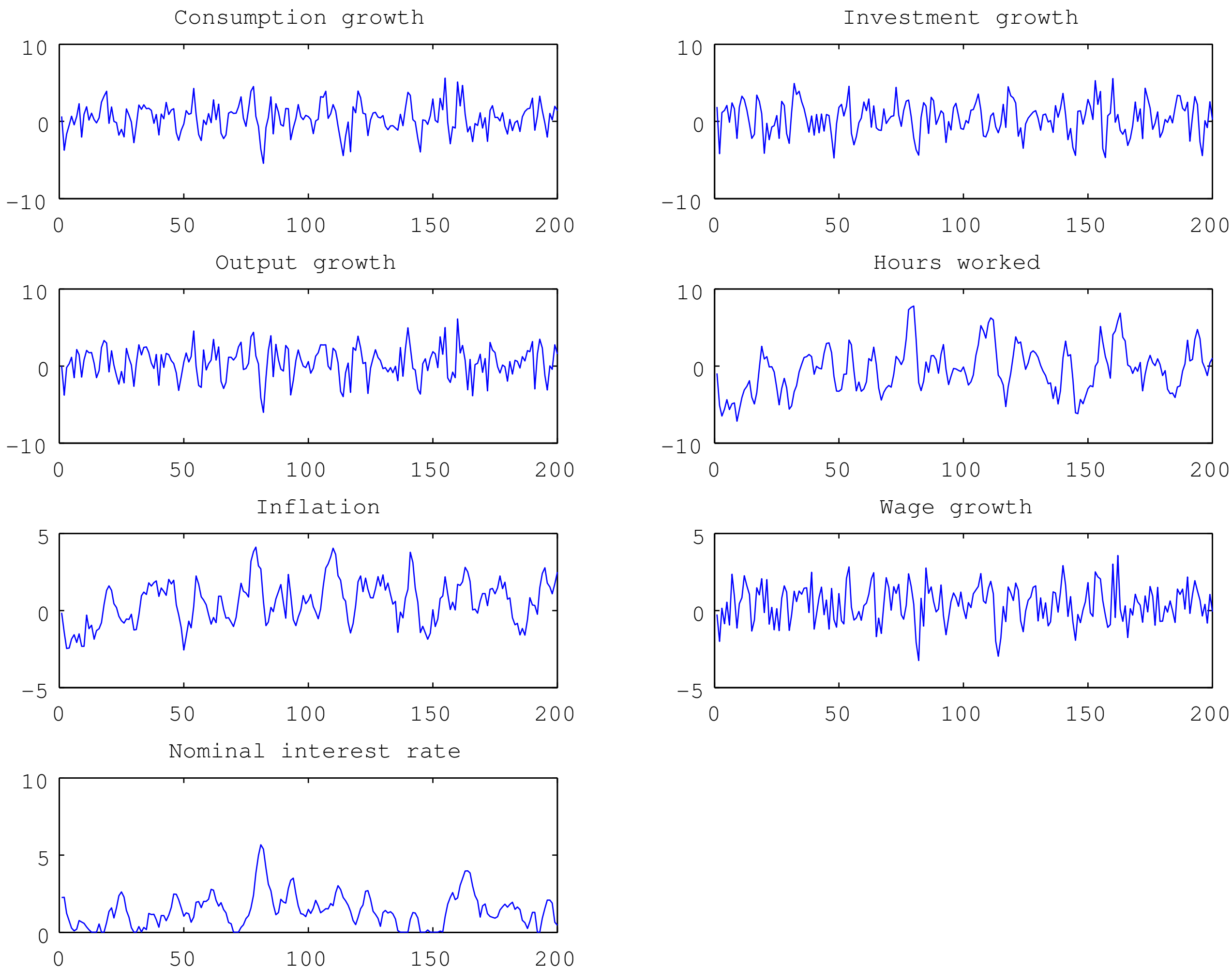


(7) Wage markup shock



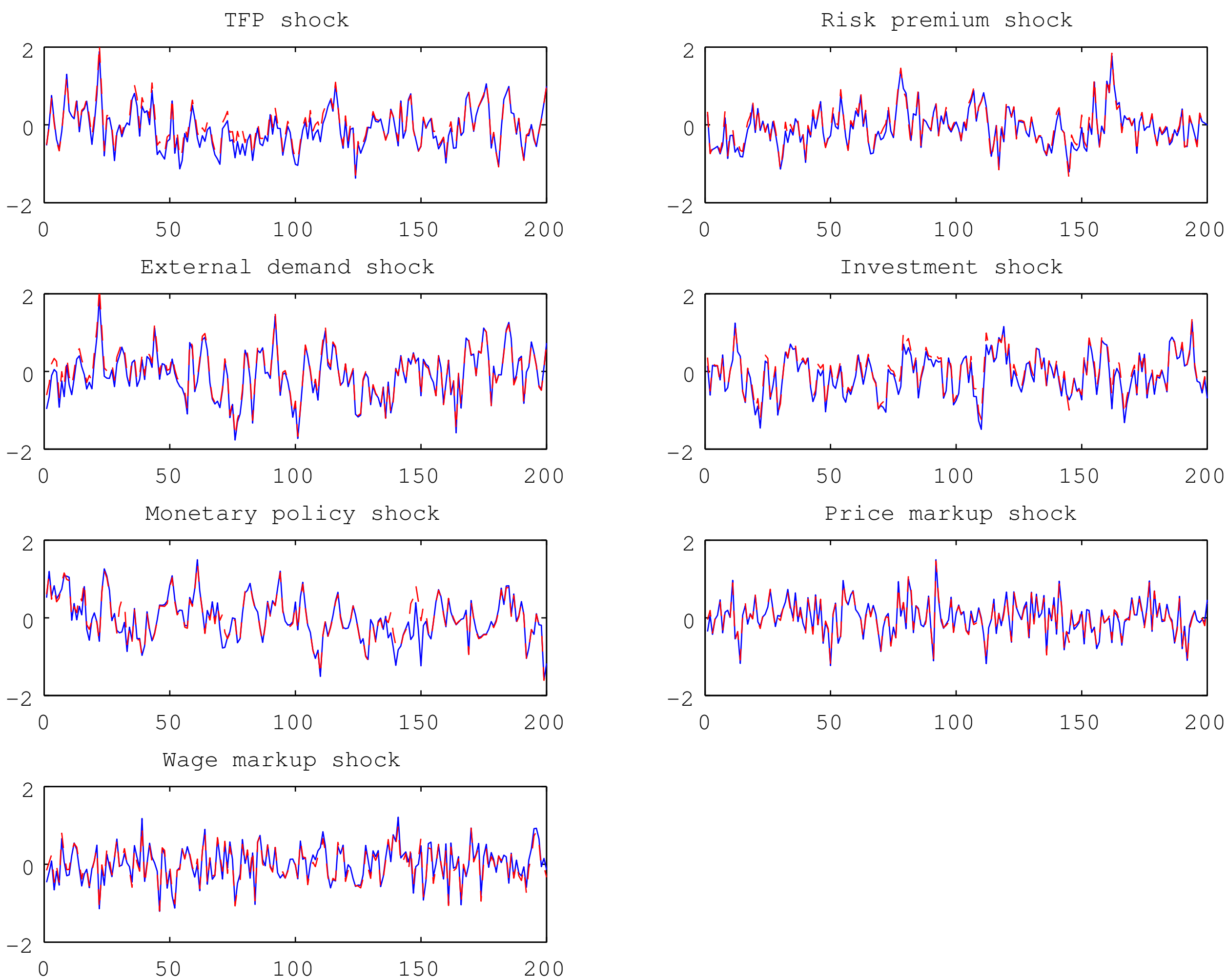
Notes: The figure shows the impulse responses of output, inflation, and the nominal interest rate (in terms of the percentage deviation from the steady state) to one standard deviation shocks in the TFP, the risk premium, external demand, investment, monetary policy, the price markup, and the wage markup. The solid thick lines represent the true responses, and the solid thin lines and dashed lines are respectively the average of the posterior means and 90% credible intervals for the estimated responses, where the averages are based on 200 Monte Carlo replications.

Figure 10: Simulated sample in robustness analysis



Note: The figure shows a sample of consumption growth, investment growth, output growth, hours worked, inflation, wage growth, and the nominal interest rate simulated from the DGP in the robustness analysis.

Figure 11: Smoothed estimates of shocks in robustness analysis



Note: The figure compares the true shocks (solid lines) in the TFP, the risk premium, external demand, investment, monetary policy, the price markup, and the wage markup, with the posterior mean estimates of the Kalman smoothed shocks (dashed lines) based on the sample presented in Figure 10.