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Hiroshi Fujiki*

Abstract

Fujiki (2003, 2006) extended the Freeman (1996) model to a two-country model, demonstrating that elastic money supplies in foreign exchange markets and the domestic credit market yield efficiency gains in monetary equilibrium, and that several institutional designs equally achieve the desired elastic money supplies. The present paper considers four institutional designs using a model similar to Fujiki (2003): a combination of central bank discount window policy and the CLS Bank; a central bank intervention both in the domestic credit market and the foreign exchange market; cross-border collateral arrangements; and foreign currency liquidity swap lines. These institutional designs yield the same efficiency gains in our model.

Keywords: Foreign exchange market; CLS; Cross-border collateral arrangements; Liquidity swap lines
JEL Classification: E58, F31, F33

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1. Introduction

In this paper, we show how central banks improve the efficiency of equilibrium under liquidity shortages in the domestic credit market and the foreign exchange market due to the mismatch of the timing to arrival in these markets. We consider four institutional designs: a combination of central bank discount window policy and the CLS Bank; a central bank intervention both in the domestic credit market and the foreign exchange market; cross-border collateral arrangements; and foreign currency liquidity swap lines. We show that these four institutional designs have the same welfare-improving effects as those in an extended model by Fujiki (2003), which is a two-country version of Freeman (1996). We begin with a review of these models.

Freeman (1996) considers an economy in which agents are spatially separated and private debt incurred between two parties can only be redeemed with fiat currency in a central clearing area. Suppose that the departure rate of creditors from the central clearing area is higher than the arrival rate of debtors. In this case, the amount of currency available in the central clearing area is less than the par value of debt, and late-departing creditors can buy the risk-free asset of early-departing creditors at discounted prices in exchange for fiat money. A central bank can issue additional fiat money to purchase the risk-free asset of early-departing creditors and can receive fiat money from the debtors in the central clearing area. Then the risk-free asset of early-departing creditors can be cleared at par value. Freeman (1996) shows that such an elastic money supply enhances the efficiency of monetary equilibrium.

Fujiki (2003) considers a two-country version of the Freeman model under a gold standard and assumes that old domestic creditors wish to consume young foreign debtors' goods at their second stage of life with a small probability. Suppose that old creditors know their preference for foreign goods only after their debt is settled by their domestic creditors. Suppose further that they must pay foreign currency to obtain goods from young foreign debtors and thus old domestic creditors with a taste shock meet foreign old creditors with a taste shock in the foreign exchange market to exchange their fiat money. Suppose that the rate at which old creditors with a taste shock arrive in the foreign exchange market is not equal. Then, even after the central banks intervene to clear domestic debt at par value, the foreign exchange rate will depart from the fair value determined by the gold reserves in each country. Fujiki (2003) shows that when combined, an elastic money supply in the foreign exchange market and an elastic supply in the domestic credit market yield efficiency gains in monetary equilibrium. Fujiki (2006) discusses three other institutional designs that yield the same efficiency gain: a private arrangement based on a payment versus a payment settlement standard supported by central banks' free intraday credit; a financial institution that provides a negotiable certificate of deposit; and a currency union.

We extend the model of Fujiki (2003, 2006) by discussing the welfare-improving effects of two new temporary policy measures adopted during the recent financial crisis: domestic liquidity operations using cross-border collateral or operations based on standing swap lines that supply foreign currency among central banks. Domestic market operations using cross-border collateral mean that a central bank accepts high-quality marketable collateral denominated in foreign currencies or held in foreign locations for its domestic currency loan. For example, on November 28, 2011 the Bank of Japan (BOJ) and the Bank of Thailand (BOT) began liquidity provision in Thai baht with the BOT utilizing Japanese government securities as collateral.[1] Domestic liquidity operations based on standing swap

[1] The original idea behind the cross-border use of the collateral was a shift toward real-time gross settlement of central bank payment systems, which requires large overdraft facilities and thus collateral. The few studies on this issue include Manning and Willison (2006). The European Central Bank, a notable example of a cross-border central bank, mitigates borrowers' mismatch between the location of its collateral holdings and its

lines that supply foreign currency among central banks mean that a central bank accepts high-quality marketable collateral for its foreign currency loan. A central bank obtains the foreign currency needed for the loan from the other central bank using standing swap lines. To take a recent example, in May 2010 the Federal Open Market Committee (FOMC) authorized that it had established U.S. dollar liquidity swap lines with the Bank of Canada, the Bank of England, the European Central Bank, the BOJ, and the Swiss National Bank.[2]

To examine the welfare-improving effects of these two temporary policy measures, we add the following six assumptions to Fujiki (2006): (1) all old creditors wish to consume young domestic and foreign debtors' goods; (2) all young creditors lend to both domestic debtors and foreign debtors; (3) due to national laws regulating foreign loans, young debtors must repay the loan in creditors' currency when old; (4) due to national laws regulating foreign goods transactions, young debtors accept only domestic currency to sell their goods; (5) old creditors exchange their currency among themselves to purchase foreign debtors' goods; and (6) young debtors exchange their currency among themselves to repay foreign creditors' loan in foreign currencies when they grow old.

With these assumptions, this paper supports the main findings by Fujiki (2003, 2006), that is, several institutional designs equally support efficient resource allocation free from liquidity shortages, and alleviate the inefficiency due to the mismatch of the timing of arrival in markets. In particular, this paper shows that four institutional designs support the efficient allocation: a combination of central bank discount window policy and the CLS Bank; a central bank intervention both in the domestic credit market and the foreign exchange

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liquidity needs through a correspondent central banking model (CCBM) within the euro area. The U.S. Federal Reserve accepts several foreign government bonds as collateral. See the explanation on CCBM in ECB (2013).

2 Goldberg, Kennedy, and Miu (2011) provide an excellent overview of the evolution of reciprocal currency arrangements or dollar swap facilities established by the Federal Reserve with foreign central banks from 2007 to 2010. Fujiki (2013) is a two-country extension of Chapman and Martin (2012), which adds the moral hazard of debtor to Freeman (1999). Freeman (1999) adds an aggregate default shock to Freeman (1996).

market; cross-border collateral arrangements; and foreign currency liquidity swap lines. Below I will explain the departure from the previous research.

The welfare-improving effects of two institutional designs, a combination of central bank discount window policy and the CLS Bank, and central bank intervention both in the domestic credit market and the foreign exchange market, are examined by Fujiki (2006). Unlike Fujiki (2006), in which only a fraction of old creditors with a taste shock consumes foreign goods, this paper assumes that all old creditors consume foreign goods.

The welfare-improving effects of cross-border collateral arrangements and foreign currency liquidity swap lines are examined by Fujiki (2013). Unlike Fujiki (2013), in which the creditors' monitoring efforts reduce the probability of the debtors' default, this paper assumes that all agents honor their contracts. While Fujiki (2013) focuses on the welfare-improving effects of the institutional designs under a collapse of the private international collateral swap market similar to the Eurodollar market) subject to the moral hazard of debtors, this paper focuses on their welfare-improving effects to alleviate the inefficiency due to the mismatch of the timing of arrival in the foreign exchange market and the Eurodollar market, without considering the effects of the moral hazard of debtors.

The rest of the paper is organized as follows. Section 2 explains the environment and trading patterns. Section 3 analyzes the market equilibrium and defines the symmetric laissez-faire equilibrium where the liquidity constraints of two economies are binding. Section 4 defines the benchmark efficient resource allocation and considers the effects of the four institutional designs to mitigate the effects of liquidity shortage in the domestic market and the foreign exchange market due to the mismatch of the timing of arrival in these markets. Section 5 concludes with reservations.

2. Environment and trading patterns

2.1 The environment

This section explains our model, which extends Fujiki (2003). There are two types of agents, called creditors and debtors, in the domestic country (hereafter Japan) and the foreign country (hereafter the United States). In both countries, creditors and debtors are scattered and live in small villages (for the sake of simplicity, the term “villages” is used for both Japan and the United States). Their populations are normalized to one, and their lifetime is divided into two periods. We refer to agents in their first period of life as young and agents in their second period of life as old. Japanese and U.S. creditors and debtors are endowed with nonstorable goods specific to their villages in their first period of life, in the amounts of $\bar{y}, \bar{x}, \bar{Y}$, and $\bar{X}$, respectively. Lowercase letters represent Japanese variables, and uppercase letters represent U.S. variables. The superscripts *c*, *C*, *d*, and *D* on the choice variable denote the individual type, Japanese and U.S. creditors, or Japanese and U.S. debtors. Both Japanese and U.S. laws require that financial transactions in each economy must be made in their national currencies, the yen and the U.S. dollar. Central banks in the Japanese market and the U.S. market issue currency with initial stocks of $\overline{m}$ yen and $\overline{M}$ dollars to each initial old creditor, whose mass is one and who lives only in the first period, hereafter period 0. The initial old creditors purchase goods by fiat money from the young debtors born in period 1. As for the transactions of agents born after period 1, all agents can issue unfalsifiable IOUs that identify the issuer. Legal authorities exist in the Japanese market and the U.S. market and enforce the agreements between the parties currently in the villages. Legal authorities do not exist to enforce agreements in the foreign exchange market and at the agents’ final destination.

The sequence of travel and trading patterns of debtors and creditors in each country during their lifetimes is summarized in Figures 1, 2, and 3. In the figures, the solid line shows the Japanese resident, the dotted line shows the U.S. resident, and the double solid line shows the national border. We will explain the trading pattern below. Thanks to the symmetry, the trading patterns of the U.S. residents are easily understood by replacing lowercase letters with uppercase letters, and thus we summarize them in the Appendix.

2.2 Trips by Japanese debtors

Japanese debtors consume their own endowment and Japanese creditor goods and U.S. creditor goods in the first period. In Figure 1, solid lines illustrate these trading patterns. At the beginning of the period, Japanese young debtors travel to the Japanese creditor village (Figure 1 (1)) and the U.S. creditor village with which they are paired (Figure 1 (2)). In the Japanese (U.S.) creditor village, they consume Japanese (U.S.) creditor village goods in exchange for an IOU to pay in yen (U.S. dollars) in the second period in the Japanese (U.S.) market, where all IOUs denominated in yen (dollars) are repaid. The reason for the necessity of IOUs in these transactions is as follows. The young debtors wish to consume goods from creditors' villages but do not have national currency at hand. The young debtors offer creditors a promise to pay a sum of money in the next period in the domestic market, IOUs. Because of the extent of legal authority, we assume that debtors can only promise to pay in creditors' national currency. Afterward, they return to their village of origin (Figure 1 (3)) and sell their remaining endowment to old creditors in exchange for yen to prepare their payments of IOUs in the market at the beginning of the second period (Figure 1 (4)). After growing old, Japanese debtors and U.S. debtors meet in the debtors' foreign exchange market and exchange a part of their currencies for the currency of the other economy to pay the IOU that is payable in foreign currency only (Figure 2 (1)), visiting the Japanese market (Figure 2 (2)) and then moving on to the U.S. market (Figure 2 (3)).

The function $v(c_{xt}^d, c_{yt}^d, c_{Yt}^d)$ shows the expected utility of Japanese debtors born in time period *t*, where c_{xt}^d, c_{yt}^d, and c_{Yt}^d show the consumption of the Japanese debtor village goods, Japanese creditor village goods, and U.S. creditor village goods, and subscript t shows time period t. The function v is additively separable, continuous and continuously differentiable strictly increasing concave functions, and its first derivatives have infinite marginal utilities when their arguments are zero.

2.3 Trips by Japanese creditors

Japanese creditors born in time period t consume c^c_{yt} units of their own endowment when young. In Figure 3, solid lines illustrate their trading patterns. After growing old, they first travel to the Japanese market to exchange their IOU issued by Japanese old debtors and U.S. old debtors for yen (Figure 3 (1)). Second, they travel to the foreign exchange market to exchange a part of the yen for dollars (Figure 3 (2)). Third, they travel to the Japanese debtor village to consume c^c_{xt+1} units of Japanese debtor goods in exchange for yen (Figure 3 (3)), and finally they travel to the U.S. debtor village to consume c^c_{Xt+1} units of U.S. debtor goods in exchange for dollars (Figure 3 (4)). The utility of Japanese creditors is $u(c^c_{xt+1}, c^c_{Xt+1}, c^c_{yt})$, and the function u is additively separable, continuous, continuously differentiable, strictly increasing, and concave in each argument.

2.4 Transactions in the Japanese market and the U.S. market

Arrival in the Japanese market takes place in two stages. In the first stage, all old Japanese creditors (Figure 3 (1), solid line), λ ($0 \leq \lambda \leq 1$) old Japanese debtors (Figure 2 (2), solid lines), and Λ ($0 \leq \Lambda \leq 1$) old U.S. debtors arrive (Figure 2 (2), dotted lines). At the end of the first stage, debt that has arrived in the market is settled and (1 – α) old Japanese creditors leave for the foreign exchange market (Figure 3 (2)). The rest of the creditors remain until the end of the second stage. The remaining 1 – λ old Japanese debtors (Figure 2 (2), solid lines) and 1 – Λ old U.S. debtors arrive (Figure 2 (2), dotted lines) in the Japanese market, and all debts are settled. We assume that domestic debt and foreign debt are settled in different areas of the market. All old debtors move on to the U.S. market, and old Japanese creditors move on to the foreign exchange market.

Because of the time-zone difference between Japan and the United States, the first stage of the U.S. market begins after the second stage of the Japanese market. Arrival in the U.S. market takes place in two stages. In the first stage, all old U.S. creditors (Figure 3 (1), dotted line), λ old Japanese debtors (Figure 2 (3), double solid line) and Λ old U.S. debtors arrive (Figure 2 (3), dotted line). At the end of the first stage, debt that has arrived in the market is settled and $(1 - A)$ old U.S. creditors leave for the foreign exchange market (Figure 3 (2), dotted line). In the second stage, all debts are settled and the remaining $1 - \lambda$ old Japanese debtors and $1 - \Lambda$ old U.S. debtors arrive. Old creditors move on to the foreign exchange market.

All creditors face the same probability of leaving the Japanese market and the U.S. market early ($(1 - \alpha)$ for Japanese creditors and $(1 - A)$ for U.S. creditors), and all debtors face the same probability of arriving early or late. Each learns his/her arrival or departure time as soon as he/she grows old.

2.5 *Transactions in the foreign exchange market*

Figure 4 summarizes the trading sequence in the foreign exchange market. We have two foreign exchange markets, the market among debtors and the market among creditors.

Regarding the market among debtors, old Japanese debtors and old U.S. debtors meet in the foreign exchange market and exchange a part of their national currencies for the currency of the other economy at the nominal exchange rate of e_t (yen per dollar) to pay for the IOU that is payable in foreign currency, visiting the Japanese market and then moving on to the U.S. market (Figure 4, in the boxes labeled the old debtors' market). They repay their debt in each national currency.

The old creditors who wish to consume foreign goods in the second period need foreign currency. To obtain foreign currency, the old creditors must exchange their national currency for foreign currency in the creditors' foreign exchange market (Figure 4, in the box labeled the old creditors' market). Note that the arrival in the creditors' foreign exchange market takes place in two stages, reflecting the departure from the U.S. market. Specifically, in the first stage, $(1 - \alpha)$ Japanese early-departing creditors and $(1 - A)$ U.S. early-departing creditors trade with each other at the nominal exchange rate of $\tilde{e}_{t+1}$ yen per dollar. In the second stage, α Japanese late-departing creditors and A U.S. late-departing creditors trade with each other at the nominal exchange rate of $\hat{e}_{t+1}$ yen per dollar. After exchanging dollars and yen to purchase foreign debtors' goods, old creditors randomly scatter to a selected Japanese debtor island and a selected U.S. debtor island.

3. Equilibrium

This section first examines the optimization problem by debtors and creditors and then moves on to present the market equilibrium conditions before defining a symmetric laissez-faire equilibrium with liquidity constraints.

3.1 Debtor's problem

Let p_t be the yen price of Japanese debtor goods in the Japanese debtor villages at time t. Because only debtor goods are sold in exchange for money in the current period, the yen price of Japanese debtor goods is a measure of the Japanese price level. Let m_t be the acquisition of yen from old Japanese creditors in exchange for debtor goods, m^*_t be the acquisition of yen from old U.S. creditors in exchange for debtor goods, h_t be the nominal value at t of the Japanese debtor's indebtedness in yen, and h^*_t be the nominal value at t of the

Japanese debtor's indebtedness in dollars. Note that Japanese debtors must sell goods in exchange for yen. Moreover, Japanese debtors know that they must repay their debt to U.S. creditors in dollars in the next period. The Japanese debtor born in period t maximizes $v(c^d_{xt}, c^d_{yt}, c^d_{Yt})$ subject to the following budget constraints:

$$(\bar{x} - c^d_{xt})\, p_{xt} = m_t + m^*_t\,, \tag{1}$$

$$m_t = h_t\,, \quad m^*_t = h^*_t e_t\,, \tag{2}$$

$$h_t = p_{yt} c^d_{yt}\,, \quad h^*_t = P_{Yt} c^d_{Yt}\,. \tag{3}$$

Equation **(1)** shows that a debtor splits his/her endowment between consumption and sales to creditors in exchange for yen and dollar currencies. Equation **(2)** shows that a debtor acquires money to repay his/her debt in each currency. Equation **(3)** states that the amount of creditor goods consumed is equal to the real value of the debt.

Inserting these three constraints into the utility function, the Japanese debtor maximizes

$$v(\bar{x} - \frac{m_t + m^*_t}{p_{xt}}, \frac{m_t}{p_{yt}}, \frac{m^*_t}{P_{Yt} e_t})\,, \tag{4}$$

by the choice of m_t and m^*_t.

The resulting first-order condition will be

$$\frac{v_x}{v_y} = \frac{p_{xt}}{p_{yt}}, \frac{v_x}{v_Y} = \frac{p_{xt}}{P_{Yt} e_t}, \tag{5}$$

where primes indicate first derivatives, and subscripts to the utility function show the variable that obtains derivatives, incorporating the fact that utility function v is additive separable. Equation **(5)** shows that the marginal cost of having one more unit of money and giving up the consumption of debtor goods today must equal the benefit of consuming domestic creditor goods today in each economy.

The U.S. debtor's problem is a mirror image of the Japanese debtor's problem. The U.S. debtor's budget constraints and optimization problem are mirror images of the Japanese debtor's problem, which is shown in the Appendix. The resulting first-order condition is

$$\frac{V_X}{V_Y} = \frac{P_{Xt}}{P_{Yt}}, \frac{V_X}{V_y} = \frac{P_{Xt}e_t}{p_{yt}} . \qquad \textbf{(6)}$$

Finally, because of the legal restriction, exchange rate e_t must satisfy $h_t^* = P_{Yt}c_{Yt}^d = \frac{p_{yt}C_{yt}^D}{e_t} = \frac{H_t^*}{e_t}$ in equilibrium.

3.2 *Creditor's problem*

Let l_t be the nominal value of a Japanese creditor's loans to Japanese debtors in yen at time *t*. Let l^*_t be the nominal value of a Japanese creditor's loans to U.S. debtors in yen at time *t*. Let q_{t+1} (yen) and q^*_{t+1} (yen) be the par value of nominal debt purchased by Japanese late-leaving creditors from early-leaving Japanese creditors' loans to Japanese debtors and U.S. debtors at time *t* + 1, respectively. Let $\rho_{t+1} \le 1$ represent the nominal price at which ¥1 of the loan to the Japanese debtor is exchanged among Japanese creditors, and let $\rho^*_{t+1} \le 1$ represent the nominal price at which ¥1 of the loan to U.S. debtors is exchanged among Japanese creditors at the first stage of visits in the Japanese market at *t* + 1.

Japanese creditors born in period t maximize $u(c^c_{xt+1}, c^c_{Xt+1}, c^c_{yt})$ subject to the budget constraint when young:

$$p_{yt}(\bar{y} - c^c_{yt}) = l_t + l^*_t . \qquad \textbf{(7)}$$

Depending on whether he/she departs the market early or late, a Japanese creditor has the following budget constraints when old.

First, if he/she departs early (with a probability of (1 – α)), he/she consumes Japanese goods and U.S. goods subject to equations **(8)** and (9):

$$\rho_{t+1}(1-\lambda)l_t + \lambda l_t = \hat{c}^c_{xt+1} p_{xt+1} \text{,} \tag{8}$$

$$\rho^*_{t+1}(1-\Lambda)l^*_t + \Lambda l^*_t = \hat{e}_{t+1}\hat{c}^c_{Xt+1} P_{Xt+1} \text{.} \tag{9}$$

The left-hand sides of these equations show that with probability 1 – λ and 1 – Λ, their loan will not be settled and thus sold to the late-departing creditors at the discount value, and with probability λ and Λ, their loan will be settled and recovered at full value. The right-hand side of equation (9) incorporates the fact that they exchange their yen-denominated loans into dollar-denominated loans at the beginning of time t + 1 at the exchange rate $\hat{e}_{t+1}$.

Second, if he/she departs late (with a probability of α), he/she consumes Japanese goods and U.S. goods subject to equations **(10)** and **(11)**:

$$l_t + (1-\rho_{t+1})q_t = \tilde{c}^c_{xt+1} p_{xt+1} \text{,} \tag{10}$$

$$l^*_t + (1-\rho^*_{t+1})q^*_t = \tilde{e}_{t+1}\tilde{c}^c_{Xt+1} P_{Xt+1} \text{.} \tag{11}$$

The left-hand sides of these equations show that their consumption will be financed through their settled loans to the debtors and their profit from discounting early-departing creditors' second-hand loans. The right-hand side of equation **(11)** incorporates the fact that they exchange their yen-denominated loans into dollar-denominated loans at the beginning of time t + 1 at the exchange rate $\tilde{e}_{t+1}$.

Late-departing creditors face two more liquidity constraints:

$$\lambda l_t \geq \rho_{t+1} q_t \text{,} \tag{12}$$

$$\Lambda l^*_t \geq \rho^*_{t+1} q^*_t \text{.} \tag{13}$$

Equation **(12)** (or **(13)**) states that the nominal value of IOUs issued by Japanese (or U.S.) debtors purchased by late-departing Japanese creditors from early-departing Japanese creditors, $\rho_{t+1}q_t$ (or $\rho^*_{t+1}q^*_t$), is less than the amount of yen available at the end of the first stage, λl_t (or Λl^*_t). In formulating equations **(12)** and **(13)**, we assume that Japanese late-departing creditors separate the transaction of IOUs issued by Japanese debtors and those issued by U.S. debtors, and thus the interest rates can differ. The discrimination between Japanese debtors and U.S. debtors by the Japanese creditor is consistent with the observation of a "home bias" in international portfolio investment.

Using the budget constraints at times t and $t+1$ yields the following optimization problem for the young Japanese creditors with respect to l_t, l^*_t, q_t, and q^*_t at time t:

$$Max\, u(\bar{y}-\frac{l_t+l_t^*}{p_{yt}})+(1-\alpha)\left\{\hat{u}(\frac{\rho_{t+1}(1-\lambda)l_t+\lambda l_t}{p_{xt+1}})+\hat{u}(\frac{\rho_{t+1}^*(1-\Lambda)l_t^*+\Lambda l_t^*}{P_{Xt+1}\hat{e}_{t+1}})\right\} \quad \textbf{(14)}$$
$$+\alpha\left\{\tilde{u}(\frac{l_t+(1-\rho_{t+1})q_t}{p_{xt+1}})+\tilde{u}(\frac{l_t^*+(1-\rho_{t+1}^*)q_t^*}{P_{Xt+1}\tilde{e}_{t+1}})\right\}+\mu(\lambda l_t-\rho_{t+1}q_t)+\mu^*(\Lambda l_t^*-\rho_{t+1}^*q_t^*).$$

We here assume that μ and μ^* are Lagrangian multipliers for the liquidity constraints, equations **(12)** and **(13)**.

The first-order conditions of this problem for l_t and l^*_t are

$$u_y(\frac{1}{p_{yt}})=(1-\alpha)\hat{u}_x\left\{\frac{\rho_{t+1}(1-\lambda)+\lambda}{p_{xt+1}}\right\}+\alpha\tilde{u}_x(\frac{1}{p_{xt+1}})+\lambda\mu \,. \quad \textbf{(15)}$$

$$u_y(\frac{1}{p_{yt}})=(1-\alpha)\hat{u}_X\left\{\frac{\rho_{t+1}^*(1-\Lambda)+\Lambda}{P_{Xt+1}\hat{e}_{t+1}}\right\}+\alpha\tilde{u}_X(\frac{1}{P_{Xt+1}\tilde{e}_{t+1}})+\Lambda\mu^* \,. \quad \textbf{(16)}$$

The first-order conditions for q_t and q^*_t are

$$\alpha\tilde{u}_x(\frac{1-\rho_{t+1}}{p_{xt+1}})=\mu\rho_{t+1}\,, \quad \textbf{(17)}$$

$$\alpha\tilde{u}_X(\frac{1-\rho_{t+1}^*}{P_{Xt+1}\tilde{e}_{t+1}})=\mu^*\rho_{t+1}^*\,. \quad \textbf{(18)}$$

The combination of equations (15) and (17) yields equation **(19)**, and the combination of equations (16) and (18) yields equation **(20)**.

$$u_y(\frac{p_{xt+1}}{p_{yt}})=(1-\alpha)\hat{u}_x\{\rho_{t+1}(1-\lambda)+\lambda\}+\alpha\tilde{u}_x\left\{1+\lambda(\frac{1}{\rho_{t+1}}-1)\right\}. \quad \textbf{(19)}$$

$$u_y(\frac{P_{Xt+1}}{p_{yt}})=(1-\alpha)\hat{u}_X\{\rho_{t+1}^*(1-\Lambda)+\Lambda\}\frac{1}{\hat{e}_{t+1}}+\alpha\tilde{u}_X\left\{1+\Lambda(\frac{1}{\rho_{t+1}^*}-1)\right\}\frac{1}{\tilde{e}_{t+1}}. \quad \textbf{(20)}$$

These equations show that the marginal benefit of extending one more unit of loans at the sacrifice of the consumption of the creditor good at *t* (that is, the left-hand sides) is equal to the probability-weighted average of utility of consuming Japanese debtor goods or U.S. debtor goods as an early-departing creditor or a late-departing creditor (that is, the right-hand sides).

Note that when the liquidity constraints **(17)** and **(18)** do not bind, $\mu = 0$ and $\mu^* = 0$, and thus $\rho_{t+1} = 1$ and $\rho^*_{t+1} = 1$. In this case, equation **(19)** and **(20)** are simplified to equations **(21)** and **(22)**.

$$\frac{u_y}{u_x} = \frac{p_{yt}}{p_{xt+1}}, u_x = \hat{u}_x = \tilde{u}_x \quad \text{(21)}$$

$$u_y\left(\frac{P_{Xt+1}}{p_{yt}}\right) = (1-\alpha)\hat{u}_X \frac{1}{\hat{e}_{t+1}} + \alpha\tilde{u}_X \frac{1}{\tilde{e}_{t+1}} . \quad \text{(22)}$$

Equation **(21)** shows that the marginal rate of substation between good *x* and good *y* is equal to the relative price of these two goods, and the consumption of early-departing creditors and that of late-departing creditors are equal. Equation **(22)** shows that the marginal rate of substitution between good *y* and good *X* is equal to a weighted average of the marginal utility of consuming good *X* as early-departing creditors and that of late-departing creditors multiplied by the relevant real exchange rate evaluated at the probability of becoming an early-departing creditor or a late-departing creditor. If $\hat{e}_{t+1} = \tilde{e}_{t+1}$, then the consumption of early-departing creditors and that of late-departing creditors are equal, and the marginal utility of consuming good *y* and good *X* is equal to the unique real exchange rate. Therefore, if a policymaker wishes to achieve equal consumption among domestic creditors both at home and abroad, he/she needs to make the liquidity constraint unbinding in both the domestic market and the foreign exchange market.

The consumption pattern, budget constraints, and optimization problems of a U.S. creditor born at *t* can be derived similarly and are summarized in the Appendix.

3.3 Market-clearing conditions

The conditions for the clearing of the market of goods denominated in yen and dollars in debtor villages are as follows:

$$p_{xt} = \frac{\overline{m}}{\overline{x} - c_{xt}^{d}} = \frac{m_t + m_t^*}{\overline{x} - c_{xt}^{d}}, \tag{23}$$

$$P_{Xt} = \frac{\overline{M}}{\overline{X} - C_{Xt}^{D}} = \frac{M_t + M_t^*}{\overline{X} - C_{Xt}^{D}}. \tag{24}$$

The clearing of the market for loans in yen and dollars requires

$$l_t = h_t = m_t, \frac{l_t^*}{e_t} = \frac{H_t^*}{e_t} = M_t^*, \tag{25}$$

$$L_t = H_t = M_t, L_t^* e_t = h_t^* e_t = m_t^*. \tag{26}$$

Let us move on to the clearing of the foreign exchange market. First, in a market among debtors, because of the legal restriction, equilibrium condition becomes

$$h_t^* = P_{Yt} c_{Yt}^{d} = \frac{p_{yt} C_{yt}^{D}}{e_t} = \frac{H_t^*}{e_t}, \tag{27}$$

Note that the determination of the debtor's exchange rate for the initial old Japanese creditor and the initial old U.S. creditor differs. The initial old Japanese creditor maximizes the function $u(c_{x0}^c, c_{X0}^c)$ subject to the budget constraint that $p_{x0} c_{x0}^c + P_{X0} c_{X0}^c e_0 = \overline{m}$, where p_0 and P_0 show a nominal price level in yen and dollars in period 0, and e_0 is a nominal exchange rate (yen per dollar) in period 0. The initial old U.S. creditor maximizes $U(C_{X0}^C, C_{x0}^C)$ subject to the budget constraint that $P_{X0} C_{X0}^C + p_{x0} C_{x0}^C \frac{1}{e_0} = \overline{M}$. Because of the legal restriction, the exchange rate e_0 must satisfy $p_{x0} C_{x0}^C = P_{X0} c_{X0}^c e_0$ in equilibrium.

Equilibrium conditions in the foreign exchange market for early-departing creditors and late-departing creditors become

$$(1-\alpha)\hat{c}^{c}_{Xt+1}P_{Xt+1}\hat{e}_t = (1-\mathrm{A})\hat{C}^{c}_{xt+1}p_{xt+1}, \tag{28}$$

$$\alpha\tilde{c}^{c}_{Xt+1}P_{Xt+1}\tilde{e}_t = \mathrm{A}\tilde{C}^{c}_{xt+1}p_{xt+1}. \tag{29}$$

The clearing of second-hand debt in the Japanese market when the liquidity constraint binds requires that the total value of the second-hand loan (the left-hand side) is equal to the not yet redeemed debt from late-arriving creditors allocated to early-departing creditors (the right-hand side):

$$\alpha q_t = (1-\alpha)(1-\lambda)l_t, \tag{30}$$

$$\alpha q_t^* = (1-\alpha)(1-\Lambda)l_t^*. \tag{31}$$

Combining equations **(30)** and **(31)** with liquidity constraints **(12)** and **(13)**, we have

$$\rho_{t+1} = \frac{\alpha\lambda}{(1-\alpha)(1-\lambda)}, \tag{32}$$

$$\rho^*_{t+1} = \frac{\alpha\Lambda}{(1-\alpha)(1-\Lambda)}. \tag{33}$$

Similarly, the clearing of second-hand debt in the U.S. market when the liquidity constraint binds yields the following discount rates in the U.S. second-hand debt market:

$$R^*_{t+1} = \frac{\mathrm{A}\lambda}{(1-\mathrm{A})(1-\lambda)}, \tag{34}$$

$$R_{t+1} = \frac{\mathrm{A}\Lambda}{(1-\mathrm{A})(1-\Lambda)}. \tag{35}$$

Money is essential to make final payment to settle the debt, and without repayment in the national currency creditors will not accept the debt. Debts are cleared in the national market, but not always bilaterally. Because the arrival rate of the old debtors is lower than the departure rate of old creditors, the early-departing old creditors sell their not yet redeemed debt to the late-departing old creditors. The amount of debt redeemed in this second-hand debt market is limited by the currency that the early-arriving old debtors bring to the market, and thus the second-hand debt may be traded at a discount value. To illustrate such a situation, we assume that discount rates, $\rho, \rho^*, R,$ and R^*, are smaller than one.

In addition, goods markets must be cleared, but we can ignore these conditions due to Walras' Law.

3.4 *Symmetric laissez-faire equilibrium with liquidity constraints*

Consider a symmetric laissez-faire equilibrium where both Japanese and U.S. liquidity constraints are binding and the same types of Japanese and U.S. creditors choose the same actions, and central banks do not intervene in the markets. Let $p_{xt+1} = p_{xt} = p_x$, $p_{yt} = p_y$, $\rho_{t+1} = \rho_s$, $\rho^*_{t+1} = \rho_s^*$, $l_t = l$, $l^*_t = l^*$, $h_t = h$, $h^*_t = h^*$, $m_t = m$, $m^*_t = m^*$, $p_s = p_y/p_x$, $l_s = l/p_x$, and $l^*_s = l^*/p_x$ for Japanese variables in a symmetric laissez-faire equilibrium. Let $P_{Xt+1} = P_{Xt} = P_X$, $P_{Yt} = P_Y$, $R_{t+1} = R_s$, $R^*_{t+1} = R_s^*$, $L_t = L$, $L^*_t = L^*$, $H_t = H$, $H^*_t = H^*$, $M_t = M$, $M^*_t = M^*$, $P_s = P_Y/P_X$, $L_s = L/P_X$, and $L^*_s = L^*/P_X$ for U.S. variables in a symmetric laissez-faire equilibrium, and let $e_t = e_s$, $\hat{e}_{t+1} = \hat{e}_s$, and $\tilde{e}_{t+1} = \tilde{e}_s$ for the exchange rate between debtors, early-departing creditors, and late-departing creditors, respectively.

Using these notations and equilibrium conditions in the loan, money, second-hand debt, and the foreign exchange market, we can simplify the equilibrium conditions as follows:

$$v_x(\bar{x} - l_s - L_s^* \frac{P_X e_s}{p_x}) = \frac{1}{p_s} v_y(\frac{l_s}{p_s}) , \tag{36}$$

$$v_x(\bar{x} - l_s - L_s^* \frac{P_X e_s}{p_x}) = \frac{1}{P_s} \frac{p_x}{p_X e_s} v_Y(\frac{L_S^*}{P_S}) , \tag{37}$$

$$V_X(\bar{X} - L_s - l_s^* \frac{p_x}{P_X e_s}) = \frac{1}{P_s} V_Y(\frac{L_S}{P_S}) , \tag{38}$$

$$V_X(\bar{X} - L_s - l_s^* \frac{p_x}{P_X e_s}) = \frac{1}{p_s} \frac{P_X e_s}{p_x} V_y(\frac{l_s^*}{p_s}) , \tag{39}$$

$$\frac{1}{p_s} u_y(\bar{y} - \frac{l_s}{p_s} - \frac{l_s^*}{p_s}) = \lambda u_x(\frac{\lambda}{1-\alpha} l_s) + (1-\lambda) u_x(\frac{1-\lambda}{\alpha} l_s) \text{ ,} \quad \textbf{(40)}$$

$$\frac{1}{p_s} u_y(\bar{y} - \frac{l_s}{p_s} - \frac{l_s^*}{p_s}) = \Lambda \frac{p_x}{P_X \hat{e}_s} u_X(\frac{\Lambda}{1-\alpha} \frac{l_s^*}{p_s} \frac{p_x}{P_X \hat{e}_s}) + (1-\Lambda) \frac{p_x}{P_X \tilde{e}_s} u_X(\frac{1-\Lambda}{\alpha} \frac{l_s^*}{p_s} \frac{p_x}{P_X \tilde{e}_s}) \quad \textbf{(41)}$$

$$\frac{1}{P_S} U_Y(\bar{Y} - \frac{L_s}{P_s} - \frac{L_s^*}{P_s}) = \Lambda U_X(\frac{\Lambda}{1-A} L_s) + (1-\Lambda) U_X(\frac{1-\Lambda}{A} L_s) \text{ ,} \quad \textbf{(42)}$$

$$\frac{1}{P_S} U_Y(\bar{Y} - \frac{L_s}{P_s} - \frac{L_s^*}{P_s}) = \lambda \frac{P_X \hat{e}_s}{p_x} U_x(\frac{\lambda}{1-A} \frac{L_s^*}{p_s} \frac{P_X \hat{e}_s}{p_x}) + (1-\lambda) \frac{P_X \tilde{e}_s}{p_x} U_x(\frac{1-\lambda}{A} \frac{L_s^*}{p_s} \frac{P_X \tilde{e}_s}{p_x}) \quad \textbf{(43)}$$

$$\frac{l_s^*}{L_s^*} = \frac{P_X e_s}{p_x} \text{ ,} \quad \textbf{(44)}$$

$$\frac{l_s^*}{L_s^*} \frac{\Lambda}{\lambda} = \frac{P_X \hat{e}_s}{p_x} \text{ ,} \quad \textbf{(45)}$$

$$\frac{l_s^*}{L_s^*} \frac{1-\Lambda}{1-\lambda} = \frac{P_X \tilde{e}_s}{p_x} \text{ ,} \quad \textbf{(46)}$$

$$p_x = \frac{\bar{m}}{\bar{x} - l_s - L_s^* \left(P_X e_s / p_x \right)} \text{ ,} \quad \textbf{(47)}$$

$$P_X = \frac{\bar{M}}{\bar{X} - L_s - l_s^* \left(p_x / P_X e_s \right)} \text{ .} \quad \textbf{(48)}$$

Equations **(36)** through **(48)** consist of four parts, as below.

First, equations **(36)** through **(39)** come from first-order conditions for debtors, which equate the marginal utility of consuming the debtor good and creditor good at home and abroad. Specifically, equations **(36)** and **(37)** come from Japanese debtors' first-order conditions, equation **(4)**. Equations **(38)** and **(39)** come from U.S. debtors' first-order conditions, equation **(6)**.

Second, equations **(40)** through **(43)** come from first-order conditions for creditors, which equate the marginal utility of consuming one more unit of creditor goods when young and the probability-weighted average of utility of consuming Japanese debtor goods or U.S. debtor goods as an early-departing creditor or a late-departing creditor when old. Specifically, equations **(40)** and **(41)** come from Japanese creditors' first-order conditions, equations **(19)** and **(20)**. Equations **(42)** and **(43)** come from similar U.S. creditors' first-order conditions at the end of the Appendix.

Third, equations **(44)** through **(46)** come from the clearing of the foreign exchange market for debtors (equation **(27)**), early-departing creditors (equation (28)), and late-departing creditors (equation (29)).

Fourth, equations (47) and (48) come from equations (23) and (24), market-clearing conditions for money and goods in Japan and the United States.

We obtain the equilibrium in the following way. First, by inserting the right-hand side of equation **(44)** into equations **(36)** through **(39)**, and **(47)** and **(48)**, we eliminate the real exchange rate for debtors. Second, by inserting equations **(45)** and **(46)** into **(41)** and **(43)**, we eliminate the real exchange rate for early-departing and late-departing creditors.

Then we use the remaining 10 equations, that is, **(36)** through **(43)**, and **(47)** and **(48)**, to solve for 10 unknowns, p_x, p_y, p_s, l_s, l^*_s, P_X, P_Y, P_s, L_s, and L^*_s. Using these results and equations **(44)** through **(46)** gives us e_s, $\hat{e}_s$, and $\tilde{e}_s$. Given p_x, l_s, and l^*_s, we find $p_x l_s = l = h = m$, and $p_x l^*_s = l^* = H^* = Me_s^*$. Given P_X, L_s, and L^*_s, we find $P_x L_s = L = H = M$, and $P_x L_s^* = L^* = h^* = m^* / e_s$.

Hereafter, we assume the existence of the solution.

4. Institutional designs to alleviate domestic and international liquidity constraints

4.1 The benchmark allocations: Pareto-efficient allocation

We consider a technically feasible allocation of resources as a benchmark. Specifically, we consider an allocation that maximizes a weighted-average utility of Japanese debtors, creditors, U.S. debtors, and creditors with weights of $\theta, 1-\theta, \Theta, \text{and } 1-\Theta$ subject to the resource constraints, where η_i shows the Lagrange multiplier for the resource constraints for good i, as below:

$$
\begin{aligned}
&Max\{\theta v(c_x^d, c_y^d, c_Y^d) + (1-\theta)u(c_x^c, c_X^c, c_y^c)\} \\
&+ \{\Theta V(C_X^D, C_Y^D, C_y^D) + (1-\Theta)U(C_X^C, C_x^C, C_Y^C)\} \\
&- \eta_x(c_x^d + c_x^c + C_x^C - \overline{x}) - \eta_y(c_y^d + c_y^c + C_y^D - \overline{y}) \\
&- \eta_X(c_X^d + C_X^D + C_X^C - \overline{X}) - \eta_Y(c_Y^d + C_Y^D + C_Y^C - \overline{Y}),
\end{aligned} \tag{49}
$$

Inspections of first-order conditions yield the following optimality conditions:

$$
\frac{v_x}{v_y} = \frac{u_x}{u_y}, \frac{V_X}{V_Y} = \frac{U_X}{U_Y}, \frac{v_x}{v_Y} = \frac{U_x}{U_Y}, \frac{V_X}{V_y} = \frac{u_X}{u_y}. \tag{50}
$$

The optimality condition says that the marginal rate of substitutions between the same pair of goods must be equal among creditors and debtors. Our symmetric laissez-faire equilibrium does not achieve these optimality conditions. For example, the marginal utilities of early-departing creditors and late-departing creditors are not equal in equation **(40)**, hence even with equation **(36)**, the symmetric laissez-faire equilibrium violates the first optimality condition shown in equation **(50)**. Only if liquidity constraints in Japan and the United States do not bind and if real exchange rates for debtors, early-departing creditors, and late-departing creditors are the same, the optimality conditions for the symmetric laissez-faire equilibrium coincide with the optimality condition for the benchmark allocation, as shown by equations **(21)** and **(22)**. Therefore, we can improve the efficiency of the symmetric laissez-faire equilibrium by achieving these three conditions stated above. We consider four institutional designs to improve the efficiency of the symmetric laissez-faire equilibrium by resolving these conditions in turn.

4.2 Effects of a central bank's discount window policy and the CLS Bank

Consider a central bank's discount window policy as in Freeman (1996). Suppose that two central banks can issue and lend additional fiat money for the debt presented by the late-departing creditors in each market. These central bank loans will be repaid with the fiat money upon arrival in the market of the late-arriving debtors, and the central bank should then destroy the fiat money obtained from the late-arriving debtors out of circulation to keep the aggregate supplies of money constant. Suppose that the two central banks conduct the policy above with the nominal discount rate at one. Such policies allow the late-departing creditors to purchase the second-hand debt from the early-departing creditors at par value, which eliminates the effects of liquidity constraints. In these situations, equations **(40)** and **(42)** collapse to equation **(21)**, which corresponds to the optimality conditions in equation **(50)**. However, equations **(45)** and **(46)** (coming from equations **(28)** and **(29)**) become equations **(51)** and **(52)** below, because the timing of the arrival in the foreign exchange market reflects the timing of the departure from the U.S. market, as in Fujiki (2003).

$$\frac{l_s^*}{L_s^*}\frac{1-\alpha}{(1-\mathrm{A})} = \frac{P_X \hat{e}_s}{p_x}, \qquad \textbf{(51)}$$

$$\frac{l_s^*}{L_s^*}\frac{\alpha}{\mathrm{A}} = \frac{P_X \tilde{e}_s}{p_x}. \qquad \textbf{(52)}$$

How could policymakers achieve the condition that $\hat{e}_s = \tilde{e}_s$ to obtain the same optimality condition as the benchmark allocations? As a practical solution to this problem, the CLS Bank makes the settlements in the foreign exchange market on the basis of delivery versus payments. In particular, the CLS Bank eliminates the risk of counterparty default due to the mismatch of the timing of the payments such as a time-zone difference. Since the total amount of fiat monies delivered to the foreign exchange market is given, the use of the CLS Bank changes both equations **(51)** and **(52)** into equation **(44)**. With this condition both equations **(41)** and **(43)** are equal to the optimal condition of equation **(50)**. In this way, the combination of discount window policy and the CLS Bank achieves the benchmark resource allocation, as proposed by Fujiki (2006) in a similar setting.

4.3 Effects of foreign exchange intervention policy

This section shows that the mixture of discount window policy and the intervention in the foreign exchange market also achieves the benchmark allocation, as Fujiki (2003) observed.

If $1-\alpha < 1-\mathrm{A}$, equations **(51)** and **(52)** show that compared to the benchmark equation **(44)**, the amount of yen in the foreign exchange market for early-departing creditors is insufficient. The BOJ should issue $(\alpha-\mathrm{A})l^* p_x$ of yen, and purchase dollars at an exchange rate consistent with **(44)**. In this operation, the Japanese central bank obtains the dollar amount of $(\alpha-\mathrm{A})L^* P_x = (\alpha-\mathrm{A})l^* p_x e_s$, and sells it in the foreign exchange market for late-departing creditors at the exchange rate consistent with **(44)**. In this way, the mixture of discount window policy and the intervention in the foreign exchange market also achieves the benchmark allocation. The operation would expand the balance sheet of the BOJ at the first stage of the transaction, and shrink it at the second stage of transaction, as shown in Figure 5.

Since the parameter α, A measures the fraction of creditors who do not need to be repaid immediately before all debtors arrive, this parameter measures the size of the banking sector relative to the total number of creditors. $1-\alpha < 1-\mathrm{A}$ means that $\alpha > \mathrm{A}$; therefore, in the above example, a central bank in an economy with a relatively large banking sector must intervene in the foreign exchange market. In other words, the foreign exchange intervention is asymmetric, and only one of the central banks needs to intervene.

4.4 Effects of cross-border collateral arrangements

On November 28, 2011, the BOJ and the BOT began liquidity provision in Thai baht with the BOT utilizing Japanese government securities as collateral. Specifically, the BOJ opened securities custody accounts enabling the BOT to accept Japanese government securities as collateral. Becoming effective on the same day was the BOT's notification to accept Japanese government securities as eligible collateral for part of its liquidity provisioning measures for financial institutions operating in Thailand, including Japanese banks. Thanks to this arrangement, a Japanese bank lending in Thai baht could obtain loans in Thai baht from the BOT using Japanese government securities as eligible collateral. Such an arrangement made it possible to address the problem of liquidity shortage in the domestic market and the foreign exchange market in our model, as shown in Figure 6.

Similar to our analysis of foreign exchange market intervention discussed in the previous section after the domestic monetary operation, if $1-\alpha < 1-\mathrm{A}$, equation **(44)** shows that compared to the benchmark, the amount of yen in the foreign exchange market for early-departing creditors is insufficient. The Japanese central bank should issue yen and accept $(\alpha - \mathrm{A})L^{*}P_{x}$ of the U.S. creditor's loan to Japanese debtors denominated in dollars at an exchange rate consistent with **(44)** as collateral. Note that thanks to the domestic market operations, these loans would already be turned into dollar, and thus the Japanese bank essentially would accept dollars as collateral and lend yen to U.S creditors.[3] In the market of late-departing creditors, the amount of dollars would now be insufficient. The Federal Reserve should issue $(\alpha - \mathrm{A})L^{*}P_{x}$ of dollars and accept $(\alpha - \mathrm{A})l^{*}p_{x}e_{s}$ of Japanese creditors' loans (already turned into yen) to U.S. debtors denominated in yen at an exchange rate consistent with **(44)**.

Figure 7 summarizes changes in the two central banks' balance sheets during these operations. Unlike the changes in the central bank balance sheets discussed in foreign exchange operations, cross-border collateral increases the amount of circulation of dollars and yen. However, the amounts of dollars and yen held by the debtors and creditors are unchanged, because central banks expand their balance sheets taking other central banks' fiat money as the collateral for the additional issue of fiat money. If these additional fiat monies are not used for the purchase of goods and increase only as the result of foreign exchange rate stabilization, thee additional fiat monies would not be inflationary, because the price level

[3] See Keane (2013) about the risks of securities loans collateralized by cash. In our model, the central banks will not reinvest cash obtained as collateral, and thus the risks mentioned in Keane (2013) cannot be examined.

would depend on the total amount of money used for consumption and supply of goods, as equations **(47)** and **(48)** show.

4.5 Effects of foreign currency liquidity swap lines

Domestic old creditors have incentives to exchange their loans to foreign old debtors with foreign old creditors' loans to domestic old debtors, because both domestic old creditors and foreign old creditors need foreign currency to purchase young foreign debtors' goods. If the cost is small for an offshore transaction, for example, in the Eurodollar market to swap old creditors' loans to foreign old debtors, then such transactions may occur as shown in Figure 8. The details of transactions are explained below.

Suppose that domestic old creditors exchange their foreign loans with foreign old creditors in the Eurodollar market, which is not subject to the application of each national law. Old debtors born at t, who have only domestic currency at hand, still need to exchange their cash into foreign currency at time t to repay their IOUs to foreign creditors at time $t + 1$. However, old debtors born at t do not have to travel to foreign credit market at time $t + 1$ to repay their IOUs to foreign creditors. Instead, they repay their IOUs to foreign creditors denominated in the foreign currency to domestic old creditors at time $t + 1$ in the domestic credit market, as the large curved arrows in Figure 8 show. Note that our assumption is that the young debtor must repay the loan in the creditor's currency, and thus the repayment in the foreign currency is valid in this case. Moreover, the transaction is between money and debt, which is not regulated, rather than the transaction between money and goods subject to the country-specific currency usage. Hence, the transaction would not violate any legal restriction.

Do these transactions support the benchmark efficient allocation? It depends on whether or not the arrival rate of domestic debtors at time t is smaller than the departure rate of domestic creditors. Note that domestic central bank credit policy in the domestic market supplies only domestic currency. Therefore, if the supply of foreign currency is not sufficient for the early-departing creditors, those departing early will consume a smaller amount of foreign goods relative to those departing late. Therefore, without policy intervention, such transactions would not achieve the benchmark allocation. What kind of policy interventions are then needed to resolve the inefficiency?

Foreign currency liquidity swap lines between central banks are one means of resolving the inefficiency. First, the domestic central bank borrows foreign currency through a swap agreement with the other central bank at the exchange rate of e_s. Second, the domestic central bank lends foreign currency to domestic old creditors, taking the late-arriving debtors' repayment of foreign currency as the collateral at the exchange rate of e_s. Finally, when the late-arriving debtors arrive in the domestic market and repay their loans to the domestic central bank in foreign currency as well as domestic currency, the domestic central bank remits the foreign currency obtained from the late-arriving debtors to the foreign central bank to repay the borrowing obtained from the swap line. Note that throughout the operation just described, the foreign central bank operates in way that mirrors the operation of the domestic central bank. In this sense, the foreign currency swap lines in this model should be interpreted as symmetric and standing swap lines.

Note that the discussion above hinges on the assumptions that all agents honor contracts, which allows us to safely ignore the issues of moral hazard, while in practice the existence of a stand-by facility would change the behavior of creditors and debtors depending on the degree of asymmetric information. The next section states reservations on interpreting literally the policy implication of our analysis.

5. Conclusion and reservations

We showed that central banks improve the efficiency of equilibrium under domestic and foreign currency liquidity shortages through four institutional designs: a combination of central bank discount window policy and the CLS Bank; a central bank intervention both in the domestic credit market and the foreign exchange market; cross-border collateral arrangements; and foreign currency liquidity swap lines.

Our result that the four institutional designs have the same welfare-improving effects depends on several assumptions in the model, particularly that all agents honor contracts. In practice, however, transaction costs required by these four institutional designs would differ, and thus there is no reason to believe that they would achieve the same welfare gain. One might wonder whether the creditors' cost of enforcing a contract for domestic lenders and foreign lenders would differ. One might also wonder whether central banks would prefer foreign currency liquidity swap lines compared to cross-border collateral arrangements, because the former means that they would lend to foreign central banks but the latter means that they would lend to foreign creditors regulated under the foreign legal system. In this context, Mills (2006) demonstrates that the existence of an opportunity cost of collateral for central bank lending might distort efficient allocation with the possibility of an endogenous default decision by agents in a closed-economy model. On this point, further research is needed on a two-country model.

Appendix: Details on U.S. residents' trading patterns and optimization problems

Trips by U.S. debtors

U.S. debtors consume their own endowment, U.S. creditor goods, and Japanese creditor goods in the first period. In Figure 1, dotted lines illustrate these trading patterns. At the beginning of the period, U.S. young debtors travel to the creditor village and Japanese creditor village with which they are paired (Figure 1 (1)). In the U.S. (Japanese) creditor village, they may consume U.S. (Japanese) creditor village goods in exchange for the IOU to pay in U.S. dollars (yen) the second period in the U.S. (Japanese) market, where all IOUs denominated in dollars (yen) are repaid (Figure 1 (2)). They return to their debtor villages later in the period (Figure 1 (3)) and sell their remaining endowment to old creditors in exchange for dollars to prepare their payments of IOUs in the market in the beginning of the second period (Figure 1 (4)).

When old, U.S. debtors and Japanese debtors meet in the debtors' foreign exchange market and exchange a part of their currencies for the currency of the other economy to pay the IOU that is payable in foreign currency only (Figure 2 (1)), visiting the Japanese market (Figure 2 (2)), and then moving on to the U.S. market (Figure 2 (3)).

The function $V(C^D_{Xt}, C^D_{Yt}, C^D_{yt})$ shows the expected utility of U.S. debtors born at time period *t*, where C^D_{Xt}, C^D_{Yt}, and C^D_{yt} show the consumption of the U.S. debtor village, U.S. creditor village goods, and Japanese creditor village goods when young. The function *V* has the same properties as the utility function of *v*.

Trips by U.S. creditors

U.S. creditors born at time period *t* consume C^C_{Yt} units of their own endowment when young. In Figure 3, dotted lines illustrate these trading patterns. When old, first, they travel to the U.S. market to exchange their IOU issued by the Japanese old debtor and U.S. old debtors for dollars (Figure 3 (1)). Second, they travel to the foreign exchange market to exchange part of their dollars for yen (Figure 3 (2)). Third, they travel to the U.S. debtor village to consume C^C_{Xt+1} units of U.S. debtor goods in exchange for dollars (Figure 3 (3)), and finally they travel to the Japanese debtor village to consume C^{C}_{xt+1} units of Japanese debtor goods (Figure 3 (4)). The utility of U.S. creditors is $U(C^{C}_{Xt+1}, C^{C}_{xt+1}, C^{C}_{Yt})$, which has the same properties as utility function *u*.

The U.S. debtor's problem

The U.S. debtor born in period *t* provides a mirror image of the Japanese debtor's problem. Let P_t be the dollar price of U.S. debtor goods in the U.S. debtor village at time *t*. Let M_t be the acquisition of dollars from old U.S. creditors in exchange for debtor goods, M^*_t be the acquisition of dollars from old Japanese creditors in exchange for debtor goods, H_t be the nominal value at *t* of the U.S. debtor's indebtedness in dollars, and H^*_t be the nominal value at *t* of the U.S. debtor's indebtedness in yen. The U.S. debtor maximizes $V(C^{D}_{Xt}, C^{D}_{Yt}, C^{D}_{yt})$ subject to the following budget constraints:

$$(\overline{X} - C^{D}_{Xt})P_{Xt} = M_t + M^*_t,$$

$$M_t = H_t,\ \ M^*_t = \frac{H^*_t}{e_t},$$

$$H_t = P_{Yt}C^{D}_{Yt},\ \ H^*_t = p_{yt}C^{D}_{yt}.$$

Inserting these three constraints into the utility function, the U.S. debtor maximizes

$$V(\overline{X} - \frac{M_t + M^*_t}{P_{Xt}}, \frac{M_t}{P_{Yt}}, \frac{M^*_t e_t}{p_{yt}}),$$

by the choice of M_t and M^*_t.

The resulting first-order condition will be

$$\frac{V_X}{V_Y} = \frac{P_{Xt}}{P_{Yt}}, \frac{V_X}{V_y} = \frac{P_{Xt}e_t}{p_{yt}}.$$

Finally, because of the legal restriction, exchange rate e_t must satisfy $h^*_t = P_{Yt}c^{d}_{Yt} = \frac{p_{yt}C^{D}_{yt}}{e_t} = \frac{H^*_t}{e_t}$ in equilibrium.

The U.S. creditor's problem

Let L_t be the nominal value of a U.S. creditor's loans to U.S. debtors in dollars at time t. Let L^*_t be the nominal value of a U.S. creditor's loans to Japanese debtors in dollars at time t. Let Q_t (dollars) and Q^*_t (dollars) be the par value of nominal debt to the U.S. debtor and Japanese debtors purchased by late-leaving U.S. creditors from early-leaving U.S. creditors at time t + 1. Let $R_{t+1} \leq 1$ represent the nominal price at which $1 of debt to the U.S. debtor is exchanged among U.S. creditors and $R^*_{t+1} < 1$ represent the nominal price at which $1 of the debt to the Japanese debtor is exchanged among U.S. creditors at the first stage of visits in the U.S. market at $t + 1$.

U.S. creditors born in period t maximize $U(C^C_{Xt+1}, C^C_{xt+1}, C^C_{Yt})$ subject to the following budget constraint when young:

$$P_{Yt}(\overline{Y} - C^C_{Yt}) = L_t + L^*_t.$$

Depending on whether he/she departs the market early or late, a U.S. creditor has the following four budget constraints when old.

First, if he/she departs early (with a probability of (1 – Λ)), he/she consumes U.S. goods and Japanese goods subject to the equations below:

$$R_{t+1}(1-\Lambda)L_t + \Lambda L_t = \hat{C}^C_{Xt+1}P_{Xt+1}.$$

$$R^*_{t+1}(1-\lambda)L^*_t + \lambda L^*_t = \frac{\tilde{C}^C_{xt+1}p_{xt+1}}{\hat{e}_{t+1}}.$$

Second, if he/she departs late (with a probability of Λ), he/she consumes Japanese goods and U.S. goods subject to the equations below:

$$L_t + (1-R_{t+1})Q_t = \tilde{C}^C_{Xt+1}P_{Xt+1}.$$

$$L^*_t + (1-R^*_{t+1})Q^*_t = \frac{\tilde{C}^C_{xt+1}p_{xt+1}}{\tilde{e}_{t+1}}.$$

Late-departing creditors face two more constraints:

$$\lambda L_t \geq R^*_{t+1} Q^*_t \, .$$

$$\Lambda L^*_t \geq R_{t+1} Q_t \, .$$

Using the budget constraints at times t and t + 1 yields the following optimization problem for the young U.S. creditors with respect to L_t, L^*_t, Q_t, and Q^*_t at time t:

$$Max\, U(\overline{Y} - \frac{L_t + L^*_t}{P_{yt}}) + (1-\mathrm{A})\left\{\hat{U}(\frac{R_{t+1}(1-\Lambda)L_t + \Lambda L_t}{P_{Xt+1}}) + \hat{U}(\frac{R^*_{t+1}(1-\lambda)L^*_t + \lambda L^*_t}{p_{xt+1}}\hat{e}_{t+1})\right\}$$
$$+ \mathrm{A}\left\{\tilde{U}(\frac{L_t + (1-R_{t+1})Q_t}{P_{Xt+1}}) + \tilde{U}(\frac{L^*_t + (1-R^*_{t+1})Q^*_t}{p_{xt+1}}\tilde{e}_{t+1})\right\} + \mathrm{M}(\Lambda L_t - R_{t+1}Q_t) + \mathrm{M}^*(\lambda L^*_t - R^*_{t+1}Q^*_t).$$

We here assume that M and M*are Lagrangian multipliers for the liquidity constraints above.

The first-order conditions of this problem for L_t and L^*_t are

$$U_Y(\frac{1}{P_{Yt}}) = (1-\mathrm{A})\hat{U}_X\left\{\frac{R_{t+1}(1-\Lambda)+\Lambda}{P_{Xt+1}}\right\} + \mathrm{A}\tilde{U}_X(\frac{1}{P_{Xt+1}}) + \Lambda\mathrm{M}.$$

$$U_Y(\frac{1}{P_{Yt}}) = (1-\mathrm{A})\hat{U}_x\left\{\frac{R^*_{t+1}(1-\lambda)+\lambda}{p_{xt+1}}\hat{e}_{t+1}\right\} + \alpha\tilde{U}_x(\frac{1}{p_{xt+1}}\tilde{e}_{t+1}) + \lambda\mathrm{M}^*.$$

The first-order conditions for Q_t and Q^*_t are

$$\mathrm{A}\tilde{U}_X(\frac{1-R_{t+1}}{P_{Xt+1}}) = \mathrm{M}R_{t+1}.$$

$$\mathrm{A}\tilde{U}_X(\frac{1-R^*_{t+1}}{P_{xt+1}}\tilde{e}_{t+1}) = \mathrm{M}^*R^*_{t+1}.$$

Combining these equations yields the following first-order conditions for domestic loans and foreign loans:

$$U_Y(\frac{P_{Xt+1}}{P_{Yt}}) = (1-\mathrm{A})\hat{U}_X\{R_{t+1}(1-\Lambda)+\Lambda\} + \mathrm{A}\tilde{U}_X\left\{1+\Lambda(\frac{1}{R_{t+1}}-1)\right\}.$$

$$U_Y(\frac{p_{xt+1}}{P_{Yt}}) = (1-\mathrm{A})\hat{U}_X\{R^*_{t+1}(1-\lambda)+\lambda\}\hat{e}_{t+1} + \mathrm{A}\tilde{U}_x\left\{1+\lambda(\frac{1}{R^*_{t+1}}-1)\right\}\tilde{e}_{t+1}.$$

References

Chapman, James, and Antoine Martin, "Rediscounting under Aggregate Risk with Moral Hazard," *Journal of Money, Credit and Banking*, 2012 (forthcoming).

European Central Bank, "Correspondent Central Banking Model (CCBM) – Procedures for Eurosystem Counterparties," 2013.

Freeman, Scott, "The Payments System, Liquidity and Rediscounting," *American Economic Review*, 86 (5), 1996, pp. 1126–1138.

——, "Rediscounting under Aggregate Risk," *Journal of Monetary Economics*, 43 (1), 1999, pp. 197–216.

Fujiki, Hiroshi, "A Model of the Federal Reserve Act under the International Gold Standard System," *Journal of Monetary Economics*, 50 (6), 2003, pp. 1333–1350.

——, "Institutions of Foreign Exchange Settlement in a Two-Country Model," *Journal of Money, Credit and Banking*, 38 (3), 2006, pp. 697–719.

——, "Policy Measures to Alleviate Foreign Currency Liquidity Shortages under Aggregate Risk with Moral Hazard," *The Japanese Economic Review*, doi: 10.1111/jere.12015, 2013.

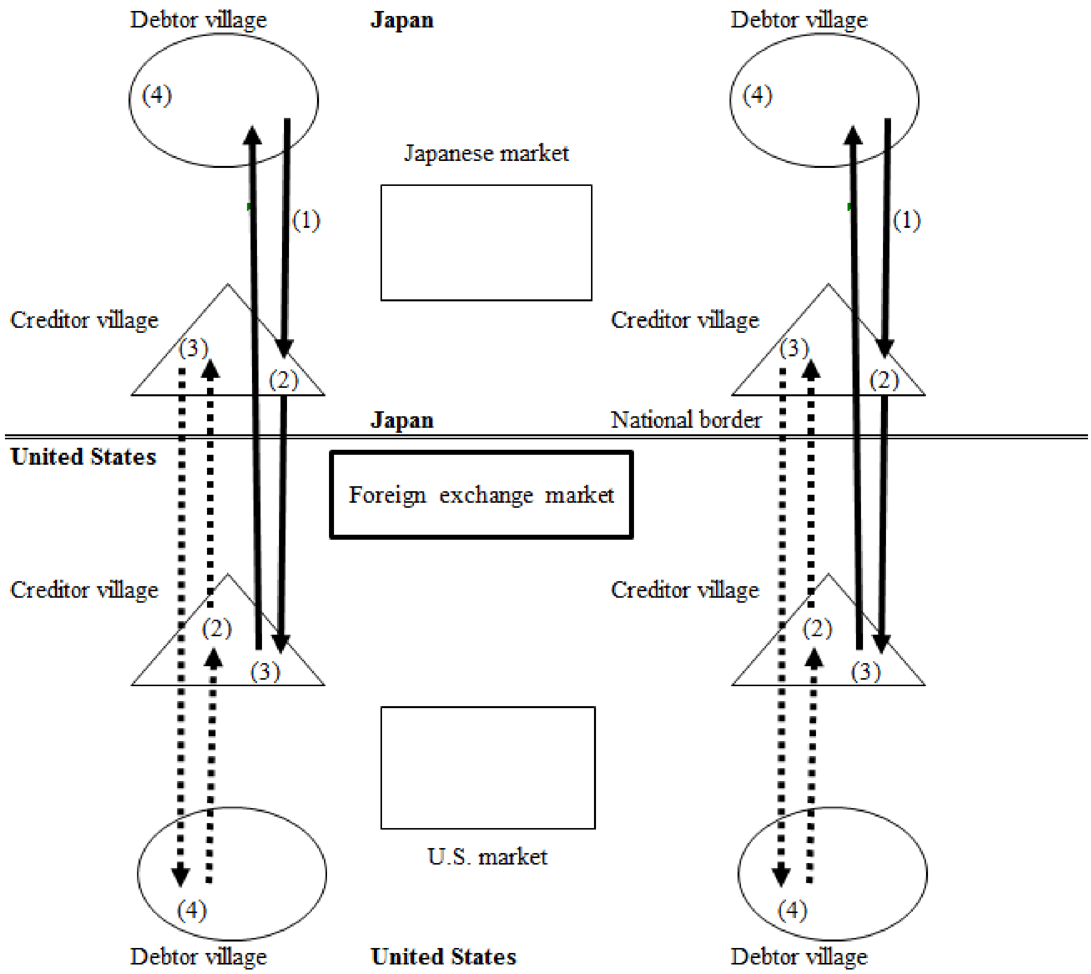
Goldberg, Linda S., Craig Kennedy, and Jason Miu, "Central Bank Dollar Swap Lines and Overseas Dollar Funding Costs," *Economic Policy Review*, Federal Reserve Bank of New York, 17 (1), 2011, pp. 3–20.

Keane, Frank M., "Securities Loans Collateralized by Cash: Reinvestment Risk, Run Risk, and Incentive Issues," *Current Issues in Economics and Finance*, Federal Reserve Bank of New York, 19 (3), 2013, pp.1–8.

Manning, Mark J., and Matthew Willison, "Modelling the Cross-Border Use of Collateral in Payment Systems," BOE Working Paper No. 286, Bank of England, 2006.

Mills, David C., Jr., "Alternative Central Bank Credit Policies for Liquidity Provision in a Model of Payments," *Journal of Monetary Economics*, 53 (7), 2006, pp. 1593–1611.

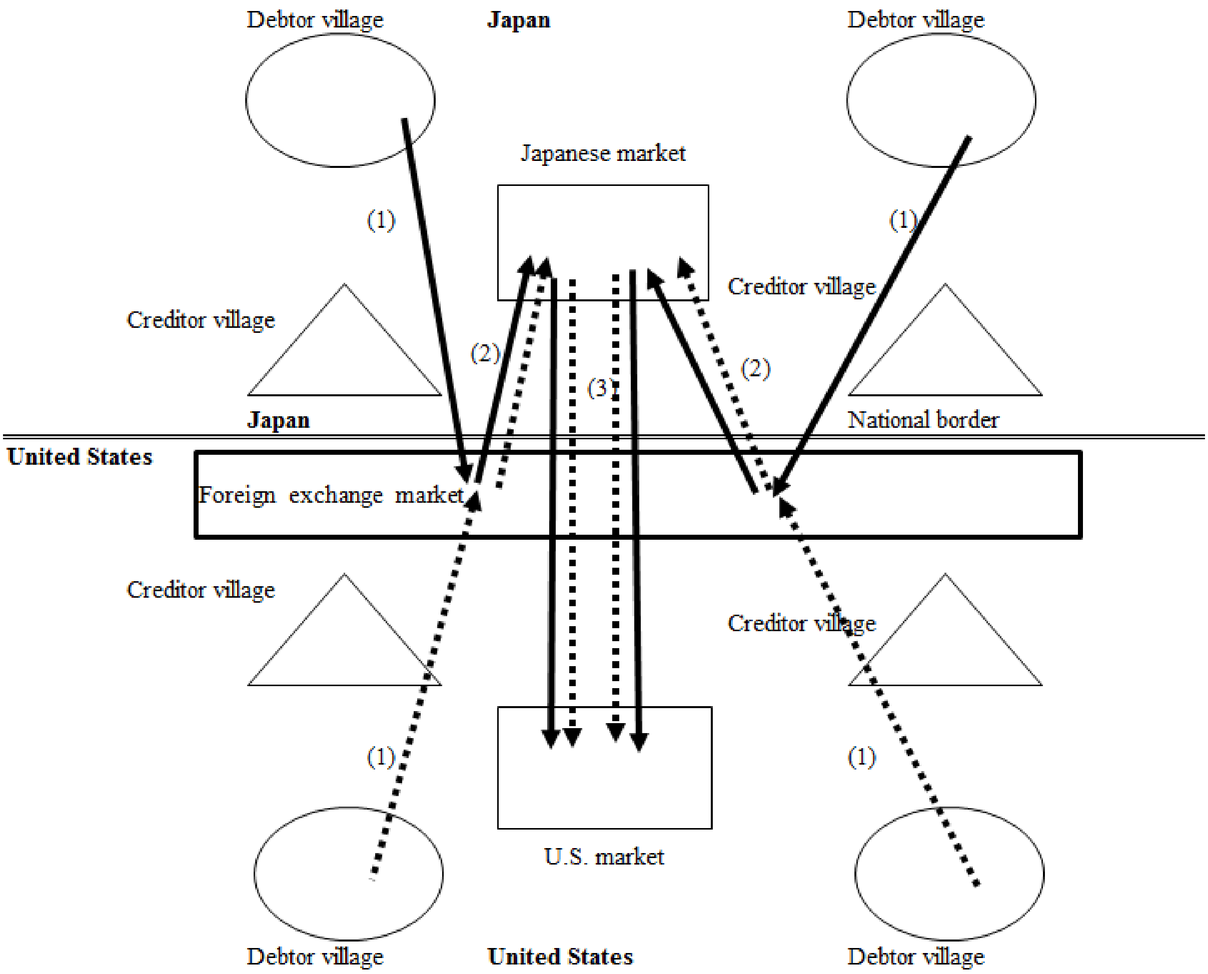
Figure 1 Pattern of debtor's travel when young



(1) Young debtor goes to the creditor village.
(2) Young debtor meets young creditor to obtain goods in exchange for debt.
(3) Young debtor returns to his/her village.
(4) Young debtor meets old creditor or defaulted old debtor to obtain currency in exchange for goods.

Note: The solid line shows the Japanese resident, the dotted line shows the U.S. resident, and the double solid line shows the national border.

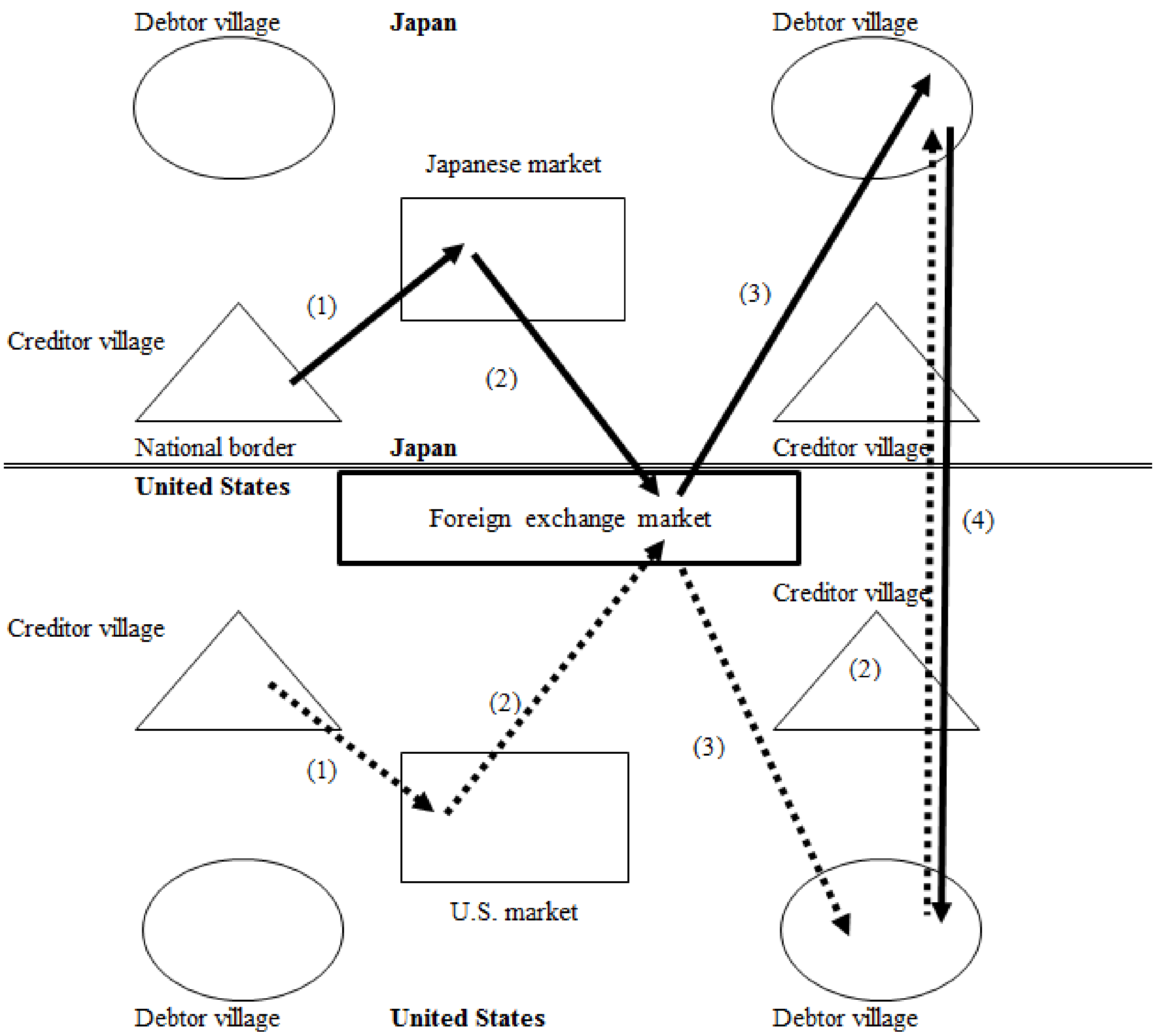
Figure 2 Pattern of debtor's travel when old



(1) Old debtor goes to the foreign exchange market and obtain foreign currency.
(2) Old debtor goes to the Japanese market.
(3) Old debtor goes to the U.S. market.

Note: The solid line shows the Japanese resident, the dotted line shows the U.S. resident, and the double solid line shows the national border.

Figure 3 Pattern of creditor's travel when old



(1) Old creditor goes to each economy's market, and settles their domestic loans.
(2) Old creditor goes to foreign exchange market to obtain foreign currency.
(3) Old creditor goes to each economy's debtor village to obtain goods for money.
(4) Old creditor goes to foreign debtor's village to obtain goods for money.

Note: The solid line shows the Japanese resident, the dotted line shows the U.S. resident, and the double solid line shows the national border.

Figure 4 Transactions in the foreign exchange market

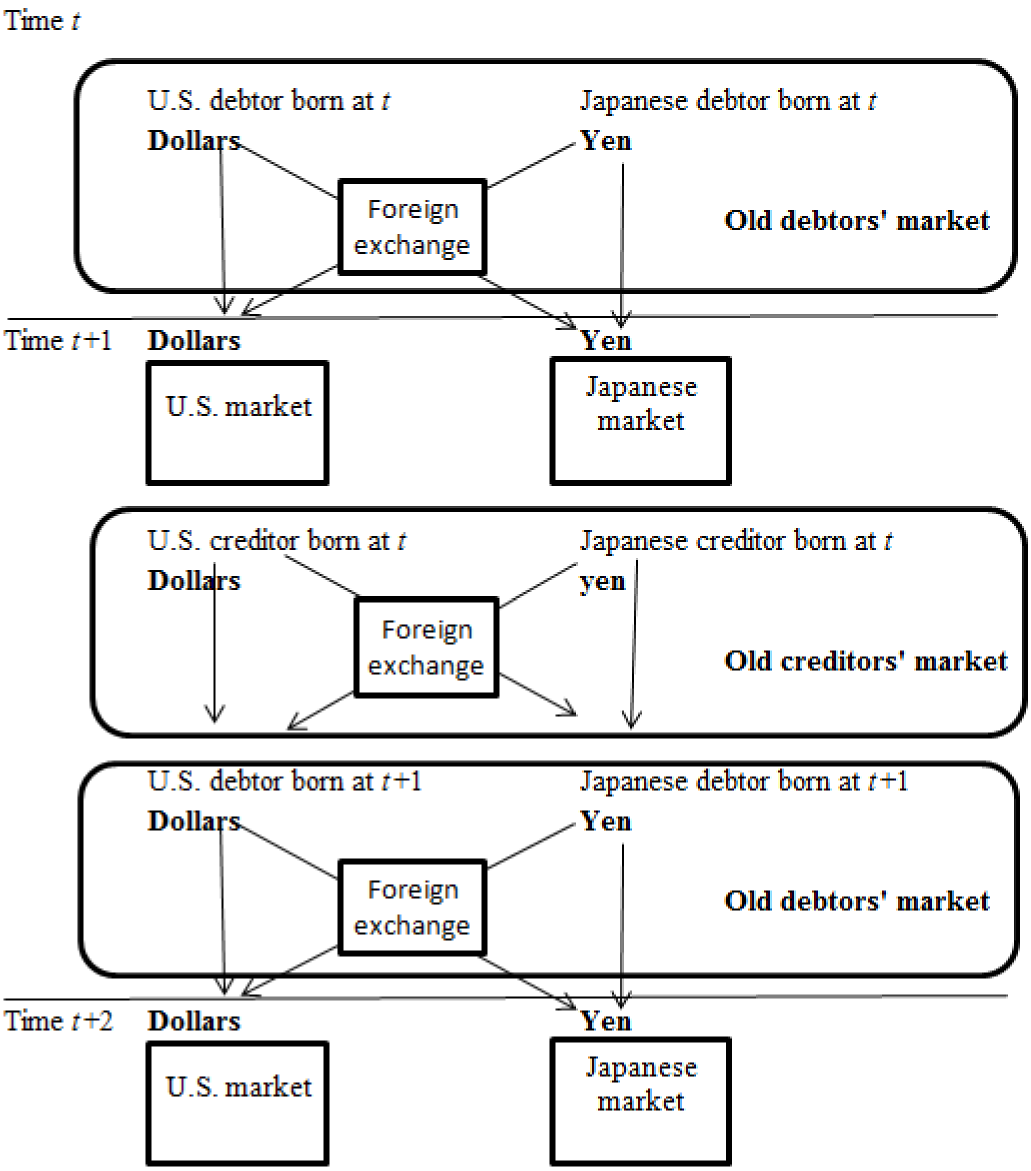


Figure 5 Changes in central bank balance sheets during foreign exchange market intervention

The Fed's balance sheet

The BoJ's balance sheet

Foreign exchange market; first wave

Assets	Liability
Dollars	yen
$+(\alpha - A)I^{*} p_x e_s$	$+(\alpha - A)I^{*} p_x$

Foreign exchange market; second wave

Assets	Liability
Dollars	yen
$-(\alpha - A)I^{*} P_x$	$(\alpha - A)I^{*} p_x e_s$

End of period

Assets	Liability
0	0

Figure 6 Pattern of settlement with cross-border cash collateral

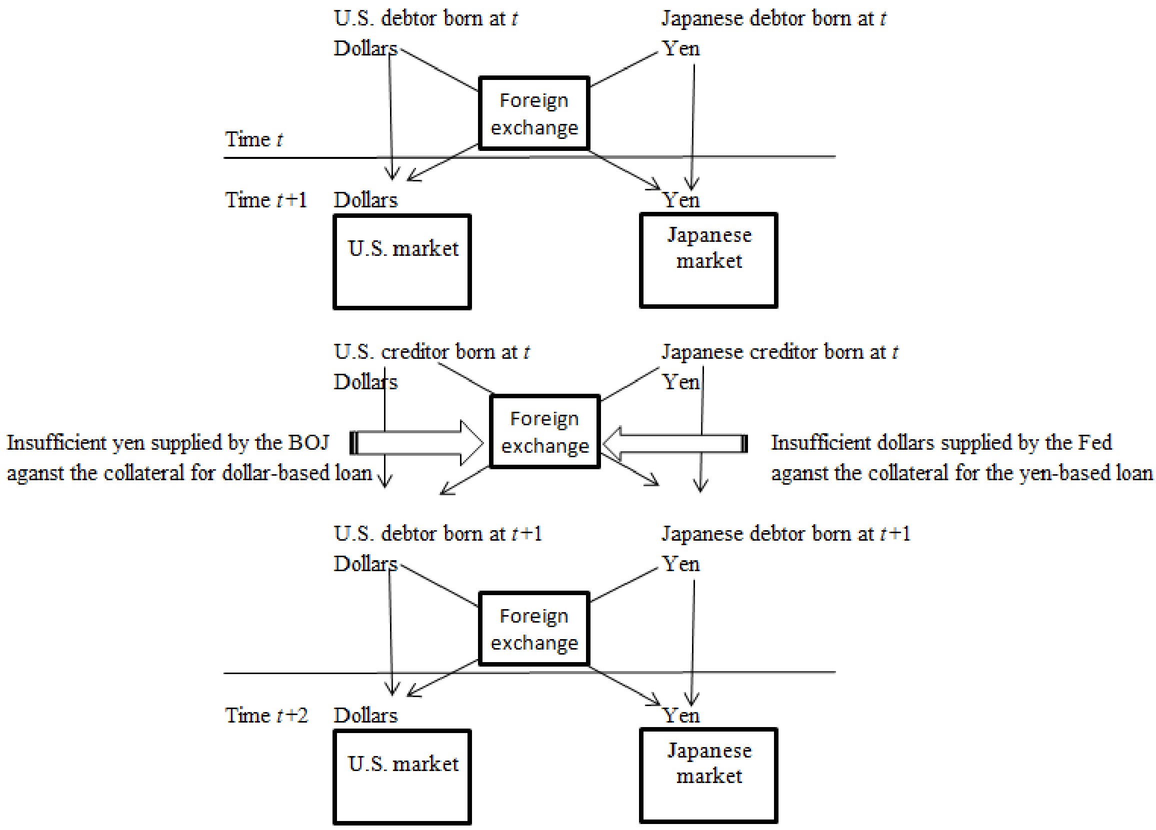


Figure 7 Changes in central bank balance sheets with cross-border cash collateral

The Fed's balance sheet

Foreign exchange market; second wave

Assets	Liability
Loan yen	(dollars)
$(\alpha - A)l^* p_x e_s$	$(\alpha - A)L^* P_x$

Foreign exchange market; settlement period

Assets	Liability
Loan yen	(dollars)
$(\alpha - A)l^* p_x e_s$	$(\alpha - A)L^* P_x$

The BoJ's balance sheet

Foreign exchange market; first wave

Assets	Liablity
Loan (dollars)	yen
$(\alpha - A)L^* P_x$	$(\alpha - A)l^* p_x$

Foreign exchange market; settlement period

Assets	Liability
Loan (dollars)	yen
$(\alpha - A)L^* P_x$	$(\alpha - A)l^* p_x$

Figure 8 Pattern of settlement with foreign currency liquidity swap lines