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Naohisa Hirakata* and Takeki Sunakawa**

Abstract

We develop a two-sector growth model with financial frictions to examine the effects of a decline in the working population ratio and change in the structure of household demand on sectoral TFP and structural change. Our findings are twofold. First, with financial frictions, a decline in labor input reduces the real interest rate and increases excess demand for borrowing, tightening collateral constraints at a given credit-to-value ratio and generating capital misallocation and lower sectoral TFP. Second, compared to the case with no financial frictions, such changes in sectoral TFP impede structural changes driven by the change in the structure of household demand.

Keywords: Financial frictions; heterogeneous firms; capital misallocation; total factor productivity; structural change
JEL classification: E23, E44, O41, O47

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1 Introduction

Total factor productivity (TFP) is one of the primary factors that determine a nation's per capita income and welfare. Not only measuring TFP but also grasping the reasons for differences in sectoral TFP are undoubtedly important issues in economics. Some recent studies link capital misallocation among firms to aggregate or sectoral TFP as one of the reasons behind such differences.[1] Other studies, such as Moll (2012), emphasize the role of financial frictions as a cause of capital misallocation. Furthermore, intra-sectoral capital misallocation and financial frictions may affect inter-sectoral capital misallocation (i.e., structural change), a shift in sectoral employment or output share taking place over a long period of time.[2]

Financial frictions and stuructural change may be affected by changes in the working population ratio. For example, Poterba (2001) discusses the effects of demographic change on financial markets and asset prices.[3] Changes in the working population ratio, on the other hand, affect the household demand structure. The effects of changes in the working population ratio on sectoral TFP would be important since many countries have experienced or will experience a decline in the working population ratio.

This paper analyzes the effects of a decline in the working population ratio on capital misallocation, sectoral TFP, and structural change in the presence of financial frictions. We develop a two-sector growth model with financial frictions and two household types: workers and retirees. We introduce two types of firms to analyze financial frictions: borrowers and savers. Borrowers have large financial needs, but their borrowing is limited by collateral constraints (c.f., Kiyotaki and Moore, 1997). Savers have less need for financing, and their borrowing is unconstrained. Our model has two sectors, new and old. The new sector is more credit-constrained (i.e., the new sector has more borrowers than the old sector). As the number of retirees increases—that is, as the working population ratio declines—demand for goods produced in the new sector increases. This is the driving force of structural change explored in the paper.

Our findings are twofold. First, in the presence of financial frictions, a decline in the working population ratio distorts the allocation of capital, which lowers TFP in both new and old sectors. The decline in the working population ratio lowers real interest rates, increasing borrowing demand. Increased demand for borrowing tightens the collateral constraint at

[1] For example, see Hsieh and Klenow (2009); Restuccia and Rogerson (2008). *Review of Economic Dynamics* edited a special issue in January 2013 on "Misallocation and Productivity" (Restuccia and Rogerson, 2013).

[2] For an excellent, extensive survey, see Matsuyama (2008). Buera, Kaboski, and Shin (2011) develop a two-sector model with financial friction and investigate the differential effects of financial frictions on capital misallocation and TFPs across sectors.

[3] See also Ikeda and Saito (2012).

a constrant credit-to-value ratio, since the value of collateral does not rise at a one-for-one pace. Borrowers have too little capital, whereas savers have too much capital, and borrowers produce fewer goods than they would do without financial frictions. Such capital misallocation lowers sectoral TFP in both sectors.

Second, the TFP in the new sector is more and disproportionately affected by financial frictions than the TFP in the old sector, impeding the structural change driven by changes in the structure of household demand. The number of borrowers in the new sector exceeds that in the old sector, which means the new sector has a greater need for financial resources and is more vulnerable to financial frictions than the old sector. Thus, capital misallocation in the new sector is more severe than in the old sector. Such a difference in the effect of financial frictions between the sectors makes TFP in the new sector lower than that in the old sector, impeding structural change. That is, it leads to too short a supply of goods produced in the new sector compared to the case without financial frictions.

Previous studies suggest various reasons for resource misallocation: regulations, taxes, labor market institutions and credit market imperfections. For example, Lagos (2006) examines the effects of labor market institutions on aggregate TFP. Melitz (2003) argues that trade reforms contribute to aggregate TFP increases through the inter-firm reallocations towards more productive firms. Schmitz (2001) demonstrates that low TFP stems from government policy supporting inefficient public enterprises. Numerous other recent studies on financial frictions and TFP include Aoki, Benigno, and Kiyotaki (2009), Moll (2012) and Buera, Kaboski, and Shin (2011), which show that financial frictions lead to resource misallocation and low TFP.

In a paper that discusses issues closely related to those explored in our own paper, Buera, Kaboski, and Shin (2011) develop a quantitative framework to explain the relationship between TFP, structural change measured by the ratio of employment or output in the service sector to that in the manufacturing sector, and financial development across countries. They analyze the effects of exogenous change in financial frictions on sectoral TFP (for manufacturing and services) to explain cross-country differences in TFP and structural change.[4] In contrast, the present paper focuses on tightening collateral constraints caused by a decline in labor input. We also focus on changes in the structure of household demand as a driving force for structural change, rather than traditional sector-biased technological progress or non-homothetic preferences (Kongsamut, Rebelo, and Xie, 2001; Ngai and Pissarides, 2007).

Unlike Buera, Kaboski, and Shin (2011), we build a prototype two-sector economy with wedges whose allocations and prices are equivalent to those in the original two sector econ-

[4] Arellano, Bai, and Kehoe (2012); Khan and Thomas (2011) studies the effect of exogenous changes of financial frictions on the measured TFP in heterogeneous agent models.

omy introduced above in order to analytically investigate the relationship among financial frictions, capital misallocation and structural change.[5] This method was originally proposed by Chari, Kehoe, and McGrattan (2007). Although our approach employs a simplified modeling of heterogeneity across firms, we obtain analytical results as well as numerical ones.[6]

We present the original two-sector economy with financial frictions and households' demand structure in the following section. Section 3 analyzes certain correspondences from financial frictions and demand structure to the wedges in the prototype economy. Section 4 presents numerical exercises using the prototype economy. Section 5 presents a conclusion. The details of the prototype economy with wedges and the proofs of propositions are presented in Appendix A.

2 A two-sector model with financial frictions and demand structure

We build a two-sector neoclassical growth model with financial frictions. On the firm side, there are two sectors, the old and the new. In each sector, there are a final-good producing firm and multiple intermediate-good producing firms. The final-good producing firm uses intermediate goods as inputs to produce final goods, and has a constant elasticity of substitution (CES) production function. We assume that financial needs are different among intermediate-good producing firms; a fraction of them have a lower discount factor. They are *borrowers*, who are bound by collateral constraints (Kiyotaki and Moore, 1997). The other firms with a higher discount factor are *savers*, who are unconstrained. Even though borrowers and savers have the same Cobb-Douglas production function, their levels of holding capital are different depending on their financial frictions. Therefore, the final-good producing firm with the CES function in each sector has lower sectoral TFP and hence output. We also assume that the number of borrowers is larger in the new sector, which leads to differences in sectoral TFP.

On the household side, there are workers and retirees. Workers supply labor inelastically and consume final goods in each sector, whereas retirees only consume final goods in each sector. Workers and retirees have different preferences over final goods produced. When the number of workers declines, there are two effects; one is a decline in labor input, and the other is a change in the structure of household demand.

[5]The prototype two-sector economy with wedges is used in other papers, such as Hayashi and Prescott (2008), Esteban-Pretel and Sawada (2009), and Cheremukhin, Golosov, Guriev, and Tsyvinski (2013).

[6]See also Buera, Kaboski, and Shin (2011); Midrigan and Xu (2012); Moll (2012).

2.1 Firms

There are two sectors, the old and the new. Let 1 denote the new sector, and 2 denote the old. The final-good producing firm in each sector $i = \{1, 2\}$ minimizes its expenditure:

$$p_t^i y_t^i = \int_0^1 p_{jt}^i y_{jt}^i dj, \tag{1}$$

subject to

$$y_t^i = \left[\int_0^1 (y_{jt}^i)^{\varrho} dj \right]^{1/\varrho}, \tag{2}$$

The final-good producing firm in sector i purchases $p_{jt}^i y_{jt}^i$ of the intermediate good $j \in [0, 1]$ and sells $p_t^i y_t^i$ of the final good to households, where p_{jt}^i and y_{jt}^i are the price and output of intermediate goods, and p_t^i and y_t^i are the price and output of final goods. The firm produces final goods by a CES production function (2), where $\varrho \in (0, 1)$ is the degree of substitution between intermediate goods. The first-order condition (FOC) is given by

$$y_{jt}^i = (p_{jt}^i / p_t^i)^{1/(\varrho - 1)} y_t^i. \tag{3}$$

This is the demand function of each intermediate good. The intermediate-good producing firm $j \in [0, 1]$ maximizes its discounted sum of future profits:

$$\sum_{t=0}^{\infty} \beta_j^t \left(\frac{\lambda_t}{\lambda_0} \right)^{-1} \left\{ p_{jt}^i y_{jt}^i - w_t n_{jt}^i - p_t^2 (k_{jt+1}^i - (1 - \delta) k_{jt}^i) - q_t b_{jt+1}^i + b_{jt}^i \right\},$$

subject to

$$y_{jt}^i = (k_{jt}^i)^{\alpha} (n_{jt}^i)^{1-\alpha}, \tag{4}$$

$$y_{jt}^i = (p_{jt}^i / p_t^i)^{1/(\varrho - 1)} y_t^i,$$

$$-b_{jt+1}^i \leq \theta p_t^2 k_{jt}^i, \tag{5}$$

Each intermediate-good producing firm j in sector i sells $p_{jt}^i y_{jt}^i$ of intermediate goods to the final-good producing firm in the same sector, pays wage bill $w_t n_{jt}^i$ to workers, and purchases capital goods produced in sector 2. w_t is the real wage, and we assume that labor is freely mobile between firms and sectors. $\beta_j^t (\lambda_t / \lambda_0)^{-1} = \prod_{s=1}^{t} \beta (\lambda_s / \lambda_{s-1})^{-1}$ is the cumulative stochastic discount factor, where $\beta_j \in (0, 1)$ and λ_t^{-1} is the Lagrange multiplier on the household's budget constraint. Each firm produces using a Cobb-Douglas production function (4) with capital share $\alpha \in (0, 1)$ subject to the demand function (3). Each firm also borrows or lends b_{jt+1}^i bonds priced at the risk-free bond price q_t, and faces a collateral constraint (5) with a parameter for the credit-to-value ratio $\theta \geq 0$. Let μ_{jt}^i be the Lagrange multiplier on the collateral constraint. The FOCs are

$$\partial k^i_{jt+1}: \quad p^2_t \lambda_{t+1} = \beta_j \lambda_t \left\{ \varrho \alpha p^i_{jt+1} y^i_{jt+1} / k^i_{jt+1} + p^2_{t+1}(1 - \delta + \theta \mu^i_{jt+1}) \right\}, \tag{6}$$

$$\partial n^i_{jt}: \quad w_t = \varrho(1-\alpha) p^i_{jt} y^i_{jt} / n^i_{jt}, \tag{7}$$

$$\partial b^i_{jt+1}: \quad (q_t - \mu^i_{jt}) \lambda_{t+1} = \beta_j \lambda_t. \tag{8}$$

The complementary slackness conditions are

$$\mu^i_{jt}(b^i_{jt+1} + \theta p^2_t k^i_{jt}) = 0,$$
$$\mu^i_{jt} \geq 0.$$

Note that firms have different discount factors β_j, which determine their financial needs and bond positions. We assume that there are only two types of intermediate-good firms, *borrowers* and *savers*, denoted by $j = \{b, s\}$. We further assume that

Assumption 1. *$\beta_b < \beta_s = \beta$ so that only the borrowers' collateral constraint binds.*

From Assumption 1, we immediately obtain $b^i_{bt+1} = -\theta p^2_t k^i_{bt} < 0 < b^i_{st+1}$, $\mu^i_{bt} > 0$ and $\mu^i_{st} = 0$. Note that different financial needs and discount factors are the only source of heterogeneity among firms, and as long as the borrowers' discount factor β_b is common among sectors, $\mu^i_{bt} \equiv \mu_t > 0$ does not depend on i.

Sectoral output y^i_t, capital k^i_t and labor n^i_t are given by

$$y^i_t = \left[\chi^i (y^i_{bt})^\varrho + (1-\chi^i)(y^i_{st})^\varrho \right]^{1/\varrho}, \tag{9}$$

$$k^i_t = \chi^i k^i_{bt} + (1-\chi^i) k^i_{st}, \tag{10}$$

$$n^i_t = \chi^i n^i_{bt} + (1-\chi^i) n^i_{st}, \tag{11}$$

where $\chi^i \in (0,1)$ is the ratio of borrowers in each sector. We assume that

Assumption 2. *$\chi^1 > \chi^2$ so that sector 1 is the more constrained sector.*

The TFP in each sector is defined as

$$z^i_t \equiv \frac{y^i_t}{(k^i_t)^\alpha (n^i_t)^{1-\alpha}}. \tag{12}$$

From Assumption 2, the more intermediate-good producing firms that are constrained, the more allocation between the two types of firms is distorted; therefore, the sectoral TFP in the more constrained sector is lower; $z^1_t < z^2_t$ holds.[7]

[7]See also Section 3.

2.2 Households

There are two types of households, workers and retirees, denoted by $k = \{w, r\}$. They have different preferences over different final goods produced in each sector. Each household k minimizes its expenditure:

$$\begin{aligned} & p_t^k c_t^k = p_t^1 c_t^{1k} + p_t^2 c_t^{2k}, \\ & \text{subject to} \\ & c_t^k = \left[(\mu^k)^{1-\rho} (c_t^{1k})^\rho + (1-\mu^k)^{1-\rho} (c_t^{2k})^\rho \right]^{1/\rho}, \end{aligned} \tag{13}$$

where p_t^k is the price of composition goods c_t^k consumed by household k. Composition goods are produced by a CES function (13) of c_t^{1k} and c_t^{2k}, which are the consumption of each sector's goods by household k. The function has parameters for the degree of substitution $\rho \in (0,1)$ and the sector-bias effect $\mu^k \in (0,1)$ for each type of household k. The FOCs are

$$c_t^{1k} = \mu^k (p_t^1 / p_t^k)^{1/(\rho-1)} c_t^k, \tag{14}$$

$$c_t^{2k} = (1-\mu^k)(p_t^2 / p_t^k)^{1/(\rho-1)} c_t^k. \tag{15}$$

The sector-bias parameters μ^w and μ^r determine each household's demand for goods produced in sector 1. We assume that

Assumption 3. $\mu^w < \mu^r$ *so that retirees demand for goods produced in sector 1 more than workers do.*

From Assumption 3, the decline in labor input implies a shift in the structure of household demand from the old sector to the new sector. An increase in the number of retirees leads to an increase in demand for goods produced in the new and more constrained sector (sector 1).

We assume that there is a utilitarian who insures risk that each household faces. The utilitarian chooses the allocation of composition goods to consume c_t^w and c_t^r, and the amount of the risk-free bond B_t. Given the prices of composition goods p_t^w and p_t^r, and the risk-free bond price q_t, the utilitarian maximizes the joint life-time utility of workers and retirees

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t \left\{ n_t^w \log c_t^w + (1-n_t^w) \log c_t^r \right\}, \\ & \text{subject to} \\ & n_t^w p_t^w c_t^w + (1-n_t^w) p_t^r c_t^r + q_t B_{t+1} \le \omega_t n_t^w + B_t + \pi_t, \end{aligned}$$

where $n_t^w = n_t^1 + n_t^2$ is the total number of workers and π_t is the sum of transfers from firms.

Let λ_t^{-1} be the Lagrange multiplier. The FOCs are

$$\partial c_t^w: \quad \lambda_t = p_t^w c_t^w, \tag{16}$$

$$\partial c_t^r: \quad \lambda_t = p_t^r c_t^r, \tag{17}$$

$$\partial B_{t+1}: \quad q_t \lambda_t^{-1} = \beta \lambda_{t+1}^{-1}. \tag{18}$$

Note that a decline in the number of workers n_t^w has two effects; (i) a decline in labor input and the wage bill, and (ii) an increase in the number of retirees and demand for goods produced in sector 1.

2.3 Market-clearing conditions

The good market in each sector, the capital market, labor market and bond market all clear:

$$c_t^1 = n_t^w c_t^{1w} + (1 - n_t^w) c_t^{1r}, \tag{19}$$

$$c_t^2 = n_t^w c_t^{2w} + (1 - n_t^w) c_t^{2r}, \tag{20}$$

$$c_t^1 = y_t^1, \tag{21}$$

$$c_t^2 = (1 - \psi_t) y_t^2 + (1 - \delta) k_t - k_{t+1}, \tag{22}$$

$$k_t = k_t^1 + k_t^2, \tag{23}$$

$$n_t^w = n_t^1 + n_t^2, \tag{24}$$

$$\sum_i \left[\chi^i b_{b,t+1}^i + (1 - \chi^i) b_{s,t+1}^i \right] + B_{t+1} = 0. \tag{25}$$

c_t^1 and c_t^2 are the total amounts of consumption of each sectoral good. goods produced in sector 1 are only consumed, whereas goods produced in sector 2 are also used for each firm's investment and the government expenditure. $\psi_t = g_t / y_t^2$ is the ratio of government expenditure to output in sector 2. There is an integrated bond market to which all firms and households have access, with a unique market-clearing bond price q_t.

We denote the price of goods produced in sector 1 $p_t^1 = p_t$ and normalize $p_t^2 = 1$ hereafter. p_t is the relative price, i.e., the ratio of the price of goods produced in sector 1 to those in sector 2. A competitive equilibrium is defined as the set of prices and allocations satisfying the relevant equations.

3 Analysis

In this section, we present on analysis using the detailed model presented in the previous section. In neoclassical growth models, a decline in labor input leads to temporally lower real interest rates, as the capital-labor ratio rises. We analytically show that such a drop in real interest rates, i.e., higher risk-free bond prices, in turn leads to tighter collateral constraints and capital misallocation among firms, which is also linked to sectoral TFP. Regarding structural change, we present a prototype two-sector model with wedges (Chari, Kehoe, and McGrattan, 2007) whose allocations and prices are equivalent to the detailed two-sector model with financial frictions and structure of households demand. By using this equivalence, we derive the demand and supply curves in terms of the relative price and output.

3.1 Capital misallocation and sectoral TFPs

There is a relationship between the tightness of collateral constraints and capital misallocation among firms. In Proposition 1, we analytically show that, given the relative price, a tighter collateral constraint leads to capital misallocation. Also, capital misallocation leads to lower sectoral TFP.

Note that the FOC of the risk-free bond held by the utilitarian (18) determines the risk-free bond price q_t. Combining it with the FOC of borrowers (8), we have

$$\mu_t = (1 - \beta_b/\beta)q_t. \tag{26}$$

From Assumption 1, $\beta_b < \beta$ and $\mu_t > 0$ holds; only borrowers' collateral constraint binds. There is a one-to-one relationship between the risk-free bond price and the tightness of collateral constraints; that is, the higher risk-free bond price is, the tighter collateral constraint is. As the risk-free bond price increases, investment returns are relatively higher than bond returns, and borrowers have more incentive to borrow, but the collateral constraint prevents them from doing so; therefore, the collateral constraint becomes tighter.[8]

The risk-free bond price, or the tightness of collateral constraints is linked to misallocation among borrowers and savers. We show that

Lemma 1. *(i) The ratio of marginal revenue product of capital (MRPK) among borrowers and savers is given by*

$$\begin{aligned} x_{t+1} &\equiv \frac{p^i_{bt+1} y^i_{bt+1} / k^i_{bt+1}}{p^i_{st+1} y^i_{st+1} / k^i_{st+1}}, \\ &= \frac{\beta}{\beta_b} \frac{1 - (\beta_b/\beta) q_t [1 - \delta + \theta(1 - \beta_b/\beta) q_{t+1}]}{1 - q_t(1 - \delta)}. \end{aligned} \tag{27}$$

[8]Note that borrowers want to borrow infinite amount as their discount factor is less than savers' discount factor, the risk-free bond price in steady state, i.e., $\beta_b < \beta = q$.

(ii) If the credit-to-value ratio is smaller than the threshold,

$$\theta < \bar{\theta}_{t+1} = \frac{\beta}{\beta_b} \frac{1}{q_t q_{t+1}},$$

then $x_{t+1} > 1$ holds.

Proof. See Appendix B.1. $\square$

If $x_{t+1} = 1$, MRPKs of borrowers and savers are equalized, and capital is efficiently allocated among borrowers and savers. If $x_{t+1} > 1$, MRPK of borrowers is greater than that of savers, which implies capital misallocation among borrowers and savers. We assume θ is far enough below $\bar{\theta}_{t+1}$ so that $x_{t+1} > 1$ holds. As we assume that the borrowers' discount factor β_b is common among sectors, and it is the only source of heterogeneity among firms, the ratio of MRPK is also common among sectors.

Also, capital misallocation among firms is linked to sectoral TFP. We show the following proposition

Proposition 1. *(i) The sectoral TFP z^i_{t+1} is a function of the ratio of MRPK x_{t+1} and the ratio of borrowers χ^i,*

$$z^i_{t+1} = \left[\frac{\left(1 - \chi^i + \chi^i x^{\gamma}_{t+1}\right)^{1-\nu}}{\left(1 - \chi^i + \chi^i x^{\gamma-1}_{t+1}\right)^{\tilde{\alpha}}} \right]^{1/\varrho}, \tag{28}$$

where $\tilde{\alpha} = \varrho\alpha$, $\nu = \varrho(1-\alpha)$ and $\gamma = \tilde{\alpha}/(\tilde{\alpha} + \nu - 1) < 1$.

(ii) $\frac{\partial x_{t+1}}{\partial \mu_t} \frac{\mu_t}{x_{t+1}} > 0$ and $\frac{\partial z^i_{t+1}}{\partial x_{t+1}} \frac{x_{t+1}}{z^i_{t+1}} < 0$ for $i = \{1, 2\}$ hold.

(iii) If $\chi^1 > \chi^2$, then $\left| \frac{\partial z^1_{t+1}}{\partial x_{t+1}} \frac{x_{t+1}}{z^1_{t+1}} \right| > \left| \frac{\partial z^2_{t+1}}{\partial x_{t+1}} \frac{x_{t+1}}{z^2_{t+1}} \right|$ holds.

Proof. See Appendix B.2. $\square$

Note that if $x_t = 1$, then $z^i_t = 1$ holds; if resources are efficiently allocated among firms, the sectoral TFP is constant. Otherwise, the sectoral TFP drops. An increase in the bond price leads to a tighter collateral constraint. As the collateral constraint is tighter, borrowers cannot produce a sufficient amount of their intermediate goods and sell them to the final-good producing firm; borrowers have too little capital, whereas savers have too much capital. Such a capital misallocation hurts the efficiency of final goods production and lowers sectoral TFP. Also, the sectoral TFP in the more constrained sector is dampened more by capital misallocation among firms.

3.2 Structural change

In the previous subsection and Proposition 1, we showed that the sectoral TFP (we also call it the efficiency wedge hereafter, and these two terms are used interchangeably) z_t^i is a function of the MRPK ratio, x_t, which measures the degree of capital misallocation. In this subsection, we will show that the relative price p_t, which is defined by the relative efficiency of each sector, is also a function of x_t. The relative price of goods produced in the new sector increases as the new sector is more constrained than the old sector. Such a change impedes the structural change.

Structural change is measured by the relative price and output. To derive the relative demand and supply curves of the relative price and output, we write down the prototype two-sector economy with wedges, which is described in details in Appendix A. We show the following "equivalence result" (Chari, Kehoe, and McGrattan, 2007):

Lemma 2. *In the prototype model with wedges, allocations and prices are equivalent to those in the detailed model, if and only if the wedges* $\{z_{t+1}^i, (1+\tau_{kt+1}^i)^{-1}, \varphi_t\}$ *satisfy*

$$\begin{aligned} z_{t+1}^i &= \left[\frac{\left(1-\chi^i+\chi^i x_{t+1}^{\gamma}\right)^{1-\nu}}{\left(1-\chi^i+\chi^i x_{t+1}^{\gamma-1}\right)^{\tilde{\alpha}}}\right]^{\frac{1}{\varrho}}, \\ (1+\tau_{kt+1}^i)^{-1} &= \varrho\left[\frac{1-\chi^i+\chi^i x_{t+1}^{\gamma-1}}{1-\chi^i+\chi^i x_{t+1}^{\gamma}}\right], \\ \varphi_t &= n_t^w\left[1+(1/\mu^w-1)(p_t)^{\rho/(1-\rho)}\right]^{-1} \\ &\quad +(1-n_t^w)\left[1+(1/\mu^r-1)(p_t)^{\rho/(1-\rho)}\right]^{-1}. \end{aligned} \tag{29}$$

Proof. See Appendix B.3. □

Two things are worth noting here. Firstly, the efficiency and capital wedges z_{t+1}^i and $(1+\tau_{kt+1}^i)^{-1}$ depend on the MRPK ratio x_{t+1} and the ratio of borrowers χ^i; these wedges represent financial frictions and capital misallocation. If allocation among the sectors is efficient, $x_t = 1$, $z_t = 1$ and $(1+\tau_{kt}^i)^{-1} = \varrho$ hold, which is the inverse of a gross markup stemming from monopolistic competition of intermediate-good producing firms in the original detailed economy. Secondly, the preference wedge φ_t is a utility-based weight on consumption in sector 1 in the prototype economy (see also equation (32)), and depends on the demand structure of households, i.e., the number of workers n_t^w, the sector-bias parameters μ^k, and the relative price p_t. As the number of workers declines and $\mu^w < \mu^r$, φ_t increases and a change in demand structure from the old sector (sector 2) to the new sector (sector 1) occurs.

In the prototype economy with wedges, we can easily derive the relative supply curve

(Buera and Kaboski, 2009). From firms' profit maximization,

$$p_t = \frac{[(1+\tau_{kt}^1)r_t]^\alpha \omega_t^{1-\alpha}}{z_t^1 \alpha^\alpha (1-\alpha)^{1-\alpha}},$$
$$1 = \frac{[(1+\tau_{kt}^2)r_t]^\alpha \omega_t^{1-\alpha}}{z_t^2 \alpha^\alpha (1-\alpha)^{1-\alpha}},$$

Then we have

$$p_t = \frac{z_t^2}{z_t^1}\left(\frac{1+\tau_{kt}^1}{1+\tau_{kt}^2}\right)^\alpha. \tag{30}$$

Note that the supply curve is horizontal, as the production function has constant returns to scale. The relative price is a function of the wedges z_t^i and $(1+\tau_{kt}^i)^{-1}$, which in turn depend on the MRPK ratio x_t and the ratio of borrowers χ^i. Next, we show the following proposition:

Proposition 2. *(i) The relative price p_t is a function of the ratio of MRPK x_t and the ratio of borrowers χ^i,*

$$p_t = \left[\frac{1-\chi^1+\chi^1 x_t^\gamma}{1-\chi^2+\chi^2 x_t^\gamma}\right]^{\frac{\varrho-1}{\varrho}}. \tag{31}$$

(ii) If $\chi_1 > \chi_2$ and $x_t > 1$, then $p_t > 1$ holds.
(iii) If $\chi_1 > \chi_2$, then $\frac{\partial p_t}{\partial x_t}\frac{x_t}{p_t} > 0$ holds.

Proof. See Appendix B.4. □

The price of goods in the new sector (sector 1), which is more constrained, becomes higher than that in the old sector (sector 2), as its sectoral TFP is more dampened and there is too little supply in the new sector.

We can also derive the relative demand curve. From the representative household's utility maximization,

$$p_t c_t^1/\lambda_t = \varphi_t,$$
$$c_t^2/\lambda_t = (1-\varphi_t).$$

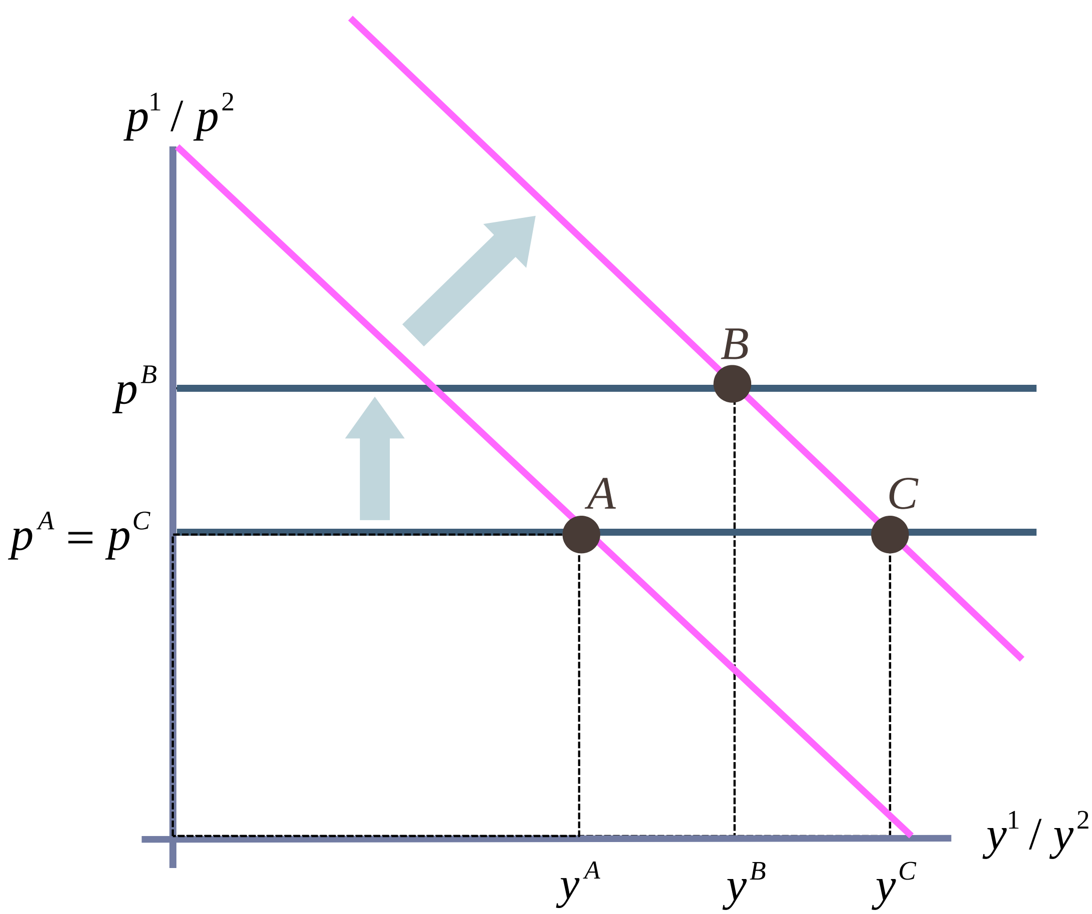
Then we have

$$p_t = \frac{1-\varphi_t}{\varphi_t}\left(\frac{c_t^1}{c_t^2}\right)^{-1}. \tag{32}$$

Note that the demand curve has a 45-degree downward slope, as the elasticity of substitution in the household's utility function is one.

Figure 1 graphically shows the relative demand and supply curves, (30) and (32). Let us suppose there is a decline in labor input. The demand curve shifts upward. As the number of workers declines and the number of retirees increases, there is a change in households' demand structure from the old sector (sector 2) to the new sector (sector 1). Also, with financial frictions, the decline in labor input leads to endogenously strengthened collateral constraints and capital misallocation, which lowers sectoral TFP (efficiency wedges), as shown in Proposition 1. In the more constrained sector, the efficiency and capital wedges decrease more, the relative price increases and the supply curve shifts upward, as shown in Proposition 2. In sum, after a decline in labor input, the equilibrium shifts from A to B, with financial frictions. On the other hand, in the case with no financial frictions, the supply curve is unchanged and only the demand curve shifts upward. Equilibrium shifts from A to C. Comparing the two equilibria, equilibrium B with financial frictions and equilibrium C with no financial frictions, the relative price is higher and the relative output is lower in B than C. That is, financial frictions and capital misallocation impede the structural change driven by the change in households' demand structure.

Figure 1: Equivalent problem.



4 Numerical exercise

In this section, we numerically demonstrate the analytical results we have shown in the previous section with calibrated parameters.

4.1 Calibration

The capital share $\alpha = 0.362$, discount factor for savers and households $\beta = 0.96$, and depreciation rate $\delta = 0.089$, are set following Hayashi and Prescott (2002), which calibrate a one-sector neoclassical growth model to the 1980s Japanese economy. These values are also very close to the ones in the U.S. economy. The elasticity of final goods ρ is set to 1/2, a conventional value. The credit-to-value ratio is 70%, $\theta = 0.7$, which is consistent with the literature, such as Iacoviello (2005).

The other parameters are calibrated so as to match the model moments to the data moments. The elasticity of intermediate goods is set to $\varrho = 3/4$, which implies the steady state markup $\tau_k = 1/\varrho - 1 = 1/3$.

(β_b, χ^1) are calibrated to match the ratio of the output-labor ratio and the output-capital ratio. We set $(\beta_b, \chi^1) = (0.6, 0.75)$ so that $(y^2/n^2)/(y^1/n^1) = p = 1.24$ and $(y^2/k^2)/(y^1/k^1) = p[(1+\tau_k^2)/(1+\tau_k^1)] = 0.91$, which implies $x = 3.85$.[9] Note that $y^2/n^2 > y^1/n^1$ and $y^1/k^1 > y^2/k^2$ hold. Labor cost is lower in sector 1, whereas capital return is higher in sector 1. The bias for sector 1 goods in each household's preference is $(\mu^w, \mu^r) = (0.75, 0.95)$ so as to match sectoral output and consumption shares. Specifically, the nominal consumption share of sector 1 is $\varphi = c^1/(pc^1 + c^2) = 0.78$, and the nominal output (or employment) share of sector 1 is $\varphi^y = y^1/(py^1 + y^2) = 0.58$, which is largely consistent with the Japanese data after the 1980s. Parameters are summarized in Table 1.

4.2 Steady state

Table 2 summarizes the steady state values in both cases, with and without financial frictions.[10] In the case without financial frictions, i.e., $\theta = \bar{\theta} = 1/(\beta\beta_b)$, capital allocation among firms is efficient, the tightness of collateral constraint is $\mu = 0$ and the ratio of MRPK is $x = 1$. The efficiency wedges are $z^1 = z^2 = 1$, the capital wedges are $(1+\tau_k^1)^{-1} = (1+\tau_k^2)^{-1} = \varrho = 3/4$, and the relative price is $p = 1$. While the efficiency and capital wedges and related variables are independent of the size and structure of the economy, some aggregate variables and sectoral composition depend on the size and demand

[9] We have only two targets (p, x), whereas we have three free parameters $(\beta_b, \chi^1, \chi^2)$. We set $\chi^2 = 1 - \chi^1$. We loosely set the targets for demonstration purposes.

[10] The steady state values are analytically obtained in Appendix A.3.

Table 1: Parameter values.

	description	value	source or target
parameters cited from other papers			
β	discount factor	.96	Hayashi and Prescott (2002)
δ	depreciation rate	.089	Hayashi and Prescott (2002)
α	capital share	.362	Hayashi and Prescott (2002)
ρ	elasticity of final goods	.50	conventional value
θ	credit-to-value ratio	.7	Iacoviello (2005)
parameters calibrated to data			
ϱ	elasticity of intermediate goods	.75	markup: $\tau_k = 1/\varrho - 1 = 0.33$
β_b	discount factor for borrowers	.6	MRPK ratio: $x = 3.85$
χ^1	ratio of borrowers in sector 1	.75	relative price: $p = 1.24$
χ^2	ratio of borrowers in sector 2	.25	
μ^w	workers bias for sector-1 goods	.75	consumption share
μ^r	retirees bias for sector-1 goods	.95	(preference wedge): $\varphi = 0.78$

structure of households. The aggregate capital decreases as the working population ratio and labor input declines. At the same time, a demand shift from sector 2 to sector 1 occurs, and the preference wedge (equivalent to consumption share, $c^1/(pc^1 + c^2)$) and relative output both increase.

In the case with financial frictions, as $\theta < \bar{\theta}$ holds in our calibration, only the borrowers are constrained and there is capital misallocation among firms, resulting in a tightness of collateral constraints of $\mu = \beta - \beta_b = 0.36$ and a ratio of MRPK of $x = 3.85$. As a consequence, the sectoral TFP (efficiency wedges) is smaller than one, $z^1 = 0.86$ and $z^2 = 0.95$ respectively. Note that $z^1 < z^2$ as we assume $\chi^1 > \chi^2$ (see Proposition 1). Similarly, $(1+\tau_k^1)^{-1} = 0.52$, $(1+\tau_k^2)^{-1} = 0.71$ and $p = 1.24$, showing capital allocation between sectors is inefficient compared to the case without financial frictions. The values of the aggregate capital, relative output and preference wedge also decrease with financial frictions and capital misallocation.

4.3 Transition dynamics

We assume that the number of workers declines by 10% from 0.7 to 0.63 in period 1. We perform a perfect foresight simulation[11] to see the transition dynamics as the labor input permanently declines and the economy migrates to the new steady state.

Figure 2 shows the transition dynamics within each sector. When the number of workers drops in period 1, the collateral constraint is tighter as borrowers want to borrow more in

[11] We use Dynare developed by Cepremap.

Table 2: Steady state values.

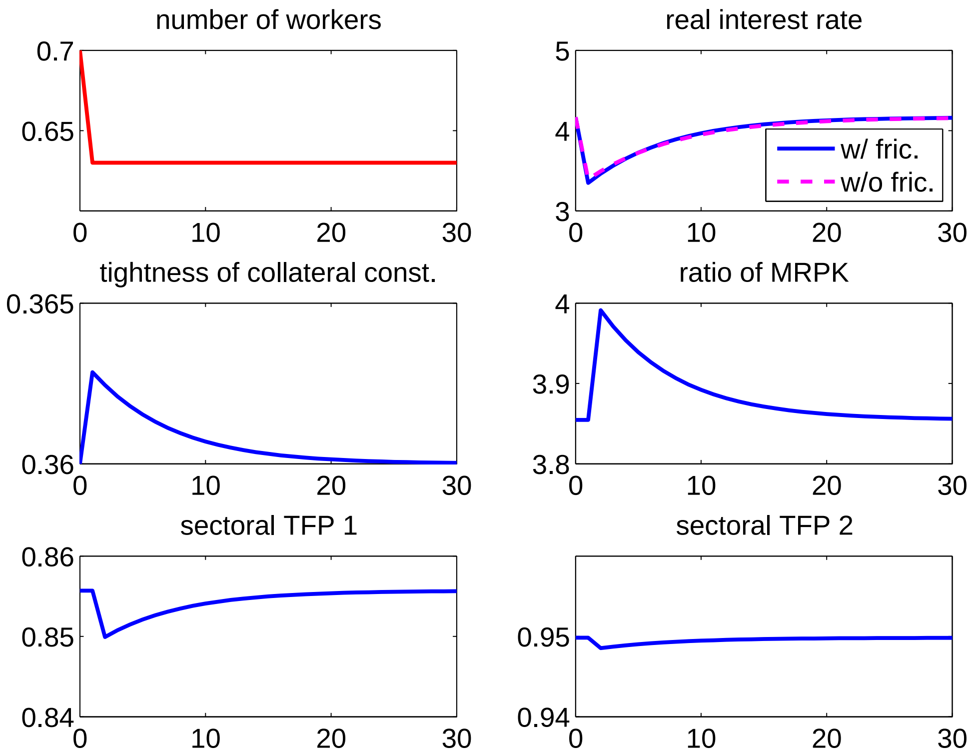
	description	value			
		w/ fric.		w/o fric.	
n^w	working population ratio	0.70	0.63	0.70	0.63
μ	tightness of borrowing constraint	0.36	0.36	0	0
x	MRPK ratio	3.85	3.85	1	1
z^1	efficiency wedge in sector 1	0.86	0.86	1	1
z^2	efficiency wedge in sector 2	0.95	0.95	1	1
$(1+\tau_k^1)^{-1}$	capital wedge in sector 1	0.52	0.52	0.75	0.75
$(1+\tau_k^2)^{-1}$	capital wedge in sector 2	0.71	0.71	0.75	0.75
p	relative price	1.24	1.24	1	1
φ	preference wedge	0.78	0.79	0.81	0.82
y^1/y^2	relative output	1.60	1.71	1.94	2.05
k	capital	1.54	1.38	2.20	1.98

face of low real interest rates, the inverse of the risk-free bond price. The MRPK ratio increases in the next period as there is a one-period lag for new capital to be installed. The sectoral TFP (efficiency wedges) also decrease in the next period, and more so in the more constrained sector (sector 1), as shown in Proposition 1. Note that as the real interest rate goes back to the steady state, all the variables within each sector also revert to their steady state values. There is almost no difference in the real interest rate between the case with and without financial frictions, as there is little feedback from the efficiency wedges to the real interest rate via financial frictions.

The responses of the relative price, i.e., the ratio of the price of goods produced in the new sector (sector 1) to that in the old sector (sector 2), is shown in the upper panel of Figure 3. As the new sector (sector 1) is more constrained, its efficiency wedge (sectoral TFP) and capital wedge decrease more. Therefore, the relative price increases and the supply curve shifts upward, which dampens the relative output. As shown in Figure 1, without financial frictions the supply curve does not shift and the relative output is higher than in the case with financial frictions. The lower panel of Figure 3 shows the difference in relative output from the efficient level without financial frictions. Although relative output is higher than the pre-shock level in the long run, as we saw in Table 2, the increase in the relative price dampens relative output in the short run.

Finally, Figure 4 shows the case of rejuvenation. In particular, labor input increases and workers have preferences over goods produced in the new and more constrained sector (sector 1). We set $\mu_w = 0.95$ and $\mu_r = 0.75$, and assume that the number of workers increases from 0.63 to 0.7 in period 1. There is a change in the demand structure of households from

Figure 2: Responses of sectoral variables.



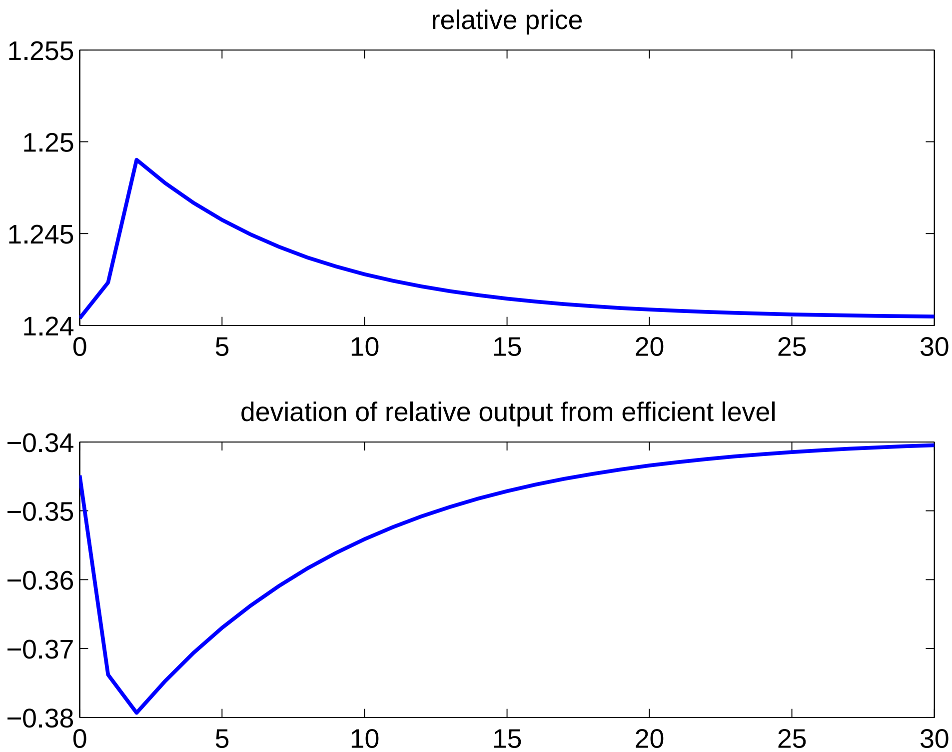
the less constrained sector (sector 2) to the more constrained sector (sector 1), but with an increase in labor input. As there are more workers, capital allocation among firms become less inefficient, which improves the efficiency and capital wedges, and the relative price decreases. Therefore, the relative output overshoots in the short run and is higher than the pre-shock level in the long run.

5 Concluding Remarks

We examined the effects of a decline in the working population ratio inducing changes in labor input and the structure of household demand in a two-sector model with financial frictions. We assumed that firms have differing financial needs and that the new sector in which demand increases due to structural changes is more constrained than the old sector in which demand declines. The original two-sector model with financial frictions is equivalent to a prototype two-sector model with wedges in terms of allocations and prices, while the wedges depend on the degree of capital misallocation among firms and the structure of household demand.

We found that the decline in labor input strengthens the collateral constraints of borrowers and generates capital misallocation and lower sectoral TFP. With respect to implications

Figure 3: Responses of relative price and output.

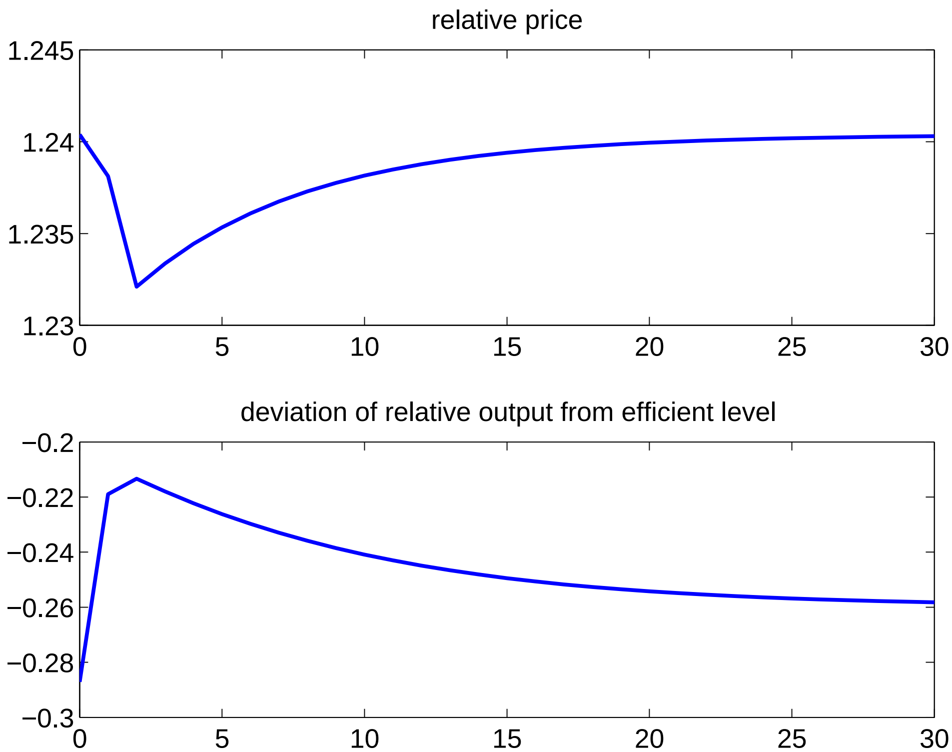


for structural change, since the new sector is more constrained than the old sector, the new sector's TFP is lower than the old sector's TFP, and the supply of goods produced in the new sector is too short compared to the case with no financial frictions, impeding structural change.

We leave two topics for investigation in the future work. First, the two-sector economy with *endogenous* wedges in the present paper is stylized enough to take the model to the data. Endogenizing the wedges and examining the role of capital misallocation in the structural changes considered in the previous papers are areas worthy of future investigation. For example, Buera and Kaboski (2009) apply a multi-sector model with *exogenous* wedges, showing that the wedges are needed to explain the structural change that occurred in the U.S. in the twentieth century. Hayashi and Prescott (2008) show that in a two-sector model similar to the prototype model in the present paper, exogenous sectoral TFP and labor immobility from rural to urban areas can explain the structural change from the agricultural to the manufacturing sector in Japan in the prewar period.

Further, one can investigate aggregate technology shocks and/or non-homothetic preferences (Kongsamut, Rebelo, and Xie, 2001; Ngai and Pissarides, 2007), which is commonly used in the literature as a main driver of structural changes. It would be useful to investigate the effects of technology shocks and structures in our stylized model.

Figure 4: Responses of relative price and output: rejuvenation.



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Appendix

A Prototype two-sector model with wedges

A.1 Model

In the prototype two-sector model, there is a representative firm in each sector, a capital owner, and a representative household. The firm in each sector maximizes its profit:

$$
\begin{aligned}
&p_t^i y_t^i - w_t n_t^i - (1+\tau_{kt}^i) r_t k_t^i, \\
&\text{subject to} \\
&y_t^i = z_t^i (k_t^i)^{\alpha} (n_t^i)^{1-\alpha}.
\end{aligned}
$$

Each firm sells $p_t^i y_t^i$ value of goods to the household, pays wage bill $w_t n_t^i$ to the household and rental cost $(1+\tau_{kt}^i) r_t k_t^i$ to the capital owner. w_t is real wage, and we assume that labor is freely mobile between firms and sectors. $1+\tau_{kt}^i$ is a time-varying tax on the rental cost of capital. Each firm produces using a Cobb-Douglas production function with capital share $\alpha \in (0,1)$. z_t^i is a wedge on the efficiency of production. The FOCs are

$$
\begin{aligned}
\partial k_t^i: &\quad (1+\tau_{kt}^i) r_t = \alpha p_t^i y_t^i / k_t^i, \\
\partial n_t^i: &\quad w_t = (1-\alpha) p_t^i y_t^i / n_t^i.
\end{aligned}
$$

The representative household chooses an allocation of consumption of sectoral goods c_t^1 and c_t^2, and the amount of risk-free bond B_t. Given the prices of sectoral goods p_t^1 and p_t^2, and the risk-free bond price q_t, the household maximizes its life-time utility:

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t \left\{ \varphi_t^1 \log c_t^1 + \varphi_t^2 \log c_t^2 \right\}, \\
&\text{subject to} \\
&p_t^1 c_t^1 + p_t^2 c_t^2 + q_t B_{t+1} \leq w_t n_t^w + B_t + \pi_t,
\end{aligned}$$

where n_t^w is the number of workers and π_t is a sum of transfers from firms. φ_t^i is a wedge on utilities from consumption of each sectoral goods, which is also a weight on sectoral goods in each sector. Let λ_t^{-1} be the Lagrange multiplier. The FOCs are

$$\begin{aligned}
\varphi_t^1 \lambda_t &= p_t^1 c_t^1, \\
\varphi_t^2 \lambda_t &= p_t^2 c_t^2, \\
q_t \lambda_{t+1} &= \beta \lambda_t.
\end{aligned}$$

The capital owner, who owns the aggregate capital $k_t = k_t^1 + k_t^2$, and rents it to each firm, maximizes his profit:

$$\sum_{t=0}^{\infty} \beta^t \lambda_t^{-1} \left\{ r_t k_t - p_t^2 (k_{t+1} - (1-\delta) k_t) \right\},$$

The FOC is

$$p_t^2 \lambda_{t+1} = \beta \lambda_t \left\{ r_{t+1} + p_{t+1}^2 (1-\delta) \right\}.$$

Finally, the goods market in each sector, the capital market, labor market and bond market all clear:

$$\begin{aligned}
c_t^1 &= y_t^1, \\
c_t^2 &= (1-\psi_t) y_t^2 + (1-\delta) k_t - k_{t+1}, \\
k_t &= k_t^1 + k_t^2, \\
n_t^w &= n_t^1 + n_t^2, \\
B_{t+1} &= 0.
\end{aligned}$$

Sector-1 goods are only consumed, whereas sector-2 goods are also used for investment by each firm and government expenditure. $\psi_t = g_t/y_t^2$ is the ratio of government expenditure to output in sector 2. We denote the sector-1 goods price $p_t^1 = p_t$ and normalize $p_t^2 = 1$.

A.2 Equilibrium conditions

The equilibrium conditions in the prototype model are given by

$$
\begin{aligned}
&\lambda_{t+1} = \beta\lambda_t(r_{t+1} + 1 - \delta),\\
&\varphi_t\lambda_t = p_t y_t^1,\\
&(1-\varphi_t)\lambda_t = y_t^2 - g_t + (1-\delta)k_t - k_{t+1},\\
&(1+\tau_{kt}^1)r_t = \alpha p_t y_t^1/k_t^1,\\
&(1+\tau_{kt}^2)r_t = \alpha y_t^2/k_t^2,\\
&p_t y_t^1/n_t^1 = y_t^2/n_t^2,\\
&y_t^1 = z_t^1(k_t^1)^\alpha(n_t^1)^{1-\alpha},\\
&y_t^2 = z_t^2(k_t^2)^\alpha(n_t^2)^{1-\alpha},\\
&k_t = k_t^1 + k_t^2,\\
&n_t^w = n_t^1 + n_t^2,\\
&q_t = \beta\lambda_t/\lambda_{t+1}.
\end{aligned}
$$

Note that in the proof of Lemma 2, we show that $\varphi_t \equiv \varphi_t^1 = 1 - \varphi_t^2$. Also, the prices and wedges are given by

$$
\begin{aligned}
&\mu_t = (1-\beta_b/\beta)q_t,\\
&x_{t+1} = \frac{\beta}{\beta_b}\frac{1-(q_t-\mu_t)(1-\delta+\theta\mu_{t+1})}{1-q_t(1-\delta)},\\
&z_{t+1}^i = \left[\frac{\left(1-\chi^i+\chi^i x_{t+1}^\gamma\right)^{1-\nu}}{\left(1-\chi^i+\chi^i x_{t+1}^{\gamma-1}\right)^{\tilde{\alpha}}}\right]^{\frac{1}{\varrho}},\\
&(1+\tau_{kt+1}^i)^{-1} = \varrho\left[\frac{1-\chi^i+\chi^i x_{t+1}^{\gamma-1}}{1-\chi^i+\chi^i x_{t+1}^{\gamma}}\right],\\
&\varphi_t = n_t^w\left[1-(1-1/\mu^w)(p_t)^{\rho/(1-\rho)}\right]^{-1} + (1-n_t^w)\left[1-(1-1/\mu^r)(p_t)^{\rho/(1-\rho)}\right]^{-1},
\end{aligned}
$$

where $\tilde{\alpha} = \varrho\alpha$, $\nu = \varrho(1-\alpha)$ and $\gamma = \tilde{\alpha}/(\tilde{\alpha}+\nu-1) < 1$. Given n_t^w, there are 18 equations and $\{y_t^1, k_t^1, n_t^1, y_t^2, k_t^2, n_t^2, k_t, \lambda_t, p_t, q_t, r_t, \mu_t, x_t, z_t^1, z_t^2, \tau_{kt}^1, \tau_{kt}^2, \varphi_t\}$ 18 variables. k_t is the only endogenous state variable. Note that if $\beta = \beta_b$, $\mu_t = 0$, $x_t = z_t^i = 1$, $(1+\tau_{kt}^i)^{-1} = \varrho$. It also implies $p_t = 1$.

A.3 Steady state

We can analytically solve for the steady state of the model. First, the prices and wedges in the steady state are given by

$$
\begin{aligned}
&q = \beta,\\
&\mu = (1 - \beta_b/\beta)q = \beta - \beta_b,\\
&x = \frac{\beta}{\beta_b}\frac{1-(q-\mu)(1-\delta+\theta\mu)}{1-q(1-\delta)},\\
&z^i = \left[\frac{\left(1-\chi^i+\chi^i x^\gamma\right)^{1-\nu}}{\left(1-\chi^i+\chi^i x^{\gamma-1}\right)^{\tilde{\alpha}}}\right]^{\frac{1}{\varrho}},\\
&(1+\tau_k^i)^{-1} = \varrho\left[\frac{1-\chi^i+\chi^i x^{\gamma-1}}{1-\chi^i+\chi^i x^\gamma}\right],\\
&p = \left[\frac{1-\chi^1+\chi^1 x^\gamma}{1-\chi^2+\chi^2 x^\gamma}\right]^{\frac{\varrho-1}{\varrho}},\\
&r = \beta^{-1} - 1 + \delta,\\
&\varphi = n_t^w\left[1-(1-1/\mu^w)(p)^{\rho/(1-\rho)}\right]^{-1} + (1-n_t^w)\left[1-(1-1/\mu^r)(p)^{\rho/(1-\rho)}\right]^{-1}.
\end{aligned}
$$

Given the prices and wedges, we have the output-capital ratio in each sector

$$
\begin{aligned}
py^1/k^1 &= (1+\tau_k^1)r/\alpha,\\
y^2/k^2 &= (1+\tau_k^2)r/\alpha.
\end{aligned}
$$

Then we have

$$
\begin{aligned}
(1+\tau_k)py^1/k^1 &= y^2/k^2,\\
py^1/n^1 &= y^2/n^2.
\end{aligned}
$$

where $1+\tau_k \equiv (1+\tau_k^2)/(1+\tau_k^1)$. From the production function, we have

$$
\begin{aligned}
(py^1/n^1)/(y^2/n^2) &= p(z^1/z^2)(k^1/k^2)^\alpha(n^1/n^2)^{-\alpha} = 1,\\
(py^1/k^1)/(y^2/k^2) &= p(z^1/z^2)(k^1/k^2)^{\alpha-1}(n^1/n^2)^{1-\alpha} = (1+\tau_k)^{-1}.
\end{aligned}
$$

Then we have

$$k^1/k^2 = (1+\tau_k)n^1/n^2.$$

Then we can solve for the sectoral labor

$$\begin{aligned}
n^1/n^2 &= \frac{(1-\psi) - \delta(k^2/y^2)}{(1-\varphi)/\varphi + \delta(1+\tau_k)(k^2/y^2)}, \\
n^2 &= n^w/(1+n^1/n^2).
\end{aligned}$$

Given the sectoral labor, we have the sectoral output and capital

$$\begin{aligned}
y^i &= (z^i)^{\frac{-1}{\alpha-1}}(y^i/k^i)^{\frac{\alpha}{\alpha-1}}n^i, \\
k^i &= y^i/(y^i/k^i).
\end{aligned}$$

B Proofs

B.1 Lemma 1

From Equation (6) and (8), the MRPK is

$$\begin{aligned}
\frac{\varrho\alpha p^i_{jt+1}y^i_{jt+1}}{k^i_{jt+1}} &= \frac{\lambda_{t+1}}{\beta_j\lambda_t} - (1-\delta+\theta\mu^i_{jt+1}), \\
&= (q_t - \mu^i_{jt})^{-1} - (1-\delta+\theta\mu^i_{jt+1}).
\end{aligned}$$

Note that $\mu^i_{bt} = \mu_t = (1-\beta_b/\beta)q_t > 0$ and $\mu^i_{st} = 0$. Then we have the ratio of MRPK,

$$\begin{aligned}
x_{t+1} &\equiv \frac{p^i_{bt+1}y^i_{bt+1}/k^i_{bt+1}}{p^i_{st+1}y^i_{st+1}/k^i_{st+1}}, \\
&= \frac{(q_t-\mu_t)^{-1} - (1-\delta+\theta\mu_{t+1})}{(q_t)^{-1} - (1-\delta)}, \\
&= \frac{q_t}{q_t-\mu_t}\frac{1-(q_t-\mu_t)(1-\delta+\theta\mu_{t+1})}{1-q_t(1-\delta)}, \\
&= \frac{\beta}{\beta_b}\frac{1-(\beta_b/\beta)q_t[1-\delta+\theta(1-\beta_b/\beta)q_{t+1}]}{1-q_t(1-\delta)}.
\end{aligned}$$

Then,

$$
\begin{aligned}
& x_{t+1} > 1, \\
\Leftrightarrow \quad & 1 - (q_t - \mu_t)(1 - \delta + \theta\mu_{t+1}) - 1 + q_t(1-\delta) > 0, \\
\Leftrightarrow \quad & \mu_t - q_t(q_t - \mu_t)\theta\mu_{t+1} > 0, \\
\Leftrightarrow \quad & \frac{1}{(\beta_b/\beta)q_t q_{t+1}} \equiv \bar{\theta}_{t+1} > \theta.
\end{aligned}
$$

If $\theta < \bar{\theta}$, $x_{t+1} > 1$ holds; the MRPK ratio is greater than one. Note that $1 - q_t(1-\delta) > 0$ is a necessary condition, which holds in steady state as $1 - \beta(1-\delta) > 0$.

B.2 Proposition 1

From Equation (3), (4) and (7), we have

$$
\begin{aligned}
\frac{p^i_{bt} y^i_{bt}}{p^i_{st} y^i_{st}} &= \left(\frac{y^i_{bt}}{y^i_{st}}\right)^{\varrho}, \\
&= \left(\frac{k^i_{bt}}{k^i_{st}}\right)^{\tilde{\alpha}} \left(\frac{n^i_{bt}}{n^i_{st}}\right)^{\nu}, \\
&= \frac{n^i_{bt}}{n^i_{st}},
\end{aligned}
$$

where $\tilde{\alpha} = \varrho\alpha$ and $\nu = \varrho(1-\alpha)$. Then we have

$$
\begin{aligned}
\frac{k^i_{bt}}{k^i_{st}} &= \left(\frac{n^i_{bt}}{n^i_{st}}\right)^{(\gamma-1)/\gamma}, \\
x_t = \frac{p^i_{bt} y^i_{bt}/k^i_{bt}}{p^i_{st} y^i_{st}/k^i_{st}} &= \left(\frac{n^i_{bt}}{n^i_{st}}\right)^{1/\gamma},
\end{aligned}
$$

where $\gamma = \tilde{\alpha}/(\tilde{\alpha} + \nu - 1) < 1$. Then we have

$$
\frac{p^i_{bt} y^i_{bt}}{p^i_{st} y^i_{st}} = \frac{n^i_{bt}}{n^i_{st}} = \left(\frac{y^i_{bt}}{y^i_{st}}\right)^{\varrho} = x_t^{\gamma}, \tag{33}
$$

$$
\frac{k^i_{bt}}{k^i_{st}} = x_t^{\gamma-1} \tag{34}
$$

From Equation (9)-(12), (33) and (34), we have

$$
\begin{aligned}
z_t^i &= \frac{y_t^i}{(k_t^i)^\alpha (n_t^i)^{1-\alpha}}, \\
&= \frac{\left[\chi^i (y_{bt}^i)^\varrho + (1-\chi^i)(y_{st}^i)^\varrho\right]^{1/\rho}}{\left[\chi^i k_{bt}^i + (1-\chi^i) k_{st}^i\right]^\alpha \left[\chi^i n_{bt}^i + (1-\chi^i) n_{st}^i\right]^{1-\alpha}}, \\
&= \frac{y_{st}^i}{(k_{st}^i)^\alpha (n_{st}^i)^{1-\alpha}} \frac{\left[\chi^i (y_{bt}^i/y_{st}^i)^\varrho + (1-\chi^i)\right]^{1/\rho}}{\left[\chi^i (k_{bt}^i/k_{st}^i) + (1-\chi^i)\right]^\alpha \left[\chi^i (n_{bt}^i/n_{st}^i) + (1-\chi^i)\right]^{1-\alpha}}, \\
&= \frac{\left[\chi^i x_t^\gamma + (1-\chi^i)\right]^{1/\rho}}{\left[\chi^i x_t^{\gamma-1} + (1-\chi^i)\right]^\alpha \left[\chi^i x_t^\gamma + (1-\chi^i)\right]^{1-\alpha}}, \\
&= \left[\frac{\left(1-\chi^i+\chi^i x_t^\gamma\right)^{1-\nu}}{\left(1-\chi^i+\chi^i x_t^{\gamma-1}\right)^{\tilde{\alpha}}}\right]^{1/\varrho}.
\end{aligned}
$$

Note that if $\chi^i = 0$, then $z_t^i = 1$ holds. Also, if $\chi^i = 1$, then $z_t^i = (x_t)^{\gamma(1-\nu)/\varrho - (\gamma-1)\tilde{\alpha}/\varrho} = 1$ holds.

From Equation (27) and (8), the elasticity of x_t to μ_t is

$$
\begin{aligned}
\varepsilon_{x\mu} &= \frac{\partial x_t}{\partial \mu_t}\frac{\mu_t}{x_t}, \\
&= \frac{\beta}{\beta_b}\frac{-(q_{t-1}-\mu_{t-1})\theta}{1-q_{t-1}(1-\delta)}\frac{\mu_t}{x_t}, \\
&= \frac{-(q_{t-1}-\mu_{t-1})\theta\mu_t}{1-(q_{t-1}-\mu_{t-1})(1-\delta+\theta\mu_t)} < 0.
\end{aligned}
$$

Also, the elasticity of x_t to μ_{t-1} is

$$
\begin{aligned}
\varepsilon_{x\mu_{-1}} &= \frac{\partial x_t}{\partial \mu_{t-1}}\frac{\mu_{t-1}}{x_t}, \\
&= \frac{\beta}{\beta_b}\frac{(1-\delta+\theta\mu_t)}{1-q_{t-1}(1-\delta)}\frac{\mu_{t-1}}{x_t}, \\
&= \frac{(1-\delta+\theta\mu_t)\mu_{t-1}}{1-(q_{t-1}-\mu_{t-1})(1-\delta+\theta\mu_t)} > 0.
\end{aligned}
$$

From Equation (28), the elasticity of z_t^i to x_t is

$$\begin{aligned}
\varepsilon_{zx}^i &= \frac{\partial z_t^i}{\partial x_t}\frac{x_t}{z_t^i} \\
&= \left[\frac{1-\nu}{\varrho}\left(1-\chi^i+\chi^i x_t^\gamma\right)^{\frac{1-\nu}{\varrho}-1}\gamma\chi^i x_t^{\gamma-1}\left(1-\chi^i+\chi^i x_t^{\gamma-1}\right)^{\frac{\tilde{\alpha}}{\varrho}}\right. \\
&\quad \left.-\frac{\tilde{\alpha}}{\varrho}\left(1-\chi^i+\chi^i x_t^\gamma\right)^{\frac{1-\nu}{\varrho}}\left(1-\chi^i+\chi^i x_t^{\gamma-1}\right)^{\frac{\tilde{\alpha}}{\varrho}-1}(\gamma-1)\chi^i x_t^{\gamma-2}\right]\frac{x_t}{z_t^i}, \\
&= \frac{\chi^i x_t^{\gamma-1}}{\varrho}\left[\frac{(1-\nu)\gamma x_t}{1-\chi^i+\chi^i x_t^\gamma}-\frac{\tilde{\alpha}(\gamma-1)}{1-\chi^i+\chi^i x_t^{\gamma-1}}\right], \\
&= \frac{\chi^i x_t^{\gamma-1}}{\varrho}\frac{(1-\chi^i+\chi^i x_t^\gamma)[(1-\nu)\gamma x_t-\tilde{\alpha}(\gamma-1)]}{\left(1-\chi^i+\chi^i x_t^\gamma\right)\left(1-\chi^i+\chi^i x_t^{\gamma-1}\right)}, \\
&= -\frac{\alpha(1-\gamma)\chi^i x_t^{\gamma-1}(x_t-1)}{\left(1-\chi^i+\chi^i x_t^{\gamma-1}\right)} < 0.
\end{aligned}$$

Also,

$$\begin{aligned}
\frac{\varepsilon_{zx}^1}{\varepsilon_{zx}^2} &= \frac{\chi^1\left(1-\chi^2+\chi^2 x_t^{\gamma-1}\right)}{\chi^2\left(1-\chi^1+\chi^1 x_t^{\gamma-1}\right)} > 1, \\
&\Leftrightarrow \chi^1\left(1-\chi^2+\chi^2 x_t^{\gamma-1}\right)-\chi^2\left(1-\chi^1+\chi^1 x_t^{\gamma-1}\right) > 0, \\
&\Leftrightarrow \chi^1-\chi^2 > 0.
\end{aligned}$$

If $\chi^1 > \chi^2$, then $\varepsilon_{zx}^1 > \varepsilon_{zx}^2$ holds.

B.3 Lemma 2

For z_{t+1}^i, see Appendix B.2. For $1+\tau_{kt+1}^i$, from Equation (9), (10), (33) and (34), we have

$$\begin{aligned}
p_t^i y_t^i &= \chi^i p_{bt}^i y_{bt}^i + (1-\chi^i) p_{st}^i y_{st}^i, \\
&= p_{st}^i y_{st}^i\left[1-\chi^i+\chi^i x_t^\gamma\right], \\
k_t^i &= \chi^i k_{bt}^i + (1-\chi^i) k_{st}^i, \\
&= k_{st}^i\left[1-\chi^i+\chi^i x_t^{\gamma-1}\right].
\end{aligned}$$

Note that from Equation (6) with $\mu_{st}^i = 0$ and the Euler equation in the equivalent economy,

$$\begin{aligned}
\frac{\alpha p_{st+1}^i y_{st+1}^i}{k_{st+1}^i} &= \varrho^{-1}(q_t^{-1}+1-\delta), \\
&= \varrho^{-1} r_{t+1}.
\end{aligned}$$

Then we have

$$\begin{aligned} 1+\tau^i_{kt+1} &= \frac{\alpha p^i_{t+1} y^i_{t+1}}{r_{t+1} k^i_{t+1}}, \\ &= \frac{\alpha p^i_{st+1} y^i_{st+1}}{r_{t+1} k^i_{st+1}} \left[\frac{1-\chi^i+\chi^i x^{\gamma}_{t+1}}{1-\chi^i+\chi^i x^{\gamma-1}_{t+1}}\right], \\ &= \frac{1}{\varrho}\left[\frac{1-\chi^i+\chi^i x^{\gamma}_{t+1}}{1-\chi^i+\chi^i x^{\gamma-1}_{t+1}}\right]. \end{aligned}$$

For ϕ^i_t, from Equation (19) and (14), we have

$$\begin{aligned} c^1_t &= n^w_t c^{1w}_t + (1-n^w_t) c^{1r}_t, \\ &= n^w_t \mu^w (p^1_t/p^w_t)^{1/(\rho-1)} c^w_t + (1-n^w_t)\mu^r (p^1_t/p^r_t)^{1/(\rho-1)} c^r_t, \\ p^1_t c^1_t &= p^1_t n^w_t \mu^w (p^1_t/p^w_t)^{1/(\rho-1)} c^w_t + p^1_t (1-n^w_t)\mu^r (p^1_t/p^r_t)^{1/(\rho-1)} c^r_t, \\ &= n^w_t \mu^w (p^1_t/p^w_t)^{\rho/(\rho-1)} p^w_t c^w_t + (1-n^w_t)\mu^r (p^1_t/p^r_t)^{\rho/(\rho-1)} p^r_t c^r_t, \end{aligned}$$

Similarly, from Equation (20) and (15), we have

$$\begin{aligned} c^2_t &= n^w_t c^{2w}_t + (1-n^w_t) c^{2r}_t, \\ &= n^w_t (1-\mu^w)(p^2_t/p^w_t)^{1/(\rho-1)} c^w_t + (1-n^w_t)(1-\mu^r)(p^2_t/p^r_t)^{1/(\rho-1)} c^r_t, \\ p^2_t c^2_t &= p^2_t n^w_t (1-\mu^w)(p^2_t/p^w_t)^{1/(\rho-1)} c^w_t + p^2_t (1-n^w_t)(1-\mu^r)(p^2_t/p^r_t)^{1/(\rho-1)} c^r_t, \\ &= n^w_t (1-\mu^w)(p^2_t/p^w_t)^{\rho/(\rho-1)} p^w_t c^w_t + (1-n^w_t)(1-\mu^r)(p^2_t/p^r_t)^{\rho/(\rho-1)} p^r_t c^r_t, \end{aligned}$$

From Equation (16) and (17), we have

$$\begin{aligned} &\varphi^1_t = p^1_t c^1_t/\lambda_t = n^w_t \mu^w (p^1_t/p^w_t)^{\rho/(\rho-1)} + (1-n^w_t)\mu^r (p^1_t/p^r_t)^{\rho/(\rho-1)}, \\ &\varphi^2_t = p^2_t c^2_t/\lambda_t = n^w_t (1-\mu^w)(p^2_t/p^w_t)^{\rho/(\rho-1)} + (1-n^w_t)(1-\mu^r)(p^2_t/p^r_t)^{\rho/(\rho-1)}. \end{aligned}$$

Note that from Equation (13), (14) and (15), prices of composition goods are given by

$$\begin{aligned} p^k_t &= \left[(\mu^k)(p^1_t)^{\rho/(\rho-1)} + (1-\mu^k)(p^2_t)^{\rho/(\rho-1)}\right]^{(\rho-1)/\rho}, \\ 1 &= (\mu^k)(p^1_t/p^k_t)^{\rho/(\rho-1)} + (1-\mu^k)(p^2_t/p^k_t)^{\rho/(\rho-1)}. \end{aligned}$$

for $k = \{w, r\}$. Then we have

$$\begin{aligned}
\varphi_t^1 &= n_t^w \mu^w (p_t^1/p_t^w)^{\rho/(\rho-1)} + (1-n_t^w)\mu^r (p_t^1/p_t^r)^{\rho/(\rho-1)}, \\
&= n_t^w \mu^w (p_t^1)^{\rho/(\rho-1)} \left[(\mu^w)(p_t^1)^{\rho/(\rho-1)} + (1-\mu^w)(p_t^2)^{\rho/(\rho-1)}\right]^{-1} \\
&+ (1-n_t^w)\mu^r (p_t^1)^{\rho/(\rho-1)} \left[(\mu^r)(p_t^1)^{\rho/(\rho-1)} + (1-\mu^r)(p_t^2)^{\rho/(\rho-1)}\right]^{-1}, \\
&= n_t^w \left[1 + (1/\mu^w - 1)(p_t^2/p_t^1)^{\rho/(\rho-1)}\right]^{-1} + (1-n_t^w)\left[1 + (1/\mu^r - 1)(p_t^2/p_t^1)^{\rho/(\rho-1)}\right]^{-1}.
\end{aligned}$$

Note that

$$\begin{aligned}
\varphi_t^1 + \varphi_t^2 &= n_t^w \mu^w (p_t^1/p_t^w)^{\rho/(\rho-1)} + (1-n_t^w)\mu^r (p_t^1/p_t^r)^{\rho/(\rho-1)} \\
&+ n_t^w (1-\mu^w)(p_t^2/p_t^w)^{\rho/(\rho-1)} + (1-n_t^w)\mu^r (p_t^2/p_t^r)^{\rho/(\rho-1)}, \\
&= n_t^w \left[\mu^w (p_t^1/p_t^w)^{\rho/(\rho-1)} + (1-\mu^w)(p_t^2/p_t^w)^{\rho/(\rho-1)}\right] \\
&+ (1-n_t^w)\left[\mu^r (p_t^1/p_t^r)^{\rho/(\rho-1)} + (1-\mu^r)(p_t^2/p_t^r)^{\rho/(\rho-1)}\right], \\
&= 1.
\end{aligned}$$

In other words, the sum of weights is equal to one and we denote $\varphi_t = \varphi_t^1 = 1 - \varphi_t^2$. Then we have

$$\begin{aligned}
p_t^1 c_t^1 + p_t^2 c_t^2 &= (\varphi_t^1 + \varphi_t^2)\lambda_t, \\
&= \lambda_t.
\end{aligned}$$

Note that in the original economy, from Equation (16), (17), (19) and (20), we have

$$\begin{aligned}
p_t^1 c_t^1 + p_t^2 c_t^2 &= n_t^w p_t^1 c_t^{1w} + (1-n_t^w) p_t^1 c_t^{1r} \\
&+ n_t^w p_t^2 c_t^{2w} + (1-n_t^w) p_t^2 c_t^{2r}, \\
&= n_t^w p_t^w c_t^w + (1-n_t^w) p_t^r c_t^r, \\
&= \lambda_t.
\end{aligned}$$

B.4 Proposition 2

By plugging the wedges (28) and (29) into the supply function (30), we have

$$
\begin{aligned}
\frac{p_t^1}{p_t^2} &= \frac{z_t^2}{z_t^1}\left(\frac{1+\tau_{kt}^1}{1+\tau_{kt}^2}\right)^{\alpha}. \\
&= \left[\left(\frac{1-\chi^2+\chi^2 x_t^{\gamma}}{1-\chi^1+\chi^1 x_t^{\gamma}}\right)^{1-\nu}\left(\frac{1-\chi^1+\chi^1 x_t^{\gamma-1}}{1-\chi^2+\chi^2 x_t^{\gamma-1}}\right)^{\tilde{\alpha}}\right]^{1/\varrho} \\
&\times \left[\left(\frac{1-\chi^1+\chi^1 x_t^{\gamma}}{1-\chi^2+\chi^2 x_t^{\gamma}}\right)\left(\frac{1-\chi^2+\chi^2 x_t^{\gamma-1}}{1-\chi^1+\chi^1 x_t^{\gamma-1}}\right)\right]^{\alpha}, \\
&= \left(\frac{1-\chi^1+\chi^1 x_t^{\gamma}}{1-\chi^2+\chi^2 x_t^{\gamma}}\right)^{\alpha-\frac{1-\nu}{\varrho}}\left(\frac{1-\chi^1+\chi^1 x_t^{\gamma-1}}{1-\chi^2+\chi^2 x_t^{\gamma-1}}\right)^{\frac{\tilde{\alpha}}{\varrho}-1}, \\
&= \left(\frac{1-\chi^1+\chi^1 x_t^{\gamma}}{1-\chi^2+\chi^2 x_t^{\gamma}}\right)^{\frac{\varrho-1}{\varrho}}.
\end{aligned}
$$

Note that $\tilde{\alpha} = \varrho\alpha$ and $\nu = \varrho(1-\alpha)$. Then we have

$$
\begin{aligned}
& \frac{p_t^1}{p_t^2} > 1, \\
\Leftrightarrow\ & -\chi^1+\chi^2+(\chi^1-\chi^2)x_t^{\gamma} > 0, \\
\Leftrightarrow\ & (\chi^1-\chi^2)(x_t^{\gamma}-1) > 0.
\end{aligned}
$$

If $\chi^1 > \chi^2$, then $p_t^1 > p_t^2$ holds. Also, we have

$$
\begin{aligned}
\varepsilon_{px} = \frac{\partial p_t}{\partial x_t}\frac{x_t}{p_t} &= \frac{x_t}{p_t}\frac{\varrho-1}{\varrho}\gamma\chi^1 x_t^{\gamma-1}\left(1-\chi^1+\chi^1 x_t^{\gamma}\right)^{\frac{\varrho-1}{\varrho}-1}\left(1-\chi^2+\chi^2 x_t^{\gamma}\right)^{\frac{1-\varrho}{\varrho}}, \\
&+ \frac{x_t}{p_t}\frac{\varrho-1}{\varrho}\gamma\chi^2 x_t^{\gamma-1}\left(1-\chi^1+\chi^1 x_t^{\gamma}\right)^{\frac{\varrho-1}{\varrho}}\left(1-\chi^2+\chi^2 x_t^{\gamma}\right)^{\frac{1-\varrho}{\varrho}-1}, \\
&= \frac{\varrho-1}{\varrho}\gamma\chi^1 x_t^{\gamma}\left(1-\chi^1+\chi^1 x_t^{\gamma}\right)^{-1} + \frac{\varrho-1}{\varrho}\gamma\chi^2 x_t^{\gamma}\left(1-\chi^2+\chi^2 x_t^{\gamma}\right)^{-1}, \\
&= \frac{\varrho-1}{\varrho}\frac{\gamma x_t^{\gamma}\left[\chi^1\left(1-\chi^2+\chi^2 x_t^{\gamma}\right)-\chi^2\left(1-\chi^1+\chi^1 x_t^{\gamma}\right)\right]}{\left(1-\chi^1+\chi^1 x_t^{\gamma}\right)\left(1-\chi^2+\chi^2 x_t^{\gamma}\right)}, \\
&= \frac{\varrho-1}{\varrho}\frac{\gamma x_t^{\gamma}(\chi^1-\chi^2)}{\left(1-\chi^1+\chi^1 x_t^{\gamma}\right)\left(1-\chi^2+\chi^2 x_t^{\gamma}\right)}.
\end{aligned}
$$

If $\chi^1 > \chi^2$, then $\varepsilon_{px} > 1$ holds.