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# A Macroeconomic Approach to Corporate Capital Structure

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**Abstract**

In this paper, I investigate the cross-sectional determinants of corporate capital structure using a general equilibrium model with endogenous firm dynamics, a realistic tax environment, and financial frictions. I find that the equilibrium firm distribution in the model replicates fairly well the distribution of corporate capital structure as well as the relationship between capital structure, profitability, and firm size in the data. The key mechanisms here are economies of scale and two types of productivity shocks: persistent and transitory. The counterfactual experiment using the model implies, among other things, that tax benefits have relatively small effects on corporate capital structure choice compared with default costs and the costs of outside equity, including the dividend tax. It also reveals that the effects of those frictions on corporate capital structure choice are highly interrelated with each other.

**Keywords:** Corporate Capital Structure; Dynamic Trade-off Theory; Heterogeneous Firm Model
**JEL classification:** G32, G31, E22

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# 1 Introduction

Many theoretical and empirical works in the corporate finance literature have investigated the logic behind corporate capital structure choice. Modigliani and Miller (1958), a seminal classic paper in capital structure theory, argued that the dispersion of corporate capital structure among firms has nothing to do with their optimization. However, numerous empirical works have established clear relationships between capital structure and other characteristics of firms such as size and profitability, which suggest that their capital structure choice is related to their profit maximization.[1] Given the stylized facts established by the empirical works, the theoretical works following Modigliani and Miller (1958) have investigated the cross-sectional determinants of corporate capital structure. Among other theoretical works, the dynamic trade-off theory, which describes firms' dynamic choice of their capital structure under the trade-off between tax benefits and financial distress costs, has succeeded in quantitatively accounting for the stylized facts. While most papers based on the dynamic trade-off theory are very recent and are still not sufficiently well developed to explain some stylized facts, this theory is now the most promising theory among the theoretical models that quantitatively account for corporate capital structure.

In this paper, I construct a structural model based on the dynamic trade-off theory and investigate the following quantitative questions, which have not been fully investigated by previous works. First, I examine whether the dynamic trade-off theory can explain the distribution of corporate capital structure observed in the real economy. This question cannot be answered by standard dynamic trade-off models because most of them are partial equilibrium models, focusing on a certain firm's optimal behavior. Therefore, deriving a cross-sectional distribution in equilibrium is outside their scope. In order to overcome this shortcoming, I extend the model to a dynamic general equilibrium model with heterogeneous firms and endogenous entry/exit a la Hopenhayn and Rogerson (1993). By doing so, I obtain not only an optimal policy for each firm, but also an equilibrium cross-sectional distribution regarding firms' characteristics such as firm size and capital structure.[2] Then, I use the distribution as a natural counterpart of the empirical distribution in data for comparison.

Second, I examine whether the trade-off theory accounts for the relationship between corporate capital structure, firm size, and profitability. I focus on the relationship between those three vari-

[1]Bernanke et al. (1990) and Frank and Goyal (2008) discuss the distribution of leverage based on US data. Rajan and Zingales (1995) use the G7 countries' cross-sectional data and investigate the cross-sectional relationships between corporate capital structure and other characteristics of firms such as profitability and firm size. Fama and French (2002) and Frank and Goyal (2009) use US firm panel data and obtain similar results. Lemmon et al. (2008) also uses US panel data and emphasizes the fixed effect of each firm. Graham and Harvey (2001) collect extensive survey data from the chief financial officers (CFOs) of the US firms and explore the key determinants of their capital structure decisions.

[2]Another way to obtain a cross-sectional distribution in a structural model is to generate simulated data and construct a distribution using them [e.g., Strebulaev (2007)]. This approach does not consider the distribution itself as an equilibrium, but it is conceptually very similar to the stationary equilibrium approach in this paper.

ables because there is little disagreement on the relationship in the empirical works.[3] In particular, I focus on the following stylized facts about the relationship.

1. The correlation between profitability and firm size is **positive**.

2. The correlation between leverage and firm size is **positive**.

3. The correlation between leverage and profitability is **positive**, but it becomes **negative** if the data are limited to large firms.

4. The correlation between leverage and profitability is **negative** after controlling for firm size.

As far as I know, no structural model *simultaneously* accounts for these stylized facts. In particular, although the size dependency of the relationship between leverage and profitability has been investigated empirically, its microfoundations are not even mentioned in existing theoretical papers.[4] As the potential mechanisms to explain those stylized facts, I incorporate the following two features into the dynamic trade-off model. One feature is idiosyncratic productivity shocks of two types (transitory and persistent) and the other feature is economies of scale. While these features are common in other literature and justified by empirical works, they are not usually incorporated in dynamic trade-off models. In the quantitative part of this paper, I test whether the combination of those two features and the trade-off between tax benefits and financial distress costs quantitatively account for those stylized facts.

Finally, through counterfactual experiments, I provide an answer to one of the most recurrent questions in the corporate finance literature: Which cross-sectional determinants are relevant to corporate capital structure choice? In the experiments, I drop frictions from the baseline model one by one and recalculate the equilibrium. Then, I measure the effect of the friction on corporate capital structure by comparing average and aggregate leverage in the new equilibrium with average and aggregate leverage in the equilibrium of the baseline model.

The main findings of this paper are summarized as follows. First, I find that the model's equilibrium distribution accounts for the features of the distribution of corporate capital structure in the US economy. In particular, it accounts for the fact that (1) more than 30% of firms are almost-zero leverage firms, and (2) leverage of nonzero leverage firms considerably differs from firm to firm.

Second, I find that the equilibrium distribution also accounts for the stylized facts regarding the relationship between capital structure, firm size, and profitability. In particular, it accounts

[3] In the empirical works, a growth expectation measured by the market-to-book ratio is often considered as one of the determinants, but there is no agreement on the sign of the effect on book leverage in the empirical works. For example, while Fama and French (2002) argue that it is positive, Rajan and Zingales (1995) and Lemmon et al. (2008) argue that it is negative. Frank and Goyal (2009) show the sign of the effect varies over time and conclude that it is not stable over time.

[4] For references to empirical papers that address size dependency, see Rajan and Zingales (1995) and Covas and DenHaan (2011).

for the four stylized facts stated above. The logic in the model is as follows. The first stylized fact is explained by economies of scale. In this model, the economies of scale emerge because a fixed cost dampens small firms' profitability, measured by ROA, more significantly than it dampens large firms' profitability. The second stylized fact emerges in the model as a combination of two correlations. In the model, productivity and leverage are positively correlated for two reasons. First, they expand the financing deficit (the gap between investment and internal funds) because they have good investment opportunities, and the financing gap is mainly filled by debt. Second, the debt market is more accessible to the firms under the trade-off between tax benefits and financial distress costs. Productivity and firm size are also positively correlated because the optimal size of firms with high productivity is large. Those two positive correlations result in the positive correlation between size and leverage. The key mechanism behind the third stylized fact is the difference between responses to the persistent and transitory productivity shocks. An intuitive explanation is as follows. On the one hand, as a combination of the first and the second stylized facts, a *persistent* productivity shock causes a positive correlation between leverage and profitability. On the other hand, a *transitory* productivity shock causes a negative correlation between leverage and profitability. That is, while the transitory productivity shock decreases leverage by pushing up the level of the firm's internal funds, it increases profitability measured by ROA by increasing profit (the numerator) without affecting firm size (the denominator). When I measure the correlation between profitability and leverage, the effect of the persistent productivity is more relevant for corporate capital structure choice on average, inducing the positive correlation between leverage and profitability (the first part of the third stylized fact). However, the effect of the transitory productivity shock becomes more relevant *among large firms* because the economies of scale caused by fixed costs are very weak among them, inducing the negative correlation between leverage and profitability (the second part of the third stylized fact). Finally, the fourth stylized fact is interpreted as follows. When controlling for firm size, firm size absorbs the effect of the persistent productivity because firms with high persistent productivity become large. Thus, profitability in the regression just captures the effect of transitory productivity, and the sign of the coefficient on profitability becomes negative.

Finally, I discover the following implications about the relative importance between the cross-sectional determinants of capital structure through counterfactual experiments. First, even if the tax benefit created by corporate income tax does not exist, the aggregate and average leverages would not significantly change. This is in contrast to previous works. This contrast stems from the difference in the assumptions about firms' entry/exit. That is, in the case *without* firms' entry/exit in the standard dynamic trade-off model, all firms would eventually use 100% equity by accumulating their internal funds when the tax benefit does not exist. However, in the case *with* firms' entry/exit, young firms always exist and use debt in the process of accumulating their internal funds as long as outside equity is more costly than debt.[5] Second, when the costs of

[5]This may answer the question of why debt finance was a popular funding method before corporate income tax

outside equity such as the dividend tax and the flotation cost of equity are eliminated, in addition to the tax benefit, all firms use 100% equity finance. Third, when only the costs of outside equity are eliminated, the sign of the effect on leverage depends on the firm size, profitability, and age. Big, rich, and old firms increase their leverage whereas small, poor, and young firms decrease their leverage. The nonlinear results obtained up to this point imply that the effects of the tax benefit and the costs of outside equity are highly interrelated with each other. Fourth, the default cost has a large effect on corporate capital structure choice. Fifth, the investment irreversibility magnifies the disadvantage of debt finance, but it has no effect on leverage when the costs of outside equity do not exist.

### Related Literature

After the Modigliani–Miller theorem claimed that corporate capital structure is irrelevant to a firm's optimization, many theoretical papers have explored what makes the firm's capital structure relevant to its optimization.[6] While the trade-off theory, which explains corporate capital structure by the trade-off between tax benefits and financial distress costs, is one of the most accepted theories, another well-accepted theory is called the pecking order theory [e.g., Myers (1984)]. This theory claims that asymmetric information makes the capital structure relevant, and implies that firms prefer internal funds, debt, and outside equity, in that order. There are also other models to explain corporate capital structure using asymmetric information. Tirole (2006) reviews those models. Stiglitz (1973) argues that the cost of outside equity generated by the tax on stock returns makes the capital structure relevant. He showed that, with the tax, a firm's behavior would be similar to the pecking order situation.

The most closely related literature to this paper is the dynamic trade-off theory. In particular, Hennessy and Whited (2005, 2007) are the most related papers in the literature. They assume endogenous investment and payout policy as well as endogenous capital structure choice, as in this paper, and account for the negative relationship between leverage and profitability. The most important difference between my model and theirs is that their model is a *partial* equilibrium model, focusing on a certain firm's optimal capital structure choice, whereas this paper is a *general* equilibrium model with endogenous entry/exit. In addition, because their model does not consider the decomposition of productivity and economies of scale, it seems unable to account for the relationship between leverage, profitability, and firm size simultaneously. Tserlukevich (2008) uses a model with endogenous investment and shows that the negative relationship between leverage and profitability emerges if firms face very severe investment irreversibility. DeAngelo et al. (2011)

was introduced. Frank and Goyal (2008) comment in their conclusion that "The US corporate income tax did not begin until 1909, when it was introduced at a 1% rate. The use of debt contracts by businesses has a much longer history than does the corporate income tax. Thus, while taxes probably play an important role, there must be more to it."

[6] See Frank and Goyal (2008) for a recent survey of this literature. It surveys several stylized facts regarding corporate capital structure as well as a vast number of empirical and theoretical papers.

account for a conservative firm's financing behavior by incorporating an exogenous debt capacity into a dynamic trade-off model with endogenous investment and payout.

Another type of dynamic trade-off model is a continuous time model with a stochastic profit process and adjustment costs, which was pioneered by Fischer et al. (1989). Strebulaev (2007) focuses on the relationship between leverage and profitability, as this paper does, and accounts for it by adjustment costs for rebalancing capital structure. Gorbenko and Strebulaev (2010) investigate the effect of a temporary shock in addition to a persistent shock, as this paper does, and argue that it induces a more conservative financing behavior. Kurshev and Strebulaev (2006) incorporate a fixed adjustment cost for capital structure and account for the positive relationship between leverage and firm size. These recent papers adopt a similar quantitative method to this paper in order to obtain cross-sectional implications, but the most striking difference is that a firm's profit and investment as well as payout policy are totally exogenous in those papers. Therefore, it is not obvious whether the results shown in those papers are still valid under endogenous investment and the payout assumption because they cannot consider the effect of the financing deficit (the gap between investment and internal funds), which is said to be an important determinant of corporate capital structure.

I use a dynamic equilibrium model with heterogeneous firms as a baseline model. An influential classic paper in this literature is Hopenhayn (1992). He proposes an economic model with firm heterogeneity and shows the existence of a competitive equilibrium with a stationary distribution of firms' characteristics. Hopenhayn and Rogerson (1993) extend the Hopenhayn model to a dynamic general equilibrium model, and use the model to obtain the aggregate implications of a firing tax. Cooley and Quadrini (2001) introduce financial intermediaries and a debt contract into a heterogeneous firm model. While they do not focus on capital structure, but on the size and age dependency of firms' characteristics, this paper adopts the same contractual environment used in their model. Gomes (2001) introduces capital accumulation into a general equilibrium model with heterogeneous firms and endogenous entry/exit. While he focuses on the logic behind the cash flow effect on investment, I use the same quantitative method proposed in his model to test cross-sectional stylized facts. Gourio (2008) shows that it is important to decompose idiosyncratic productivity into persistent and transitory parts. He assumes that firms simultaneously face several types of productivity shocks and then structurally estimates the parameters of the productivity process, using a structural model similar to this paper. Miao (2005) is the only paper investigating corporate capital structure using a general equilibrium model with entry/exit. He accounts for some characteristics of corporate capital structure and entry/exit behavior using a heterogeneous firm model, but he makes a number of simplifications to obtain a closed form solution, whereas this paper focuses on a quantitative solution under more realistic circumstances.[7]

The rest of paper is organized as follows. Section 2 presents the economic model. Section 3

[7]For example, Miao assumes a perpetual bond that pays a fixed amount of coupons, and the amount is fixed after they enter the economy.

calibrates the model and computes a stationary equilibrium. Section 4 describes some empirical implications of the model and does counterfactual experiments, and Section 5 concludes the paper.

## 2 The Model

The model is based on a dynamic general equilibrium model with heterogeneous firms and endogenous entry/exit, as in Hopenhayn and Rogerson (1993) and Gomes (2001). Because firms are assumed to be hit by idiosyncratic productivity shocks, but not aggregate productivity shocks, the model has a competitive equilibrium with a stationary distribution regarding firms' characteristics.[8]

The economy consists of three types of agents: firms, households, and financial intermediaries (FIs). The firm produces consumption goods using assets and labor in every period. It finances the asset by three financing sources: internal funds, outside equity, and debt. Note that the first two sources are listed as "equity" and the third one is listed as "debt" in its balance sheet. As a result of the optimal choice between the three financing sources, the capital structure is determined endogenously in the model.

The household is homogeneous and infinitely lived, and maximizes its lifetime utility given consumption and labor supply. The household's financial asset consists of the shares of firms and a risk-free deposit at the FI. Its income consists of wages, dividends on the shares, and interests on the deposit. The household uses the income to buy consumption goods and new shares, and the rest is deposited at the FI at the risk-free rate.

The last agent in the model is the FI. It collects deposits from the households at a risk-free rate and lends them to the firm as a business loan. Because I assume a competitive FI market, the FI's expected profit is zero.[9] As to the financial contract between the FI and the firm, I assume a standard one-period debt contract with default costs as in Cooley and Quadrini (2001) and Hennessy and Whited (2007).[10]

[8]As Frank and Goyal (2008) describe, the aggregate leverage ratio is very stable over time. This fact justifies the assumption that there is no aggregate shock.

[9]Actually, because I focus on a stationary economy, the ex post FI's profit is also always equal to zero owing to the law of large numbers.

[10]The assumption to limit the contract space to a one-period debt contract significantly simplifies the model, and the fact that the one-period debt contract is one of the most common financial contracts in the real economy justifies the assumption. However, the assumption excludes the following more general contract forms from the contract space. First, I exclude a dynamic lending contract under asymmetric information as in Quadrini (2004) and Clementi and Hopenhayn (2006). Second, I exclude one-period financial contracts outside a debt contract. Gale and Hellwig (1985) show that a debt contract would be optimal among general one-period contracts if information frictions between lenders and borrowers and a monitoring cost (or a default cost) exist. Unfortunately, I cannot directly utilize their result because the current model is a dynamic model with *persistent* idiosyncratic shocks whereas their model is a static model.

## 2.1 Firms

There is a continuum of firms producing final goods using assets and labor. In every period, after the firm produces final goods, it has the following three choices: continue, exit, or default. There is also a continuum of new entrants. In a stationary equilibrium, the distribution of the firms' characteristics is "stationary" in the sense that their entry and exit offset each other.

### 2.1.1 Technology, Labor Choice, and Profit

The firm uses two inputs, assets, $k$, and labor, $l$, to produce consumption goods. As to its technology, I assume a standard Cobb–Douglas production function:

$$y = zk^{\alpha_k}l^{\alpha_l},$$

where $z$ is a productivity idiosyncratic to each firm. I assume diminishing returns to scale, $\alpha_k+\alpha_l < 1$, to make firm size matter.[11] As a competitive consumption goods market is assumed, the price level of the consumption goods is the same for all firms. When normalizing the price level to one, the revenue (i.e., the price times the amount of sales) becomes just the amount of sales, $zk^{\alpha_k}l^{\alpha_l}$.

Next, I formulate the optimal labor choice of the firm as a static problem. A salient empirical regularity about the labor choice is that the autocorrelation process of labor is much more persistent than that of profit and leverage. One interpretation is that productivity shocks that affect the labor choice are more persistent than those that affect the firm's profit and leverage. In order to replicate those processes, I make the following two assumptions. First, I assume that the idiosyncratic productivity consists of two parts: a persistent component, $z_p$, and a transitory component, $\eta$:

$$z \equiv z_p \cdot \eta.$$

The persistent productivity follows the AR(1) process after log-transformation:

$$\log(z_p') = \rho \log(z_p) + \epsilon, \text{where} \ \ \epsilon \sim N(\mu_\epsilon, \sigma_\epsilon), \tag{1}$$

whereas the transitory component is independent and identically distributed (IID) and follows a normal distribution after log-transformation:

$$\log(\eta) \sim N(0, \sigma_\eta).$$

Then, the next period's labor choice becomes independent of the current period's transitory productivity shock, $\eta$, because it does not affect the next period's productivity. Second, I assume that the firm chooses the current period's labor *after* the realization of the persistent productivity, $z_p$, but *before* the realization of the transitory productivity, $\eta$. Then, the current period's labor choice

[11] If the technology involves constant returns to scale and there is heterogeneity in firms' productivity, it would be efficient that the firm with the highest productivity uses all assets and labor, and the firm size distribution would be degenerate.

is also independent of $\eta$. As a result of these two assumptions, the firm's labor process is affected only by the process of the persistent productivity shock, $z_p$, whereas the firm's profit and leverage choice are affected by both the persistent and transitory productivity shocks, $z_p$ and $\eta$. Therefore, the process of labor becomes more persistent than that of profit and leverage.[12]

Let $l^*$ be the optimal level of labor:

$$l^*(k; z_p, w) = \text{argmax}_l \Big\{ z_p k^{\alpha_k} l^{\alpha_l} - \underbrace{wl}_{labor\ cost} \Big\}, \tag{2}$$

where $w$ is a wage rate. Note that $z_p k^{\alpha_k} l^{\alpha_l}$ is the expected revenue of the firm at the moment it chooses the level of labor input because $\mathbb{E}[\eta] = 1$ and $Cov(z_p, \eta) = 0$.

Given the optimal choice of labor, $l^*(k; z_p, w)$, I define the firm's profit before an interest payment, a tax payment, and depreciation (so called EBITDA) as follows:

$$\pi(k; z, w) = z k^{\alpha_k} l^{*\alpha_l} - wl^* - \underbrace{c_f}_{fixed\ cost}. \tag{3}$$

I assume that the firm must pay a fixed cost, $c_f$, in every period as in Hopenhayn (1992). The fixed cost has two roles in the model. First, it induces economies of scale, which are observed in the real economy. Without the fixed cost, the firm's profitability measured by ROA would be almost independent of its size (that is, there would be no economies of scale), because productive firms become large (i.e., increase the denominator) and profitable (i.e., increase the numerator) simultaneously. Second, it gives unproductive firms an incentive to shut down their business and exit from the economy. Without the fixed cost, no firms would have an incentive to exit from the economy because the lower bound of the firm's profit would be zero.

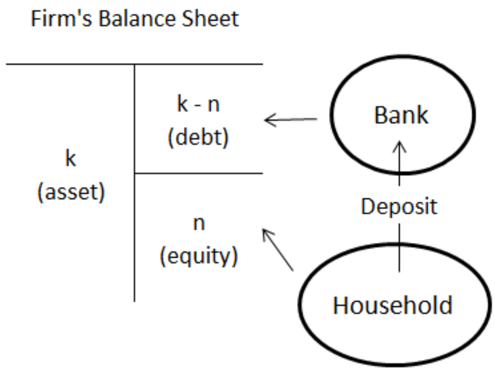
### 2.1.2 Evolution of the Firm's Balance Sheet

Figure 1 represents a firm's balance sheet at the *beginning* of the period, where $k$ represents physical assets and $n$ is the amount of equity at the beginning of the period. When the amount of assets is more than that of equity, i.e., $k - n > 0$, then $k - n$ is the amount of debt. The firm signs a debt contract, which is defined as a combination of the amount of debt and the interest rate assigned on the debt $(k - n, r)$, with the FI. The firm pays interests to the FI and dividends to the household. When the amount of assets is less than that of equity, i.e., $k - n < 0$, then $k - n$ is the firm's deposit at the FI. In this case, I assume that the return of the firm's deposit is equal to the risk-free rate, $r_f$.[13]

[12] This idea is based on the identification strategy in Gourio (2008). In the quantitative part of this paper, I calibrate the parameters of the two productivity processes so that the model replicates the difference in persistency between the labor process and the leverage process.

[13] Under this setting, the firm cannot have both debt and deposits simultaneously. As most firms have both of these in the real economy, it would be an interesting extension of the model to allow it.

Figure 1: Balance Sheet at the *Beginning* of the Period




Given the firm's EBITDA, which is denoted by $\pi(k; z, w)$ in (3), the firm's equity at the *end* of the period, $e(k, n; z, r, w)$, is determined as follows:

$$e(k, n; z, r, w) = (1 - \tau_c)\Big[\pi(k; z, w) - \delta k - r(k - n)\Big] + n, \tag{4}$$

where $\delta$ is the depreciation rate of the physical assets and $\tau_c$ is the corporate income tax rate. Intuitively, Equation (4) says that the equity at the *end* of the period is the sum of the firm's equity at the *beginning* of the period, $n$, plus its current profit, $(1 - \tau_c)\left[\pi(k; z, w) - \delta k - r(k - n)\right]$.

An important point here is that the corporate income tax system gives the firm a big incentive to use debt rather than equity due to the tax deductibility of interest payments. That is, the tax system enables the firm to decrease its corporate income tax payment by using more debt, $(k - n)$, because the corporate income tax is levied on the firm's income after the interest payments. The benefit of using debt is called the "tax benefit" in the capital structure literature.

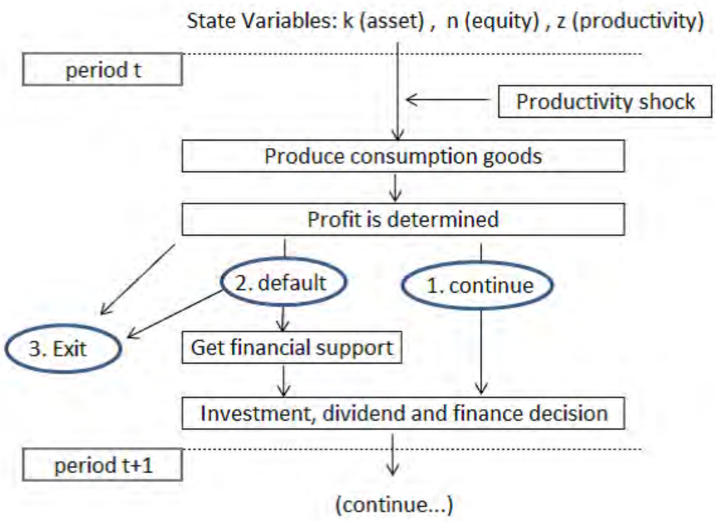
### 2.1.3 Dynamic Optimization

Figure 2 summarizes the timing of the firm's decision. Given the amount of equity at the end of the period, $e(k, n; z, r, w)$, the firm solves two dynamic optimization problems. The first one is the continue/exit/default decision and the second one is an investment decision. In the rest of this subsection, I first define the dividend in this model, and then explain the two dynamic optimization problems step by step.

Let me define the firm's dividend, $d(k', n'; e, k)$, as follows:

$$d(k', n'; e, k) = \begin{cases} (1 - \tau_d)[e(k, n; z, r) - (1 - \tau_c)g(k', k) - n'], & d \geq 0 \\ (1 + \lambda)[e(k, n; z, r) - (1 - \tau_c)g(k', k) - n'], & d < 0, \end{cases} \tag{5}$$

Figure 2: Timing of the Firm's Decision




where $k'$ and $n'$ are assets and equity in the next period, respectively. $g(k', k)$ is a downward adjustment cost, which the firm has to pay when it decreases the amount of assets from $k$ to $k'$:

$$g(k', k) = \max\{\xi((1-\delta)k - k'), 0\} \quad \text{where} \quad 0 \leq \xi < 1.$$

This type of adjustment cost is often called a partial investment irreversibility and it is included in many corporate finance and macroeconomics papers, including Abel and Eberly (1994) and Veracierto (2002). The above definition of the firm's dividend says that the dividend, $d$, is defined as what the firm owns at the end of the period, $e(k, n; z, r)$, minus what the firm keeps for the next period as its equity, $n'$, and the adjustment cost, $(1 - \tau_c)g(k', k)$. That is, the dividend is determined as a residual when the firm chooses its investment and financing sources, i.e., it chooses $k'$ and $n'$. When the amount of the dividend is positive, the firm has to pay the dividend tax. Its tax rate is denoted by $\tau_d$. On the other hand, when it is negative, it means that the amount of outside equity finance is positive. In this case, the firm has to pay a proportional flotation cost of equity, $\lambda$.[14]

### Exit Decision

As Figure 2 describes, after its profit is determined, the firm has three choices: continue, exit, or

[14] The flotation cost of equity can be interpreted as the fees paid to securities companies, the cost of asymmetric information, and so forth.

default. The discrete choice problem is formulated as follows:

$$\hat{v}(e,k;z_p) = \max\Big\{ \underbrace{v(e,k;z_p)}_{continue},\ \underbrace{d(0,0;e,k)}_{exit},\ \underbrace{0}_{default} \Big\},$$

where $v(e,k;z_p)$ is the firm's value when it decides to continue doing business. When the firm chooses to exit, it would sell all of its assets and payout $d(0,0;e,k)$, the dividend when $k' = n' = 0$. This means that the firm distributes the rest of the money to households *after* it pays back all of its debt to the FI. When the firm chooses to default on its loan, it gets nothing, but does not have to pay anything due to the limited liability assumption. Therefore, if the firm's liquidation value is lower than the amount of debt, the firm would choose to default rather than exit. I will explain the response of the FI towards the defaulting firms in the next section.

Let $h(e,k;z_p)$ be the policy function of the discrete decision problem above. $h(e,k;z_p) = 1$ if the firm continues doing business, $h(e,k;z_p) = 2$ if the firm exits, and $h(e,k;z_p) = 3$ if the firm defaults on its loan.

I assume that the firm that has chosen to continue doing business in the above *endogenous* exit/continue decision is hit by an *exogenous* exit shock with probability $\chi$. When the firm is hit by the exogenous exit shock, it must exit.[15] Given the exogenous exit shock, the value function $v(e,k;z_p)$ is defined as follows:

$$v(e,k;z_p) = (1-\chi)\cdot \tilde{v}(e,k;z_p) + \chi \cdot d(0,0;e,k),$$

where $\tilde{v}(e,k;z_p)$ is the value of the firm given that it continues doing business.

**Investment and Financing Decision**

The second maximization problem for the firm is the investment and financing decision. The firm faces this second problem only when it chooses to continue doing business and it is not hit by the exogenous exit shock. In this second problem, the firm simultaneously chooses the size of its balance sheet (i.e., the amount of assets, $k'$) and its capital structure (i.e., the amount of equity, $n'$) for the next period. The firm signs a one-period debt contract $(k'-n', r')$ with the FI to use debt financing. The value function, $\tilde{v}(e,k;z_p)$, is defined as follows:

$$\tilde{v}(e,k;z_p) = \max_{n',k',r'} \Big\{ d(k',n';e,k) + \beta \mathbb{E}_{z'_p|z_p,\eta}\left[\hat{v}(e(k',n';z',r'),k';z')\right] \Big\} \tag{6}$$

s.t. FI's zero profit condition,

where $d(k',n';e,k)$ is the amount of dividends, which is defined by (5), and $\beta$ is a discount factor. Note that I formulate the contractual problem as if the firm chooses the lending rate in a debt

[15] This shock is introduced in order to capture the fact that large firms as well as small firms also exit. Without the *exogenous* exit shock, only small firms would exit from the economy in the model, because firm size and profitability are strongly correlated and profitability is the only reason to exit in the model. The exogenous exit is also supposed to capture the fact that some firms exit for exogenous reasons (scandals, disasters, no successors, and so on) in the real economy.

contract, $r'$, subject to the FI's zero profit condition (i.e., an individual rationality condition for the FI). The dividend is determined as a residual when the firm chooses $k'$ and $n'$ given $e$, $k$, and $z_p$. The future value of the firm in this problem is $\hat{v}(e, k; z_p)$, the firm value before the continue/exit/default decision, because the maximization problem in the next period starts with the discrete choice again.

In this maximization problem, I assume that the firm must choose the asset size and capital structure so that the liquidation value of the asset plus its deposit must be more than the sum of the fixed cost, $c_f$, and the corporate income tax. That is, I assume that:

$$(1-\xi)(1-\delta)k' > c_f + \text{ corporate income tax}, \tag{7}$$

where $\xi$ is a downward adjustment const. This assumption prevents firms having a "wait and see" attitude. Without this assumption, unproductive firms would wait one period without producing anything in order to see their productivity in the next period rather than immediately exit, and then the firm distribution would have a strange shape.[16]

### 2.1.4 New Entrants

I assume that the potential entrants enter the economy by paying an entry cost, $c_e$. Their initial productivity follows a cumulative distribution function, $\zeta(z)$. The potential entrants enter the economy if:

$$\int_{\{z_p : v(0,0;z_p)>0\}} v(0,0;z_p) d\zeta(z_p) \geq c_e. \tag{8}$$

That is, they enter if the expected value of entry is higher than the entry cost. When the mass of entrants is positive, this condition should be satisfied with equality. Otherwise, an infinite number of entrants would enter the economy.

## 2.2 Financial Intermediary

I assume a competitive market for FIs. By doing so, the FI in the model can be reduced to the following zero profit condition:

$$\begin{aligned}(1+r_f)(k'-n') \quad = \quad \mathbb{E}_{z'_p|z_p}\Bigg[\int_{\underline{\eta}}^{x(z'_p)} & \Big[\pi(k'; z'_p, \eta', w') + (1-\gamma)(1-\delta)k' \\ & - \min\big\{b(k'; z'_p), g(0, k')\big\}\Big] d\Pi(\eta') \\ & + \big[1 - \Pi(x(z'_p))\big] \cdot (1+r')(k'-n')\Bigg]. \end{aligned} \tag{9}$$

[16]The firm prefers such an attitude because it does not have to pay anything, including the fixed cost and the corporate income tax, when the firm declares a default. The firm pays little cost for waiting one period compared with its option value to wait. This assumption is a technical one, but it can be interpreted as follows. The firm would not be trusted by business partners and would not be able to continue doing business unless it has enough physical assets to cover at least the fixed cost and the corporate income tax.

The FI's zero profit condition in the model is relevant to the real economy because the firm solves the contract problem (6) subject to the condition.[17]

The left-hand side of the equation is the FI's funding cost. The FI receives deposits from the households at the risk-free rate, $r_f$. The right-hand side of the equation is the FI's expected earnings from the debt contract $(k' - n', r')$. $\Pi(\eta')$ is a cdf of the transitory productivity shocks, $\eta'$. $x(z_p')$ is a default threshold of $\eta'$. That is, when the persistent productivity level is equal to $z_p'$, the firm chooses to default if and only if the transitory productivity shock, $\eta'$, is lower than $x(z_p')$.[18] The threshold is implicitly defined by:

$$\max\left\{v(e(k,n;x\cdot z_p,r),k;z_p)\ ,\ d(0,0;e(k,n;x\cdot z_p,r),k)\right\}=0.$$

The first and second lines of the right-hand side of Equation (9) represent the case of default. In the case of default, the FI first takes the firm's profit and assets, $\pi(k';z',w')+(1-\delta)k'$. Next, the FI has to pay a default cost, which is proportional to the amount of the assets, $\gamma(1-\delta)k$. Finally, the FI has two options about the defaulting firm's future: provide the minimum financial support (i.e., debt forgiveness), $b(k;z_p)$, in order for the firm to remain a going concern, or liquidate the firm by paying a liquidation cost, $g(0,k)$.

Intuitively, the first and second choices for the defaulting firm correspond to Chapters 11 and 7, respectively. When the FI chooses the first option (i.e., Chapter 11), the amount of the financial support, which is denoted by $b(k;z_p)$, is implicitly determined by the value function of the firm before the exogenous exit, $v(e,k,z_p)$, as follows:

$$v(b(k;z_p),k;z_p)=0.$$

This means that the FI gives $b(k;z_p)$ back to the defaulting firm so that the firm is indifferent between defaulting on the loan and continuing to do business. On the other hand, when the FI chooses the second option (i.e., Chapter 7), it has to pay a liquidation cost, $g(0,k)$, which is the downward adjustment cost to decrease the amount of the firm's assets to zero. The FI selects either of the two options by comparing the cost of each option, $b(k;z_p)$ and $g(0,k)$.

The FI's decision regarding the future of the defaulting firm depends on $z_p$ and $k$ in general, but under reasonable parameter values, the FI keeps the firm as a going concern (i.e., it chooses Chapter 11) in most cases in the model. The reason is that the firm voluntarily exits if it expects to default and it exits in the next period with high probability because the firm and the FI can reduce the default cost, $\gamma$. The FI chooses to liquidate the defaulting firm only when $b(k';z_p')$ unexpectedly reaches a high value due to a drastic decrease in productivity, $z_p'$.

[17] Under standard parameter values, the profit for the firm and the FI move in opposite directions with respect to the interest rate. Therefore, the zero profit condition is always binding when the firm optimally solves the contract problem.

[18] As both $v(e(k,n;z,r),k;z_p)$ and $d(0,0;e(k,n;z,r),k)$ are increasing functions with respect to $\eta$, the firm adopts such a threshold policy rule.

The third line of the right-hand side of the equation represents the case where the firm pays back the loan and the interest as promised in the debt contract. It contains the following two choices for the firm: to continue running the business or to exit from the economy. Note that the firm has to pay back all of its debt in both cases. The expected return in this case is equal to $[1 - \Pi(x(z'_p))] \cdot (1 + r')(k' - n')$ because its probability is $[1 - \Pi(x(z'_p))]$.

### 2.3 Household

I assume a representative household. It supplies the labor force, $L^s_t$, to the firm and obtains wages, $w_t L^s_t$. In addition, because the household owns all firms in the economy as a stockholder, it also obtains the aggregate dividend, $D_t$, as another source of its income. The household allocates the incomes to consumption, $C_t$, and savings at the FI, $S_t$, at the risk-free rate, $r_f$. The household faces the budget constraint:

$$C_t + S_{t+1} = [1 + r_f(1 - \tau_i)]S_t + D_t + w_t L^s_t + T_t, \tag{10}$$

where $\tau_i$ is the interest income tax rate and $T_t$ is a lump sum transfer from the government. The household maximizes its lifetime utility by consumption and labor supply. I assume log-utility for consumption and linear disutility for labor supply as in Hopenhayn and Rogerson (1993) and Gomes (2001). Then, the household maximization problem becomes:

$$\max_{L^s_t, S_{t+1}, C_t} \mathbb{E} \sum_{t=0}^{\infty} \beta^t [\log(C_t) - A L^s_t, ] \tag{11}$$

subject to (10). $\beta$ is a discount factor. The first-order conditions with respect to $L^s_t$ and $S_{t+1}$ are:

$$\frac{w_t}{C_t} = A \qquad \text{and} \qquad \frac{1}{C_t} = \beta \mathbb{E} \frac{1 + r_f(1 - \tau_i)}{C_{t+1}}. \tag{12}$$

In the quantitative part of this paper, I will focus only on the stationary equilibrium. As all aggregate variables and prices are constant in a stationary equilibrium, those first-order conditions are rewritten as:

$$\frac{w}{C} = A \qquad \text{and} \qquad \beta = \frac{1}{1 + r_f(1 - \tau_i)}, \tag{13}$$

and the budget constraint is:

$$C = r_f(1 - \tau_i)S + D + wL^s + T. \tag{14}$$

Under this budget constraint, $D$ and $T$ are exogenously given to the household. Then, given these two values and the wage, $w$, the household chooses $C$, $S$, and $L^s$. I will use the first-order conditions and the budget constraint to compute a stationary equilibrium.[19]

[19] In the above formulation of the household problem, the household obtains the aggregate dividend and does not choose the amount of the shares. Alternatively, I can formulate the household problem so that the household chooses

## 2.4 Aggregation and Market Clearing Conditions

Now that I have completed the description of the individual firm behavior and the household decision, I aggregate all firms and characterize a stationary equilibrium. In the stationary equilibrium, because all prices and aggregate variables are constant by definition, they are dropped from the list of state variables. Then, each firm can be specified by the following three variables, $(e, k, z_p)$: the amount of equity, the amount of assets, and the level of persistent productivity. Let $\mu(e, k, z_p)$ be the mass of firms at the state $(e, k, z_p)$. The law of motion of the firm distribution is:

$$\begin{aligned}
&\mu_{t+1}(e', k', z'_p) \\
= &\int_{(e,k,z_p,\eta,z'_p)} \left\{ \begin{array}{c} \mathbb{I}_{\{\eta \geq x(z'_p)\}} \cdot \mathbb{I}_{\{e'=e^*(n^*,k^*;\eta \cdot z'_p)\}} \cdot \mathbb{I}_{\{k'=k^*(e,k;z_p)\}} \cdot (1 - s_1(e^*, k^*; z'_p)) \\ +\mathbb{I}_{\{\eta < x(z'_p)\}} \cdot \mathbb{I}_{\{e'=b(k';z'_p)\}} \cdot \mathbb{I}_{\{k'=k^*(e,k;z_p)\}} \cdot (1 - s_2(k^*; z'_p)) \end{array} \right\} \\
&\qquad \times \mu_t(e, k, z) \cdot \Pr(z'_p | z_p) \cdot d\Pi(\eta) dz_p dk de \\
&+ M \int_{z_p} \mathbb{I}_{\{e'=e^*(n^*(0,0;z'_p),k^*;z'_p)\}} \cdot \mathbb{I}_{\{k'=k^*(0,0;z'_p)\}} \cdot (1 - s_3(z'_p)) d\zeta(z'_p), \qquad (15)
\end{aligned}$$

where $\mathbb{I}$ is an indicator function that $\mathbb{I} = 1$ if the inside of the brace is true. $k^*$ and $n^*$ are the firm's optimal policy functions for assets and equity at the state $(e, k, z_p)$. $s_1(e, k'; z'_p)$ and $s_2(k'; z'_p)$ are indicator functions that are equal to one when the firm at state $(e, k'; z'_p)$ chooses to exit in the case of default and no default, respectively. Similarly, $s_3(z'_p)$ is equal to one when the entrant chooses not to enter the economy. $e^*(n^*, k^*, \eta \cdot z'_p)$ is the amount of equity at the end of the period when the firm optimally chooses the amount of assets, $k^*$, and the amount of equity, $n^*$. The first line of the inside of the integral represents the case of default and the second line represents the case of no default. The last term on the right-hand side represents new entrants. $M$ is the mass of new entrants. Note that the amounts of equity and assets for the new entrants are zero. A stationary distribution is a distribution $\mu^*$ satisfying $\mu_{t+1} = \mu_t = \mu^*$. Practically, it is derived by repeatedly applying the above law of motion to an arbitrary distribution until the distribution converges to the stationary distribution.

Once we derive the stationary distribution, $\mu^*$, the aggregate asset, equity, labor demand, and

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the amount of shares in every period. As the household's behavior in the alternative formulation eventually gives the same allocation *in a stationary equilibrium*, the difference between the two formulations does not matter for the quantitative results. However, by explicitly formulating the endogenous choice regarding the shares by the household, the following two things can be derived as a result of the household's optimization. First, the return on equity is equal to the risk-free rate, $1 + r_f(1 - \tau_i)$, in equilibrium. While this sounds a little strange because it means that an equity premium is equal to zero, it is a natural consequence of the household's optimal portfolio choice *without* aggregate uncertainty. Second, the discount rate, $\beta$, for the household is equal to that for the firm. In general, the firm's discount rate should be stochastic because it depends on the household consumption level. However, in this paper, because I assume that there is no aggregate uncertainty and focus just on a stationary equilibrium, the discount factor for the firm becomes constant and equal to $\beta$.

output is defined as follows:

$$\begin{aligned}
\text{Asset} \quad &: \quad \mathbf{K} = \int k^*(e,k;z_p)\mu(e,k;z_p)dedkdz_p, && (16)\\
\text{Equity} \quad &: \quad \mathbf{N} = \int n^*(e,k;z_p)\mu(e,k;z_p)dedkdz_p, && (17)\\
\text{Dividend} \quad &: \quad \mathbf{D} = \int d(k^*(e,k;z_p),n^*(e,k;z_p);e,k)\mu(e,k;z_p)dedkdz_p, && (18)\\
\text{Labor Demand} \quad &: \quad \mathbf{L}^d = \int l^*\big(k^*(e,k;z_p);z_p'\big)\mu(e,k;z_p)Pr(z_p'|z_p)d\Pi(\eta)dedkdz, && (19)\\
\text{Output} \quad &: \quad \mathbf{Y} = \int \big[\eta z_p' k^*(e,k;z_p)^{\alpha_k} l^*(k^*(e,k;z_p);z_p')^{\alpha_l}\\
&\qquad - c_f\big]\mu(e,k;z_p)Pr(z_p'|z_p)d\Pi(\eta)dedkdz. && (20)
\end{aligned}$$

In addition, the aggregate adjustment cost caused by frictions is as follows:

$$\begin{aligned}
\text{Adj. Cost} \quad : \quad \mathbf{G} = &\int g(k^*(e,k;z_p),k)\mu(e,k;z_p)dedkdz_p\\
&+ \int g(0,k^*(e,k;z_p))\Big[\mathbb{I}_{\{\eta\geq x(z_p')\}}\cdot s_1(e^*,k^*;z_p') + \mathbb{I}_{\{\eta<x(z_p')\}}\cdot s_2(k^*;z_p')\Big]\\
&\quad \times Pr(dz_p'|z_p)d\Pi(\eta)\mu(e,k,z)dedkdz\\
&+ \int \mathbb{I}_{\{e^*<n^*(e^*,k^*;z_p')\}}\lambda(e^*-n^*(e^*,k^*;z_p'))\mu(e,k;z_p)Pr(z_p'|z_p)d\Pi(\eta)dedkdz\\
&+ \int \mathbb{I}_{\{\eta<x(z_p')\}}\gamma k^*\mu(e,k;z_p)Pr(z_p'|z_p)d\Pi(\eta)dedkdz. \qquad (21)
\end{aligned}$$

The first line is the downward adjustment cost of assets. The second line is the liquidation cost for exiting firms. The third line is the flotation cost of equity and the fourth line is the default cost. All costs are aggregated using the stationary distribution and assumed to be thrown away into the sea. The last aggregate variable is the tax revenue:

$$\begin{aligned}
\text{Tax Revenue} \quad : \quad \mathbf{T} = &\int \tau_c\cdot(\pi(k^*;\eta z_p',w) - \delta k^* - r^*(k^*-n^*) - g(k^*(e^*,k^*;\eta z_p'),k^*))\\
&\quad \times Pr(dz_p'|z_p)d\Pi(\eta)\mu(e,k,z)dedkdz\\
&+ \int \tau_d\cdot\mathbb{I}_{\{d(k^*,n^*;e,k)>0\}}\cdot d(k^*(e,k;z_p),n^*(e,k;z_p);e,k)\mu(e,k,z)dedkdz\\
&+ \int \tau_i\cdot r_f\cdot(\mathbf{K}-\mathbf{N})\mu(e,k;z_p)dedkdz. \qquad (22)
\end{aligned}$$

The first, second, and third lines represent the corporate income tax, the dividend tax, and the interest income tax, respectively.

Next, I characterize the market clearing conditions for the following three markets: labor, consumption goods, and savings. First, the market clearing condition for the savings market is:

$$S = \mathbf{K} - \mathbf{N}. \qquad (23)$$

The left-hand side is the savings by the representative household and the right-hand side is the aggregate debt owned by firms. This equation means that all savings are used as debt in the firm's balance sheet. Next, the market clearing condition for the labor market is:

$$L^s = \mathbf{L}^d. \tag{24}$$

Finally, the market clearing condition for the consumption goods market is:

$$C = \mathbf{Y} - \delta\mathbf{K} - \mathbf{G}. \tag{25}$$

This condition says that the aggregate consumption is equal to the aggregate output minus the depreciation of assets and the adjustment costs. Note that $\delta\mathbf{K}$ is equal to the amount of investment in the stationary equilibrium, and then the gross domestic product (GDP) is:

$$\text{GDP} = C + \delta\mathbf{K}.$$

## 2.5 Stationary Competitive Equilibrium

I close the model by characterizing a stationary competitive equilibrium as follows:

**Definition 1** *A stationary competitive equilibrium is a set of (i) allocation rules for labor, saving, and consumption for the household, $L^s(\boldsymbol{D}, \boldsymbol{T}; w)$, $S(\boldsymbol{D}, \boldsymbol{T}; w)$, and $C(\boldsymbol{D}, \boldsymbol{T}; w)$, (ii) allocation rules for labor, assets, and equity for each firm, $l^*(k; z_p, w)$, $k^*(e, k; z_p, w)$, and $n^*(e, k; z_p, w)$, (iii) a continue/exit/default decision for each firm, $h(e, k; z_p)$, (iv) value functions for each firm, $\hat{v}(e, k; z_p)$, $v(e, k; z_p)$, and $\tilde{v}(e, k; z_p)$, (v) aggregate variables, $\boldsymbol{K}, \boldsymbol{N}, \boldsymbol{L}^d, \boldsymbol{D}$, $\boldsymbol{Y}, \boldsymbol{G}$, and $\boldsymbol{T}$, (vi) a wage rate, $w$, and a lending rate, $r$, and (vii) a stationary distribution, $\mu^*(e, k, z_p)$, and the mass of entrants, $M$, such that the following apply.*

1. *The household decision rules satisfy its FOCs and the budget constraint.*
2. *The firm's decision rules, a lending rate, and value functions solve the maximization problems for each firm.*
3. *The market clearing conditions are satisfied.*
4. *The free entry condition is satisfied.*
5. *The aggregation rules (i.e., consistency) are satisfied.*
6. *The stationary distribution, $\mu^*$, satisfies the law of motion with $\mu_{t+1} = \mu_t = \mu^*$.*

Table 1: Calibration 1

| Parameters | Values | |
|---|---|---|
| Discount rate, $\beta$ | 0.966 | Risk-free rate = 0.04 |
| Labor disutility, $A$ | 1.28 | Labor supply = 0.6 |
| Return to scale, $\alpha_k + \alpha_l$ | 0.85 | Atkeson and Kehoe (2005) |
| Technology, $\alpha_k$ | 0.25 | Investment/Output = 0.16 |
| Depreciation, $\delta$ | 0.078 | Capital/Output = 2.0 |
| Scrapping cost, $\xi$ | 0.41 | Hennessy and Whited (2005) |
| Equity funding cost, $\lambda$ | 0.059 | Hennessy and Whited (2005) |
| Default cost, $\gamma$ | 0.07 | Estimate by World Bank |
| Exogenous exit rate, $\chi$ | 0.02 | Exit rate for large firms |

# 3 Stationary Equilibrium

In this section, I compute a stationary competitive equilibrium. To begin with, I calibrate the model based on the US data. After the calibration, I numerically compute a stationary equilibrium, and then investigate the corporate capital structure choice in the stationary equilibrium using the following two steps. First, I describe the dispersion of leverage in the model and compare it with the data. Second, I explore the relationship between leverage, firm size, and profitability. I conduct some regressions using both real economic data and artificial data generated from the stationary distribution, and compare the regression results.

The algorithm to compute a stationary equilibrium is based on Hopenhayn and Rogerson (1993) and it is summarized in Appendix A. As for the data source, I use COMPUSTAT in this paper. See Appendix C for more detail about the data and definitions of variables.

## 3.1 Calibration

I set one period in the model to one year, and then I set the risk-free rate, $r_f$, to 4%.[20] By the Euler equation of the household, the discount rate for the household, $\beta$, is equal to $1/(1+r_f(1-\tau_i))$ because I focus solely on a stationary equilibrium. In the baseline model, the wage, $w$, is set to 1.0 and the labor disutility parameter, $A$, is chosen so that an equilibrium labor supply is equal to 0.6, which is the average employment rate in the US, as in Hopenhayn and Rogerson (1993). For technology parameters, I set the degree of diminishing returns, $\alpha_k + \alpha_l$, to 0.85 as in Veracierto (2002) and Atkeson and Kehoe (2005). Then, I choose $\alpha_k$ so that the aggregate investment–output

[20]This rate is a little higher than the risk-free rate in the real economy but because there is no aggregate uncertainty and the equity premium is equal to zero in this model, this return is interpreted as a more general return in the economy.

Table 2: Calibration 2

| Parameters | Calibration target |
|---|---|
| Fixed cost, $c_f$ | Total exit rate = 0.07 |
| The persistent productivity shocks, $z_p$ | |
| AR(1) parameter, $\rho$ | Autocorrelation of employee size |
| Mean , $\mu_\epsilon/1-\rho$ | Average employee size |
| Standard deviation, $\sigma_\epsilon$ | Std. of residuals in employment process |
| Std. of the transitory shocks, $\sigma_\eta$ | Autocorrelation of leverage |
| New entrant's distribution, $\zeta(z_p)$ | |
| Mean | Unconditional mean of $z_p$ |
| Standard deviation | Entrants' size distribution |

ratio is equal to 16%. $\alpha_k = 0.25$ gives the target value of the aggregate investment–output ratio, and then $\alpha_l = 0.85 - 0.25 = 0.6$. Finally, I set $\delta = 0.078$ so that the aggregate capital–output ratio in the stationary equilibrium is equal to 2.0. The target investment–output ratio and the target capital–output ratio are taken from NIPA data for the last 15 years in the US.

Next, I calibrate the friction parameters, which are relatively specific to this model. First, I set the flotation cost of equity and the downward adjustment cost of assets to $\lambda = 0.059$ and $\xi(1-\tau_c) = 0.41$ according to Hennessy and Whited (2005). I choose the default cost, $\gamma = 0.07$, according to the "Doing Business" database at the World Bank, which states that the default cost in the US is about 7% of a defaulting firm's estate value. I set the exogenous exit rate, $\chi$, to 2%, which is the exit rate for firms with more than 150,000 employees in COMPUSTAT because the exogenous exit is introduced to capture the fact that some large firms choose to exit. Given this exogenous exit rate, I calibrate the fixed cost, $c_f$, so that the total exit rate, including the exogenous one, is equal to 7%, which is the total exit rate calculated by COMPUSTAT in the last five years.[21] The parameters up to this point are summarized in Table 1.

For the stochastic processes of productivity, the unconditional mean of the persistent productivity shock, $\mu_\epsilon$, is first chosen so that the average firm size measured by the number of employees matches that in COMPUSTAT. I can use firm size as a calibration target because firm size and productivity are strongly correlated. Next, I calibrate the AR(1) parameter $\rho$ and the standard deviation $\sigma_\epsilon$ so that the autocorrelation of the labor process and the standard deviation of residuals

[21] It may be observed that this is lower than the exit rate computed by the US Census data, which is around 9%. This difference stems from the fact that COMPUSTAT consists of relatively high performing firms because this database contains only listed firms in the US. I assume that firms exit from the economy at period $t$ if they existed in period $t-1$, but they do not exist in period $t$. Of course, they disappear from the database for other reasons such as mergers or no longer listing, but I think that the value is a rough proxy for the exit rate.

Table 3: Tax Rates

| Parameters | Values |
|---|---|
| Corporate incomes, $\tau_c$ | |
| Current profit $> 0$, $\tau_c^h$ | 0.35 |
| Current profit $< 0$, $\tau_c^h$ | 0.20 |
| Dividends, $\tau_d$ | 0.12 |
| Interest incomes, $\tau_i$ | 0.296 |

in the autocorrelation process of labor match those in COMPUSTAT.[22] I can use the autocorrelation process of labor as a calibration target because the firm's labor choice is affected only by the persistent productivity and is independent of the transitory productivity in the model. According to the procedure, I choose $\rho = 0.97$ and $\sigma_\epsilon = 0.115$.

For the transitory productivity shocks, because the firm's leverage is affected by both persistent and transitory productivity in this model, I set the standard deviation of transitory productivity, $\sigma_\eta$, so that the model accounts for the autocorrelation process of leverage. Given the parameter values for the persistent productivity process, $\sigma_\eta = 0.35$ makes the model's leverage process match closely to the data.[23]

For the productivity distribution of entrants, $\zeta(z_p)$, I assume that the distribution is the normal distribution. Then, the parameters to be specified are the mean and variance of the distribution. First, I set the mean of the entrant's productivity distribution to the unconditional mean of productivity, $\mu_\epsilon/1-\rho$, as in Gomes (2001). Given this value of the mean, I calibrate the variance of the entrant's productivity distribution so that the size distribution of entrants matches that in COMPUSTAT. The calibration targets of parameter values up to this point are summarized in Table 2.

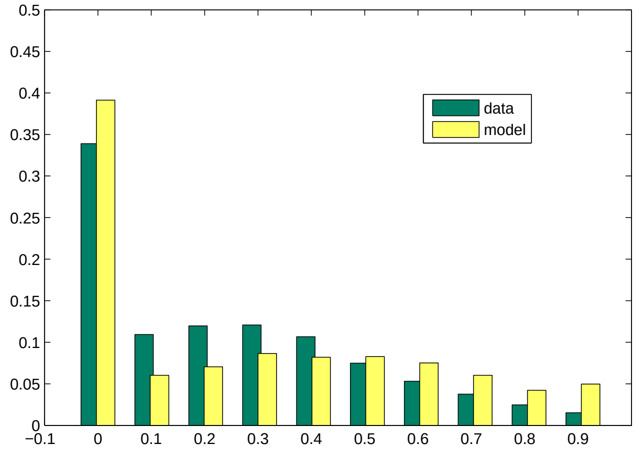
For the tax rates in the model, I set the dividend tax rate, $\tau_d$, and the interest income tax rate, $\tau_i$, to 12% and 29.6%, respectively, based on Graham (2000). In addition, I set the corporate income tax rate, $\tau_c^h$, to 35% for firms with positive profits based on Graham (2000). However, on the other hand, I set the corporate income tax rate for firms with negative profit, $\tau_c^l$, to 20% because the corporate income tax system in many countries adopts a progressive tax rate system.[24] This corporate income tax system is still simple compared with the tax system in the real economy, but it captures the progressivity in the corporate income tax system and affects the quantitative

[22]This calibration strategy is the same as in Hopenhayn and Rogerson (1993) except that they use establishment data rather than firm data.

[23]The autocorrelation and the standard deviation of residuals are 0.83 and 0.14 in the data and 0.83 and 0.15 in the model.

[24]I choose a nonzero value for $\tau_c^l$ because the loss in the current period will be deducted from the future taxable income. Thus, I set the corporate income tax rate for exiting firms with negative profits to zero.

Figure 3: Histogram of Leverage




Note: The figure shows the histograms of firms' leverage in COMPUSTAT and in the model. It shows that the stationary distribution in the model accounts for the fact that (1) more than 30% of firms are almost-zero leverage firms, and (2) leverage of nonzero leverage firms differs considerably from firm to firm. The simple average of leverage is 0.29 in COMPUSTAT and it is 0.29 in the model.

results in the model. The tax rates used in the model are summarized in Table 3.

### 3.2 Result 1: Distribution of Leverage

Figure 3 illustrates the histograms of firms' leverage in the data and in the model. The stationary distribution in the model accounts for the fact that (1) more than 30% of firms are almost-zero leverage firms, and (2) leverage of nonzero leverage firms differs considerably from firm to firm. As I do not use cross-sectional moments of leverage as calibration targets, I conclude that the model captures the mechanism to generate the dispersion of leverage fairly well.

### 3.3 Result 2: Relationship between Leverage, Firm Size, and Profitability

Next, I investigate the empirical relationship between leverage, firm size, and profitability. I conduct several regressions using artificial data generated from the stationary distribution, and compare the estimation results with the estimations using the real economic data.

I estimate the following reduced form equations, which are familiar in empirical corporate finance papers such as Rajan and Zingales (1995) and Fama and French (2002):

$$\begin{aligned}
\text{Leverage}_i &= \beta_0 + \beta_1 \log(\text{Employee}_i) + \epsilon_i, && (26)\\
\text{Leverage}_i &= \beta_0 + \beta_1 \text{ROA}_i + \epsilon_i, && (27)\\
\text{Leverage}_i &= \beta_0 + \beta_1 \text{ROA}_i + \beta_2 \log(\text{Employee}_i) + \epsilon_i, && (28)
\end{aligned}$$

where $i$ represents an individual firm. In the first (second) equation, I estimate a simple relationship between leverage and firm size (profitability). The number of employees and ROA are proxies for firm size and profitability, respectively. In the last equation, I estimate a marginal effect of firm size and profitability on leverage after controlling for the other variable. As econometricians usually control for many variables to measure a marginal effect, the last equation is the most familiar equation in empirical corporate finance literature.

**Estimation Using Real Data**

Table 4 shows the results of the estimation using the real economic data. First, it shows that the simple correlation between leverage and firm size is positive. Second, it shows that the simple correlation between leverage and profitability is also positive. This positive correlation between leverage and profitability may be a little surprising because previous empirical papers do not focus on such a simple correlation. Third, it shows that the coefficient on ROA turns out to be *negative* when I limit the data to firms larger than the average. This result is consistent with Rajan and Zingales (1995), who point out that the relationship between leverage and profitability tends to be negative as firm size becomes large. Finally, the estimation shows that when I estimate the relationships of leverage with firm size and profitability simultaneously, the coefficient on profitability is negative. Many academic researchers have tried to theoretically explain the negative relationship between leverage and profitability after controlling for other characteristics of firms because the negative relationship is difficult to be justified on the grounds of the trade-off between the tax benefit and the financial distress cost.

**Estimation Using the Model Output**

Next, I estimate equations (26)–(28) using the model output by conducting the following two steps. First, I randomly draw artificial data from the equilibrium stationary distribution. Second, I conduct the regressions using the artificial data. This procedure is the same as in Gomes (2001).

Table 4 shows the estimation results using the artificial data generated from the stationary distribution. It shows that while the magnitudes of coefficients are slightly different from those in the estimation using the real economic data, the artificial data from the model accounts for the sign of the relationship between leverage, firm size, and profitability. The coefficient on firm size and profitability is positive when I estimate the simple correlation between leverage and those variables, but the coefficient on profitability turns out to be negative once I limit the data to large firms or once I control for firm size.

Table 4: Estimation Results

**Data**

| Variable | Book Leverage | | | |
|---|---|---|---|---|
| log(Employee) | 0.026 | – | (Large firms only) | 0.030 |
| | [0.025 0.027] | | | [0.029 0.031] |
| ROA | – | 0.068 | -0.334 | -0.066 |
| | | [0.059 0.077] | [-0.356 -0.312] | [-0.076 -0.057] |
| adj. $R^2$ | 0.16 | 0.11 | 0.14 | 0.16 |
| Obs. | 53,874 | 50,642 | 29,981 | 50,642 |

**Model**

| Variable | Book Leverage | | | |
|---|---|---|---|---|
| log(Employee) | 0.019 | – | (Large firms only) | 0.021 |
| | [0.017 0.021] | | | [0.019 0.024] |
| ROA | – | 0.024 | -0.109 | -0.037 |
| | | [0.013 0.036] | [-0.137 -0.082] | [-0.050 -0.024] |

Note: I conduct the estimate using ordinary least squares. The contents of the brackets show the 95% confidence intervals. For the real economic data, I use the pooling panel data of COMPUSTAT from the last 20 years. See Appendix C for more detail. I drop observations as outliers if their ROA is more than the upper 3% tile or less than the lower 3% tile in both the data and the model output. "Large firms only" means that I drop the observation from the dataset if the firm size is smaller than the average. When I conduct the estimate using the real economic data, I add a time dummy for each year and an industry dummy based on SIC codes. When I conduct the estimate using the model output, I randomly draw 50,000 samples from the stationary distribution, which is close to the number of samples in the real economic data. The number of draws does not change the estimation results significantly.

This estimation result is one of the main results of the paper. Some other structural models also succeed in accounting for the negative relationship between leverage and profitability, which has been a big puzzle in the corporate finance literature. However, as far as I know, a structural model to *simultaneously* account for the sign of the relationship between leverage, profitability, and firm size does not exist. In particular, while the size dependency of the relationship between leverage and profitability is pointed out by empirical works, its microfoundation has not been investigated or even mentioned in previous theoretical papers.

## 3.4 The Logic Behind the Results

What is the logic behind the distribution of leverage and the relationship between leverage, firm size, and profitability? In this subsection, I first describe a firm's leverage behavior by looking at an individual firm's policy functions. Then, given the leverage behavior, I interpret the results that I have obtained above.

### 3.4.1 The Firm's Leverage Behavior

To investigate a firm's leverage behavior, I separately show a firm's optimal policy in relation to its equity (the numerator) and assets (the denominator). I then describe the leverage behavior as a combination of these two policy functions.

Figure 4 shows the policy function of equity, $n_{t+1}$, with respect to the amount of equity at the end of the previous period, $e_t(= n_t + \pi_t)$. There are two notable features in this figure. First, it shows that the optimal amount of equity is on the 45 degree line in most cases for both low and high productivity firms. This behavior implies that the firm uses its internal funds as much as possible for investment, and when the internal funds become insufficient for the investment, the firm fills its financing deficit (the gap between investment and the amount of internal funds) mainly by debt. That is, the firm's preference is:

$$\text{Internal Fund} \succeq \text{Debt} \succeq \text{Outside Equity}.$$

This behavior is known as the "pecking order" and it is consistent with the empirical findings of many papers.[25] The pecking order behavior stems from the costs of outside equity such as the flotation cost of equity and the dividend tax.[26] While these costs obviously make firms prefer debt to outside equity, they also make firms prefer internal funds to debt because the costs operate as a form of financial distress costs. That is, because the outside equity is one of the important financing sources for dealing with financial distress, the costs associated with outside equity make debt unattractive and give the firm an incentive to use its internal funds to prepare for future

[25] For example, see Shyam-Sunder and Myers (1999) and Leary and Roberts (2005).

[26] Stiglitz (1973) is the first paper showing the costs of outside equity inducing the pecking order preference.

financial distress.[27] Second, Figure 4 also shows that the firm uses the outside equity only if its productivity is low *and* its internal funds are small.[28] The equilibrium lending rates by firm productivity in Figure 5 give an intuition behind this behavior.[29] The figure shows that the lending rate is increasing with respect to leverage particularly for a low productivity firm. Such a high lending rate for a low productivity firm forces it to decrease its leverage by increasing the outside equity up to the point where the firm can use debt at a reasonable lending rate.

Next, I describe a firm's optimal behavior in regard to its asset size, $k_{t+1}$. Figure 6 is a three-dimensional graph showing the policy function for assets. It shows that the optimal size of assets is monotonically increasing with respect to the firm's productivity, but it is barely affected by the amount of internal funds, except in the case of firms with very low productivity.[30] It implies that the outside financing constraint is relevant only to the investment decisions of very low productivity firms and it is irrelevant to others.[31]

Given the policy function for equity, $n_{t+1}$, and asset size, $k_{t+1}$, I describe a firm's leverage behavior, $(k_{t+1} - n_{t+1})/k_{t+1}$. Figure 7 shows the policy function of leverage. The figure has the following two noticeable features, which have an implication about the relationship between leverage and productivity.

The first feature is that firms with high persistent productivity are more leveraged in equilibrium (the solid line is above the dotted line). This implies:

$$\mathrm{Corr}(z_p, \mathrm{lev}) > 0,$$

where "lev" is a firm's leverage and $z_p$ is the persistent productivity. There are two reasons for the relationship. The first reason is that the optimal asset size (and the financing deficit, as a result) of high productivity firms is large, and the financing deficit is usually filled by debt, according to

[27] This conservative behavior by firms in relation to debt finance seems to correspond to the criteria of "financial flexibility" that CFOs of US firms nominate as one of the most important determinants of corporate capital structure in the survey by Graham and Harvey (2001). That is, the behavior relates to the desire of firms to "remain flexible in the sense of minimizing interest obligations, so that they do not need to shrink their business in case of an economic downturn."

[28] Because firms with low productivity and low internal funds are usually small, less profitable, and much less leveraged, this behavior is consistent with the empirical findings by Leary and Roberts (2010), who argue that firms issuing new outside equity in violation of the pecking order are small, less profitable, and much less leveraged. See also Frank and Goyal (2003) and Lemmon and Zender (2009) for similar results.

[29] It may be noticed that the level of credit spreads is small for all firms compared with the actual data. The tight credit spreads in this model are not surprising, as I do not include aggregate uncertainty in this model. Papers such as Chen (2010) and Gomes and Schmid (2010) argue that aggregate uncertainty is a key component accounting for a plausible level of credit spread. Incorporating aggregate uncertainty to account for the level of credit spreads is an interesting potential extension of this model.

[30] Note that there is an inaction area where the optimal amount of assets is flat, which is common for a model with investment irreversibility.

[31] This is consistent with the recent findings about the firm size distribution. Angelini and Generale (2008) use Italian firm data and argue that financial constraints are important for small firms, but play little role in determining the firm size distribution as a whole.

Figure 4: Policy Function of Equity

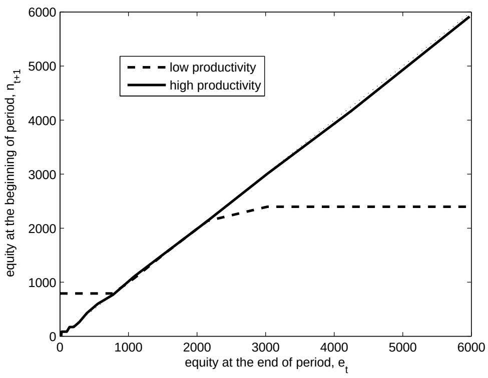



Figure 5: Policy Function of the Lending Rate

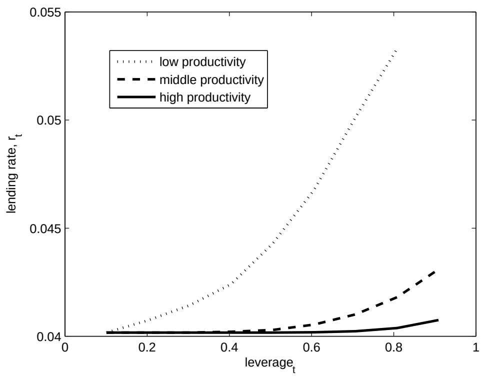



Figure 6: Policy Function of Assets

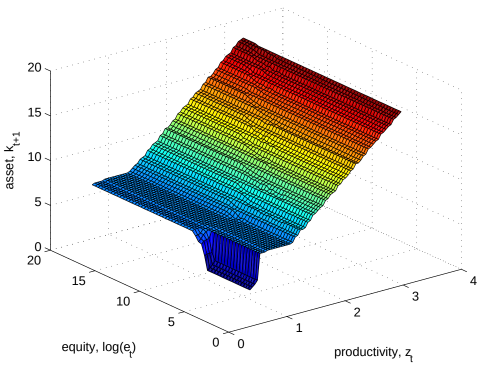
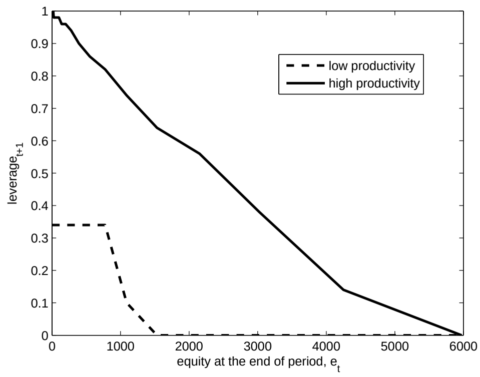



Figure 7: Policy Function of Leverage




Note: Figure 4 shows the policy function of equity for the next period, $n_{t+1}$, with respect to the amount of equity at the end of the current period, $e_t$. The solid line is the policy function for a high productivity firm and the dotted one is that for a low productivity firm. Figure 5 shows the equilibrium lending rates with respect to the firm's leverage. The horizontal axis is the firm's leverage, and the dotted, dashed, and solid lines are the lending rates for the low, middle, and high productivity firms, respectively. Figure 6 shows the three-dimensional policy function of assets, $k_{t+1}$. The x- and y-axes indicate the amount of equity at the end of period, $e_t$, and the productivity, $z_{p,t}$. Figure 7 shows the policy function of leverage. The horizontal axis shows the amount of equity, $e_t$. Again, the solid line is the policy function for a high productivity firm and the dotted one is that for a low productivity firm.

the pecking order. The second reason is that debt is less accessible for low productivity firms than it is for high productivity firms because of the high equilibrium lending rates for low productivity firms, as shown in Figure 5.

The second feature is that the policy functions of leverage are decreasing with respect to the amount of equity, $e_t$. The reason is as follows. When $e_t$ increases, firms tend to use more internal funds (i.e., equity), according to the pecking order, whereas they do not change the optimal size of their assets, as is shown in Figure 6. This feature implies that the transitory productivity, $\eta$, decreases a firm's leverage because it just increases the firm's internal funds through an increase in its profit, $\pi$. That is:

$$\mathrm{Corr}(\eta, \mathrm{lev}) < 0.$$

In the next subsection, I use the relationship between leverage and the two types of productivity for interpreting the result that I have obtained.

### 3.4.2 Interpretation of the Results

First, given the behavior of leverage, I investigate the logic behind the stationary distribution of leverage. In particular, I describe the logic behind the following two notable features of the distribution.

The first feature is that many firms, more than 30%, are almost-zero leverage firms. There are two reasons for this. First, because the average firm profit must be positive (otherwise, the firms would choose to exit from the economy), the pecking order behavior implies that firms accumulate their internal funds and as a result decrease their leverage as time goes on. Second, as some low productivity firms cannot use debt due to high lending rates, they remain less leveraged.

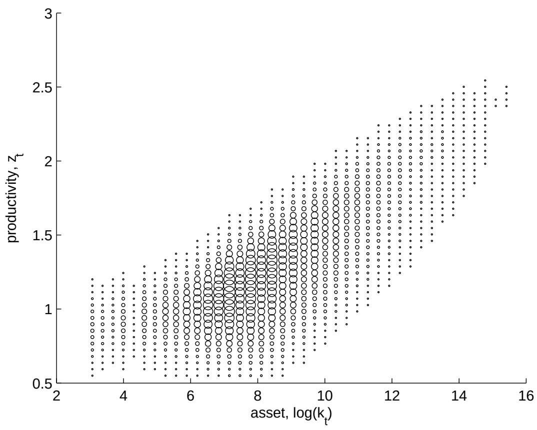
The second feature is that the leverage of nonzero leverage firms considerably differs from firm to firm. Because I have already explained why some firms are less leveraged, I will explain why some firms are highly leveraged. The logic is as follows. As the optimal asset size is barely affected by the amount of internal funds, as shown in Figure 6, firms with low internal funds increase their assets just by adjusting the leverage. For example, if firms are suddenly hit by a good productivity shock or if firms are too young to accumulate their internal funds, they would be highly leveraged.

Next, I describe the logic behind the second result: the relationship between leverage, firm size, and profitability. First, I explain why leverage and firm size are positively correlated. I claim that this positive correlation is a combination of the two correlations. That is:

$$\mathrm{Corr}(z_p, \mathrm{lev}) \ \ \& \ \ \mathrm{Corr}(z_p, \mathrm{size}) > 0 \quad \Rightarrow \quad \mathrm{Corr}(\mathrm{lev}, \mathrm{size}) > 0,$$

where "lev" is a firm's leverage and $z_p$ is the persistent productivity. I have already explained the logic behind $\mathrm{Corr}(z_p, \mathrm{lev}) > 0$. Also, I can establish easily that $\mathrm{Corr}(z_p, \mathrm{size}) > 0$ because the optimal firm size becomes almost proportional to the productivity according to a firm's profit maximization. Figure 8 represents the stationary joint distribution of productivity and asset size. The figure shows a clear positive relationship between them.

Figure 8: Distribution of Productivity and Assets




Note: This represents the stationary joint distribution of productivity and asset size. The horizontal axis shows the amount of assets, $k$, in log-scale and the vertical axis shows the persistent part of productivities, $z_p$. The figure shows a clear positive relationship between them.

Next, I consider the logic behind the relationship between leverage and profitability measured by ROA. In the model, it could be either positive or negative. On the one hand, leverage and profitability could be *positively* correlated as the combination of the two positive correlations:

$$\text{Corr(lev, size)} > 0 \;\;\&\;\; \text{Corr(size, ROA)} > 0 \;\;\Rightarrow\;\; \text{Corr(lev, ROA)} > 0.$$

I already explained the logic behind Corr(lev, size) $> 0$. I can establish Corr(size, ROA) $> 0$ by the economies of scale, which emerges as a result of the fixed cost, $c_f$, in this model.[32] Figures 9 and 10 are the joint distributions of ROA and the log of employment size in the data and the model, respectively. Both figures show similar "economies of scale", which seem to be caused by the fixed cost: ROAs of large firms are higher than those of small firms, but the larger the firm size, the smaller the economies of scale.

On the other hand, a firm's leverage and profitability could be *negatively* correlated through the transitory productivity, $\eta$:

$$\text{Corr}(\text{lev}, \eta) < 0 \;\;\&\;\; \text{Corr}(\eta, \text{ROA}) > 0 \;\;\Rightarrow\;\; \text{Corr(lev, ROA)} < 0.$$

The logic behind Corr(lev, $\eta$) $< 0$ was explained in the previous subsection. The transitory productivity, $\eta$, and ROA are positively correlated because the transitory productivity increases the

[32]Without the fixed cost, the relationship between ROA and firm size would be ambiguous and almost uncorrelated, because both the denominator and the numerator of ROA (i.e., firm size and EBITDA) would increase at the same pace as the persistent productivity, $z_p$, increases. With the fixed cost, however, ROA and firm size are positively correlated because the firm's EBITDA would increase faster than its size as the persistent productivity increases.

Figure 9: ROA and Employment Size (Data)

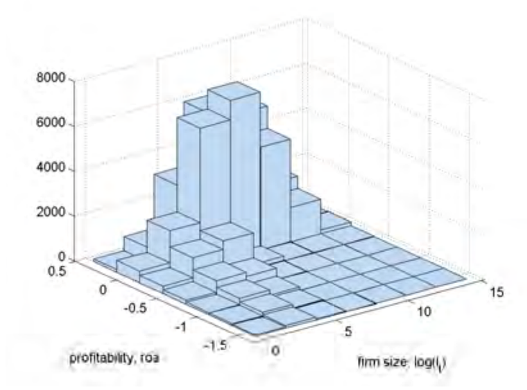
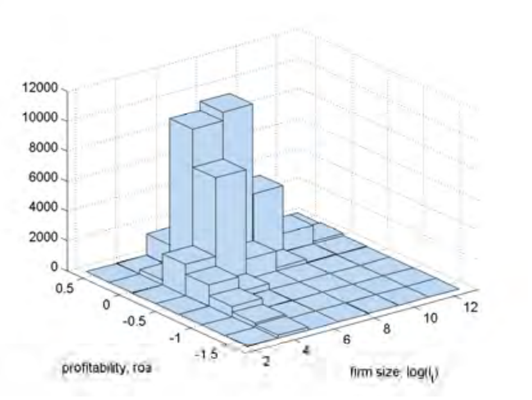



Figure 10: ROA and Employment Size (Model)




Note: These figures are the joint distributions of ROA and the log of employment size in COMPUSTAT and the model. Both figures show similar "economies of scale", which seem to be caused by the fixed cost: ROAs of large firms are higher than those of small firms, but the larger the firm size, the smaller the economies of scale.

firm's EBITDA (the numerator) but does not affect the firm size (the denominator).

Because of the potential positive and negative correlations, the sign of correlation between leverage and profitability is ambiguous in general in this model. However, when I estimate the simple correlation between leverage and profitability, the positive correlation is dominant because the persistent productivity affects a firm's behavior more than the transitory productivity. Then, we obtain:

$$\text{Corr}(\text{lev, ROA}) > 0,$$

both in the data and the model.

However, when I limit the data to *large firm data*, ROA is almost independent of firm size (i.e., no economies of scale) because the fixed cost is less relevant to large firms. Therefore, only the negative correlation between leverage and ROA through the transitory productivity remains. Then, I obtain:

$$\text{Corr}(\text{lev, ROA} \mid \text{Large firms only}) < 0,$$

both in the data and the model.

Similarly, when I estimate the relationship between leverage, firm size, and ROA simultaneously, the firm size absorbs the positive effect of ROA on leverage. Therefore, ROA in this regression captures only the negative effect through the transitory productivity, and so the coefficient on profitability turns out to be negative:

$$\text{Corr}(\text{lev, ROA} \mid \text{size}) < 0,$$

both in the data and the model.

Table 5: Changes in Average and Aggregate Leverage

| | Corporate tax (pos. profit) ($\tau_c^h$) | Corporate tax (neg. profit) ($\tau_c^l$) | Flotation cost & dividend tax ($\lambda + \tau_d$) | Default cost ($\gamma$) | Capital adjustment cost ($\xi$) | Average leverage ratio | Aggregate leverage ratio |
|---|---|---|---|---|---|---|---|
| Baseline | 0.35 | 0.20 | 0.179 | 0.07 | 0.41 | 0.34 | 0.29 |
| | **0.28** | 0.20 | 0.179 | 0.07 | 0.41 | 0.33 | 0.26 |
| | **0.28** | 0.20 | **0.0** | 0.07 | 0.41 | 0.00 | 0.00 |
| | 0.35 | 0.20 | **0.0** | 0.07 | 0.41 | 0.23 | 0.51 |

Note: The bold letters represent the values that are different from the baseline model. The average leverage ratio means the simple average of firms' leverage and the aggregate leverage ratio means the aggregate equity divided by the aggregate debt plus equity in the new stationary equilibrium.

# 4 Counterfactual Experiment

What is the key determinant of corporate capital structure? This is a recurrent question in the corporate finance literature. In this section, I try to answer the question through counterfactual experiments in two parts. First, I explore what makes the firm use debt. To answer the question, I drop the benefits of using debt one by one, and see how the average and aggregate leverages would change. Second, I explore what makes the firm use equity. To answer this question, I drop the benefits of using equity one by one. The procedure of the counterfactual experiment is given in Appendix B.

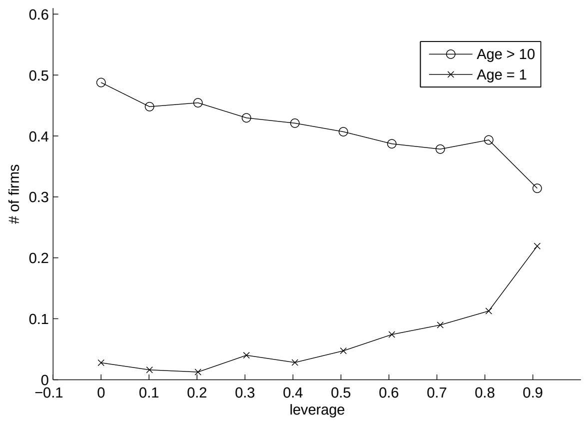
## 4.1 What Makes Firms Use Debt?

What gives the firm an incentive to use debt? A natural guess is the tax benefit generated by the gap between the corporate income tax rate and the interest income tax rate. As the first experiment, I eliminate the tax benefit by lowering the corporate income tax rate to 28%, which is lower than the interest income tax rate, 29.6%, and recalculate the stationary equilibrium. The second row of Table 5 shows the firm's average and aggregate leverage in the new stationary equilibrium. It shows that without the tax benefit, the firm's leverage would decrease a little, but the magnitude of the change in leverage is very small. This result is actually in contrast to previous works. For example, Hennessy and Whited (2005) state in their counterfactual experiment that:

> When we lower the maximal corporate tax rate below the tax rate on interest income, we find that the firm always retains funds and only finances with equity.

This contrast stems from the assumption about firms' entry/exit. *Without* firms' entry/exit as a standard dynamic trade-off model, all firms would eventually use 100% equity by accumulating

Figure 11: Firm Age and Leverage




Note: The figure plots the fraction of entrants and firms older than 10 years in each category of leverage. It shows that while the fraction of entrants increases as the leverage increases, the fraction of firms older than 10 years decreases as the leverage increases.

internal funds, according to the pecking order, if the tax benefit does not exist. However, *with* firms' entry/exit as in this paper, young firms always exist and use debt in the process of their evolution because they prefer debt to outside equity.[33] Figure 11 plots the fraction of entrants and firms older than 10 years in each category of leverage. It shows that young firms tend to use more debt. That is, while the fraction of entrants increases as the leverage increases, the fraction of firms older than 10 years decreases as the leverage increases. For example, while half of zero leveraged firms are firms older than 10 years, less than 5% of them are entrants. The result of the first counterfactual experiment is consistent with the fact that debt finance was a popular financing tool before the corporate income tax was introduced.

Next, as the second experiment, I eliminate the outside equity costs (the dividend tax and the flotation cost of equity) in addition to the tax benefit. That is, I set $\lambda + \tau_d = 0$ in addition to $\tau_c^h = 28\%$.[34] The third row of the table shows the result. The average and aggregate leverages in equilibrium become zero because the firm has no incentive to use debt.

Do the results up to this point imply that the costs of outside equity such as the flotation cost of equity and the dividend tax are the main reasons to be leveraged? Unfortunately, it is not so simple. As the third experiment, I set the costs of outside equity to zero, $\lambda + \tau_d = 0$, but change the corporate income tax rate back to $\tau_c^h = 35\%$. I obtain a somewhat puzzling result, as is shown

[33]This feature is not new to this paper, but many papers, including Cooley and Quadrini (2001), argue that young firms use more debt than old firms.

[34]It is easy to show that the dividend tax and the flotation cost of equity mathematically have the same effect and so the relevant value is just the sum of them, $\lambda + \tau_d$.

Table 6: Changes in Average and Aggregate Leverages

| | Corporate tax (pos. profit) ($\tau_c^h$) | Corporate tax (neg. profit) ($\tau_c^l$) | Flotation cost & dividend tax ($\lambda + \tau_d$) | Default cost ($\gamma$) | Capital adjustment cost ($\xi$) | Average leverage ratio | Aggregate leverage ratio |
|---|---|---|---|---|---|---|---|
| Baseline | 0.35 | 0.20 | 0.179 | 0.07 | 0.41 | 0.34 | 0.29 |
| | 0.35 | 0.20 | 0.179 | **0.0** | 0.41 | 0.61 | 0.39 |
| | 0.35 | **0.35** | 0.179 | 0.07 | 0.41 | 0.50 | 0.46 |
| | 0.35 | **0.35** | 0.179 | **0.0** | 0.41 | 0.75 | 0.62 |
| | 0.35 | **0.35** | **0.0** | **0.0** | 0.41 | 0.91 | 0.99 |
| | 0.35 | 0.20 | 0.179 | 0.07 | **0.21** | 0.48 | 0.44 |

Note: The bold letters represent the values that are different from the baseline model. The average leverage ratio refers to the simple average of firms' leverage positions and the aggregate leverage ratio refers to the aggregate equity divided by the aggregate debt plus equity in the new stationary equilibrium.

in the last row of Table 5: the average leverage decreases, but the aggregate leverage increases. This means that small firms are less leveraged whereas large firms are more leveraged. The logic behind the result is as follows. As I explained in Section 3.4.1, the costs of outside equity cause the pecking order preference:

$$\text{Internal Fund} \succeq \text{Debt} \succeq \text{Outside Equity}.$$

Therefore, the costs of outside equity have an ambiguous effect on leverage in general because they encourage the firm to use more internal funds rather than debt, but they encourage the firm to use debt rather than outside equity. However, their relative magnitude depends on the firm's productivity and age. The costs of outside equity tend to increase the leverage of productive and well-established firms because the choice between internal funds and debt is more relevant for them. On the other hand, those costs tend to decrease leverage for unproductive and new firms because the choice between debt and outside equity is more relevant for them. As the large firms are usually productive and well established, the costs of outside equity cause the different responses between the average and aggregate leverage in the experiment.

## 4.2 What Makes Firms Use Equity?

The second question in the experiment is why the firm uses equity rather than debt. To answer this question, I drop the frictions that make the firm use equity from the model, and recalculate the stationary equilibrium.

A natural guess is that firms use equity rather than debt because of the default cost, $\gamma$. It makes debt unattractive because the equilibrium lending rate is determined by taking into account endogenous default and its cost. As the first experiment, I set the default cost equal to zero, $\gamma = 0$. The second row of Table 6 shows the result: when the default cost is eliminated, the firm's leverage is almost doubled (average: 34% $\rightarrow$ 61%). The result of the experiment shows that the default cost is an important determinant of capital structure. It implies that it is not a good approach for capital structure analysis to assume that a risk-free bond without default is the only choice of debt.

The tax system in this paper gives the tax *dis*benefit to unprofitable firms because the corporate income tax rate for them is *lower* than the interest income rate as a result of the progressive tax rate system. As the second experiment, I increase the corporate tax rate for firms with negative corporate income, $\tau_c^l$, from 20% to 35%. By doing so, I eliminate the tax disbenefit for unprofitable firms. The third row of Table 6 shows that firms are more leveraged when the tax disbenefit is eliminated (average: 34% $\rightarrow$ 50%). This implies that the tax disbenefit is also an important factor making firms use equity.

Even when I eliminate both the default cost and the tax *dis*benefit, the average leverage increases to 75%. What else makes firms use equity? The answer is the costs of outside equity such as the flotation cost of equity and the dividend tax, $\lambda+\tau_d$, which operate as types of financial distress costs. When I eliminate the costs of outside equity in addition to the default cost and the tax *dis*benefit as the next experiment, the average and aggregate leverage become almost equal to one.[35] The results up to this point show that a firm's leverage would become close to one once the default cost, the tax *dis*benefit, and the costs of outside equity are eliminated. It is a little surprising because some papers, including Hennessy and Whited (2005), emphasize investment irreversibility as a major financial distress cost. As the last experiment, I eliminate only the investment irreversibility. The last row of Table 6 shows that if investment irreversibility is mitigated, firms would be much more leveraged (average: 34% $\rightarrow$ 48%). This implies that the investment irreversibility has a strong effect on the corporate capital structure *as long as* it coexists with the costs of outside equity. The intuition is as follows. Without the costs of outside equity, the firm does not have to conduct any fire sale of assets to deal with financial distress because outside equity is a cheap way to deal with it. However, with the costs of outside equity, the fire sale becomes the cheapest way to deal with the financial distress and the degree of investment irreversibility becomes a relevant determinant of corporate capital structure.

# 5 Conclusion

In this paper, I investigate the cross-sectional determinants of corporate capital structure using a general equilibrium model with endogenous firm dynamics, a realistic tax environment, and financial frictions. The main findings are as follows. First, I find that the equilibrium firm distribution in the model replicates the distribution of corporate capital structure. Second, I find that the model's stationary equilibrium accounts for the relationship between capital structure, profitability, and

[35] It is not equal to one because of the constraint (7). Without the constraint, the average leverage becomes one.

firm size. In particular, the model accounts for the negative relationship between leverage and profitability, and its size dependency. The key mechanisms here are economies of scale and two types of productivity shocks, persistent and transitory.

Finally, the counterfactual experiments conducted using the model provide the following implications. First, even if the tax benefit does not exist, the corporate capital structure would not significantly change, which is in contrast to previous works. This contrast stems from the assumption about firms' entry/exit. Second, the effects of the costs of outside equity (the dividend tax and the flotation cost of equity) on corporate capital structure depend on a firm's productivity and age. With those costs, productive, well established, and large firms would decrease their leverage whereas unproductive, new, and small firms increase their leverage. Third, when I eliminate both the tax benefit and the costs of outside equity, firms do not use debt at all. These nonlinear results imply that the effects of those frictions are highly interrelated with each other. Fourth, the default cost makes debt finance unattractive and has a strong effect on corporate capital structure. This implies that it is important to model endogenous default and its cost when we discuss corporate capital structure. Fifth, the tax *dis*benefit stemming from the gap between $\tau_i$ and $\tau_c^l$ also makes firms prefer equity to debt, implying that the progressive tax rate system in the corporate income tax affects corporate capital structure. Sixth, the investment irreversibility magnifies the unattractiveness of debt, but it would have no effect on leverage without the costs of outside equity.

As a future work, it is an interesting extension to incorporate aggregate uncertainty into the model and account for the capital structure behavior over business cycles. Chugh (2010) and Covas and DenHaan (2011) describes some interesting stylized facts. In addition, Jermann and Quadrini (2010), using a more parsimonious model, argue that corporate capital structure may play an important role in explaining business cycle fluctuations. Incorporating an aggregate uncertainty into this model may provide a more microfounded description about the role of corporate capital structure over business cycles. Another interesting future work is to investigate the role of cash holdings by extending the model so that firms can hold deposits and debt simultaneously.

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### Appendix A: Algorithm to Compute a Stationary Equilibrium

In this subsection, I briefly explain the numerical algorithm that I use to compute the stationary equilibrium. As mentioned, I set $w = 1.0$ and $L^s = 0.6$ in the baseline model as in Hopenhayn and Rogerson (1993). The basic algorithm to compute the stationary equilibrium in the baseline model is as follows.

1. Solve the Bellman equations for each firm under $w = 1.0$.
2. By the free entry condition, set $c_e = \int V(0, z)d\lambda(z)$.
3. Calculate the stationary distribution.
4. Using the stationary distribution, we can calculate the equilibrium aggregate labor supply $L^s$. Set the mass of entrants $M$ so that the aggregate labor supply is equal to 0.6.
5. Using this mass of the new entrants $M$ and the stationary distribution, we can calculate the aggregate consumption $C$. Then, set $A$ so that the first-order condition of the households is satisfied.

### Appendix B: Algorithm for the Numerical Experiment

1. Guess the equilibrium wage $w^*$.
2. Solve the Bellman equations for each firm under $w^*$.
3. Compare $c_e$ and $\int V(0, z)d\lambda(z)$. If the entry cost is equal to the value for the entrants (i.e., the free entry condition holds), go to the next step. If not, adjust $w^*$ and go back to the previous step.
4. Calculate the stationary distribution.
5. Using the stationary distribution, we can calculate the equilibrium aggregate labor supply $L^s$ and aggregate consumption $C$. Set the mass of entrants $M$ so that the first-order condition of the households is satisfied.

### Appendix C: Data

I use COMPUSTAT data for the 10-year period from 1988 to 2008. As other papers using this dataset have done, I drop some data based on the following criteria. First, I drop firms in the financial sector and regulated industries because the capital structure in those industries is quite different from other industries.[36] I drop observations from the dataset if their SIC code is from

[36] Chapter 2 of Tirole (2006) reviews these differences. In addition, Adrian and Shin (2008) show that financial institutions' behavior in relation to the leverage ratio is quite different from that of nonfinancial corporations.

4900 to 4999 or from 6000 to 6999. Second, I drop the observations if the number of employees, book assets, book equity, or book debt is zero or negative.

I use the firm's ROA as a proxy for its profitability. ROA in this paper is defined as:

$$\frac{\text{Operating Income Before Depreciation (item 13)}}{\text{Assets (item 6)}}.$$

In previous papers, some definitions of the firm's leverage are proposed. Among them, I adopt the following definition:

$$\frac{\text{Debt in Current Liabilities (item 34)} + \text{Long-term Debt (item 9)}}{\text{Debt in Current Liabilities (item 34)} + \text{Long-term Debt (item 9)} + \text{Stockholders Equity (item 216)}}.$$

Rajan and Zingales (1995) examine several definitions of leverage and discuss the advantages and disadvantages of each definition. Then, they argue that the definition that I adopt in this paper is closest to the one supposed in the economic model. See Rajan and Zingales (1995) and Frank and Goyal (2009) for more detail about the definitions.