IMES DISCUSSION PAPER SERIES

# Accounting for the Decline in the Velocity of Money in the Japanese Economy

Nao Sudo

Discussion Paper No. 2011-E-16

IMES

INSTITUTE FOR MONETARY AND ECONOMIC STUDIES

BANK OF JAPAN

2-1-1 NIHONBASHI-HONGOKUCHO
CHUO-KU, TOKYO 103-8660
JAPAN

**IMES Discussion Paper Series 2011-E-16**
**July 2011**

# Accounting for the Decline in the Velocity of Money in the Japanese Economy

**Nao Sudo***

**Abstract**

A notable feature of the Japanese economy following the banking crisis of the late 1990s is the drastic decline in the velocity of money and the consequent decline in the price level. Based on the inventory model of money demand à la Alvarez, Atkeson, and Edmond (2009), we explore how macroeconomic shocks affect the velocity. Households in the model are subject to a multiple-period cash-in-advance constraint in which the portion of the payment in cash, which we call the liquidity requirement, varies according to the credit service supply in the economy. Extracting various shocks underlying the velocity variations from 1990 to 2010, we find that an increase in the liquidity requirement is the key driver of the decline in velocity. Particularly important is the channel stemming from households' expectations about the future liquidity requirement. During the Japanese banking crisis and the global financial crisis, credit service is disrupted and households expect the disruption to last long. Since they demand additional money for a higher liquidity requirement for current and future transactions, the velocity and the price level decrease, even though the growth rate of money stock then exceeds that of consumption.

**Keywords:** Velocity of Money; Liquidity Requirement; Financial Crises
**JEL classification:** E4, E5

***** Deputy Director, Institute for Monetary and Economic Studies (currently Research and Statistics Department), Bank of Japan (E-mail: nao.sudou@boj.or.jp)

The author would like to thank Kosuke Aoki, Anton R. Braun, Yasuo Hirose, Robert G. King, Keisuke Otsu, Akihisa Shibata, Takayuki Tsuruga, Aleh Tsyvinski and seminar participants of the GRIPs, SWET2010, Kyoto University, and the staff of the Institute for Monetary and Economic Studies (IMES), the Bank of Japan, for their useful comments. Views expressed in this paper are those of the author and do not necessarily reflect the official views of the Bank of Japan.

# 1 Introduction

Since the mid-1990s, the Japanese velocity of money has continuously declined. The upper panel of Figure 1 displays the time paths of the monetary aggregate stock (M2+CDs),[1] the real consumption expenditure, the consumption deflator, and the velocity of money from 1990Q1 to 2010Q4.[2] While the money stock growth exceeds the consumption growth, a large decline in the velocity offsets the money stock growth, preventing the price level from rising one-for-one with the money stock. The negative growth of the velocity is prominent particularly during the financial crisis that occur in this period. As the lower panel of Figure 1 shows, during the two financial crises, the Japanese banking crisis that began in late 1997 and the global financial crisis that began in mid-2007, the velocity growth is drastically reduced.

This secular decline in velocity, notably during the financial crisis periods, cannot be reconciled smoothly with standard monetary models. For example, in a one-period cash-in-advance (CIA) model, velocity is almost constant (Lucas and Stokey [1987] and Hodrick, Kocherlakota, and Lucas [1991]) and the price level is given by the relative quantity of money stock with respect to the quantity of goods.[3] In the other workhorse model of the money-in-utility function, the velocity increases with the nominal interest rate (McGrattan [1998] and Fujiki and Watanabe [2004]).[4] By contrast, as shown in Figure 2, the Japanese nominal interest rate gradually decreases in the first half of the 1990s, and remains nearly unchanged after the Japanese banking crisis. Clearly, an alternative explanation is called for to explain the reduction in the velocity after the Japanese banking crisis.[5]

In this paper, we provide an explanation for the velocity decline that is consistent with the movements of the money stock growth, the consumption growth, and the nominal interest rate, using a multi-period CIA constraint model à la AAE (2009).[6] In the model, households hoard money because they spend their money holdings for goods purchase

[1] In this paper, we choose M2+CDs as our measure of the aggregate money stock, following Alvarez, Atkeson, and Edmond (AAE) (2009).

[2] To construct the velocity of money, we divide the nominal private final consumption expenditure series by the M2+CDs series.

[3] Bordo and Jonung (1987) describe the long-run trend in velocity in the modern economy as influenced by technical innovations in the financial sector, in particular those that economize the money balances. According to this claim, velocity and price level should increase for given amounts of money stock and goods supply.

[4] Fujiki and Watanabe (2004) study the relationship between the log of several measures of M1 velocity and the log of the call rate based on the sample period from 1980 to 2003, and find cointegration between them.

[5] Admittedly, as indicated in Fujiki and Watanabe (2004), the nominal interest rate movements, to some extent, account for the Japanese velocity variations in the periods particularly before the nation's "lost decades." In the current paper, however, we concentrate our analysis on the periods after the Japanese banking crisis of the late 1990s, where the stable nominal interest rate and falling velocity coexist.

[6] The related model includes Baumol (1952), Tobin (1956) and Grossman and Weiss (1983).

over multiple periods.

Our paper's contribution is that it sheds light on the importance of the households' intertemporal decisions to the velocity variations; such a channel has not been investigated extensively in the literature. Based on the model, we first show theoretically that the households' need for liquidity for future monetary expenses is an important driving force in the money demand, velocity, and price level. We next show quantitatively that this channel is in fact the key determinant of the velocity decline in the Japanese economy. In particular, during the two financial crises, the Japanese banking crisis and the global financial crisis, the expected shortage of the credit service strengthens the households' money demand, reducing the velocity and the price level.

Our model differs from AAE (2009) in two respects. First, there are two means of exchange in the economy-money and credit service-whose supplies are exogenous and time-varying. The households utilize the credit service in paying a certain portion of their expenses, and settle the rest of the payment with money holdings. Similar to Benk, Gillman, and Kejak (BGK) (2008), the portion of payment paid in cash, which we call the liquidity requirement, is driven by the exogenous shocks to the credit service supply. Once the adverse shock occurs in the credit service supply, it is caught as a positive shock to the liquidity requirement. With a high current liquidity requirement, the households must spend additional money for a transaction, reducing the velocity and the price level. In addition, if the households expect that the increase in the liquidity requirement is persistent, they hoard money holdings for the future transaction, reducing the velocity and the price level further.

Second, the households are exposed to changes in various macroeconomic environments, in addition to those in the money stock growth rate. These include changes in the productivity growth, the population growth, and the discount factor. While the effects of these economic changes on the velocity dynamics have not been investigated thoroughly in the literature, our model sheds light on the theoretical relationship between them. The key working mechanism is the households' intertemporal decisions. Since the households hoard money for future consumption, a change in the macroeconomic environment affects the velocity whenever it involves a change in the their intertemporal consumption decision.

Based on our model calibrated to the Japanese economy, we first investigate the theoretical implication of changes in the macroeconomic environment to the velocity dynamics. We show that the velocity decline can be induced by a tightening of the liquidity requirement, an increase in the growth rate of the population and that of the money stock, a decrease in the growth rate of productivity, and a rise in the households' discount factor. Because the channel through which each of these changes affects the velocity differs, however, the responses of the other variables, such as the nominal interest rate or the price level, also differs. For instance, when the liquidity requirement rises, the nominal interest rate rises to clear the households' money demand for the current transaction. By contrast, when the households' discount factor rises, since all of the households rush into saving, the nominal interest rate falls to clear the asset market.

Employing the Japanese data from 1990Q1 to 2010Q4, we next explore the quantitative contributions of each macroeconomic shock in the Japanese economy.[7] We obtain shocks to the money stock growth, the productivity growth, and the population growth based on the vector autoregression (VAR) approach similar to AAE (2009). We then estimate the time paths of the discount factor and the liquidity requirement as well as those of their corresponding shocks, drawing on the time series of the Japanese velocity and the nominal interest rate following the methodology proposed in BGK (2008). According to the decomposition of the velocity together with the nominal interest rate into the structural shocks, we find that shocks to the liquidity requirement and the households' discount factor are the key drivers explaining the velocity reduction.

The general developments in the estimated liquidity requirement illustrate their close relationship to the performance of the financial sector and the financial stress in the Japanese economy. The series shows a drastic increase during the two financial crises and comoves with the indices of the financial sector's lending attitude to the firms and firms' financial positions reported in the Short-Term Economic Survey of Enterprises in Japan (Tankan). In addition, the estimated series exhibits high persistency, suggesting that the households are motivated to hoard a large amount of money to prepare for their future monetary expenditures during these periods.

Our finding is also supported by the narrative episodes of the financial crises. During the Japanese banking crisis, major Japanese financial institutions such as Sanyo Securities and Hokkaido Takushoku Bank collapsed, and concerns about a repetition of the crisis lasted into the early 2000s (Baba et al. [2005]). During the global financial crisis, the financial turmoil starting in 2007 led to malfunctioning of the credit markets in the United States and the rest of the world, as illustrated by the collapse of Lehman Brothers in September 2008. Consistent with these episodes, the estimated liquidity requirement starts suddenly to grow from 1998Q1, reaching its peak of growth in 2000. After about five years of moderate growth, it again displays an upsurge in 2008.

The liquidity requirement shocks play the essential role in reducing the velocity after the Japanese banking crisis. In particular, the bulk of the velocity declines during the two financial crises is accounted for by the liquidity shocks. The estimated autoregressive parameter of the shock is nearly unity, indicating that a positive shock to the liquidity requirement lasts a lengthy period of time for subsequent periods. In the wake of this shock, therefore, the households expect the future needs of higher liquidity, hoarding more money than otherwise. Consequently, the velocity and the price level fall drastically, offsetting the effect of other shocks.[8]

[7] See also Otsu (2011) for an analysis of shocks to money stock growth in the Japanese economy using the flexible price model.

[8] Considering that the higher current and expected liquidity requirement is equivalent to a shortage of credit service current and in the future in the model, our result is in line with Hayakawa and Maeda (2000), who stress the role of "financial anxiety" in lowering the velocity during the Japanese banking

By contrast, the discount factor varies cyclically, reducing the velocity and the price level in the four recessions of the last two decades.[9] The shocks to the discount factor are important in accounting for the variations in the velocity during pre-financial-crisis period, but become less important thereafter. They contribute to the bulk of the nominal interest rate variations, affecting them positively in the first half of 1990s and negatively in the later periods. During the two financial crises, while shocks to the liquidity requirement put upward pressure on the nominal interest rate, the shocks to the discount factor cause downward pressure, leaving the interest rate barely changed.

The rest of the paper is organized as follows. In Section 2, we describe the model and demonstrate how the macroeconomic variables, particularly velocity, respond to the various shocks including shocks to the discount factor and liquidity requirement. In Section 3, based on the Japanese data, we extract macroeconomic shocks and evaluate the significance of each shock in explaining the variations of velocity. In Section 4, we conclude.

# 2 The Model

## 2.1 Model setting

### 2.1.1 Environment

The economy consists of $s = 0, \ldots, \overline{S} - 1$ equally divided types of households. Each type of households consists of $N_t(h^t)$ number of agents. Here $h^t$ is the history of the economy up to period $t$. The number of agents grows each period by growth rate $\mu_{N,t}(h^t)$. A coalition is characterized by the initial period type, and each fraction consists of $1/\overline{S}$ measure of households. In each period, households move forward: type 0 becomes type 1, type 1 becomes type 2, ..., and type $\overline{S} - 1$ becomes type 0. The economy is composed of the two separate markets, and each household has two financial accounts that are attached to the markets. One market is the financial market where households trade interest bearing assets for money, using their "brokerage account." The other market is the goods market where households trade goods for money, using their "bank account." The two markets are segmented in the sense that only the type $s = 0$ household can rebalance asset between the two accounts, and other households have no means to rebalance their assets. When agents of one household become type $s = 0$ and access the

crisis.

[9] Our finding is consistent with the arguments that shocks to the discount factor are a driving force behind deflation in Japan. In fact, Braun and Korber (2010), based on a cash-less sticky-price model, analyze the role of shocks to the discount factor in a way parallel to ours in attempting to explain Japan's lost decades. In this type of New Keynesian framework, a rise in the discount factor induces deflation by dampening the current demand and reducing the real marginal cost. (See also Eggertson and Woodford [2003] and Christiano, Eichenbaum, and Rebelo [2009].) In these studies, however, the implication of the shocks to the discount factor for the velocity dynamics is not analyzed.

brokerage account in period $t = t_0$, they transfer nominal money $x_{t_0}(h^{t_0})$ from the brokerage account to the bank account. Once they have rebalanced their assets, the agents $s = 0$ in $t = t_0$ waits another $\overline{S} - 1$ quarters before the next rebalancing.

Every agent in a household receives one unit of labor input every period exogenously. Each household owns a production technology that is common in the economy, and produces $N_t(h^t)\, y(h^t)$ amount of outputs. Here, $y_t(h^t)$ is the labor productivity, growing at the rate $y_t(h^t)/y_{t-1}(h^{t-1}) = \mu_{Y,t}(h^t)$.

Each household is then divided into a shopper-seller pair. The sellers sell their production output to the shoppers of the other households while the shoppers purchase goods from the sellers of the other households.

The households are subject to liquidity constraints. Namely, they must pay a certain portion of goods expense with money holdings in their bank accounts. The rest is paid on the basis of assets in the brokerage account that do not involve the transfer of money holdings in the bank account. In what follows, we refer to the former manner of payment as "payment by money" and the latter manner of payment as "payment by credit." This portion is exogenously given by the liquidity requirement parameter $\theta_t(h^t) \in [0,1]$. In the terminology of Lucas and Stokey (1987), $\theta_t(h^t) = 0$ indicates that the goods are credit goods, whereas $\theta_t(h^t) = 1$ indicates the goods are cash goods. For each unit purchase of goods, $\theta_t(h^t)$ of the payment is deducted from the shopper's bank account, and $1 - \theta_t(h^t)$ of the payment is deducted from the shopper's brokerage account.

Similar to BGK (2008), this liquidity requirement parameter $\theta_t(h^t)$ varies over time. A change in the liquidity requirement affects households' money demand through the intra-temporal channel and the intertemporal channel. When parameter values for the current liquidity requirement $\theta_t(h^t)$ are high, the households' liquidity constraints are tighter and the households need to spend a larger amount of money for current monetary expenses. In addition, when the households in period $t$ expect that the future value of the liquidity requirement $\theta_{t+l}(h^{t+l})$ for $l \geq 1$ will also be high, they hoard a larger amount of money for their future monetary expenses.

There is a close relationship between the financial activities and the liquidity requirement. Although the financial sectors are not explicitly modeled, a higher liquidity requirement is equivalent to a shortfall in the credit service supply in our model. Our preferred interpretation of a rise in the liquidity constraint is an exogenous disruption of the credit service provided by the financial sector. A negative productivity shock to the financial sector analyzed in BGK (2008) is a possible source of such disruption.[10] Alternatively, an exogenous deterioration in the households' financial condition, such as their debt position, can trigger a lower credit service supply by the financial sector, or equivalently, a higher liquidity requirement for the goods transaction.

In fact, as shown below, the liquidity requirement series distilled from the Japanese

[10] Along the same line, Hamilton (1989) constructs a model that includes cash goods and credit goods. Technological progress in producing credit goods results in the reduction of cash goods consumption, leading to a change in money demand.

data increases drastically during the two financial crises (the Japanese banking crisis that began in the late 1990s and the global financial crisis that began in the late 2000s), suggesting its close relationship with the financial activities. In addition, the series tracks well the time path of indices of both the financial sector's lending attitude to firms and firms' financial positions reported in the Tankan.

### 2.1.2 Households problem

Each type of households maximizes utility over the consumption of goods. The utility function is additively separable with respect to time:

$$\sum_{t=0}^{\infty}\sum_{h^t}\beta^t\prod_{j=0}^{t}d_j\left(h^j\right)\left[\Pr(h^t)N_t\left(h^t\right)u(c_t(s,h^t))\right],$$

where $\Pr(h^t)$ is the probability of realizing a particular history $h^t$, $\beta$ is the discount factor, $d_j\left(h^j\right)$ is an exogenous shock to the discount factor, $c_t(s,h^t)$ is per capita goods consumption of the type $s$ household, and $u(c(s,h^t))$ is the temporal utility function with the constant relative risk aversion (CRRA) form:

$$u(c(s,h^t)) = \frac{c_t(s,h^t)^{1-\sigma}}{1-\sigma}. \tag{1}$$

The households are subject to two constraints: the first is the CIA constraint (or "bank account" constraint),

$$\theta_t(h^t)P_t(h^t)C_t(s,h^t) + Z_t(s,h^t) = M_t(s,h^t), \tag{2}$$

where

$$M_t(s,h^t) = Z_{t-1}(s-1,h^{t-1})+\gamma\theta_{t-1}(h^t)P_{t-1}(h^{t-1})N_{t-1}\left(h^{t-1}\right)y_{t-1}(h^{t-1})+\mathbf{1}\cdot P_t(h^t)x_t(h^t). \tag{3}$$

$P_t(h^t)$ denotes the price level of the consumption goods, $C_t(s,h^t)$ is total consumption expenditure of type $s$ agents, defined by $C_t(s,h^t)\equiv c_t(s,h^t)N_t(h^t)$, $Z_t(s,h^t)$ is the money holding unspent at the end of period $t$, $M_t(s,h^t)$ is the money holding at the beginning of period $t$, and $\mathbf{1}$ is an index function taking 1 if $s=0$ and 0 otherwise.

The type $s$ household carries the money holdings from period $t$ to $t+1$, following equations (2) and (3). In period $t$, the shopper of type $s$ household pays a portion of the money holding for his or her goods purchase $\theta_t(h^t)P_t(h^t)C_t(s,h^t)$, and carries the remaining portion $Z_t(s,h^t)$ to period $t+1$. The seller of type $s$ household sells his or her production output and receive the payment $P_t(h^t)N_t(h^t)y_t(h^t)$. Following AAE (2009), we assume that portion $1-\gamma$ of their income is deposited into the brokerage account and the rest is deposited into the bank account. Since the transfer from the brokerage account is positive only for $s = 0$, if $s \neq 0$, the household's money holding in the next period is the sum of the money holdings unspent and the portion of the income.

The second constraint is a brokerage account constraint given by the following equation:

$$\begin{aligned}&\sum_{h_{t+1}} q_{t+1}^{t}(h^{t+1})B_t(s,h^{t+1}) + A_t(s,h^t) + \mathbf{1}\cdot P_{1,t}(h^t)x_t(h^t) + \left(1-\theta_t(h^t)\right)P_t(h^t)C_t(s,h^t)\\ &= B_{t-1}(s-1,h^t) + A_{t-1}(s-1,h^{t-1}) + \left[1-\theta_t(h^t)\right]P_t(h^t)N_t(h^t)y_t(h^t)\\ &+ [1-\gamma]\,\theta_{t-1}(h^{t-1})P_{t-1}(h^{t-1})N_{t-1}(h^{t-1})y_{t-1}(h^{t-1}) - P_t(h^t)\tau_t(h^t), \qquad (4)\end{aligned}$$

where $B_t(s,h^{t+1})$ is the amount of bond holding of the type $s$ household, $q_{t+1}^{t}(h^{t+1})$ is the price of a one-period state contingent bond returning one dollar in period $t+1$, and $A_t(s,h^t)$ is the money holding at the brokerage account.[11] Notice that the portion of expenditure paid by credit (the third term on the right-hand side of the equation), the portion of income (the fourth term on the right-hand side of the equation), and the lump-sum tax/transfer by the government are deposited in the brokerage account.

### 2.1.3 Government sector, exogenous variables, and the equilibrium conditions

**Government budget constraint**

The government faces the following government budget constraint:

$$B_{t-1}(h^t) = M_t(h^t) - M_{t-1}(h^{t-1}) + P_t(h^t)\tau_t(h^t) + \sum_{h_{t+1}} q_{t+1}^{t}(h^{t+1})B_t(h^{t+1}), \qquad (5)$$

where $M_t(h^t)$ and $B_t(h^t)$ are the aggregate values of money stock and bonds in the economy, respectively. We also define the growth rate of money stock by[12]

$$\mu_{M,t}\left(h^t\right) = M_t(h^t)/M_{t-1}(h^{t-1}). \qquad (6)$$

**Law of motion for exogenous variables**

The five exogenous variables, $\mu_{M,t}\left(h^t\right)$, $\mu_{Y,t}\left(h^t\right)$, $\mu_{N,t}\left(h^t\right)$, $d_t\left(h^t\right)$ and $\theta_t\left(h^t\right)$, evolve in the following way:

[11] While we allow money holding in the brokerage account, we focus on the equilibrium such that $A_t(s,h^t) = 0$.

[12] In the following simulation section, we employ M2+CDs as the model's measure of $M_t\left(h^t\right)$. We assume that all of the disaggregated components of M2+CDs, including the monetary base and money multiplier, are exogenous to the households. We, do not, however, explore how the monetary base and money multiplier are determined or to what extent a monetary authority can affect these variables, since they are out of the current paper's scope. See, for example, Freeman and Kydland (2000), who study the endogenous response of the money multiplier to an exogenous shock to the monetary base.

$$\ln \mu_{M,t}\left(h^t\right) = \rho_M \ln \mu_{M,t-1}\left(h^{t-1}\right) + \varepsilon_{M,t}\left(h^t\right), \tag{7}$$

$$\ln \mu_{Y,t}\left(h^t\right) = \rho_Y \ln \mu_{Y,t-1}\left(h^{t-1}\right) + \varepsilon_{Y,t}\left(h^t\right), \tag{8}$$

$$\ln \mu_{N,t}\left(h^t\right) = \rho_N \ln \mu_{N,t-1}\left(h^{t-1}\right) + \varepsilon_{N,t}\left(h^t\right), \tag{9}$$

$$\ln d_t\left(h^t\right) = \rho_d \ln d_{t-1}\left(h^{t-1}\right) + \varepsilon_{d,t}\left(h^t\right), \tag{10}$$

$$\ln \theta_t\left(h^t\right) = \rho_\theta \ln \theta_{t-1}\left(h^{t-1}\right) + \varepsilon_{\theta,t}\left(h^t\right), \tag{11}$$

where $\rho_M$, $\rho_Y$, $\rho_N$, $\rho_d$, and $\rho_\theta \in (0,1)$ are autoregressive roots of the exogenous variables, and $\varepsilon_{M,t}\left(h^t\right), \varepsilon_{Y,t}\left(h^t\right), \varepsilon_{d,t}\left(h^t\right)$, and $\varepsilon_{\theta,t}\left(h^t\right)$ are innovations that are mutually independent, serially uncorrelated, and normally distributed with mean zero and variances $\sigma_M^2$, $\sigma_Y^2$, $\sigma_N^2$, $\sigma_d^2$, and $\sigma_\theta^2$, respectively.

**Equilibrium conditions**

**Definition**: A *competitive equilibrium* of the economy is a sequence of prices $\left\{P_t(h^t), q_{t+1}^t(h^{t+1})\right\}_{t=0}^\infty$ and the allocations $\{\{c_t(s,h^t),\ x_t(h^t),\ B_t(s,h^{t+1}),\ A_t(s,h^t),\ M_t(s,h^t),\ Z_t(s,h^t)\}_{s=0}^{\overline{S}-1}\}_{t=0}^\infty$ for a given government policy $\{\tau_t(h^t),\ M_t(h^t),\ B_t(h^t)\}_{t=0}^\infty$, process for population and labor productivity $\{N_t\left(h^t\right),\ y_t(s,h^t)\}_{t=0}^\infty$ and initial conditions $\{B_{-1}(s-1,.),\ A_{-1}(s-1,.),\ Z_{-1}(s-1,.)\}_{s=0}^{\overline{S}-1}$ such that for all $t, h^t$ :
(i) household maximizes utility given the prices;
(ii) the government budget constraint holds; and
(iii) markets clear:

$$\frac{1}{\overline{S}}\sum_s c_t(s,h^t) = y_t(h^t)N_t(h^t). \tag{12}$$

$$\frac{1}{\overline{S}}\sum_s B_t(s,h^{t+1}) = B_t(h^{t+1}) \tag{13}$$

$$\frac{1}{\overline{S}}\sum_s [M_t(s,h^t) + A_t(s,h^t)] = M_t(h^t). \tag{14}$$

We focus on an economy with a positive interest rate, so that $A_t(s,h^t) = Z_t(\overline{S}-1,h^t) = 0$.[13] We add a few more variables that describe the aggregate economy. The

[13] AAE (2009) show that in an economy with positive steady state inflation rate, the type $s = N-1$ agents do not carry money into the next period. This is because they can rebalance their assets between the bank account and brokerage account in the subsequent period.

nominal interest rate is represented by

$$\frac{1}{1+i_t(h^t)} = \sum_{h_{t+1}} q^t_{t+1}(h^{t+1}), \tag{15}$$

and the aggregate velocity of money, $v_t(h^t)$, is given by

$$v_t(h^t) = \frac{P_t(h^t)\sum_{s=0}^{\overline{S}-1} c_t(s,h^t)}{\overline{S}M_t(h^t)} = \frac{P_t(h^t)y_t(h^t)N_t(h^t)}{M_t(h^t)}, \tag{16}$$

respectively.

## 2.2 First order conditions

The optimality conditions for type $s$ household in period $t$ are given by the appropriate transversality condition ($\lim_{T\to\infty}\lambda_T B_T = 0$) and the following first order optimality conditions (FOCs):

$$\begin{aligned}
x_t(h^t) &: \eta_t(0,h^t) = \lambda_t(0,h^t), && (17)\\
c_t(s,h^t) &: N_t(h^t)c_t(s,h^t)^{-\sigma} + \theta_t\left(h^t\right)P_t(h^t)\lambda_t^M(s) = \theta_t\left(h^t\right)P_t(h^t)\eta_t(s,h^t) && (18)\\
&\quad + \left(1-\theta_t\left(h^t\right)\right)P_t(h^t)\lambda_t(s,h^t), &&\\
B_t(s,h^{t+1}) &: q^t_{t+1}(h^{t+1})\lambda_t(s,h^t) = \beta d_{t+1}\left(h^{t+1}\right)\Pr(h^{t+1}|h^t)\lambda_{t+1}(s+1,h^{t+1}), && (19)\\
Z_t(s,h^t) &: \eta_t(s,h^t) = \lambda_t^Z(s,h^t) + \lambda_t^M(s,h^t) + \beta d_{t+1}\left(h^{t+1}\right)\Pr(h^{t+1}|h^t)\eta_{t+1}(s+1,h^{t+1}), && (20)\\
A_t(s,h^t) &: \lambda_t(s,h^t) = \lambda_t^A(s,h^t) + \beta d_{t+1}\left(h^{t+1}\right)\Pr(h^{t+1}|h^t)\lambda_{t+1}(s+1,h^{t+1}). && (21)
\end{aligned}$$

Here, $\eta_t(s,h^t)$ and $\lambda_t(s,h^t)$ are the Lagrange multipliers of the constraint on the bank account (2), and the constraint on the brokerage account (4), respectively. $\lambda_t^Z(s,h^t)$, $\lambda_t^M(s,h^t)$, and $\lambda_t^A(s,h^t)$ are the Lagrange multipliers associated with non-negativity constraints for money carried between the bank accounts $Z_t(s,h^t)$, money holdings in the bank account $M_t(s,h^t)$, and money holdings in the brokerage account $A_t(s,h^t)$.

Combining these equations, we have the Euler equation that is given by

$$\begin{aligned}
&\beta d_{t+1}\left(h^{t+1}\right)N_{t+1}(h^{t+1})c_{t+1}(s+1,h^{t+1})^{-\sigma}\\
&-\beta d_{t+1}\left(h^{t+1}\right)\left[1-\theta\left(h^{t+1}\right)\right]N_{t+1}(h^{t+1})c_{t+1}\left(0,h^{t+1}\right)^{-\sigma}\\
=\ &\left[N_t(h^t)c_t(s,h^t)^{-\sigma} - \left[1-\theta\left(h^t\right)\right]N_t(h^t)c_t(0,h^t)^{-\sigma}\right]\\
&\times\frac{P_t\left(h^{t+1}\right)\theta\left(h^{t+1}\right)}{P_t\left(h^t\right)\theta\left(h^t\right)}.
\end{aligned} \tag{22}$$

[14]Because the household $s$ in period $t$ pays $\left(1-\theta_t\left(h^t\right)\right)$ of goods expenses with assets in the brokerage account (credit), the intertemporal consumption decision is affected by a value of a dollar in the brokerage account, which is represented by the marginal utility of active household $s=0$, $c_t\left(0,h^t\right)^{-\sigma}$, and the degree of liquidity constraint in period $t$ and $t+1$, $\theta_t\left(h^t\right)$ and $\theta_{t+1}\left(h^{t+1}\right)$.

## 2.3 Model response to exogenous shocks

In this section, we show how our economy responds to the exogenous shocks with the model parameterized to the Japanese economy. In particular, we focus on the dynamics of three variables: the velocity, the price level, and the nominal interest rate after the five exogenous shocks.

To this end, we first calculate the non-stochastic steady state of our model where all of the detrended endogenous variables are unchanged. For a variable $G_t\left(h^t\right)$, detrending is done by applying $\widetilde{g}_t\left(h^t\right)=G_t\left(h^t\right)/\left[Y_t\left(h^t\right)N_t\left(h^t\right)\right]$. The non-stochastic steady state is defined as follows.

**Definition**: *The non-stochastic steady state* of the economy is an equilibrium with $\tilde{a}_t(s)=0$, $\tilde{z}_t(\overline{S}-1)=0$, such that
$\{\tilde{c}_t(s,h^t),\tilde{x}_t\left(h^t\right),\tilde{b}_t(s,h^t),\tilde{m}_t(s,h^t),\tilde{z}_t(s,h^t),\tilde{m}_t\left(h^t\right),\pi_t\left(h^t\right)\}_{s=0}^{\overline{S}-1}$
$=\{\overline{c}(s),\overline{x},\overline{b}(s),\overline{m}(s),\overline{z}(s),\overline{m},\overline{\pi}\}_{s=0}^{\overline{S}-1}$.
We then compute a linear approximation of the model's equilibrium time path following a shock in the neighborhood of the non-stochastic steady state.

**Benchmark parameterization**

Our model's time frequency is quarterly. The values for the taste parameters $\beta$ and $\sigma$ are set to 0.995 and 1.249, respectively, following Sugo and Ueda (2008). The steady state growth rate of money $\overline{\mu}_M$, productivity $\overline{\mu}_Y$, and population $\overline{\mu}_N$ is taken from the average of the Japanese data from 1990 to 2010. See Table 1 for details.

The assumption that the households go to the brokerage only infrequently, or equivalently $\overline{S}\geq 1$, is important to our model setting. When this is the case, the households hold more money than their current monetary expenditure. In fact, the Japanese households hold a large quantity of monetary assets, including currency and deposits, regardless of the existence of a huge opportunity cost. Table 2A displays the opportunity cost of holding monetary assets, defined as the government bond rate minus its own rate. It is clear that while the opportunity cost declines over the period, it remains significantly

[14]In deriving equation (22), we impose several assumptions following AAE (2009). First, we assume that the Lagrange multiplier and the bond holding at the initial state $t=0$, $\lambda_0(s,h^0)$ and $B_0(s,h^0)$, are equal across the households so that the bond price in period $t$ $q_{t+1}^t(h^{t+1})$ is determined by the marginal utility of a dollar for the active households $s=0$. Second, in this paper, we focus on the equilibrium where the steady state nominal interest rate is above zero so that the households do not hold money in the brokerage account and the households $s=\overline{S}-1$ do not carry money holding to the next period.

different from zero. Table 2B displays the households' financial assets relative to their consumption. From 1990 to 2010, the sum of the four listed monetary assets exceeds twice the nominal consumption expenditure, implying that the households hold a larger amount of money than needed for their goods purchase.

To pin down the parameters associated with the financial friction, we use the historical average of the Japanese velocity. The frequency at which agents access their brokerage accounts $\overline{S}^{-1}$ and the paycheck parameter $\gamma$ are set to $12^{-1}$ and 0.8 so that steady-state value of velocity of circulation in the model equals 0.45, the average value of Japan's velocity. This parameterization implying that agents rebalance their portfolio once in 12 quarters approximates the setting employed in AAE (2009), where $\overline{S}$ is 38 months. The steady-state value of the liquidity requirement $\theta$ is set to .92, the average value of nominal expenditure for durable goods purchases over the total consumption expenditure during our sample period.

### 2.3.1 The velocity response under a simplified setting

Before going to the benchmark simulation, we first explain how the economy responds to shocks using a simplified setting. The key feature of the current model is that the households hoard money holding for future consumption. Here, their intertemporal decision affects how much money they spend and the velocity.

In this subsection, we assume that $\overline{S} = 2$, $\gamma = 0$, and $\mu_{M,t}(h^t) = \mu_{Y,t}(h^t) = \mu_{N,t}(h^t) = \theta_t(h^t) = d_t(h^t) = 1$ for $\forall t$, unless otherwise noted. When $\overline{S} = 2$, money stock is held either by the agents $s = 0$ or agents $s = 1$. Consequently, the aggregate money stock in period $t = 1$ is given by

$$M_1 = \overbrace{Z_1\left(0, h^1\right)}^{\text{Money holding by household } s=0} + \overbrace{Z_1\left(1, h^1\right)}^{\text{Money holding by household } s=1}.$$

Similarly, aggregate velocity in period $t = 1$ is given by the weighted average of the individual velocity of household $s = 0$ and household $s = 1$ that is written as follows.

$$\begin{aligned} v_1\left(h^1\right) &= \overbrace{\frac{Z_1(0,h^1)}{M_1(h^1)}}^{(i)\text{Portion of money holding by household } s=0} \times \overbrace{\frac{P_1(h^1)\,c_1(0,h^1)}{Z_1(0,h^1)}}^{(ii)\text{Individual velocity of household } s=0} \\ &+ \overbrace{\frac{Z_1(1,h^1)}{M_1(h^1)}}^{(iii)\text{Portion of money holding by household } s=1} \times \overbrace{\frac{P_1(h^1)\,c_1(1,h^1)}{Z_1(1,h^1)}}^{(iv)\text{Individual velocity of household } s=1} . \end{aligned} \quad (23)$$

Notice that the individual velocity of household $s = 1$ is unity, since it spends all of the money holdings. Once velocity is determined, the price level is given by equation (16).

Combing the first order conditions with the asset price equation, (15), we obtain the expression of the nominal interest rate:

$$\frac{1}{1+i_1(h^1)} = \beta d_1\left(h^1\right) \mathrm{E}_1\left\{\mu_{N,2}\left(h^2\right) \frac{c_1\left(0,h^1\right)^{\sigma} P_1\left(h^1\right)}{c_2\left(0,h^2\right)^{\sigma} P_2\left(h^2\right)}\right\}. \tag{24}$$

As we see in equation (22), since the nominal interest rate reflects a value of a dollar in the brokerage account, it is linked to the marginal utility of agents belonging to household $s=0$ that has access to the brokerage account.

**The model's response to the shock to the productivity growth rate**

Suppose that the economy is at the steady state in period $t=0$. We assume that there is an unexpected increase in the productivity growth rate $\mu_{Y,t}\left(h^t\right)$ in period $t=1$, that reverts back to the steady state thereafter. We show that the individual velocity of household $s=0$ increases, enhancing also the aggregate velocity.

To see this, using equation (22), we arrange the term $(ii)$ in equation (23) above to

$$\frac{P_1\left(h^1\right) c_1\left(0,h^1\right)}{Z_0\left(0,h^0\right)} = \frac{1}{\beta\left(\frac{c_2(1,h^2))}{c_1(0,h^1)}\right)^{1-\sigma}+1}. \tag{25}$$

This expression illustrates the relation between the individual velocity of household $s=0$ and the consumption growth. When $\sigma > (<)\ 1$, the wealth effect (the substitution effect) dominates. As the higher productivity growth leads to a higher consumption growth from $t=1$ to $t=2$, the household $s=0$ spends more (less) and the individual velocity increases (decreases). Notice that the other three terms in the equation (23), $(i)$, $(iii)$, and $(iv)$, are unaffected by these shocks. In our parameterization where $\sigma>1$, therefore, the velocity $v_1\left(h^1\right)$ increases with the productivity growth.

On one hand, an increase in productivity growth makes money stock relatively scarce, lowering the price level. On the other hand, the increasing velocity implies that households spend more money for transaction, moderating the first effect. Consequently, as indicated in (16), price level falls slowly than otherwise.

**The model's response to the shock to the money stock growth rate**

In explaining the effect of a shock to the money stock, we further assume that $\sigma=1$ for simplicity. We maintain this assumption for the other three shocks examined below. Suppose there is an unexpected one-shot increase in money stock by $\Delta M_1\left(h^1\right)$ in period $t=1$, equation (23) is arranged as follows.

$$\begin{aligned} v_1\left(h^1\right) \quad = \quad & \frac{Z_0\left(0,h^0\right)+\Delta M_1\left(h^1\right)}{M_0\left(h^0\right)+\Delta M_1\left(h^1\right)} \times \frac{P_1\left(h^1\right) c_1\left(0,h^1\right)}{Z_0\left(0,h^0\right)+\Delta M_1\left(h^1\right)} \\ & + \frac{Z_0\left(1,h^0\right)}{M_0\left(h^0\right)+\Delta M_1\left(h^1\right)} \times \frac{P_1\left(h^1\right) c_1\left(1,h^1\right)}{Z_0\left(1,h^0\right)}. \end{aligned}$$

As $\sigma$ is unity, the equation (25) is reduced to

$$\frac{P_1(h^1)\, c_1(0,h^1)}{Z_0(0,h^0)+\Delta M_1(h^1)} = \frac{1}{\beta+1} < \text{ term } (iv) = 1.$$

While the individual velocities are unaffected by the shock, the monetary shock increases a portion of the money held by agent $s=0$ (term $(i)$) and reduces that held by agent $s=1$ (term $(iii)$). Consequently, we have

$$\frac{\partial v_1(h^1)}{\partial \Delta M(h^1)} = \frac{M_0(h^0)\left(\frac{1}{1+\beta}-\frac{1+\beta}{1+2\beta}\right)}{M_1(h^1)^2} < 0. \tag{26}$$

The increase in the money stock raises the price level. Reflecting the decline in velocity, however, the price increase becomes sluggish than otherwise.

According to the equation $(24)$, we have the following expression for the nominal interest rate:

$$\begin{aligned}\frac{1}{1+i_1(h^1)} &= \beta \mathrm{E}_1\left\{\frac{c_1(0,h^1)\,P_1(h^1)}{c_2(0,h^2)\,P_2(h^2)}\right\} = \beta \mathrm{E}_1\left\{\frac{Z_1(0,h^1)}{Z_2(0,h^2)}\right\} \\ &= \beta \mathrm{E}_1\left\{\frac{Z_0(0,h^0)+\Delta M_1(h^1)}{Z_0(0,h^0)+\frac{1}{1+\beta}\Delta M_1(h^1)}\right\}.\end{aligned}$$

As shown above, the nominal interest rate is determined to clear the money demand of household $s=0$. Since the money demand of household $s=0$ in period $t=2$ is smaller relative to that in period $t=1$, the nominal interest rate falls to clear the money demand of household $s=0$ in period $t=1$ (liquidity effect).[15]

**The model's response to the shock to the population growth rate**

We consider a case where $\gamma_{N,t}(h^t)$ rises from unity to $\overline{\gamma}_N > 1$ in period $t=1$ and remains at the value for good. Similar to the productivity growth rate, the households' intertemporal decision delivers a avenue that the population growth rate affects the velocity.

Notice that the terms $(i)$, $(iii)$, and $(iv)$ of the equation $(23)$ are unaffected by this shock. By contrast, the term $(ii)$ ,the individual velocity of household $s=0$, falls because the household hoards more money holdings to prepare for the population increase. We have

$$\frac{P_1(h^1)\, c_1(0,h^1)}{Z_0(0,h^0)} = \frac{1}{1+\beta\overline{\gamma}_N} < \frac{1}{1+\beta}.$$

Because of the fall in the individual velocity, velocity $v_1(h^1)$ declines. The increase in the population makes the money stock relatively scarce, reducing the price level. As the

[15] See also Edmond and Weill (2008) and AAE (2009) for the mechanism of the liquidity effect.

equation (16) suggests, because of the endogenous decline in the velocity, a drop in the price level occurs more quickly than otherwise.

The rise in population growth rate reduces the nominal interest rate. The nominal interest rate in period $t = 1$ is given by:

$$\frac{1}{1+i_1(h^1)} = \beta\overline{\gamma}_N \mathrm{E}_1 \left\{ \frac{Z_1(0,h^1)}{Z_2(0,h^2)} \right\} = \frac{\beta\overline{\gamma}_N}{\frac{1}{1+\beta\overline{\gamma}_N} + \frac{\beta}{1+\beta}}. \tag{27}$$

Clearly, the nominal interest rate decreases with the population growth rate. A rise in the population growth makes households leave a larger quantity of money unspent for a next period consumption, reducing their current money demand. The nominal interest rate falls to clear the asset market.

**The model's response to the shock to the discount factor**

A positive shock to the discount factor has the similar impact on the velocity, the price level, and the nominal interest rate. Suppose that $d_t(h^t)$ takes unity in period $t \leq 0$ and rises permanently to $\overline{d} > 1$ in period $t = 1$. Again, the terms $(i)$, $(iii)$, and $(iv)$ in the equation (23) are unaffected by this shock. Because agents prefer future consumption to current consumption, household $s = 0$ reduces current consumption expenditure. The individual velocity of household $s = 0$ is now given by

$$\frac{P_1(h^1)\,c_1(0,h^1)}{Z_0(0,h^0)} \frac{1}{1+\beta\overline{d}} < \frac{1}{1+\beta}.$$

Since the individual velocity of household $s = 0$ declines, the velocity $v_1(h^1)$ declines.

While a change in the discount factor does not cause a change in either the goods supply or the money stock, it leads to a drop in the price level. Because the velocity endogenously declines, the amount of money stock spent for the goods transaction shrinks and money becomes scarce in the goods market, resulting in a decline in price.

The rise in the discount factor reduces the nominal interest rate. The nominal interest rate in $t = 1$ is expressed as follows:

$$\frac{1}{1+i_1(h^1)} = \beta\overline{d}\mathrm{E}_1 \left\{ \frac{Z_1(0,h^1)}{Z_2(0,h^2)} \right\} = \frac{\overline{d}\beta}{\frac{1}{1+\beta\overline{d}} + \frac{\beta}{1+\beta}}. \tag{28}$$

A rise in the discount factor increases the value of consumption tomorrow through the term $\beta\overline{d}$ as well as through the term $Z_2(0,h^2)$. Since household $s = 0$ in period $t = 2$ prefers saving to consumption, the nominal interest rate falls to clear the asset market.

**The model's response to the shock to the liquidity requirement**

Lastly, we discuss the implication of a change in the liquidity requirement for the velocity. Here, we consider a case where $\theta_t(h^t)$ takes unity at the steady state and temporarily rises by $\Delta\overline{\theta} > 1$ in period $t = 1$, reverts by $(1-\rho_\theta)\Delta\overline{\theta}$ in period $t = 2$, and becomes unity again in period $t = 3$. Other simplified assumptions made above are maintained.

Upon the positive shock to the liquidity requirement, the households need to exchange more money for the same amount of goods. The money holdings held by the household $s = 0$ and $s = 1$ in period $t = 1$, $Z_1(0, h^1)$ and $Z_1(1, h^1)$, are expressed in the following way.

$$
\begin{aligned}
Z_1\left(0, h^1\right) &= \left(1+\Delta\overline{\theta}\right) P_1\left(h^1\right) c_1\left(0, h^1\right) + \left(1+\rho_\theta\Delta\overline{\theta}\right) P_2\left(h^2\right) c_2\left(1, h^2\right), && (29) \\
Z_1\left(1, h^1\right) &= \left(1+\Delta\overline{\theta}\right) P_1\left(h^1\right) c_1\left(0, h^1\right). && (30)
\end{aligned}
$$

Other things being equal, as $\rho_\theta$ increases, the household $s = 0$ must hoard more money for the consumption in period $t = 2$.

The tightening of the liquidity requirement influences the velocity via an intra-temporal channel and an intertemporal channel in the following way. First, when a higher liquidity requirement $\theta_t(h^t)$ is realized, the households demands more money for the current transaction, enhancing a value of money and reducing the price level. Second, when $\rho_\theta > 0$, since the households expect that the future liquidity requirement is high, they leave more money for future consumption, reducing the velocity and the price level further. The second channel captures the effect that the households' "financial anxiety" brings to the economy (Hayakawa and Maeda [2000]).

To see this, from equation (22), we derive the ratio of money spending of the households $s = 0$ in period $t = 1$ and that in period $t = 2$:

$$
\frac{P_1(h^1)\, c_1(0, h^1)\left[1+\Delta\overline{\theta}\right]}{P_2(h^2)\, c_2(1, h^2)\left[1+\rho_\theta\Delta\overline{\theta}\right]} = \frac{\left[1+\Delta\overline{\theta}\right]}{\beta} - \rho_\theta\Delta\overline{\theta}\frac{P_1(h^1)\, c_1(0, h^1)\left[1+\Delta\overline{\theta}\right]}{P_2(h^2)\, c_2(0, h^2)\left[1+\rho\Delta\overline{\theta}\right]}. \qquad (31)
$$

The first term on the right hand side represents the money demand for the current transaction and the second term represents the money demand for the transaction next period. When $\rho_\theta$ is zero, the household $s = 0$ increases its spendings by a factor of $\left[1+\Delta\overline{\theta}\right]$, and its individual velocity falls to

$$
\frac{P_1(h^1)\, c_1(0, h^1)}{Z_0(0, h^1)} = \frac{1}{1+\Delta\overline{\theta}+\beta} < \frac{1}{1+\beta}.
$$

Since the individual velocity of household $s = 1$ also declines,

$$
\frac{P_1(h^1)\, c_1(1, h^1)}{Z_0(1, h^1)} = \frac{1}{1+\Delta\overline{\theta}} < 1,
$$

the velocity declines, leading to a drop of the price level. As $\rho_\theta$ increases, the agents $s = 1$ spend less money today to prepare for the future monetary expense, reducing the current velocity further.

As for the nominal interest, we have

$$\begin{aligned}\frac{1}{1+i_1(h^1)} &= \beta \mathrm{E}_1\left\{\frac{Z_1\left(0,h^1\right)(1+\beta)}{Z_2\left(0,h^2\right)\left(1+\Delta\overline{\theta}+\beta\right)}\right\} \\ &= \frac{(1+\beta)^2}{(1+\beta)+\beta\left(1+\Delta\overline{\theta}+\beta\right)}. \end{aligned} \tag{32}$$

Reflecting an increase in money demand in period $t = 2$, the nominal interest rate rises to clear the asset market.[16]

### 2.3.2 Simulation results under benchmark parameterization

We now simulate our model under benchmark parameterization. Figure 3 displays the response of productivity growth, the nominal interest rate, the price level, and velocity, to a temporary increase in the productivity grow rate in period $t = 0$. We assume that $\mu_{Y,t}\left(h^t\right)$ reverts back to its steady state level with autoregressive parameter $\rho_Y = 0.95$. As described above, this shock temporarily brings about a higher velocity than the steady state. Consequently, while the scarcity of money stock causes a fall in the price level, its decline becomes sluggish. The nominal interest rate increases since households' consumption growth rises.

Figures 4 displays the model's response to a temporary increase in the money growth rate $\mu_{M,t}\left(h^t\right)$ in period $t = 0$. Here we assume that $\rho_M = 0.1$ so that the money growth rate gradually reverts back to $\overline{\mu}_M$. The qualitative implications are unchanged from those under the simplified setting above. As goods become scarce relative to the money stock, the price level increases. Its dynamics are sluggish, reflecting a fall in velocity. As a consequence of the liquidity effect, the nominal interest rate drops to clear the asset market.

Figures 5 and 6 display the model's response to a temporary increase in the population growth rate and agents' discount factor, respectively. The autoregressive parameters are set to 0.95. As discussed above, all of the agents other than agents $s = \overline{S} - 1$ postpone their consumption, reducing money used for the current goods transaction. Since money becomes scarce in the goods market, the price level drops (16). As agents prefer saving to current consumption, the nominal interest rate declines to clear the asset market.

Figures 7 displays the model's response to a temporal positive shock to the liquidity requirement for the goods transaction (or equivalently, a negative shock in credit service) with $\rho_\theta = 0.95$. As households need more money for the goods transaction for today and tomorrow, all of the individual velocities declines. From equation (16), the price level also drops. In contrast to a positive shock to the population growth and discount

[16] Here, we derive the case for $\rho_\theta = 0$. We display the response of the nominal interest rate to a shock to the liquidity requirement for $\rho_\theta > 0$ in the numerical analysis in the next subsection.

factor, the nominal interest rate rises since current money demand is tighter than that next period.

It is important to disentangle the two channels through which the liquidity requirement shock affects the macroeconomic variables. In Figure 8, we separate the quantitative implications of the intra-temporal channel and that of the intertemporal channel.[17] The red lines with circle denote the effect stemming from the first channel and the black lines with triangle denote the total effect. The bulk of the fall in the velocity and the price level are generated from the intertemporal channel rather than the intra-temporal channel. Since a higher value for $\rho_\theta$ implies a tighter demand for the liquidity in the future transaction, the intertemporal channel effect increases with $\rho_\theta$.

# 3 Quantitative Roles of the Structural Shocks in Japanese Economy

## 3.1 Recovering the structural shocks

To see the determinant of Japan's velocity, we decompose the variations of macroeconomic variables from 1990Q1 to 2010Q4 into five structural shocks. While productivity, money stock, and population are observable series, the discount factor and the liquidity requirement cannot be directly observed. Following the methodology of BHK (2008), we make use of two data series, velocity and nominal interest rate, and recover the two unknown series. The data series used for extracting shocks in this section, the money stock growth rate, the productivity growth rate, the population growth rate, the velocity of money, and the nominal interest rate are demeaned.[18]

We first distill shocks to a money stock growth rate $\varepsilon_{\mu_M,t}(h^t)$ and productivity growth rate $\varepsilon_{\mu_Y,t}(h^t)$, following AAE (2009). To capture the interaction between the money stock series and the real household consumption and to see how these two series and shock series evolve over time, we formulate a bivariate vector autoregressive regression (VAR) with four quarter lags that consists of the growth rate of M2+CDs and that of per capita real private final consumption expenditure.[19][20] Using the estimate outcome,

[17] To calculate the equilibrium response stemming from the intra-temporal channel, we first compute the time path of deviation of $\theta_t(h^t)$ from the impact period to twenty quarters under the economy when $\rho_\theta = 0.95$. We then compute the model's initial response to the one-shot shock, or equivalently the shock with $\rho_\theta = 0$, with the shock size of twenty values of $\theta_t(h^t)$ calculated in the first step.

[18] Since the steady state values of these variables are calibrated based on their historical average, the actual data shown in Figures 1 and 2 are recovered from the model-implied deviations of these variables from their steady state plus the steady state values to the first order approximation.

[19] In this estimation, we employ the not-seasonally-adjusted money stock series, the seasonally-adjusted real private final consumption expenditure series, and the population series, which are released by the Bank of Japan, the Cabinet Office, and the Ministry of Internal Affairs and Communications, respectively. We seasonally-adjust the money stock series by X-11.

[20] The shock $\varepsilon_{\mu_M,t}(h^t)$ therefore captures shocks to the monetary base and money multiplier that are

we modify the model's law of motions of $\mu_{M,t}$ and $\mu_{Y,t}$, given by the equations (7) and (8), into the VAR system where the two variables are lagged by each other with the estimated lag coefficients. We employ the residuals of our VAR estimate as our structural shocks $\varepsilon_{\mu_M,t}(h^t)$ and $\varepsilon_{\mu_Y,t}(h^t)$. Similarly, assuming that the population grows at the independent rate of the other variables, we obtain shocks to the population growth rate $\varepsilon_{\mu_N,t}(h^t)$ from the autoregressive regression with four lags for the population growth rate.

Next, we estimate the two unknown variables, the discount factor $d_t(h^t)$ and the liquidity requirement $\theta_t(h^t)$, and their corresponding shocks $\varepsilon_{d,t}(h^t)$ and $\varepsilon_{\theta,t}(h^t)$, making use of the discrepancy between the model's implying the velocity and the nominal interest rate conditional on the three shocks already obtained and the actual data series. Assuming that the two variables $d_t(h^t)$ and $\theta_t(h^t)$ evolve as specified in equations (10) and (11), we recover the time series of shocks together with the autoregressive parameters $\rho_d$ and $\rho_\theta$, based on an approach used in Nolan and Thoenissen (2009) and BGK (2005).

We first guess the values of $\rho_d$ and $\rho_\theta$, such that $\rho_d = \rho_{d,0}$ and $\rho_\theta = \rho_{\theta,0}$. Provided with this initial guess and the model's decision rule, we compute the time path of the velocity and the nominal interest rate conditional on $\left\{\varepsilon_{\mu_M,t}(h^t), \varepsilon_{\mu_Y,t}(h^t), \varepsilon_{\mu_N,t}(h^t)\right\}_{t=1991Q1}^{2010Q4}$. The discrepancies between this model's implying series and their data counterparts are given by the linear combinations of the time series of the two shocks $\{\varepsilon_{d,t}(h^t),\ \varepsilon_{\theta,t}(h^t)\}_{t=1991Q1}^{2010Q4}$. Consequently, we obtain the initial estimates of $\{d_t(h^t),\ \theta_t(h^t),\ \varepsilon_{d,t,0}(h^t),\ \varepsilon_{\theta,t,0}(h^t)\}_{t=1991Q1}^{2010Q4}$ under the assumption that $\rho_d = \rho_{d,0}$ and $\rho_\theta = \rho_{\theta,0}$. We then estimate $\rho_{d,1}$ and $\rho_{\theta,1}$ by regressing these distilled series of $\{d_{t,0}(h^t),\ \theta_{t,0}(h^t)\}_{t=1991Q1}^{2010Q4}$ on their lags, and obtain the second-round estimate of the autoregressive parameter, such that $\rho_d = \rho_{d,1}$ and $\rho_\theta = \rho_{\theta,1}$. We repeat this procedure until we obtain the convergence for $\rho_d$ and $\rho_\theta$.

Our estimates that $\rho_d = 0.9808$ and $\rho_\theta = 0.9896$, indicate that the two shocks are considerably persistent. In particular, as discussed above, the persistence of the liquidity requirement plays an important role in varying the velocity and the price level through the households' intertemporal decision.

## 3.2 Decomposition of endogenous variables

Figure 9 displays the yearly differences of $\ln d_t(h^t)$ and $\ln \theta_t(h^t)$ that are implied from our model, respectively. For the purpose of comparison, we use shaded areas to denote the Japanese recessionary periods. In the long run, the growth rate of the discount factor $d_t(h^t)$ increases by a large amount at the beginning of 1990, and starts to grow moderately thereafter. From the viewpoint of business cycle frequency, its time path is weakly cyclical, increasing by a relatively greater amount during a recession and moderately

not accounted for by the lags of the money stock and real consumption expenditure. This specification implies that the two series are independent from the other exogenous variables.

during a boom.

In contrast to the discount factor $d_t(h^t)$, the time path of the liquidity requirement $\theta_t(h^t)$ is closely related to the two episodes of financial crises: the Japanese banking crisis that began in 1997 and the global financial crisis that began in 2007. The liquidity requirement $\theta_t(h^t)$ declines in the first half of the 1990s. This implies that credit-based transactions using the brokerage account are amply provided by the financial sector, or equivalently, less liquidity is required for the goods transaction. At the time of the first crisis following the bankruptcy by Sanyo Securities in November 1997, the credit-based transaction begins to shrink immediately. The liquidity requirement continues to increase until 2000, a shortly after the period of recapitalization of banks by the Japanese government in March 1999, when the increase becomes moderate. Following a several years of moderation, it rises again when the second financial crisis hits the global economy, including Japan. It exhibits an abrupt upsurge from 2008Q2, about the time of the failure of Lehman Brothers, reaching a peak in 2009.

In the lower panel of the figure, we plot measures of financial services: the diffusion index (DI) of the lending attitude of financial institutions (the percentage of firms that find banks' lending attitude "accommodative" minus those that find it "severe," with the scale reversed) and that of the firms' financial position (the percentage of firms that find their own financial position "easy" minus those that find it "tight," with the scale reversed) in the Tankan, together with the liquidity requirement. The three time series comove closely, particularly during the two periods of financial stress, suggesting the presence of a shortage of credit service for goods transactions during that time.

To gauge the quantitative role of each structural shock in explaining the variations in endogenous variables, we calculate the model-generated values of these variables by feeding each of the five shock series $\left\{\varepsilon_{\mu_M,t}(h^t), \varepsilon_{\mu_Y,t}(h^t), \varepsilon_{\mu_N,t}(h^t), \varepsilon_{\mu_d,t}(h^t), \varepsilon_{\mu_\theta,t}(h^t)\right\}_{t=1991Q1}^{2010Q4}$ into the model. Each simulation yields a portion of variations explained by the corresponding structural shock.

Figure 10 displays the time path of velocity for the demeaned level series and growth rate series (on a year-on-year basis) together with the contributions of the five structural shocks.[21] The two shocks—the discount factor shocks and liquidity shocks—are the key drivers behind the velocity dynamics. The effect of the other shocks are relatively minor. Although the two driving shocks both work to lower the velocity, they operate in different timings. The discount factor shocks reduce the velocity growth mainly in the first half of the 1990s. Their impacts almost diminishes greatly after the period of the Japanese financial crisis. By contrast, the liquidity shocks begin to reduce the velocity growth only after the Japanese financial crisis at the end of 1997. Their dampening effects on the velocity decrease as the financial crisis ceases to operate until mid-2007, when the global financial crisis occurs. These results imply that the two financial crises

[21] The first four observations are dropped as our VAR for money stock and consumption expenditure consists of four lags. Our results are unchanged if we employ an eight lags specification instead of a four lags specifications.

are closely associated with credit service shortages, or equivalently an increase in the cash requirement. As a result, the money demand for goods transaction increases and the velocity falls.

Figure 11 displays the time path of inflation (on a year-on-year basis) over the sample period. While a sizable portion of inflation variations is explained by the money stock and goods supply, the liquidity requirement shocks explain the bulk of deflationary pressure, in particular, during the period of the two financial crises. In these periods, credit service becomes scarce, leading to a higher demand for the liquidity as a means of exchange. In addition, since households expect a tight liquidity constraint in the subsequent periods, they hoard more money for future consumption. Consequently, the value of money is enhanced, resulting in deflation. The discount factor shocks help lower inflation during the first half of the 1990s, but their effects then are offset by the negative shocks to the liquidity requirement, maintaining inflation variations unchanged.

As depicted in Figure 12, the nominal interest variations are mainly driven by the discount factor shocks. At the beginning of the 1990s, these shocks cause a rise in the nominal interest rate. From the mid-1990s, however, their contributions turn negative, lowering the nominal interest rate up to 2010. During the two financial crisis periods where shocks to the liquidity requirement cause upward pressure on the nominal interest rate, the effect of the discounting shocks is offsetting the upward pressure, maintaining the time path of the nominal interest rate remain stable.

# 4 Conclusion

A notable feature of Japan's economy is the continuous decline in the velocity of money since the mid-1990s. In particular, during the Japanese banking crisis that began in 1997 and the global financial crisis that began in 2007, the velocity growth exhibits drastic declines. Consequently, deflationary pressure prevails while money stock grows positively.

To see the mechanism behind the velocity dynamics, we build an inventory model of money demand à la AAE (2009). In their model, the households are constrained from accessing the financial market and hoard money holdings for current and future consumption. We extend the model in two dimensions. First, households face a time-varying degree of liquidity constraints. The households determine how to spend money holdings, expecting their current and future liquidity constraints. Second, the economy grows with an increase in productivity and population. Since the households hoard money for future consumption, the growth property of the economy has a unique implication for the model's dynamics.

Based on the model calibrated to Japanese economy, we provide a theoretical linkage between the standard macroeconomic shocks and the dynamics of velocity and the nominal interest rate. Because households gradually spend their money holdings over multiple periods, the households' intertemporal consumption decision plays the key role in the velocity variations.

Employing the Japanese data from 1990 to 2010, we explore the quantitative contributions of each macroeconomic shock in Japan's economy. According to the decomposition of macroeconomic variables, we find that shocks to the liquidity requirement and the households' discount factor are the key shocks in explaining the velocity reduction. In particular, the estimated former shocks are closely related to financial activity or the financial stress in the Japanese economy, and play the essential role in the velocity reduction following the Japanese banking crisis. An increase in the liquidity requirement bolsters the households' money demand for current and future goods transactions, leading to a velocity reduction and deflation.

Extending our model into three dimensions is needed to further investigate the sources behind the velocity variations. In this regard, first, it is important to consider the feedback effect from the macroeconomy to the financial activities. For instance, economic downturns may harm the financial sector's balance sheet, leading to a disruption in the credit service supply. Second, while the current paper focuses on financial assets, it may be useful to investigate money demand from the asset portfolio perspective by incorporating real assets as well as financial assets into the model. Third, an international comparison would be useful to gain a deeper understanding of the velocity in Japan. Comparisons with the United States and the euro area might enrich the analysis of the liquidity requirement shocks and the discount factor shocks analyzed in this paper.

# References


[1] Alvarez, Fernando, Andrew Atkeson, and Chris Edmond (2009) "Sluggish Responses of Prices and Inflation to Monetary Shocks in an Inventory Model of Money Demand," *Quarterly Journal of Economics*, Vol. 124, No. 3, pp. 911-967.

[2] Benk, Szilard, Max Gillman and Michal Kejak (2005) "Credit Shocks in the Financial Deregulatory Era: Not the Usual Suspects," Review of Economic Dynamics, Vol. 8, No. 3, pp. 911-967.

[3] Benk, Szilard, Max Gillman and Michal Kejak (2008) "Money Velocity in an Endogenous Growth Business Cycle with Credit Shocks," Journal of Money, Credit and Banking, Vol. 40, No. 6, pp. 1283-1293.

[4] Baba, Naohiko, Shinichi Nishioka, Nobuyuki Oda, Masaaki Shirakawa, Kazuo Ueda, and Hiroshi Ugai (2005), "Japan's Deflation, Problems in the Financial System and Monetary Policy," Monetary and Economic Studies, Bank of Japan.

[5] Baumol, William J. (1952) "The Transactions Demand for Cash: An Inventory Theoretic Approach," *Quarterly Journal of Economics*, Vol. 66, No. 4, pp. 545-556.

[6] Braun R. Anton and Lena Mareen Korber (2010). "New Keynesian Dynamics in a Low Interest Rate Environment," IMES Discussion Paper Series 10-E-05, Institute for Monetary and Economic Studies, Bank of Japan

[7] Bordo, Michael D. and Lars Jonung (1987) "The long-run behavior of the velocity of circulation: The international evidence," Cambridge University Press, Cambridge.

[8] Chiu, Jonathan (2005) "Endogenous Segmented Asset Market in an Inventory Theoretic Model of Money Demand," Bank of Canada Working Paper, 07-46.

[9] Christiano, Lawrence, Martin Eichenbaum and Sergio Rebelo (2009) "When is the government spending multiplier large?," NBER Working Papers 15394, National Bureau of Economic Research, Inc.

[10] Edmond, Chris. and Pierre-Olivier Weill (2008) "Models of the liquidity effect" in S.N. Durlauf and L.E. Blume. (ed.), The New Palgrave Dictionary of Economics 2nd edition, Palgrave Macmillan.

[11] Eggertsson Gauti B. and Michael Woodford (2003) "The Zero Bound on Interest Rates and Optimal Monetary Policy," Brookings Papers on Economic Activity, Economic Studies Program, The Brookings Institution, Vol. 34, No. 2003-1, pp. 139-235.

[12] Freeman, Scott, Kydland, Finn.E. (2000) "Monetary Aggregates and Output," American Economic Review 90, pp. 1125–1135.

[13] Fujiki Hiroshi and Kiyoshi Watanabe (2004). "Japanese Demand for M1 and Demand Deposits:Cross-Sectional and Time-Series Evidence from Japan," Monetary and Economic Studies, Institute for Monetary and Economic Studies, Bank of Japan.

[14] Grossman, Sanford J., and Laurence Weiss (1983) "A Transactions-Based Model of the Monetary Transmission Mechanism," *American Economic Review*, Vol. 73, No. 5, pp. 871-880.

[15] Hamilton, James (1989) "The long-run behavior of the velocity of circulation: A review essay," *Journal of Monetary Economics,* Vol. 23, pp. 335-344.

[16] Hayakawa, Hideo and Eiji Maeda (2000) "Understanding Japan's Financial and Economic Developments Since Autumn 1997," BOJ Working Paper Series 00-1, Bank of Japan.

[17] Hodrick Robert J., Narayana Kocherlakota, and Deborah Lucas (1991) "The Variability of Velocity in Cash-in-Advance Models," *Journal of Political Economy*, Vol. 99, No. 2, pp. 358-384.

[18] Khan, Aubhik, and Julia K. Thomas (2007) "Inflation and Interest Rates with Endogenous Market Segmentation," Federal Reserve Bank of Philadelphia Working Paper 07-1.

[19] Kakkar, Vikas and Masao Ogaki (2002) "The Distortionary Effects of Inflation: An Empirical Investigation," mimeo.

[20] McGrattan, Ellen R. (1998) "Trends in Velocity and Policy Expectations: A Comment." Carnegie-Rochester Conference Series on Public Policy, 49.

[21] Nolan, Charles and Christoph Thoenissen (2009) "Financial Shocks and the US Business Cycle," Journal of Monetary Economics, Vol. 56, No. 4, pp. 596-04.

[22] Lucas, Robert E., Jr., and Stokey, Nancy L. (1987) "Money and Interest in a Cash-in-Advance Economy," *Econometrica*, Vol. 55, No. 3, pp. 491-514.

[23] Ogaki, Masao (1988) "Learning about Preferences from Time Trends." Unpublished Ph.D. dissertation, University of Chicago, Chicago. Distortionary Effects of Inflation: An Empirical Investigation," mimeo.

[24] Otsu Keisuke (2011). "Accounting for Japanese Business Cycles: a Quest for Labor Wedges," forthcoming, Monetary and Economic Studies, Institute for Monetary and Economic Studies, Bank of Japan.

[25] Sugo, Tomohiro and Ueda, Kozo (2008) "Estimating a dynamic stochastic general equilibrium model for Japan," Journal of the Japanese and International Economies, Vol. 22, No. 4, pp. 476-502.

[26] Tobin, James (1956) "The Interest-Elasticity of Transactions Demand For Cash," *Review of Economics and Statistics*, Vol. 38, No. 3, pp. 241-247.

Table 1: Parameters and Steady State Values[22]

| Parameter | Value | Description |
|---|---|---|
| $\beta$ | 0.995 | Discount Factor |
| $\sigma$ | 1.249 | Inverse of Elasticity of Intertemporal Substitution |
| $\overline{S}^{-1}$ | $12^{-1}$ | Frequency Agents Access to their Brokerage Accounts |
| $\gamma$ | 0.8 | Paycheck Parameter |
| $\theta$ | .92 | Liquidity Requirement at Steady State |
| $\overline{\mu}_M$ | 1.0067 | Growth Rate of Money at Steady State |
| $\overline{\mu}_Y$ | 1.0015 | Growth Rate of Labor Productivity at Steady State |
| $\overline{\mu}_N$ | 1.0011 | Growth Rate of Population at Steady State |

[22]Figures are quarterly unless otherwise noted.

Table 2A : Opportunity cost of Holding Monetary Asset[23]

| | 1990 | 1995 | 2000 | 2005 | 2008 |
|---|---|---|---|---|---|
| Deposit Rate | 3.8 | 1.63 | 1.68 | 1.09 | 0.86 |
| Certificates of Deposit Rate | -0.34 | 2.53 | 1.53 | 1.35 | 0.73 |

Table 2B : Household's Financial Asset relative to Households' Consumption[24]

| | 1990 | 1995 | 2000 | 2005 | 2010 |
|---|---|---|---|---|---|
| Currency | 0.14 | 0.15 | 0.13 | 0.18 | 0.20 |
| Transferable Deposits | 0.24 | 0.31 | 0.46 | 1.00 | 1.08 |
| Time and Savings Deposits | 1.77 | 2.00 | 2.13 | 1.62 | 1.69 |
| Certificates of Deposit | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

[23] Opportunity cost is calculated from the government bond rate less the own rate. All of the data series are shown in annual percentage point and taken from IFS data base.

[24] The data for financial asset is taken from Flow of Funds, released from the Bank of Japan. For years 1990 and 1995, demand deposits series is used for transferable deposits. All figures are devided by the nominal personal consumption expenditure.

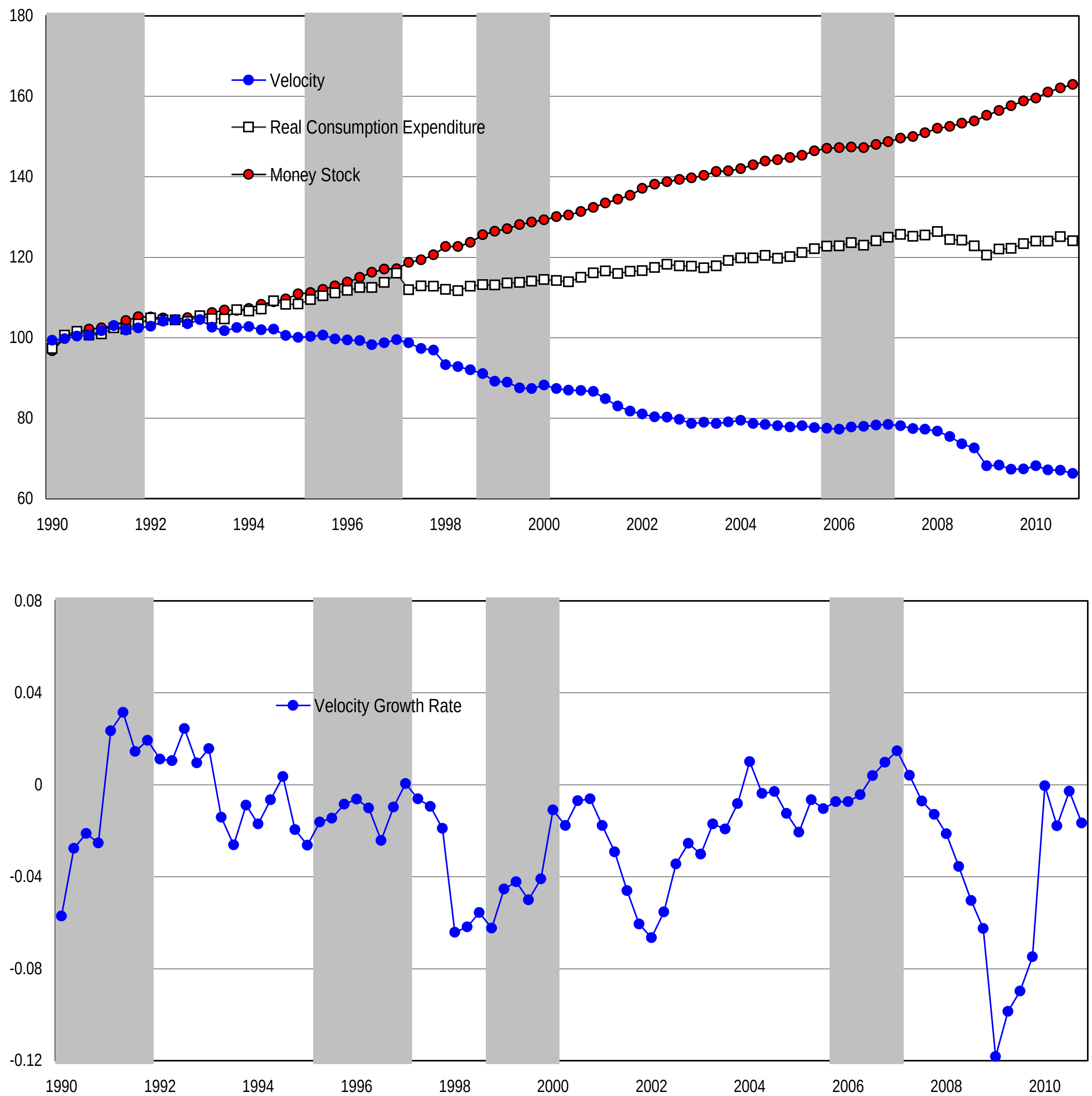



Figure 1: Upper panel displays the money stock (M2+CDs), goods supply (real personal consumption expenditure), and velocity of money in the Japanese economy since 1990Q1. All series are normalized so that values of 1990 = 100. Lower panel displays the yearly growth rate of velocity of money. Recession periods are shaded.

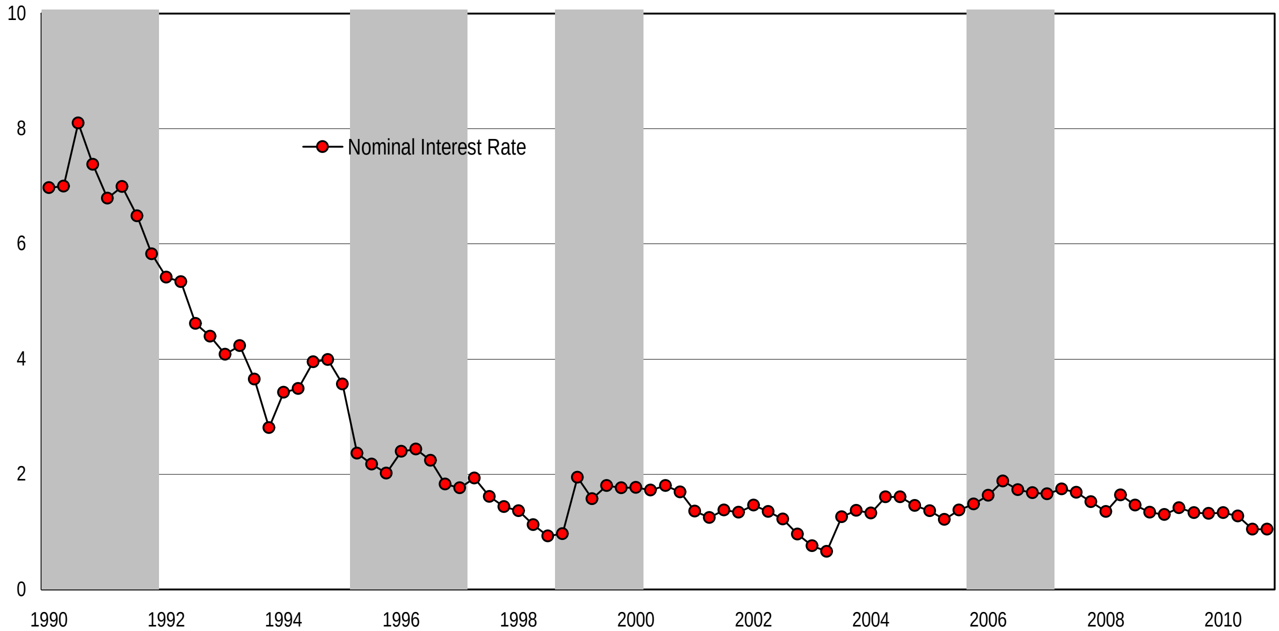



Figure 2: The nominal interest rate (10-year Japanese government bond yield) in the Japanese economy since 1990Q1.

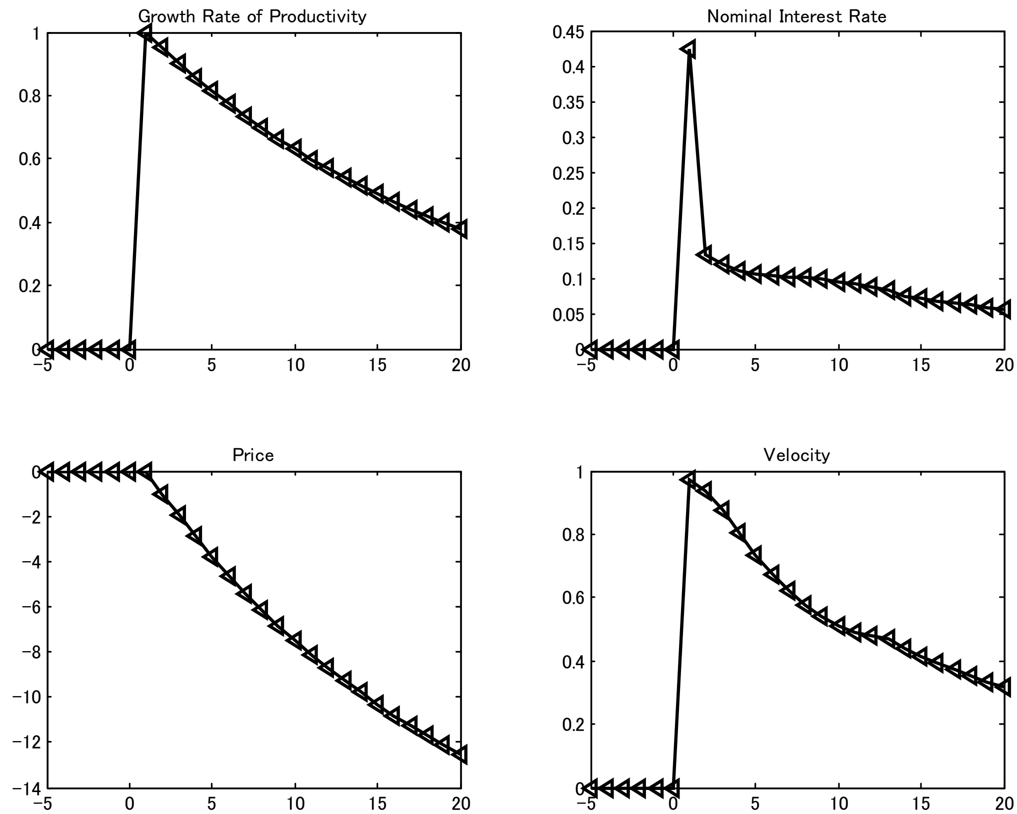



Figure 3: Impulse response of the productivity growth rate, nominal interest rate, price, and velocity to a positive shock to the productivity growth rate. The horizontal axis denotes quarters after the shock, and the vertical axis denotes a deviation from the steady state value.

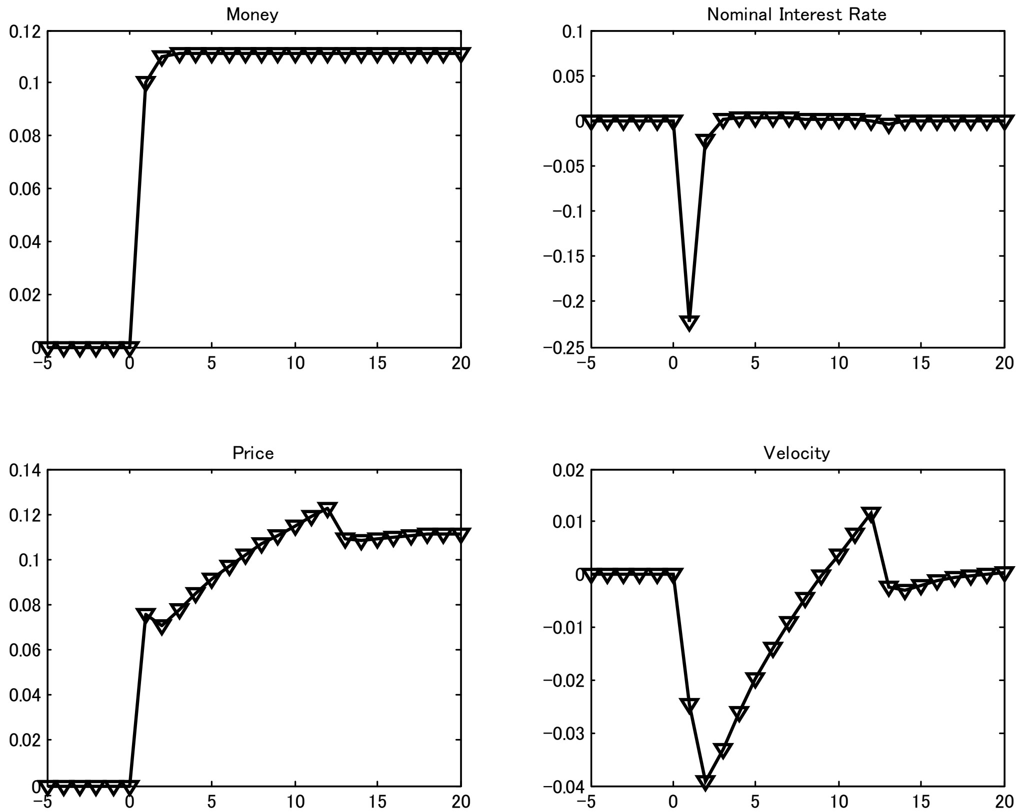



Figure 4: Impulse response of money stock, nominal interest rate, price, and velocity to a positive shock to the money growth rate. The horizontal axis denotes quarters after the shock, and the vertical axis denotes a deviation from the steady state value.

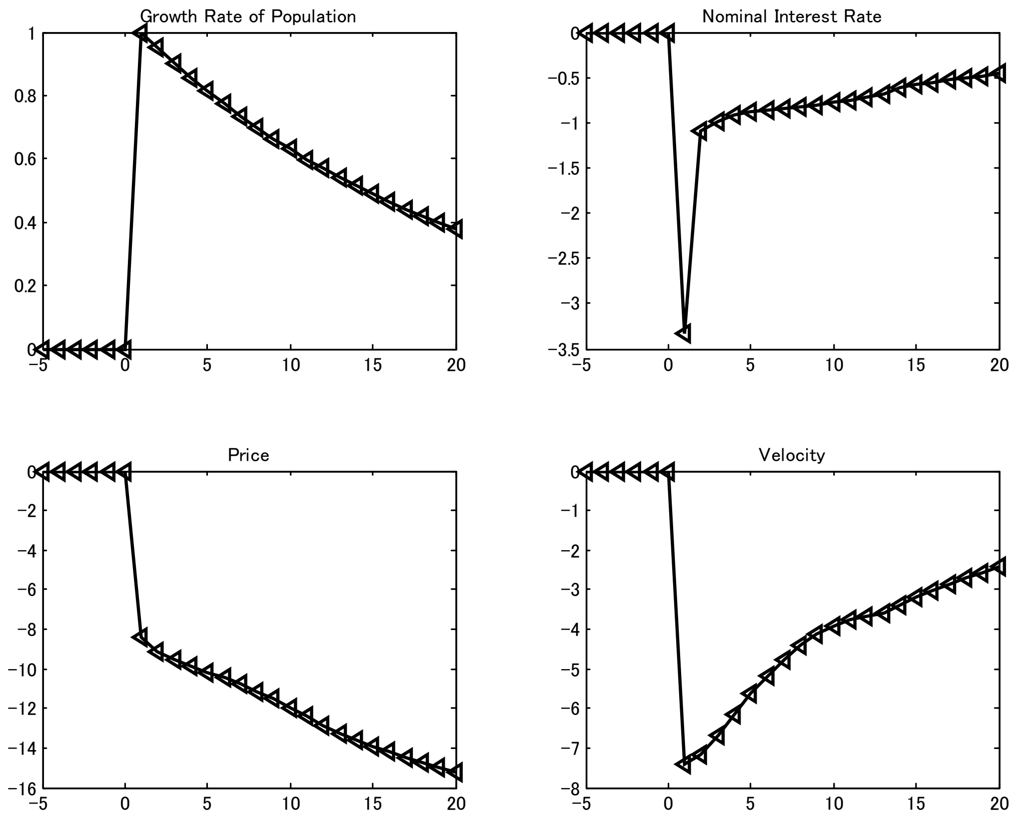



Figure 5: Impulse response of population growth rate, nominal interest rate, price, and velocity to a positive shock to the population growth rate. The horizontal axis denotes quarters after the shock, and the vertical axis denotes a deviation from the steady stata value.

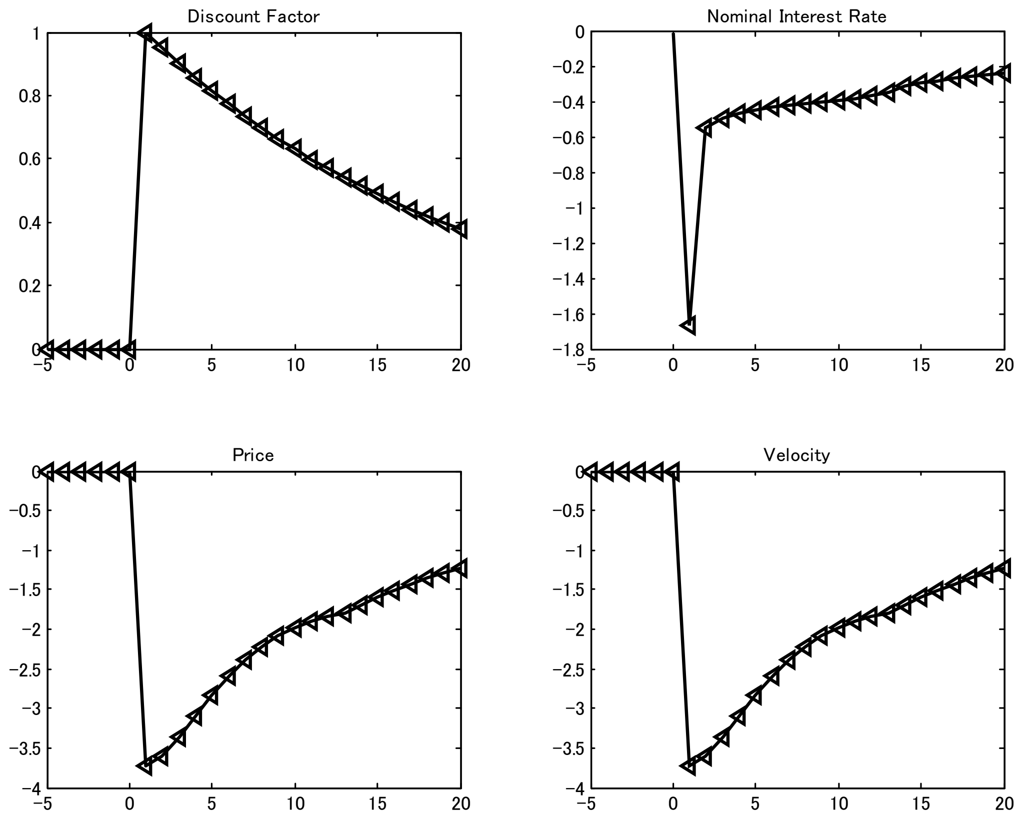



Figure 6: Impulse response of discount factor, nominal interest rate, price, and velocity to a positive shock to the discount factor. The horizontal axis denotes quarters after the shock, and the vertical axis denotes a deviation from the steady state value.

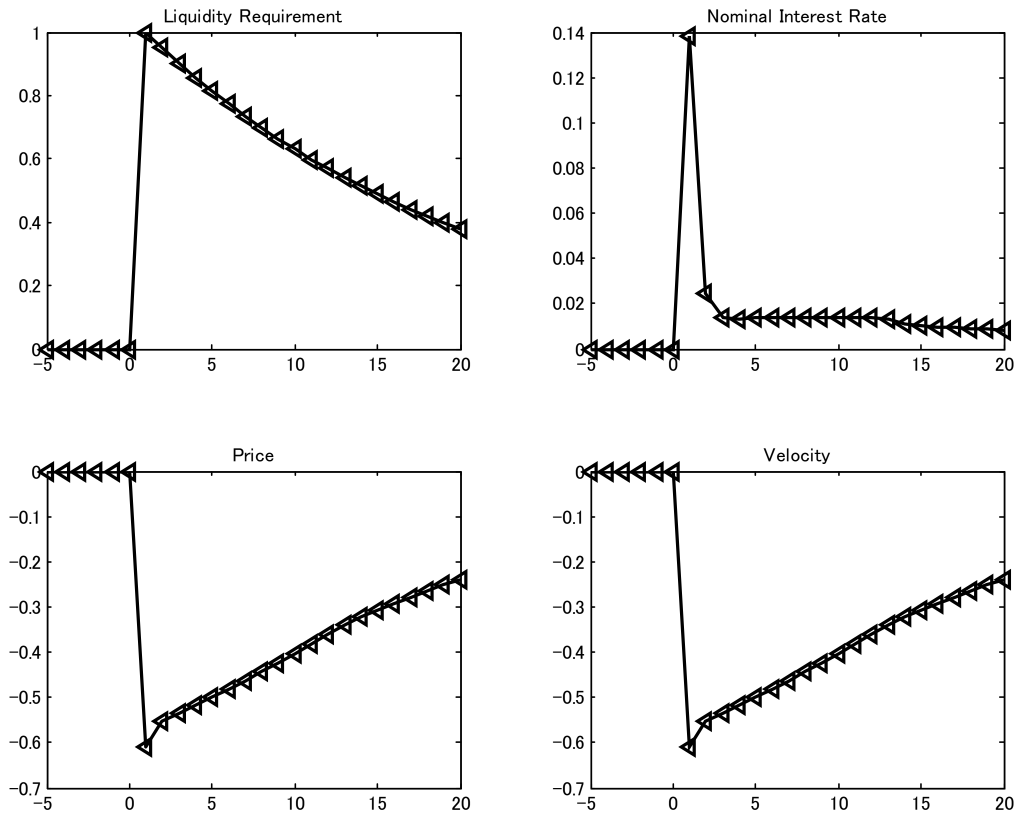



Figure 7: Impulse response of liquidity requirement, nominal interest rate, price, and velocity to a positive shock to the liquidity requirement. The horizontal axis denotes quarters after the shock, and the vertical axis denotes a deviation from the steady state value.

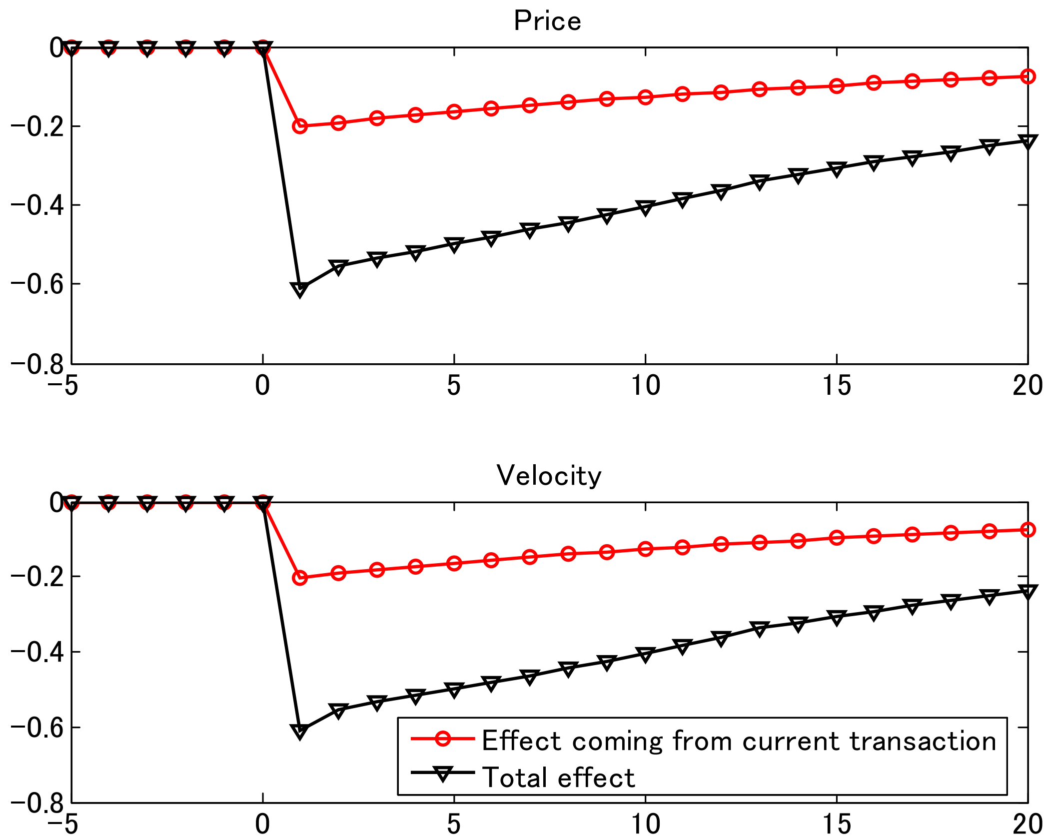
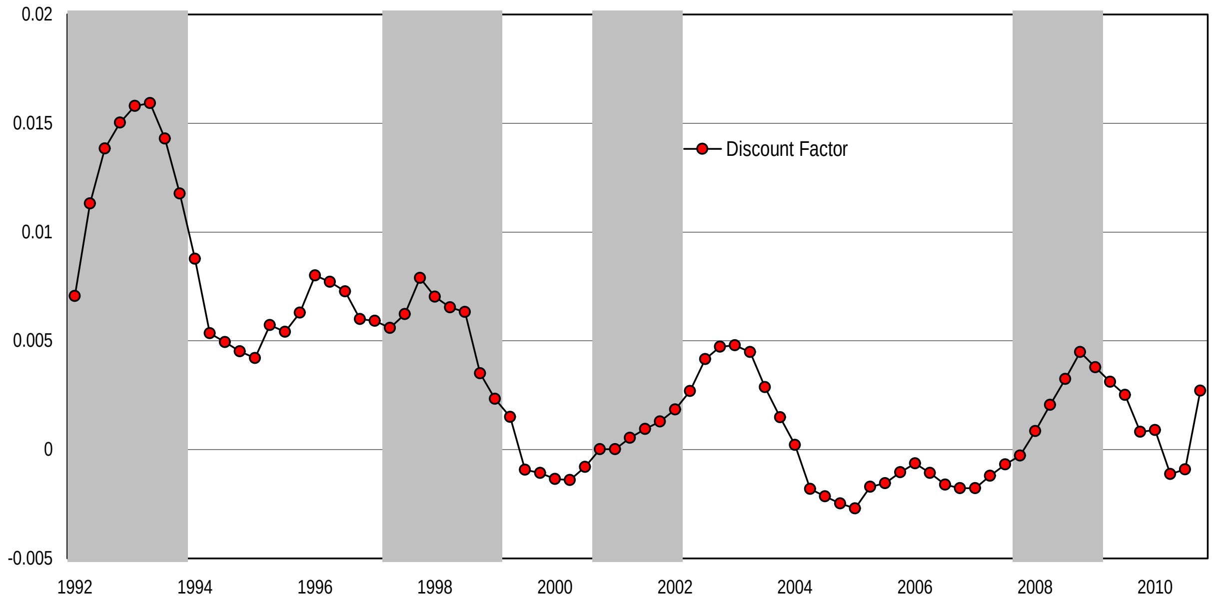



Figure 8: Impulse response of price and velocity to a positive shock to the liquidity requirement. Red lines with circle display effect of the shocks coming from the current transaction demand. The horizontal axis denotes quarters after the shock, and the vertical axis denotes a deviation from the steady state value.



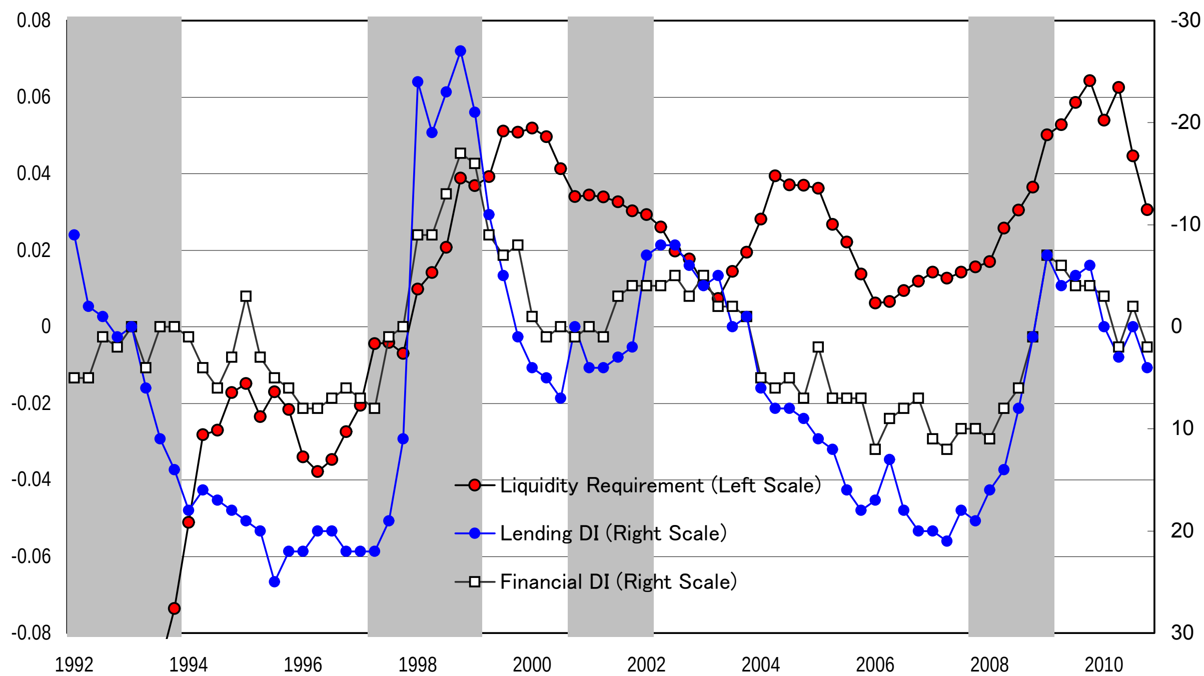




Figure 9: Yearly difference of estimated discount factor $d_t\left(h^t\right)$ (upper panel) and cash requirement $\theta_t\left(h^t\right)$ (lower panel). The left scale of the vertical axis denotes the yearly difference of the deviation from the steady state. The right scale of the vertical axis denotes the D.I. of lending attitude of financial institutions and that of financial position.

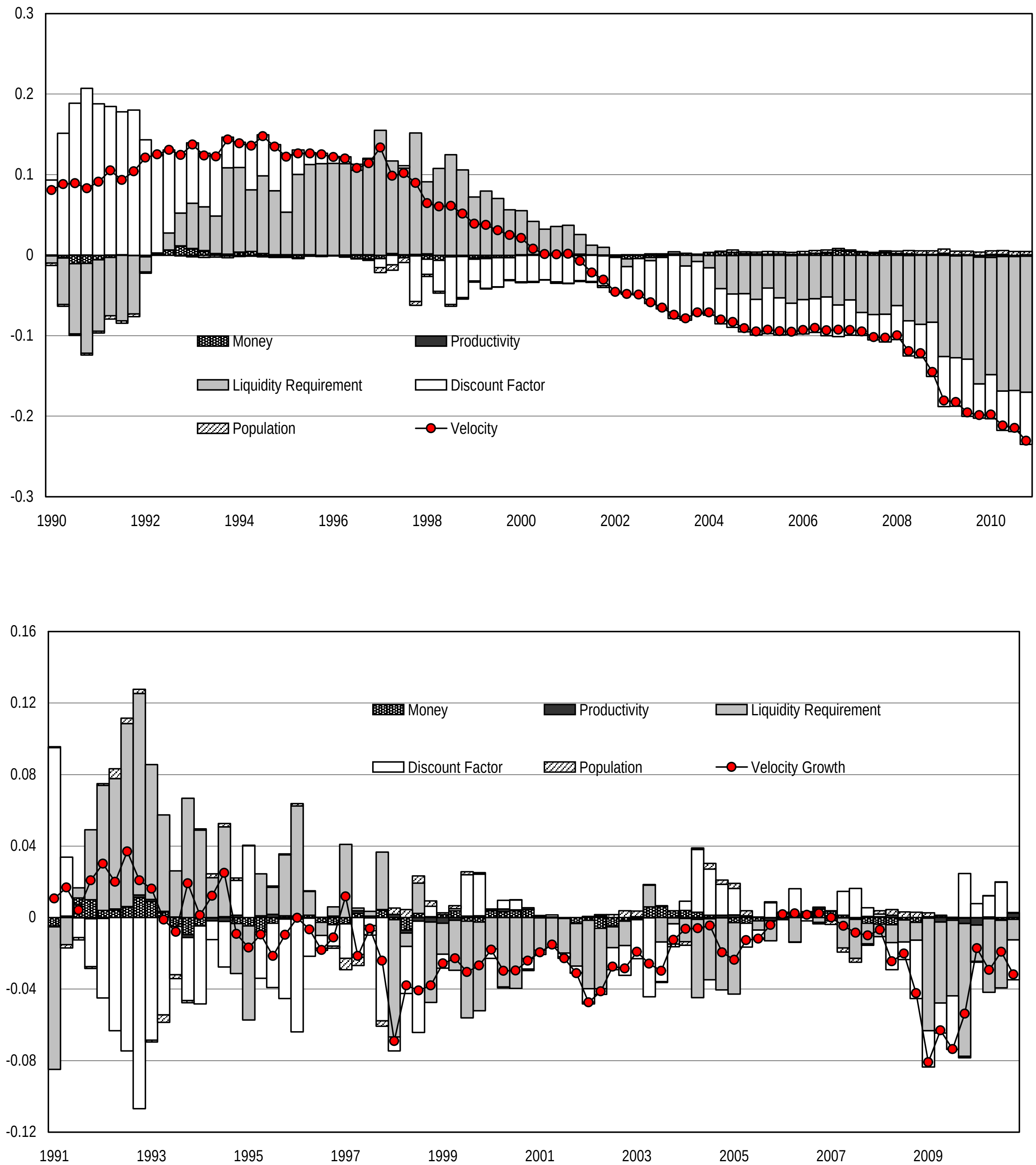



Figure 10: Historical decomposition of velocity variations for demeaned series from 1990Q1 to 2010Q4 (the upper panel) and growth rate from 1991Q1 to 2010Q4 (the lower panel) into five structural shocks. The vertical axis denotes percentage deviation from the mean and its yearly difference, respectively.

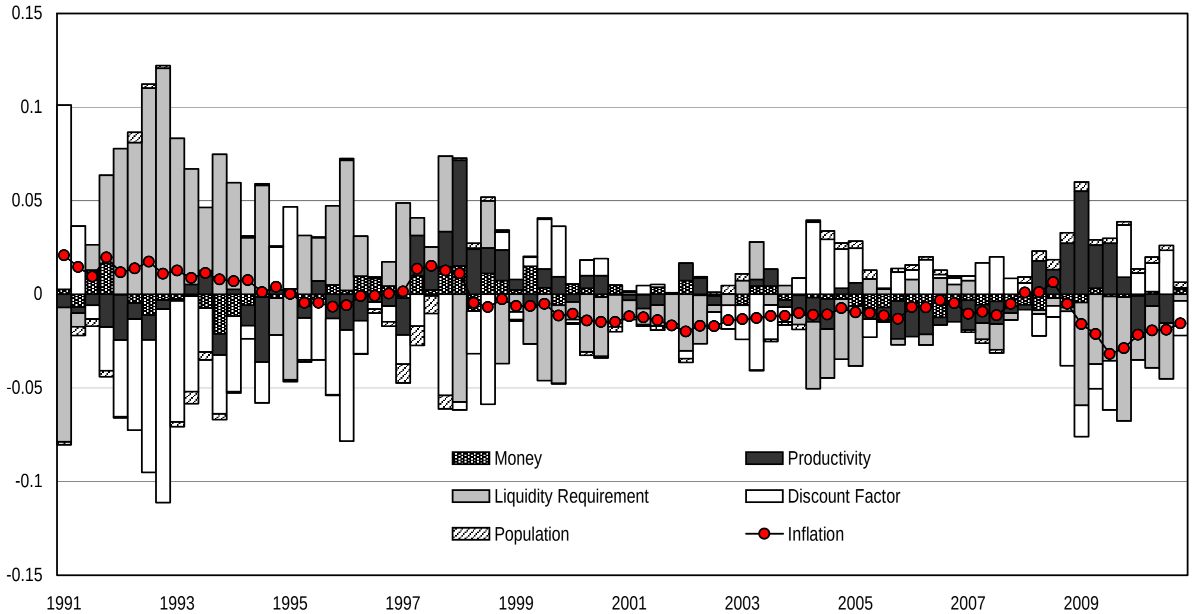



Figure 11: Historical decomposition of yearly difference of price into five structural shocks from 1991Q1 to 2010Q4. The vertical axis denotes percentage deviation.

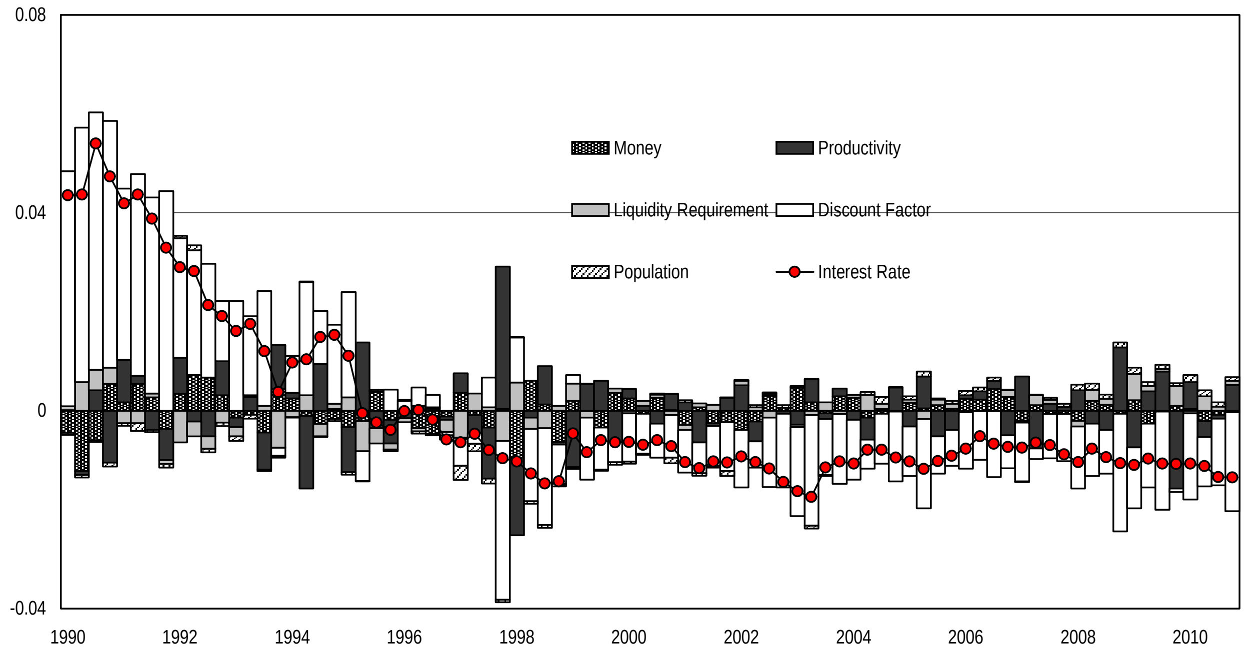



Figure 12: Historical decomposition of nominal interest rate into five structural shocks from 1990Q1 to 2010Q4. The vertical axis denotes percentage deviation from the mean.