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Abstract

The cross-country correlations of international real business cycle models depend critically on the number of countries in the models. A positive productivity shock in one country will stimulate investment in the country that has experienced the shock, while reducing internal investment in the other countries, which will then simultaneously experience a slump. This comovement mechanism is absent in two-country models.

Keywords: International Real Business Cycles; Cross-Country Correlations; Multi-Country; Country Size
JEL classification: E32, F41

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1 Introduction

This paper analyzes the effects of having more than two countries in standard international real business cycle (IRBC) models. The most important finding is that the cross-country correlations of the models depend critically on the number of countries in the models. Literature has been struggling to adequately explain positive cross-country correlations of input (investment and labor) and output in data pertaining to two-country models, which usually produce negative correlations (Backus et al., 1992; Baxter, 1995; Ambler et al., 2004). However, the well-documented, cross-country correlation problem can be explained, in part, if there are more than two countries in the IRBC models.

Behind the implications of cross-country correlations is a coherent intuition that can be understood if one compares the effects of a technology shock in simple two- and three-country models. Taking two countries, for example the United States (U.S.) and Germany, and supposing that a positive total factor productivity (TFP) shock hits the former, it causes not only the U.S. to increase domestic investment, but German savings to flow to the U.S. Germany thus experiences lower domestic investment, which creates a negative cross-country correlation of investment in the model. In the U.S., higher investment leads to a higher marginal product of labor that, in turn, increases labor in the U.S. In Germany, meanwhile, the loss of investment leads to lower labor and explains the models' negative cross-country correlation of labor. In the end, the U.S. experiences a boom and Germany a recession: negative output correlation.

In a three-country model (U.S., Germany and Japan), model cross-country correlations potentially have different values driven by the identical TFP shock. Essentially, comovement arises because responses to a shock in one country are the same across the rest of the countries. When the U.S. experiences a relatively high TFP period, both Germany and Japan decrease their internal investments, implying a *positive* investment correlation between Germany and Japan. Similarly, labor input and output decrease in both Germany and Japan. Two-country models completely exclude this potential determinant of cross-country correlations. Hence, multi-country models and two-country models plausibly have different cross-country correlations.

Finding that the number of countries matters in IRBC models has important implications for more complex two-country macro models. Despite the difficulties in explaining basic data (Obstfeld and Rogoff, 2000), various two-country macro models are the main tools used for analyzing the international interactions of macro variables (e.g., Obstfeld and Rogoff, 1996; Laxton and Pesenti, 2003; Erceg et al., 2006). Since standard IRBC models are basic components of more complex international macro models, this paper suggests that the difficulties encountered in explaining basic data are partly caused by the popular two-country assumption, and that the implications drawn from two-country models are possibly misleading.

Other findings include that: the effects of a multiplicity of countries diminish as the number of countries in the model increases; the size of countries in multi-country models changes models' cross-country correlations; the intra-country business cycle properties of the models depend more on the size of the countries than on their number.

After a brief description of the data and model facts in Section 2, the models and calculation methodology are explained in Section 3, the results of business-cycle moments and a detailed examination of the mechanism using impulse response functions are provided in Section 4, and the last section presents concluding points.

2 Literature and facts

After the successful use of closed-economy real business cycle models to explain basic business cycle moments (Kydland and Prescott, 1982; King et al., 1988), researchers extend the framework to two-country settings to explain the international comovement of business cycles. Prototypical two-country international real business cycle (IRBC) models (Backus et al., 1992; Baxter and Crucini, 1993, 1995; Kollmann, 1996) successfully replicate the intra-country business cycle moments, although they fail to show empirically consistent cross-country business cycle facts.

Notable gaps exist between the data and models in contemporaneous cross-country correlations (Baxter, 1995; Obstfeld and Rogoff, 2000; Ambler et al., 2004).[1] Table 1 compares data and prototypical model international business cycle statistics. The figures comprise the quarterly data of seven developed countries, while the model is a two-country version of King et al. (1988) in which two countries trade complete state contingent claims.[2]

A comparison of the data and model columns indicates the following: (1) Cross-country correlations of output are positive in the data, but the model indicates strong negative correlations. (2) Cross-country correlations of consumption tend to be less than those of output correlations in the data, yet the model predicts much higher positive consumption correlations than output correlations. The risk-sharing behavior under the complete market assumption drives this high consumption correlation in the model. (3) Cross-country correlations of investment are positive across countries in the data, but the model predicts strong negative correlations. (4) Cross-country correlations of labor are positive across countries in the data, but the model predicts strong negative correlations. (5) The data indicates slightly negative correlations of net exports, whereas the model implies strong negative correlations.[3]

[1]There are two additional important problems. One is that, while investment is reasonably volatile, it is not as unstable as predicted by a simple frictionless model that implies quick international capital reallocations. The other problem is that the net exports (over output) and output of a country are negatively correlated in the data, whereas the model predicts a positive correlation. These issues are resolved in early literature (e.g., Baxter and Crucini, 1995).

[2]Appendix A provides detailed explanations.

[3]Cross-country correlations of net exports are rarely examined. In a two-country model, positive net

Over the past two decades, literature on the topic has attempted to resolve these issues, mainly by modifying production, market, and/or shock structures, while retaining the *two-country* framework (e.g., Backus et al., 1992; Baxter and Crucini, 1995; Kollmann, 1996; Kehoe and Perri, 2002; Baxter and Farr, 2005; Wen, 2007).[4] In addition to these modifications, Baxter and Crucini (1993) and Head (1995) examine the effects of changing the *size* of the model economies and reveal that changing the size of the economy basically does not change the cross-country correlations in their two-country models.

IRBC models in which there are more than two countries are rarely examined. One notable exception is the study by Head (1995), who compares symmetric two-country, symmetric five-country, and asymmetric two-country versions of the model presented by King et al. (1988). Head's hypothesis is that having more sources of consumption risk sharing (by having more countries in the "world") can change the welfare cost of business cycles. He reports that the consumption correlation is smaller and output correlation is larger in the five-country model than in the two-country one. However, he does not study the underlying mechanism affecting the correlations. Zimmermann (1997) and Kose and Yi (2006) consider three-country models for analyzing the determinants of business cycle diffusion through trade by extending the model of Backus et al. (1994). While macroeconomic models developed by central banks and international institutions sometimes include more than two countries (e.g., Laxton and Pesenti, 2003; Erceg et al., 2006), they mainly try to explain and predict macro variables of a specific country in an open-economy context, rather than analyze comovement among countries. Finally, van Wincoop (2000) and articles led by den Haan et al. (2011) compare computational methodologies of calculating multi-country IRBC models, but do not analyze the implications of the multiplicity of countries.

3 The models

The models are single-good multi-country real business cycle models. They are described as a social planner's problem, since the equilibrium allocations are (constrained) Pareto optimal. The countries potentially vary in size, but are symmetric. The Pareto weight is equivalent to a country's population size.

Time is discrete and indexed by t. The world population is a unit measure. The world comprises N countries, indexed by $i = 1, ..., N$. Each country has a π_i fraction of the population, where $\sum_i \pi_i = 1$. Only one type of good exists in the economy, and it is used for

exports in one country automatically imply negative net exports for the other. Cross-country correlation is always negative unity. If we construct a dataset wherein there are only two countries—a focus country and the aggregate of the rest of the world—then the cross-country correlation of net exports in this data is negative unity, as in the two-country model. However, the usual data set picks two of the world's countries.

[4]This paper considers the single-good IRBC setting, and these cited papers are a few examples of modified, single-good two-country models. Other modifications are to include multiple-goods (e.g., Backus et al., 1994; Stockman and Tesar, 1995) and nominal rigidity (e.g., Chari et al., 2002).

consumption as well as investment.

3.1 Households

Households enjoy consumption, $C_{i,t}$, and dislike working, $L_{i,t}$. By normalizing total time endowment to be unity, $1 - L_{i,t}$ stands for the leisure of a household in i at t. $L_{i,t}$ is used for the production in country i, i.e., labor is immobile across countries. The social planner maximizes the weighted sum of the utility of the stand-in households

$$E_0 \sum_{i=1}^{N} \pi_i \sum_{t=0}^{\infty} \beta^t \frac{(C_{i,t}^{\psi}(1 - L_{i,t})^{1-\psi})^{1-\gamma}}{1-\gamma}. \tag{1}$$

subject to a technology, a shock process and a resource constraint.

3.2 The baseline production technology and the shock process

The baseline model employs the standard neoclassical production function (King et al., 1988). Various other production technologies are considered: time-to-build (Backus et al., 1992), reduced-form adjustment friction (Baxter and Crucini, 1993), and reduced-form adjustment friction plus variable utilization of capital (Baxter and Farr, 2005) technologies. In addition, a time-to-build model with variable utilization is examined. Since the expressions of the additional technologies are straightforward extensions of two-country models, this section only shows the baseline technology. Appendix B presents other production technologies.

The output, $Y_{i,t}$, is produced using capital ($K_{i,t}$) and labor ($L_{i,t}$).

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha} L_{i,t}^{1-\alpha} \tag{2}$$

$$K_{i,t+1} = (1 - \delta) K_{i,t} + I_{i,t} \tag{3}$$

where $A_{i,t}$ is the country-specific total factor productivity (TFP) and $I_{i,t}$ is investment. TFP has a world common trend component ($\bar{A}_t$) growing at rate g, and country specific stochastic components ($\tilde{A}_{i,t}$), such that

$$A_{i,t} = \bar{A}_t \tilde{A}_{i,t} = A_0 (1 + g)^t \tilde{A}_{i,t}. \tag{4}$$

Moreover, let $\tilde{\boldsymbol{a}}_t$ be a vector of $\log \tilde{A}_{i,t}$ and

$$\tilde{\boldsymbol{a}}_t = \boldsymbol{\Omega} \tilde{\boldsymbol{a}}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{5}$$

where the diagonal element of $\boldsymbol{\Omega}_{(i,i)}$ is autocorrelation parameter ω_{ii}, and the off-diagonal element of $\boldsymbol{\Omega}_{(i,h)}$ is spillover parameter ω_{ih}. The vector of shock, $\boldsymbol{\varepsilon}_t$, is jointly normal ($\boldsymbol{\varepsilon}_t \sim N$), independently and identically distributed across t, $E(\boldsymbol{\varepsilon}_t) = \mathbf{0}$, $V(\boldsymbol{\varepsilon}_t)_{(i,i)} = \sigma_{ii}^2$, and $V(\boldsymbol{\varepsilon}_t)_{(i,h)} = \sigma_{ih}$.

3.3 Resource constraint

The resource constraint of the baseline model is that the planner collects the entire global output and allocates both consumption and investment, which is identical to the complete market situation in the decentralized model.

$$\sum_i \pi_i NX_{i,t} = 0. \tag{6}$$

where $NX_{i,t}$ is net exports and

$$NX_{i,t} = Y_{i,t} - C_{i,t} - I_{i,t}. \tag{7}$$

Appendix B presents two alternative resource constraints, one including a complete market subject to trade costs, and the other restricting assets to non-state contingent bonds (risk-free bond market).

3.4 Computation and calibration

The computation procedure is standard (Kydland and Prescott, 1982; King et al., 1988). The economy is transformed into a detrended economy, and its non-stochastic steady state is calculated. Model moments are obtained from cyclical behavior around the steady state using the log-linear rational expectation system.[5] A separate appendix presents the non-stochastic steady state and log-linear equations.

The multi-country setting does not greatly inflate the state space of the model; state variables in the system of the baseline model are $A_{i,t}$, $K_{i,t}$ for all $i = 1, ..., N$. In an $N = 10$ economy, for example, the total number of state variables is 20. The bond market assumption and the time-to-build technology require additional state variables, but the speed of an increment is a multiple of the number of countries and, hence, standard linear rational expectation solvers can handle the calculations.[6] All the moments are calculated for each selection of production technology, shock process and market structure. To compare the implications of the number of countries per se, only the number of countries in the model is changed.

[5]For example, the output is calculated by $\hat{y}_{i,t} = \log(Y_{i,t}/Y)$, where Y is the steady state output level. Net exports are calculated by:

$$\widehat{nx}_{i,t} = \hat{y}_{i,t} - \frac{C}{Y}\hat{c}_{i,t} - \frac{C}{Y}\hat{\imath}_{i,t},$$

where C and I are steady state consumption and investment respectively.

[6]The calculation employs AIM implementation of Anderson=Moore algorithm (Anderson, 2008).

The moments are obtained by 100,000-period simulations and filtered using an approximated band-path filter.[7]

The parameterization follows the literature. Table 2 summarizes the parameter values. The time period is set to a quarter of a year. The first six parameters in Table 2 are associated with the utility function, Cobb-Douglas production share, and the trend growth rate; they are standard values. Other parameters associated with production technology are set as in the benchmark parameters of Backus et al. (1992), Baxter and Crucini (1995), and Baxter and Farr (2005).

An important issue in the parameterization is the selection of the shock process. The strategy is to expand the process used in the standard two-country setting to a multi-country setting, in order to make the model properties as comparable as possible to standard two-country models.[8] The size of the shock is arbitrarily chosen because it influences the volatility level, but not other business cycle statistics (Baxter, 1995). The correlation of the shock follows Baxter (1995). Two sets of parameters associated with the diffusion structure, Ω, are tried. One shuts down the cross-country diffusion, following Baxter and Farr (2005); the other, from Backus et al. (1992), has a positive diffusion across countries. The off-diagonal term, $\omega_{ih} > 0$, is adjusted so that the largest eigenvalue of Ω is the same as for the non-diffusion model, in order to preserve the identical auto-correlation nature of the shock.

4 Results

In the first part of this section, the business cycle moments of the models are given, following which the intuitive mechanism is explored using impulse response functions. These analyses assume that each country is the same $(1/N)$, while those in the last part assume that the size of each country differs.

The results given are based on specific combinations of the technology, shock process and resource constraint, but the implications for other combinations are robust. Business cycle moments are presented only for the baseline model, a multi-country extension of King et al. (1988) with non-diffusion shock and a complete market.

[7] The parameters are $BP_K(p,q) = BP_{12}(6,32)$, following the recommendation in Baxter and King (1999). The results obtained using the HP filter (Hodrick and Prescott, 1997) are similar. If the number of countries is relatively small, the moments can be directly calculated from the system. As the number of countries increases, the computational burden prevents the use of a direct calculation method. Simulations give almost identical moments for cases with a small number (e.g., five) of countries.

[8] A potential alternative approach is to calibrate the diffusion process and variance-covariance matrix of the shock using the data for a large set of countries. However, the primary objective of the exercise is to understand the role of the number of countries in the model, not to replicate the business cycle comovement of the countries in the real world.

4.1 Intra- and cross-country moments

In Table 3, the second to sixth columns from the left record the business cycle moments for two-, three-, five-, ten- and 25-country baseline models. The moments for the two-country model are the same as given in Table 1.

The table shows that there is no great change in intra-country business cycle moments; as the number of countries increases, investment, labor and net export volatilities slightly increase relative to output, while the volatility of consumption decreases.[9] These changes, mainly driven by changes in the size of the countries, are discussed later in this section. Correlations to output are virtually constant, regardless of the number of countries; autocorrelations also are mostly unchanged. However, there are notable changes in cross-country moments, and Figures 1 to 5 graphically show the changes.

Figures 1 to 5 describe how changing the number of countries in the models influences cross-country correlations. In these figures, the number of countries (N) is represented by the X axis, and cross-country correlations by the Y axis. The largest number of countries examined in the figures is 25. In a 25-country model, each country has 4% of the world population, roughly matching the mean of G7 countries.[10] The solid lines show the model cross-country correlations. The filled symbols represent positive values whereas the open symbols negative values. The interrupted lines indicate data values. For example, the interrupted line (Y) represents the median of the G7 cross-country output correlations (0.37).

Figure 1 shows the cross-country correlations obtained by the baseline models, some of which also appear in Table 3. Figures 2, 3, and 4 are multi-country versions of Backus et al. (1992), Baxter and Crucini (1995), and Baxter and Farr (2005), respectively. Finally, Figure 5 is a multi-country time-to-build model with variable utilization.

Overall, increasing the number of countries improves the quantitative performance of the models. Figure 1 shows that strong negative cross-country correlations of output, investment, labor and net exports in the baseline model are mitigated when the number of countries is increased; this mitigation suggests that part of the cross-country correlation puzzles is caused by the conventional two-country assumption.

Figure 2 extends Backus et al. (1992) to a multi-country version, which extension attenuates strong positive consumption correlations, as well as strong negative correlations in output and investment. The investment correlation is positive if there are more than 13 countries. Figure 3 presents a multi-country version of Baxter and Crucini (1995), which uses a bond market assumption to reduce the cross-country correlation of consumption. The negative correlation of consumption in the model is not a generic result: the correlation is positive if there

[9]These specific models do not match the investment volatility and countercyclicality of net exports, but the classical modification of introducing investment friction and restricting the asset market, as done by Baxter and Crucini (1995), can resolve these two problems. A multi-country version of Baxter and Crucini (1995) performs similarly. See a separate appendix.

[10]See Appendix A.

are more than three countries. The picture also shows *positive* cross-country correlations of investment and labor if more than eight countries are included in the model. And regardless of the model, the inclusion of many countries renders cross-country net export correlations closer to the data value.

Another finding is that the effects of increasing the number of countries diminish. Compared with changes in cross-country correlations for two- and three- country models, those for 15- and 16-country models are smaller.

Note also that cross-country correlations do not converge to zero. Some might suggest that these diminishing effects are because, as the number of countries increases, cross-country moments are attenuated toward zero as a result of the weaker country-by-country interactions. However, for example, labor and consumption in Figure 3 do not move toward zero.

Finally, the multi-country extension is a complement to technology and financial market modifications, because the multiplicity preserves and occasionally enhances improvement results in intra- and cross-country moments obtained by technology and financial market modifications. Figures 4 and 5 show relatively successful two-country models. The two-country model of Baxter and Farr (2005) shows empirically consistent signs for all the cross-country correlations (Figure 4). Similarly, allowing variable utilization improves the two-country Backus et al. (1992) model. The multi-country extension helps to explain the net exports cross-country correlations in these models. At the same time, there is no great change in other cross-country correlations.

4.2 Impulse response functions

Impulse response functions (IRFs) illustrate the mechanisms behind the moment results. Figures 6 to 10 are IRFs of output, consumption, investment, labor, net exports and TFP. The X axis represents the model time period, with the shock hitting the economies at period 0. The vertical axis shows the percentage deviations of the variables from the steady state values.

Figures 6 and 7 are derived from a two-country model, and Figures 8 to 10 from a ten-country model. Figures 6 and 7 show IRFs for countries 1 and 2, respectively. Similarly, Figures 8, 9 and 10 are the IRFs of Country 1, Country 2 and Country 3, respectively.

Both models use baseline technology, a complete market assumption and a shock that excludes the cross-country diffusion component ($\omega_{ih} = 0$). The figures are drawn from correlated shocks; at period 0, a 1% positive TFP shock hits Country 1 and a 0.12% positive TFP shock hits Country 2. Other countries in the ten-country model do not experience shocks. Since the remaining countries are completely symmetric, Figure 10 shows IRFs of the remaining countries.

Figures 6 and 7 indicate that, after the positive productivity shock in the two-country model, Country 1 experiences a boom and Country 2 a recession; a large positive TFP shock stimulates investment in Country 1; and the investment spike leads to a higher labor supply in Country 1. Output rises since capital stock, labor and TFP increase. Country 2 also experiences a positive productivity shock (TFP line in Figure 7). Nonetheless, in the two-country model, investment in Country 2 decreases, since the higher marginal return to investment in Country 1 attracts investment away from Country 2 to Country 1, as indicated by large changes in net exports. The reduction in investment leads to a decrease in the labor supply in Country 2. The output then decreases, despite the positive TFP shock, because of a large decline in input. The complete market assumption ensures consumption increments in both countries and the consumption correlation is strongly positive.

The implications are drastically different in a ten-country model. As in the two-country model, Country 1 experiences a boom (Figure 8), by inducing investments from other countries. Internal investments decrease in Countries 3 to 10 (Figure 10). Cross-country correlations among these 28 ($= 8 \times 7/2$) pairs are *positive* because all these countries experience reductions in output, investment and labor. In addition, the correlated shocks in the ten-country model lead to a boom in the second-highest TFP country. Figure 9 shows that, in Country 2, there is a slight rise in investment in the ten-country model. Other variables also increase, although the magnitude of the increments is smaller than in the case of Country 1. In the ten-country model, the diminishing marginal returns for capital in the highest TFP country (Country 1) lead to an investment inflow to the second-highest TFP country (Country 2). As a result, cross-country correlations of investment, labor and output are positive between Countries 1 and 2. This particular correlated shock delivers two types of positive cross-country correlations of investment: one increasing pair and 28 decreasing pairs. The other 16 ($= 10 \times 9/2 - 1 - 28$) combinations are negatively correlated.

4.3 Number of countries and respective sizes

Given that increasing the number of countries effectively makes them smaller, it is natural to ask whether the effects of the multiplicity of countries are captured in two- or three-country models by aggregating uninterested economies as a single "rest of the world." If such two- or three-country models provide approximately the same implications as multi-country models, using simpler models reduces computational costs. Unfortunately, however, postulating a fictitious giant economy leads to potentially misleading implications.

In Table 3, the second to sixth columns from the right give moments of various size/number combinations to explore the role of the sizes.[11] Columns A and B show a "small and large" model (Baxter and Crucini, 1993; Head, 1995). The first country has 10% of the world population, whereas the second has 90%. Intra-country moments of the small country are

[11] Results using alternative production, market and shock assumptions are reported in a separate appendix. The following implications are generally robust in other cases.

shown in A, and those of the large one in B. Since there are two countries, both columns show the same cross-country correlations.

Columns C and D show a three-country model. The first two countries have 10% of the population; the third has 80% of the world population. This third country may correspond to a "rest of the world" economy (Zimmermann, 1997). Column C shows intra-country moments of a small country, and the cross-country correlations between the two small countries. Column D shows intra-country moments of the large country, and the cross-country correlations between the large country and one of the small countries. Column E considers a case in which there are 19 countries. Nine of these are small (10%) countries, and the rest are divided into ten smaller (1%) countries. The statistics given in the table are the small (10%) country's intra-country moments, and the cross-country correlations between two small countries. Cross-country technology diffusion and shock correlations are the same for each pair of countries. The purpose of presenting this research is not to replicate realistic moments to the greatest extent possible but, rather, to examine the behavior of the model under various size assumptions.

Some of the intra-country business cycle moments vary across size specifications, notably the relative volatilities of consumption, investment and net exports. As discussed by Baxter and Crucini (1993), changes in the values of a large country influence the real interest rate of the model more than those of a small country. Changing the real interest rate mitigates fluctuations in investment in a large country. Hence, as can be seen from comparing Columns A and B, and C and D, a large country generally experience less volatile investment and net exports. These implications of consumption, investment and net exports are consistent with results in the symmetric models. As size of the country decreases (by increasing the number of countries), the volatility of consumption decreases and the volatilities of investment and net exports increase. Hence, manipulating the size of the countries can mimic the effects of changing the number of countries on the intra-country moments.

As reported in Baxter and Crucini (1993) and Head (1995), the cross-country correlations of two-country models are essentially the same, whether they display size variations or are symmetrical. A comparison of symmetric two-country models and Column A indicates the effects of changing sizes in two-country models. As in the symmetric two-country model, the "small and large" model shows strongly positive consumption correlations and strongly negative correlations of output, investment and labor. These results are qualitatively the same as those for the symmetric two-country model.

The cross-country correlations obtained from both the symmetric three-country model and the three-country model in which the countries are of different sizes, differ considerably from the correlations for the two-country model. Cross-country correlations between a large country and one of the small countries (Column D) are similar to those obtained from symmetric two-country or symmetric three-country models, whereas Column C shows strong positive correlations for all the variables. In particular, the positive correlation of net exports suggests that an investment swing between the two countries plays a minor role, and that the two small countries move in tandem. A positive TFP shock to one of the small countries induces investment inflows from the two small countries. But the amount of inflow is limited owing to its small size. Moreover, the inflow is dominated by that from the large country. As a result, the degree of negative correlation between the two small countries is weakened. On the contrary, a positive TFP shock to the large economy induces investment inflows from two small countries. Consequently, the two small countries show positive correlations for all the dimensions.

The problem now is that in the real world, there is no such dominant economy, and simply taking the non-interested countries as the aggregate "rest of the world" may alter the model implications. Thus, even if our research is designed to explain the cross-country correlations of two small countries, it is more meaningful to manage the behavior of the rest of the countries explicitly.

The cross-country correlations in Column E are essentially the same as those derived from a symmetric ten-country model, bearing in mind that the Column E modification from the symmetric ten-country model divides one country into 10 smaller countries. One implication is that the specifications of small countries, compared with those of an interested pair, do not affect cross-country correlations of a pair of relatively large economies.

In summary, cross-country correlations, obtained by using IRBC models, are sensitive to the presence of relatively large economies. However, the specifications of the smaller economies, compared with the interested pair, do not influence cross-country correlations.

5 Conclusion

New insights into international macroeconomic literature have been provided by increasing the number of countries in otherwise standard international real business cycle models. The most important finding is that the number of countries is critical in the cross-country correlations in the models. Boosting the number of the countries in a model helps explain the well-documented, cross-country correlation puzzles that appear in the literature.

The existence of a relatively large country changes cross-country correlations among smaller countries. Hence, aggregating realistically sized countries into a single giant economy (the "rest of the world") leads to potentially misleading implications. Nevertheless, it is suggested that, when considering business cycle comovement of developed countries, models do not need to include hundreds of countries. In a symmetric setting, the effects of a multiplicity of countries diminish. For example, there is no significant difference between a 15-country and 16-country model. In addition, the specifications of smaller economies do not influence cross-country correlations.

Attention has been focused on the simple IRBC models. However, since they are basic components of more complex international macro models, the multiplicity of countries may alter the implications of these complex models. For this reason, it is important that other models be examined in the future.

Finally, the mechanism behind the results here presented is not necessarily specific to international models. Essentially, what has been shown is that comovement arises because responses to a shock in an agent (country) are the same across the rest of the agents (countries). Such a mechanism may exist in other contexts and, if so, employing a two-agent model for analyzing comovement property may result in misleading implications.

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A Data Appendix

A.1 G7 business cycle moments

Table 1 shows key real business cycle moments. Intra-country moments are obtained from the median (mean) of the G7 countries: Canada, France, (West) Germany, Italy, Japan, the United Kingdom and the United States. The main data sources are quarterly national accounts on "SourceOECD." If longer data is available through a country source or older SNA system, the main economic indicator data is extrapolated using the growth rates of additional data. All the variables are seasonally adjusted quarterly data. The real values in the data sets are used. Hours and employment are normalized by year 2000 values (average of four quarters in 2000), and then multiplied to obtain labor. Italian labor is calculated only using employment data, since the hours data is missing. Filtering is applied for the longest possible dataset, and then moments are calculated using a limited sample period (1970, 1st quarter to 2006, 4th quarter). The data for Germany and West Germany are combined after filtering. Excluding net exports, variables are taken from natural logs, after which filters are applied. Net exports are filtered after the figures have been divided by real GDP. Further details are explained in the dataset, which is available on request.

Cross-country correlations are the medians (means) of the possible 21 (= 7 × 6/2) combinations of cross-country correlations. Table 1 includes U.S. statistics. U.S.-foreign cross-country correlations are the median of the possible six cross-country correlations. The last column shows model moments. The model is a straightforward extension of a textbook closed-economy business cycle model (King et al., 1988) to a two-country, connecting, complete set of state contingent claims, with standard parameterizations.

A.2 Size of G7 countries

In the models, each country has an equivalent steady state per capita output, because of the symmetry of the utility and technologies. The population weight captures both population size and economic size. The economic size is used for considering business cycle properties of developed countries.

The data is from Penn World Table 6.2 (Heston et al., 2008). Country GDP is real GDP per capita chain (*rgdpch*) times total population (*POP*). Calculations are the data for 1970, 1985 and 2000. The total world GDP is the sum of the GDPs of all the available countries. The ratios for the United States's GDP to total world GDP are 26% in 1970, 24% in 1985 and 21% in 2000; for Japan's are 8% in 1970, 9% in 1985 and 7% in 2000. Similarly, the ratios for Germany's GDP to the total world GDP for the three years are: 8%, 6% and 5%; for the United Kingdom: 5%, 4% and 3%; for France: 5%, 4% and 3%; for Italy: 4%, 4% and 3%; and for Canada: 2%, 2% and 2%.

B Models

B.1 Alternative production technologies

B.1.1 Adjustment friction and variable utilization

A generalized version of the standard Cobb-Douglas model is expressed by

$$Y_{i,t} = A_{i,t}\zeta(Z_{i,t})K_{i,t}^{\alpha}L_{i,t}^{1-\alpha} \tag{B1}$$

$$K_{i,t+1} = (1-\delta(Z_{i,t}))K_{i,t} + \phi\left(\frac{I_{i,t}}{K_{i,t}}\right)K_{i,t}, \tag{B2}$$

where $I_{i,t}$ is the investment, $K_{i,t}$ is the capital stock, $Z_{i,t}$ is the degree of capital utilization, and $\phi(.)$ is a function of the capital adjustment friction. The function $\zeta(.)$ represents the possibility of variable capital utilization. If $\zeta(Z) = Z^{\alpha}$, the model is equivalent to Baxter and Farr (2005). If $\zeta(.) = 1$ and $\delta(.) = \delta$, the capital utilization and depreciation rate are constant, which is reduced to the model of Baxter and Crucini (1993) specifications. Following Baxter and Farr (2005), a set of conditions is imposed on these functions, so that the non-stochastic steady state values are irrelevant with or without friction and/or adjustment assumptions.[12] Then, only elasticity parameters are relevant for calculating log-linearized models.

B.1.2 Time-to-build

Employing a simplified version (Rouwenhorst, 1991) while allowing variable capital utilization, the technology is expressed as follows:

$$Y_{i,t} = A_{i,t}\zeta(Z_{i,t})K_{i,t}^{\alpha}L_{i,t}^{1-\alpha}, \tag{B3}$$

$$K_{i,t+1} = (1-\delta(Z_{i,t}))K_{i,t} + V_{i,t,1}, \tag{B4}$$

$$I_{i,t} = \frac{1}{J}\sum_{j=1}^{J} V_{i,t,j}, \text{ and} \tag{B5}$$

$$V_{i,t+1,j} = V_{i,t,j+1} \quad \text{for } j = 1, ..., J-1, \tag{B6}$$

where $I_{i,t}$ is the total investment in country i at t, and the investment is divided into J fractions of $V_{i,t,j}$. Each $V_{i,t,j}$ becomes the working capital stock as j period passes. The functions $\zeta(.)$ and $\delta(.)$ take one of the two alternative assumptions. If $\zeta(Z) = Z^{\alpha}$ and $\delta(.)$ satisfies the conditions in footnote 12, the model embodies variable capital utilization property. If $\zeta(.) = 1$ and $\delta(.) = \delta$, it is a version of the time-to-build structure used by Rouwenhorst (1991).

B.2 Alternative resource constraints

B.2.1 Resource constraints in case of a complete market and trade cost

Backus et al. (1992) specify a quadratic trade cost

$$\sum_i \pi_i \left(NX_{i,t} - 0.1(NX_{i,t})^2/Y\right) = 0, \tag{B7}$$

where Y is the steady state output.

[12] The conditions are

$$\phi\left(\frac{I}{K}\right) = \frac{I}{K}, \quad \phi'\left(\frac{I}{K}\right) = 1$$
$$\delta(Z) = \delta, \quad Z = 1,$$

where variables without subscripts are non-stochastic steady state values derived in the separate appendix.

B.2.2 Resource constraints in a bond market

Baxter and Crucini (1995) and Kollmann (1996) consider a model in which only risk-free bonds are available. It is expressed by

$$NX_{i,t} + B_{i,t} = P_t B_{i,t+1} \tag{B8}$$

$$\sum_i \pi_i B_{i,t} = 0, \tag{B9}$$

where $B_{i,t}$ is the bond holding of i country and P_t is the subjective "price" of a bond that the planner faces. This price is the same as the competitive equilibrium price of the non-state-contingent bond.

C Derivation

The derivation of equations is available in a separate appendix that is available on request.

D Robustness results

The robustness results are available in the separate appendix.

Table 1: G7 Business Cycle Moments

Country	G7	G7	G7	G7	USA	Model
	Median	Mean	S.D.	Median	Mean	
Filter	BP	BP	BP	HP	BP	
Standard deviation relative to standard deviation of output						
Consumption	0.91	0.92	0.12	0.92	0.81	0.29
Investment	2.60	2.63	0.35	2.67	2.82	13.21
Labor	1.04	1.30	0.53	1.67	1.07	0.56
Net exports	0.61	0.52	0.21	0.65	0.23	3.33
Correlation to output						
Consumption	0.77	0.76	0.10	0.74	0.88	0.73
Investment	0.75	0.80	0.11	0.74	0.96	0.05
Labor	0.74	0.63	0.37	0.53	0.93	0.97
Net exports	−0.36	−0.32	0.17	−0.29	−0.46	0.21
Autocorrelation						
Output	0.93	0.92	0.02	0.82	0.94	0.91
Consumption	0.94	0.94	0.01	0.77	0.94	0.91
Investment	0.93	0.93	0.01	0.86	0.95	0.79
Labor	0.93	0.93	0.03	0.59	0.92	0.91
Net exports	0.91	0.91	0.02	0.72	0.95	0.80
Cross-country correlation						
Output	0.37	0.41	0.19	0.35	0.45	−0.72
Consumption	0.28	0.21	0.29	0.28	0.38	0.64
Investment	0.23	0.23	0.23	0.25	0.30	−0.99
Labor	0.23	0.23	0.30	0.35	0.51	−0.96
Net exports	−0.02	−0.02	0.22	−0.01	−0.08	−1.00

The author's calculation is based mainly on "SourceOECD." The sample period is 1970 Q1–2006 Q4. See Appendix A for details. Intra-country moments are medians (means) of the G7 countries. Cross-country correlations are medians (means) of 21 ($= 7 \times 6/2$) statistics. Cross-country correlations of the U.S. column are medians of six pairwise correlations between the U.S. and other G7 countries. The S.D. column shows the standard deviations of seven intra-country moments and 21 cross-country correlations. The BP filter (Baxter and King, 1999) uses parameters $BP_{12}(6, 32)$. The HP filter (Hodrick and Prescott, 1997) uses parameter $\lambda = 1600$. The model is the two-country baseline model (King et al. (1988), with a complete market and no shock diffusion).

Table 2: Simulation Parameters

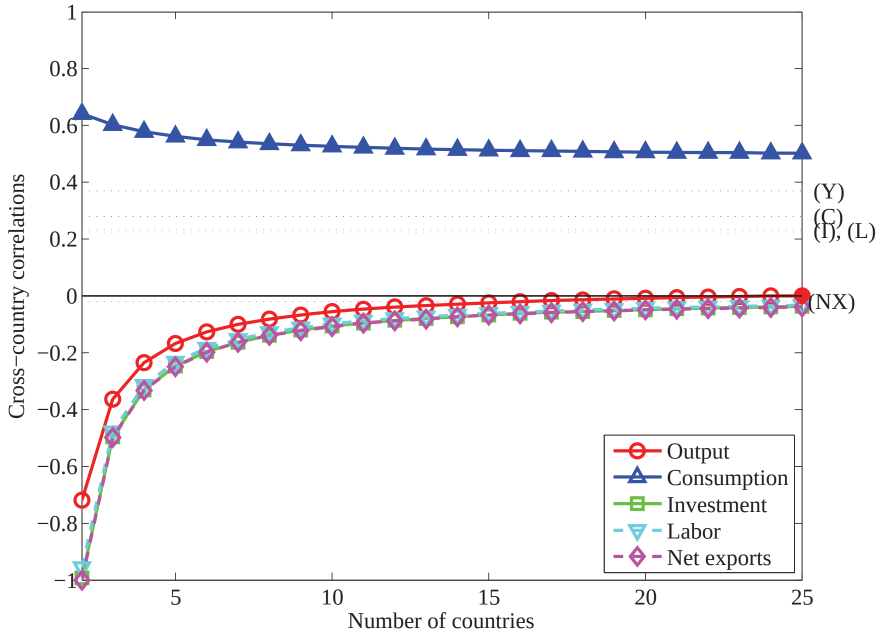
Parameter	Value	Description	Source
β	0.988	Subjective discount factor	
γ	2.00	Controlling relative risk aversion	
ψ	0.35	Utility consumption share	
α	0.36	Production capital share	
δ	0.025	Steady state depreciation	
g	0.004	TFP net growth	
J	4	Maximum time-to-build length	Backus et al. (1992)
$-\frac{\phi'' \frac{I}{K}}{\phi'}$	0.15	Adjustment friction elasticity	Baxter and Crucini (1993)
$\frac{\delta'' Z}{\delta'}$	0.1	Elasticity of marginal depreciation	Baxter and Farr (2005)
σ_{ii}^2	0.001^2	TFP shock variance	
σ_{ih}	$0.258\sigma_{ii}^2$	TFP shock covariance	Baxter (1995)
"No shock diffusion"			
ω_{ii}	0.999	TFP autocorrelation	Baxter and Farr (2005)
ω_{ih}	0	TFP spillover	Baxter and Farr (2005)
"Shock diffusion"			
ω_{ii}	0.906	TFP autocorrelation	Backus et al. (1992)
ω_{ih}	$(0.999 - 0.906)/N$	TFP spillover	Backus et al. (1992)

Table 3: Baseline Model's Business Cycle Moments

N	2	3	5	10	25	A	B	C	D	E	
Size	0.5	0.33	0.2	0.1	0.04	0.1	0.9	0.1	0.8	0.1	Data
Standard deviation relative to standard deviation of output											
Consumption	0.29	0.24	0.22	0.20	0.19	0.18	0.57	0.18	0.48	0.20	0.91
Investment	13.21	13.56	13.75	13.87	13.94	15.82	5.20	15.47	7.39	13.84	2.60
Labor	0.56	0.57	0.58	0.58	0.59	0.64	0.32	0.63	0.39	0.58	1.04
Net exports	3.33	3.42	3.47	3.50	3.51	4.01	1.19	3.92	1.78	3.49	0.61
Correlation to output											
Consumption	0.73	0.75	0.77	0.80	0.82	0.47	0.97	0.53	0.95	0.80	0.77
Investment	0.05	0.04	0.04	0.04	0.04	−0.01	0.38	−0.00	0.25	0.04	0.75
Labor	0.97	0.98	0.99	0.99	0.99	0.99	0.96	0.99	0.96	0.99	0.74
Net exports	0.21	0.21	0.21	0.22	0.22	0.25	0.08	0.24	0.11	0.21	−0.36
Autocorrelation											
Output	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.93
Consumption	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.94
Investment	0.79	0.79	0.80	0.80	0.80	0.79	0.81	0.79	0.80	0.80	0.93
Labor	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.93
Net exports	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.91
Cross-country correlation											
Output	−0.72	−0.36	−0.17	−0.06	0.00	−0.54	−0.54	0.18	−0.50	−0.06	0.37
Consumption	0.64	0.60	0.56	0.53	0.50	0.67	0.67	0.58	0.65	0.53	0.28
Investment	−0.99	−0.50	−0.25	−0.11	−0.05	−0.97	−0.97	0.31	−0.79	−0.12	0.23
Labor	−0.96	−0.48	−0.23	−0.10	−0.04	−0.83	−0.83	0.26	−0.73	−0.11	0.23
Net exports	−1.00	−0.50	−0.25	−0.11	−0.03	−1.00	−1.00	0.32	−0.81	−0.12	−0.02

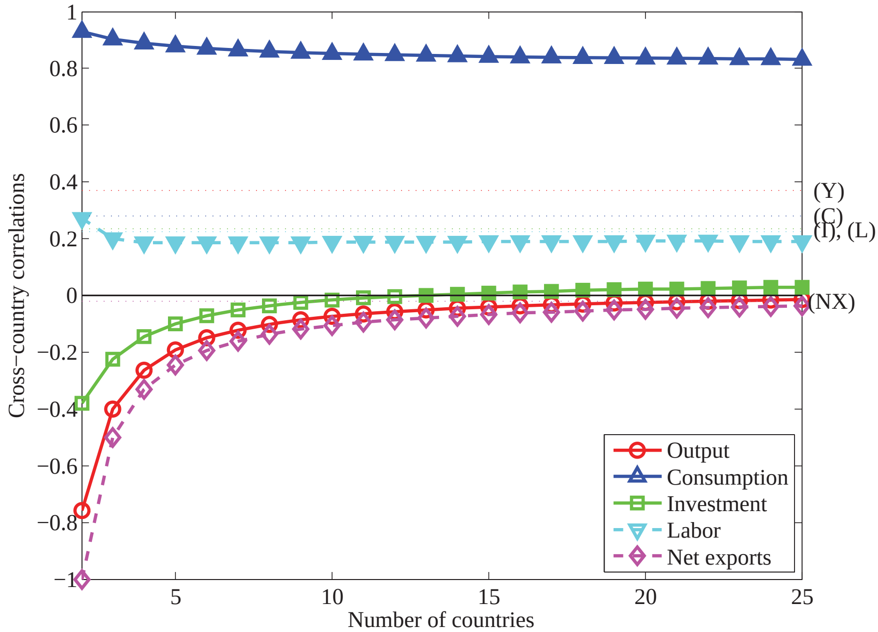
N and size patterns: The first five columns represent models of symmetric size. Sizes are $1/N$. A, B: $N = 2, \{0.1, 0.9\}$. Intra-country moments are for the first (A) and second (B) countries. C, D: $N = 3, \{0.1, 0.1, 0.8\}$. Intra-country moments are for the small (C) and large (D) countries. Cross-country correlations are of the two small countries (C), and the large and one of the small countries (D). E: $N = 19, \{\{0.1\} \times 9, \{0.01\} \times 10\}$. Cross-country correlations are of first two countries. The data (median of G7 countries) is taken from Table 1.

Figure 1: Cross-country Correlations and Number of Countries in the Baseline Model



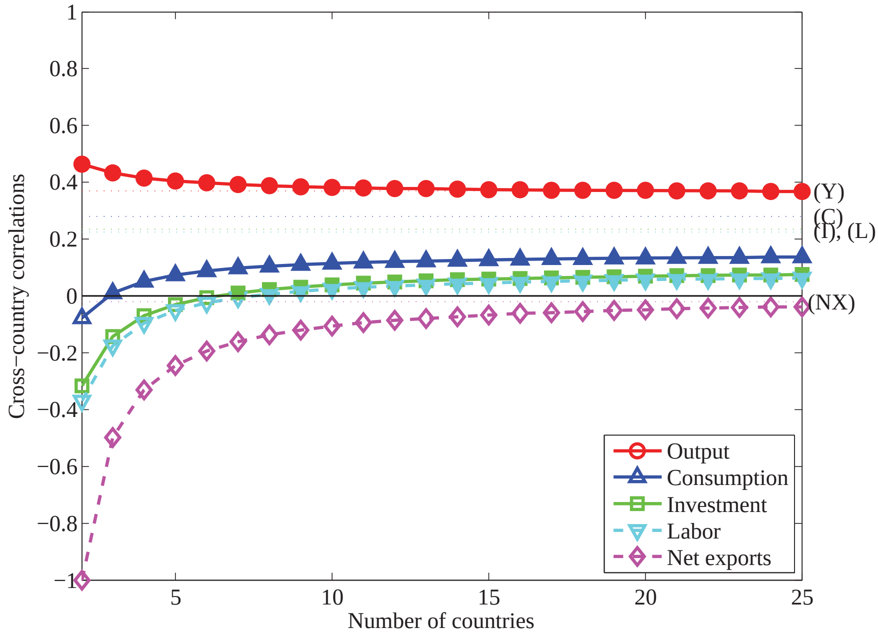
The solid lines show the model's cross-country correlations, given the number of countries in the model. The filled symbols represent positive values whereas the open symbols negative values. The interrupted lines represent data values taken from Table 1. The multi-country versions of King et al. (1988), with no shock diffusion and complete market.

Figure 2: Cross-country Correlations and Number of Countries in Backus et al. (1992) Model



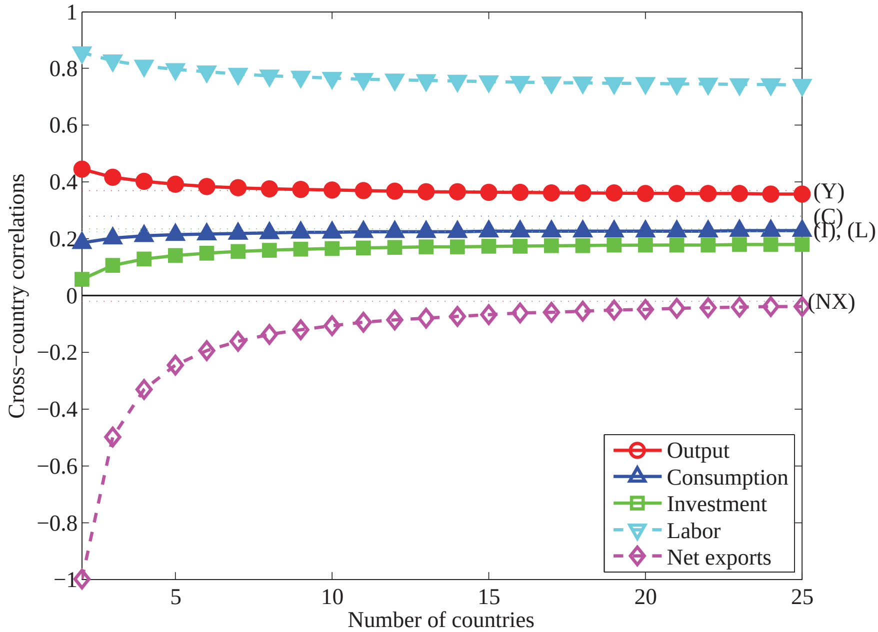
The solid lines show the model's cross-country correlations, given the number of countries in the model. The filled symbols represent positive values whereas the open symbols negative values. The interrupted lines represent data values taken from Table 1. The multi-country versions of Backus et al. (1992), with shock diffusion and a complete market with trade cost.

Figure 3: Cross-country Correlations and Number of countries in Baxter and Crucini (1995) Model



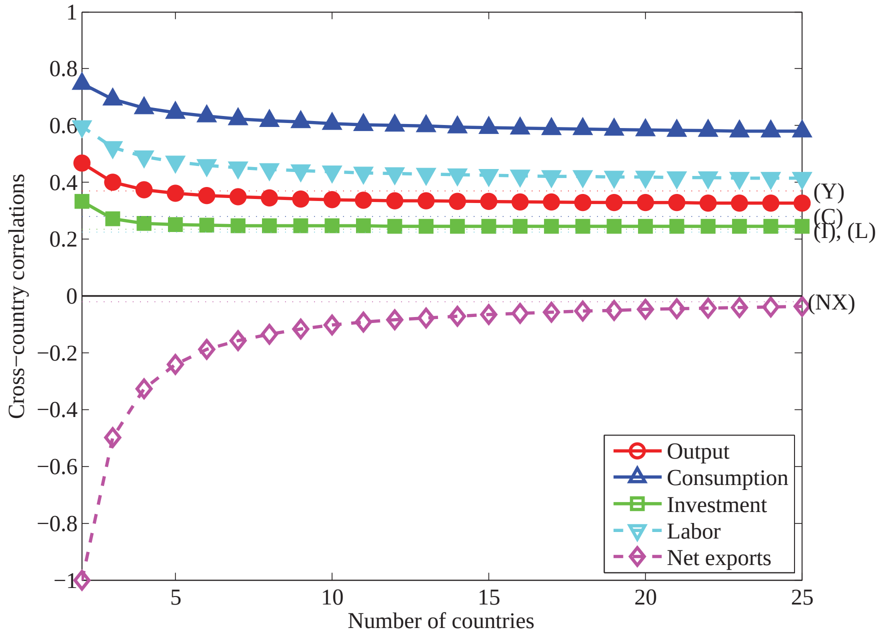
The solid lines show the model's cross-country correlations, given the number of countries in the model. The filled symbols represent positive values whereas the open symbols negative values. The interrupted lines represent data values taken from Table 1. The multi-country versions of Baxter and Crucini (1995), with no shock diffusion and a bond only market.

Figure 4: Cross-country Correlations and Number of Countries in Baxter and Farr (2005) Model



The solid lines show the model's cross-country correlations, given the number of countries in the model. The filled symbols represent positive values whereas the open symbols negative values. The interrupted lines represent data values taken from Table 1. The multi-country versions of Baxter and Farr (2005), with no shock diffusion and a bond only market.

Figure 5: Cross-country Correlations and Number of Countries in Time-to-Build and Variable Utilization Model



The solid lines show the model's cross-country correlations, given the number of countries in the model. The filled symbols represent positive values whereas the open symbols negative values. The interrupted lines represent data values taken from Table 1. The multi-country versions of Backus et al. (1992) with variable utilization, shock diffusion and complete market with trade cost.

Figure 6: Impulse Response Functions of Country 1 (Two-country Model)
(+1% TFP shock to Country 1, +.12% TFP shock to Country 2.)

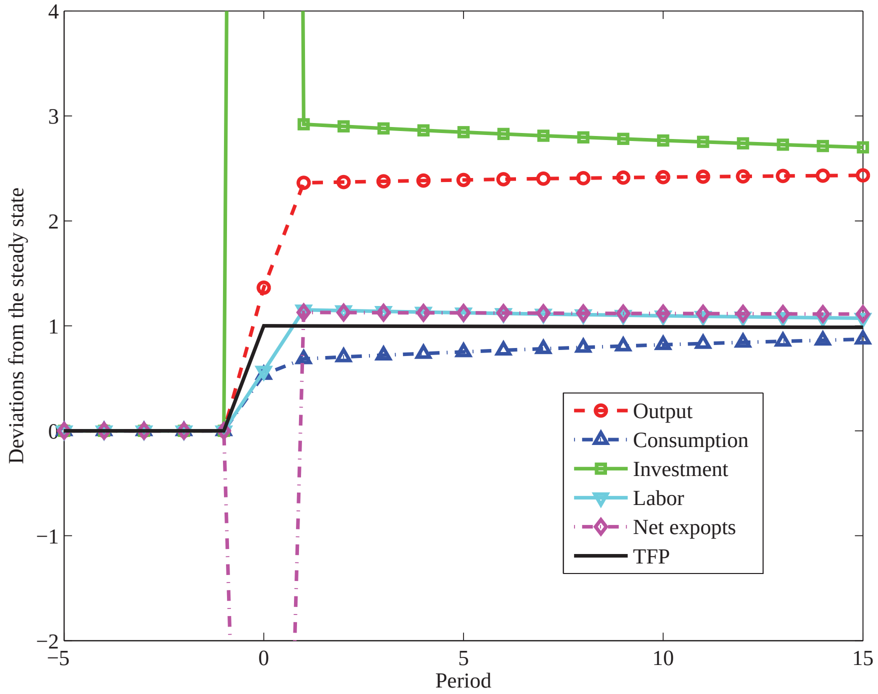


Figure 7: Impulse Response Functions of Country 2 (Two-country Model)
(+1% TFP Shock to Country 1, +.12% TFP Shock to Country 2.)

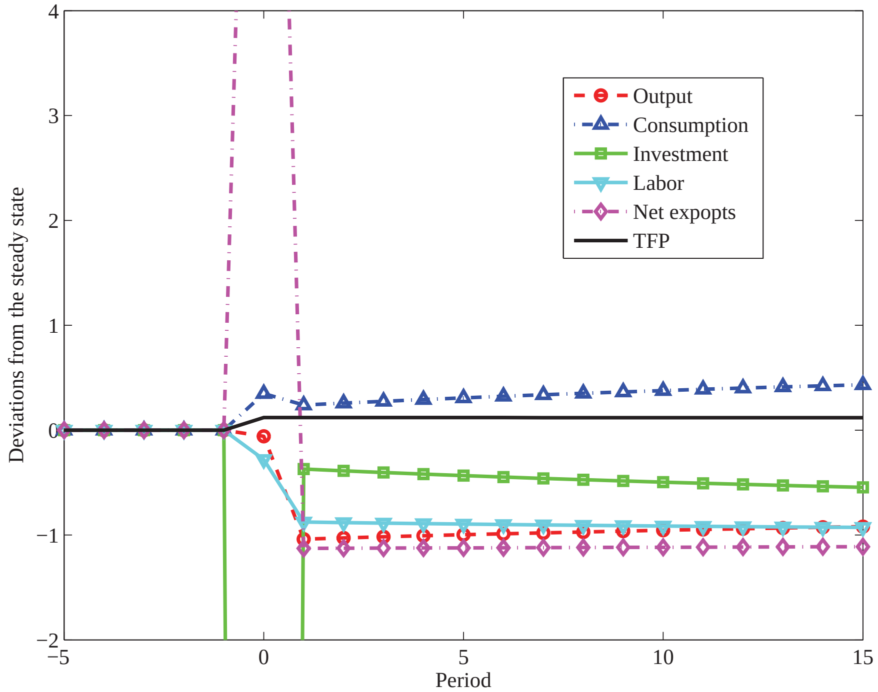


Figure 8: Impulse Response Functions of Country 1 (Ten-country Model)
(+1% TFP Shock to Country 1, +.12% TFP Shock to Country 2, No Shock to Others)

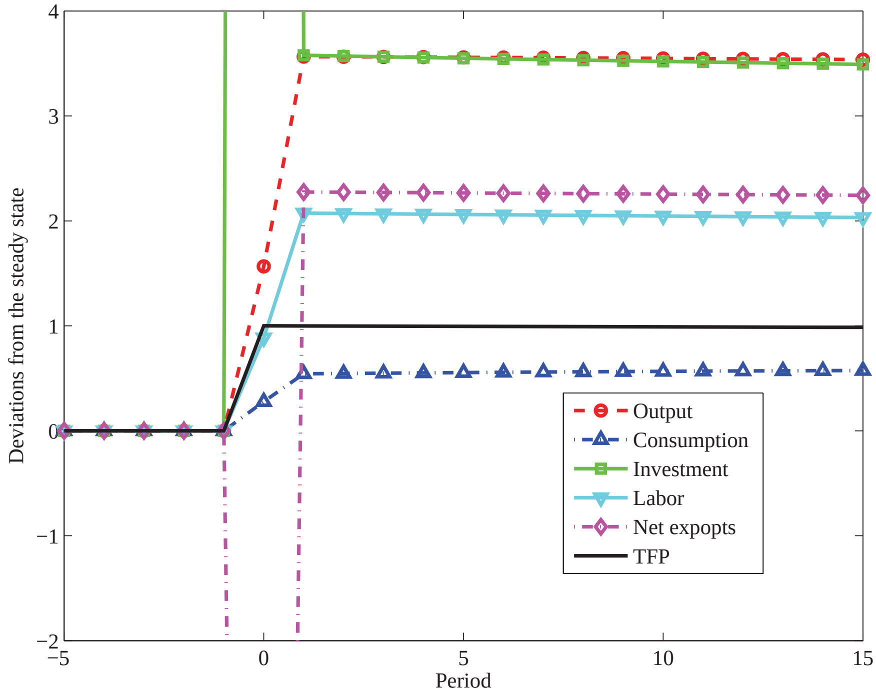


Figure 9: Impulse Response Functions of Country 2 (Ten-country Model)
(+1% TFP Shock to Country 1, +.12% TFP shock to Country 2, No Shock to Others)

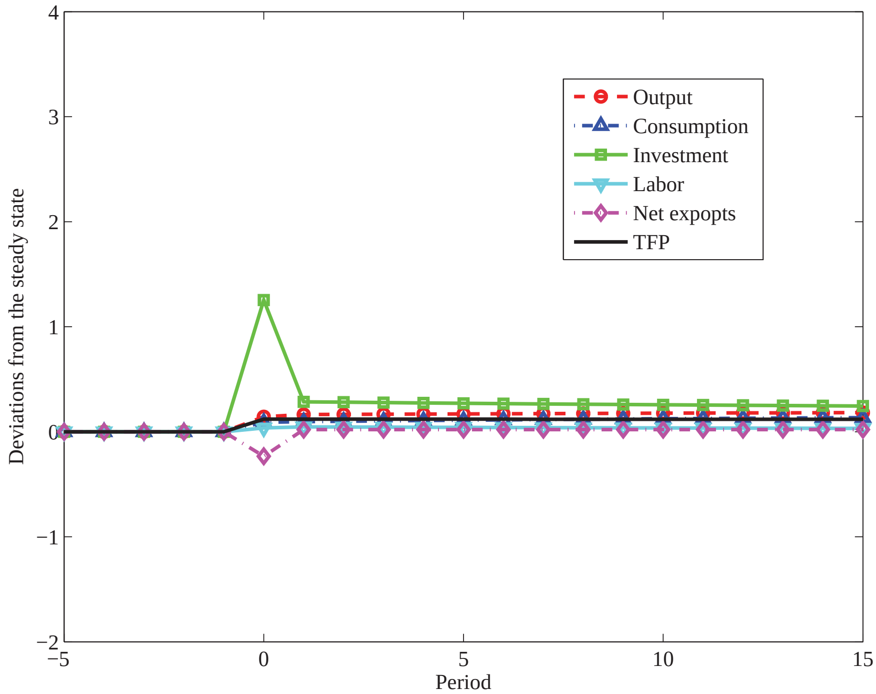


Figure 10: Impulse Response Functions of Country 3–10 (Ten-country Model)
(+1% TFP Shock to Country 1, +.12% TFP Shock to Country 2, No Shock to Others)