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Equity Sales and Manager Efficiency Across Firms and the Business Cycle

Fabio Ghironi* and Karen K. Lewis**

Abstract

Smaller firms sell more equity in response to expansions than do larger firms. Also, consumption is more pro-cyclical for high income groups than others. In this paper, we present a model that captures key features of both of these patterns found in recent empirical studies. Managers own firms with unique differentiated products and can sell ownership in these firms. Equity sales require paying consulting fees, but the resulting scrutiny also make firms more efficient. We find four main results: (1) Equity sales are pro-cylical since the benefits of efficient production outweigh the consulting fees during a boom. (2) Equity shares in smaller firms are more pro-cyclical because expansions make previously solely-owned firms to seek outside equity financing. (3) Households must absorb the increased equity sales by managers, thereby affecting their consumption response relative to managers. (4) Greater underlying managerial inefficiency induces more firms to seek outside advice and ownership in equilibrium. As a result, the cyclical impact on efficiency is mitigated by outside ownership.

Keywords: Equity Sales; Managerial Efficiency; Firm Size; Business Cycles
JEL classification: E25, E44, E21

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1 Introduction

Financing behavior over the business cycle differs significantly depending on firm size. The difference in financing according to size has been found in a number of countries. For example, Covas and Den Haan (forthcoming) show that US equity sales are more pro-cyclical for smaller firms than larger firms. While this result is specific to companies that are already listed on the US exchanges, Covas and Den Haan (2007) find similar results for Canada using data for a wider set of firms including unlisted firms. Similarly, Hayashi and Prescott (2000) provide evidence that smaller firms disproportionately shed financial assets during Japan's "Lost Decade."

In addition, a large literature has examined the importance of income inequality on consumption. Recent research has found that the cyclicality of consumption differs across groups in the economy. In particular, Parker and Vissing-Jorgensen (2010) demonstrate that high income households have greater pro-cylicality in consumption than do lower income households. They also find that the high cyclicality is linked to the top-income share across subgroups and across countries.

Overall, the evidence suggests that both firm financing and consumption behavior across the cycle differ according to size and income level. While studies typically treat these variables as independent, the nature of firm financing may affect consumption and vice versa. In equilibrium, any new financing by firms must be provided by households, thereby affecting the pro-cyclicality of consumption. New financing in the form of equity also implies an increase in ownership shares in firms. Therefore, pro-cylical equity sales imply changes in the composition of firm ownership.

In this paper, we examine the cyclical interaction between size-differentiated firm financing and consumption across groups that raise the funds, "Managers", and those that provide the funds, "Households." To focus on the role of ownership, we define our financing as "equity" but allow for a range of interpretations of equity financing.[1] Selling ownership stakes in the firm can take the form of private equity interests as is typical for small firms or can imply going fully public on a stock exchange as is typical for large firms. To be able to sell the equity, managers must pay a reporting fee and subject themselves to scrutiny by outsiders to the firm. For firms that are not fully listed on an exchange, these managers receive advice from consultants that allows them to operate more efficiently than they would in the absence of outside monitoring.[2] On the other hand, these advisors seek only to maximize current profits and thereby may induce the managers to behave myopically.

We provide a role for managers that highlight these relationships. Managers have an innate ability to produce goods of a given variety. Every period, these managers decide how much equity to sell in their company given the trade-off between the value of the outside advice and the reporting cost. The equity sales decision implies a choice between three possibilities. First, they can be sole

[1] Our main results would continue to hold if we defined our financing as long term bank financing as in Stebenovs (2008).

[2] After the company goes public, more diffuse ownership can also allow the manager to shirk and operate inefficiently as suggested by Jensen and Meckling (1976). For the present paper, we consider the business cycle impact of ownership on private companies and leave the impact of seasoned equity offerings for future research.

owners of their own firm and thereby avoid monitoring but remain inefficient. Second, they can partially sell off their firm but retain a significant stake in the company. Third, they can fully list their firm on a stock exchange.[3] Accordingly, managers endogenously fall into three categories. First, managers of low productivity firms do not sell outside shares in their company. These firms remain private property of the respective entrepreneurs because the costs associated with public ownership more than offset the private benefits to owners. Second, managers of sufficiently productive firms sell some equity, but they retain a significant ownership share. Finally, managers of high productivity firms fully list their companies.

Given this role for managers, we study the aggregate implications of these equity sales, finding five interesting implications. First, the model implies that equity finance is pro-cyclical, consistent with the empirical literature.[4] Intuitively, as the aggregate economy becomes more productive, individual firms also become more productive. As a result, the benefit of issuing equity increases relative to the costs. More entrepreneurs choose to sell equity in their firms, and those who were already listed prior to the economy's expansion choose to sell more equity.

Second, the procyclical impact of equity sales varies with firm size. Consistent with the empirical evidence, equity sales by smaller firms are more pro-cyclical than larger firms. Managers of small firms that previously had no outside ownership will choose to seek equity financing, while larger firms will not sell more equity.

Third, equity sales during an expansion increases the difference between consumption behavior by households and managers. When an expansion makes newly issued equity more valuable than previously issued shares, equity sales dampen the pro-cyclical impact on household consumption. Intuitively, household consumption is dampened both because they must absorb the new equity shares sold and because the expansion entails increases in the managerial inefficiency and reporting costs. The higher managerial inefficiency and consulting fees for equity sales reduces dividend payments available for household spending. On the other hand, equity sales by managers and the higher profits associated with consulting augments income available for managers.

Fourth, the effect of the expansion on the value of newly issued equity shares compared to existing shares depends upon the persistence in profits. Intuitively, expansions encourage more equity to be sold, implying an near term increase in future dividends on those shares. The willingness of households to purchase these new shares then depends upon the interaction of two effects. On the one hand, households expect a one time spike in dividend payments from these new shares in the next period. On the other hand, dividends are expected to be higher in all future periods. The increase in future expected consumption leads households to prefer to consume more today to smooth intertemporally. Overall, when persistence is high, consumers have a stronger preference to consume future profits today. Accordingly, they are more reluctant to reduce current consumption by buying equity and hence the price of new shares will be lower than existing shares in equilibrium. By contrast, when persistence of profits is low, consumers will be more willing to

[3] In reality, other cases are possible such as companies listed on exchanges in which a significant block of shares are owned by families of the original owners. We abstract from these issues for simplicity in the text.

[4] See for example Covas and Den Haan (forthcoming).

buy new shares in order to benefit from the near term increase in dividends. As a result, the price of new shares can by higher than existing shares when the persistence of profits is low.

Fifth, since managers tend to choose more outside consulting when the gains are particularly high, the effects of aggregate shocks on managerial inefficiency are dampened in cases when underlying distortions are highest. For example, when the underlying managerial inefficiency is high, more firms choose to receive outside monitoring in steady state and thereby react less to aggregate shocks.

Our paper represents one of the few studies of the interaction between corporate monitoring and aggregate fluctuations. Phillipon (2006) examines the impact of aggregate shocks on firms with different levels of governance. For this purpose, Phillipon (2006) builds a model in which managers may over-invest in labor and capital and therefore create an inefficiency in output. Shareholders may monitor these managers at a cost. Badly governed firms are monitored less and are therefore more sensitive to aggregate shocks than well governed firms. The study uses a cross-section of empirical measures of corporate governance in firms over the business cycle and finds that the results corroborate the model. While this paper provides important insights into the role of corporate governance and the sensitivity of firms to aggregate shocks over the business cycle, it does not examine the managerial decision to sell equity nor the macroeconomic implications of this decision.

The structure of the rest of the paper is as follows. Section 2 provides more information about equity behavior across the cycle. It also develops the equity sales decision of managers and shows that the group of firms with outside ownership versus those without depends upon an endogenous, firm-level productivity cut-off that varies with the aggregate economic cycle. Section 3 completes the solution of the model and demonstrates additional macroeconomic implications of these corporate managerial decisions. Section 4 describes results from quantitatively evaluating the model. Section 5 concludes.

2 Equity Sales and Firm Size

In this section we develop our model relating equity ownership sales to firm size and consumption. Before describing the model, we begin by providing several observations that guide our analysis.

2.1 Empirical Motivation

We draw on a number of features in the literature concerning managerial monitoring through outside ownership and financial market likely. First, the literature of stock market cross-listing has found that the equity price of a firm increases when listed in a market with more stringent disclosure requirements[5]. Thus, while monitoring can take place through many channels, we choose to focus upon equity listing for the discussion below. Second, a common observation is that larger and more

[5] Studies that have looked at the effects of corporate governance on prices at listings include Doidge, Karolyi, and Stulz (2007) and Bailey, Karolyi, and Salva (2006).

productive firms tend to be more likely to list on an exchange.[6] Thus, our model should be able to reproduce this finding. Third, managers may be inefficient in the absence of outside consulting and reporting.

While these relationships concern the overall efficiency of firms, they may also be related to the cyclical behavior of equity financing for different sized firms. Table 1 illustrates this behavior reported by Covas and den Haan (forthcoming) using annual Compustat data from 1980 to 2006 for all listed US companies excluding financial firms and utilities. The financing for these firms are categorized into size groups based upon last period's book value of assets. The first column shows the groupings of firms according to capital size measured by last period book value of assets. The first three entry rows group firms according to the first three 25 percentiles: firms in the smallest 25%, firms in the 50% to 75% category, and firms in between. The next four entry rows break down the top 25% group into finer partitions: 75% to 90%, 90% to 95%, 95% to 99%, and, finally, the top 1%.

Table 1: US Equity Issuance by Firm Size

Size Classes	$Corr(GDP_t, \Delta E_t)$	$Corr(GDP_{t+1}, \Delta E_t)$
[0; 25%]	0.53	0.45
[25%; 50%]	0.57	0.36
[50%; 75%]	0.43	0.23
[75%; 90%]	0.41	0.20
[90%; 95%]	0.30	0.03
[95%; 99%]	0.13	0.02
[99%; 100%]	-0.36	-0.39
[0; 95%]	0.46	0.23
[0; 99%]	0.35	0.16
All firms	0.17	0.01

Source: Covas and Den Haan (forthcoming), Table 3

The second and third columns restate the correlations between the change in net equity issuances, E, and the current and next period detrended GDP, respectively. The currrent correlations between GDP and equity are generally positive across the firms. However, the size of the correlations are greatest for firms in the bottom 50 percentile with a correlation above .53. This size declines for the larger firms. For the top 1% size firms, the net equity issuance is significantly negative. Covas and den Haan (forthcoming) argue that this result stems from episodes in which large firms bought back equity during booms. The correlations between equity and GDP the

[6]For evidence that firms tend to list when productivity is high, see for example Jain and Kini (1994), Pagano, Panneta & Zingales (1998), and Barh, Gulbrandsen & Schone (2005). Among other features, Chemmanur, He & Nandy (2007) find that listed firms tend to be larger.

following year shows a similar albeit smaller pattern. They also examine this relationship for other measures of equity and generally find that the equity issuance response is greater for smaller firms.

One shortcoming of the evidence in Table 1 is that the data only include US listed companies. This limits somewhat the ability to draw aggregate implications about first-time listing companies since they are not part of the sample until they in fact issue equity. Covas and den Haan (2007) analyze the financing behavior of private as well as publically traded Canadian companies that help address this ommission. Table 2 reports the same patterns as Table 1 for these firms from 1979 to 2004.

Table 2: Canadian Financing by Firm Size

Size Classes	$Corr(GDP_t, \Delta E_t)$	$Corr(GDP_{t+1}, \Delta E_t)$
[0; 25%]	0.41	0.31
[0; 50%]	0.27	0.39
[0; 75%]	0.47	0.53
[0; 99%]	0.35	0.69
[90%; 95%]	0.36	0.60
[95%; 99%]	0.19	0.59
[99%; 100%]	0.30	0.26
All firms	0.40	0.67

Source: Covas and Den Haan (2007), Table 2

The cyclical behavior of equity from this wider data set for Canada shows similar patterns to the bottom 95% size of the US distribution. In particular, the correlation between equity and GDP is positive for all size groupings and is significantly so with the exception of the below 50% group as a whole. Compared to the US data, the last columns shows somewhat less tendency for mean reversion in equity sales.

Consumption behavior over the business cycle also differs across income levels. Parker and Vissing-Jorgensen (2010) find that high income household spending behavior is more sensitive to the cycle than lower income households in the US. They also find that the covariation of the high income household share and aggregate output is higher than lower income shares across fifteen countries.

While our model addresses directly the size distribution of firms, the link to income shares is less direct. Nevertheless, our framework implies a dichotomy between groups in the economy that provide capital "Households" and those that use the capital to produce "Managers." As a result, we also discuss the impact of financing on the consumption of these two groups in our analysis below.

To incorporate the essential features of firm financing, we require a framework in which managers differ according to skill and, hence, productivity. Therefore, we assume that managers have specific skills to produce a given variety of goods. The managers differ in their productivity and, hence, in equilibrium their firms will also have different sizes. The economy is populated by atomistic, identical households, who consume, supply labor, and hold shares in firms initially owned by managers that have sought outside owners. These manager-entrepreneurs operate as monopolistic competitors. Each manager has a unique advantage in producing a firm-specific good variety ω. Each entrepreneur is also an equity holder in the firm that he manages. As inputs, the production process requires both labor and the unique managerial skill of the entrepreneur.

The entrepreneur decides whether or not to share the ownership of the firm with households by selling shares in the asset market.[7] Below we distinguish between firms in which managers maintain a significant share of ownership in the company and those that are publically held as on a stock exchange. Michaely and Roberts (2007) examine a similar grouping of firms in the United Kingdom where data exists on the ownership structure on the full cross-section of firms. They distinguish between firms that are "Wholly Owned" with highly concentrated ownership as, for examine, in family firms. They also consider firms that have more dispersed ownership, "Private Dispersed," and those that are fully public firms such as firms listed on a stock exchange, "Public." Similarly, we consider an equilibrium in which some firms remain wholly owned by the manager, some firms become fully public, and an intermediate group of firms have some outside ownership while the entrepreneur maintains a significant ownership stake.[8] Entrepreneurs obtain income from their share of firm profits and selling firm shares (if they decide to do so). They use this income to finance consumption and, possibly, repurchases of shares in their firm. To highlight the roles played by managers and households, we assume that households can hold shares in all firms that are open to outside ownership, but entrepreneurs cannot hold shares in firms other than their own.[9]

2.2 Households

Households supply labor and hold the equity in the economy. The representative household supplies L units of labor inelastically in each period and maximizes $E_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_s)$, where $\beta \in (0,1)$ and where the period utility from consumption, $U(C_t)$, has the standard properties. We restrict utility to $U(C_t) = C_t^{1-\gamma}/(1-\gamma)$, $\gamma > 0$, where convenient below.

To allow for heterogeneous production, households have a preference for variety. We assume this preference is captured by the consumption basket C_t in the utility function given by a standard Dixit-Stiglitz aggregator:

$$C_t = \left(\int_0^1 c_t(\omega)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta}{\theta-1}}. \tag{1}$$

[7] We assume that entrepreneurs have created their firms in the infinite past, and we abstract from endogenous firm entry to focus on the entrepreneurs' decision whether or not to list firms in the stock market.

[8] Roberts and Michaely (2007) examine the implications of these ownership patterns on the potential for information asymmetry and agency problems. We abstract from these issues in the present paper in order to focus on cyclical relationships.

[9] Since some agents in the economy will have access to the equity market while others do not, our paper shares some similarities to the limited participation literature as in Alvarez, Atkeson, and Kehoe (2002, 2008) and Cole, Chien, and Lustig (2008).

where $c_t(\omega)$ is the consumption of good variety ω and where θ is the elasticity of substitution with $\theta > 1$.

The household's decision within a period can be considered in two stages. In the first stage, the household receives labor income and its share of firm profits from existing shares $x_t(\omega)$ of shares in each firm ω. If the firm was wholly owned by the manager in firm ω the previous period, then $x_t(\omega) \equiv 0$ and clearly the firm receives no dividend payments from this firm. In the second stage of the period, the goods market and equity markets open. Households use income earned from dividends and from labor to consume goods and purchase new shares. Therefore, the household's budget constraint is:

$$\int_0^1 \pi_t(\omega) x_t(\omega) d\omega + w_t L \geq C_t + \int_0^1 v_t(\omega)\left[x_{t+1}(\omega) - x_t(\omega)\right] d\omega, \tag{2}$$

where $\pi_t(\omega)$ is firm ω's time-t profits, $v_t(\omega)$ is the share price, and w_t is the real wage, all in units of the consumption basket.[10] The left hand side of equation (2) represents the funds received by the household in stage one; that is, the sum of the aggregate over all firms of profits paid to shareholders, $\pi_t(\omega) x_t(\omega)$, and the labor income, $w_t L$. The right hand side of the equation is the sum of spending on goods, C_t, and the value of the net purchase of any new shares by firm ω, $v_t(\omega)\left[x_{t+1}(\omega) - x_t(\omega)\right]$. Intertemporal optimization using this budget constraint results in a set of Euler equations for holdings of shares in each firm that has outside ownership:

$$v_t(\omega) = \beta E_t\left[\frac{U'(C_{t+1})}{U'(C_t)}\left(\pi_{t+1}(\omega) + v_{t+1}(\omega)\right)\right]. \tag{3}$$

Moreover, within a period, utility maximization implies that consumption is allocated to individual good varieties according to:

$$c_t(\omega) = \rho_t(\omega)^{-\theta} C_t \tag{4}$$

where $\rho_t(\omega)$ is the price of good ω in units of consumption.

2.3 Managers and Firms

To focus on the efficiency problem, we assume a simple role for the manager. Manager-entrepreneurs are born with an innate ability to produce a good of a given variety. The good cannot be produced without the manager and the manager consumes only his share of profits from the firm.[11] As in Melitz (2003) and Ghironi and Melitz (2005), we assume that a firm producing variety ω is associated with a firm-specific productivity z, and hereafter we replace variety with productivity. This productivity is drawn from a continuous distribution $G(z)$ with support $[z_{\min}, \infty)$. Output is produced with linear technology $y_t(z) = Z_t z l_t(z)$, where $l_t(z)$ is the amount of labor employed by

[10] We assume below that an individual firm's profits depend on its ownership structure. For now, we simply refer to profits as $\pi_t(\omega)$.

[11] Many of our main conclusions below will continue to hold if we assume managers can also hold equity of other firms, but the classification of "household consumption" in our calibrated results would require re-interpretation. See the discussion in Section 4 below.

the firm and where Z_t is a stochastic process generating aggregate productivity. In our quantitative exercises in Section 4, we assume that Z_t follows an $AR(1)$ process in logs.

Managers have the same utility function as households. Thus, they consume the same bundle of products as the households given in equation (1). We define the aggregate consumption bundle for entrepreneur z as $C_t(z)$.

Managers depend upon the profit from their own firm for income. This profit in turn depends upon the efficiency of the managers relative to the natural profit of the variety produced. To understand this profit, note that the optimal consumption across goods in equation (4) implies that the firm faces demand $y_t^D(z) = \rho_t(z)^{-\theta} Y_t^A$, where Y_t^A is the economy's total absorption of consumption output. Since labor is the only production input besides the entrepreneur, a standard model with these preferences and production functions would imply the well-known form to profits given by:

$$\pi_t^V(z) \equiv \rho_t(z) y_t(z) - w_t l_t(z) = \frac{1}{\theta} \rho_t(z)^{1-\theta} Y_t^A. \tag{5}$$

Since these are the optimal profits in the absence of any managerial inefficiency, we call $\pi_t^V(z)$ "natural profits" below. The first expression simply says that profits are revenues minus labor costs. The expression to the right of the second equal sign follows from Dixit-Stiglitz preferences.

Managers each have an ability to produce a given good at productivity z. However, if they are sole owners of the company, they rely upon their own advice which implies an inherent inefficiency relative to natural profits. To characterize this inefficiency, we assume that only a portion of natural profits can be acheived in the absence of outside advice. In principle, the loss in profits due to managerial inefficiency can be assumed to arise from inefficient use of inputs. Alternatively, this profit may represent expenditures within the company that are inessential for efficient production (e.g., designer carpeting, company vacations). Below, we do not take a stand on the nature of the inefficiency, but take a reduced form approach to incorporate these standard stories.

To consider these effects, we follow related literature by assuming that the managerial inefficiency is proportional to an optimal asset of the firm. For example, Phillipon (2006) assumes that managerial inefficiency is proportional to the production from labor and capital inputs. Albuquerque and Wang (2008) assume that a controlling shareholder may divert a proportion of firm output per period.[12] In our model, we define the proportional distortion from managerial inefficiency as $1 - \tau$. That is, τ represents the proportion of natural profits that are available to households after potential diversion by managers: $\tau \pi_t^V(z)$.

Managers may choose to improve their efficiency by seeking outside advice. For simplicity, we assume that outside owners provide this advice. Thus, if managers choose outside shareholders, they will be monitored by these shareholders, thereby mitigating their inefficient management. To characterize the gain in efficiency of outside monitoring, we posit that the proportion τ of natural profits is a function of shares sold by managers to outsiders. Specifically, defining the shares of stocks sold by the manager of firm z as $x_{t+1}(z)$, we define the share of natural profits

[12] In turn, these authors build on the "stealing" technology described in Johnson, Boone, Breach, and Friedman (2000) and La Porta, Lopez-de-Silanes, Schleifer, and Vishny (2002).

available for dividends as a function $\tau(x_{t+1}(z))$. This function is continuous on $x_{t+1}(z) \in (0,1)$ and monotonically increasing to capture the improvement in entrepreneurial/managerial efficiency from listing. That is, in order for listing to provide efficiency benefits, we require that the firm become more efficient with listing; i.e., $\tau'(x_{t+1}(z)) > 0$, $\tau''(x_{t+1}(z)) < 0$. For entrepreneurs in wholly owned firms, this proportion reaches a lower bound: $\tau(0) = \overline{\tau} \in (0,1)$. Alternatively, for fully public firms as those listed on a stock exchange, $\tau(1) = 1$.

So far, we have described the benefits of outside ownership. However, outside monitoring also comes at a cost which we term "reporting costs." For example, to list on a US stock exchange, firms must abide by SEC regulations and provide annual reports to shareholders. In our calibrations below, we treat the reporting costs as representing these more overt costs. Though not modeled explicitly, these costs can also represent the more subtle loss to the manager of giving up the ability to divert profits. To capture these costs associated with selling equity we define a function $f(x_{t+1}(z))$ that depends upon equity shares sold, $x_{t+1}(z)$. We assume that the function $f(x_{t+1}(z))$ is increasing in the number of shares sold; i.e., $f'(x_{t+1}(z)) > 0$. This relationship is consistent with the idea that there are losses due to increased public ownership. Similar to the profit distortion proportion τ, we assume that the reporting cost function is monotone such that: $f(0) = 0$, $f(1) = f > 0$. The firm incurs no reporting cost if it sells no equity. It incurs reporting cost f if it is completely owned by the public, and the reporting cost increases monotonically with the amount of equity sold. This assumption captures the idea that the more public the firm, the more stringent the information requirements it must satisfy and the higher the associated costs.

Combining the profit distortion function and the listing costs, the profits available to shareholders can be written:

$$\pi_t(x_{t+1}(z); z) \equiv \tau(x_{t+1}(z))\, \pi_t^V(z) - f(x_{t+1}(z)) \tag{6}$$

As this equation shows, listing has two opposing effects on actual profits. First, selling more equity means increasing the proportion of natural profits manifested in actual profits through $\tau(x_{t+1}(z))$, but more selling equity also means increasing the costs through $f(x_{t+1}(z))$. We next describe the manager's listing decision based upon this trade-off.

2.4 The Entrepreneur's Equity Sale Problem

Each entrepreneur z is the original owner of his firm. His decision every period mirrors the household decision described above. Upon entering the period, he observes the aggregate productivity shock, Z_t. Based upon this information, he can infer his output demand, and, hence, his natural potential profits, $\pi_t^V(z)$. Thus, in the first stage of the period, he decides how much equity to sell. In order to issue this equity, he must pay the reporting costs and receive outside monitoring that provides productive advice. Given this decision, he earns profits according to equation (6), paying out wages and profits to existing shareholders.

In the second stage of the period, the asset market opens and households decide how much to purchase of any newly issued shares as described above. In equilibrium, the manager must set the ownership shares price so that households are willing to buy the newly issued shares.

2.4.1 Stage 1 Problem

As described above, a firm's profit depends on the firm's ownership structure. Time-t profits depend on whether or not the firm is listed in period t, and on the amount of shares that are sold in that period, $x_{t+1}(z)$. The manager recognizes the trade-offs between efficiency and costs and considers each period how much equity to sell. In order to sell equity, he employs outside consultants who provide advice about the amount of equity to sell. We assume that these consultants base their advice about how much equity to sell by maximizing current period profits. This assumption is consistent with investment banks that typically choose to maximize the stock price in order to earn the highest proportional fees.

Therefore, the manager's decision about how much equity to sell reduces to the following maximization problem:[13]

$$\max_{x_{t+1}(z)} \pi_t\left(x_{t+1}(z); z\right), \tag{7}$$

We assume that the entrepreneur never fully relinquishes the ownership of the firm, and we constrain $x_{t+1}(z)$ to be in the interval $[0, \bar{x}]$, $\bar{x} < 1$. We maintain this assumption for two reasons. First, as an empirical matter, entrepreneurs tend to hold at least some shares in their own firms, even after they go public. Second, for the purpose of our model, managers have an innate ability to produce their own goods and therefore they continue to make decisions about equity sales.[14] The first-order condition for problem (7) thus implies:

$$\tau'\left(x_{t+1}(z)\right)\pi_t^V(z) = f'\left(x_{t+1}(z)\right). \tag{8}$$

The entrepreneur chooses to sell the amount of equity such that the marginal benefit of selling an additional unit of equity is equal to the marginal cost. Note that $\pi_t^V(z)$ acts as a scaling factor for the marginal benefit of listing and selling equity.

Figure 1 illustrates the solution of this problem. Because of our assumptions on the function $\tau\left(x_{t+1}(z)\right)$, the marginal benefit schedule $MB\left(x_{t+1}(z)\right) = \tau'\left(x_{t+1}(z)\right)\pi_t^V(z)$ is a decreasing function of $x_{t+1}(z)$. For ease of illustration, we assume that $\tau'\left(x_{t+1}(z)\right)$ is linear over the relevant range of values of $x_{t+1}(z)$. Similarly, for analytical convenience, we assume that equity issuing costs are proportional to shares, given by the linear specification: $f\left(x_{t+1}(z)\right) = \overline{f}\ x_{t+1}(z)$ where $\overline{f} > 0$. In this case, the marginal cost schedule is fixed at $MC\left(x_{t+1}(z)\right) = \overline{f}$. Thus, the intersection of $MB(z_h)$ for given firm z_h and $\overline{f}$ determines the choice of outside equity at $x_{t+1}(z_h)$ for this manager.

Figure 1 also depicts the solution for managers with other productivity levels. A lower produc-

[13] Managers may also choose to ignore the advice of these managers and decide instead to intertemporally smooth their own consumption. In this case, they will not in general decide to sell the amount of equity that maximizes current period profits. We leave this extension for future research

[14] In our quantitative analysis below, we take $\overline{x}$ to be arbitrarily close to one, an assumption that is consistent with most large non-financial firms in the United States.

tivity firm is shown with $MB(z_l)$. For this manager, the benefits of outside ownership are lower than the reporting costs.[15] In this case, the firm decides to remain wholly owned, i.e., $x_{t+1}(z_l) = 0$. As z increases, $\pi_t^V(z)$ increases, shifting the MB schedule upward.[16] The figure shows that there is a cutoff level of z, denoted z_t^o, such that $\tau'(x_{t+1}(z_t^o))\,\pi_t^V(z_t^o) = \overline{f}$ at $x_{t+1}(z_t^o) = 0$. This cutoff productivity defines the identity of the marginal firm manager that seeks outside ownership. All firms with productivity $z > z_t^o$ will sell a positive amount of equity to be held by the public in period $t+1$, determined by the intersection of the marginal benefit and marginal cost schedules. For instance, in Figure 1, the firm with productivity $z_h > z_t^o$ sells the amount of equity $x_{t+1}(z_h)$.

For firms with increasingly higher z, the amount of equity sold also increases until it approaches the upper bound $\bar{x}$. Therefore, the model implies a second cutoff firm-specific productivity level $\bar{z}_t$ such that all firms with productivity $z \geq \bar{z}_t$ sell the maximum amount of equity, $\bar{x}$. Note that the outside ownership cutoff productivity z_t^o and the upper bound cutoff $\bar{z}_t$ are time-varying, since they are affected by cyclical conditions that contribute to determine the level of natural variable profits $\pi_t^V(z)$.

A convenient functional form for the function $\tau(x_{t+1}(z))$ that is consistent with the requirements above and delivers the implications discussed in Figure 1 is:

$$\tau(x_{t+1}(z)) = \overline{\tau} + x_{t+1}(z) - \overline{\tau} x_{t+1}(z)^2, \quad 0 < \overline{\tau} < \frac{1}{2}. \tag{9}$$

The marginal benefit schedule is then:

$$MB(x_{t+1}(z)) = (1 - 2\overline{\tau} x_{t+1}(z))\,\pi_t^V(z). \tag{10}$$

As depicted in Figure 1, increasing z shifts the schedule upwards and makes it steeper.

Solving for the productivity that implies marginal benefit equals marginal cost as in equation (8) using the MB form in equation (10) and the form for reporting costs, we obtain the outside ownership cutoff:

$$z_t^o = \frac{\theta}{\theta - 1}\frac{w_t}{Z_t}\left(\frac{\theta \overline{f}}{Y_t^A}\right)^{\frac{1}{\theta-1}}. \tag{11}$$

Similarly, using the MB and reporting costs function, and solving the first-order condition (8) for the z that implies optimal shares are just equal to the upper bound, $x(z)_{t+1} = \overline{x}$, gives the upper-bound cutoff:

$$\bar{z}_t = \frac{\theta}{\theta - 1}\frac{w_t}{Z_t}\left[\frac{\theta \overline{f}}{(1 - 2\overline{\tau}\bar{x})\,Y_t^A}\right]^{\frac{1}{\theta-1}}. \tag{12}$$

The cutoff productivities for selling the minimum and maximum amount of equity are lower the

[15] Thus, for Figure 1, we implicitly assume that the lower-bound firm productivity $z_{\min}$ is low enough to ensure $MB(z_{\min}) < f$. This relationship need not hold in general. We describe this constraint in more detail in the quantitative section below.

[16] As illustrated in Figure 1, the marginal benefit curve also becomes steeper as productivity increases. While not necessary in general, the particular functional form we use below implies the curve steepens with z.

higher aggregate output absorption Y_t^A and productivity Z_t (for given real wage). Intuitively, increases in demand or lower unit production costs boost natural variable profits, making it relatively more attractive to sell equity. Also, the effects of increasing or lowering the marginal cost of selling equity are straightforward. Since $\theta > 1$, a higher cost of listing $\overline{f}$ requires a higher profit benefit to offset it. Therefore higher costs increase both the outside ownership cut-off z_t^o and the maximum cut-off $\bar{z}_t$. We return to these results below.

At this first stage decision, it is clear that our model delivers several plausible implications. First, low productivity firms do not list. They remain private property of the respective entrepreneurs because the costs associated with public ownership more than offset the benefits. Second, sufficiently productive firms list and sell a proportion of the firm determined by equating the marginal benefits and marginal costs of listing. Third, high productivity firms sell the maximum amount of equity by going public on a stock exchange.

Figure 1 illustrates the effects of the heterogeneity of firms on the manager's decision to sell equity for a given level of aggregate productivity Z_t. More generally, however, the position of the marginal benefit schedule $MB\left(x_{t+1}\left(z\right)\right) = \tau'\left(x_{t+1}\left(z\right)\right)\pi_t^V\left(z\right)$ is also affected by the other determinants of natural variable profits $\pi_t^V\left(z\right)$. For instance, suppose there is an increase in aggregate absorption, Y_t^A, or in aggregate productivity, Z_t for given real wage w_t. Both these changes shift the marginal benefit schedule upward and make it steeper for any given firm z. This productivity increase induces some firms that were wholly owned by the entrepreneur to start selling outside shares. In other words, consistent with the discussion above, the cutoff productivity for outside ownership decreases. Higher aggregate demand or productivity induces firms that already had outstanding shares to sell more equity, inducing some of these firms to sell the maximum amount $\bar{x}$ since the cutoff $\bar{z}_t$ decreases. However, there will be little additional equity sales for firms that were already near $\bar{x}$ before the expansion of the economy, and zero additional equity sales for the firms that were already selling $\bar{x}$. These results are consistent with the evidence documented in Covas and den Haan (forthcoming) that equity sales are procyclical, but less so for the largest firms. We show that this pattern holds in our quantitative analysis below.

Figure 2 illustrates these cyclical properties of equity sales generated by increases in Z_t to Z_{t+1}.[17] First consider the decision by manager with productivity z_h. If aggregate productivity increases, then the natural profits will be higher implying a shift in the marginal benefit schedule to the right. As a result, this manager will issue more shares of equity from $x_t(z_h)$ to $x_{t+1}(z_h)$. Similarly, for managers of firms with productivity such as z_l that were previously wholly owned, the increase in benefits from $MB_t\left(z_l\right)$ to $MB_{t+1}\left(z_l\right)$ generate a decision for the firm to sell equity at $x_{t+1}(z_l)$. Finally, some firms with a high productivity, such as z_{hh} now sell the maximum amount of shares possible.

[17] In Figure 2, marginal benefit schedules before (after) the expansion of the economy are denoted with a subscript 0 (1).

2.4.2 Stage 2 Problem

In the second stage of the period, the goods and equity markets open. The entrepreneur, having decided how much equity to sell, decides on the price to offer this equity. We denote the price set by the entrepreneur at time t for the amount of equity $x_{t+1}(z)$ with $v_t(z)$.

In period t, the entrepreneur receives income in two components coming from dividends on retained shares and sales of equity:

$$(1 - x_t(z))\,\pi_t(x_{t+1}(z); z) + v_t(z)\,x_{t+1}(z).$$

In other words, the entrepreneur receives his share of period-t profits $(1 - x_t(z))\,\pi_t(x_{t+1}(z); z)$, where $1 - x_t(z)$ is the share of equity that the entrepreneur kept as his own in period $t-1$, and the value of selling the listed shares, $x_{t+1}(z)$ equal to $v_t(z)\,x_{t+1}(z)$. The entrepreneur uses this income to finance consumption and buy back the equity he had sold in period $t-1$. This yields the budget constraint

$$[1 - x_t(z)]\,\pi_t(x_{t+1}(z); z) + v_t(z)\,[x_{t+1}(z) - x_t(z)] \geq C_t(z), \tag{13}$$

where $C_t(z)$ denotes the entrepreneur's consumption of the consumption basket. Note that the entrepreneur budget constraint (13) nests all the possible scenarios of current and past listing decisions: For an entrepreneur who had no outstanding equity in period $t-1$ and does not sell equity in period t, equation (13) reduces to $\pi_t(0; z) \geq C_t(z)$. For an entrepreneur who had no outstanding equity in period $t-1$ and chose to sell equity in period t, $\pi_t(x_{t+1}(z); z) + v_t(z)\,x_{t+1}(z) \geq C_t(z)$. For an entrepreneur who had outstanding equity in $t-1$ and decided to buy back equity in the current period, $(1 - x_t(z))\,\pi_t(0; z) - v_t(z)\,x_t(z) \geq C_t(z)$. Finally, equation (13) with $x_t(z)$ and $x_{t+1}(z)$ different from zero applies to entrepreneurs who had outstanding equity in $t-1$ and t.

The entrepreneur z who engages in equity transactions sets the desired price for his firm's equity at a price that will induce households to purchase his shares in equilibrium. Thus he sets the equity price as given by the household's Euler equation (3).[18]

[18] In principle, the decision to list his firm generates an additional benefit for the entrepreneur by ameliorating the consequences of our limited participation assumption that entrepreneurs do not trade equity other than their own firm. If the manager could make equity sales decisions without advisors that maximize present period profits, he would have an Euler equation that relates equity sales to his intertemporal consumption decision. This Euler equation for the entrepreneur's pricing of his equity supply together with the household's Euler Equation on the demand side imply a restriction that relates the entrepreneur's expected marginal utility growth to the household's, thereby providing a channel for consumption risk sharing between households and listing entrepreneurs. We leave this extension for future research.

2.4.3 Household Choices Over Heterogeneous Stocks

Using profits in equation (6), the household's budget constraint in equation (2) can be re-cast by aggregating over firm-specific productivities to yield:

$$\int_{z_{\min}}^{\infty} \left(\pi_t \left(x_{t+1}(z); z\right) + v_t(z)\right) x_t(z) \, dG(z) + w_t L \geq C_t + \int_{z_{\min}}^{\infty} v_t(z) \, x_{t+1}(z) \, dG(z) \,. \tag{14}$$

The Euler equation for the representative household's holding of equity in firm z can then be rewritten in terms of equity shares as:

$$v_t(z) = \beta E_t \left[\frac{U'(C_{t+1})}{U'(C_t)} \left(\pi_{t+1}\left(x_{t+2}(z); z\right) + v_{t+1}(z)\right) \right]. \tag{15}$$

Thus, equity prices will depend upon aggregate fluctuations for two reasons. First, an increase in output increases natural profits through the usual channels. Second, an increase in output induces more listing leading to more profits available for shareholders. This second channel is a novel outcome of our framework with managerial inefficiency.

2.5 Labor Market Equilibrium and Aggregate Output

To focus on the distortion to consumption caused by managerial inefficiency, we otherwise leave the production effects unchanged. Thus, we keep the economy on its production possibilities frontier and assume the labor market is undistorted. As in the Melitz (2003) model, the production function of individual firm z is linear in labor employed $l_t(z)$. Thus, the output of firm z is: $y_t(z) = Z_t z l_t(z)$. Recalling that the price of good z is given by $\rho_t(z)$, aggregating output across firms gives total output or GDP as:

$$Y_t = \int_{z_{\min}}^{\infty} \rho_t(z) \, y_t(z) \, dG(z) = \int_{z_{\min}}^{\infty} \rho_t(z) \, Z_t z l_t(z) \, dG(z) \,. \tag{16}$$

Further, optimal price setting by the firm yields:

$$\rho_t(z) = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t z}. \tag{17}$$

Substituting (17) into (16) implies that output can be written in terms of the wage rate and aggregate labor employed:

$$Y_t = \frac{\theta}{\theta - 1} w_t \int_{z_{\min}}^{\infty} l_t(z) \, dG(z) \,. \tag{18}$$

To determine the labor demand, we equate the output demand for the firm, $y_t^D(z) = \rho_t(z)^{-\theta} Y_t^A$, to production, $y_t(z) = Z_t z l_t(z)$. We then use the requirement that total output production must be equal to absorption in equilibrium ($Y_t = Y_t^A$) and substitute the result into equation (18). Using the definition of average productivity,

$$\tilde{z} \equiv \left(\int_{z_{\min}}^{\infty} z^{\theta-1} dz \right)^{\frac{1}{\theta-1}}, \tag{19}$$

implies the equilibrium wage rate is:

$$w_t = \frac{\theta - 1}{\theta} Z_t \tilde{z} \tag{20}$$

Intuitively, monopoly power results in labor being paid a fraction $(\theta - 1)/\theta < 1$ of its overall productivity. Note that labor market clearing also implies $L = \int_{z_{\min}}^{\infty} l_t(z)\, dG(z)$. Hence, total labor income in the economy is such that $w_t L = (\theta - 1) Y_t / \theta$.

2.6 Aggregate Accounting

Given labor market equilibrium and the budget constraints of the households and managers, we can now combine these relationships to derive the aggregate constraints of the economy. We begin by aggregating the entrepreneurial budget constrant in equation (13) across firms to get:

$$\begin{aligned} &\int_{z_{\min}}^{\infty} (1 - x_t(z))\, \pi_t(x_{t+1}(z); z)\, dG(z) + \int_{z_{\min}}^{\infty} v_t(z) \left[x_{t+1}(z) - x_t(z)\right] dG(z) \\ = &\int_{z_{\min}}^{\infty} C_t(z)\, dG(z). \end{aligned}$$

Next, we sum this aggregated manager constraint and the household budget constraint in equation (14) to obtain the aggregate resource constraint of the economy:

$$\int_{z_{\min}}^{\infty} \pi_t(x_{t+1}(z); z)\, dG(z) + w_t L = \int_{z_{\min}}^{\infty} C_t(z)\, dG(z) + C_t. \tag{21}$$

The left-hand side of this equation is the economy's total income: the sum of profit income, retained by entrepreneurs or distributed to shareholders, and labor income. Aggregate accounting requires total income to be equal to total consumption spending by entrepreneurs and households.

For notational convenience, we define aggregate entrepreneurial consumption as $\tilde{C}_t \equiv \int_{z_{\min}}^{\infty} C_t(z)\, dG(z)$. Note that this variable combines consumption by entrepreneurs of whollly owned firms and managers of firms with outside ownership. Therefore, aggregate entrepreneurial consumption can be decomposed at any point is time as:

$$\tilde{C}_t = \int_{z_{\min}}^{z_t^o} C_t(z)\, dG(z) + \int_{z_t^o}^{\infty} C_t(z)\, dG(z). \tag{22}$$

Aggregate profits are distorted by both managerial inefficiency and reporting costs. Therefore, we can further decompose the aggregate resource constraint by aggregating these two distortions across firms.

As we have noted above, the managerial inefficiency by firm z is given by $1-\tau\left(x_{t+1}\left(z\right)\right)$ multiplied by the natural profit of firm z, given by $\pi_t^V\left(z\right)$. Using the solution for natural profits and aggregating across firms implies that the aggregate inefficiency is:

$$\tilde{\tau}_t \equiv \int_{z_{\min}}^{\infty} \pi_t^V\left(z\right)\left(1-\tau\left(x_{t+1}\left(z\right)\right)\right) dG\left(z\right) = \frac{1}{\theta} Y_t \int_{z_{\min}}^{\infty} \rho_t\left(z\right)^{1-\theta}\left(1-\tau\left(x_{t+1}\left(z\right)\right)\right) dG\left(z\right). \tag{23}$$

Similarly, if a firm sells equity, profits available for consumption, $\pi_t\left(z\right)$, are lower than natural profits by the reporting costs. Aggregating these reporting costs across firms gives the GDP component of reporting as:

$$\tilde{f}_t \equiv \int_{z_{\min}}^{\infty} f\left(x_{t+1}\left(z\right)\right) dG\left(z\right) = \int_{z_t^o}^{\infty} f\left(x_{t+1}\left(z\right)\right) dG\left(z\right). \tag{24}$$

Then, decomposing profits into natural profits, the deadweight loss from inefficiency in equation (23) and the cost of reporting in equation (24), and using aggregate profits and labor market equilibrium, we can rewrite equation (21) as[19]:

$$Y_t = \tilde{C}_t + C_t + \tilde{\tau}_t + \tilde{f}_t = Y_t^A. \tag{25}$$

Equation (25) states that there are four sources of output absorption in our model economy: entrepreneurial consumption, household consumption, and two sources of distortions from natural profits. The first distortion, captured by $\tilde{\tau}_t$, arises because managerial inefficiency reduces profits available for consumption below natural profits. And the second distortion, captured by $\tilde{f}_t$, derives from the monitoring cost for all listing firms $f\left(x_{t+1}\left(z\right)\right)$. The expressions for $\tilde{\tau}_t$ and $\tilde{f}_t$ define the aggregated resources diverted due to profit inefficiency and monitoring costs, respectively. Absent inefficiencies in firm management ($\tau\left(x_{t+1}\left(z\right)\right)=0$) and monitoring costs ($f\left(x_{t+1}\left(z\right)\right)=0$), all output would be available for consumption by households and entrepreneurs.

3 Assessing the Impact of Ownership Sales on Managerial Inefficiencies

We have developed the basic framework for analyzing the impact of equity sales on managerial waste and the aggregate economy above. In this section, we present analytical results of our model. We begin by solving for the steady state where aggregate productivity and the state variables are constant and then study the dynamics around this steady state.

[19] See the appendix for a derivation.

3.1 The Steady State

In the steady state, aggregate productivity Z_t is assumed constant and equal to one. All endogenous variables are constant, and we denote their steady-state levels by dropping time subscripts. Since the labor supply is inelastic, we normalize this supply to equal one. Details of the solution for the steady state of the model are provided in the appendix.

Using the equilibrium wage rate in equation (20) and the aggregate productivity in equation (19), it is straightforward to verify that $Z = L = 1$ implies:

$$w = \frac{\theta - 1}{\theta}\tilde{z} \quad \text{and} \quad Y = \tilde{z}. \tag{26}$$

Thus, steady-state wages and GDP are simply pinned down by the average firm productivity.

Equity sales imply an important relationship in our analysis below. Using the condition that the marginal benefit of listing equal the cost, $\tau'\left(x_{t+1}\left(z\right)\right)\pi_t^V\left(z\right) = \overline{f}$, and the assumed functional form for $\tau\left(x_{t+1}\left(z\right)\right)$, the steady-state share of equity sold by the entrepreneur managing listed firm z has the form:

$$x(z) = \frac{1}{2\overline{\tau}}(1 - \frac{\overline{f}}{\pi^V(z)}). \tag{27}$$

The larger $\overline{\tau}$, the smaller the share of equity sold on the stock market. The intuition is straightforward: Larger $\overline{\tau}$ implies that the firm is making actual profits closer to the natural level. Hence, the incentive to sell equity and bear the costs associated with listing is weaker. The share of equity sold is an increasing function of the firm's natural profit and thus its relative productivity. The larger are natural profits, the larger are the implied marginal benefits of equity sales, making the entrepreneur more willing to bear the costs associated with selling a larger share of equity.[20] Finally, the share of equity sold is decreasing in the marginal cost of equity sale $\overline{f}$, a straightforward implication of the first-order condition.

Since equity shares cannot exceed the full value of the firm and cannot be negative, $x(z) \in (0, 1)$. From equation (27) the feasible set of equity shares in turn has implications for the potential range of parameters. In particular, substituting the optimal goods price (17) into the natural profit equation (5) and using the equilibrium level of output and wages from equation (26), the natural profit function in steady state is:

$$\pi^V(z) = \frac{1}{\theta}z^{\theta-1}\tilde{z}^{-(\theta-2)}. \tag{28}$$

Rearranging the share of equity sold by each manager in (27) makes clear that for equity shares to be greater than or equal to zero, $x(z) \geq 0$, we require that:

$$\pi^V(z) \geq \overline{f}. \tag{29}$$

[20] For given firm productivity z, the share of equity sold decreases with average productivity $\tilde{z}$ under the realistic assumption $\theta > 2$ because this reduces natural profits via its effect on the firm's relative price (which more than offsets the effect on GDP and aggregate demand).

This condition is ensured by solving for the outside ownership cutoff above. In other words, the level of productivity where this condition holds with equality determines the steady-state outside ownership cutoff. Substituting the steady state natural profits (28) into equation (29) and solving for z implies this cutoff is:

$$z^o = \tilde{z}^{\frac{\theta-2}{\theta-1}} \left(\theta \overline{f}\right)^{\frac{1}{\theta-1}}. \tag{30}$$

Similarly, to ensure that equity sold in each firm is less than one, $x(z) \leq 1$, inspection of equation (27) shows that we require:

$$\frac{\overline{f}}{1-2\overline{\tau}} \geq \pi^V(z).$$

This condition is likely to hold the higher is $\overline{f}$, the closer is $\overline{\tau}$ to (1/2), and the lower is the firm's productivity, z. However, since the upper tail of the distribution of z is unbounded, we require an upper cutoff to keep highly productive managers from selling more than the total value of their firm.

Note that we can solve for this upper bound, $\overline{z}$, as the productivity level that implies shares are equal to $\overline{x} < 1$. For our quantitative analysis, we assume $\overline{x}$ is arbitrarily close to one. Setting $x(z) = 1$ in (27) and solving for z yields the cut-off productivity of firms for managers who want to sell less than all their firm equity. Using the solution for the outside ownership cutoff in (30) and for natural profits in (28), this upper cutoff productivity can be rewritten:

$$\overline{z} = z^o(1-2\overline{\tau})^{-\left(\frac{1}{\theta-1}\right)}.$$

This relationship highlights the impact of the managerial inefficiency on the intermediate range of stock sales. First, the upper bound is clearly always higher than z^o because $(1-2\overline{\tau})^{-\left(\frac{1}{\theta-1}\right)} > 0$. Moreover, the closer $\overline{\tau}$ is to its upper limit of one-half, the greater the difference between the two cutoffs.

3.2 Dynamics

We now describe how these profits of firms deviate in response to aggregate shocks away from the steady state. Given $L = 1$, equation (20) and $w_t L = (\theta - 1) Y_t/\theta$ imply that GDP is simply determined by the product of aggregate productivity and average firm-level productivity: $Y_t = Z_t \tilde{z}$. In turn, optimal pricing and the equilibrium wage in equation (20) imply that the relative price of good z, $\rho_t(z)$, is always constant and equal to $\tilde{z}/z$. It is then straightforward to find that firm z's natural profits are:

$$\pi_t^V(z) = \frac{1}{\theta} z^{\theta-1} \tilde{z}^{-(\theta-2)} Z_t. \tag{31}$$

Note that this equation is the dynamic counterpart to the steady state profits in equation (28). Higher aggregate productivity increases GDP and thus the equilibrium demand for the consumption basket. Hence, natural profits rise.

Equation (31) implies that profits and consumption for entrepreneurs who have wholly owned firms are determined by:

$$\pi_t(0; z) \equiv \overline{\tau}\frac{1}{\theta}z^{\theta-1}\tilde{z}^{-(\theta-2)}Z_t. \tag{32}$$

Solving for the productivity cutoff for equity sales by setting $\pi_t^V(z) = \overline{f}$ as above implies:

$$z_t^o = \tilde{z}^{\frac{\theta-2}{\theta-1}}\left(\frac{\theta\overline{f}}{Z_t}\right)^{\frac{1}{\theta-1}}. \tag{33}$$

Clearly, higher aggregate productivity Z_t increases the marginal benefit of listing, thereby lowering the maximum productivity level for optimal listing. A larger number of firms are therefore present in the stock market.

The optimality condition for equity sales and the assumed form for the function $\tau(x_{t+1}(z))$ in equation (9) imply that the amount of equity sold by each firm that sells equity shares less than the total firm is determined by:

$$x_{t+1}(z) = \frac{1}{2\overline{\tau}}(1 - \frac{f}{\pi_t^V(z)}). \tag{34}$$

The intuition is similar to the steady-state counterpart in equation (27). The difference here is that natural profits are no longer constant. Higher aggregate productivity increases natural profits and thus increases an entrepreneur's incentive to sell equity. An increase in aggregate productivity has two effects on the size of the equity market by increasing the marginal benefit of selling equity. On one side, it induces more wholly owned firms to sell equity. On the other side, it causes each firm with outstanding equity to sell more shares.

As in steady-state, we require an upper bound to this equity issuance of $x_{t+1}(\overline{z}_t) = 1$. Setting equation (34) equal to one, solving for z and using the solutions to z_t^o and $\pi_t^V(z)$ as before, we can show that:

$$\overline{z}_t = z_t^o(1 - 2\overline{\tau})^{-\left(\frac{1}{\theta-1}\right)} \tag{35}$$

Given the solution for $x_{t+1}(z)$ and the assumptions on $\tau(x_{t+1}(z))$ and $f(x_{t+1}(z))$, we can then obtain realized profits for firms with outstanding equity as:

$$\pi_t(x_{t+1}(z); z) = \tau(x_{t+1}(z))\frac{1}{\theta}z^{\theta-1}\tilde{z}^{-(\theta-2)}Z_t - f(x_{t+1}(z)). \tag{36}$$

We now use the dynamics of profits to determine the evolution of household consumption. For this purpose, we consider the household budget constraint (14) at equality and substitute the profits paid to shareholders from equation (36) to rewrite the constraint as:

$$\int_{z_{t-1}^{list}}^{\infty}\pi_t(x_{t+1}(z); z)\,x_t(z)\,dG(z) + w_t L = C_t + \int_{z_t^{list}}^{\infty}v_t(z)\,x_{t+1}(z)\,dG(z) - \int_{z_{t-1}^{list}}^{\infty}v_t(z)\,x_t(z)\,dG(z). \tag{37}$$

To clarify the household budget constraint relationship, we define the following variables:

$$\Pi_{t,t-1} \equiv \int_{z_{t-1}^{list}}^{\infty} \pi_t\left(x_{t+1}(z);z\right) x_t(z)\, dG(z), \tag{38}$$

$$V_{t,t-1} \equiv \int_{z_{t-1}^{list}}^{\infty} v_t(z)\, x_t(z)\, dG(z), \tag{39}$$

$$V_{t,t} \equiv \int_{z_{t}^{list}}^{\infty} v_t(z)\, x_{t+1}(z)\, dG(z). \tag{40}$$

Thus, $\Pi_{t,t-1}$ is the aggregate dividend pay-out at time t to equity investors holding the outstanding shares from time $t-1$. That is, for the set of shares outstanding at time $t-1$, i.e., $x_t(z)$ for all $z \geq z_{t-1}^{list}$, the investors receive the current profit earnings as dividends. $V_{t,t-1}$ is the current market value of the equities held by investors entering period t. However, since new shares are issued or repurchased within the period, the market value of equities at the beginning of the period differs from the end of the period. The end of period market value of outstanding shares at time t is given by $V_{t,t}$. Recalling that $w_t L = (\theta - 1) Y_t/\theta$ and $Y_t = Z_t \tilde{z}$, the household budget constraint (37) can be rewritten using the definitions in equations (38), (39) and (40) as:

$$\Pi_{t,t-1} + \frac{\theta - 1}{\theta} Z_t \tilde{z} = C_t + (V_{t,t} - V_{t,t-1}). \tag{41}$$

Intuitively, equation (41) says that household income given on the left hand side must equal household spending given on the right-hand side. Household income equals dividends, $\Pi_{t,t-1}$, plus labor income. Household income is then spent on consumption and the net inflow of new stocks listed. Households are the net absorbers of changes in equity listings, a role that significantly affects consumption dynamics below.

To solve for the dynamics of stock values, consider next the Euler equation (15). Multiplying both sides by $x_t(z)$ and aggregating across firms yields the aggregated Euler equation of the value of outstanding shares:

$$\int_{z_{\min}}^{\infty} v_t(z)\, x_t(z)\, dG(z) = \beta E_t \left\{ \frac{U'(C_{t+1})}{U'(C_t)} \begin{bmatrix} \int_{z_{\min}}^{\infty} \pi_{t+1}\left(x_{t+2}(z);z\right) x_t(z)\, dG(z) \\ + \int_{z_{\min}}^{\infty} v_{t+1}(z)\, x_t(z)\, dG(z) \end{bmatrix} \right\}.$$

Combining this relationship with the definition of profits paid to shareholders in equation (38) and of the listed stock values in equation (40), and recognizing that integrals between $z_{\min}$ and the listing cutoffs are zero, implies:

$$V_{t,t-1} = \beta E_t \left[\frac{U'(C_{t+1})}{U'(C_t)} \left(\Pi_{t+1,t-1} + V_{t+1,t-1}\right)\right] \tag{42}$$

$$V_{t,t} = \beta E_t \left[\frac{U'(C_{t+1})}{U'(C_t)} \left(\Pi_{t+1,t} + V_{t+1,t}\right)\right] \tag{43}$$

In equation (42), $\Pi_{t+1,t-1}$ and $V_{t+1,t-1}$ are, respectively, the time $t+1$ aggregate dividend payout and market value of firms that had listed at $t-1$. In equation (43), $\Pi_{t+1,t}$ and $V_{t+1,t}$ are the corresponding counterparts for firms that list in period t.

We can then solve for the value of the change in total shares in the economy. For this purpose, we define $V_{t,\Delta} \equiv (V_{t,t} - V_{t,t-1})$ and $\Pi_{t,\Delta} = (\Pi_{t,t} - \Pi_{t,t-1})$ and then combine equations (42) and (43) to write difference between the value of the new shares and the existing shares as:

$$V_{t,\Delta} = \beta E_t \left\{ \frac{U'(C_{t+1})}{U'(C_t)} (\Pi_{t+1,\Delta} + V_{t+1,\Delta}) \right\} \tag{44}$$

where the difference in profits from these shares is:

$$\Pi_{t+1,\Delta} = \int_{z_t^o}^{\infty} \pi_{t+1}(x_{t+2}(z); z) x_{t+1}(z) dG(z) - \int_{z_{t-1}^o}^{\infty} \pi_{t+1}(x_{t+2}(z); z) x_t(z) dG(z)$$

This difference can be further decomposed depending on the relative positions of the cutoffs z_t^o and z_{t-1}^o. For instance, assume that an expansion of the economy caused z_t^o to fall below z_{t-1}^o. Then:

$$\begin{aligned} \Pi_{t+1,\Delta} &= \int_{z_t^o}^{z_{t-1}^o} \pi_{t+1}(x_{t+2}(z); z) x_{t+1}(z) dG(z) \\ &\quad + \int_{z_{t-1}^o}^{\infty} \pi_{t+1}(x_{t+2}(z); z) [x_{t+1}(z) - x_t(z)] dG(z). \end{aligned} \tag{45}$$

Equation (45) shows that the difference in future market dividend payments generated by equity sales in period t in response to economic expansion can be decomposed into two components. The first is the aggregated future profits of newly listed firms on the interval $\left[z_t^o, z_{t-1}^o\right]$. The second component is generated by the change in shares sold by firms with equity outstanding on the interval $[z_{t-1}^o, \infty)$.

Solving the model requires computing the integrals in the definitions (38)-(40). For this purpose, we can write the aggregate profits paid off as dividends to investors entering period t in two components using the interior solution for equity shares $x(z)_t$ in equation (34), yielding:

$$\begin{aligned} \Pi_{t,t-1} &\equiv \int_{z_{t-1}^o}^{\infty} \pi_t(x_{t+1}(z); z) x_t(z) dG(z) \\ &= \frac{1}{2\overline{\tau}} \left[\int_{z_{t-1}^o}^{\infty} \pi_t(x_{t+1}(z); z) dG(z) - \overline{f} \int_{z_{t-1}^o}^{\infty} \left(\frac{\pi_t(x_{t+1}(z); z)}{\pi_t^V(z)} \right) dG(z) \right] \\ &= \frac{1}{2\overline{\tau}} \left(\tilde{\pi}_{t,t-1} - \overline{f} \tilde{\pi}_{t,t-1}^* \right). \end{aligned} \tag{46}$$

Here $\tilde{\pi}_{t,t-1}$ is the aggregate of profits for all listed firms and would represent pay-outs if these firms were fully public. By contrast, $\tilde{\pi}_{t,t-1}^*$ is the ratio of actual profits to natural profits. If there were no managerial inefficiencies, this ratio would equal one, $\overline{f} = 0$, and $\overline{\tau} = 1/2$, implying .$\Pi_{t,t-1} = \tilde{\pi}_{t,t-1}$. These components of dividend pay-outs depend in turn upon moments of the natural profits

$\left(\pi_t^V\left(z\right)\right)^n$. Therefore, it is useful to define the following generalized productivity averages:

$$\tilde{z}_n \equiv \left(\int_{z_{\min}}^{\infty} z^{n(\theta-1)}dz\right)^{\frac{1}{n(\theta-1)}}, \tag{47}$$

$$\tilde{z}_{n,t}^o \equiv \left[\frac{1}{1-G\left(z_t^o\right)}\int_{z_t^o}^{\infty} z^{n(\theta-1)}dz\right]^{\frac{1}{n(\theta-1)}}, \tag{48}$$

where n can take any value on the real axis. The definitions (47) and (48) generalize the market-share weighted average productivity for all firms (19) and the corresponding definition of market-share weighted average productivity for firms with outside ownership by allowing adjustment of the weighting by any real number n. Thus, the market-share weighted productivity average for these firms, $\tilde{z}_t^o$, is simply given by $\tilde{z}_t^o = \tilde{z}_{1,t}^o$.

Given these definitions we prove the following results in the Appendix:

$$\begin{aligned}\int_{z_{\min}}^{\infty}\left(\pi_t^V\left(z\right)\right)^n dG\left(z\right) &= \left(\pi_t^V\left(\tilde{z}_n\right)\right)^n,\\ \int_{z_t^o}^{\infty}\left(\pi_t^V\left(z\right)\right)^n dG\left(z\right) &= \left(1-G\left(z_t^o\right)\right)\left(\pi_t^V\left(\tilde{z}_{n,t}^o\right)\right)^n.\end{aligned}$$

Clearly, the aggregated profits of listed firms differ from total profits according to the probability of the firm lying above the listing cutoff, or $(1-G\left(z_t^o\right))$.

Using the results obtained above, we can reduce the model to a system of six equations in six endogenous variables. For this purpose, we use the household budget constraint (41), and its components. These components are the dividends paid, $\Pi_{t,t-1}$, in equation (46), wages (codetermined by productivity), and the Euler equation for new equity relative to existing equity $V_{t,\Delta}$ in (44). We also use the solutions for the managerial efficiency loss $\tilde{\tau}_t$ in equation (23) and the aggregate reporting costs $\widetilde{f}_t$ in equation (24). Together with the aggregate resource constraint in equation (25), these relationships provide six equations that determine the six endogenous variables: household consumption C_t, managerial consumption $\widetilde{C}_t$, dividends paid $\Pi_{t,t-1}$, efficiency loss $\tilde{\tau}_t$, reporting costs $\widetilde{f}_t$, and the change in market capitalization due to new issues, $V_{t,\Delta}$.

4 Quantitative Implications

Given the dynamic evolution of the model described above, we can now consider its quantitative implications. We begin by describing the log-linear approximation of the model around the steady state. To demonstrate how the steady state depends upon the parameters, we provide numerical solutions for alternative parameter values. We then evaluate the model's aggregate implications by showing impulse responses to productivity shocks and simulations.

4.1 Log-linearized Model

We solve the system obtained in Section 3 by log-linearization around the steady state. As we show in the appendix, the system can be expressed as:

$$\widehat{\Pi}_{t,t-1} + \left(\frac{\theta-1}{\theta}\right)\frac{\widetilde{z}}{\Pi}\widehat{Z}_t = \frac{C}{\Pi}\widehat{C}_t + \frac{V_\Delta}{\Pi}\widehat{V}_{t,\Delta} \tag{49}$$

$$\widehat{\Pi}_{t,t-1} = h_0\widehat{Z}_{t-1} + h_1\widehat{Z}_t \tag{50}$$

$$\widehat{\tilde{\tau}}_t = \frac{1}{\tilde{\tau}}\left\{\frac{\widetilde{z}(1-\overline{\tau})}{\theta} - \frac{\psi k}{(\theta-1)}\right\}\widehat{Z}_t \tag{51}$$

$$\widehat{\tilde{f}}_t = \frac{k}{\theta-1}\widehat{Z}_t \tag{52}$$

$$\widehat{\widetilde{C}}_t = \left[\frac{Z}{\widetilde{C}}\right]\widehat{Z}_t - \left[\frac{C}{\widetilde{C}}\right]\widehat{C}_t - \left[\frac{\widetilde{\tau}}{\widetilde{C}}\right]\widehat{\tilde{\tau}}_t - \left[\frac{\widetilde{f}}{\widetilde{C}}\right]\widehat{\tilde{f}}_t \tag{53}$$

$$\widehat{V}_{t,\Delta} = \gamma E_t\left[\widehat{C}_{t+1} - \widehat{C}_t\right] + \beta E_t\widehat{V}_{t+1,\Delta} + E_t\left[\widehat{\Pi}_{t+1,t} - \widehat{\Pi}_{t+1,t-1}\right] \tag{54}$$

where we use hats over the variables to denote percent deviations from steady state. Also, the h's and ψ are constants that depend on structural parameters and are detailed in the appendix. In log-linearizing the model, we assume that the distribution of firm-level productivity $G(z)$ is Pareto, with curvature parameter $k > \theta - 1$. This restricion is a standard assumption in models with heterogeneous productivity.

Equations (49) to (54) have a straightforward interpretation. Equation (49) simply restates the household budget constraint normalized by steady-state profit pay-outs, Π. The evolution of these profits in turn is given by equation (50) and depends upon both lagged productivity, Z_{t-1}, and current productivity, Z_t. Profits to shareholders depend upon both Z_t and Z_{t-1}. These profits depend upon lagged productivity because Z_{t-1} determines how many shares from each firm are owned by households. The profits also depend upon current productivity because Z_t determines profits in the current period. These two effects on payouts to shareholders are captured by the coefficients h_0 and h_1. In the appendix we show that the solution to these coefficients depend upon whether productivity expands or contracts relative to steady state. Intuitively, an expansion will increase the number of firms seeking outside ownership while dividends on these shares are not paid out yet. On the other hand, a contraction means some firms will choose to leave the financial market, but our timing implies that they must pay out liquidating dividends before doing so.

Equation (51) shows that the evolution of the efficiency loss, $\tilde{\tau}$, depends upon the interplay of two opposing effects. When the economy expands, profits rise for all firms implying a proportional increase in the resource lost through $(1-\overline{\tau})$ times the effect on aggregate profits $\widetilde{z}/\theta$. On the other hand, an expansion induces more wholly owned firms to seek outside ownership. The proportion of these firms in the distribution is $k/(\theta-1)$. The effects of greater equity ownership on the resource loss is measured by ψ. Similarly, the evolution of reporting costs given in equation (52) depends directly upon the increase in newly open firms according to $k/(\theta-1)$. Since only firms with outside equity holders pay these costs and equity sales are pro-cyclical, reporting costs unambiguosly increase with output.

Given the solution for all other uses of output, we can calculate entrepreneurial consumption as the residual as in (53). Finally, the value of newly issued equity relative to the previously issued equity, $V_{t,\Delta}$, in equation (54) is just given by the difference between the expected marginal utility of future profits of these shares.

To close this system, we assume that aggregate productivity shocks follows the process:

$$\widehat{Z}_t = \phi \widehat{Z}_{t-1} + \xi_t, \tag{55}$$

where $0 \leq \phi \leq 1$ and ξ_t is an i.i.d. normal innovation with zero mean and variance σ^2.

4.2 Quantitative Implications on Steady State

Before studying the dynamics generated by the log-linearized model, we examine the implications of the steady state of the economy. For this purpose, we consider several sets of parameters we can later use to describe the annual response of the economy as in empirical studies. These different parameter values correspond to four different scenarios as detailed in Table 3.

In the baseline case, we use the measures of k and θ from Ghironi and Melitz (2005), picked to match features of US firm-level data. The baseline magnitude of reporting costs is 10%.[21] We do not have direct evidence for the value of $\overline{\tau}$. However, we have shown above that given our functional form it must be less than (1/2). Also, the lower is $\overline{\tau}$, the greater is the underlying managerial inefficiency. To be conservative, therefore, we assume this parameters is close to the upper bound and set it equal to 0.4 in our baseline case. For the other parameters in the baseline and other scenarios we keep the numbers as consistent as possible with other business cycle models. We set the preference parameters of γ to two and β to 0.95. For the standard deviation of the productivity shock, we back out σ to match the standard deviation of approximately 2% reported for output in Backus, Kehoe, and Kydland (1992).[22] Finally, we set the persistence parameter to the technology shock, ϕ, equal to 0.9. Following the literature, we normalize the minimum z to be one in all of our scenarios.

[21] In practice, issuing costs such as IPO fees are expressed as a proportion of the value of the issue, not the quantity of shares per se. However, in our model the equilibrium quantity sold is related directly to the price since it depends upon the firm's natural profits. Also, note that the issuing costs would be one-time initial costs while f represents on-going reporting costs so that we are combining both fees into this single measure.

[22] Backus, Kehoe, and Kydland (1992) report 1.7% as the standard deviation of output in their Table 5. For their quantitative model, they input a smaller standard deviation in the innovation to the technology shock to match quarterly data.

Table 3: Calibration Scenarios				
Parameters	Baseline	High k	Low $\overline{f}$	Low $\overline{\tau}$
k	3.4	6	3.4	3.4
θ	3.8	3.8	3.8	3.8
$\overline{f}$	.10	.10	.05	.10
$\overline{\tau}$	.40	.40	.40	.20
Preferences: $\gamma = 2$, $\beta = 0.95$				
Technology: $StdDev(Z_t) = .02$, $\phi = 0.9$, $z_{\min} = 1$				

In addition to the baseline case, we consider three other scenarios. In the first scenario, we consider a higher value of the parameter in the cross-sectional distribution of firms by setting k equal to 6. This parameter controls how much of the probability mass of firms is concentrated near the bottom. Thus, a higher k means that there are more small firms and hence more firms on the margin that might enter the equity market if the economy expands. Though this number is too high to be realistic, this scenario provides insights into what a larger share of small firms imply for the model.

The second scenario considers the impact of a lower reporting cost at 5%. This variable is important because lower costs of reporting reduce the marginal cost of equity sales and therefore reduce the cutoff of outside ownership.

The last scenario we examine is a higher level of managerial waste in the absence of listing. Higher waste corresponds to a lower proportion of natural profits available to owners, $\overline{\tau}$. We consider halving this proportion relative to the baseline by setting $\overline{\tau} = 0.2$.

Using these parameters, we examine the steady-state shares of household consumption, entrepreneurial consumption, reporting costs, and managerial inefficiency as a proportion of GDP. These numbers are reported in Table 4. The baseline scenario implies that household consumption is 89% while managerial consumption is .4 basis points of GDP. The aggregate reporting costs of the firms with outstanding equity is 8% and our baseline parameters produce a benchmark level of managerial inefficiency of 2.5%. The lower four rows of the table report the implied outside ownership cutoff productivity z^o and upper bound of productivity where managers retain partial ownership, $\overline{z}$. These levels in combination with the density parameter k determine the proportion of firms that are fully public and those that have some outside ownership. In the baseline scenario, the proportion of firms that have some outside ownership but are not fully public is 72% while the proportion of firms that are fully public is 12%.

The "High k" scenario increases the number of small firms. In this scenario, the lower bound z^o at 0.82 is less than the minimum of one so that all firms have some outside ownership. However, most of these firms continue to have a significant ownership stake by the entrepreneur since the proportion of firms between z^o and $\overline{z}$ is 89% and only 11% of the firms are fully public. Since the managers of these firms have sold more equity shares generating greater efficiency, the share of manager consumption increases to 12%. Moreover, the larger number of shares implies greater aggregate reporting costs at 26% of GDP. The combination of higher managerial consumption and reporting costs crowds out household consumption which declines to 58%.

Table 4: Steady State Absorption Shares and Firm Listing Distribution				
Shares	Baseline	High k	Low $\overline{f}$	Low $\overline{\tau}$
Household Consn $\left(\frac{C}{Y}\right)$	0.890	0.584	0.914	0.788
Managerial Consn $\left(\frac{\widetilde{C}}{Y}\right)$	0.004	0.124	0.002	0.137
Reporting Costs $\left(\frac{f}{Y}\right)$	0.081	0.260	0.069	0.040
Managerial Inefficiency $\left(\frac{\widetilde{\tau}}{Y}\right)$	0.025	0.032	0.015	0.034
Listing Lower Bound z^o	1.05	0.82	0.82	1.05
All-Public Ownership Bound $\overline{z}$	1.87	1.45	1.78	1.27
Proportion Firms Partial Outside Ownership $\Pr(z^o < z < \overline{z})$	0.72	0.89	0.73	0.39
Proportion Firms All-Public $\Pr(\overline{z} < z)$	0.12	0.11	0.14	0.45

The scenario generated by lower marginal reporting costs, $\overline{f}$, implies more outstanding equity. Compared to the baseline case, the "Low $\overline{f}$" case generates a lower cutoffs for both outside ownership z^o and for "All-Public Ownership" $\overline{z}$. The intuition is clear. Lower reporting costs means the net benefit to outside ownership is higher at each productivity level. As a result the bounds are lower in equilibrium. The higher level of outside ownership generates more consulting which reduces inefficiency to 1.5% of GDP relative to 2.5% in the baseline. While the higher amount of outside ownership would in principle imply more reporting costs as in the "High k" case, the reduction of reporting costs to 5% from 10% means a reduction in the GDP share of reporting to 7%. Overall, the combination of lower reporting costs and managerial inefficiency raises household consumption to 91%.

Finally, the "Low $\overline{\tau}$" scenario represents a lower proportion of profits by wholly owned firms available for consumption by households. In response, more firms choose to go fully public as the "All-Public" cutoff decreases to 1.27 to compensate for this greater efficiency loss. Despite seeking outside ownership and the concomitant consulting to offset this inefficiency, the equilibrium amount of inefficiency increases by 35% relative to the baseline case to 3.4% of GDP. When managers are inefficient household consumption is lower at 79% of GDP while aggregate managerial consumption increases to 14%.

4.3 Impulse Responses

Given these relationships, we now study the properties of these scenarios around the steady state. For each of our scenarios, we consider the impact of a 1% increase in productivity, Z_t. While we focus on an expansion for parsimony, the appendix describes the solution for a contraction. We examine the effects of this shock on two sets of variables. In the first group, we analyze the effect of the shock on consumption responses across households and different managers. In the second group, we consider the impact of the expansion on the distortions such as managerial inefficiency and reporting costs. Before showing the results, we use our model above to discuss the theoretical effects on these groups.

4.3.1 Impulse Responses: Theoretical Relationships

Below we consider the impulse responses of the consumption by households, by entrepreneurs of wholly owned companies, and a representative entrepreneur that retains a significant ownership stake. In our simulations below, we also consider a wider range of representative entrepreneurs.

To understand this set of consumption responses, it is useful to understand how a productivity shock will affect each group of consumers. An increase in productivity will always reduce the outside ownership cutoff (equation (33)), thereby increasing the number of shares of stocks in the market. As the household constraint in equation (41) makes clear, households must absorb this increase in shares. Therefore, if the value of new shares is greater than listing shares, household consumption will increase by less than it would in a world without managerial waste or reporting costs. At the same time, aggregate managerial consumption will tend to increase by more. To see this effect, we can decompose the expression for aggregate managerial consumption in (22) using the profits to wholly owned firms in equation (32) and the budget constraint to managers in equation (13) to obtain:

$$\begin{aligned}\tilde{C}_t &= \int_{z_{\min}}^{z_{t-1}^o} \pi_t\left(0; z\right) dG\left(z\right) + \int_{z_{t-1}^o}^{\infty} \pi_t\left(x_{t+1}\left(z\right); z\right) dG\left(z\right) - \int_{z_{t-1}^o}^{\infty} \pi_t\left(x_{t+1}\left(z\right); z\right) x_t\left(z\right) dG\left(z\right) \\ &\quad + \int_{z_t^o}^{\infty} v_t\left(z\right) x_{t+1}\left(z\right) dG\left(z\right) - \int_{z_{t-1}^o}^{\infty} v_t\left(z\right) x_t\left(z\right) dG\left(z\right).\end{aligned}$$

Comparing this aggregate budget constraint to that of households in equation (37) makes clear the relationship between the two groups. For this purpose, we use the definitions of aggregate payouts and the change in stock market values, $\Pi_{t,t-1}$ and $V_{t,\Delta}$, respectively, to rewrite aggregate entrepreneurial consumption as:

$$\tilde{C}_t = \int_{z_{\min}}^{z_{t-1}^o} \pi_t\left(0; z\right) dG\left(z\right) + \int_{z_{t-1}^o}^{\infty} \pi_t\left(x_{t+1}\left(z\right); z\right) dG\left(z\right) - \Pi_{t,t-1} + V_{t,\Delta}. \tag{56}$$

At the same time, we have shown that the household budget constraint implies:

$$C_t = w_t L + \Pi_{t,t-1} - V_{t,\Delta}. \tag{57}$$

Thus, the two groups of consumptions are negatively related by payouts to shareholders $\Pi_{t,t-1}$ and the difference in market value of the new issues relative to existing shares, $V_{t,\Delta}$. The intuition is clear. Dividend payouts to shareholders are a source of income to households but a relative loss to managers. Similarly, households must reduce consumption to acquire any new shares, but these sales represent an increase in revenue to managers.

On the other hand, $\tilde{C}_t$ and C_t also differ according to other components. First, only households supply labor and therefore earn labor income, $w_t L$. Also, for managers of firms with no existing equity shares (i.e. for $z < z_t^o$) the managers only consume profits. Finally, managers of firms with outside equity shares consume their share of profits in addition to gains from equity sales.

While equation (56) gives the aggregate consumption by all the managers in the economy, the behavior and, hence, consumption of managers differ depending upon their productivity levels. To examine the behavior of an individual manager with productivity z, we use the manager's budget constraint in equation (13) to write his consumption as:

$$C_t(z) = [1 - x_t(z)]\, \pi_t(x_{t+1}(z); z) + v_t(z)\, [x_{t+1}(z) - x_t(z))] \tag{58}$$

Note that this equation subsumes the case when firms have no outside ownership since in this case, $x_t(z) = 0$ so that $C_t(z) = \pi_t(x_{t+1}(z); z) = \overline{\tau}\pi_t^V(z)$ as noted above. A log-linear expansion of this budget constraint implies that the dynamics of consumption for these individual managers have the form:

$$\begin{aligned} \widehat{C}_t(z) \quad = \quad & [1 - x(z)]\, q \left(\frac{\partial \pi_t(z)}{\partial Z_t}\right) \widehat{Z}_t - \pi(z) q \left(\frac{\partial x_t(z)}{\partial Z_{t-1}}\right) \widehat{Z}_{t-1} \\ & v(z)\, q \left[\left(\frac{\partial x_{t+1}(z)}{\partial Z_t}\right) \widehat{Z}_t - \left(\frac{\partial x_t(z)}{\partial Z_{t-1}}\right) \widehat{Z}_{t-1}\right] \end{aligned} \tag{59}$$

where $q = (Z/\ C(z))$ and where $x(z)$, $\pi(z)$, $C(z)$, and $v(z)$ denote the steady state levels of equity sales, profits, managerial consumption, and equity sales, respectively, for firms with productivity level z. For expositional clarity, we have suppressed the dependence of profits on equity sales. Equation (59) shows the main components of dynamics in manager consumption. The first term shows that consumption positively comoves with aggregate productivity through the profits of the firm, according to the amount of ownership the manager has retained in the firm, $[1 - x_0(z)]$. However, managerial consumption is negatively related with lagged aggregate productivity Z_{t-1} since prior expansions imply a lower level of manager ownership and, hence, lower current period payouts. The last term illustrates the effects of equity sales on consumption. In the appendix, we show the solution to this dynamic equation in terms of the underlying parameters of the model.

In addition to the consumption responses, we also depict the impulse responses of the managerial inefficiency, τ_t, the reporting costs, f_t, and aggregate profits available for dividends to shareholders, $\Pi_{t,t-1}$. Since stock sales, and hence outside monitoring is pro-cyclical, both $\widetilde{\tau}$ and $\widetilde{f}$ are also pro-cyclical. Clearly, profits increase with productivity, but whether the response is greater or less than proportional to the productivity shock depends upon how efficiently the economy responds to this shock.

4.3.2 Impulse Responses: Results

Figures 3 to 7 illustrate the effects on the endogenous variables from a 1% increase in productivity when the parameters are by the four scenarios described in Table 2. As we describe below, the effects on consumption depend strongly upon the degree of persistence of productivity. Therefore, we also consider the baseline scenario assuming lower persistence parameter ϕ.

Figure 3 depicts the effects on some key endogenous variables for the baseline case. Figure 3a shows the effects on the consumption of households and managers with firms that have no outside ownership, the Wholly Owned firms. As equation (58) shows, when $x_t(z) = 0$ consumption is simply proportional to natural profits which is in turn proportional to productivity Z_t. As a result, managerial consumption of wholly owned firms simply move in proportion to the market as shown by the short dashed line. By contrast, consumption of households rises by more than aggregate productivity. To understand why, note that household consumption depends upon the three components given in equation (57): labor income $w_t L$, dividend payouts on current holdings of equity $\Pi_{t,t-1}$, and the value of net purchases of new equity $V_{t,\Delta}$. As in the standard model, labor income increases proportionally with output. However, the effects of dividend payouts and purchases of new equity are different from the standard model.

Figure 3d shows how these variables respond to the expansion as well as the profits earned by firms with outside ownership, $\Pi_{t,t}$. When the economy expands, the cutoff for outside ownership, z^0, and the cutoff for fully public firms, $\overline{z}$, both decline and total profits, $\Pi_{t,t}$, increase by more than proportional to the shock. However, since payouts are only made on existing shares, dividend payments $\Pi_{t,t-1}$ lag one period as shown by the boxed line. Since the persistence on this expansion of profits is high, households were prefer to consume more today. At the same time, managers are selling more equity shares and in equilibrium these must be bought by households out of current period consumption. With high persistence, households are only willing to buy these shares if the price of the newly issued shares, $V_{t,t}$, is lower than the price of previously issued shares, $V_{t,t-1}$, or $V_{t,\Delta}$. Thus, as Figure 3d shows the value of new equity issues initially declines and then returns toward zero over time. As a result of the lower price of new issues relative to previously held shares, households receive a capital gain on their equity portfolio so that consumption increases by more than the productivity shock.

While consumption behavior is identical across households, the response of managerial consumption will depend upon the product variety and, hence, productivity of the firm. As equation (59) shows, this response will depend upon how much managers respond to aggregate shocks by selling equity. For managers with either wholly owned firms or managers in fully public firms, there is no response; in other words, $(\partial x_{t+1}(z)/\partial Z_t) = 0$. Thus for these managers either below the level for outside ownership z^o or above the cutoff for fully public firms $\overline{z}$, consumption varies in proportion to the productivity shock. Figure 3b shows this response in the long-dashed line labeled C-Public. By contrast, the behavior of consumption for managers of firms on the interval

$z \in (z^o, \overline{z})$ depends upon how much they sell equity in response to the shock. To consider a candidate manager within this set, we arbitrarily pick the manager at the midpoint between the upper and lower cutoffs. Figure 3b depicts this response with the short dashed line labeled C-Outside. As noted earlier, the consultants pick equity sales to maximize current period profits so that there is a spike in managerial consumption. Thereafter, the managers consumption evolves similarly with the fully public managers. Although we have arbitrarily chosen the manager in the middle of the interior distribution of firms partially owned by households, we consider a wider range of managers in our simulations below.

So far we have focused upon the consumption shares in GDP. However, the responses of both aggregate managerial inefficiency $\widetilde{\tau}$ and reporting costs $\widetilde{f}$ given in equations (51) and (52), respectively, also drive a wedge between GDP and consumption. Figure 3c shows that managerial inefficiency given by the triangle-marked line increases with profits as expected since this inefficiency is proportional to natural profits. The figure also illustrates the aggregate reporting costs in the circle-marked line. The expansion prompts more equity sales which in turn increase reporting costs.

To highlight the impact of the productivity persistence on consumption behavior, Figure 4 reports the impulse responses for the same variables as Figure 3 under the baseline parameter case except that the autocorrelation coefficient is lowered so that $\phi = 0.5$. Figure 4a shows that household consumption is now attenuated after the expansion so that it only increases by about 0.7% in sharp contrast to Figure 3a. As before, the role of households as equilibrium purchasers of equity plays a key role. Figure 4d shows that the initial response of profits and payouts to shareholders is identical to the high persistence case as must hold since consultants only consider initial period profits. But now that the persistence of these profits is lower, consumers have a weaker desire to intertemporally smooth consumption and are willing to buy the new shares at a higher price than older shares. Figure 4d shows this effect since $V_{t,\Delta}$ now rises on impact and then declines over time to zero in steady state. The expenditure on the new purchases reduces current period consumption by households so that the response to the output shock is dampened.

Similarly, Figure 4b illustrates the effects of lower persistence on managerial consumption for both the fully public manager and the candidate manager with partial outside ownership. As before, the consumption of the fully public manager moves in proportion to the aggregate shock. However, the candidate manager with outside ownership exhibits quite different behavior. The initial increase in consumption due to listening to the advice of consultants is identical to the high persistence case. But since managers have less shares of their own firm the following period and these profits decline more rapidly due to the faster mean reversion, the manager's consumption declines. Thereafter it rises toward the steady state.

Figure 5 demonstrates the effects of an expansion when the parameter values are given by the High k case. While this level of k implies an unrealistically large number of small firms in the distribution compared to US data, it provides a useful counterexample for considering the effects of the cross-sectional distribition on responses. As Figure 5a shows, the effect of more small firms would imply that household consumption declines in response to an output shock. The reason for this decline again stems from the response of payouts to shareholders and the resources taken from current consumption to purchase new equity. Figure 5c depicts these variables. Since there are more small firms, the expansion implies a larger number of new equity shares, both from new and existing firms. As a result current profits increase significantly more to twice the size of the initial output shock. However, aggregate payouts to shareholders, $\Pi_{t,t-1}$, decline.

This result stems from the impact of newly issued equity on the aggregate reporting costs and managerial inefficiency, as reported in Figure 5b. As before, reporting costs rise commensurate with the increase in profits and the initial costs are paid out of current profits implying an initial decline in profits paid to shareholders. In addition, the increase in profits by more small firms implies a more significant increase in managerial inefficiency. As a result the response of dividend payouts increase toward steady state but remain below trend. Consequently, the value of new equity shares relative to existing equity initially declines and then rises toward steady state as well. The impact of lower dividends dominates the capital gain to existing equity shares and household consumption declines. Despite this effect on households, the consumption of the manager that is midway between wholly owned and public remains the same as in Figure 3b, and is therefore not reported.

The "low $\overline{f}$" case is given in Figure 6. Lower marginal reporting costs increases the tendency to sell equity. Therefore while the per share reporting costs decline, the amount of equity outstanding increases so that the effect on aggregate reporting costs depicted in Figure 6c are relatively unchanged from the baseline case in Figure 3c. However, the increase in efficiency from more consulting arising from equity issuance means that the managerial inefficiency given in Figure 6c does not increase by as much as the baseline. As in the baseline case, the high persistence of output makes households reluctant to purchase the new equity shares relative to existing share driving the price of new shares down relative to old. Household consumption therefore expands by more than the output shock.

Figure 7 illustrates the economy's dynamic response when $\overline{\tau}$ is very low at .2. Table 2 reported that in the steady state only 84% of the firms have outside ownership but 45% are fully listed, much higher than the 12% when $\overline{\tau}$ is 0.4. As a result, a much higher proportion of firms have insulated themselves from managerial inefficiency. Therefore, in response to a productivity shock, managerial inefficiency increases only slightly as Figure 7b shows. Also since increased equity shares imply more efficiency, dividends available for current shareholders exceed the profits available to new shareholders and managers.

Overall, the dynamics of the system show that the equity sales decisions have a significant impact on consumption. Household consumption always responds to productivity shocks according to the effect on shareholder payouts and the value of newly issued shares relative to their existing portfolio. When the productivity persistence is low, then households recognize that increases in future profits are relatively temporary and are more willing to consume less today. As a result, they are willing to buy new equity shares in exchange for future consumption. In this case, the value of newly issued equity is higher than that of existing shares of equity. On the other hand, when persistence is high, future marginal utility is expected to be low. In this case, households are less willing to make this substitution and will only do so if the price of new shares is lower than existing shares. Households earn a capital gain on existing equity shares and consumer more today. Moreover, these equity sales decisions depend upon the relative benefits of more equity including the reporting costs and the inherent market inefficiency.

The dynamics also have clear implications for managerial consumption. So far, we have considered a candidate manager located midway between the cutoff for any outside ownership and the cutoff for fully listed firms. In the next section, we report simulations that analyze the impact on different managers on the equity sales decision and consumption.

4.4 Simulations

Smaller firms tend to sell more equity in response to an expansion than do larger firms, a relationship we described in the introduction. We now use simulations to analyze the relationship between a productivity shock and the sales according to firm size. These simulations also allow us to examine the interaction between the consumption of managers of different firm sizes and the households.

To examine the response of equity and managerial consumption, we consider four different representative managers for firms. The first manager is the "Outside" manager located at the steady state cutoff productivity for seeking outside owners, z^o, and hence is just willing to sell equity in response to an expansion. The next three managers have alternatively low, medium, and high productivities within the interior range $z \in \{z^o, \overline{z}\}$. We choose the "Low", "Medium", and "High" Managers as those one fourth, one half, and three fourths of the distance between z^o and $\overline{z}$.[23] Table 5 shows the steady state levels of natural profits π^V, actual profits π, and equity sold by the manager z for each of the scenarios described in Table 3.

Table 5: Steady State Firm Variables for Representative Managers

Scenario	Baseline			High k			Low $\overline{f}$			Low $\overline{\tau}$		
Variable	π^V	π	x	π^V	π	x	π^V	π	x	π^V	π	x
Outside	0.10	0.04	0.00	0.18	0.09	0.54	0.09	0.04	0.53	0.10	0.02	0.00
Low	0.16	0.08	0.49	0.24	0.14	0.72	0.12	0.07	0.72	0.11	0.03	0.32
Medium	0.25	0.08	0.75	0.31	0.21	0.85	0.15	0.11	0.85	0.13	0.04	0.59
High	0.36	0.26	0.90	0.40	0.30	0.94	0.20	0.15	0.94	0.15	0.07	0.81

In the three columns under "Baseline," the table reports the productivity level z and the corresponding natural profits, actual profits and equity sold for each manager. For the marginal "Outside" manager, natural profits are 0.10, but after accounting for inefficiency the actual profits

[23] Thus, $z^{Low} = z^o + \left(\frac{1}{4}\right)(\overline{z} - z^o)$, $z^{Med} = z^o + \left(\frac{1}{2}\right)(\overline{z} - z^o)$, and $z^{High} = z^o + \left(\frac{3}{4}\right)(\overline{z} - z^o)$.

are only 0.04. This manager does not sell equity in steady state. However, as the level of productivity increases to Low, Medium, and High, the amount of equity sold increases from 0.5 to 0.9 of the firm and both the level of profits and the proportion of inefficiency decreases. In both the "High k" and "Low $\overline{f}$ " cases, the outside ownership cutoff is less than the $z_{\min}$. Therefore, even at this minimum level of productivity, managers prefer to sell equity. In these cases, the minimum level of equity sold is about 0.5 for both of these cases. For both scenarios, profits and the level of inefficiency increase as firms become more productive and the amount of outside equity expands. Finally, in the "Low $\overline{\tau}$" scenario, a higher proportion of managers choose to sell outside equity as previously shown in Table 4. As a result, the levels of productivity z are lower for each representative manager. Steady state profits are therefore lower across the managers.

Based upon these steady state levels, we simulate the model by generating a time series for the productivity process Z_t and using this process to calculate the variables in the system. We then repeat this process 10,000 times. From these simulated data, we calculate the means, correlations, and variance-covariance matrix of all the endogenous variables.

Table 6 reports the mean response for the initial period denoted "t" and the subsequent period denoted "$t+1$".

Table 6: Mean Equity Sales for Representative Managers

Scenario	Baseline		High k		Low $\overline{f}$		Low $\overline{\tau}$	
Period	t	$t+1$	t	$t+1$	t	$t+1$	t	$t+1$
Outside	1.25	1.13	0.71	0.64	0.72	0.65	2.50	2.25
Low	0.76	0.69	0.53	0.47	0.53	0.48	2.18	1.96
Medium	0.50	0.45	0.40	0.36	0.40	0.36	1.91	1.72
High	0.35	0.31	0.31	0.28	0.31	0.28	1.69	1.52

As the table shows, the equity sales response declines with the size of the firm for all four scenarios. For the "Baseline" case, a mean 1% increase in productivity on average leads to an average 1.25% increase in equity listing by the marginal "Outside" manager. This response declines to 0.76% for the "Low" firm manager and finally 0.35% for the "High" firm manager. The subsequent period the effects of mean reversion n the productivity shock implies a decline in equity sales relative to the steady state. Interestingly, the greatest equity response arises in the "Low $\overline{\tau}$" case with share issuances rising from 1.7 times the productivity shock for the "High" firm manager to 2.5 times this shock for the marginal "Outside" firm manager. Equity sales are very high both in steady state and in response to aggregate shocks because managerial inefficiency is very high and managers try to offset the losses from this inefficiency by selling equity.

Another empirical finding described in the introduction is that the consumption of high income households tend to be more pro-cyclical than low income households. While we do not have a direct measure of income separate from consumption, Parker and Vissing-Jorgensen (2010) also relate their results to high income professionals including top management. If we loosely group managers into the high income category, we can use our simulations to consider the cyclical response of consumption across groups. Table 7 reports the mean responses of the four representative managers along with household consumption as reference.

Table 7: Mean Consumption of Representative Managers and Households								
Scenario	Baseline		High k		Low $\overline{f}$		Low $\overline{\tau}$	
Period	t	$t+1$	t	$t+1$	t	$t+1$	t	$t+1$
Outside	4.84	0.46	4.74	1.04	3.18	1.24	5.30	0.34
Low	3.94	1.14	0.40	1.02	2.67	1.19	6.01	1.50
Medium	3.01	1.08	0.31	0.94	2.28	1.10	5.96	1.84
High	2.41	0.92	2.70	0.79	2.00	0.99	5.56	1.65
Household	3.95	3.56	-1.60	-1.41	4.01	3.62	5.89	5.30

For almost all cases, consumption increases across all groups in response to the productivity shock. As described above, the "High k" case implies a negative household consumption due to the increase in equities sold and managerial inefficiency. Table 7 also shows that the response of managerial consumption is not monotonic with size. For example, for the "Low $\overline{\tau}$" case, the consumption response for the marginal Outside manager is 5.3% and this response increases to 6% for the Low firm manager before declining to 5.6% for the High firm manager. This response reflects the difference in effects of higher profits as well as lower ownership of the existing equity by the manager.

The initial mean responses are also the betas of consumption on output. Since the simulation is based upon the first-order approximation of the model, by definition the consumption responses are certainty equivalent and cannot be used to directly infer risk properties. However, the second moment behavior of these consumption movements help illustrate the impact of productivity changes on these groups. Under the baseline case, the betas of consumption for these groups are generally higher than one since managers of these firms and households buy and sell equity, creating greater variation in consumption. Managers that are Wholly Owned and Fully Public do not sell equity and therefore have betas equal to one in our model.[24]

In Table 8, we report the correlations between household consumption and consumption of the other groups. Since we only have one aggregate shock and the contemporaneous correlations are one by definition, we report the correlations for one and two periods after the shock. Also, since we have argued above that the "High k" case implies unrealistic responses of household consumption, we report the other three scenarios alone. As the table shows, the contemporaneous correlation between household consumption and managers is high one and two periods after the shock. In all cases, the correlations increase with the productivity of the firm. This pattern reflects the declining ownership by the manager in his own firm. When the manager has less equity to sell, he is less exposed to the variability of consumption due to the sale of equity.

[24] This property can be verified by inspecting the managers budget constraint in equation (58).

Table 8: Household and Manager Consumption Correlations						
Scenario	Baseline		Low $\overline{f}$		Low $\overline{\tau}$	
Period	$t+1$	$t+2$	$t+1$	$t+2$	$t+1$	$t+2$
Outside	0.80	0.73	0.94	0.92	0.78	0.70
Low	0.90	0.87	0.95	0.94	0.88	0.85
Medium	0.92	0.91	0.96	0.95	0.91	0.89
High	0.93	0.92	0.96	0.96	0.90	0.88

Overall, in this section, we have shown that equity sales are procyclical and that this response declines with the size of firms. This pattern is consistent with the empirical literature. Moreover, while our model is too stylized to address differences in income across groups in the economy, we have illustrated some cross-sectional patterns in consumption due to cyclical equity sales. Since managers sell equity and households purchase these sales, the endogenous equity sales drive a wedge between the equilibrium consumption of these two groups. However, as managers sell off more of the equity in their own firms, their consumption responses become more correlated with households.

5 Concluding Remarks

Outside monitoring has often been touted as a vehicle for improving managerial efficiency. Despite the importance of this relationship, research has not considered the aggregate effects of managers seeking outside financing. In this paper, we have begun to fill this gap. We developed a framework to consider the implications of equity sales on managerial efficiency and, hence, the profitability of the firm, and the aggregate economy. We have incorporated a cross-section of firms as in some of the recent literature on models with heterogeneous producers.

Given this framework, we have shown several effects. First, the willingness to be monitored, represented in our model as net equity sales, is pro-cyclical. Moreover, larger and more productive firms are the most likely to list. Both of these results are consistent with the empirical evidence. Second, the pro-cyclical equity sales do not necessarily manifest as greater aggregate managerial efficiency. An expansion induces higher profits for all firms in the economy, implying a concomitant proportional efficiency loss. An expansion can only offset this loss when the equilibrium of the economy already contains a sufficiently high inefficiency and when enough firms decide to sell equity on the margin. Third, equity sales have different effects on the consumption of households relative to managers. Listings and other equity sales during expansions depress consumption spending as households must curtail spending to purchase newly listed firms when the price of these firms exceed those of existing firms. On the other hand, managers may be able to increase consumption by more or less, depending upon how significantly the advice associated with the equity sales improves their firm's available profit.

Since this paper represents a first step toward considering the interaction of managerial behavior and the macroeconomy, it leaves open a number of important issues. First, we have focused on the implications of improved efficency through outside advice instead of financing. In reality, firms largely seek outside financing during expansions to fund their investments. Second, we have discussed our efficiency-improving vehicle as the outside consultation required for equity issuance. However, this vehicle can take many forms, and need not derive only through equity. Moreover, while many of the relationships are likely to hold with other liabilities, the specific macroeconomic effects may differ. Third, for tractability, we have assumed that the manager must listen to advisors who do not necessarily optimize his consumption over time. In a richer model, the manager would likely use some outside advise but also intertemporally optimize his own consumption decisions. Fourth, to focus on the key relationships and innovations of the model, we have kept its structure as simple as possible, abstracting from important features such as capital accumulation and endogenous labor supply. Despite these caveats, the approach presented in this paper provides a starting point for considering these remaining issues.

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Appendix

A Production Equilibrium

Our model focuses upon the effects of managerial inefficiencies on the consumption side of the economy. Therefore, the production equilibrium is standard as in this class of models. Specifically, aggregate absorption implies an equilibrium output demand. This assumption implicitly means that even distorted absorption such as the managerial inefficiencies and the reporting costs are spent in the same proportion as managers and households.

Therefore, as in the standard model, output demand is given by $y_t^D(z) = \rho_t(z)^{-\theta} Y_t^A$. Together with the production $y_t(z) = Z_t z l_t(z)$ these together imply

$$l_t(z) = \frac{\rho_t(z)^{-\theta} Y_t^A}{Z_t z} = \left(\frac{\theta}{\theta-1}\right)^{-\theta} w_t^{-\theta} (Z_t z)^{\theta-1} Y_t^C,$$

where we used (17). Thus, we have:

$$Y_t = \left(\frac{\theta}{\theta-1}\right)^{1-\theta} \left(\frac{w_t}{Z_t}\right)^{1-\theta} Y_t^A \int_{z_{\min}}^{\infty} z^{\theta-1} dG(z).$$

In equilibrium, total output of the consumption basket must be equal to the economy's total absorption, or $Y_t = Y_t^A$. Hence,

$$1 = \left(\frac{\theta}{\theta-1}\right)^{1-\theta} \left(\frac{w_t}{Z_t}\right)^{1-\theta} \int_{z_{\min}}^{\infty} z^{\theta-1} dG(z).$$

Solving this equation for w_t implies equation (20).

B Model Solution Set-up

We now combine the standard production solution above with our novel consumption side of the economy to solve the model. As described in the text, the model can be fully solved using five equations describing the state variables and two Euler equations. In particular, the five equations are:

- Aggregate profits distributed to existing shareholders $\Pi_{t,t-1}$: equation (38)
- Aggregate managerial inefficiency $\widetilde{\tau}$: equation (23)
- Aggregate reporting costs $\widetilde{f}$: equation (24)
- Household budget constraint: equation (14)
- Aggregate economy resource constraint: equation (25)

In addition, the model requires solving for the stock market value of the existing shares, $V_{t,t-1}$, and of the current shares including the newly issued shares, $V_{t,t}$, defined in equations (39) and (40), respectively. These share prices are determined by the two Euler equations:

- Aggregate price of existing shares $V_{t,t-1}$: equation (42)
- Aggregate price of total current shares $V_{t,t}$: equation (43)

The household budget constraint and aggregate economy resource constraint are straightforward to calculate given the aggregate profits, managerial inefficiency, reporting costs, and pricing of shares. We describe the solution of each in turn below.

Before doing so, note that all these variables depend upon integrals of the profit of firms,

$$\pi_t\left(x_{t+1}\left(z\right);z\right) \equiv \tau\left(x_{t+1}\left(z\right)\right)\pi_t^V\left(z\right) - f\left(x_{t+1}\left(z\right)\right) \tag{A.1}$$

In turn, these profits depend upon the natural profits $\pi_t^V\left(z\right)$ both directly and indirectly through the equilibrium effect on equity sold, $x_{t+1}\left(z\right)$ given by equation (34). These natural profits using the production equilibrium in the appendix above can be written:

$$\pi_t^V\left(z\right) = \frac{1}{\theta}\rho_t\left(z\right)^{1-\theta}Y_t^A = \frac{1}{\theta}\tilde{z}^{-(\theta-2)}Z_t z^{\theta-1} \tag{A.2}$$

Clearly, then, since all other parameters including the aggregate state variable Z_t are independent of z, the aggregate variables depend upon integrals of the moments of the distribution of $z^{\theta-1}$. Since this distribution is Pareto, we define two general forms of these moments as given by either the integral over the full distribution as in (47) or conditional on a time varying lower bound as exemplified by (48)in the text. Defining this time varying lower bound more generally as z_t^b, we can restate (48) as:

$$\tilde{z}_{n,t}^b \equiv \left[\frac{1}{1-G\left(z_t^b\right)}\int_{z_t^b}^{\infty} z^{n(\theta-1)}dz\right]^{\frac{1}{n(\theta-1)}},$$

where using the definition of a Pareto distribution:

$$G\left(z_t^{lb}\right) = 1 - \left(\frac{z\min}{z_t^{lb}}\right)^k$$

Using properties of the moment-generating function, we can then rewrite

$$\begin{aligned} \tilde{z} &= \nu_n z_{\min} &\quad (A.3)\\ \tilde{z}_{n,t}^{lb} &= \nu_n z_t^{lb} &\quad (A.4)\end{aligned}$$

where:

$$\nu_n \equiv \left[\frac{k}{k-n\left(\theta-1\right)}\right]^{\frac{1}{n(\theta-1)}}$$

To show the relationships given in the text, substitute natural profits in equation (A.2) into the moments of natural profits across all firms, $\int_{z_{\min}}^{\infty}\left(\pi_t^V(z)\right)^n dG(z)$ and across the conditional group of firms above z_t^{lb} given by $\int_{z_t^{lb}}^{\infty}\left(\pi_t^V(z)\right)^n dG(z)$. Using the definitions of $\tilde{z}$ in equation(60) and $\tilde{z}_{n,t}^{lb}$ in equation(60), the solution to these integrals is immediate:

$$\int_{z_{\min}}^{\infty}\left(\pi_t^V(z)\right)^n dG(z) = \left(\pi_t^V(\tilde{z}_n)\right)^n, \tag{A.5}$$

$$\int_{z_t^b}^{\infty}\left(\pi_t^V(z)\right)^n dG(z) = \left(1-G\left(z_t^b\right)\right)\left(\pi_t^V\left(\tilde{z}_{n,t}^b\right)\right)^n. \tag{A.6}$$

Equation (60) gives the solution for moments of natural profits when profits and the lower bound both depend upon the current state of the economy, Z_t. However, the profits paid out to existing shareholders, $\Pi_{t,t-1}$ depends upon the number of firms with outside ownership at $t-1$. Therefore, we also require solutions of moments of the distribution of the form:$\int_{z_{t-1}^b}^{\infty}\left(\pi_t^V(z)\right)^n dG(z)$. In this case, the profits are driven by the current aggregate state, Z_t, but the set of existing shares was previously determined by the lagged aggregate state, Z_{t-1}. In this case, aggregate natural profit moments conditional on firms with previous outside ownership becomes

$$\int_{z_{t-1}^o}^{\infty}\left(\pi_t^V(z)\right)^n dG(z) = (1-G(z_t^o))\left(\pi_t^V\left(\tilde{z}_{n,t-1}^o\right)\right)^n$$

or more generally, for lower bound z_{t-1}^b

$$\int_{z_{t-1}^b}^{\infty}\left(\pi_t^V(z)\right)^n dG(z) = \left(1-G\left(z_{t-1}^b\right)\right)\left(\pi_t^V\left(\tilde{z}_{n,t-1}^b\right)\right)^n \tag{A.7}$$

B.1 Model Solution: Profit Payouts to Share-holders

As described in the text, the profit payouts to existing share-holders is given by equation (46) repeated here for convenience:

$$\begin{aligned}
\Pi_{t,t-1} &\equiv \int_{z_{t-1}^o}^{\infty}\pi_t\left(x_{t+1}(z);z\right)x_t(z)\,dG(z) \\
&= \frac{1}{2\tau}\left\{\int_{z_{t-1}^o}^{\infty}\pi_t\left(x_{t+1}(z);z\right)dG(z) - \overline{f}\int_{z_{t-1}^{list}}^{\infty}\left(\frac{\pi_t\left(x_{t+1}(z);z\right)}{\pi_t^V(z)}\right)dG(z)\right\} \\
\Pi_{t,t-1} &= \frac{1}{2\tau}\left\{\tilde{\pi}_{t,t-1}-\overline{f}\tilde{\pi}_{t,t-1}^*\right\}
\end{aligned}$$

In order to use our moment relationships and still account for the fact that $x_{t+1}=1$ for $z>\overline{z}$, we rewrite profits as:

$$\Pi_{t,t-1} \equiv \int_{z_{t-1}^o}^{\infty}\pi_t\left(x_{t+1}(z);z\right)x_t(z)\,dG(z) + \int_{\overline{z}_{t-1}}^{\infty}\pi_t^V(z)\,dG(z) - \int_{\overline{z}_{t-1}}^{\infty}\pi_t\left(x_{t+1}(z);z\right)x_t(z)\,dG(z) \tag{A.8}$$

We first determine profits for each firm, $\pi_t(x_{t+1}(z); z)$, by substituting into equation (A.1) the solution for equity sold in equation (34), and the function forms for $\tau(x_{t+1}(z))$ and $f(x_{t+1}(z))$. We then note that as given by equation (35), $\overline{z}_t = z_t^o(1-2\tau)^{-\left(\frac{1}{\theta-1}\right)}$. Then using the relationships from the moments of natural profits in equation (A.7) along with the fact that for the Pareto distribution, $1 - G\left(z_t^b\right) = (z_{\min}/z_t^b)^k$ we can write $\tilde{\pi}_{t,t-1}$ and $\tilde{\pi}^*_{t,t-1}$ in terms of the aggregate state variable Z in the form of:

$$\tilde{\pi}_{t,t-1} = g_0 Z_{t-1}^{k/(\theta-1)} \left(\nu_1^{(\theta-1)}\overline{f}(1-2\tau)^{\left(\frac{k}{\theta-1}-1\right)} + S_{t,t-1} - \overline{S}_{t,t-1}(1-2\tau)^{\left(\frac{k}{\theta-1}\right)}\right)$$

where:

$$g_0 = (z_{\min}^k / z_t^o) Z_{t-1}^{-k/(\theta-1)} = z_{\min}^k \tilde{z}^{-k\left(\frac{\theta-2}{\theta-1}\right)} \left(\theta\overline{f}\right)^{-k/(\theta-1)}$$

and

$$S_{t,t-1} = \overline{f}\left(\overline{\tau} + \frac{1}{4\overline{\tau}}\right)\nu_1^{(\theta-1)}\left(\frac{Z_t}{Z_{t-1}}\right) + \left(\frac{\overline{f}}{4\overline{\tau}}\right)\nu_{-1}^{-(\theta-1)}\left(\frac{Z_t}{Z_{t-1}}\right)^{-1} - \left(\frac{\overline{f}}{2\overline{\tau}}\right)$$

and

$$\overline{S}_{t,t-1} = (1-2\tau)^{-1}\overline{f}\left(\overline{\tau} + \frac{1}{4\overline{\tau}}\right)\nu_1^{(\theta-1)}\left(\frac{Z_t}{Z_{t-1}}\right) + (1-2\tau)\left(\frac{\overline{f}}{4\overline{\tau}}\right)\nu_{-1}^{-(\theta-1)}\left(\frac{Z_t}{Z_{t-1}}\right)^{-1} - \left(\frac{\overline{f}}{2\overline{\tau}}\right).$$

Intuitively, g_0 measures the effects on aggregate profits from the conditional probability of listed firms based on z_t^o. $S_{t,t-1}$ accounts for the evolution of the aggregate state on profits of listed firms while $\overline{S}_{t,t-1}$ adjusts for the fact that $x = 1$ for firms with productivity above $\overline{z}$.

Following the same steps, we find that:

$$\tilde{\pi}^*_{t,t-1} = g_0 Z_{t-1}^{k/(\theta-1)} \left(S^*_{t,t-1} - \overline{S}^*_{t,t-1}(1-2\tau)^{\left(\frac{k}{\theta-1}\right)}\right)$$

where:

$$S^*_{t,t-1} = \left(\overline{\tau} + \frac{1}{4\overline{\tau}}\right)\left(\frac{Z_t}{Z_{t-1}}\right) + \left(\frac{1}{4\overline{\tau}}\right)\nu_{-2}^{-2(\theta-1)}\left(\frac{Z_t}{Z_{t-1}}\right)^{-1} - \left(\frac{1}{2\overline{\tau}}\right)\nu_{-1}^{-(\theta-1)}$$

and

$$\overline{S}^*_{t,t-1} = \left(\overline{\tau} + \frac{1}{4\overline{\tau}}\right)\left(\frac{Z_t}{Z_{t-1}}\right) + \left(\frac{1}{4\overline{\tau}}\right)\nu_{-2}^{-2(\theta-1)}\left(\frac{Z_t}{Z_{t-1}}\right)^{-1}(1-2\tau)^2 - \left(\frac{1}{2\overline{\tau}}\right)\nu_{-1}^{-(\theta-1)}(1-2\tau)^2.$$

Substituting the solutions for $\tilde{\pi}_{t,t-1}$ and $\tilde{\pi}^*_{t,t-1}$ back into equation (46) gives the evolution of profits paid to shareholders according to the current and lagged aggregate productivity, Z_t, Z_{t-1}.

B.2 Model Solution: Managerial Inefficiency

Next we determine the solution to the aggregate managerial inefficiency in equation (23) repeated here for convenience:

$$\tilde{\tau}_t \equiv \int_{z_{\min}}^{\infty} \pi_t^V(z)\left(1-\tau\left(x_{t+1}(z)\right)\right) dG(z)$$

Substituting for the equilibrium $\tau\left(x_{t+1}(z)\right)=\tau\left(x\left(\pi_t^V(z)\right)\right)$ as above, we can rewrite the managerial inefficiency as:

$$\tilde{\tau}_t \equiv \int_{z_{\min}}^{\infty} \pi_t^V(z)\left\{1-\left(\overline{\tau}+x_{t+1}(z)-\overline{\tau}\left[x_{t+1}(z)\right]^2\right)\right\} dG(z) \tag{A.9}$$

where

$$\begin{aligned}
x_{t+1}(z) &= 0, \text{ for } z<z_t^o \\
x_{t+1}(z) &= 1, \text{ for } z>\overline{z}_t
\end{aligned}$$

To use the moments of the aggregate profit, we decompose this integral by writing the integral in (A.9) as:

$$\begin{aligned}
\tilde{\tau}_t \equiv & \int_{z_{\min}}^{\infty} \pi_t^V(z)(1-\overline{\tau}) dG(z)-\int_{\overline{z}_t}^{\infty} \pi_t^V(z)(1-\overline{\tau}) dG(z) \\
& -\left\{\int_{z_t^o}^{\infty} \pi_t^V(z) x_{t+1}(z) dG(z)-\int_{\overline{z}_t}^{\infty} \pi_t^V(z) x_{t+1}(z)) dG(z)\right\} \\
& +\overline{\tau}\left\{\int_{z_t^o}^{\infty} \pi_t^V(z)\left[x_{t+1}(z)\right]^2 dG(z)-\int_{\overline{z}_t}^{\infty} \pi_t^V(z)\left[x_{t+1}(z)\right]^2 dG(z)\right\}
\end{aligned}$$

Using the moments of π_t^V as above, and collecting terms, the managerial inefficiency can be rewritten as:

$$\tilde{\tau}_t=\left(\frac{1}{\theta}\right) \tilde{z}(1-\overline{\tau}) Z_t-\psi Z_t^{\left(\frac{k}{\theta-1}\right)} \tag{A.10}$$

where:

$$\begin{aligned}
\psi = & \; g_0 \overline{f}\left\{(1-\overline{\tau}) \nu_1^{(\theta-1)}(1-2 \tau)^{\left(\frac{k}{\theta-1}\right)-1}-\left(\frac{1}{2 \overline{\tau}}\right)\left[\left(\nu_1^{(\theta-1)}-1\right)-\left(\nu_1^{(\theta-1)}(1-2 \tau)^{-1}-1\right)(1-2 \tau)^{\left(\frac{k}{\theta-1}\right)}\right]\right. \\
& \left.+\left(\frac{1}{4 \overline{\tau}}\right)\left[\left(\nu_1^{(\theta-1)}+\nu_{-1}^{-(\theta-1)}-2\right)-\left(\nu_1^{(\theta-1)}(1-2 \tau)^{-1}+\nu_{-1}^{-(\theta-1)}(1-2 \tau)-2\right)(1-2 \tau)^{\left(\frac{k}{\theta-1}\right)}\right]\right\}
\end{aligned}$$

Though messy looking, the equilibrium managerial waste has an intuitive interpretation. The first term, $\left(\frac{1}{\theta}\right) \tilde{z}(1-\overline{\tau}) Z_t$ corresponds to $\int_{z_{\min}}^{\infty} \pi_t^V(z)(1-\overline{\tau}) dG(z)$. In other words, it is the effect of managerial inefficiency in the absence of outside monitoring. Without this monitoring, managerial inefficiency is clearly pro-cyclical and increases with the aggregate economy through Z_t. On the other hand, the second term, $\psi Z_t^{\left(\frac{k}{\theta-1}\right)}$ corresponds to the offsetting effects of outside ownership, both through concentrated managerial stakes from z_t^o to $\overline{z}_t$ and from full listing for firms with productivity above $\overline{z}_t$. It is straightforward to verify that $\psi > 0$ for $k > (\theta - 1)$, our maintained assumption. Therefore, outside owners always mitigate the procyclical impact of managerial inefficiency.

B.3 Model Solution: Reporting Costs

We now solve for reporting costs in equation (24) repeated here for convenience:

$$\tilde{f}_t \equiv \int_{z_{\min}}^{\infty} f\left(x_{t+1}\left(z\right)\right) dG\left(z\right) = \int_{z_t^o}^{\infty} f\left(x_{t+1}\left(z\right)\right) dG\left(z\right).$$

Using the assumed form of $f\left(x_{t+1}\left(z\right)\right) = \overline{f} x_{t+1}\left(z\right)$, we can rewrite the aggregate reporting costs as:

$$\tilde{f}_t \equiv \overline{f} \int_{z_t^o}^{\infty} x_{t+1}\left(z\right) dG\left(z\right)$$

To write this relationship in terms of the aggregate state, we use the interior solution of equity sold in equation (34) and adjust for the range where $x_{t+1}\left(z\right) = 1$ for $z > \overline{z}_t$. Thus, we rewrite the integrals as:

$$\begin{aligned} \tilde{f}_t \quad &= \quad \frac{\overline{f}}{2\overline{\tau}} \int_{z_t^o}^{\infty} dG\left(z\right) - \frac{\overline{f}^2}{2\overline{\tau}} \int_{z_t^o}^{\infty} \left(\pi_t^V\right)^{-1} dG\left(z\right) \\ &\quad - \left[\frac{\overline{f}}{2\overline{\tau}} \int_{\overline{z}_t}^{\infty} dG\left(z\right) - \frac{\overline{f}^2}{2\overline{\tau}} \int_{\overline{z}_t}^{\infty} \left(\pi_t^V\right)^{-1} dG\left(z\right)\right] \\ &\quad + \overline{f} \int_{\overline{z}_t}^{\infty} dG\left(z\right) \end{aligned}$$

Using the moments of natural profits in equation (60) and rearranging, aggregate reporting costs can be rewritten as:

$$\tilde{f}_t = \overline{f} g_0 q Z_t^{k/(\theta-1)} \tag{A.11}$$

where

$$q = \left(1 - \nu_{-1}^{-(\theta-1)}\right) \frac{1}{2\overline{\tau}} \left[\left(1 - 2\tau\right)^{-\left(\frac{k}{\theta-1}\right)-1} - 1\right] \left(1 - 2\tau\right)^{\left(\frac{k}{\theta-1}\right)+1}$$

It is straightforward to verify that $q > 0$ and thus aggregate reporting costs are pro-cyclical. The intuition is clear. An increase in productivity increases the number of firms seeking outside ownership, measured by this probability through g_0. In turn, intensity of equity sales is captured in the difference in shares from firms in the intermediate listing range $z^o < z < \overline{z}$ and those fully listed above $\overline{z}$. These offsetting effects are captured by q.

B.4 Model Solution: Euler Equations

Households enter every period holding the existing shares of equity the previous period. During the period, managers issue new shares. Some of these managers may have been wholly owned the previous period. Thus, households consider in their budget constraint the value of two different aggregate sets of shares. We have defined these aggregate market values in the text as in equations (39) and (40) restated here for convenience

$$\begin{aligned} V_{t,t-1} &\equiv \int_{z^o_{t-1}}^{\infty} v_t(z)\, x_t(z)\, dG(z), \\ V_{t,t} &\equiv \int_{z^o_t}^{\infty} v_t(z)\, x_{t+1}(z)\, dG(z). \end{aligned}$$

Note that $V_{t,t}$ differs from $V_{t,t-1}$ for two reasons. First, in period t there is a new set of firms with outside equity. For example, if the productivity shock is positive, then $z^o_t < z^o_{t-1}$ so that some new firms begin issue equity on the margin. Second, for the firms that had outside equity at $t-1$, the number of shares issued will differ. Again, if the productivity shock is positive, then for existing firms, $x_{t+1}(z) > x_t(z)$ unless firms were previously fully public as above $\overline{z}$ in which case the number of shares do not respond.

When the asset market opens at time t, households will absorb any additional stock issues in their portfolio. Therefore, they will price both $V_{t,t-1}$ and $V_{t,t}$. These are priced according to the Euler equations given in (42) and (43) given here as:

$$\begin{aligned} V_{t,t-1} &= \beta E_t \left\{ \frac{U'(C_{t+1})}{U'(C_t)} \left(\Pi_{t+1,t-1} + V_{t+1,t-1} \right) \right\} \\ V_{t,t} &= \beta E_t \left\{ \frac{U'(C_{t+1})}{U'(C_t)} \left(\Pi_{t+1,t} + V_{t+1,t} \right) \right\} \end{aligned}$$

where $\Pi_{t+1,t}$ are the profits paid to shareholders at time $t+1$ holding the shares issued at time t and $\Pi_{t+1,t-1}$are the profits paid to shareholders at time $t+1$ holding the shares issued at time $t-1$. In other words, $\Pi_{t+1,t}$ is the same as $\Pi_{t,t-1}$ from equation (38) led one period or:

$$\Pi_{t+1,t} \equiv \int_{z^o_t}^{\infty} \pi_{t+1}\left(x_{t+2}(z); z\right) x_{t+1}(z)\, dG(z)$$

while $\Pi_{t+1,t-1}$ is the same integral but based upon shares at $t-1$:

$$\Pi_{t+1,t-1} \equiv \int_{z^o_{t-1}}^{\infty} \pi_{t+1}\left(x_{t+2}(z); z\right) x_t(z)\, dG(z)$$

Iterating the Euler equations forward imply that:

$$
\begin{aligned}
V_{t,t-1} &= E_t \left\{ \sum_{j=1}^{\infty} \beta^j \frac{U'(C_{t+j})}{U'(C_t)} \Pi_{t+1,t-1} \right\} \qquad \text{(A.12)} \\
V_{t,t} &= E_t \left\{ \sum_{j=1}^{\infty} \beta^j \frac{U'(C_{t+j})}{U'(C_t)} \Pi_{t+1,t} \right\}
\end{aligned}
$$

C Steady State

We can now solve for the steady-state of the economy using the sequence of equations described in the Model Set-up section. We drop time subscripts to all steady state variables and impose the steady state production, $Y = \tilde{z}$. Then, aggregate profits distributed to existing shareholders from equation (A.8) using the solution in terms of aggregate Z_t becomes:

$$
\begin{aligned}
\Pi &= g_0 Z^{k/(\theta-1)} \{ \nu_1^{(\theta-1)} \overline{f} (1-2\overline{\tau})^{\left(\frac{k}{\theta-1}-1\right)} + \frac{1}{2\overline{\tau}} \left[S_0 - \overline{S}_0 (1-2\overline{\tau})^{\left(\frac{k}{\theta-1}\right)} \right] \\
&\quad - \frac{\overline{f}}{2\overline{\tau}} \left[S_0^* - \overline{S}_0^* (1-2\overline{\tau})^{\left(\frac{k}{\theta-1}\right)} \right] \}
\end{aligned}
$$

where S_0, $\overline{S}_0$, S_0^*, and $\overline{S}_0^*$ are the steady state counterparts to $S_{t,t-1}$, $\overline{S}_{t,t-1}$, $S_{t,t-1}^*$, and $\overline{S}_{t,t-1}^*$ where $Z_t = Z_{t-1} = Z = 1$. Or more succinctly,

$$
\Pi = g_0 Z^{k/(\theta-1)} X_0 \qquad \text{(A.13)}
$$

Similarly, by setting the aggregate shocks to one in equation (A.10), we get that in the steady state managerial inefficiency is:

$$
\tilde{\tau} = \left(\frac{1}{\theta} \right) \tilde{z} (1 - \overline{\tau}) - \omega \qquad \text{(A.14)}
$$

Steady state aggregate reporting costs as determined from equation (A.11) as:

$$
\tilde{f} = \overline{f} g_0 q \qquad \text{(A.15)}
$$

Next, using the household budget constraint in equilibrium, equation (41)

$$
\Pi + \frac{\theta - 1}{\theta} \tilde{z} = C \qquad \text{(A.16)}
$$

where we have used the fact that in steady state there is no net equity issuance so that :$(V_{t,t} - V_{t,t-1}) = V_{t,\Delta} = V_{0,\Delta} = 0$.

Finally, the aggregate steady-state resource constraint implies $\tilde{z} = \tilde{C} + C + \tilde{\tau} + \tilde{f}$ which determines managerial consumption:

$$\tilde{C} = \int_{z_{\min}}^{z^o} \pi\left(0;z\right) dG\left(z\right) + \int_{z^o}^{\infty} C(z)dG(z). \tag{A.17}$$

In addition, the steady state market values are given by setting $Z_t = Z_{t-1} = Z = 1$ in equations (60) implying that:

$$V_{0,-1} = V_{0,0} = \frac{\beta}{1-\beta}\Pi_0 \tag{A.18}$$

Thus, in the absence of growth, the steady state value of the equity market is the β-discounted present value of aggregate profits distributed to shareholders.

D Model Dynamics: Solution Details

Given the solution to the model above and the steady state, we can now consider the dynamic responses of all the state variables by first-order log-linearization. We again follow the same sequence as above.

First, we consider the response of profits paid to shareholders. Using the solution to (A.8) and the definitions of $S_{t,t-1}$, $\overline{S}_{t,t-1}$, $S^*_{t,t-1}$, and $\overline{S}^*_{t,t-1}$, the log-linear approximation of these profits becomes:

$$\widehat{\Pi}_{t,t-1} = h_0^r \widehat{Z}_{t-1} + h_1^r \widehat{Z}_t \tag{A.19}$$

where h_0^r, h_1^r for $r = u, d$ are coefficients that depend upon whether output expands ("up") or contracts ("down"). When $\widehat{Z} > 0$, the response includes the effect of the expansion on the new firms that are selling equity. In this case, $h_i^r = h_i^u$ for $i = 0, 1$ where:

$$\begin{aligned} h_0^u &= \frac{g_0 Z^{\left(\frac{k}{\theta-1}\right)}}{\Pi_{0,0}} \left\{ \left(\frac{k}{\theta-1}\right) X_{0,-1}^{n,u} + U_0 - U_{-1} \right\} \\ h_1^u &= \frac{g_0 Z^{\left(\frac{k}{\theta-1}\right)}}{\Pi_{0,0}} \left\{ \left(\frac{k}{\theta-1}\right) X_{0,0}^{n,u} - U_0 + U_{-1} \right\} \end{aligned}$$

for

$$U_0 = \frac{\overline{f}}{2\overline{\tau}}(1-2\overline{\tau})^{\left(\frac{k}{\theta-1}\right)} \left\{ \begin{array}{c} 1 + \left(\overline{\tau} + \frac{1}{4\overline{\tau}}\right) \nu_1^{(\theta-1)} (1-2\overline{\tau})^{-1} \\ + \left(\frac{1}{2}\right) \nu_{-1}^{-(\theta-1)} (1-2\overline{\tau}) \end{array} \right\}$$

$$\begin{aligned} U_{-1} &= \frac{\overline{f}}{2\overline{\tau}}(1-2\overline{\tau})^{\left(\frac{k}{\theta-1}\right)} \left[(1-2\tau)^{-1} \nu_1^{(\theta-1)} - 1 \right] \\ &+ \frac{\overline{f}}{2\overline{\tau}} \left\{ \begin{array}{c} \left(\overline{\tau} + \frac{1}{4\overline{\tau}}\right) \nu_1^{(\theta-1)} - \left(\frac{1}{4\overline{\tau}}\right) \nu_{-1}^{-(\theta-1)} \\ - \left(\overline{\tau} + \frac{1}{4\overline{\tau}}\right) + \left(\frac{1}{4\overline{\tau}}\right) \nu_{-2}^{-2(\theta-1)} \end{array} \right\} \end{aligned}$$

$$
\begin{aligned}
X_{0,0}^{n,u} &= (1-2\overline{\tau})^{\left(\frac{k}{\theta-1}\right)}\frac{1}{2\overline{\tau}} \\
&\left\{\left[\nu_1^{(\theta-1)}(1-2\overline{\tau})^{-1}-1\right]\overline{f}+\overline{S}_{0,0}-\overline{fS}_{0,0}^{*}\right\}
\end{aligned}
$$

$$
\begin{aligned}
X_{0,-1}^{n,u} &= (1-2\overline{\tau})^{\left(\frac{k}{\theta-1}\right)}\frac{1}{2\overline{\tau}} \\
&\left\{\left[\nu_1^{(\theta-1)}(1-2\overline{\tau})^{-1}-1\right]\overline{f}+S_{0,0}-\overline{f}S_{0,0}^{*}\right\}
\end{aligned}
$$

$S_{0,0}^{*}$ is the steady state level of $S_{t,t}^{*}$ and $X_{0,0}^{n,d}$ is given by:

$$
X_{0,0}^{n,d} = (1-2\overline{\tau})^{\left(\frac{k}{\theta-1}\right)}\left\{(1-\left(\frac{1}{2\overline{\tau}}\right))\overline{S}_{0,0}+\left(\frac{\overline{f}}{2\overline{\tau}}\right)\overline{S}_{0,0}^{*}\right\}+\overline{\tau}\nu_1^{(\theta-1)}\overline{f}
$$

Similarly, when $\widehat{Z}<0$, the response includes the effect of the contraction on firms as they delist and some become wholly owned. In this case, $h_i^r = h_i^d$ for $i=0,1$ where:

$$
\begin{aligned}
h_0^d &= \frac{g_0 Z^{\left(\frac{k}{\theta-1}\right)}}{\Pi_{0,0}}\left\{\left(\frac{k}{\theta-1}\right)X_{0,-1}^{n,d}+\left(\frac{\overline{f}}{2\overline{\tau}}\right)S_{0,0}^{*}-U_{-1}\right\} \\
h_1^d &= \frac{g_0 Z^{\left(\frac{k}{\theta-1}\right)}}{\Pi_{0,0}}\left\{\left(\frac{k}{\theta-1}\right)X_{0,0}^{n,d}-\left(\frac{\overline{f}}{2\overline{\tau}}\right)S_{0,0}^{*}+U_{-1}\right\}
\end{aligned}
$$

for

$$
\begin{aligned}
U_{-1} &= \frac{\overline{f}}{2\overline{\tau}}(1-2\overline{\tau})^{\left(\frac{k}{\theta-1}\right)}\left[(1-2\tau)^{-1}\nu_1^{(\theta-1)}-1\right] \\
&+\frac{\overline{f}}{2\overline{\tau}}\left\{\begin{array}{c}\left(\overline{\tau}+\frac{1}{4\overline{\tau}}\right)\nu_1^{(\theta-1)}-\left(\frac{1}{4\overline{\tau}}\right)\nu_{-1}^{-(\theta-1)} \\ -\left(\overline{\tau}+\frac{1}{4\overline{\tau}}\right)+\left(\frac{1}{4\overline{\tau}}\right)\nu_{-2}^{-2(\theta-1)}\end{array}\right\}
\end{aligned}
$$

$S_{0,0}^{*}$ is the steady state level of $S_{t,t}^{*}$ and $X_{0,0}^{n,d}$ is given by:

$$
\begin{aligned}
X_{0,0}^{n,d} &= \left[(1-2\overline{\tau})^{\left(\frac{k}{\theta-1}\right)-1}-\overline{\tau}\right]\nu_1^{(\theta-1)}\overline{f} \\
&-\left[(1-2\overline{\tau})^{\left(\frac{k}{\theta-1}\right)}-\frac{1}{2\overline{\tau}}\right]S_{0,0}-\frac{\overline{f}}{2\overline{\tau}}S_{0,0}^{*}
\end{aligned}
$$

$$
X_{0,-1}^{n,d} = (1-2\overline{\tau})^{\left(\frac{k}{\theta-1}\right)}\left\{(1-\left(\frac{1}{2\overline{\tau}}\right))\overline{S}_{0,0}+\left(\frac{\overline{f}}{2\overline{\tau}}\right)\overline{S}_{0,0}^{*}\right\}+\overline{\tau}\nu_1^{(\theta-1)}\overline{f}
$$

The profit dynamics in equation (A.19) has two components. The first component depends only upon lagged Z_{t-1}. This effect arises because a lagged shock increases firms that sell equities.

The second component depends upon the current productivity, Z_t. This component captures the increase in profits available for distribution to current shareholders arising from the increase in current profits after managerial inefficiency and reporting costs. The effect includes the endogenous equity sales decisions of managers in the three regions of z truncated at z_t^o and $\overline{z_t}$. Clearly, equation (A.19) provides the same equation as given in the text as equation (50).

Next, straightforward log-linearization of the managerial inefficiency equation (A.10) yields:

$$\widehat{\tilde{\tau}} = \frac{1}{\tilde{\tau}} \left[\left(\frac{1}{\theta} \right) \tilde{z}(1 - \overline{\tau}) - \psi \frac{k}{\theta - 1} \right] \mathsf{Z}_t \tag{A.20}$$

Again, as we described above, managerial inefficiency has two components. Greater output increases managerial inefficiency for all firms according to natural profits by $\left(\frac{1}{\theta}\right) \tilde{z}(1 - \overline{\tau})$. On the other hand, greater output increases the proportion of aggregate profits from listed firms with this effect measured as: $\frac{k}{\theta-1}$. The degree to which firms respond depend upon the differences in aggregate natural profit moments, detailed above in ψ. Clearly, equation (A.20) establishes equation (51) in the text.

Similarly, the response of aggregate reporting costs as determined by log-linearizing equation (A.11) implying:

$$\tilde{f} = \frac{k}{\theta - 1} \mathsf{Z}_t \tag{A.21}$$

The intuition for this result is clear. An increase in output increases the number of firm shares by $\frac{k}{\theta-1}$ as described above, increasing reporting costs in this proportion. Equation (A.21) establishes equation (52) in the text. Next, log-linearizing the household budget constraint in equation (41) immediately yields equation (49) in the text. Finally, log-linearizing the aggregate steady-state resource constraint, $\tilde{z} = \tilde{C} + C + \tilde{\tau} + \tilde{f}$ directly verifies equation (53) in the text.

To determine the evolution of the household budget constraint we must solve for the dynamics of the market values of existing and new equity shares given in equations (39) and (40). For this purpose, we log-linearize the Euler equations (42) and (43) implying:

$$\begin{aligned} \widehat{V}_{t,t-1} &= E_t \left\{ -\gamma \left(\widehat{C}_{t+1} - \widehat{C}_t \right) + \widehat{\Pi}_{t+1,t-1} + \widehat{V}_{t+1,t-1} \right\} \\ \widehat{V}_{t,t} &= E_t \left\{ -\gamma \left(\widehat{C}_{t+1} - \widehat{C}_t \right) + \widehat{\Pi}_{t+1,t} + \widehat{V}_{t+1,t} \right\} \end{aligned}$$

or

$$\widehat{V}_{t,\Delta} = E_t \left\{ -\gamma \left(\widehat{C}_{t+1} - \widehat{C}_t \right) + \widehat{\Pi}_{t+1,\Delta} + \widehat{V}_{t+1,\Delta} \right\} \tag{A.22}$$

Intuitively, newly listed firms and previously listed firms will pay out according to the aggregate state in all future periods.

Figure 1 Listing Decision Given Productivity

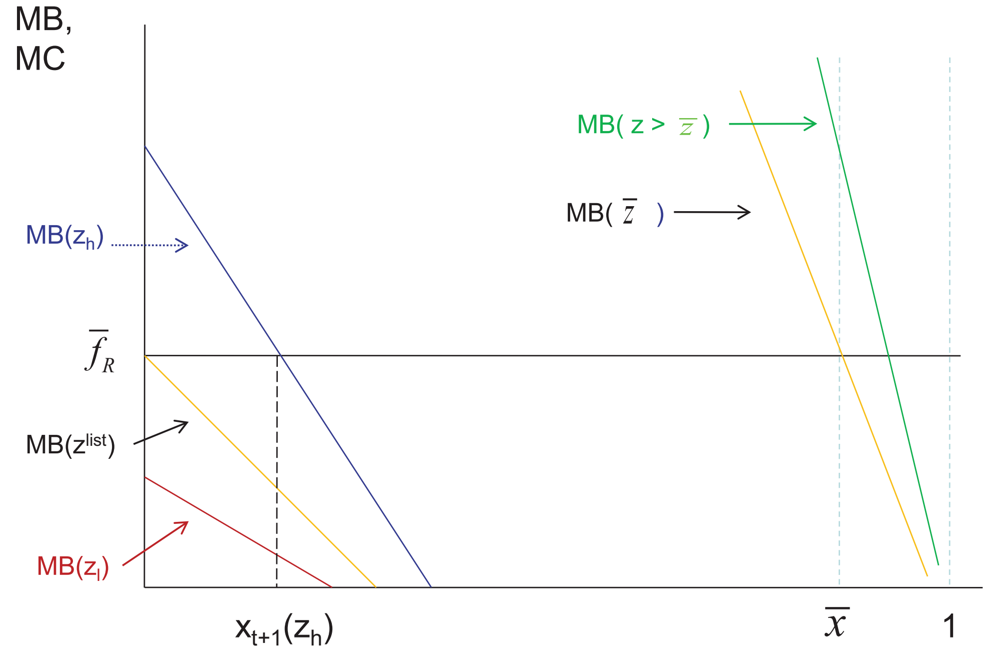
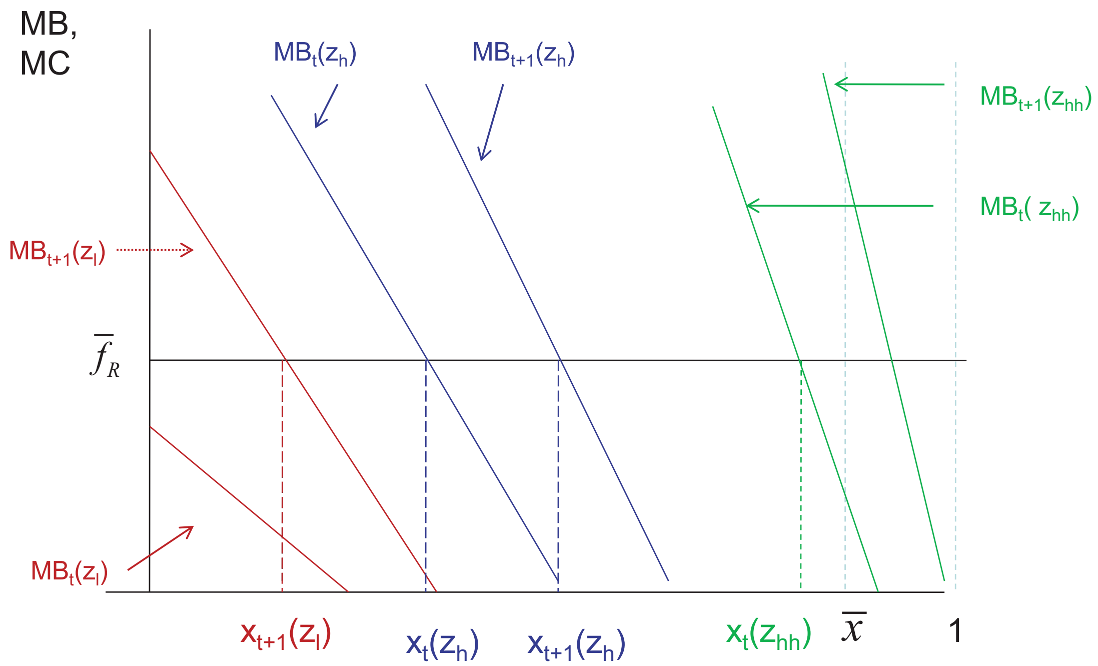


Figure 2 Effects of Productivity Shock

$Z_{t+1} > Z_t$



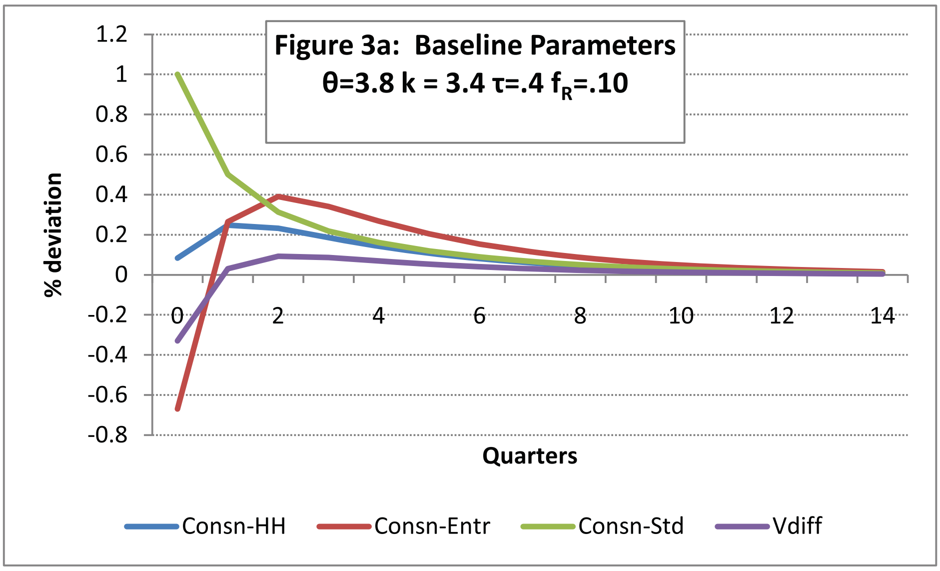


Figure 3a: Baseline Parameters
θ=3.8 k = 3.4 τ=.4 f_R=.10
% deviation
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
2
4
6
8
10
12
14
Quarters
Consn-HH
Consn-Entr
Consn-Std
Vdiff

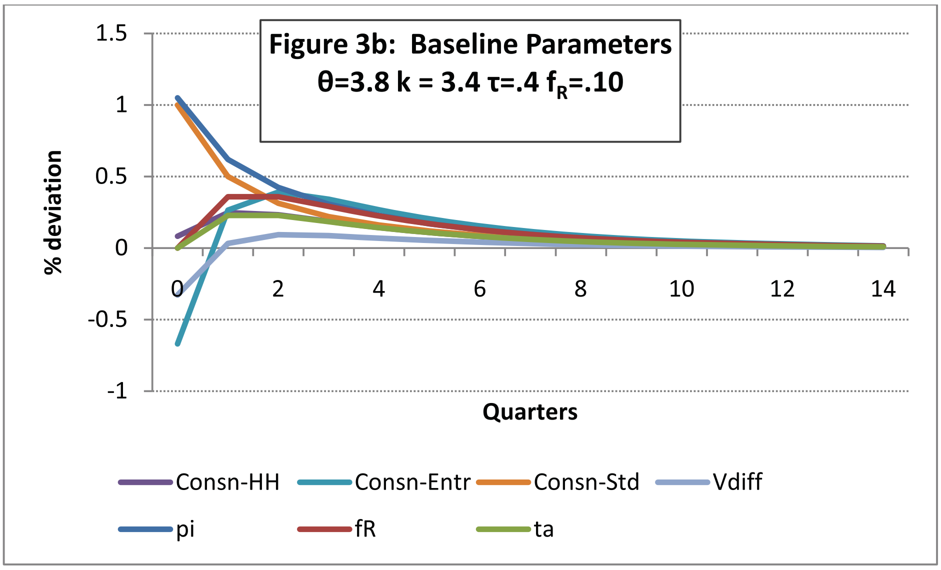


Figure 3b: Baseline Parameters
θ=3.8 k = 3.4 τ=.4 f_R=.10
% deviation
1.5
1
0.5
0
-0.5
-1
0
2
4
6
8
10
12
14
Quarters
Consn-HH
Consn-Entr
Consn-Std
Vdiff
pi
fR
ta

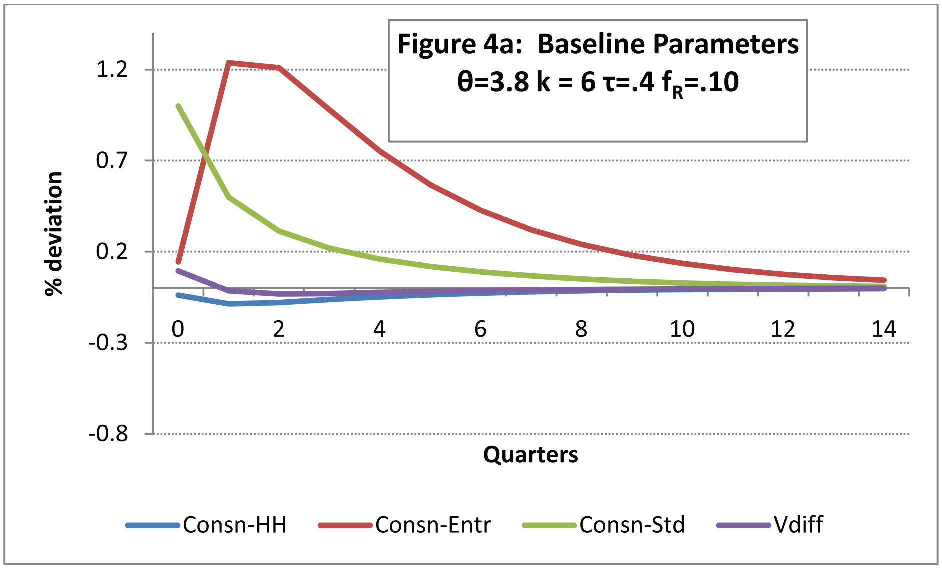


Figure 4a: Baseline Parameters
θ=3.8 k = 6 τ=.4 fR=.10
% deviation
1.2
0.7
0.2
-0.3
-0.8
0
2
4
6
8
10
12
14
Quarters
Consn-HH
Consn-Entr
Consn-Std
Vdiff

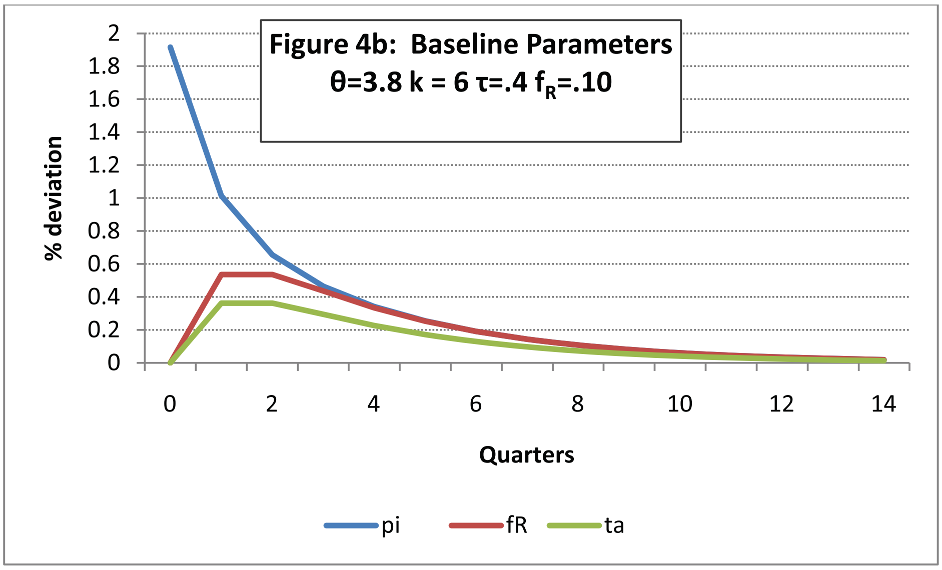


Figure 4b: Baseline Parameters
θ=3.8 k = 6 τ=.4 fR=.10
% deviation
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
Quarters
pi
fR
ta

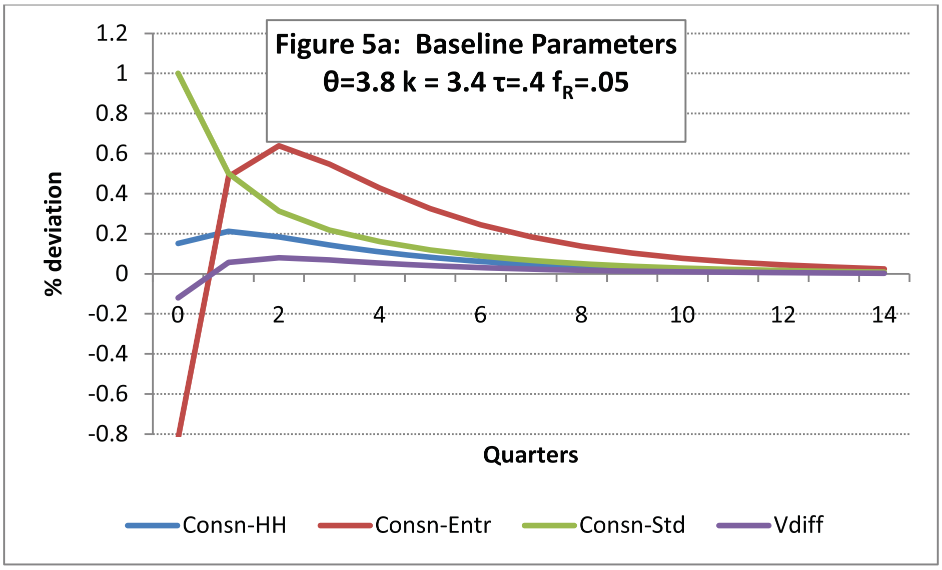


Figure 5a: Baseline Parameters
θ=3.8 k = 3.4 τ=.4 f_R=.05
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
% deviation
0
2
4
6
8
10
12
14
Quarters
Consn-HH
Consn-Entr
Consn-Std
Vdiff

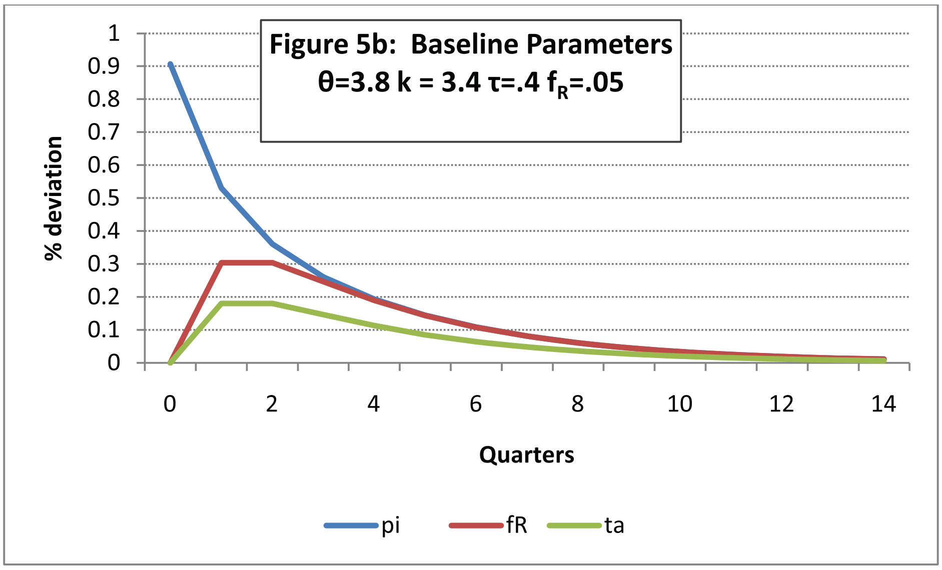


Figure 5b: Baseline Parameters
θ=3.8 k = 3.4 τ=.4 f_R=.05
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
% deviation
0
2
4
6
8
10
12
14
Quarters
pi
fR
ta

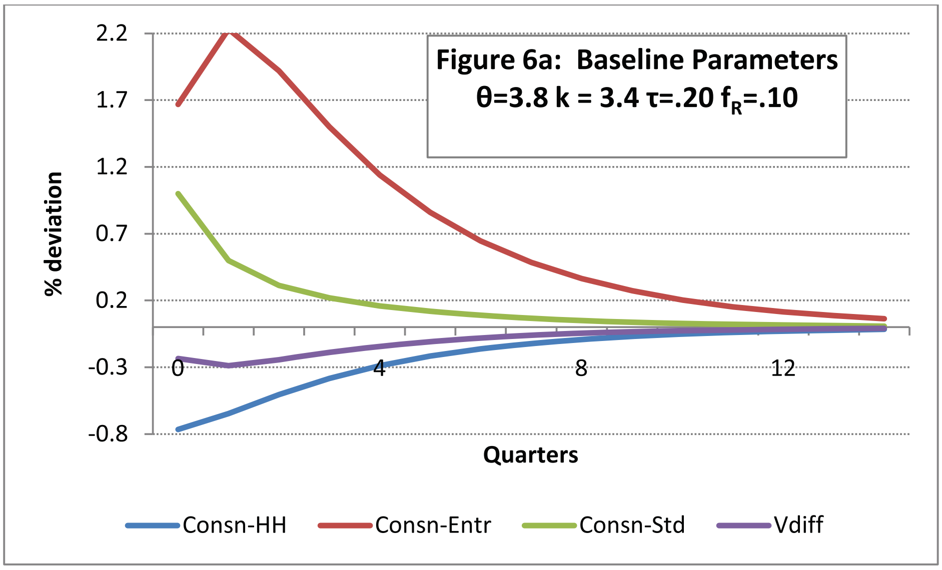


Figure 6a: Baseline Parameters
θ=3.8 k = 3.4 τ=.20 f_R=.10
% deviation
2.2
1.7
1.2
0.7
0.2
-0.3
-0.8
0
4
8
12
Quarters
Consn-HH
Consn-Entr
Consn-Std
Vdiff

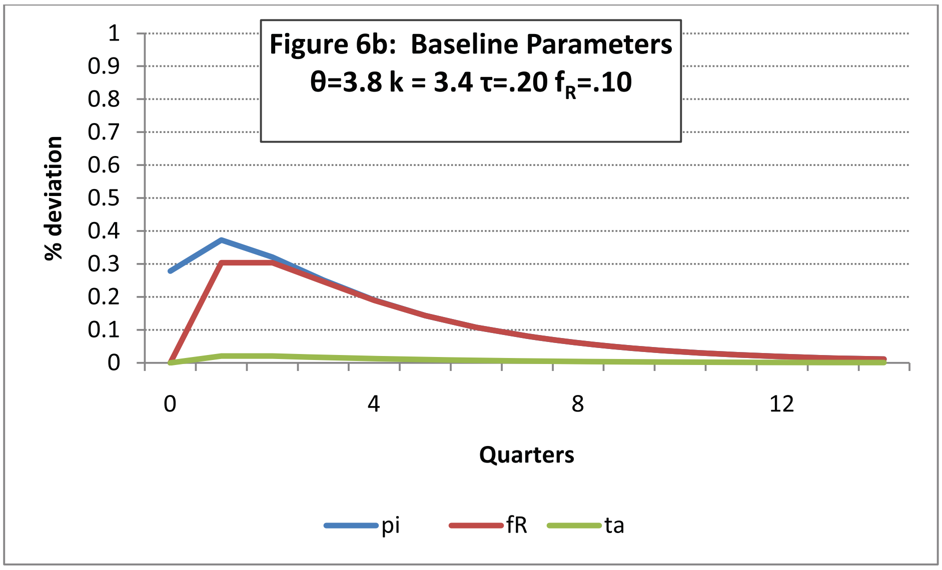


Figure 6b: Baseline Parameters
θ=3.8 k = 3.4 τ=.20 f_R=.10
% deviation
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
4
8
12
Quarters
pi
fR
ta