IMES DISCUSSION PAPER SERIES

New Keynesian Dynamics in a Low Interest Rate Environment

R. Anton Braun and Lena Mareen Körber

Discussion Paper No. 2010-E-5

IMES

INSTITUTE FOR MONETARY AND ECONOMIC STUDIES

BANK OF JAPAN

2-1-1 NIHONBASHI-HONGOKUCHO
CHUO-KU, TOKYO 103-8660
JAPAN

You can download this and other papers at the IMES Web site:

http://www.imes.boj.or.jp

IMES Discussion Paper Series 2010-E-5
March 2010

New Keynesian Dynamics in a Low Interest Rate Environment

R. Anton Braun* and Lena Mareen Körber**

Abstract

Recent research has found that the dynamics of the New Keynesian model are very different when the nominal interest rate is zero. Improvements in technology shocks and reductions in the labor tax rate lower economic activity and the size of the government purchase multiplier can be as large as four. We consider the empirical relevance of these dynamics using Japanese data. Japan is interesting because it experienced a protracted period of zero nominal interest rates. A prototypical New Keynesian model calibrated to Japan and solved using nonlinear methods exhibits orthodox dynamics with a government purchase multiplier that is less than one.

Keywords: Government purchases, zero nominal interest rates, monetary policy
JEL classification: E3, E5, E6

*****University of Tokyo (E-mail: R.Anton.Braun@gmail.com)
******German Institute for Economic Research (E-mail: Lenakoerber@gmail.com)

The authors would like to thank the participants at the JEDC conference "Frontiers in Structural Macroeconomic Modeling", the Bank of Japan and GRIPS for their useful comments. We wish to particularly thank our discussant Tsutomu Watanabe for his many helpful comments. Views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan.

1 Introduction

Recent research has found that the dynamics of the New Keynesian (NK) model can be quite different when the nominal interest rate is zero. A reduction in the labor tax or an improvement in technology can lower output and the size of the government purchase multiplier can be much larger than one. To understand why the dynamics can be so different consider the case of a positive, transitory shock to technology. If the central bank keeps the nominal interest rate constant output may fall. In the presence of costly price adjustment of goods the arrival of a positive technology shock today has a depressing effect on economic activity. Firms experience temporarily high markups and profits. But, households realize that prices will be lower tomorrow and choose to defer their consumption and investment activities.

One situation where monetary policy cannot respond to technology, or any other shocks for that matter, is when the nominal interest rate is constrained by its lower bound of zero. Braun and Waki (2006) find in this situation that output falls in response to a persistent but transitory improvement in technology using a NK model calibrated to Japanese data. Eggertsson (2010) illustrates that a reduction in the labor tax has a depressing effect on hours and output. Christiano, Eichenbaum and Rebelo (2010) and Woodford (2010) find that the size of the government purchases multiplier can be much *much* larger than one.

Taken together these results might lead one to conclude that the NK model works very differently when the nominal rate is zero.

In this paper, we argue that this focus on the unorthodox properties of the NK model in a liquidity trap may be misplaced. We start by demonstrating a surprising result. When we fit a standard version of the NK model to Japanese data we find that it has completely orthodox properties during Japan's eight year episode with zero interest rates. An increase in the labor tax reduces output and an improvement in the state of technology raises output. Moreover, the government purchase output multiplier is less than one. We also produce several specifications that exhibit unorthodox properties. We then show that the specification with orthodox properties is consistent with economic conditions in Japan during the period of zero nominal interest rates but that the specifications with unorthodox properties are not.

The model that we consider is a medium scale New Keynesian model with quadratic price adjustment costs as in Rotemberg (1996), labor supply, capital accumulation and shocks to preferences, technology, taxes, monetary policy and government purchases. In this setting the duration of the period of zero interest rates is endogenous as in Braun and Waki (2006) or Erceg and Linde (2010).

Research by Braun and Waki (2010) has found that the common method of solving the model by log-linearizing all equilibrium conditions except the zero bound constraint about a steady state with stable prices can produce large approximation errors in the aggregate resource constraint. In this paper we use an extended path solution method that avoids this problem.

This solution technique can easily handle models with multiple endogenous state variables and multiple shocks. However, it is difficult to automate and thus is not easily amenable to estimation which involves solving the model for many different configurations of the parameters. For this reason we calibrate the model's parameters and some of the shocks that cannot be measured directly. The resulting specification does a reasonable job of reproducing the paths of real and nominal variables in a sample period that extends from 1990-2007. We then conduct and impulse response analysis in years before and during the period of zero interest rates. Surprisingly, the model exhibits orthodox output responses to shocks to technology and the labor tax. In addition, the maximum government purchase output multiplier is less than 0.9.

The single most important factor for why our baseline specification produces orthodox results is household expectations about the duration of the period of zero interest rates. Our baseline specification sets these expectations in a way that renders them consistent with estimates reported in Ichiue and Ueno (2007). They use yield curve data to estimate the expected duration of zero interest rates and find that the expected duration of zero interest rates between 1999 and 2006 was 2.3 years or less.

If instead our model is calibrated so that the number of periods that households expect the interest rate to be zero is 5 years or longer it also exhibits unorthodox responses.

Japan's episode with zero nominal interest rates was a period of tranquility. Japanese output and real marginal cost volatility dropped by about 1/2 and inflation volatility declined by more than 70 percent during the period of zero nominal interest rates which extended from 1999-2006 when compared to the years 1988-1998.

We use these facts as a device to assess the relative plausibility of specifications with orthodox properties and specifications with unorthodox properties. Our baseline specification with orthodox properties predicts a large decline in both real and nominal volatility. The specifications with unorthodox properties, however, all predict counterfactual increases in real and nominal volatility between 1999 and 2006. With a longer expected duration of zero interest rates, price and markup variability in response to a variety of shocks are large and this leads these models to predict that the period of zero interest rates should have been associated with an increase in economic volatility.

The unorthodox specifications have another troubling property. They imply that the resource costs of price adjustment are very large and range from 2.5 to over 7 percent of output. The resource costs of price adjustment in the baseline specification are much smaller and well less than one percent of output.

We conclude that Japanese data from 1999-2006 is most consistent with a New Keynesian model that has the following properties:

1. A lower labor tax rate increases output.

2. An improvement in neutral technology increases output.

3. The government purchases multiplier less than one.

The remainder of the paper is organized as follows. Section 2 describes the economy. Section 3 explains how the model is calibrated and solved. The results are reported in Section 4 and Section 5 contains our concluding remarks.

2 Economy

We consider a prototypical New Keynesian economy. The economy is populated by a representative household, a representative final good producer, a continuum of intermediate good producing monopolists that face quadratic costs of adjusting prices, a government and a central bank. We discuss the problems of these agents in turn.

Households

The representative household chooses sequences of consumption $\{c_t\}_{t=0}^{\infty}$ and leisure $\{1-h_t\}_{t=0}^{\infty}$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \prod_{j=0}^{t} d_j \left\{ \frac{(c_t^{\nu}(1-h_t)^{1-\nu})^{1-\sigma}}{1-\sigma} \right\} \tag{1}$$

where c_t is consumption of the composite good and h_t is hours worked expressed as a fraction of a time endowment of one. β denotes the discount factor, ν is the preference weight a household attaches to consumption and σ determines risk aversion. Finally, d_t is a shock to the subjective discount rate with the law of motion

$$\ln(d_t) = \rho_d \ln(d_{t-1}) + \epsilon_{d,t} \tag{2}$$

where ϵ_t is an I.I.D, mean zero Gaussian random variable. The household's period t budget constraint is given by

$$(1+\tau_{c,t})c_t + x_t + \frac{B_t}{P_t} = (1+R_{t-1})\frac{B_{t-1}}{P_t} +$$
$$\int_0^1 \frac{\Pi_t(i)}{P_t} di + T_t + (1-\tau_{t,K})r_t k_{t-1} + (1-\tau_{t,W})w_t h_t + \tau_{t,K}\delta k_{t-1} \tag{3}$$

where P_t is the price level, w_t is the wage rate and r_t is the real interest rate. B_t is the household's holdings of nominal debt at the end of period t, k_{t-1} is the level of capital chosen in period $t-1$ and x_t is investment. Households hold equal shares in each intermediate goods firm so that $\Pi_t(i)$ is per capita nominal profits from intermediate firm indexed i. Households pay taxes $\tau_{c,t}$, $\tau_{t,k}$ and $\tau_{t,w}$ on consumption, capital income and labour income, and receive lump-sum transfers of size T_t from the government. Ponzi schemes are ruled out by limiting attention to solutions that satisfy the standard transversality condition for bonds and capital. Capital is subject to adjustment costs and is accumulated according to

$$k_t = (1-\delta)k_{t-1} + x_t - \frac{\phi}{2}\left(\frac{x_t}{k_{t-1}} - \mu_k + 1 - \delta\right)^2 k_{t-1} \tag{4}$$

where μ_k is the growth rate of capital in the balanced growth path and δ is the depreciation rate. Let $\lambda_{c,t}$ and $\lambda_{k,t}$ be the Lagrangian multipliers on the household's budget constraint (3) and on the law of motion for capital (4), respectively. The optimal choices of the representative household satisfy

$$\frac{\nu(c_t^{\nu}(1-h_t)^{1-\nu})^{1-\sigma}}{c_t} = \lambda_{c,t}(1+\tau_{c,t}) \tag{5}$$

$$\frac{(1-\nu)(c_t^{\nu}(1-h_t)^{1-\nu})^{1-\sigma}}{1-h_t} = \lambda_{c,t}(1-\tau_{w,t})w_t \tag{6}$$

$$\lambda_{c,t} = \lambda_{k,t}\left[1 - \phi\left(\frac{x_t}{k_{t-1}} - \mu_k + 1 - \delta\right)\right] \tag{7}$$

$$\begin{aligned} 0 &= \beta E_t d_{t+1}\lambda_{c,t+1}\left[(1-\tau_{k,t+1})r_{t+1} + \tau_{k,t+1}\delta\right] - \lambda_{k,t} \\ &\quad + \beta E_t d_{t+1}\lambda_{k,t+1}\left[1 - \delta + \phi\left(\frac{x_{t+1}}{k_t} - \mu_k + 1 - \delta\right)\frac{x_{t+1}}{k_t} - \frac{\phi}{2}\left(\frac{x_{t+1}}{k_t} - \mu_k + 1 - \delta\right)^2\right] \end{aligned} \tag{8}$$

$$-\lambda_{c,t}/P_t + \beta E_t d_{t+1}\lambda_{c,t+1}(1+R_t)/P_{t+1} = 0 \tag{9}$$

Final Good Firm

Perfectly competitive final good firms use a continuum of intermediate goods $i \in 0,1$ to produce a single final good that can be used for consumption and investment. The final good is produced using the following production technology

$$y_t = \left(\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}} \tag{10}$$

The profit maximizing input demands of the final good firm are

$$y_t(i)^d = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} y_t \tag{11}$$

where $p_t(i)$ denotes the price of the good produced by firm i. The price index P_t is defined as

$$P_t = \left(\int_0^1 p_t(i)^{1-\theta} di\right)^{1/(1-\theta)} \tag{12}$$

Intermediate Goods Firms

There is a continuum of monopolistically competitive firms each producing one differentiated, intermediate good according to the technology

$$y_t(i) = k_{t-1}(i)^{\alpha}(A_t h_t(i))^{1-\alpha} \tag{13}$$

We assume that there are permanent shocks, $\psi_{A,t}$ and transitory shocks, $\epsilon_{A,t}$ to technology. Both $\psi_{A,t}$ and $\epsilon_{A,t}$ are I.I.D, mean zero Gaussian random variables. Technology evolves according to

$$A_t = Z_{A,t} e^{v_{A,t}} \tag{14}$$

$$v_{A,t} = \rho_A v_{A,t-1} + \epsilon_{A,t} \tag{15}$$

$$Z_{A,t}/Z_{A,t-1} = \mu_{A,t} \tag{16}$$

$$\ln \mu_{A,t} = \ln \mu_A + \psi_{A,t} \tag{17}$$

Each intermediate firm solves a dynamic profit maximization problem that can be broken down into two parts: The choice of the cost minimizing level of inputs and the choice of the optimal sequence of prices of output. There are two inputs: Labor and capital. Cost minimization implies

$$r_t = \alpha \chi_t k_{t-1}(i)^{\alpha-1}(A_t h_t(i))^{1-\alpha} \tag{18}$$

$$w_t = (1-\alpha)\chi_t A_t^{(1-\alpha)} k_{t-1}(i)^{\alpha} h_t(i)^{-\alpha} \tag{19}$$

where $\chi_t = \frac{r_t^{\alpha} w_t^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}A_t^{1-\alpha}}$ is real marginal cost.

Price rigidity is introduced using a convex cost of price adjustment as in Rotemberg (1996). Define gross inflation $1+\pi_t(i)$ as $p_t(i)/p_{t-1}(i)$. Given the optimal choice of labor and capital, a typical intermediate goods producer chooses a sequence of prices $p_t(i)$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \prod_{j=0}^{t} d_j \lambda_{c,t} \left[p_t(i) y_t(i) - P_t s \chi_t y_t(i) - \frac{\gamma}{2} P_t(\pi_t(i)-\pi)^2 y_t \right] / P_t \tag{20}$$

subject to the input demands (11). We assume a subsidy $s = \theta/(\theta-1)$ is in place that corrects the static inefficiency due to monopolistic competition. This subsidy isolates the dynamic distortion caused by the variation in the markup which is the distortion that monetary policy corrects in a New Keynesian model. Introducing a subsidy is also very convenient because it allows us to nest a real business cycle model as a special case by setting the adjustment costs on prices to zero.

The first order condition for the firms' price setting problem reads

$$\beta E_t \frac{d_{t+1}\lambda_{c,t+1} y_{t+1}}{\lambda_{c,t} y_t} \gamma(\pi_{t+1}-\pi)(1+\pi_{t+1}) = -\left[1-\theta+\theta s \chi_t - \gamma(\pi_t-\pi)(1+\pi_t)\right] \tag{21}$$

Monetary Policy

Interest rate targeting rules have been found to be good empirical specifications of monetary policy in e.g. Taylor (1993) and we refer to monetary policy rules of this form as Taylor rules. The particular Taylor rule considered here is

$$R_t = \max\left[(1+R)\left(\frac{1+\pi_t}{1+\pi}\right)^{\rho_\pi}\left(\frac{1+R_{t-1}}{1+R}\right)^{\rho_P} e^{u_{M,t}} - 1, 0\right] \tag{22}$$

where $u_{M,t}$ is an I.I.D, mean zero, Gaussian random variable. One special feature of this rule is that it output does not appear. We will consider a sample period during which Japan experienced a long and persistent departure from its trend growth rate. It is not clear how to define the target level of output in this type of situation.[1]

Fiscal Policy

The fiscal authority finances its expenditures by collecting distortionary taxes and lump-sum transfers and by issuing nominal bonds. Fiscal policies satisfy the period budget constraint

$$g_t + (1+R_{t-1})\frac{B_{t-1}}{P_t} + S_t = \frac{B_t}{P_t} - T_t + \tau_{w,t}w_t h_t + \tau_{c,t}c_t + \tau_{k,t}k_{t-1}(r_t - \delta) \tag{23}$$

where S_t is a subsidy to intermediate monopolists. Defining $b_t \equiv \frac{B_t}{P_t}$, we can rewrite the government budget constraint as

$$g_t + (1+R_{t-1})b_{t-1}\frac{1}{1+\pi_t} = b_t - T_t + \tau_{w,t}w_t h_t + \tau_{c,t}c_t + \tau_{k,t}k_{t-1}(r_t - \delta) \tag{24}$$

The tax rates on capital, consumption and labor and government purchases have the following laws of motion

$$\tau_{c,t} = (1-\rho_c)\tau_c + \rho_c\tau_{c,t-1} + \epsilon_{c,t} \tag{25}$$

$$\tau_{k,t} = (1-\rho_k)\tau_k + \rho_k\tau_{k,t-1} + \epsilon_{k,t} \tag{26}$$

$$\tau_{w,t} = (1-\rho_w)\tau_w + \rho_w\tau_{w,t-1} + \epsilon_{w,t} \tag{27}$$

$$log\left(\frac{g_t}{gnp_t}\right) = (1-\rho_g)log\left(\frac{g}{gnp}\right) + \rho_g log\left(\frac{g_{t-1}}{gnp_{t-1}}\right) + \epsilon_{w,t} \tag{28}$$

where the shocks to each variable are I.I.D, mean zero Gaussian random variables. Lump-sum transfers are assumed to adjust to satisfy the government budget constraint.

To close the model, the aggregate resource constraint is given by

$$g_t + c_t + x_t = y_t(1 - \frac{\gamma}{2}(\pi_t - \pi)^2) \tag{29}$$

[1] One possibility is to introduce the growth rate of output in the Taylor rule. Results for that specification are discussed briefly in Section 4.5 below.

Equilibrium

The notion of equilibrium considered here is an imperfectly competitive general equilibrium in which the markets for the final good, intermediate goods, labor, capital and government debt clear in each period. The model developed above admits a symmetric equilibrium and we limit attention to that equilibrium. We start by defining a perfect foresight equilibrium.

Definition A *perfect foresight symmetric monopolistic competitive equilibrium* consists of a sequence of allocations $\{c_t, h_t, x_t, k_t, \lambda_{c,t}, \lambda_{k,t}, y_t\}_{t=0}^{\infty}$, a set of policies $\{R_t\}_{t=0}^{\infty}$, a sequence of prices $\{r_t, w_t, \chi_t, \pi_t\}_{t=0}^{\infty}$ and a finite set of integers I_B that satisfies the

- Households' optimality conditions
- Firms' optimality conditions
- Monetary policy rule:
  - $\forall$ t $\notin I_B$ the zero constraint on interest rates is not binding and the Central Bank follows the Taylor rule
  - $\forall$ t $\in I_B$ the zero constraint on interest rates is binding and the Central Bank sets $R_t = 0$
- Aggregate resource constraint and market clearing

given initial conditions (P_{-1}, R_{-1}, k_0), and sequences of shocks to the rules for $\{A_t, d_t, \tau_{k,t}, \tau_{c,t}, \tau_{w,t}, g_t\}_{t=0}^{\infty}$.[2]

Two points are worth mentioning. First, the definition of equilibrium is sequential. Second, the definition of equilibrium includes a statement of specific intervals where the zero lower bond on the nominal rate is binding.

3 Solution Method and Calibration

3.1 Solution Method

Our choice of solution method is motivated by four considerations. First, we choose a nonlinear solution method because recent research by Braun and Waki (2010) has found that the common practice of log-linearizing all of the equilibrium conditions except for the Taylor rule around a steady state with a stable price level can produce large approximation errors. For large or persistent shocks, these approximation errors can result in sign reversals

[2]Because we assume that the government adjusts lump-sum transfers such that its budget constraint is satisfied, we omit the government budget constraint from the equilibrium conditions and we omit government bonds and transfers from the list of variables determined in equilibrium.

of e.g. the response of hours to a change in the labor tax rate, upward biases in the size of the government purchases multiplier and implausibly large implied costs of price adjustment.[3]

The second motivation for our choice relates to finding the interval when the nominal interest rate is zero. Braun and Waki (2006) consider the problem of computing an equilibrium for an economy similar to ours in a perfect foresight setting. They limit attention to equilibria of the form where the interest rate is zero for only one finite and contiguous number of periods. Even with this restriction they find that there can be multiple equilibria and they impose two further equilibrium selection devices. First, they impose the restriction that the nominal interest rate in the model hits zero in a specific year that is dictated by Japanese data. Second, they select the equilibrium where the nominal interest rate is zero for the shortest interval of time. We use the same strategy for selecting an equilibrium here.

Third, we want to relax the perfect foresight assumption maintained in e.g. Braun and Waki (2006, 2010) and allow for new shocks/news to arrive each period.

Fourth, we want to analyze an empirically relevant model of the Japanese economy. To do that one needs at a minimum to model capital formation and multiple shocks. Global solution methods used in e.g. Wollman (2005), Adam and Billi (2006) and Nakov (2008) have the distinct advantage that agents forecasts are probability distributions over future outcomes and not degenerate. Unfortunately, these methods are subject to a curse of dimensionality that limits their usefulness in empirical settings where models have multiple shocks and endogenous state variables.

These four considerations led us to use an extended path solution.[4] Starting from the initial period, agents solve the set of nonlinear equations that describe their respective decision rules forward for 100 periods. We assume that our economy is at its steady state in period 101.[5] In these future periods, shocks are set to 0. We then move time forward by one period. Agents experience a new set of shocks and have a new set of initial conditions. They once again solve forward for 100 periods. This is repeated for each year from 1988 to 2007.

Because our solution method is sequential, we can limit the problem of dealing with the zero bound constraint to a small set of periods. Prior to 1999, households assign zero probability to the constraint binding in equilibrium. In the periods where households anticipate or experience a binding constraint we solve the model by hand using guess and verify methods to find the interval where the nominal interest rate is zero.

[3] We will provide an illustration of this final point below.

[4] See Heer and Maussner (2008) for a description of the algorithm. They refer to it as an extended shooting algorithm.

[5] We have also experimented with longer transitions and found that our results are qualitatively very similar.

3.2 Calibration of Parameters

Recently, Bayesian MLE estimation has become popular for parameterizing models like ours. Bayesian MLE estimation is very convenient if one can solve the model using a loglinearized solution technique. However, as we have already noted above this solution technique can break down when considering periods where the nominal interest rate is zero.[6] One could in principal estimate the model parameters using an earlier sample period when the nominal interest rate is positive. However, previous research by e.g. Chen, Imrohoroglu and Imrohoruglu (2006) and Braun, Ikeda and Joines (2009) show that Japan was undergoing large transitional adjustments between 1960 and 1990. This was the period of Japan's growth miracle and it is difficult to derive a stationary representation in the presence of large one off transitional dynamics induced by e.g. a low capital stock. For these reasons we chose to calibrate the parameters of our economy by matching model variables to calibration targets in Japanese data between 1981 and 2007.

Table 1 reports the model parameterization. Most of the parameters are computed using averages from Japanese data over the sample period 1981-2007. The data used for calibrating the model are updated versions of the data employed by Hayashi and Prescott (2002).[7] The capital share parameter α is calibrated to match capital's share of income. The depreciation rate δ reproduces average depreciation in Japanese data. The steady state nominal rate is the average of the Japanese overnight call-rate. The coefficients for the laws of motion of the taxes on, consumption, labor and capital are estimated using Japanese data on average tax rates for these three variables. The parameters for the law of motion of government's share of output are estimated in the same manner.

The preference discount factor β is set to 0.995, a rather high level for a model in which the length of a period is one year. This choice implies that the inflation rate associate with a steady state nominal interest rate of 2.9% is zero. Conditional on the rest of the parameterization, a lower value of β would imply that the steady state inflation rate associated with a nominal interest rate of 2.9% is negative. We set the curvature parameter in preferences to 2. The weight on leisure in the utility function, ν, is calibrated to match the average labor input between 2000 and 2007. We choose this period because prior to 2000, labor input exhibits a significant downward trend. The resulting value of ν is 0.27.

Other parameters are set in a more informal way. The parameter controlling the size of adjustment costs on investment is set to 4 which is a bit larger than the value of 2 used by Braun and Waki (2006).

The average markup is set to 15% as in Braun and Waki (2006). It then follows that the

[6]One promising alternative strategy is pursued in Adjemian and Juillard (2009). They estimate a NK model using an extended shooting solution technique and a simulated GMM estimation strategy.

[7]We wish to thank Nao Sudou of the Bank of Japan for providing us with an updated version of the Hayashi and Prescott (2002) dataset.

value of the subsidy is 1.15. We assume that technology, A_t, advances at an average rate of 2% per annum.

The coefficient on inflation, ρ_π, and the lagged nominal rate in the Taylor rule, ρ_R, are set to 1.7 and 0.4 respectively. The adjustment cost parameter γ is set to 80.[8] These parameter choices imply that the nominal rate increases on impact by 0.4% in response to a 1% shock to monetary policy. This response is a bit lower than the response of 0.6% estimated by Sugo and Ueda (2006) for the Japanese economy.

Finally, we start simulating our economy from 1987 and set the initial capital stock in our economy to the same value as its counterpart in Japanese data in 1987.

3.3 Calibration of Shocks

We use Japanese data to derive sequences of innovations to technology, government purchases, and capital and labour taxes. However, we calibrate the innovations to the preference discount factor, consumption tax and monetary policy to reproduce particular targets. We next describe how this was done.

We started out by simulating our economy using the parameterization described above setting the shocks on the consumption tax, the preference discount rate and monetary policy to zero in all periods. That specification preformed reasonably well in terms of its implications for most real variables. However, the model did not produce a large secular decline in labor input after 1987. Between 1987 and 1991, there were some important institutional changes in labor market arrangements in Japan. The number of national holidays were increased and the length of the work week was reduced. However, labor input continues to decline throughout the 1990s. Miyazawa (2010) shows that the secular decline in labor input during the 1990s can partially be attributed to a change in the composition of jobs from full-time to part-time work. We do not explicitly model these factors here and instead treat them as altering the labor wedge as in e.g. Kobayashi and Inaba (2006).[9]

Simulation results for this parameterization are reported in Figure 1. Inspection of Figure 1 indicates that the model does a reasonable job of reproducing some of the basic secular movements in the real side of the Japanese economy. It captures the capital deepening that occurred between 1990 and 2007. The model also captures the decline in output and consumption relative to their trends during the 1990s. However, it does not reproduce the secular decline in labor input after 1991. In addition, the decline in the nominal interest rate

[8]When log-linearized, introducing nominal rigidities via Rotemberg price adjustment costs produces a New Keynesian Phillips curve identical to the one obtained from a Calvo model of nominal rigidities. The Calvo parameter of price stickiness associated with our parameterization is 0.75.

[9]We accomplished this by altering $\tau_{c,t}$ in the years 1987 to 1991 to reproduce movements in Japanese labor input during this sub-sample. $\tau_{c,t}$ also affects the intertemporal first order condition. However, in our experience this effect is quantitatively very small.

and inflation rate is counterfactually small during the 1990s. The model, most importantly, does not predict a period of zero nominal interest rates.

Shocks to the preference discount rate play an important role in getting the interest rate to fall to zero in the work of Eggertsson and Woodford (2003), Taehun, Teranishi and Watanabe (2005) and Christiano Eichenbaum and Rebelo (2010). We follow their approach and introduce shocks to the preference discount factor as follows: for 1993 to 1995, 2%, 1%, and 1%, respectively and for 1999, 2%. This final shock makes the zero lower bound bind in 1999. The value of the log discount rate in 1999 implied by the above shocks is 0.044.

Introducing shocks to d_t gets the nominal interest rate to hit its lower bound of zero but these shocks also result in a deterioration in the fit for output and labour input. To counteract the stimulative effect that shocks to the preference discount rate have on these variables, we introduced simultaneous variations in the labor wedge by shocking $\tau_{c,t}$. With some experimentation we found that using a fixed factor of 5 works well.

Preference discount rate shocks produce counterfactually low inflation in the second half of the 1990s too. To counteract the deflationary pressure due to these shocks, we introduced negative monetary policy shocks in the late 1990s. In our economy, a negative shock to monetary policy lowers the nominal interest rate and increases the inflation rate. In other research Sugo and Ueda (2006) have found that negative monetary policy shocks are important for understanding the Japanese economy during this period. The shocks to monetary policy are -0.5% in the years 1993, 1996 and 1998 and -1% in 1997.

These shocks bring the nominal interest rate and the inflation rate down in the 1990s and in particular get the nominal interest rate to hit its lower bound of zero in 1999. However, once the nominal interest rate is zero we were left with a question of how to choose these shocks during the period of zero nominal interest rates. In our baseline specification, we assume that in each period between 1999 to 2005 households expect that the nominal interest rate will be zero for two years. This assumption is based on evidence reported in Ichiue and Ueno (2007). They find using an affine model of the yield curve that the maximum expected duration of zero nominal interest rates during this period was 2.3 years. In 2006, the nominal rate is zero in the current period but agents expect positive nominal interest rates for 2007 and beyond. Then in 2007 the nominal interest rate turns positive.

Results for the baseline simulation are reported in Figure 2. A comparison of Figure 2 with Figure 1 reveals that the shocks we have added after 1991 achieve the desired goal of bringing inflation and the nominal rate down during the second half of the 1990s. Moreover, the level of inflation during the period of zero nominal interest rates is about of the same level as we observe in Japanese data. Relative to Figure 1, there is some deterioration in the fit of the model for real allocations. The baseline economy understates consumption and overstates the extent of capital deepening. The reason for these changes in the fit of the model for real variables is the preference shock. On the one hand, a d_t shock brings the nominal rate down but it also stimulates current labor input and output. We compensate for these effects using a shock to $\tau_{c,t}$. This improves the fit for these variables but also induces households to consume less and save more.

Overall, the baseline model captures some of the principal features of Japan's experience between 1990 and 2007. It provides us with a quantitatively relevant NK framework for analyzing the effects of changes in technology, the labor tax rate and government purchases on output both before and during Japan's episode with zero interest rates.

4 Results

4.1 Dynamic Responses of the Baseline Specification

This section contains one of the principal results in this paper. Previous research has found that the dynamic properties of the NK model are very different when the nominal interest rate is zero. We now show that a quantitatively relevant specification calibrated to the Japanese economy exhibits orthodox responses to labor tax and technology shocks both when the nominal rate is positive and also when it is zero. Moreover, the government purchase output multiplier is always less than one.

Table 2 reports impulse responses for the baseline specification. The first row shows the year in which the shocks are perturbed. The second row reports the number of years that agents expect the nominal interest rate to be zero. The third row reports the resource costs of price adjustment as a percent of output. The remaining rows report impact responses of output and the markup to various shocks. Results are reported for permanent and transitory shocks to technology, shocks to the labor tax rate and shocks to government purchases. In all instances, the sign of the shock is positive.

The upper panel of Table 2 reports the percentage change in output to a 1% impulse in the variable that is shocked for the first three shocks. For the shock to government purchases though the results are expressed as government purchase multipliers which are defined as the ratio of the change in output to the change in government purchases. The first column of results is for shocks that arrive in 1995, which is representative of years in which the current nominal interest rate is positive and expected nominal interest rates are positive in all future years. We also report impulse responses for shocks that arrive in 1999 and 2004. These are both years in which the nominal interest rate is zero. In 1999 agents continue to expect the nominal interest to be zero for two periods after each of the shocks arrives. In 2004, in contrast, three of the shocks reduce the expected number of periods of zero nominal interest rates by one period.

Inspection of Table 2 reveals a surprising fact. Output exhibits orthodox responses to technology shocks and labor tax shocks both in 1995, a year when the nominal rate is positive but also in 1999 and 2004, years when the nominal rate is zero. Output increases when technology improves and output falls when the labor tax is increased. Moreover, the government purchase output multiplier is less than one in all periods. In 1995 it is 0.65. It increases to 0.87 in 1999 but never rises above 0.9 in any period. Although not reported in Table 2 we wish to point out that private consumption falls when government purchases are increased in 1995, 1999 or 2004.

The most significant difference between the results for 1995 and the results for 1999 and 2004 relates to the markup response. The markup response to each of the shocks is about three times larger in the years where the nominal interest rate is zero.

What is the economic mechanism responsible for the approximately threefold increase in markup volatility? It is known from previous work by e.g. Khan, King and Wolman (2002) that optimal government policy in a model with imperfectly competitive intermediate goods markets is to smooth the dynamic response of the markup to shocks. Schmitt-Grohe and Uribe (2007) find that a monetary policy that stabilizes the price level is an effective way to achieve this objective. In the New Keynesian model prices have a close link to the value of the markup via the New Keynesian Philips curve and stabilizing prices acts to limit the size of the response of the markup to shocks to government purchases and other exogenous variables. In practice, a simple Taylor (1993) rule with a large inflation elasticity also works very well. Once the nominal interest rate is zero though, the Taylor rule is no longer operative and monetary policy ceases to stabilize the response of the markup to shocks. This is the mechanism triggering the larger markup responses in Table 2. However, what is noteworthy about our results is that the level of markup variability prior to 1999 is very small. Thus, increasing its variability by a factor of three only has small quantitative effects on the dynamic response of the economy to shocks.

On the one hand, the results reported in Table 2 for the period of zero interest rates are reassuring. They imply that we don't have to change the way we think about the world when the nominal interest rate is constrained by its lower bound of zero. On the other hand, our results are surprising in light of the previous literature. The value of the government purchase output multiplier reported in Table 2 is less than one. Christiano, Eichenbaum and Rebelo (2010) and Woodford (2010), in contrast, find that the government purchases multiplier is much larger than one when the nominal interest rates is zero. In addition, the sign of the output response to either type of technology shock is positive in Table 2. Braun and Waki (2006) and Christiano, Eichenbaum and Rebelo (2010) find that it is negative when the nominal interest rate is zero. Finally, we find that output (and hours) fall when the labor tax is increased during the period of zero nominal interest rates. Eggertsson (2010) finds that hours increase in this situation. We turn now to present some variants of our model that produce these types of unorthodox responses.

4.2 Unorthodox Responses

The most important reason why the baseline calibration produces orthodox responses is that the shocks to d_t are calibrated to deliver an expected duration of zero interest rates of two years in each year between 1999-2005. If we increase the expected duration of zero interest rates enough the model yields specifications that have unorthodox properties. In practice, there are two patterns of unorthodox results that emerge. Some specifications exhibit a government purchase output multiplier that is much larger than one but the response of output to shocks to technology and the labor tax are orthodox. A second type of specification has the property that the government purchase multiplier is larger than one and the responses of output to technology and labor tax shocks are also unorthodox.

We consider two general strategies for increasing the expected duration of zero interest rates. The first increases the serial correlation coefficient in the law of motion for the preference discount factor shifter (d_t) while holding the shocks fixed at their baseline values. The second strategy varies the pattern and size of shocks while holding fixed the serial correlation coefficient ρ_d.

Before discussing the results in Table 3 we wish to emphasize the distinction in our model between *production* which we denote by y and *output*. Let output in the model be denoted by GDP.[10] GDP is defined as:

$$GDP_t \equiv c_t + g_t + x_t = y_t(1 - \frac{\gamma}{2}(\pi_t - \pi)^2)$$

The distinction between production and GDP plays an important role in the subsequent analysis. Any shock that increases the difference between current and steady state inflation also raises the resource costs of price adjustment. This, in turn, increases the gap between production and GDP.

The scenario considered in Table 3 corresponds to the 1999 scenario reported in Table 2. Column 1 restates impulse responses for the baseline specification and also reports the responses of production. The results reported in columns 2 and 3 under the heading *High serial correlation discount factor* increase the serial correlation coefficient from the baseline value of 0.9 to respectively 0.94 and 0.95. Consider the results reported in the top panel of column 2 under the heading impact response of output. Recall that output is defined as the sum of consumption, investment and government purchases as in the NIPA accounts. The government purchases multiplier is 1.55. Increasing the serial correlation of the discount factor from 0.9 to 0.94 nearly doubles the size of the government purchase multiplier.[11] Notice that this magnification of the government purchase output multiplier is due to a

[10] We use the term GDP because most national accounts data used in economic research in recent years is based on GDP. Formally, though the measure of output in our dataset is based on the Hayashi and Prescott (2002) methodology and they use GNP as their measure of output.

[11] Most of the increase occurs as the discount factor is increased from 0.93 to 0.94. For instance, if we simulate the model setting $\rho_d = 0.93$ instead, the government purchase multiplier is just above one.

longer expected duration of zero interest rates. From row 2 of Table 3 we see that increasing ρ_d from 0.9 to 0.94 increases the expected duration of zero interest rates from two to six years. A longer expected duration of zero interest rates produces a larger drop in the markup. The decline in the markup with $\rho = 0.94$ is more than twice as large as the baseline. When the Taylor feedback rule is active it acts to smooth the price and thereby the markup response. However, when the nominal interest rate is zero an increase in the expected duration of zero rates means that this mechanism is absent longer and both prices and the markup respond by more.

Interestingly, the response of production to an increase in government purchases is much smaller and only about one. Why is the output multiplier so much larger than the production multiplier?

To provide some intuition for this difference in the response of production and output we totally differentiate the resource constraint with respect to a change in government purchases to get:

$$\frac{\partial GDP}{\partial g} = (1-\Psi)\frac{\partial y}{\partial g} - y\frac{\partial \Psi}{\partial g} \tag{30}$$

where Ψ denotes the resource costs of price adjustment and where we have suppressed time subscripts for ease of exposition. Equation (30) decomposes the GDP response to a government purchase shock into two terms.[12] The first term consists of the response of production to a government purchases shock weighted by one minus the resource costs of price adjustment, $(1-\Psi)$. The second term is the response of the price adjustment costs to a government purchases shock weighted by production, y.

Table 4 reports this decomposition for the various specifications of the model. The most important factor is the response of the resource costs to a change in government purchases. A shock to government purchases puts upward pressure on prices. When individuals anticipate a longer period of zero interest rates, the price increase is larger and this produces a bigger saving in resources. With less production taken up by price adjustment more is available for consumption and investment which implies in turn a larger response of GDP. This effect is so pronounced that consumption now increases with the increase in government purchases.

Returning to Table 3 we see that the results reported in column 2 have a second notable feature. The response of both output and production to shocks in either type of technology or the labor tax are orthodox. An improvement in technology increases production and output and a higher labor tax lowers both production and output. However, there is also a distinction between the production and output response for these variables. The reasoning works in an analogous way to the case of a government purchase shock. Suppose that the labor tax is increased. This shock acts to reduce the costs of price adjustment. When the

[12] Strictly speaking, the GDP response to a transitory government purchase shock, $\partial GDP/\partial g$ has a third second order term too. However, for small changes the infinitesimals in (30) are a valid approximation.

shock to the discount factor is more persistent the price level is also lower and the associated savings in price adjustment costs are larger.

Given the mechanisms we have identified it is not surprising then that increasing ρ_d further changes the sign of the response of output to a shock in the labor tax. The results reported in column 3 of Table 3, which set $\rho_d = 0.95$, illustrate this point. Now households expect the interest rate to be zero for seven years when hit with the baseline model shocks. The government purchase output multiplier is now nearly two. The response of output to a labor tax shock or a transitory technology shock is unorthodox. output increases when the labor tax is increased and output falls when the transitory technology shock is increased.[13] If one, however, considers the response of production instead to these same shocks its properties are orthdodox. Production increases by less than one to a one unit increase in government purchases, production increases when technology improves and production falls when the labor tax is increased.

A second way to increase the expected duration of zero interest rates is to hold fixed the model parameters and instead to increase the size of the shocks. We now show that when the shocks to d_t are sufficiently large, the model also produces unorthodox results.

The *persistent expectations* specification sets the sequence of preference shocks hitting the economy between 1999 and 2007 so that agents expect zero nominal rates for 5 years as new information arrives in each year between 1999 to 2003. All parameters of the model are held fixed. After 2003, the shocks are adjusted so that agents expect that the nominal rate will become positive sooner in a way that is consistent with Japan's experience. The nominal rate becomes positive in 2007. To implement this scenario, a shock to the preference discount rate of size 3% hits the economy in 1999. From 2000 to 2007, the size of preference discount rate shocks ranges between 0.6% and -0.6%.

Results for the persistent expectations specification are reported in column 4 of Table 3. For this specification there is a 3% shock to d_t and a simultaneous 15% shock to $\tau_{c,t}$ in 1999. Consider the results reported in column 3. Even though the particular strategy used to increase the expected duration of zero interest rates is different, the message from these results is qualitatively quite similar to the results reported in column 2.

This choice of shocks induces a much larger response in the markup as compared to the baseline specification. Notice also that the resource costs are much larger here (2.51% as compared to 0.59% for the baseline). The government purchase GDP multiplier increases to 1.33. However, the government purchase multiplier for production is only moderately larger and less than one.[14] Finally, note that the output responses to a positive transitory

[13] We do not report the consumption responses here due to space considerations but we wish to mention that for all simulations we performed consumption moved in the same direction as output when technology or labor taxes were shocked.

[14] One way to see the key role played by expectations about the duration of zero interest rates is to compute the multiplier for another year. In the year 2000 there are no new shocks to d_t yet, the government purchase

technology shock or a high labor tax rate shock are also smaller in absolute value than the production responses but still have the conventional signs.

For the *large preference shock* specification, we assume that the preference discount shock arriving in 1999 is equal to 3.5% and that the shock to τ_c is 0.175. These shocks lead agents to expect that nominal rates will be zero in each year between 1999 and 2006. After 1999, no other shocks to d_t or $\tau_{c,t}$ arrive. In this specification the equilibrium value of the nominal interest rate becomes positive in 2007.

We will report two sets of simulations for this specification. The shocks in 1999 are $d_t = 0.035$ and $\tau_{c,t} = 0.175$ under each scenario. However, they differ in the treatment of the resource costs of price adjustment in the aggregate resource constraint. In column 6 they are included in the resource constraint, in column 7 they are omitted. When the resource costs of price adjustment are reflected in the budget constraint we find that the government purchase output multiplier is 1.70. The government purchase production multiplier though is less than one. This specification also produces anomalous output responses to an increase in the labor tax and an temporary improvement in technology shock. But the response of output is very muted.

Finally, consider the results in column 7. This scenario provides the reader with an indication of how the answer changes if one solves the model using a log-linearized solution, centered at a steady state with price stability and thereby abstracts from the resource costs of price adjustment. Omitting this term from the aggregate resource constraint acts to magnifies all of the responses. The response of the markup to any shock is now many orders of magnitude larger than the baseline. This results in a government purchase multiplier of 2 and large and anomalous responses of output to shocks in the labor tax and transitory technology shocks. Using the equilibrium prices one can compute the "implied" resource costs of price adjustment. They are implausibly large and exceed 18% of output.

4.3 Assessing the Plausibility of Orthodox and Unorthodox Responses

We have seen that by varying the expected duration of zero interest rates it is possible to produce specifications that have very different dynamic properties. We now and propose and implement a strategy for assessing which of these specifications is most relevant for understanding Japan's experience with zero interest rates.

One of the messages from Table 3 is that as the expected duration of zero interest rates is increased, the response of the markup to a variety of shocks increases. Each of the specifications we have considered has distinct implications for the volatility of the markup, prices, GDP and other aggregate variables. What is interesting about Japan is that the

output multiplier is 1.35 which is about the same as its value of 1.33 in 1999.

period of zero interests was associated with a sharp decline in real and nominal volatility.

Table 5 reports relative volatility statistics for Japanese data and alternative specifications of the model. For each variable we report the standard deviation from 1988 to 1998 relative to the same variable's standard deviation between 1999 and 2006. A relative volatility statistic of less than one means that the respective variable was less volatile during the period of zero nominal interest rates.

The first row of Table 5 reports relative volatility statistics for Japanese data.[15] The volatility of GDP and real marginal costs both fall by about half. The declines in the volatility of consumption and inflation are even larger. Consumption volatility falls be over 70% and both measures of inflation volatility (CPI and Consumption deflator) fall by 65%. one. Labor input is the only variable for which volatility actually increases during the 1999-2007 sample period.

The baseline specification successfully predicts the qualitative pattern of declines in nominal and relative volatility observed in Japanese data. It predicts declines in output, consumption, real marginal costs and inflation. The baseline specification also, counterfactually, predicts a decline in labor input volatility. Some of the magnitudes are off but, overall it is our view that this parsimonious model of the Japanese economy does a surprisingly good job of reproducing the evidence of tranquility in Japanese data.

For purposes of comparison we also report volatility statistics for some other specifications that have orthodox properties. The moderate price adjustment cost model uses a value of $\gamma = 10$.[16] This specification also successfully reproduces the evidence of tranquility. With lower adjustment costs, the relative volatility of inflation increases to 0.27 which brings it closer to the data value of 0.35. The flexible price specification, reported in row two of Table 5 does about as well as the moderate price adjustment cost specification for the real variables but produces too much volatility in the price level.

Consider next the results reported in rows below the baseline specification. These specifications all produce unorthodox results. Both specifications with higher serial correlation in d_t predict that volatility in consumption, real marginal cost and inflation increased during the period of zero interest rates. This occurs even though these two specifications have lower variability in the preference discount shock than the baseline specification between 1999 and 2006 and the same sequences of all other shocks as the baseline specification (see column 6 in Table 5). The match between the model and Japanese data is particularly poor when $\rho_d = 0.95$. Recall that this specification has the property that the response of output to labor tax and transient technology shocks is unorthodox. Here output volatility increases

[15] We measure real marginal costs in Japanese data using a labor share measure that accounts for employees in self-employed firms. Following Muto (2009), we calculate real marginal costs as $\frac{1}{\alpha}$Compensation of employees/(National income - households' operating surplus). We would like to thank Ichiro Muto from the Bank of Japan for his helpful comments on the measurement of real marginal costs.

[16] The associated figure under Calvo price adjustment is 0.45

and the volatility of real marginal cost more than doubles between 1999 and 2006.

The *persistent expectations* and *large preference shock* specifications also predict that the period of zero interest rates in Japan should have been a period of high economic volatility. The increase in volatility is particularly dramatic in the final row of Table 5, which omits the resource costs of price adjustment from the aggregate resource constraint. Output volatility nearly triples and labor input volatility goes up by a factor of over 5.

There is other evidence in favor of the specifications that produce low government purchase multipliers and orthodox responses of output to labor tax and productivity shocks. Ichiue and Ueno (2007) find that the maximum expected duration of zero interest rates between 1999 and 2007 was not more than 2.3 years. The specifications that produce unorthodox results all require households to expect zero interest rates for a much longer period of time.

A final reason in favor of the baseline specification is that all of the specifications that exhibit unorthodox responses also have quite large resource costs of price adjustment. As reported in Table 3, they range from 1.53% of output for the high serial correlation discount rate specification with $\rho_d = 0.93$ to a massive 18.7% of output for the specification where the resource costs are omitted from the aggregate resource constraint. In this final case the resource costs of price adjustment are "imputed" using the equilibrium value of the price level. The baseline specification exhibits much more moderate resource costs of price adjustment. The value of 0.6% lies in the range of estimates that emerge from analyzing the costs of adjusting prices from firms. Levy et al. (1997) find that menu costs constitute 0.7% of revenues in supermarket chains.

4.4 Robustness

In this section, we briefly describe the robustness of our conclusions. As regards the choice of the shock processes, we wish to first mention that our assumption that technology follows a unit root process does have an impact on some of our results. Under our current assumption that shocks to technology are permanent agents best guess of tomorrow's state of technology is today's state of technology plus drift. The past is of no help in forming expectations about the future. Technology shocks play an important role in the dynamics of the model and under the assumption of a unit root process in technology agents never expect the zero lower bound to bind in advance of 1999. If instead technological progress is deterministic and shocks to technology are serially correlated agents start to predict zero nominal interest rates several years before the nominal interest rate falls to zero and this acts to change the dynamics of the model before the nominal interest rate is zero. The dynamics start to change as soon as agents expect zero nominal rates in the future. This finding is significant in the sense that it is not necessary for the nominal interest rate to be zero in order for the dynamics of the model to start to shift. All that is necessary is that agents expect the nominal interest rate to be zero at some point in the future.

We have chosen to model quadratic price adjustment costs and not Calvo style price adjustment. Braun and Waki (2010) compare Calvo and Rotemberg models of price adjustment. They find that the increase in marginal costs associated with a given change in government purchases is larger under Calvo price adjustment. This results in a larger government purchase multiplier under Calvo price setting. On the basis of this result it is our conjecture specifications with Calvo price setting will exhibit even more excess volatility as compared to the Rotemberg specification we have considered here.

We have also performed other experiments in which we reduced the relative persistence of the shock to government purchases or the shock to the labor tax rate by lowering the serial correlation coefficient. Changing the parameters in this way reduced the size of the government purchase multiplier and yielded output responses to the labor tax that were always orthodox.

We have also conducted simulations in which we kept the tax rate on consumption constant.[17] This leads to a deterioration in the fit of the model for GDP and labor input. However, the magnitudes of the GDP impulse responses and the government purchase multiplier are very close to those reported for our baseline specification.

Finally, the qualitative nature of our results is robust to the parameterization of the Taylor rule. We have obtained qualitatively similar results using Taylor rules that set the coefficient on the lagged value of the nominal rate to zero, use a different coefficient on inflation or include output growth.

5 Concluding Remarks

In this paper we have conducted a quantitative investigation aimed at assessing the dynamics of the New Keynesian model in a low interest rate environment.

We produced a baseline specification that does a reasonable job of reproducing some basic stylized facts from the Japanese economy between 1988 and 2007. An investigation of the dynamic properties of that specification implies that the response of output to a range of shocks is consistent with standard theory. Moreover, the size of the government purchase output multiplier is less than one.

We also considered specifications of the model that have larger government purchase multipliers and some which also exhibit unorthodox predictions for the response of output to labor tax and technology shocks. We found that these specifications are difficult to square with the fact that the period of zero interest rates in Japan between 1999 and 2006 was a period of low economic volatility. All of the specifications predict the opposite should have

[17]Under this assumption a 3% shock to the discount factor is needed to induce a binding zero nominal 1999 that agents expect to last for two years.

occurred. The specifications with unorthodox properties also have other problems. They predict large resource costs of price adjustment which are difficult to reconcile with empirical evidence that menu costs are small and they require that households expect the period of zero interest rates to be counterfactually long.

References

[1] Adam, Klaus and Roberto Billi (2006), "Optimal Monetary Policy under Commitment with a Zero Lower Bound on Nominal Interest Rates", *Journal of Money, Credit and Banking*, 38, 1877-1905.

[2] Adjemian, Stephane and Michel Juillard (2009), "Dealing with ZB in DSGE Models an Application to the Japanese Economy", mimeo.

[3] Braun, R. Anton, Daisuke Ikeda and Douglas H. Joines (2009), "The Saving Rate in Japan: Why It Has Fallen and Why It Will Remain Low", *International Economic Review*, 50, 291-321.

[4] Braun, R. Anton and Yuichiro Waki (2006), "Monetary Policy during Japan's Lost Decade", *The Japanese Economic Review*, 57, 324-344.

[5] Braun, R. Anton and Yuichiro Waki (2010), "On the Fiscal Multiplier When the Zero Bound on Nominal Interest Rates is Binding", mimeo.

[6] Chen, Kaiji, Ayse Imrohoroglu and Selahattin Imrohoroglu (2006), "The Japanese Saving Rate", *American Economic Review*, 96, 1850-1858.

[7] Christiano, Lawrence, Martin Eichenbaum and Sergio Rebelo (2010), "When is the Government Spending Multiplier Large?", mimeo.

[8] Eggertsson, Gauti (2010), "The Paradox of Toil", Federal Reserve Bank of New York Staff Report 433.

[9] Eggertsson, Gauti and Michael Woodford (2003), "The Zero Interest Rate Bound and Optimal Monetary Policy", Brookings Panel on Economic Activity.

[10] Erceg, Christopher and Jesper Lindé (2010), "Is There a Fiscal Free Lunch in a Liquidity Trap?" , CEPR Discussion Paper 7624.

[11] Hayashi, Fumio and Edward Prescott (2002), "The 1990s in Japan: A Lost Decade", *Review of Economic Dynamics*, 5, 206-235.

[12] Herr, Burkhard and Alfred Maussner (2008), *Dynamic General Equilibrium Modelling*, Springer Berlin, Heidelberg, New York.

[13] Ichiue, Hibiki and Yoichi Ueno (2007), "Equilibrium Interest Rate and the Yield Curve in a Low Interest Rate Environment", Bank of Japan Working Paper.

[14] Khan, Aubhik and Robert King and Alexander Wolman (2002), "Optimal Monetary Policy," NBER Working Paper 9402.

[15] Kobayashi, Keiichiro and Masuru Inaba (2006), “Business Cycle Accounting for the Japanese Economy”, *Japan and the World Economy*, 18, 418-440.

[16] Levy, Daniel, Mark Bergen, Shantanu Dutta and Robert Venable (1997), “The Magnitude of Menu Costs: Direct Evidence from Large U.S. Supermarket Chains”, *Quarterly Journal of Economics*, 112, 791-825.

[17] Miyazawa, Kensuke (2010), “Japan’s Lost Decade and the Decline in Labor Input”, mimeo.

[18] Muto, Ichiro (2009), ”Estimating a New Keynesian Phillips Curve with a Corrected Measure of Real Marginal Cost: Evidence in Japan”, *Economic Inquiry*, 47, 667-684.

[19] Nakov, Anton (2008), “Optimal and Simple Monetary Policy Rules with Zero Floor on the Nominal Interest Rates”, *International Journal of Central Banking*, 4, 73-127.

[20] Rotemberg, Julio (1996), “Prices, Output and Hours: An Empirical Analysis based on a Sticky Price Model”, *Journal of Monetary Economics*, 37, 505-33.

[21] Schmitt-Grohe, Stephanie, and Martin Uribe (2007), “Optimal Simple and Implementable Monetary and Fiscal Rules”, *Journal of Monetary Economics*, 54, 1702-1725.

[22] Sugo, Tomohiro and Kozo Ueda (2006), ”Estimating a DSGE Model for Japan: Evaluating and modifying a CEE/SW/LOWW Model”, mimeo.

[23] Taehun, Jung, Yuki Teranishi and Tsutomu Watanabe (2005), ”Optimal Monetary Policy at the Zero-Interest-Rate Bound”, *Journal of Money, Credit, and Banking*, 37, 813-835.

[24] Taylor, John B. (1993), ”Discretion versus Policy Rules in Practice”, Carnegie-Rochester Conference Series on Public Policy, 39, 195-214.

[25] Wolman, Alexander (2005), ”Real Implications of the Zero Bound on Nominal Interest Rates”, *Journal of Money, Credit and Banking*, 37, 273-96.

[26] Woodford, Michael (2010), ”Simple Analytics of the Government Expenditure Multiplier”, NBER Working Paper 15714.

Table 1: Model Parameterization

Symbol	*Value*	*Description*
α	0.362	Capital share
δ	0.085	Depreciation rate
ϕ	4	Adjustment costs on capital
β	0.995	Discount factor
ν	0.27	Preference consumption share
σ	2	Preference curvature
γ	80	Adjustment costs on prices
$\theta/(\theta-1)$	1.15	Steady state gross markup
R	0.029	Steady state nominal rate
ρ_R	0.4	Elasticity of the nominal rate with respect to the lagged nominal rate
ρ_π	1.7	Elasticity of the nominal rate with respect to inflation
μ_A	1.02	Steady state growth rate of technology
G/Y	0.19	Steady state government share
τ_w	0.27	Steady state labor income tax
τ_k	0.41	Steady state capital tax
τ_c	0.05	Steady state consumption tax
ρ_A	0.92	Autocorrelation coefficient of transient technology shocks
ρ_G	0.89	Autocorrelation coefficient of government spending
ρ_w	0.9	Autocorrelation coefficient of labor income tax
ρ_k	0.9	Autocorrelation coefficient of capital income tax
ρ_c	0.9	Autocorrelation coefficient of consumption tax

Table 2
Impact responses of output and markup baseline specification

Year	1995	1999	2004
Years expected nominal rate is zero	none	1999-2000	2004-2005
Resource costs of price adjustment**	0.22	0.59	0.54
Impact response of output (GDP) to a positive shock in:			
Neutral technology (transitory)	0.64	0.57	0.59
Neutral technology (permanent)	0.62	0.68	0.71*
Labor tax	-0.62	-0.56	-0.57*
Government purchases	0.65	0.87	0.87*
Impact response of the markup to a positive shock in:			
Neutral technology (transitory)	0.06	0.21	0.19
Neutral technology (permanent)	-0.03	-0.14	-0.17*
Labor tax	-0.06	-0.23	-0.21*
Government purchases	-0.20	-0.64	-0.57*

*For this shock the zero bound constraint applies only in 2004

**Resource costs of price adjustment are reported in percentage terms of output

Table 3

Impulse responses to shocks that arrive in 1999 for alternative specifications of the model

	Baseline	Baseline Shocks, high serial correlation discount factor shock (0.94)	Baseline Shocks, high serial correlation in discount rate shock (0.95)	Persistent Expectations	Large preference shock	Large preference shock without price adjustment costs in the resource constraint
Years expected nominal rate is zero	1999-2000	1999-2004	1999-2005	1999-2003	1999-2006	1999-2006
Resource costs of price adjustment*	0.59	1.53	3.62	2.51	7.08	18.70
Impact response of output (GDP) to a positive shock in:						
Neutral technology (transitory)	0.57	0.08	-0.38	0.29	-0.04	-0.39
Neutral technology (permanent)	0.68	0.97	1.27	0.88	1.15	1.42
Labor tax	-0.56	-0.08	0.33	-0.28	0.05	0.42
Government purchases	0.87	1.55	1.92	1.33	1.70	2.02
Impact response of production to a positive shock in:						
Neutral technology (transitory)	0.60	0.39	0.42	0.52	0.60	-0.39
Neutral technology (permanent)	0.66	0.79	0.79	0.73	0.71	1.42
Labor tax	-0.59	-0.36	-0.36	-0.50	-0.58	0.42
Government purchases	0.78	1.03	0.87	0.87	0.67	2.02
Impact response of markup to a positive shock in:						
Neutral technology (transitory)	0.21	0.94	1.34	0.58	0.79	2.78
Neutral technology (permanent)	-0.14	-0.60	-0.89	-0.42	-0.61	-2.00
Labor tax	-0.23	-0.96	-1.35	-0.63	-0.86	-2.95
Government purchases	-0.64	-1.79	-2.07	-1.35	-1.58	-4.66

*Resource costs of price adjustment are reported in percentage terms of output.

For the specification without price adjustment costs in the resource constraint, implied resource costs are reported.

Table 4
Decomposition of the GDP response to a government purchases shock

	$\partial GDP/\partial g$	$= (1-\psi)\ \partial y/\partial g$	- y $(\partial\Psi/\partial g)$
Specification			
Baseline	0.87	= (1 - 0.006) x 0.78	- 1.02 x (- 0.09)
Higher discount factor serial correlation (0.94)	1.55	= (1 - 0.015) x 1.03	- 1.03 x (- 0.52)
Higher discount factor serial correlation (0.95)	1.92	= (1 - 0.036) x 0.87	- 1.03 x (- 1.05)
Persistent expectations	1.33	= (1 - 0.025) x 0.87	- 1.00 x (- 0.46)
Large preference shock	1.70	= (1 - 0.071) x 0.67	- 0.99 x (- 1.03)

Table 5

Relative standard deviations for Japanese data and alternative specifications of the model*

	Output	Consumption	Labor Input	Real Marginal Cost	Consumption deflator	CPI	Preference discount shock	Consumption tax shock
Japanese Data	0.52	0.28	1.33	0.51	0.35	0.35	-	-
Model Specifications with Orthodox Properties								
Flexible Price	0.4	0.69	0.38	0.00	1.02		0.29	0.11
Moderate Price adjustment cost	0.41	0.71	0.44	0.15	0.27		0.25	0.11
Baseline	0.50	0.75	0.53	0.21	0.07		0.29	0.14
Model Specifications with Unorthodox Properties								
Higher discount factor serial correlation (0.94)	0.74	1.05	0.47	1.86	1.09		0.13	0.14
Higher discount factor serial correlation (0.95)	1.40	1.3	0.65	2.25	1.68		0.11	0.14
Persistent Expectations	1.09	1.36	1.14	1.84	0.78		0.62	0.28
Large Preference Shock	2.14	1.87	1.8	3.01	1.47		0.74	0.34
Large Preference Shock (alt)	2.84	2.16	5.64	5.56	2.46		0.74	0.34

* All statistics are calculated as the standard deviation of the variable from 1999 to 2006 relative to its standard deviation from 1988 to 1998.

All variables except marginal cost are expressed in terms of log growth rates.

(alt) refers to the specification where the resource costs of price adjustment are omitted from the resource constraint.

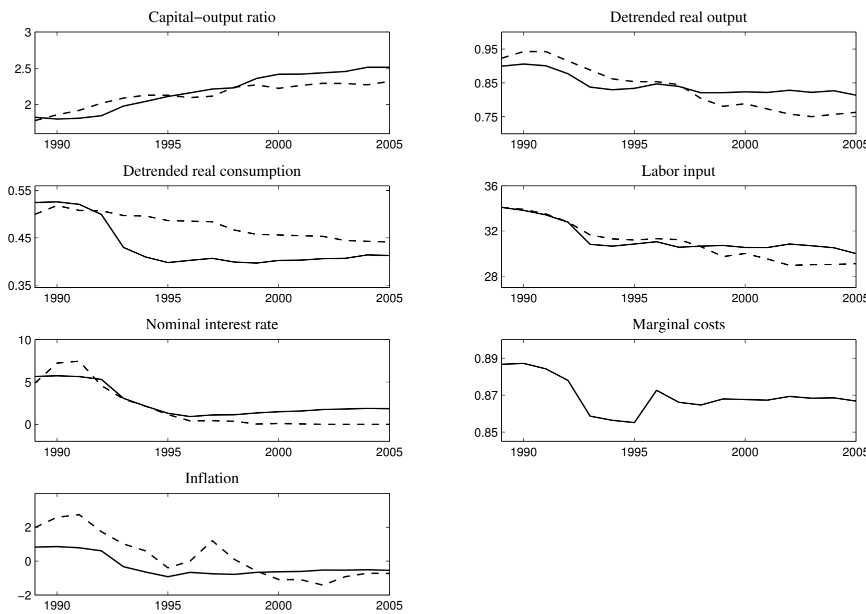

Figure 1: Baseline economy if neither preference nor monetary policy shocks arrive after 1991

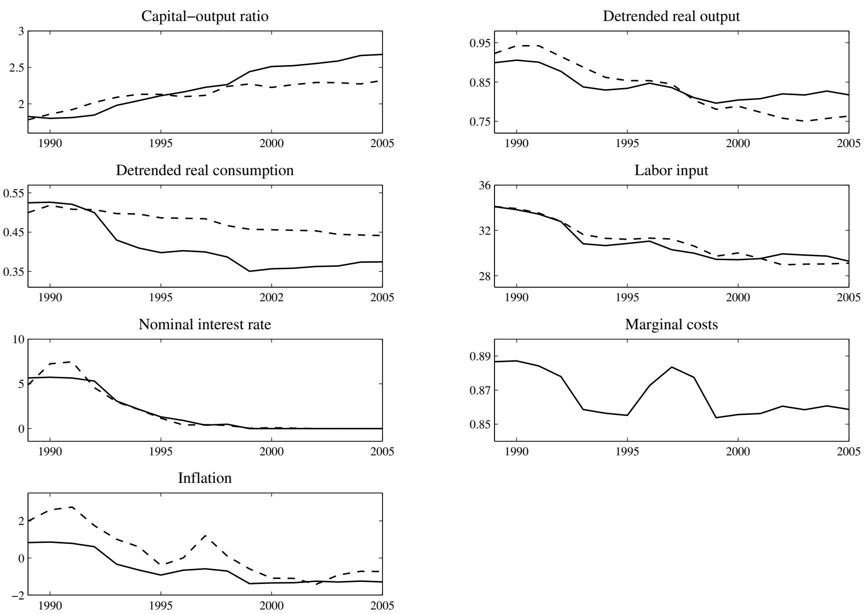

Figure 2: Baseline economy