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Hiroshi Fujiki*

Abstract

During the recent global financial crisis, some central banks introduced two innovative cross-border operations to deal with the problems of foreign currency liquidity shortages: domestic liquidity operations using cross-border collaterals and operations for supplying foreign currency based on standing swap lines among central banks. We show theoretically that central banks improve the efficiency of equilibrium under foreign currency liquidity shortages by those two innovative temporary policy measures.

Keywords: Standing swap lines; Operations supplying US dollar funds outside the US; Cross-border collateral arrangements
JEL classification: E58, F31, F33

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1. Introduction

In this paper, we show that central banks improve the efficiency of equilibrium under liquidity shortages in foreign currency markets through two temporary policy measures: either domestic liquidity operations using cross-border collaterals or operations for supplying foreign currency based on standing swap lines among central banks. We show this improvement in a two-country extension of Chapman and Martin's (2007) model.

The motivation behind our focus on these two temporary policy measures is the recent central bank innovative cross-border operations to deal with the problems of foreign currency liquidity shortages. Before discussing the details of the models, let us briefly explain why these two temporary policy measures were introduced.[1]

Since the announcement of BNP Paribas on August 9, 2007 to suspend withdrawals from three funds that had invested in the US subprime market, central banks have adopted two kinds of cross-border operations to cope with the global financial crisis.

First, central banks have widened the range of collateral to maintain control of money markets during the current crisis. These include the acceptance of high-quality marketable collateral denominated in foreign currencies or held in foreign locations.[2] For example, on May 22, 2009, the Bank of Japan (BOJ) decided to accept bonds issued by the governments of the United States, the United Kingdom, Germany and France as eligible collateral.

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[1] We focus on the US dollar fund-supplying operations conducted by several central banks associated with the US Federal Reserve's term auction facility. We do not discuss many other important issues for central banks, such as the role of monetary policy or lender of last resort policy, highlighted in the recent crisis. Freixas (2009) provides a survey of the recent literature. Obstfeld (2009) focuses on the lender of last resort role of central banks in a global economy.

[2] The original idea behind the cross-border use of the collateral was a shift towards real-time gross settlement of central bank payment systems, which requires large overdraft facilities and thus collateral. The European Central Bank, a notable example of a cross-border central bank, mitigates borrowers' mismatch between the location of its collateral holdings and its liquidity needs through a correspondent central banking model (CCBM) within the euro area. The US Federal Reserve accepts several foreign government bonds as collateral.

Second, regarding the supply of foreign currency, the US Federal Reserve, the European Central Bank (ECB) and the Swiss National Bank (SNB) established swap lines that enabled ECB and SNB to provide dollar funds to their counterparties in December 2007.[3] The actions of ECB and SNB were associated with the Federal Reserve's establishment of the term auction facility (TAF), which lent dollars obtained through swap lines arranged with the US Federal Reserve.[4] After the failure of Lehman Brothers in September 2008, the standing swap lines with the US Federal Reserve expanded to the Bank of England (BOE) and the BOJ. During this period, liquidity in the US dollar money markets outside the US quickly evaporated because of the sudden increase in the concerns over counterparty risk. LIBOR–OIS spreads increased substantially (Figure 1), and the financial strains were transmitted to the FX swap markets, as shown by the unusual increase in the spread of the FX-swap-implied dollar rate and LIBOR (3-month data) in Figure 2. Under such a situation, BOE, ECB, SNB and BOJ commenced the operations supplying US dollar funds at fixed rates within their appropriate collateral in October 2008. As Figure 3 shows, the amount of outstanding operations supplying US dollar funds increased sharply through the end of 2008.[5] Thanks to the substantial provision of dollars in the markets outside the US, tightness in money markets and FX swap markets eased in 2009, as shown in Figure 1 and Figure 2. Faced with these improvements in the financial markets, the rates applied to the operations supplying US dollar funds rose above the market rates, and the amount of outstanding operations supplying US dollar funds declined through the first half of 2009 as Figure 3

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[3] Goldberg, Kennedy and Miu (2010) provide an excellent overview of the evolution of reciprocal currency arrangements or dollar swap facilities that the Federal Reserve established with foreign central banks from 2007 to 2010.

[4] Regarding empirical studies on the effectiveness of the TAF to reduce the strain in the short-term dollar markets, see, for example, McAndrews et al. (2008), Taylor and Williams (2009), and Wu (2008). They reach different conclusions depending on the econometric identification assumptions.

[5] Baba and Packer (2009) use data of foreign exchange swap pairs between the US dollar, euro, Swiss franc, and British pound from August 9, 2007 to January 30, 2009 and find that the operations supplying US dollar

shows. On January 27, 2010, the Federal Open Market Committee announced that the temporary liquidity swap arrangements between the Federal Reserve and other central banks would expire on February 1. BOE, BOJ, ECB, and SNB announced the end of their US dollar-fund supplying operations the following day.

Given those developments, we show how the two temporary policy measures, the standing swap lines with the foreign central bank to supply foreign currency in the domestic market, and the acceptance of cross-border collateral arrangements, helped alleviate the liquidity shortages in the foreign currencies and improve the efficiency of the equilibrium in a two-country version model of Chapman and Martin (2007), which extends Freeman (1999).

Freeman (1999) examines the implication of aggregate default risk for the general equilibrium welfare effects of open market operations and discount windows. His model shows that open market operations not only reduce the risk to which the purchaser of the debt is exposed, but also spread aggregate risk more broadly and efficiently if all agents are risk averse, even if the central bank incurs losses related to default.

Chapman and Martin (2007) extend Freeman's model by introducing moral hazard. Specifically, they point out that creditors' incentives to monitor their loans would be reduced if the creditors knew that the central bank, which cannot monitor the quality of loans, would absorb losses under a "market-insensitive policy," a policy whereby a central bank purchases the loans at a preannounced uniform discount rate, which Freeman (1999) recommends. Chapman and Martin (2007) show that a "market-sensitive policy," a policy where a central bank randomly chooses a restricted number of creditors to compete for central bank funds, cures the shortcomings of the market-insensitive policy. By restricting the number of agents, the central bank limits the moral hazard problem because the creditors know that the creditors

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funds help diminish the deviation of the foreign exchange swap market from the covered interest party conditions after the failure of Lehman Brothers.

are more likely to accept funds from other creditors who can check the quality of loans. By letting the creditors compete for central bank funds, the central bank exploits market information to infer the state of the economy because creditors bid up the price at which they borrow until the expected value of the loan is equal to the expected value of the collateral.

We extend the model of Chapman and Martin (2007) by adding the following five aspects: (1) some creditors want to consume goods produced by foreign debtors, (2) these creditors need foreign currency to purchase foreign debtors' goods because foreign debtors only accept foreign currency, (3) to obtain foreign currency, these creditors first get second-hand debt denominated in foreign currency in exchange for their own currency-denominated loans in an active offshore international market (hereafter we call that offshore market the Eurodollar market, essentially a market for collateral swaps), (4) these creditors obtain foreign currency by presenting the second-hand debt to the issuer of the debt in the foreign credit market or by open market purchase by the foreign central bank, and (5) these creditors purchase foreign debtors' goods using foreign currency (i.e. they undertake cross-border activity). These extensions allow us to examine two issues.

First, we examine the effects of domestic open market operations, which include lending to foreigners against domestic collateral. The optimal policy in normal times is the market-sensitive policy that Chapman and Martin (2007) propose. However, in our case, the market-sensitive policy in one country has positive spillover effects to risk averse creditors in the other country by mitigating the risk of liquidity shortages and default.

Second, we examine the effects of two temporary policy measures: standing swap lines with the foreign central bank to supply foreign currency in the domestic market, and the acceptance of cross-border collateral arrangements. Consider a sudden increase in the perceptions of counterparty risk in the Eurodollar market, which leads to a shutdown of all transactions, as we experienced in the Eurodollar market in early October 2008. Our model shows central banks in two economies should take counterparty risks in the collateral swap market in one of the two temporary policy measures to improve the efficiency of market equilibrium. Specifically, the central banks should use the market-insensitive policy to substitute the transactions in the Eurodollar market when the creditors are reluctant to transact in this market. The market-insensitive policy could be either lending foreign currency to domestic agents against domestic collateral or lending domestic currency to foreign agents against foreign collateral. The central banks should supplement their market-insensitive policy in the Eurodollar market by their market-sensitive policy in their domestic credit markets. When the creditors resume transactions in the Eurodollar market, the central banks can exit from the market-insensitive policy in the Eurodollar market automatically. The automatic exits become possible because under the market-insensitive policy, the central banks set their discount rates slightly below the discount rates that would prevail under no monitoring effort by the creditors. These discount rates are inexpensive for the creditors in a situation in which a sudden increase in the perceptions of counterparty risk in the Eurodollar market leads to a shutdown of all transactions, but expensive for the creditors under normal market conditions.

The two temporary policy measures improve the welfare of both economies in a situation in which a sudden increase in the perceptions of counterparty risk in the Eurodollar market leads to a shutdown of all transactions, and thus these two temporary policy measures are incentive compatible to both central banks. The two temporary policy measures, however, cannot achieve the same level of efficiency as achieved by the foreign currency market transactions under normal conditions because central banks cannot use market-sensitive policy.

To the best of our knowledge, we are the first to show how the two temporary policy measures help alleviate liquidity crises in the foreign currencies in a theoretical model based on Freeman (1999) and Chapman and Martin (2007).[6]

The rest of the paper is organized as follows. Section 2 explains the environment and trading patterns. Section 3 analyzes equilibrium with liquidity constraints. Section 4 considers the role of the market-sensitive policy and market-insensitive policy to mitigate the liquidity constraints in normal market conditions. Section 5 considers the role of the standing swap lines with the foreign central bank to supply foreign currency in the domestic market when the Eurodollar market is shut down. Section 6 studies the effects of cross-border collateral arrangements when the Eurodollar market is shut down. Section 7 discusses practical issues related to the two temporary policy measures, and Section 8 concludes.

2. Environment and trading patterns

2.1 The environment

This section explains our model, which extends Chapman and Martin (2007) to a two-country model. There are two types of agents, called creditors and debtors, in the domestic country (hereafter Japan) and the foreign country (hereafter US). In both countries, creditors and debtors are scattered and live in small villages. Their populations are normalized to one, and their lifetime is divided into two periods. Japanese and US creditors and debtors are endowed with nonstorable goods specific to their villages in their first period of life, in the

[6] There are many other ways to analyze the effects of liquidity crises on consumption in a multicountry model. For example, Castiglionesi, Feriozzi and Lorenzoni (2009) use a multiregion version of the model of Diamond and Dybvig (1983) and show that financial integration reduces aggregate uncertainty and increases welfare, but induces banks to reduce their liquidity holdings, and that this increases the severity of extreme events. Regarding the cross-border use of collateral in payment systems, Manning and Willison (2006) examine the extent to which the liquidity risk arising from such a mismatch may be mitigated by allowing cross-border use of collateral in a two-country, two-bank model in which risk-neutral banks minimize expected costs with respect to their collateral choice in each country.

amounts of $\bar{y}, \bar{x}, \bar{Y}$, and $\bar{X}$ respectively. Lowercase letters represent Japanese variables and uppercase letters represent US variables. The sequence of travel and trading patterns of debtors and creditors in each country during their lifetimes are summarized in Figure 4 and Figure 5, and we will explain them in turn below.

2.2 Trips by Japanese debtors and creditors

Japanese debtors consume their own endowment and Japanese creditor goods in the first period. At the beginning of the period, Japanese young debtors travel to the Japanese creditor village with which they are paired, where they may consume creditor village goods in exchange for the IOU to pay in yen in the second period at the Japanese market, where all IOUs denominated in yen are repaid. They return to their village of origin later in the period.

At the beginning of the second period, with probability θ, default occurs. Specifically, $(1-e_t)\eta$ debtors do not travel to the Japanese market, but scatter to Japanese debtor villages where they are free to consume, and only $1-(1-e_t)\eta$ debtors travel to the Japanese market to pay their debt to the creditors, where e_t is the monitoring effort exerted by the Japanese creditors, which will be explained in the next paragraph. The default shock is independently and equally distributed over time and its realization is not known in advance. $(1-e_t)\eta$ is the same for all debtors as long as creditors choose the same monitoring effort level e_t. With probability $1-\theta$, all debtors travel to the Japanese market to pay their debt to creditors. The function $v_c(c_t^d)+v_d(d_t^d)+\theta(1-e_t)\eta \cdot v_d(d_{t+1}^d)$ shows the expected utility of Japanese debtors, where c_t^d, d_t^d and d_{t+1}^d show the consumption of debtor and creditor village goods when young, and of debtor village goods when old. The superscripts *c* and *d* on the choice variable denote the individual type. The functions v_c and v_d are continuous and continuously differentiable strictly increasing concave functions. v_c and v_d have infinite marginal utilities when their arguments are zero.

Japanese creditors consume c_t^c units of their own endowment when young, and when old, consume either d_{t+1}^c units of Japanese debtor goods with probability $(1-\tau)$ or D_{t+1}^c units of US debtor goods with probability τ. The utility of Japanese creditors is $u_c(c_t^c)+(1-\tau)\cdot u_d(d_{t+1}^c)+\tau\cdot u_D(D_{t+1}^c)$, where the subscripts on the utility function denote the types of goods consumed. The continuous and continuously differentiable utility functions u_c and u_d are strictly increasing and concave in each argument. Japanese creditors invest effort e_t in monitoring the loan they extended to the debtor. Effort linearly decreases the probability of default, and thus $(1-e_t)\eta$ is the default probability they face if they invest e_t. All creditors can verify the monitoring effort of the other creditors, but central banks cannot. Japanese creditors get disutility from their effort of monitoring $\varphi(e_t)$, where φ is an increasing concave function and $\varphi(0)=0$.

Japanese creditors know the types of goods to consume when old at the beginning of their second time period. $(1-\tau)$ creditors travel to the Japanese market. τ creditors first meet with T old US creditors, whose travel before their arrival to the Eurodollar market will be explained in the next paragraph, in the Eurodollar market, exchange the IOUs they have, and then travel to the US market. These US creditors travel from the Eurodollar market to the Japanese market holding Japanese IOUs.

Arrival at the Japanese market takes place in two stages. In the first stage, $(1-\tau)$ old Japanese creditors and T old US creditors, and λ old Japanese debtors arrive, where $0\le\lambda\le 1$ and $1-\lambda \ge (1-e_t)\eta$. At the end of the first stage, $(1-\alpha)(1-\tau)$ old Japanese creditors and $(1-A)T$ old US creditors leave for their final destination, while the rest remain until the end of the second stage. In the second stage, the remaining $1-\lambda-(1-e_t)\eta$ old debtors arrive if a default shock occurs, and $1-\lambda$ old debtors arrive otherwise. The remaining creditors know whether a default occurs or not at this time.

All creditors face the same probability of leaving the Japanese market early ($(1-\alpha)$ for Japanese creditors and $(1-A)$ for US creditors), and all debtors face the same probability of arriving early, late or not at all. Each learns his/her arrival or departure time as soon as he/she turns old. After the visit to the Japanese market, $(1-\tau)$ Japanese creditors, T US creditors and old debtors who have not gone to the Japanese market (and thus defaulted) randomly scatter to a selected Japanese debtor island. They arrive at the debtor island after the trips made by the creditors are completed.

2.3 Trips by US debtors and creditors

US debtors consume their own endowment and US creditor goods in the first period. At the beginning of the period, US young debtors travel to the creditor village with which they are paired, where they may consume creditor village goods in exchange for the IOU to pay in dollar the second period at the US market, where all IOUs denominated in dollar are repaid. They return to their debtor villages later in the period. With probability Θ, $(1-E_t)$ H debtors do not travel to the US market, but scatter to US debtor villages where they are free to consume. E_t is the monitoring effort exerted by the US creditors, which will be explained in detail in the next paragraph. With probability $1-\Theta$, all debtors travel to the US market. The default shock is independently and equally distributed over time and its realization is not known in advance. The function $V_C(C_t^D)+V_D(D_t^D)+\Theta(1-E_t)\mathrm{H}\cdot V_D(D_{t+1}^D)$ shows the expected utility of US debtors.[7]

US creditors consume C_t^C units of their own endowment when young and either D_{t+1}^C units of US debtor goods with probability (1–*T*) or d_{t+1}^C units of Japanese debtor goods when old with probability *T*. The utility of US creditors is $U_C(C_t^C)+(1-T)\cdot U_D(D_{t+1}^C)+T\cdot U_d(d_{t+1}^C)$.[8] US creditors invest effort E_t in monitoring the loan they extended to the debtor. Effort linearly decreases the probability of default, and thus $(1-E_t)\mathrm{H}$ is the default probability they face if they invest E_t. All creditors can verify the effort of monitoring, but central banks cannot. US creditors get disutility from their monitoring effort $\Phi(E_t)$, where Φ is an increasing concave function and $\Phi(0)=0$.

US creditors know the types of goods to consume when old at the beginning of their second time period. (1–*T*) creditors travel to the US market. *T* creditors first meet with τ old Japanese creditors in the Eurodollar market, exchange the IOUs they have, and then travel to the Japanese market.

After the visit to the US market, (1–*T*) old US creditors, along with τ old Japanese creditors and defaulted old US debtors randomly select US debtor islands.

Arrival at the US market takes place in two stages. In the first stage, (1–*T*) old US creditors, τ old Japanese creditors and Λ old US debtors arrive, where $0\le\Lambda\le1$ and $1-\Lambda\ge(1-E_t)\mathrm{H}$. At the end of the first stage, $(1-\alpha)\tau$ old Japanese creditors and $(1-\Lambda)(1-T)$ old US creditors leave for their final destination, while the rest remain until the end of the second

[7] C_t^D, D_t^D and D_{t+1}^D represent their consumption of debtor and creditor village goods when young and of debtor village goods when old. The functions V_C and V_D are continuous and continuously differentiable strictly increasing concave. V_C and V_D have infinite marginal utilities when their arguments are zero.

[8] The continuous and continuously differentiable utility functions U_C and U_D are strictly increasing and concave in each argument.

stage. In the second stage, the remaining $1-\Lambda-(1-E_t)\mathrm{H}$ old debtors arrive if a default shock occurs, and $1-\Lambda$ old debtors arrive otherwise. The remaining creditors know whether a default occurs or not at this time.

All creditors face the same probability of leaving early in the US market ((1−α) for Japanese creditors and (1–A) for US creditors), and all debtors face the same probability of arriving early, late or not at all. Each learns his/her arrival or departure time as soon as he/she turns old.

2.4 Trading patterns

There exist central banks in the Japanese market and US market that issue currency with initial stocks of $\overline{m}$ yen and $\overline{M}$ dollars to each initial old creditor, whose mass is one and who lives only in the first period.

All agents can issue unfalsifiable IOUs that identify the issuer. Regal authorities exist in the Japanese market and US market and enforce the agreements between the parties currently in the villages. Regal authorities do not exist to enforce agreements in the Eurodollar market and at the agents' final destination.

To consume when old, creditors must bring something of value to the debtor island. Currency will be accepted by young debtors if it helps them to pay their debts and thereby to acquire the goods they desire. If it is accepted in equilibrium, creditors will require that debts be repaid with currency.

The young debtors wish to consume goods from creditors' villages but do not have national currency at hand. The young debtors offer creditors a promise to pay a sum of money in the next period in the domestic market. Because of the extent of legal authority, we assume that debtors can only promise to pay in national currency. The young debtor will acquire this money by selling some of his/her endowment to an old creditor who brings domestic currency to the village later in the period.

Given this sequence of trade, the old creditors who want to consume foreign goods in the second period need foreign currency. To obtain foreign currency, the old creditors must exchange their national IOUs for foreign IOUs in the Eurodollar market. While the exchange of IOUs in this model is for the sake of consumption by the old creditors in foreign economies, the exchange could be interpreted as foreign currency funding in the offshore money market by multinational banks. Money is essential to make final payment to retire debt, and without repayment in the national currency, creditors will not accept debt.

Debts are cleared at the national market but not always bilaterally. Because the arrival rate of the old debtors is lower than the departure rate of old creditors, the early-departing old creditors sell their yet-unredeemed debt to the late-departing old creditors. The amount of debt redeemed in this second-hand debt market is limited by the currency that the early-arriving old debtors bring to the market, and thus the second-hand debt may be traded at a discount value.

In summary, creditors need to think about eight trading patterns, depending on whether they consume foreign goods or not, whether they depart the market early or late, and whether default occurs or not. Debtors need to consider two trading patterns, depending on whether default occurs. Note that whether a debtor arrives in the market early or late when old does not affect his/her consumption. Only whether he/she defaults or not affects his/her additional consumption when old. The probability of default of debtors in Japan and US is independent, and we have four states: state 00, with probability $(1-\theta)(1-\Theta)$ of no default in both economies; state 10, with probability $\theta(1-\Theta)$ that only Japanese debtors default; state 01, with probability $((1-\theta)\Theta)$ that only US debtors default; state 11, with probability $(\theta\Theta)$ that both Japanese and US debtors default. Those events are identified only after the arrival of all debtors in each market. The situation of default in one economy will be known to the other economy immediately. When we need to distinguish the level of choice variables in these four states, we will use the expressions x[00], x[01], x[10], x[11] to represent the value of variable x in states 00, 01, 10, and 11.

3. Equilibrium

This section first examines the optimization problem by debtors and creditors and then moves on to present the market equilibrium conditions and finally defines a symmetric laissez-fare equilibrium with liquidity constraints.

3.1 The Japanese debtor's problem

Let p_t be the yen price of Japanese debtor goods in Japanese debtor villages at time t. Because only debtor goods are sold in exchange for money in the current period, the yen price of Japanese debtor goods is a measure of the price level. Let m_t be the acquisition of yen from old creditors in exchange for debtor goods, let π_t be the price of creditor island goods at t of a promise to pay one yen on the Japanese market at t+1, and let h_t be nominal value at t of the Japanese debtor's indebtedness. The Japanese debtor faces the following budget constraints:

$$p_t \bar{x} = p_t d_t^d + m_t , \tag{1}$$

$$m_t = h_t , \tag{2}$$

$$c_t^d = h_t \pi_t , \tag{3}$$

$$p_{t+1}[10] e_{t+1}^d [10] = p_{t+1}[11] e_{t+1}^d [11] = m_t . \tag{4}$$

Note that equations (1) through (3) take into account the fact that all yen will be acquired by the young debtor in states 00, 10, 01 and 11 and thus his/her consumption decision at time t does not depend on the states of default However, his/her consumption at time t+1 takes place only when default occurs in Japan (state 10, and 11) as can be seen in equation (4). Inserting those constraints into the utility function, the Japanese debtor maximizes:

$$v_c(m_t\pi_t) + v_d(\bar{x} - \frac{m_t}{p_t}) + \theta\,(1\text{-}e_t)(1\text{-}\Theta)\,\eta \cdot v_d(\frac{m_t}{p_{t+1}[10]}) + \theta(1\text{-}e_t)\Theta\eta \cdot v_d(\frac{m_t}{p_{t+1}[11]}), \quad \textbf{(5)}$$

by the choice of m_t.

The resulting first-order condition will be:

$$v_c'\pi_t - \frac{v_d'}{p_t} + \theta\,(1-e_t)(1\text{-}\Theta)\,\eta \cdot \frac{v_d'}{p_{t+1}[10]} + \theta\,(1-e_t)\Theta\eta \cdot \frac{v_d'}{p_{t+1}[11]} = 0, \quad \textbf{(6)}$$

where primes indicate first derivatives.

3.2 The US debtor's problem

The US debtor's problem is a mirror image of the Japanese debtor's problem. Let P_t be the dollar price of US debtor goods in the US debtor village at time t. Let M_t be the acquisition of dollars from old creditors in exchange for debtor goods, let Π_t be the price in creditor island goods at t of a promise to pay one dollar in the US market at t+1, and let H_t be the nominal value at t of the US debtor's indebtedness. The US debtor faces the following budget constraints:

$$P_t\overline{X} = P_t D_t^D + M_t, \quad \textbf{(7)}$$

$$M_t = H_t, \quad \textbf{(8)}$$

$$C_t^D = H_t \Pi_t, \quad \textbf{(9)}$$

$$P_{t+1}[01]E_{t+1}^D[01] = P_{t+1}[11]E_{t+1}^D[11] = M_t. \quad \textbf{(10)}$$

Inserting these constraints into the utility function, the US debtor maximizes:

$$V_C(M_t\Pi_t)+V_D(\overline{X}-\frac{M_t}{P_t})+(1-\theta)\Theta(1-E_t)\,\mathrm{H}\cdot V_D(\frac{M_t}{P_{t+1}[01]})+\theta\Theta(1-E_t)\,\mathrm{H}\cdot V_D(\frac{M_t}{P_{t+1}[11]}), \qquad \textbf{(11)}$$

by the choice of M_t.

The resulting first-order condition will be:

$$V_C{}'\Pi_t-\frac{V_D{}'}{P_t}+(1-\theta)\Theta\,(1-E_t)\mathrm{H}\cdot\frac{V_D{}'}{P_{t+1}[01]}+\theta\Theta(1-E_t)\mathrm{H}\cdot\frac{V_D{}'}{P_{t+1}[11]}=0 \qquad \textbf{(12)}$$

3.3 The Japanese creditor's problem

Let l_t be the nominal value of a creditor's loans to debtors at time t. Let q^c_{t+1} (yen) and q^C_{t+1} (yen) be the par value of nominal debt purchased by Japanese and US late-leaving creditors at time t+1. Let $\rho_{t+1}\le 1$ represent the nominal price at which one yen of that debt is exchanged in the first stage of visits in the Japanese market at t+1. If default occurs, $(1-\lambda-(1-e_t)\eta)/(1-\lambda)$ is the proportion of the remaining debt that will be repaid. Finally, let s_t be the foreign exchange rate of yen per dollar that prevails in the Eurodollar market.

Similarly, let L_t be the nominal value of a creditor's loans to debtors at time t. Let Q^C_{t+1} (dollars) and Q^c_{t+1} (dollars) be the par value of nominal debt purchased by US and Japanese late-leaving creditors at time t+1. Let $R_{t+1}\le 1$ represent the nominal price at which one dollar of that debt is exchanged in the first stage of visits in the US market at t+1. If default occurs, $(1-\Lambda-(1-E_t)H)/(1-\Lambda)$ is the proportion of the remaining debt that will be repaid. Those notations should be defined here because some of the Japanese creditors participate in the US second-hand debt market.

Japanese creditors consume c^c_t units of their own endowment when young. When old, they consume either d^c_{t+1} units of Japanese debtor goods with probability $(1-\tau)$ or D^c_{t+1} units of US debtor goods with probability τ. The budget constraint of a Japanese creditor born at t when young is:

$$\bar{y} = c_t^c + l_t \pi_t \, . \tag{13}$$

Depending on whether he/she departs the market early or late, whether he/she consumes foreign goods or not, and whether the event of default occurs or not, a Japanese creditor has the following four budget constraints when old.

First, if he/she departs early and consumes Japanese goods (with probability of $(1-\alpha)(1-\tau)$) then:

$$\rho_{t+1}(1-\lambda)l_t + \lambda l_t = p_{t+1}[j]d_{t+1}^c[j], \;\; j = 00,10,01,11 \, . \tag{14}$$

Second, if he/she departs early and consumes US goods (with probability of $(1-\alpha)\tau$) then:

$$R_{t+1}(1-\Lambda)l_t s_{t+1} + \Lambda l_t s_{t+1} = P_{t+1}[j]D_{t+1}^c[j], \;\; j = 00,10,01,11 \, . \tag{15}$$

Equation (15) incorporates the fact that they exchange their yen-denominated loans into dollar-denominated loans at the beginning of time t+1 at the exchange rate s_{t+1}.

Third, if he/she departs late and consumes Japanese goods (with probability of $\alpha(1-\tau)$) then, without default and with default, respectively, we have:

$$l_t + (1-\rho_{t+1})q_{t+1}^c = p_{t+1}[j]d_{t+1}^c[j], \;\; j = 00,01 \text{, if no default in Japan,} \tag{16}$$

$$(1-(1-e_t)\eta)l_t + q_{t+1}^c \frac{1-\lambda-(1-e_t)\eta}{1-\lambda} - \rho_{t+1}q_{t+1}^c = p_{t+1}[j]d_{t+1}^c[j], \;\; j = 10,11 \, , \tag{17}$$

if default occurs in Japan.

Finally, if he/she departs late and consumes US goods (with probability of $\alpha\tau$) then, without default and with default, respectively, we have:

$$l_t s_{t+1} + (1-R_{t+1})Q_{t+1}^c = P_{t+1}[j]D_{t+1}^c[j], \;\; j = 00,10 \text{, if no default in the US,} \tag{18}$$

$$(1-(1-E_t)\mathrm{H})l_t s_{t+1} + Q_{t+1}^c \frac{1-\Lambda-(1-E_t)\mathrm{H}}{1-\Lambda} - R_{t+1}Q_{t+1}^c = P_{t+1}[j]D_{t+1}^c[j], \;\; j = 01,11 \, , \tag{19}$$

if default occurs in the US.

Using the budget constraints at times t and t+1 yields the following optimization problem for the young Japanese creditors with respect to l_t, q^c_{t+1}, Q^c_{t+1} and e_t at time t:

$$\begin{aligned}
&\underset{l_t, q^c_{t+1}, Q^c_{t+1}, e_t}{Max} \quad u_c(\overline{y} - l_t\pi_t) + (1-\tau)(1-\alpha)\cdot \sum_{j=00,10,01,11} \text{Prob[j]} u_d(\frac{\rho_{t+1}(1-\lambda)l_t + \lambda l_t}{p_{t+1}[j]}) \\
&+ \tau(1-\alpha)\cdot \sum_{j=00,10,01,11} \text{Prob[j]} u_D(\frac{R_{t+1}(1-\Lambda)l_t s_{t+1} + \Lambda l_t s_{t+1}}{P_{t+1}[j]}) \\
&+ (1-\tau)\alpha \cdot \left\{ \sum_{j=00,01} \text{Prob[j]} u_d(\frac{l_t + (1-\rho_{t+1})q^c_{t+1}}{p_{t+1}[j]}) + \right. \\
&\left. \sum_{j=10,11} \text{Prob[j]} u_d(\frac{(1-(1-e_t)\eta)l_t + (1-\lambda-(1-e_t)\eta/1-\lambda)q^c_{t+1} - \rho_{t+1}q^c_{t+1}}{p_{t+1}[j]}) \right\} \\
&+ \tau\alpha \cdot \left\{ \sum_{j=00,10} \text{Prob[j]} u_D(\frac{l_t s_{t+1} + (1-R_{t+1})Q^c_{t+1}}{P_{t+1}[j]}) + \right. \\
&\left. \sum_{j=01,11} \text{Prob[j]} u_D(\frac{(1-(1-E_t)\mathrm{H})l_t s_{t+1} + (1-\Lambda-(1-E_t)\mathrm{H}/1-\Lambda)Q^c_{t+1} - R_{t+1}Q^c_{t+1}}{P_{t+1}[j]}) \right\} - \varphi(e_t)
\end{aligned} \tag{20}$$

Therefore, the first-order condition of this problem for l_t is:

$$\begin{aligned}
&(-\pi_t)u_c'(\overline{y} - l_t\pi_t) \\
&+ (1-\tau)(1-\alpha)\cdot \sum_{j=00,10,01,11} \text{Prob[j]} u_d'(\frac{\rho_{t+1}(1-\lambda)l_t + \lambda l_t}{p_{t+1}[j]}) \frac{\rho_{t+1}(1-\lambda) + \lambda}{p_{t+1}[j]} \\
&+ \tau(1-\alpha)\cdot \sum_{j=00,10,01,11} \text{Prob[j]} u_D'(\frac{R_{t+1}(1-\Lambda)l_t s_{t+1} + \Lambda l_t s_{t+1}}{P_{t+1}[j]}) \frac{R_{t+1}(1-\Lambda)s_{t+1} + \Lambda s_{t+1}}{P_{t+1}[j]} \\
&+ (1-\tau)\alpha \cdot \left\{ \sum_{j=00,01} \text{Prob[j]} u_d'(\frac{l_t + (1-\rho_{t+1})q^c_{t+1}}{p_{t+1}[j]}) \frac{1}{p_{t+1}[j]} \right. \\
&\left. + \sum_{j=10,11} \text{Prob[j]} u_d'(\frac{(1-(1-e_t)\eta)l_t + (1-\lambda-(1-e_t)\eta/1-\lambda)q^c_{t+1} - \rho_{t+1}q^c_{t+1}}{p_{t+1}[j]}) \frac{1-(1-e_t)\eta}{p_{t+1}[j]} \right\} \\
&+ \tau\alpha \cdot \left\{ \sum_{j=00,10} \text{Prob[j]} u_D'(\frac{l_t s_{t+1} + (1-R_{t+1})Q^c_{t+1}}{P_{t+1}[j]}) \frac{s_{t+1}}{P_{t+1}[j]} \right. \\
&\left. + \sum_{j=01,11} \text{Prob[j]} u_D'(\frac{(1-(1-E_t)\mathrm{H})l_t s_{t+1} + \frac{(1-\Lambda-(1-E_t)\mathrm{H}}{1-\Lambda} Q^c_{t+1} - R_{t+1}Q^c_{t+1}}{P_{t+1}[j]}) \frac{(1-(1-E_t)\mathrm{H})s_{t+1}}{P_{t+1}[j]} \right\} = 0
\end{aligned} \tag{21}$$

The first-order condition for q^c_{t+1} is:

$$\begin{aligned}
0 = &\sum_{j=00,01} \text{Prob[j]} u_d'(\frac{l_t + (1-\rho_{t+1})q^c_{t+1}}{p_{t+1}[j]}) \frac{(1-\rho_{t+1})}{p_{t+1}[j]} + \\
&\sum_{j=10,11} \text{Prob[j]} u_d'(\frac{(1-(1-e_t)\eta)l_t + \frac{1-\lambda-(1-e_t)\eta}{1-\lambda} q^c_{t+1} - \rho_{t+1}q^c_{t+1}}{p_{t+1}[j]})(\frac{\frac{1-\lambda-(1-e_t)\eta}{1-\lambda} - \rho_{t+1}}{p_{t+1}[j]})
\end{aligned} \tag{22}$$

The first-order condition for Q^c_{t+1} is:

$$0 = \sum_{j=00,10} \text{Prob}[j] u_D'\left(\frac{l_t s_{t+1} + (1-R_{t+1})Q_{t+1}^c}{P_{t+1}[j]}\right)\frac{(1-R_{t+1})}{P_{t+1}[j]}$$

$$+ \sum_{j=01,11} \text{Prob}[j] u_D'\left(\frac{(1-(1-E_t)\mathrm{H})l_t s_{t+1} + \frac{1-\Lambda-(1-E_t)\mathrm{H}}{1-\Lambda}Q_{t+1}^c - R_{t+1}Q_{t+1}^c}{P_{t+1}[j]}\right)\frac{\left(\frac{1-\Lambda-(1-E_t)\mathrm{H}}{1-\Lambda} - R_{t+1}\right)}{P_{t+1}[j]} \quad \textbf{(23)}$$

The first-order condition for e_t is:

$$(1-\tau)\alpha \cdot \sum_{j=10,11} \text{Prob}[j] u_d'\left(\frac{(1-(1-e_t)\eta)l_t + \frac{1-\lambda-(1-e_t)\eta}{1-\lambda}q_{t+1}^c - \rho_{t+1}q_{t+1}^c}{p_{t+1}[j]}\right)\cdot \eta\left(\frac{l_t + \frac{q_{t+1}^c}{1-\lambda}}{p_{t+1}[j]}\right) = \varphi'(e_t) \quad \textbf{(24)}$$

3.4 The US creditor's problem

US creditors consume C_t^C units of their own endowment when young. When old, they consume either D_{t+1}^C units of US debtor goods with probability (1–T) or d_{t+1}^C units of Japanese debtor goods with probability T. The budget constraint of a US creditor born at t when young is:

$$\overline{Y} = C_t^C + L_t \Pi_t . \quad \textbf{(25)}$$

Depending on whether he/she departs the market early or late, whether he/she consumes foreign goods or not, and whether default occurs or not, a US creditor has the following budget constraints when old.

First, if he/she departs early and consumes US goods (with probability of (1–A)(1–T)) then:

$$R_{t+1}(1-\Lambda)L_t + \Lambda L_t = P_{t+1}[j]D_{t+1}^C[j],\ j = 00,10,01,11. \quad \textbf{(26)}$$

Second, if he/she departs early and consumes Japanese goods (with probability of (1–A)T) then:

$$\frac{\rho_{t+1}(1-\lambda)L_t}{s_{t+1}} + \frac{\lambda L_t}{s_{t+1}} = p_{t+1}[j]d_{t+1}^c[j],\ j = 00,10,01,11. \quad \textbf{(27)}$$

Third, if he/she departs late and consumes US goods (with probability of A(1–T)) then, without default and with default, respectively, we have:

$$L_t + (1-R_{t+1})Q^C_{t+1} = P_{t+1}[j]D^C_{t+1}[j],\ j = 00,10 \text{, if no default occurs in the US,} \quad \textbf{(28)}$$

$$(1-(1-E_t)\mathrm{H})L_t + Q^C_{t+1}\frac{1-\Lambda-(1-E_t)\mathrm{H}}{1-\Lambda} - R_{t+1}Q^C_{t+1} = P_{t+1}[j]D^C_{t+1}[j],\ j = 01,11\text{,} \quad \textbf{(29)}$$

if default occurs in the US.

Finally, if he/she departs late and consumes Japanese goods (with probability of AT) then, without default and with default respectively, we have:

$$\frac{L_t}{s_{t+1}} + (1-\rho_{t+1})q^C_{t+1} = p_{t+1}[j]d^C_{t+1}[j],\ j = 00,01 \text{, if no default occurs in Japan,} \quad \textbf{(30)}$$

$$(1-(1-e_t)\eta)\frac{L_t}{s_{t+1}} + q^C_{t+1}\frac{1-\lambda-(1-e_t)\eta}{1-\lambda} - \rho_{t+1}q^C_{t+1} = p_{t+1}[j]d^C_{t+1}[j],\ j = 10,11\text{,} \quad \textbf{(31)}$$

if default occurs in Japan.

Using the budget constraints at times t and t+1 yields the following optimization problem for the young US creditors with respect to L_t, Q^C_{t+1}, q^C_{t+1} and E_t at time t:

$$\begin{aligned}
&\underset{L_t, Q^C_{t+1}, q^C_{t+1}, E_t}{Max} \quad U_C(\overline{Y} - L_t\Pi_t) + (1-T)(1-A)\cdot \sum_{j=00,10,01,11} \mathrm{Prob[j]}\ U_D\left(\frac{R_{t+1}(1-\Lambda)L_t + \Lambda L_t}{P_{t+1}[j]}\right) \quad \textbf{(32)}\\
&+ T(1-A)\cdot \sum_{j=00,10,01,11} \mathrm{Prob[j]}\ U_d\left(\frac{\rho_{t+1}(1-\lambda)L_t + \lambda L_t}{p_{t+1}[j]s_{t+1}}\right)\\
&+ (1-T)A\cdot\left\{ \sum_{j=00,10} \mathrm{Prob[j]}\ U_D\left(\frac{L_t + (1-R_{t+1})Q^C_{t+1}}{P_{t+1}[j]}\right) + \right.\\
&\left. \sum_{j=01,11} \mathrm{Prob[j]}\ U_D\left(\frac{(1-(1-E_t)\mathrm{H})L_t + \frac{1-\Lambda-(1-E_t)\mathrm{H}}{1-\Lambda}Q^C_{t+1} - R_{t+1}Q^C_{t+1}}{P_{t+1}[j]}\right)\right\}\\
&+ TA\cdot\left\{ \sum_{j=00,01} \mathrm{Prob[j]}\ U_d\left(\frac{\frac{L_t}{s_{t+1}} + (1-\rho_{t+1})q^C_{t+1}}{p_{t+1}[j]}\right) + \right.\\
&\left. \sum_{j=10,11} \mathrm{Prob[j]}\ U_d\left(\frac{(1-(1-e_t)\eta)\frac{L_t}{s_{t+1}} + \frac{1-\lambda-(1-e_t)\eta}{1-\lambda}q^C_{t+1} - \rho_{t+1}q^C_{t+1}}{p_{t+1}[j]}\right)\right\} - \Phi(E_t)
\end{aligned}$$

Therefore, the first-order condition for L_t is:

$$
\begin{aligned}
&-\Pi_t U_C'(\overline{Y}-L_t\Pi_t) \\
&+(1-T)(1-A)\cdot \sum_{j=00,10,01,11} \text{Prob[j]}\, U_D'\left(\frac{R_{t+1}(1-\Lambda)L_t+\Lambda L_t}{P_{t+1}[j]}\right)\frac{R_{t+1}(1-\Lambda)+\Lambda}{P_{t+1}[j]} \\
&+T(1-A)\cdot \sum_{j=00,10,01,11} \text{Prob[j]}\, U_d'\left(\frac{\rho_{t+1}(1-\lambda)L_t+\lambda L_t}{p_{t+1}[j]s_{t+1}}\right)\frac{\rho_{t+1}(1-\lambda)}{p_{t+1}[j]s_{t+1}} \\
&+(1-T)A\cdot\left\{ \sum_{j=00,10} \text{Prob[j]}\, U_D'\left(\frac{L_t+(1-R_{t+1})Q_{t+1}^C}{P_{t+1}[j]}\right)\frac{1}{P_{t+1}[j]}+\right. \\
&\left.\sum_{j=01,11} \text{Prob[j]}\, U_D'\left(\frac{(1-(1-E_t)\mathrm{H})L_t+\frac{1-\Lambda-(1-E_t)\mathrm{H}}{1-\Lambda}Q_{t+1}^C-R_{t+1}Q_{t+1}^C}{P_{t+1}[j]}\right)\frac{(1-(1-E_t)\mathrm{H})}{P_{t+1}[j]}\right\} \\
&+TA\cdot\left\{ \sum_{j=00,01} \text{Prob[j]}\, U_d'\left(\frac{\frac{L_t}{s_{t+1}}+(1-\rho_{t+1})q_{t+1}^C}{p_{t+1}[j]}\right)\frac{1}{p_{t+1}[j]s_{t+1}}+\right. \\
&\left.\sum_{j=10,11} \text{Prob[j]}\, U_d'\left(\frac{(1-(1-e_t)\eta)\frac{L_t}{s_{t+1}}+\frac{1-\lambda-(1-e_t)\eta}{1-\lambda}q_{t+1}^C-\rho_{t+1}q_{t+1}^C}{p_{t+1}[j]}\right)\frac{(1-(1-e_t)\eta)}{p_{t+1}[j]s_{t+1}}\right\}=0 \quad \textbf{(33)}
\end{aligned}
$$

The first-order condition for Q_{t+1}^C is:

$$
\begin{aligned}
0=&\sum_{j=00,10}\text{Prob[j]}U_D'\left(\frac{L_t+(1-R_{t+1})Q_{t+1}^C}{P_{t+1}[j]}\right)\frac{(1-R_{t+1})}{P_{t+1}[j]}+ \\
&\sum_{j=01,11}\text{Prob[j]}U_D'\left(\frac{(1-(1-E_t)\mathrm{H})L_t+\frac{1-\Lambda-(1-E_t)\mathrm{H}}{1-\Lambda}Q_{t+1}^C-R_{t+1}Q_{t+1}^C}{P_{t+1}[j]}\right)\frac{\frac{1-\Lambda-(1-E_t)\mathrm{H}}{1-\Lambda}-R_{t+1}}{P_{t+1}[j]} \quad \textbf{(34)}
\end{aligned}
$$

The first-order condition for q_{t+1}^C is:

$$
\begin{aligned}
0=&\sum_{j=00,01}\text{Prob[j]}U_d'\left(\frac{\frac{L_t}{s_{t+1}}+(1-\rho_{t+1})q_{t+1}^C}{p_{t+1}[j]}\right)\frac{(1-\rho_{t+1})}{p_{t+1}[j]}+ \\
&\sum_{j=10,11}\text{Prob[j]}U_d'\left(\frac{(1-(1-e_t)\eta)\frac{L_t}{s_{t+1}}+\frac{1-\lambda-(1-e_t)\eta}{1-\lambda}q_{t+1}^C-\rho_{t+1}q_{t+1}^C}{p_{t+1}[j]}\right)\frac{\frac{1-\lambda-(1-e_t)\eta}{1-\lambda}-\rho_{t+1}}{p_{t+1}[j]} \quad \textbf{(35)}
\end{aligned}
$$

The first-order condition for E_t is:

$$
(1-T)A\cdot\sum_{j=01,11}\text{Prob[j]}\,U_D'\left(\frac{(1-(1-E_t)\mathrm{H})L_t+\frac{1-\Lambda-(1-E_t)\mathrm{H}}{1-\Lambda}Q_{t+1}^C-R_{t+1}Q_{t+1}^C}{P_{t+1}[j]}\right)\mathrm{H}\frac{L_t+\frac{Q_{t+1}^C}{1-\Lambda}}{P_{t+1}[j]}=\Phi'(E_t) \quad \textbf{(36)}
$$

3.5 Market clearing conditions

The conditions for the clearing of the market of goods denominated in yen and dollars in debtor villages are:

$$\overline{m} = p_t(\overline{x} - d_t^d), \quad \textbf{(37)}$$

$$\overline{M} = P_t(\overline{X} - D_t^D). \quad \textbf{(38)}$$

Note that young debtors at time t are not affected by whether old debtors have defaulted, which implies the existence of an equilibrium in which a debtor's demands for real balances do not depend on the realization of the default shock. Moreover, the same number of nominal yen and dollars arrive in each village (either brought by old creditors or old debtors) whether or not a default has occurred. Hence, equations (37) and (38) imply that the price levels are the same whether a default shock has occurred or not.

The clearing of the market for loans in yen and dollars requires:

$$h_t = l_t, \quad \textbf{(39)}$$

$$H_t = L_t. \quad \textbf{(40)}$$

These equations determine the price levels p_t and P_t, the period t goods' price of debt payable in period t+1, π_t and Π_t.

In the Eurodollar market, T US creditors and τ Japanese creditors exchange their loans. We assume that the exchange will occur on a pro rata basis, because creditors supply their loans inelastically. The nominal exchange rate will be determined to equate the nominal value of the transactions:

$$l_t \cdot s_{t+1} \cdot \tau = L_t \cdot T. \quad \textbf{(41)}$$

Per capita holding of foreign loans in domestic currency becomes TL_t/τ dollars for Japanese creditors and $\tau l_t/T$ yen for US creditors.

Equation (42) below shows that in the Japanese resale market for loans, the sum of the debt purchased by the $\alpha(1-\tau)$ Japanese late-leaving creditors, q_t^c, and AT US late-leaving creditors, q_t^C, must equal the unredeemed debt, which is the sum of the proportions (1–λ) of debt owned by the (1–α)(1–τ) Japanese early-leaving creditors, (1–λ)(1–α)(1–τ)l_t, and the proportion (1–λ) of debt owned by the (1–A)T US early-leaving creditors, (1–λ)(1–A)T(τ/T)l_t = (1–λ)(1–A)τl_t:

$$\alpha(1-\tau)q_{t+1}^c + ATq_{t+1}^C = [(1-\alpha)(1-\tau)+(1-A)\tau](1-\lambda)l_t\,. \qquad \textbf{(42)}$$

The other constraint in the Japanese resale market for loans is that the amount of yen available to repurchase the debt, $\rho_{t+1}^c q_{t+1}^c$ and $\rho_{t+1}^C q_{t+1}^C$, is limited by the amount of yen delivered by the early-arriving debtors, λl_t as follows:

$$\lambda l_t \geq \rho_{t+1}^c q_{t+1}^c + \rho_{t+1}^C q_{t+1}^C\,. \qquad \textbf{(43)}$$

In the US resale market for loans, the sum of the debt purchased by the US A(1–T) late-leaving creditors, Q_t^C, and $\alpha\tau$ Japanese late-leaving creditors, Q_t^c, must equal the unredeemed debt, which is the sum of the proportion (1–Λ) of debt owned by the (1–A)(1–T) US early-leaving creditors, (1–Λ)(1–A)(1–T)L_t, and the proportion (1–Λ) of debt owned by the (1–α)τ Japanese early-leaving creditors, (1–Λ)(1–α)τ (TL_t/τ) = (1–Λ)(1–α) TL_t, as shown in equation (44) below:

$$\alpha\tau Q_{t+1}^c + A(1-T)Q_{t+1}^C = [(1-A)(1-T)+(1-\alpha)T](1-\Lambda)L_t\,. \qquad \textbf{(44)}$$

The other constraint in the US resale market for loans is that the amount of dollars available to repurchase the debt, $R_{t+1}^C Q_{t+1}^C$ and $R_{t+1}^c Q_{t+1}^c$, is limited by the amount of dollars delivered by the early-arriving debtors, ΛL_t as follows:

$$\Lambda L_t \geq R_{t+1}^C Q_{t+1}^C + R_{t+1}^c Q_{t+1}^c\,. \qquad \textbf{(45)}$$

3.6 Symmetric laissez-fare equilibrium with liquidity constraints

Consider a symmetric laissez-fare equilibrium where both Japanese and US liquidity constraints (equations (43) and (45)) are binding and all the same types of Japanese and US creditors choose the same actions, and central banks do not intervene in the markets. Note that equations (37) and (38) imply that the price levels are the same whether or not a default shock has occurred. Taking these properties into consideration, let $p_{t+1}[j] = p_{t+1} = p$, $\rho_{t+1} = \rho$, $p_s = 1/\pi p$, $l_s = l/p$, $h_s = h/p$, $m_s = m/p$, $m\pi = (m/p)(\pi p) = m_s/p_s$, $q_s^c = q^c/p_s$, $q_s^C = q^C/p_s$ and $e_t = e_s$ for Japan, and let $P_{t+1}[j] = P_{t+1} = P$, $R_{t+1} = R$, $P_s = 1/\Pi P$, $L_s = L/P$, $H_s = H/P$, $M_s = M/P$, $M\Pi = N_s/P_s$, $Q_s^c = Q^c/P_s$, $Q_s^C = Q^C/P_s$ and $E_t = E_s$ for the US. For the sake of expressing the discount rate (the nominal price at which one dollar of that debt is exchanged in the first stage of visits on the domestic market at t+1) in the demand schedule of creditors, we will use the notation ρ^c, ρ^C, R^c, and R^C.

Using these notations and equilibrium conditions in the loan and money markets, equations (2) and (39), and $l_s = h_s = m_s$, we can simplify the first-order conditions for Japanese agents as follows. In particular, equations (6), (21), (22), (23), and (24) would be simplified as follows:

$$\frac{1}{p_s} v_c'(\frac{l_s}{p_s}) - v_d'(\bar{x} - l_s) + \theta (1 - e_s)\eta v_d'(l_s) = 0 \quad , \tag{46}$$

$$\begin{aligned}
&(\frac{1}{p_s}) u_c'(\bar{y} - \frac{l_s}{p_s}) = \\
&(1-\tau)(1-\alpha) \cdot u_d'[\rho^c (1-\lambda) l_s + \lambda l_s][\rho^c (1-\lambda) + \lambda] \\
&+ \tau(1-\alpha) \cdot u_D'[\{R^C (1-\Lambda) + \Lambda\} l_s (\frac{ps}{P})]\{R^C (1-\Lambda) + \Lambda\}(\frac{ps}{P}) \\
&+ (1-\tau)\alpha \cdot \left\{(1-\theta) u_d'(l_s + (1-\rho^c) q_s^c) + \theta u_d'[(1-(1-e_s)\eta) l_s + (1-(1-e_s)\eta/(1-\lambda) - \rho^c) q_s^c](1-(1-e_s)\eta)\right\} \\
&+ \tau\alpha \cdot \left\{(1-\Theta) u_D'[l_s + (1-R^c) Q_s^c](\frac{ps}{P}) + \Theta u_D'[(1-(1-E_s)\mathrm{H}) l_s + (1-(1-E_s)\mathrm{H}/(1-\Lambda) - R^c) Q_s^c](1-(1-E_s)\mathrm{H})\frac{ps}{P}\right\}
\end{aligned} \tag{47}$$

$$\begin{aligned}
0 = &(1-\theta) u_d'[l_s + (1-\rho^c) q_s^c](1-\rho^c) \\
&+ \theta u_d'[(1-(1-e_s)\eta) l_s + (1 - \frac{(1-e_s)\eta}{1-\lambda} - \rho^c) q^c](1 - \frac{(1-e_s)\eta}{1-\lambda} - \rho^c) \quad ,
\end{aligned} \tag{48}$$

$$0=(1-\Theta)u_D'[l_s+(1-R^c)Q_s^c\frac{ps}{P}](1-R^c)$$
$$+\Theta u_D'[(1-(1-E_s)\mathrm{H})l_s+(1-\frac{(1-E_s)\mathrm{H}}{1-\Lambda}-R^c)Q_s^c\frac{ps}{P}](1-\frac{(1-E_s)\mathrm{H}}{1-\Lambda}-R^c) ,\qquad \textbf{(49)}$$

$$(1-\tau)\alpha\cdot\theta u_d'[(1-(1-e_s)\eta)l_s+(\frac{1-\lambda-(1-e_s)\eta}{(1-\lambda)}-\rho^c)q_s^c]\eta(l_s+\frac{q_s^c}{1-\lambda})=\varphi'(e_s)\ . \qquad \textbf{(50)}$$

Equation (42), the equilibrium condition of the Japanese resale market for loans, becomes:

$$\alpha(1-\tau)q_s^c+ATq_s^C=\left[(1-\alpha)(1-\tau)+(1-A)\tau\right](1-\lambda)l_s\ . \qquad \textbf{(51)}$$

The liquidity constraint in the Japanese resale market for loans becomes:

$$\lambda l_s\geq\rho^c q_s^c+\rho^C q_s^C\ . \qquad \textbf{(52)}$$

Next, we can simplify the first-order conditions for US agents as follows. In particular, equations (12), (33), (34), (35), and (36) after using the equilibrium conditions in the loan and money markets, equations (8) and (40), and $L_s = H_s = M_s$, become:

$$\frac{1}{P_s}V_C'(\frac{L_s}{P_s})-V_D'(\overline{X}-L_s)+\Theta(1-E_s)\mathrm{H}V_D'(L_s)=0 ,\qquad \textbf{(53)}$$

$$\frac{1}{P_s}U_C'(\overline{Y}-\frac{L_s}{P_s})= \qquad \textbf{(54)}$$
$$(1-T)(1-A)\cdot U_D'[\{R^C(1-\Lambda)+\Lambda\}L_S]\{R^C(1-\Lambda)+\Lambda\}$$
$$+T(1-A)\cdot U_d'[\{\rho^C(1-\lambda)+\lambda\}L_S\frac{P}{ps}]\{\rho^C(1-\lambda)+\lambda\}\frac{P}{ps}$$
$$+(1-T)A\cdot\left\{(1-\Theta)U_D'[L_S+(1-R^C)Q_S^C]+\Theta U_D'[(1-(1-E_s)\mathrm{H})L_S+(1-\frac{(1-E_s)\mathrm{H}}{1-\Lambda}-R^C)Q_S^C])(1-(1-E_s)\mathrm{H})\right\}$$
$$+TA\cdot\left\{(1-\theta)U_d'[\{L_s+(1-\rho^C)q_s^C\}\frac{P}{ps}]\frac{P}{ps}+\theta U_d'[\{(1-(1-e_s)\eta)L_s+(1-\frac{(1-e_s)\eta}{1-\lambda}-\rho^C)q_s^C\}]\frac{P}{ps})(1-(1-e_s)\eta)\frac{P}{ps}\right\} ,$$

$$0=(1-\Theta)U_D'[L_S+(1-R^C)Q_S^C](1-R^C)+ \qquad \textbf{(55)}$$
$$\Theta U_D'[(1-(1-E_s)\mathrm{H})L_S+(1-\frac{(1-E_s)\mathrm{H}}{1-\Lambda})Q_S^C](1-\frac{(1-E_s)\mathrm{H}}{1-\Lambda}-R^C) ,$$

$$0=(1-\theta)U_d'[\{L_S+(1-\rho^C)q_S^C\}(\frac{P}{ps})(1-\rho^C)+ \qquad \textbf{(56)}$$
$$\theta U_d'[(1-(1-e_s)\eta)L_S+(1-\frac{(1-e_s)\eta}{1-\lambda}-\rho^C)q_S^C](\frac{P}{ps})(1-\frac{(1-e_s)\eta}{1-\lambda}-\rho^C) ,$$

$$(1-T)A\cdot\Theta U_D'[(1-(1-E_s)\mathrm{H})L_S+(\frac{1-\Lambda-(1-E_s)\mathrm{H}}{1-\Lambda}-R^C)Q_S^C]\mathrm{H}(L_s+\frac{Q_S^C}{1-\Lambda})=\Phi'(E_S)\ . \qquad \textbf{(57)}$$

Equation (44), the equilibrium condition of the US resale market for loans becomes:

$$\alpha\tau Q_s^c + A(1-T)Q_S^C = \left[(1-A)(1-T)+(1-\alpha)T\right](1-\Lambda)L_S \quad \textbf{(58)}$$

Equation (45), the liquidity constraint in the US resale market for loan becomes:

$$\Lambda L_S \geq R^C Q^C + R^c Q^c \quad \textbf{(59)}$$

Finally, the equilibrium condition in the Eurodollar market, equation (41), becomes:

$$\frac{ps}{P} = \frac{L_s \cdot T}{l_s \cdot \tau} \quad \textbf{(60)}$$

We obtain the equilibrium in the following way. First, by inserting the left-hand side of equation (60) into equations (47), (49), (54), and (56), we can eliminate the terms of trade, (ps/P). Then we use 14 equations, i.e. (46) through (59), to solve for 14 unknowns, p_s, l_s, q_s^c, Q_s^c, ρ^c, R^c, e_s, P_s, L_s, q_s^C, Q_s^C, ρ^C, R^C and E_s. Second, using equilibrium conditions in the money market $m_s = m/p = \overline{m}/p$ and $M_s = M/P = \overline{M}/P$, we solve for p and P given m_s, M_s, $\overline{m}$, and $\overline{M}$. Finally, using equation (60), l_s, L_s, p and P, we obtain s.

Note that competition in the resale market for loans implies three possibilities: $\rho^c = \rho^C$, $q_s^c > 0$ and $q_s^C > 0$ or $\rho^c > \rho^C$; $q_s^c > 0$ and $q_s^C = 0$, or $\rho^c < \rho^C$, $q_s^c = 0$; and $q_s^C > 0$. Similar conditions apply to the US resale market for loans. Furthermore, equations (48), (49), (54), and (55) show that the discount rates have upper bounds given the level of the effort of monitoring as follows:

$$\rho^c = 1 - \frac{\theta u_d'(d^c[11])(\frac{(1-e_s)\eta}{1-\lambda})}{u_d'(d^c[00]) + \theta\{u_d'(d^c[11]) - u_d'(d^c[00])\}} \leq 1 - \frac{\theta(1-e_s)\eta}{1-\lambda}, \quad \textbf{(61)}$$

$$R^c = 1 - \frac{\Theta u_D'(D^c[11])(\frac{\Theta(1-E_s)H}{1-\Lambda})}{u_D'(D^c[00]) + \Theta\{u_D'(D^c[11]) - u_D'(D^c[00])\}} \leq 1 - \frac{\Theta(1-E_s)H}{1-\Lambda}, \quad \textbf{(62)}$$

$$R^C = 1 - \frac{\Theta U_D'(D^c[11])(\frac{\Theta(1-E_s)H}{1-\Lambda})}{U_D'(D^c[00]) + \Theta\{U_D'(D^c[11]) - U_D'(D^c[00])\}} \leq 1 - \frac{\Theta(1-E_s)H}{1-\Lambda}, \quad \textbf{(63)}$$

$$\rho^C = 1 - \frac{\theta U_d'(d^C[11])(\frac{(1-e_s)\eta}{1-\lambda})}{U_d'(d^C[00]) + \theta\{U_d'(d^C[11]) - U_d'(d^C[00])\}} \leq 1 - \frac{\theta(1-e_s)\eta}{1-\lambda}. \tag{64}$$

The first inequalities in these equations come from the fact that $u_d'(d^c[11]) > u_d'(d^c[00])$ and $U_D'(D^c[11]) > U_D'(D^c[00])$. The equalities hold when creditors have constant marginal utilities from consumption, and the price of debt reflects only the debt's default risk, given the effort of monitoring loans. Our assumption is that binding liquidity constraints, and equations (52) and (59), reduce the rate of discount below these upper bounds.

For the sake of illustration, to obtain equilibrium, consider a case where the marginal utilities of consumption from debtors' goods are constant. Then, equations (47), (50), (54) and (57) become:

$$(\frac{1}{p_s})u_c'(\bar{y} - \frac{l_s}{p_s}) = (1-\tau)(1-\theta(1-e_s)\eta) \cdot u_d' + \tau(1-\Theta(1-E_s)\mathrm{H})u_D'(\frac{ps}{P}), \tag{65}$$

$$(1-\tau)\alpha\theta \cdot u_d'\eta(l_s + \frac{q_s^c}{1-\lambda}) = \varphi'(e_s), \tag{66}$$

$$\frac{1}{P_s}U_C'(\bar{Y} - \frac{L_s}{P_s}) = (1-T)(1-\Theta(1-E_s)\mathrm{H}) \cdot U_D' + T(1-\theta(1-e_s)\eta) \cdot U_d'\frac{P}{ps}, \tag{67}$$

$$(1-T)\mathrm{A}\Theta \cdot U_D'\mathrm{H}(L_s + \frac{Q_S^C}{1-\Lambda}) = \Phi'(E_s). \tag{68}$$

Equations (65) and (67) indicate that creditors equate the marginal utility of consuming their own endowment goods with the expected utility of consuming domestic debtors' goods and foreign debtors' goods, taking into account the probability of taste shocks and debtors' default when they exercise the monitoring efforts e_s and E_s. Equations (66) and (68) says that creditors' disutility from the effort of one more unit of monitoring equals the marginal utility of consumption from their original loan and second-hand loan when the creditors depart late, consume domestic debtors' goods, and default occurs. Equations (48), (56), (49) and (55) lead to the results that $\rho^c = \rho^C = 1 - \frac{\theta(1-e_s)}{1-\lambda}\eta$ and $R^c = R^C = 1 - \frac{\Theta(1-E_s)}{1-\Lambda}\mathrm{H}$.

Therefore, the debtor and creditors' 15 conditions (46), (48), (49), (51), (52), (53), (55), (56), (58), (59), (60), and (65) through (68), we have 15 unknowns, p_s, l_s, $q_s^{\ c}$, $Q_s^{\ c}$, ρ^c, R^c, e_s, P_s, L_s, $q_s^{\ C}$, $Q_s^{\ C}$, ρ^C, R^C, E_s and s.

4. Effects of open market purchase to alleviate domestic liquidity constraints

4.1 The benchmark allocations

We consider the effects of open market purchases and central bank discount window policy following Chapman and Martin (2007). To evaluate the effects of these policy options on resource allocation, we need a benchmark resource allocation in which liquidity constraints do not bind. In such a situation, Japanese creditors' and US creditors' budget constraints would change in two ways. First, if there is no default, all creditors receive a return of 100% from their loans. Second, if there is default, early-leavers receive a return of 100% from their loans, and late-leavers receive returns that are affected by the default shock from their loans. For example, if Japanese creditors leave late and consume Japanese goods, without default, then (16) becomes $l_t = p_{t+1}[j]d_{t+1}^c[j],\ j = 00{,}01$ because $\rho_{t+1} = 1, q_{t+1}^c = 0$. With default, early-arriving debt λl_t is paid in full value, and of the $(1-\lambda)l_t$ late-arriving debt, only $(1-\lambda-(1-e_t)\eta)l_t$ arrives, hence the left-hand side of equation (17) is $\lambda l_t + (1-\lambda)[\{1-\lambda-(1-e_t)\eta\}/(1-\lambda))]l_t = \{1-\lambda-(1-e_t)\eta\}l_t$. Moreover, no creditor will use the resale market for loans in this case, and thus $q_b^{\ c} = Q_b^{\ c} = q_b^{\ C} = Q_b^{\ C} = 0$.

These considerations will change the Japanese equilibrium condition as follows:

$$\frac{1}{p_b}v_c'(\frac{l_b}{p_b}) - v_d'(\bar{x} - l_b) + \theta(1-e_b)\,\eta v_d'(l_b) = 0\,, \tag{69}$$

$$(\frac{1}{p_b})u_c'(\bar{y}-\frac{l_b}{p_b}) = \{(1-\tau)(1-\alpha)+(1-\tau)\alpha(1-\theta)\}\cdot u_d'[l_b]$$
$$+\{\tau(1-\alpha)+\tau\alpha(1-\Theta)\}\cdot u_D'[l_b(\frac{ps}{P})](\frac{ps}{P})$$
$$+(1-\tau)\alpha\cdot\{\theta u_d'[(1-(1-e_b)\eta)l_b](1-(1-e_b)\eta)\}$$
$$+\tau\alpha\cdot\left\{\Theta u_D'[(1-(1-E_b)\mathrm{H})l_b](1-(1-E_b)\mathrm{H})\frac{ps}{P}\right\} , \quad \textbf{(70)}$$

$$(1-\tau)\alpha\cdot\theta u_d'[(1-(1-e_b)\eta)l_b]\eta l_b = \varphi'(e_b)\,. \quad \textbf{(71)}$$

Similarly, US debtors' first-order conditions become:

$$\frac{1}{P_b}V_C'(\frac{L_b}{P_b}) - V_D'(\bar{X}-L_b) + \Theta(1-E_b)\,\mathrm{H}V_D'(L_b) = 0\,, \quad \textbf{(72)}$$

$$\frac{1}{P_b}U_C'(\bar{Y}-\frac{L_b}{P_b}) = \{(1-T)(1-A)+(1-T)A\cdot(1-\Theta)\}U_D'[L_b]+$$
$$\{T(1-A)+TA(1-\theta)\}\cdot U_d'[L_b\frac{P}{ps}]\frac{P}{ps}$$
$$+(1-T)A\cdot\Theta U_D'[(1-(1-E_b)\mathrm{H})L_b](1-(1-E_b)\mathrm{H})$$
$$+TA\cdot\theta U_d'[(1-(1-e_s)\eta)L_b\frac{P}{ps}](1-(1-e_s)\eta)\frac{P}{ps} , \quad \textbf{(73)}$$

$$(1-T)A\cdot\Theta U_D'[(1-(1-E_b)\mathrm{H})L_b])\mathrm{H}L_b = \Phi'(E_b)\,. \quad \textbf{(74)}$$

Finally, the equilibrium condition in the Eurodollar market becomes:

$$\frac{ps}{P} = \frac{L_b\cdot T}{l_b\cdot\tau}\,. \quad \textbf{(75)}$$

With the seven equations (69) through (75) we solve for seven unknowns in the benchmark allocation: p_b, l_b, e_b, P_b, L_b, E_b and s. We evaluate the effects of policies to the extent that a particular policy gets an equilibrium close to the above benchmark values: p_b, l_b, e_b, P_b, L_b, E_b and s.

We first verify that the benchmark allocation and the symmetric laissez-fare equilibrium defined in Section 3.6 achieve different resource allocations.

Suppose that $l_b = l_s$ and $e_b = e_s$, and therefore $\varphi'(e_b) = \varphi'(e_s)$ Then, the right hand sides of equations (50) and (71) must be equal, and thus so are the right hand sides of these equations: $u_d'[(1-(1-e_b)\eta)l_b]l_b = u_d'[(1-(1-e_s)\eta)l_s + \{1-(1-e_s)\eta/(1-\lambda)-\rho_c\}q_s]\{l_s + q_s/(1-\lambda)\}$, which holds only if $q_s = 0$. This leads to a contradiction because $\{1-(1-e_s)\eta/(1-\lambda)\} > \{1-(1-e_s)\theta\eta/(1-\lambda)\} \geq \rho_c$ as equation (61) shows and $q_s > 0$.

4.2 Market-sensitive policy and market-insensitive policy

Freeman (1999) proposes a policy that achieves equal consumption for late-leaving and early-leaving creditors, and equal risk-sharing for default by all creditors and debtors. Specifically, he proposes a commitment by the central bank to purchase all of the late-arriving debt, $[(1-\alpha)(1-\tau)+(1-A)\tau](1-\lambda)l$, in a closed economy model assuming that the default is exogenous and the level of effort of monitoring loans is always zero. This commitment, by letting q_{t+1}^{C} and q_{t+1}^{c} equal zero, equalizes the consumption of the late-leaving and early-leaving creditors.

However, following Chapman and Martin (2007), we show that the market-insensitive policy, which sets the discount rate before the market opens as Freeman (1999) suggest, does not achieve the benchmark allocation in our setup for the following reasons. First, optimal policy by a central bank needs a commitment to purchase the late-arriving debt at a discount rate of 1 if no default shock occurs, and at discount rates of $\rho^{cb}=1-\frac{\theta(1-e_b)}{1-\lambda}\eta$ and $R^{CB}=1-\frac{\Theta(1-E_b)}{1-\Lambda}\mathrm{H}$ if a default shock occurs, where subscripts *cb* and *CB* show the discount rate offered by the two central banks. Because the central bank commits to the discount rate, it cannot set the price depending on whether a default shock occurs or not. If it chooses discount rate of 1, and default shock occurs, all creditors ask for a central bank loan and then default. If it chooses ρ^{cb} and R^{CB} but a default shock does not occur, the central bank does not resolve the liquidity constraint completely. Moreover, if a central bank allows all creditors to access freely the central bank lending, creditors choose either no effort or no borrowing from the central bank. To see this point, the value of a diversified loan portfolio in a default situation with no effort involved will be equal to $\rho^{no}=\frac{(1-\lambda)(1-\eta)}{(1-\lambda)(1-\eta)+\eta}$ and

$R^{NO} = \frac{(1-\Lambda)(1-\mathrm{H})}{(1-\Lambda)(1-\mathrm{H})+\mathrm{H}}$. If $\rho^{no} < \rho^{cb}$, all creditors with unsettled debt prefer to sell it to the central bank without making effort. If $\rho^{no} > \rho^{cb}$, no creditors would like to sell their unsettled debt to the central bank, and the discount window policy does not work. Note that Freeman's (1999) commitment requires the central bank to choose the rate at which all creditors are willing to sell their unsettled debt, and the central bank chooses a discount rate of 1. If there is a liquidity shock, all creditors will ask for a central bank loan and then default. This is because under this commitment policy, $\rho^{no} < \rho^{cb} = 1$, and thus all creditors exert no effort, and the benchmark allocations cannot be supported by the market-insensitive policy.

To cope with the problem of market-insensitive policy, Chapman and Martin (2007) propose a market-sensitive policy, where a central bank randomly chooses a restricted number of creditors to compete for central bank funds (hereafter the creditors are known as "dealers"). By restricting the number of dealers, the central bank limits the moral hazard problem because the creditors know that they are more likely to accept funds from other creditors who can check the quality of loans by examining the monitoring effort undertaken by the other creditors. By letting the creditors compete for central bank funds, the central bank exploits market information to infer the state of the economy because creditors bid up the price at which they borrow until the expected value of the loan is equal to the expected value of the collateral. Chapman and Martin (2007) show that the market-sensitive policy induces all creditors to provide the effort levels e_b and E_b, and that the benchmark allocations can be supported if the probability that a creditor is chosen to be a dealer is very small.[9] The intuition is that the benefit for creditors of deviating from the effort levels e_b and E_b would be outweighed by the expected loss from the creditors not being chosen to be a dealer and they would then have to sell their loans to the other dealers, who verify the suboptimal level of effort and sharply discount their loans. The expected losses from the deviation from the effort levels e_b and E_b could be high if the probability that a creditor is chosen to be a dealer is very small.[10]

In our framework, as long as the central banks are willing to pick up some old domestic creditors and some old foreign creditors who are subject to taste shock and participating in the domestic credit market to be a dealer, we can implement the market-sensitive policy in each credit market. The market-sensitive policy, if applied in both the US and Japan, improves the welfare of equilibrium toward the benchmark allocation through two effects as Chapman and Martin (2007) suggest. First, the market-sensitive policy, if applied in both the US and Japan, mitigates the risk of liquidity shortages in domestic credit markets and the risk of default. Second, the market-sensitive policy has spillover effects to the foreign creditors. The spillover effect comes from the fact that the creditors consuming foreign debtors' goods benefit from the operations of the foreign central bank.

4.3 Market-sensitive policy in domestic credit markets and market-insensitive policy in the Eurodollar market under the financial crisis

Consider a situation in which a sudden increase in the perception of counterparty risk occurs, and the Eurodollar market shuts down. The central banks cannot choose dealers in the Eurodollar market. We assume that the domestic credit markets in the two economies

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[9] Kahn and Roberds (2009) show that restricted access to the central bank settlement system (so-called tiering) induces the selected agents to monitor the other agents. The market-sensitive policy here is an example that tiering improves the efficiency of central bank liquidity provision.

[10] Chapman, Chiu and Molico (2008) provide a theoretical model in which agents endogenously choose the degree of tiering.

function normally. Remember that Chapman and Martin (2007) consider a case of market collapse in which there are no late-leaving creditors. In their case, a central bank can restore some of the market transactions by setting a discount rate less than the rate corresponding to the no-monitoring effort level when markets work, namely, ρ^{no}. In our model, by similar reasoning, the central banks can apply the market-insensitive policy to restore some of the transactions in the Eurodollar market and apply the market-sensitive policy to transactions in the domestic credit market. We will discuss two styles of market-insensitive policies in the following sections: lending foreign currency to domestic agents against domestic collateral or lending domestic currency to foreign agents against foreign collateral.

5. Standing swap lines with the foreign central bank to supply foreign currency in the domestic market

Suppose that a change in the perception of counterparty risk occurs, for example, because of the sudden increase in the perceptions in the default risk of foreign creditors as we have seen in September–October 2008, and the Eurodollar market shuts down. Then, τ Japanese creditors and T US creditors cannot exchange their IOUs at the Eurodollar market, and thus cannot consume foreign goods. The closure of the Eurodollar market clearly reduces welfare in both economies.

In such a situation, central banks can introduce standing swap lines, and supply their own currencies abroad against their domestic collateral, to replicate the transactions in the Eurodollar market. They can proceed using the following steps, as summarized in Figure 6 and Figure 7.

First, τ Japanese old creditors and T US old creditors identify their taste shock, and they go to the domestic market, rather than the Eurodollar market.

Second, domestic central banks accept all of the national IOUs in exchange for foreign currency, using the foreign currency obtained from the swap agreements with the other central banks. The exchange rate must satisfy equation (60). However, the discount rate for the acceptance of national IOUs must be a discount rate that corresponds to the market-insensitive policy; namely, a discount rate slightly lower than $\rho^{no} = \frac{(1-\lambda)(1-\eta)}{(1-\lambda)(1-\eta)+\eta}$ and $R^{NO} = \frac{(1-\Lambda)(1-\mathrm{H})}{(1-\Lambda)(1-\mathrm{H})+\mathrm{H}}$.[11]

Third, τ Japanese old creditors and T US old creditors visit the debtor island, without participating in the US market and Japanese market for loans. Central banks use the market-sensitive policy in their domestic credit markets, in which some of the domestic creditors work as dealers for central bank loans to encourage monitoring.

Finally, central banks collect debts by obtaining domestic currency from old debtors. Note that, without default, central banks collect larger amounts of currency than they lend to the other central bank through the swap agreement as Figure 7 shows. With default, central banks would collect smaller amounts of currency than they lend to the other central banks through the swap agreement. In this way, the domestic price level would be affected by the entry of foreign creditors, and the changes in the price level distribute the default risk to all agents consuming domestic goods.

Nonetheless, the standing swap lines and supply of domestic currencies abroad by central banks against their domestic collateral provide higher welfare than for the situation of shutdown of the Eurodollar market, which leads to no consumption of foreign goods. However, central banks' intervention cannot achieve the level of efficiency that the foreign

[11] Antoine (2009, p.411) discusses the effect of penalty rates to screen banks' incentive to borrow too many international reserves under the commodity standard. In our model, discount rates slightly lower than ρ^{no} and R^{NO} screen agents' incentive to trade with central banks or trade with each other in the Eurodollar market.

currency market transactions under normal conditions achieve because central banks cannot use the market-sensitive policy in the Eurodollar market.

6. Cross-border collateral arrangements

This section considers how the acceptance of cross-border collateral arrangements under the closure of the Eurodollar market improves welfare. This acceptance can proceed in the following way, as summarized in Figure 8 and Figure 9.

First, τ Japanese old creditors and T US old creditors identify their taste shock, deposit their IOUs in their domestic central banks, and go to the foreign market.

Second, foreign central banks exchange these old creditors' IOUs into their own currency. These operations became possible by cross-border collateral arrangements, such as the correspondent central banking model (CCBM), links between securities settlement systems and remote access to a securities settlement system. Under the cross-border collateral arrangements, each central bank serves as a custodian of international collateral, and provides the other central bank with information on the discount rate for the acceptance of national IOUs. The discount rate must be the one that corresponds to the market-insensitive policy; namely, a discount rate slightly lower than $\rho^{no} = \frac{(1-\lambda)(1-\eta)}{(1-\lambda)(1-\eta)+\eta}$ and $R^{NO} = \frac{(1-\Lambda)(1-\mathrm{H})}{(1-\Lambda)(1-\mathrm{H})+\mathrm{H}}$ and an exchange rate that satisfies equation (60).

Third, τ Japanese old creditors and T US old creditors visit the debtor island, without participating in the US market for loans and Japanese market for loans, respectively. Central banks use the market-sensitive policy in their domestic credit markets, in which some of the domestic creditors work as dealers for central bank loans to encourage monitoring.

Finally, central banks collect debts by obtaining domestic currency from old domestic debtors, on behalf of the other central banks. Note that without default, central banks collect larger amounts of currency than they lend to the other central banks through the swap agreement. With default, central banks collect smaller amounts of currency than they lend to the other central banks through the swap agreement. The domestic price level would be affected by the entry of foreign creditors, and the changes in the price level distribute the default risk to all agents consuming domestic goods.

In this way, the acceptance of cross-border collateral arrangements provides higher welfare than the situation in which shutdown of the Eurodollar market leads to no consumption of foreign goods. However, central banks' intervention cannot achieve the level of efficiency that the foreign currency market transactions under normal conditions achieve because central banks cannot use the market-sensitive policy in the Eurodollar market.

7. Discussion

7.1 Policy coordination

The two temporary policy measures, the standing swap lines with the foreign central bank to supply foreign currency in the domestic market and the acceptance of cross-border collateral arrangements, are Pareto improving for both economies, and thus both central banks are willing to cooperate. The results in Sections 5 and 6 show that policy coordination between central banks in the area of money market operations could yield substantial welfare gains in a financial crisis.[12] The results contrast sharply with the influential results of Obstfeld and Rogoff (2002), stating that the marginal welfare gains in the policy coordination of monetary policies are small, and thus central banks do not have incentive to cooperate.

[12] See Shirakawa (2009) on this point.

7.2 *Are the welfare gains from the two temporary policy measures different?*

As a comparison of Figure 7 and Figure 9 shows, the two temporary policy measures differ only on one point: whether central banks swap domestic currencies or collateral in step (2). For both temporary policy measures, the ultimate risk of default comes from the domestic debtors' default risk and the moral hazard of domestic creditors. Therefore, in our framework, as long as the nature of the domestic default risk and moral hazard is well known, and the two central banks are willing to share this information, these two temporary measures have the same welfare-improving effects.

7.3 *Entry into and exit from the market-insensitive policy in practice*

Do financial institutions have incentive not to rely on the market-insensitive policy as market conditions improve? To think about this question, suppose that the confidence of the counterparty is revived, and both the market transactions explained in Section 5 and the central bank measures explained in Section 6 are available. As we have explained in Section 4.2, the market-insensitive policy that replicates the transactions in the Eurodollar market; namely, a discount rate slightly lower than $\rho^{no} = \frac{(1-\lambda)(1-\eta)}{(1-\lambda)(1-\eta)+\eta}$ and $R^{NO} = \frac{(1-\Lambda)(1-\mathrm{H})}{(1-\Lambda)(1-\mathrm{H})+\mathrm{H}}$, would not be used if the market transactions recover because it is cheaper for creditors to exchange their loans bilaterally and join the domestic credit market, than to get central bank loans in foreign currency. Therefore, the market-insensitive policy induces market participants to trade with each other in the Eurodollar market if they think the Eurodollar market has recovered to function normally.

While in practice a central bank may not be sure whether or not the appropriate discount rate remains the same before and after the financial crisis, as long as the origins of crisis lie in the sudden increase in the counterparty risks because of temporary changes in the perception of risks by the market participants, our model is consistent with the practice taken by the non-US central banks. They set rates applied to the US dollar funds-supply operations above the market rates before the financial crises, and discourage the use of the operations supplying US dollar funds as the situation within financial markets improves. For example, BOJ's operations supplying US dollar funds began with the market-sensitive policy in September 2008, and then moved on to the market-insensitive policy in October 2008, and achieved an exit without returning to market-sensitive policy in February 2010.[13]

7.4 Reservations

Our result that both the standing swap lines with the foreign central bank supplying foreign currency in the domestic market and the acceptance of cross-border collateral arrangements have the same welfare-improving effects during the financial crisis, depends on several assumptions of the model.

First, accepting high-quality marketable collateral denominated in foreign currencies or held in foreign locations has been a permanent measure in some central banks. The acceptance is motivated by an irreversible global shift towards real-time gross settlement of central bank payment systems, which requires large amounts of collateral for internationally active banks. In our model, the acceptance of cross-border collateral arrangements is a temporary measure to replace the transactions in the Eurodollar market during the liquidity

[13] The BOJ conducted the following operations supplying US dollar funds. First, on September 18, 2008, BOJ announced operations supplying US dollar funds against pooled eligible collateral with loan rates determined by multiple-rate competitive auctions, which could be interpreted as a market-sensitive policy. The rate should not fall below the rate set by the Federal Reserve Bank of New York as a prevailing USD Overnight Indexed Swap market rate that corresponds to the duration of the loan. Second, as financial market conditions worsened, on October 14, 2008, the BOJ, introduced operations supplying US dollar funds whereby funds are provided at a fixed rate set for each operation for unlimited amounts against pooled collateral (a market-insensitive policy). The BOJ's operations supplying US dollar funds supplied as much as 122,716 million dollars as of December 31, 2008, at its peak. Later, as financial market functioning improved, the amount of US dollars supplied decreased dramatically. In December 2009, the amount of US dollars outstanding from the BOJ's US dollar fund-supplying operations fell to 545 million dollars. The last scheduled operation by the BOJ on January 12, 2010, provided as little as 100 million dollars. Given those developments, on January 28, 2010, the BOJ announced that it would stop operations supplying US dollar funds against pooled collateral, because the swap lines were no longer needed given the improvements in financial market functioning.

shortages which take the form of a shutdown of the collateral swap market. If the central bank interventions are permanent, the stationary equilibrium and the corresponding central bank policies could change forever.

Second, even restricting our attention to temporary measures in the financial crisis, two measures require different transaction costs in practice, and there are no reasons to believe that they coexist. For example, in the case of acceptance of cross-border collateral arrangements, the transaction costs may include the cost of understanding foreign legal systems among the economies, to evaluate the value of collateral.

Third, our model assumes that the use of cross-border collateral requires a central bank only their knowledge about the appropriate domestic discount rate for the market insensitive policy that reflects domestic default risks and the moral hazard by the domestic creditors, and supposes that central banks could easily share these information relevant for the market insensitive policy with other central banks. In practice, the acceptance of cross-border collateral requires risk management of foreign bonds by a central bank, and understanding the correct value of collateral or the discount rate, and the equilibrium nominal exchange rate s on a real-time basis could be difficult for central banks.

Finally, our model assumes symmetric supply of foreign currency loans made by the central banks and symmetric acceptance of cross-border collateral by the central banks. In practice, only US dollars and euros are supplied by foreign central banks. Some central banks unilaterally accept collateral denominated in foreign currency and issued in foreign countries, for example, the US Federal Reserve.

8. Conclusion

We showed central banks improve the efficiency of equilibrium under foreign currency liquidity shortages through two temporary policy measures: lending foreign currency to domestic creditors against domestic collateral or lending domestic currency to foreign creditors against foreign collateral, in a two-country model. Consider a situation in which the liquidity shortages take the form of a shutdown of the collateral swap market, which is the essential market for obtaining financial assets denominated in foreign currency. The central banks in the two economies should take counterparty risks in the collateral swap market in one of the two temporary policy measures. Those temporary policy measures help central banks in the two economies replicate a part of the functioning of the collateral swap market. These two temporary policy measures, supplemented by these central banks' market-sensitive policy in their domestic credit markets, clearly improve the welfare of both economies and thus are incentive compatible to both central banks. The two temporary policy measures, however, cannot achieve the level of efficiency that the foreign currency market transactions under normal conditions achieve because central banks cannot use market-sensitive policy.

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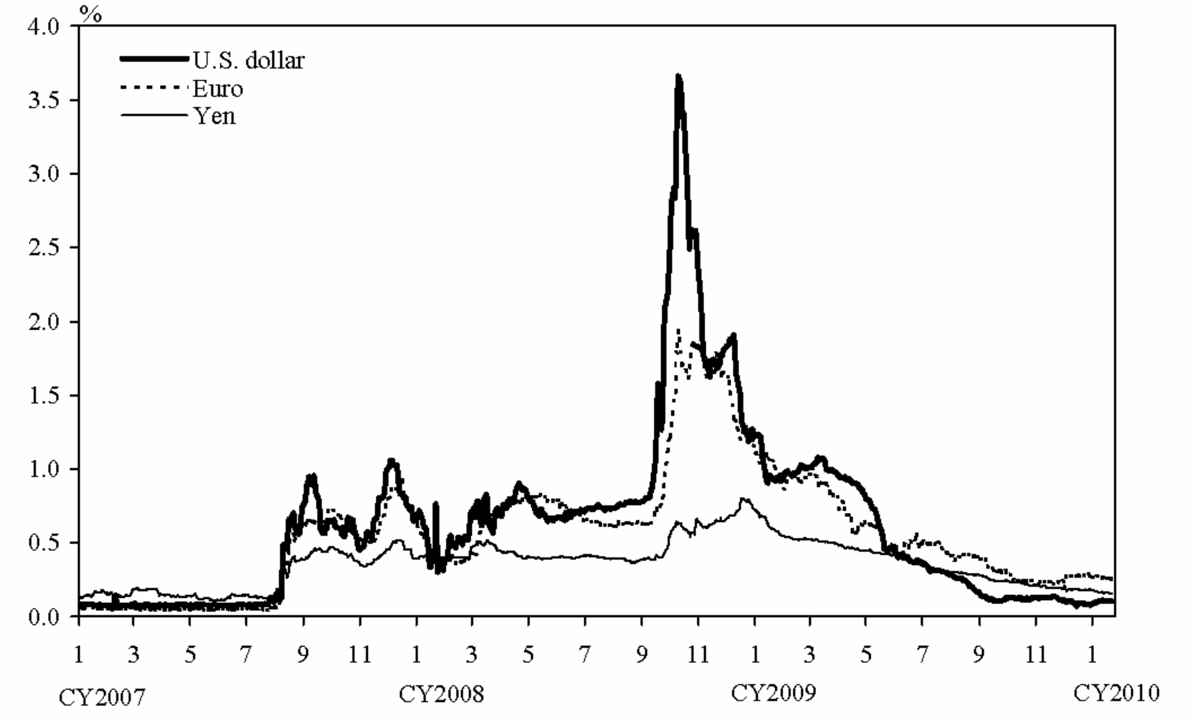
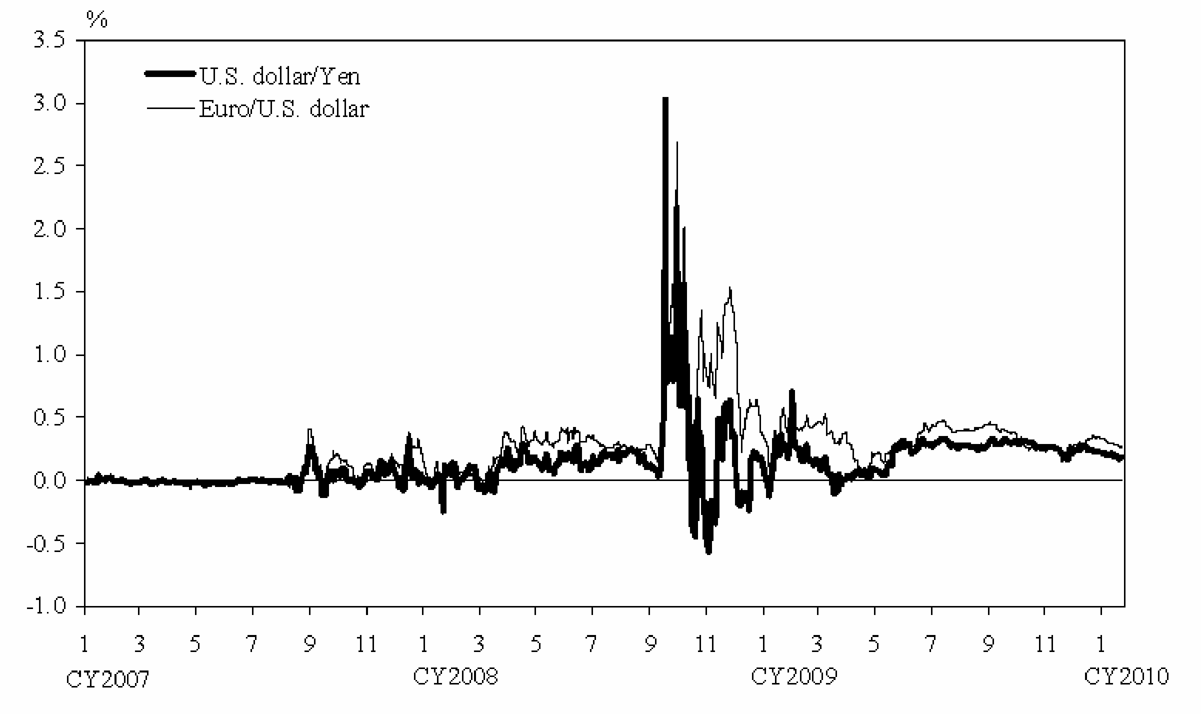
Figure 1 LIBOR–OIS spreads



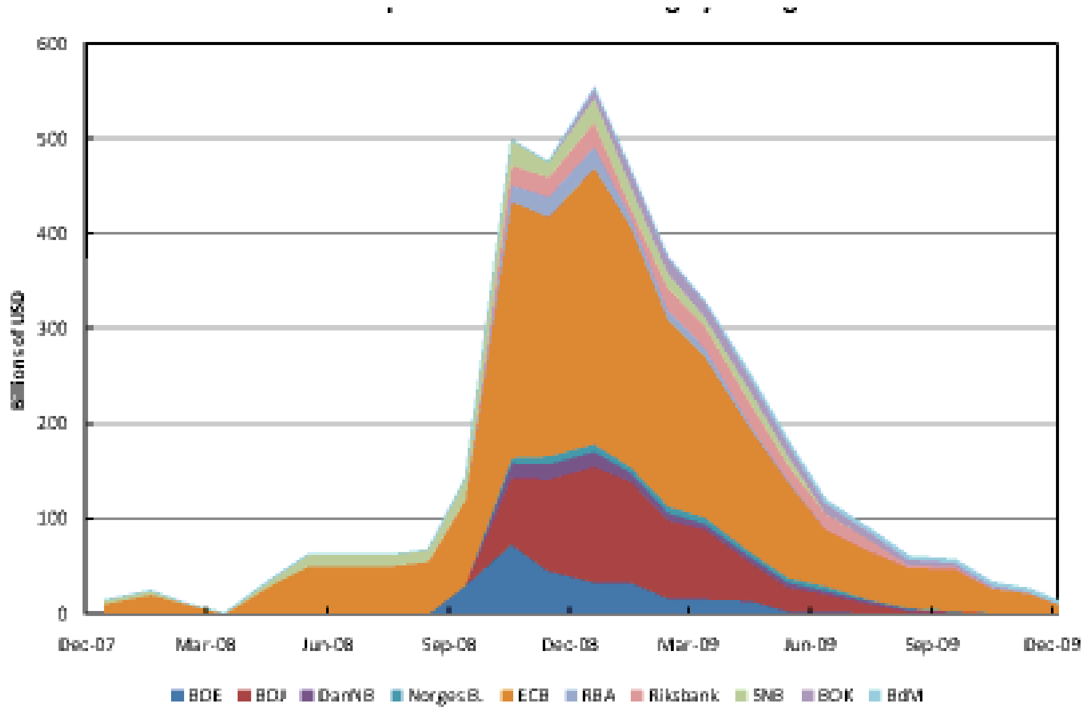
Source: The Bank of Japan, Monthly Report of Recent Economic and Financial Developments, January 2010, Chart 29.

Figure 2 FX-swap-implied dollar rate–LIBOR spreads (3-month data)



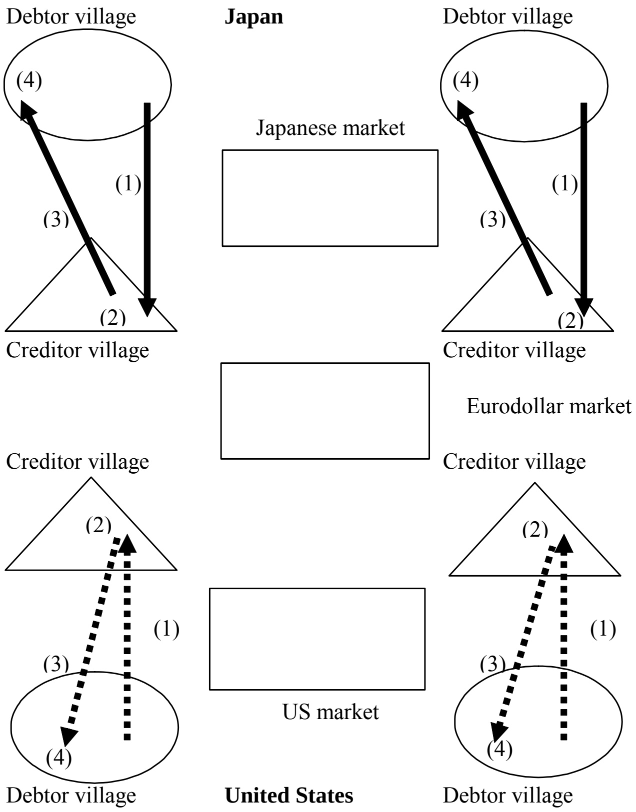
Source: The Bank of Japan, Monthly Report of Recent Economic and Financial Developments, January 2010, Chart 29.

Figure 3 Central bank dollar swap amounts outstanding by foreign central banks



Source: Goldberg, Kennedy and Miu (2010), Chart 1.

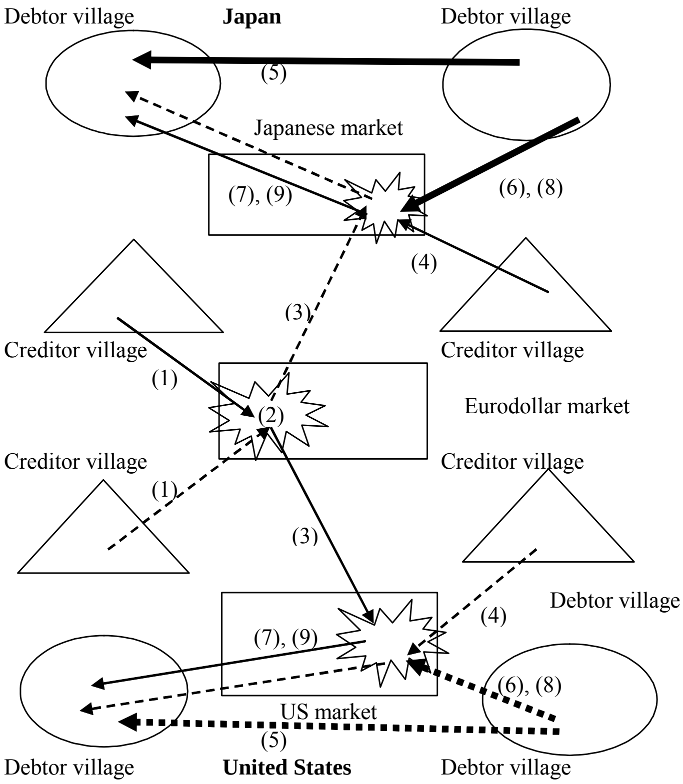
Figure 4 Pattern of travel when young



(1) Young debtor goes to creditor village.
(2) Young debtor meets young creditor to obtain goods in exchange for debt.
(3) Young debtor goes back his/her village.
(4) Young debtor meets old creditor or defaulted old debtor to obtain currency in exchange for goods.

Note: Thick line shows debtors, solid line shows Japanese, and dotted line shows US resident.

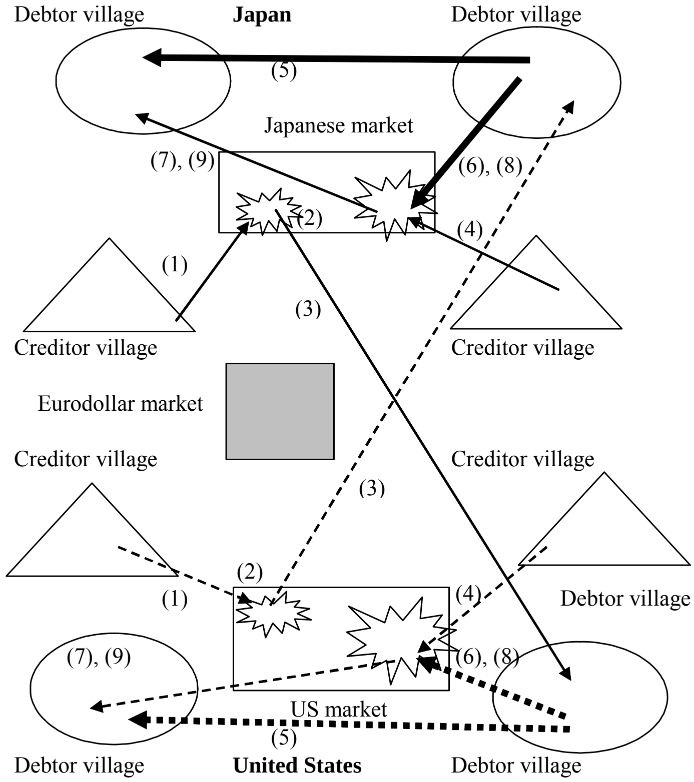
Figure 5 Pattern of travel when old



(1) τ and T of old creditors identify their taste shock, go to the Eurodollar market.
(2) They exchange their domestic IOUs for foreign IOUs.
(3) τ and T of old creditors go to the foreign debtor village.
(4) All other creditors visit their home market.
(5) With probability θ and Ξ, $(1-\varepsilon)\eta$ and $(1-E)H$ old debtors default, go to young debtors village.
(6) Arrival rate of honest debtor in the home market (λ) is lower than the departure rate of creditors $(1-\alpha)(1-\tau)+(1-A)T$.
(7) Debts are repaid with domestic currency, and early departing old creditors visit debtor village.
(8) $(1-\lambda)$ or $(1-\lambda-(1-e)\eta)$ debtors arrive at the home market, and late departing old creditors identify if default occurred.
(9) Debts are repaid with domestic currency, and late departing old creditors visit debtor village.

Note: Thick line shows debtors, solid line shows Japanese, and dotted line shows US resident.

Figure 6 Pattern of travel when old with foreign currency denominated funds-supplying operations



(1)	τ and T of old creditors identify their taste shock, go to the domestic market.
(2)	Domestic central banks exchange their domestic IOUs into foreign currency.
(3)	τ and T of old creditors go to the foreign debtor village.
(4)	All other creditors visit their home market.
(5)	With probability θ and Ξ, $(1-\varepsilon)\eta$ and $(1-E)H$ old debtors default, go to young debtors village.
(6)	Arrival rate of honest debtor in the home market (λ) is lower than the departure rate of creditors $(1-\alpha)(1-\tau)+(1-A)T$.
(7)	Debts are repaid with domestic currency, and early departing old creditors visit debtor village.
(8)	$(1-\lambda)$ or $(1-\lambda-(1-e)\eta)$ debtors arrive at the home market, and late departing old creditors identify if default occurred.
(9)	Debts are repaid with domestic currency, and late departing old creditors visit debtor village.

Note: Thick line shows debtors, solid line shows Japanese, and dotted line shows US resident.

Figure 7 Pattern of settlement with foreign currency denominated funds-supplying operations

(1) τ and T of old creditors identify their taste shock, go to the domestic market.

(2) Domestic central banks exchange their domestic IOUs into foreign currency.
Central banks swap their domestic currencies into foreign currencies.

Japanese IOU US $ US IOU
Japanese old creditor ⇄ BOJ ⇄ Fed ⇄ US old creditors
US $ Yen Yen

(3) τ and T of old creditors go to the foreign debtor village.

Japanese old creditors —US $→ US young debtors
US old creditors —Yen→ Japanese young debtors

(4) All other creditors visit their home market.

(5) With probability θ and Ξ, $(1\text{-}\varepsilon)\eta$ and $(1\text{-}E)H$ old debtors default, go to young debtors village.

(6) Arrival rate of honest debtor in the home market (λ) is lower than the departure rate of creditors (1-α)(1-τ)+(1-A)T.

(7) Debts are repaid with domestic currency, and early departing old creditors visit debtor village.

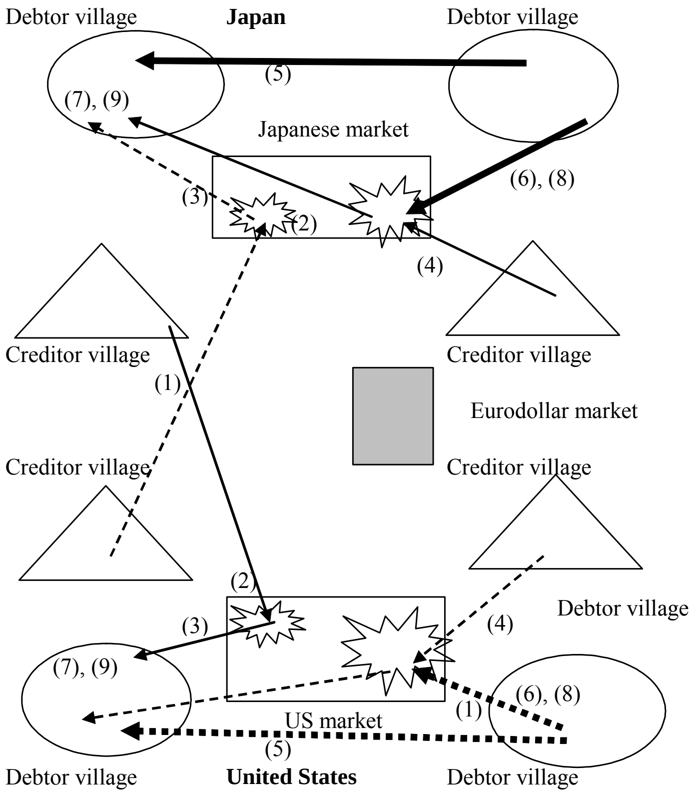
(8) (1-λ) or (1-λ-(1-e)η) debtors arrive at the home market, and late departing old creditors identify if default occurred.

(9) Debts are repaid with domestic currency, and late departing old creditors visit debtor village.

BOJ Fed
JPY ↑ ↓ Japanese IOU US $ ↑↓ US IOU
Japanese young debtors US young debtors

Currencies swapped in (1) and (2) will be returned to central banks in (5) in the next period.

Figure 8 Pattern of travel when old with cross-border collateral arrangements



(1) τ and T of old creditors identify their taste shock, go to the foreign market.
(2) Foreign central banks exchange their domestic IOUs into foreign currency.
(3) τ and T of old creditors go to the foreign debtor village.
(4) All other creditors visit their home market.
(5) With probability θ and Ξ, $(1-\varepsilon)\eta$ and $(1-E)H$ old debtors default, go to young debtors village.
(6) Arrival rate of honest debtor in the home market (λ) is lower than the departure rate of creditors $(1-\alpha)(1-\tau)+(1-A)T$.
(7) Debts are repaid with domestic currency, and early departing old creditors visit debtor village.
(8) $(1-\lambda)$ or $(1-\lambda-(1-e)\eta)$ debtors arrive at the home market, and late departing old creditors identify if default occurred.
(9) Debts are repaid with domestic currency, and late departing old creditors visit debtor village.

Note: Thick line shows debtors, solid line shows Japanese, and dotted line shows US resident.

Figure 9 Pattern of settlement with cross-border collateral arrangements

(1) τ and T of old creditors identify their taste shock, go to the foreign market.

(2) Foreign central banks exchange their domestic IOUs into foreign currency.
Central banks swap their domestic IOUs into foreign IOUs.

Japanese old creditors US $ Yen US old creditors
Japanese IOU BOJ Fed US IOU

(3) τ and T of old creditors go to the foreign debtor village.

Japanese old creditors US $ Yen US old creditors
Japanese young debtors US young debtors

(4) All other creditors visit their home market.

(5) With probability θ and Ξ, $(1\text{-}\varepsilon)\eta$ and $(1\text{-}E)H$ old debtors default, go to young debtors village.

(6) Arrival rate of honest debtor in the home market (λ) is lower than the departure rate of creditors (1-α)(1-τ)+(1-A)T.

(7) Debts are repaid with domestic currency, and late departing old creditors visit debtor village.

(8) (1-λ) or (1-λ-(1-e)η) debtors arrive at the home market, and late departing old creditors identify if default occurred.

(9) Debts are repaid with domestic currency, and late departing old creditors visit debtor village.

BOJ Fed
JPY Japanese IOU US $ US IOU
Japanese young debtors US young debtors

IOUs swapped in (1) and (2) will be settled by central banks.