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Abstract

How should monetary policy cooperation be designed when more than one country simultaneously faces zero lower bounds on nominal interest rates? To answer this question, we examine monetary policy cooperation with both optimal discretion and commitment policies in a two-country model. We reach the following conclusions. Under discretion, monetary policy cooperation is characterized by the intertemporal elasticity of substitution (IES), a key parameter measuring international spillovers, and no history dependency. On the other hand, under commitment, monetary policy features history dependence with international spillover effects.

Keywords: Optimal Monetary Policy Cooperation; Zero Lower Bound
JEL classification: E52, F33, F41

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1 Introduction

The world economy now faces the largest economic downturn since World War II. To prevent the economy from deteriorating further, most central banks in developed economies simultaneously reduced policy interest rates to unprecedented low levels at speeds not previously seen. As shown in Figure 1, the Bank of Japan (BOJ), the Bank of England (BOE), and the Federal Reserve Board (FRB) have virtually cut their policy rates to the lowest possible level.[1] Under such circumstances, with very little room for further monetary easing, how should monetary policy cooperation be designed? Is there any role for the home (foreign) central bank to assist the inactive foreign (home) monetary policy in the presence of the zero lower bound?

Reflecting on the recent economic experience in Japan, there have been several studies on monetary policy in the presence of a liquidity trap. Most, however, focus on the closed economy. For example, Reifschneider and Williams (2000), Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005), Adam and Billi (2006, 2007), and Nakov (2008) outline the characteristics of desirable monetary policy under a zero lower bound on nominal interest rates for a closed economy. Regarding the liquidity trap in the open economy, Svensson (2001, 2003) and Coenen and Wieland (2003) investigate the zero interest rate policy in open economies and stress the importance of the depreciation of nominal exchange rates for a country caught in a liquidity trap. On the other hand, Nakajima (2008) shows that nominal exchange rates should appreciate for a country adopting a zero interest rate policy under optimal commitment. These studies, however, only consider the situation where a single country is at the zero lower bound. They do not provide a framework capable of examining the current global situation. As far as we know, there

[1] "Lowering of the Bank's target for the uncollateralized overnight call rate by 20 basis points; it will be encouraged to remain at around 0.1 percent" (December 12, 2008, Statements on Monetary Policy, BOJ); "The Bank of England's Monetary Policy Committee today voted to reduce the official Bank Rate paid on commercial bank reserves by 0.5 percentage points to 0.5% ..." (March 5, 2009, News Release, BOE); "The Committee will maintain the target range for the federal funds rate at 0 to 1/4 percent and anticipates that economic conditions are likely to warrant exceptionally low levels of the federal funds rate for an extended period" (March 18, 2009, Press Release, FRB). The European Central Bank (ECB) and the Bank of Canada (BOC) have also set their policy rates at very low levels.

have been no studies on the desirable conduct of monetary policy when more than one country is simultaneously facing zero lower bounds on nominal interest rates.

The design of optimal monetary cooperation in the presence of the zero lower bound is more complicated in an open economy under the zero lower bound. For example, we need to consider possible gains from policy cooperation when only one country moves away from the zero lower bound. We set up a two-country dynamic general equilibrium model where both countries are at the zero lower bound because of temporary decreases in the natural rate of interest. We provide a tractable framework for the analysis of monetary policy cooperation with both discretion and commitment under the Markov equilibrium used in Eggertsson and Woodford (2003). We consider a case where central banks set their policy interests depending only on the state of the economy. Consequently, the dynamic model considered in this paper is reduced to a finite number of linear simultaneous equations. In our paper, the optimal monetary policy, which is obtained by minimizing the quadratic social loss under policy cooperation, is characterized by an optimally chosen home policy interest rate when the home country is free from the zero lower bound while the foreign country is subject to the zero lower bound.

Our main conclusions are as follows. Under discretion, monetary policy cooperation is influenced by the intertemporal elasticity of substitution (IES), a key parameter measuring international spillover. The optimal exit policy becomes different greatly depending on whether IES is larger or smaller than unity. When home goods and foreign goods are complements (substitutes), that is, when the inverse of IES is smaller (greater) than unity, the country that has escaped the liquidity trap earlier chooses expansionary (contractionary) monetary policy to boost economic activity in the country that is still caught in the liquidity trap. However, discretionary policy does not have history dependence regardless of the value of IES. Under commitment, optimal policy cooperation has history dependence. By committing to easing future monetary conditions, the two central banks mitigate the effects of the adverse shocks. Meanwhile, the level of policy interest rates change with IES.

Admittedly, our conclusion is not independent from several important assumptions. First, preferences of agents in the home country and foreign country are identical. Second, we assume that there are only home goods and foreign goods in the economy, and that there are no nontradeable goods. Third, there is a complete international financial market so that agents in both countries can achieve perfect risk sharing. Fourth, agents in both countries set prices in their own currency ("producer-currency pricing"). Thanks to these assumptions that are used commonly in the literature, such as in Clarida, Gali, and Gertler (2002), we can provide an intuitive description of the monetary policy.

The remainder of the paper is as follows. In Section 2, we describe the two-country model used for analysis in this paper. Section 3 clarifies the equilibrium concept and how a dynamic two-country model can be represented by analytically tractable static equations. Section 4 investigates the nature of the optimal monetary policy under discretion, and Section 5 inquires into that under commitment. Finally, Section 6 summarizes the findings in this paper and refers to future possible extensions.

2 The Model

The two-country model considered in this paper is very standard, as found in Clarida, Gali, and Gertler (2002) and Benigno and Benigno (2003). Because we adopt the assumption of complete international financial markets and assume a Markov equilibrium, it is convenient to make use of history notation. That is, let $s_t \in S$ denote all the possible states of the world that can occur in period t. Let

$$s^t = (s_0, s_1, ..., s_t)$$

denotes the history up until period t of the realized states of the world. The set of states, S, contains only two elements. One is associated with a low level of the natural rate of interest and the other is associated with a "normal" level of the natural rate of interest. We denote the probability of history s^t by $\mu\left(s^t\right)$.

2.1 Households

A representative household in the home country H has the following preferences:

$$\sum_{t=0}^{\infty} \beta^t \sum_{s^t} \mu\left(s^t\right) \left\{u\left[C\left(s^t\right)\right] - v\left[h\left(s^t\right)\right]\right\},$$

where $0 < \beta < 1$. $C\left(s^t\right)$ and $h\left(s^t\right)$ denote consumption and the supply of labor in history s^t, respectively. The household budget constraint is given by

$$W\left(s^t\right) h\left(s^t\right) + \Pi\left(s^t\right) + B\left(s^{t-1}\right) \geq \sum_{s_{t+1}} Q\left(s_{t+1}, s^t\right) B\left(s_{t+1}, s^t\right) + P\left(s^t\right) C\left(s^t\right),$$

where $W\left(s^t\right)$ and $\Pi\left(s^t\right)$ denote the wage rate and lump-sum profits and taxes in home currency units. Furthermore, the object $B\left(s_{t+1}, s^t\right)$ is an Arrow security. It is the quantity of home currency to be delivered in period $t+1$ if state s_{t+1} is realized, conditional on history s^t. The associated price is $Q\left(s_{t+1}, s^t\right)$. Finally, $P\left(s^t\right)$ denotes the price of consumption goods.

Aggregate consumption is given by

$$C\left(s^t\right) = \left[\frac{C_H\left(s^t\right)}{n}\right]^n \left[\frac{C_F\left(s^t\right)}{1-n}\right]^{1-n}, \tag{1}$$

where $0 \leq n \leq 1$ is the relative size of the home country. $C_H\left(s^t\right)$ and $C_F\left(s^t\right)$ denote the consumption of home and imported goods in the home country, respectively. Furthermore, $C_H\left(s^t\right)$ and $C_F\left(s^t\right)$ are defined as

$$C_H\left(s^t\right) = \left[\int_0^1 C_H\left(s^t, i\right)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

and

$$C_F\left(s^t\right) = \left[\int_0^1 C_F\left(s^t, i\right)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $\varepsilon > 1$.

Similarly, the lifetime utility of a representative household in the foreign country, F, is defined by

$$\sum_{t=0}^{\infty} \beta^t \sum_{s^t} \mu\left(s^t\right) \left\{u\left[C^*\left(s^t\right)\right] - v\left[h^*\left(s^t\right)\right]\right\},$$

where superscript $*$ denotes foreign variables. The budget constraint is given by

$$W^*\left(s^t\right) h^*\left(s^t\right) + \Pi^*\left(s^t\right) + \frac{B^*\left(s^{t-1}\right)}{\mathcal{E}\left(s^t\right)} \geq \sum_{s_{t+1}} \frac{Q\left(s_{t+1}, s^t\right) B^*\left(s_{t+1}, s^t\right)}{\mathcal{E}\left(s^t\right)} + P^*\left(s^t\right) C^*\left(s^t\right).$$

In the above expression, $\mathcal{E}\left(s^t\right)$ denotes the exchange rate, namely, units of home currency per unit of foreign currency.

The household maximizes utility subject to its budget constraint, taking as given prices, wages, exchange rates, and rates of return.

2.2 Firms

The i^{th}, $i \in (0,1)$, intermediate good is produced by a monopolist using the following technology:

$$Y\left(s^t,i\right) = Z\left(s^t\right) h\left(s^t,i\right),$$

where $Z\left(s^t\right)$ is the common technology and the only stochastic disturbance. The marginal cost of production for the i^{th} monopolist is given by

$$MC\left(s^t\right) = \left[1-\tau\left(s^t\right)\right] \frac{W\left(s^t\right)}{Z\left(s^t\right) P_H\left(s^t\right)}, \tag{3}$$

where $\tau\left(s^t\right)$ denotes a tax subsidy associated with the supply of labor, financed by a lump-sum tax on households. The i^{th} monopolist maximizes profits subject to its demand curve derived from consumer preferences as in equation (2), and the Calvo (1983) price frictions. In particular, the monopolist may optimize its price, $P\left(s^t,i\right)$, with probability $1-\theta$, and with probability θ it sets its price as follows:

$$P\left(s^t,i\right) = P\left(s^{t-1},i\right).$$

We assume similar production technology for the foreign country as well.

2.3 Market Clearing

Clearing in the labor market requires

$$h\left(s^t\right) = \int_0^1 h\left(s^t,i\right) \mathrm{d}i.$$

Clearing in the home homogeneous goods market requires

$$nY\left(s^t\right) = nC_H\left(s^t\right) + (1-n)\,C_H^*\left(s^t\right). \tag{4}$$

Clearing in financial markets requires

$$B\left(s_{t+1},s^t\right) + B^*\left(s_{t+1},s^t\right) = 0.$$

Following the argument in Yun (2005), output of the homogeneous home good is related to aggregate employment by

$$Y\left(s^t\right) = \Delta\left(s^t\right) Z\left(s^t\right) h\left(s^t\right), \tag{5}$$

where the relative price distortion term $\Delta\left(s^t\right)$ is

$$\Delta\left(s^t\right) = \int_0^1 \left[\frac{P_H\left(s^t, i\right)}{P_H\left(s^t\right)}\right]^{-\varepsilon} di.$$

Under the Calvo price frictions, its dynamics are defined by

$$\Delta\left(s^t\right) = \frac{1}{(1-\theta)\left\{\frac{1-\theta[1+\pi_H(s^t)]^{\varepsilon-1}}{1-\theta}\right\}^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta[1+\pi(s^t)]^{\varepsilon}}{\Delta(s^{t-1})}},$$

where

$$\pi_H\left(s^t\right) = \frac{P_H\left(s^t\right)}{P_H\left(s^{t-1}\right)} - 1.$$

We have similar clearing conditions for the foreign country.

2.4 Financial Market Equilibrium Condition

From the first-order necessary conditions with respect to holdings of Arrow securities, we have

$$\frac{u'\left[C^*\left(s^{t+1}\right)\right] q\left(s^t\right)}{u'\left[C^*\left(s^t\right)\right] q\left(s^{t+1}\right)} = \frac{u'\left[C\left(s^{t+1}\right)\right]}{u'\left[C\left(s^t\right)\right]},$$

where the real exchange rate $q\left(s^t\right)$ is defined by

$$q\left(s^t\right) = \frac{\mathcal{E}\left(s^t\right) P^*\left(s^t\right)}{P\left(s^t\right)}.$$

Under the symmetric preferences assumption, the real exchange rate is always unity. As a result, equilibrium relative consumption also equals unity with suitable initial wealth conditions under a unit elasticity of substitution between home and foreign goods,[2] namely,

$$C\left(s^t\right) = C^*\left(s^t\right). \tag{6}$$

2.5 Equilibrium Conditions

We adopt a standard sequence-of-markets equilibrium concept, using the following functional forms:

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma},\ v(h) = \frac{h^{1+\omega}}{1+\omega},$$

where $\sigma, \omega > 0$. It is important to note that, in our model, σ equals to the inverse of the intertemporal elasticity of substitution (equivalently the degree of risk aversion) following

[2] For a formal proof of this point, see the proposition in Nakajima (2008).

the notation of Rotemberg and Woodford (1997) and not the notation of Eggertsson and Woodford (2003) where σ is the intertemporal elasticity of substitution. From the first-order necessary conditions in the optimization problem mentioned above, we can derive the system of log-linearized equations as follows.[3] The aggregate supply conditions are given by the new Keynesian Phillips curves:

$$\pi_H\left(s^t\right) = \gamma_H x_H\left(s^t\right) + \gamma_{H,F}\left(1-n\right) x_F\left(s^t\right) + \beta \sum_{s^{t+1}} \mu\left(s^{t+1}\right) \pi_H\left(s^{t+1}\right), \tag{7}$$

for the home country, and

$$\pi_F^*\left(s^t\right) = \gamma_{H,F} n x_H\left(s^t\right) + \gamma_F x_F\left(s^t\right) + \beta \sum_{s^{t+1}} \mu\left(s^{t+1}\right) \pi_F^*\left(s^{t+1}\right), \tag{8}$$

for the foreign country, where

$$\begin{aligned}
\gamma_H &= \frac{\left(1-\theta\right)\left(1-\beta\theta\right)\left[1+\omega+\left(\sigma-1\right)n\right]}{\theta\left(1+\omega\varepsilon\right)}, \\
\gamma_F &= \frac{\left(1-\theta\right)\left(1-\beta\theta\right)\left[1+\omega+\left(\sigma-1\right)\left(1-n\right)\right]}{\theta\left(1+\omega\varepsilon\right)},
\end{aligned}$$

and

$$\gamma_{H,F} = \frac{\left(1-\theta\right)\left(1-\beta\theta\right)\left(\sigma-1\right)}{\theta\left(1+\omega\varepsilon\right)}.$$

For aggregate demand conditions, we derive the dynamic IS curves as follows:

$$\begin{aligned}
i_H\left(s^t\right) &= \sum_{s^{t+1}} \mu\left(s^{t+1}\right) \left\{ \begin{array}{c} \left[1+\left(\sigma-1\right)n\right] x_H\left(s^{t+1}\right) \\ +\left(\sigma-1\right)\left(1-n\right) x_F\left(s^{t+1}\right) + \pi_H\left(s^{t+1}\right) \end{array} \right\} \\
&\quad -\left[1+\left(\sigma-1\right)n\right] x_H\left(s^t\right) - \left(\sigma-1\right)\left(1-n\right) x_F\left(s^t\right) + r_n\left(s^t\right),
\end{aligned} \tag{9}$$

for the home country. For the foreign country, we have

$$\begin{aligned}
i_{F,t}\left(s^t\right) &= \sum_{s^{t+1}} \mu\left(s^{t+1}\right) \left\{ \begin{array}{c} \left[1+\left(\sigma-1\right)\left(1-n\right)\right] x_F\left(s^{t+1}\right) \\ +\left(\sigma-1\right) n x_H\left(s^{t+1}\right) + \pi_F^*\left(s^{t+1}\right) \end{array} \right\} \\
&\quad -\left[1+\left(\sigma-1\right)\left(1-n\right)\right] x_F\left(s^t\right) - \left(\sigma-1\right) n x_H\left(s^t\right) + r_n^*\left(s^t\right),
\end{aligned} \tag{10}$$

Here, the inflation rates of the home goods and the foreign goods, $\pi_H\left(s^t\right)$ and $\pi_F^*\left(s^t\right)$, are defined as follows:

$$\begin{aligned}
\pi_H\left(s^t\right) &\equiv \ln P_H\left(s^t\right) - \ln P_H\left(s^{t-1}\right), \\
\pi_F^*\left(s^t\right) &\equiv \ln P_F^*\left(s^t\right) - \ln P_F^*\left(s^{t-1}\right).
\end{aligned}$$

[3] For details of the derivations, see, for example, Clarida, Gali, and Gertler (2002), Benigno and Benigno (2003), and Nakajima (2008).

The output gaps $x_H\left(s^t\right)$ and $x_F\left(s^t\right)$ are defined by the log deviation of outputs from their flexible price levels $Y_n\left(s^t\right)$ and $Y_n^*\left(s^t\right)$:[4]

$$\begin{aligned} x_H\left(s^t\right) &\equiv \log\left[Y\left(s^t\right)\right] - \log\left[Y_n\left(s^t\right)\right], \\ x_F\left(s^t\right) &\equiv \log\left[Y^*\left(s^t\right)\right] - \log\left[Y_n^*\left(s^t\right)\right]. \end{aligned}$$

$r_n\left(s^t\right)$ and $r_n^*\left(s^t\right)$ are called the natural rates of interest. They are defined as the real interest rates that arise when both the home goods price and foreign goods price are flexible. Namely, we have

$$\begin{aligned} \frac{1}{1+r_n\left(s^t\right)} &\equiv \sum_{s^{t+1}}\left[\frac{\beta u_H\left(Y_n\left(s^{t+1}\right), Y_n^*\left(s^{t+1}\right)\right)}{u_H\left(Y_n\left(s^t\right), Y_n^*\left(s^t\right)\right)}\right], \\ \frac{1}{1+r_n^*\left(s^t\right)} &\equiv \sum_{s^{t+1}}\left[\frac{\beta u_F\left(Y_n\left(s^{t+1}\right), Y_n^*\left(s^{t+1}\right)\right)}{u_F\left(Y_n\left(s^t\right), Y_n^*\left(s^t\right)\right)}\right]. \end{aligned}$$

Here, u_H and u_F are the marginal utilities of the household with respect to home goods consumption and foreign goods consumption, respectively. It is notable that the natural rate of interest in either of the countries is a function of the flexible level outputs in both countries, $Y_n\left(s^t\right)$ and $Y_n^*\left(s^t\right)$. Therefore, each of the natural rates of interest is affected by the exogenous shocks occurring in both countries, such as technology shocks or government expenditure shocks.[5] In the following analysis, we examine the equilibrium response of the economy when these natural rates of interest follow the law of motion, such that they fall below the steady-state in a particular period and revert back to the steady-state in subsequent periods with fixed probabilities. The probabilities are assumed to be independent of each other for analytical convenience.

The equilibrium conditions are equations (7) to (10) with home and foreign monetary policy equations, which are formalized to maximize social welfare. From the second-order approximation of the nonlinear equilibrium conditions and welfare of the households in the

[4] $Y_n\left(s^t\right)$ and $Y_n^*\left(s^t\right)$ are the natural levels of output that arise when prices of both the home goods and foreign goods $P_H\left(s^t\right)$ and $P_F^*\left(s^t\right)$ are flexible. See Clarida, Gali, and Gertler (2002) for related discussions.

[5] Thus, fiscal policy is included in the natural interest rate shock in our model.

two countries, we can derive the periodic social loss function:[6]

$$\begin{aligned} L\left(s^t\right) &= \frac{\gamma_H n}{\theta} x_H\left(s^t\right)^2 + \frac{2\gamma_{H,F} n\left(1-n\right)}{\theta} x_H\left(s^t\right) x_F\left(s^t\right) + \frac{\gamma_F\left(1-n\right)}{\theta} x_F\left(s^t\right)^2 \\ &\quad + n\pi_H\left(s^t\right)^2 + \left(1-n\right)\pi_F^*\left(s^t\right)^2 . \end{aligned} \tag{11}$$

We set the parameters as follows. One period in the model corresponds to a quarter. We set $\beta = 0.99$, $\varepsilon = 7.88$, $\theta = 0.66$, $\omega = 0.47$, and $n = 0.5$. For σ, three values are considered: $\sigma = 0.5$, 1, and 5.988.[7]

2.6 International Spillover

Let us discuss international spillover. We show how international spillover is related to σ. For this, we derive two equilibrium conditions. The first equation is the optimality condition for the home country households' labor supply:

$$\begin{aligned} v'\left[h\left(s^t\right)\right] &= u'\left[C\left(s^t\right)\right]\frac{W\left(s^t\right)}{P\left(s^t\right)} \\ &= C\left(s^t\right)^{-\sigma}\frac{MC\left(s^t\right)Z\left(s^t\right)}{1-\tau\left(s^t\right)}\left[\frac{P_H\left(s^t\right)}{P_F\left(s^t\right)}\right]^{1-n}, \end{aligned} \tag{12}$$

where we used equation (3) and the definition of the consumer price index derived as the Hicksian demand function from equation (1). The other is the equation that relates consumption of the home country household to the home country output, which is derived from equations (4), (5), and (6), together with the Marshallian demand functions derived from equation (1):

$$C\left(s^t\right) = \left[\frac{P_H\left(s^t\right)}{P_F\left(s^t\right)}\right]^{1-n}\Delta\left(s^t\right)Z\left(s^t\right)h\left(s^t\right). \tag{13}$$

Note that only $P_F\left(s^t\right)$, which is expressed under the law of one price as

$$P_F\left(s^t\right) = \mathcal{E}\left(s^t\right)P_F^*\left(s^t\right),$$

enters equations (12) and (13) as the foreign variables. When $\sigma = 1$, $P_F\left(s^t\right)$ and therefore the terms of trade, defined as $P_F\left(s^t\right)/P_H\left(s^t\right)$, disappears from equations (12) and (13).

[6] For the derivation of the welfare loss function under policy cooperation, see Clarida, Gali, and Gertler (2002) and Benigno and Benigno (2003).

[7] For comparison with the literature, we choose $\sigma^{-1} = 2$ and $\sigma^{-1} = 0.167$, that are used by Eggertsson and Woodford (2003) and Jung, Teranishi, and Watanabe (2005), respectively.

In this case, the marginal cost in the home country is determined only by variables of the home country and there is no spillover from the foreign country. When $\sigma \neq 1$, however, the foreign variable affects the home variables through the terms of trade. As shown by Clarida, Gali, and Gertler (2002), spillover takes place through the two channels. The first channel is the "terms of trade effect," in which a rise in foreign output reduces the marginal cost of the home country, via appreciation of the terms of trade, working through the terms of trade, in equation (12). The second channel is the "risk sharing effect," in which an increase in foreign output increases the marginal cost of the home country by raising the consumption of the home country household, working through the terms of trade in equation (13). Note that these two cancel out when $\sigma = 1$, and the two countries become insular.

3 A Markov Equilibrium under a Global Liquidity Trap

Following Eggertsson and Woodford (2003), we analytically investigate optimal monetary policy under a Markov equilibrium for both discretionary policy and commitment policy. To see the properties of the policies, we consider an experiment in which the two natural rates of interest $r_n\left(s^t\right), r_n^*\left(s^t\right) \in \{\underline{r}, r\}$ in the home and foreign country, respectively, fall unexpectedly and simultaneously at period 0 from their steady-state value r to a negative value $\underline{r}$, and then simulate the optimal response of the policy rates to these adverse shocks. Here, we set $r = \frac{1-\beta}{\beta} > 0$ and $\underline{r} = -0.04/4 < 0$. We assume that each natural rate of interest reverts back to its steady-state value with a fixed probability in every subsequent period, and that it stays there for good once it returns to the steady-state. More precisely, for the home natural rate of interest $r_n\left(s^t\right)$, when $r_n\left(s^t\right) = \underline{r}$ at period t, $r_n\left(s_{t+1}|s^t\right)$ remains $\underline{r}$ at period $t+1$ with constant probability p and returns to its steady-state value r with constant probability $1-p$. For the foreign natural rate of interest $r_n^*\left(s^t\right)$, we assume that the foreign country shock $r_n^*\left(s^t\right) = \underline{r}$ does not revert back to the steady-state until the home country shock disappears. The foreign natural rate of interest returns to its steady-state value r with constant probability $1-q$ after the home country shock $r_n\left(s_{t+1}|s^t\right)$ returns to its steady-state value r. Thus, in our setting, the home country shock always disappears earlier than the foreign country shock does.

Here, the state of the economy is characterized by the signs of the natural rates of interest in the two countries. We employ the subscript zz for the state where $r_n\left(s^t\right)=\underline{r}$ and $r_n^*\left(s^t\right)=\underline{r}$ hold, nz for the state where $r_n\left(s^t\right)=r$ and $r_n^*\left(s^t\right)=\underline{r}$ hold, and nn for the state when $r_n\left(s^t\right)=r$ and $r_n^*\left(s^t\right)=r$ hold.[8] Table 1 shows the transition probability of the states in our model economy. For each row, the column reports the transition probability that the state changes from the current state to the other state. Notice that the state nn is an absorbing state, and the economy stays at the state nn with probability one once it reaches this state.

For the tractability of our analysis on optimal commitment policy, we further assume that central banks fix their policy rates within a state,[9] but that they can change their policy interest rates across different states.[10] For sufficiently large adverse shocks to the natural rate of interest, it is natural to predict the following. (1) The two central banks cooperatively set the policy interest rate of the country in which the adverse shock still prevails to zero; that is, $i_{Hzz}=i_{Fzz}=i_{Fnz}=0$, and (2) they set both of the policy interest rates to the steady-state level of the natural rates of interest when both of the two adverse shocks die out; that is, $i_{H,nn}=i_{F,nn}=r$. In this experiment, we assume these conditions (1) and (2) actually hold by making $\underline{r}$ sufficiently negative to ensure that these conditions are consistent with the optimality of the monetary policy implementation. Thanks to condition (2), we have one other condition: (3) $x_{H,nn}=x_{F,nn}=\pi_{H,nn}=\pi^*_{F,nn}=0$, because the economy is perfectly stabilized in this case.[11] Given these conditions (1), (2), and (3), optimal monetary policy is characterized by the optimal choice of $i_{H,nz}$ that maximizes social welfare. Admittedly, the choice set differs between discretionary policy

[8] Note that the state zn defined as $r_n\left(s^t\right)=\underline{r}$ and $r_n^*\left(s^t\right)=r$ does not exist.

[9] Because there are no endogenous state variables, dependency on the lagged variables stems solely from the history dependent policy. Therefore, as long as we impose this condition, the solution below is consistent with the nonlinear equilibrium conditions derived in the previous section.

[10] Fujiwara, Nakajima, Sudo, and Teranishi (2009) relax this condition and analyze the case in which the central banks can also adjust their policy rates even within a state.

[11] Another way to justify this condition is to set the Ramsey planner's discount factor very close to unity. Together with the condition that there is no variation in the policy interest rates within the state, this condition yields $i_{H,nn}=i_{F,nn}=r$.

and commitment policy.

Under this Markov equilibrium, the dynamic system of equations consisting of equations (7) to (10) collapses to a system of eight static equations. As for the equilibrium conditions during the state zz, we obtain

$$\pi_{H,zz} = \gamma_H x_{H,zz} + \gamma_{H,F} (1-n) x_{F,zz} + \beta \left[p\pi_{H,zz} + (1-p) \pi_{H,nz} \right], \tag{14}$$

$$\pi^*_{F,zz} = \gamma_{H,F} n x_{H,zz} + \gamma_F x_{F,zz} + \beta \left[p\pi^*_{F,zz} + (1-p) \pi^*_{F,nz} \right], \tag{15}$$

$$\begin{aligned} 0 \;=\; & [1 + (\sigma - 1) n] \left[p x_{H,zz} + (1-p) x_{H,nz} - x_{H,zz} \right] \\ & + (\sigma - 1)(1-n) \left[p x_{F,zz} + (1-p) x_{F,nz} - x_{F,zz} \right] \\ & + p\pi_{H,zz} + (1-p) \pi_{H,nz} + \underline{r}, \end{aligned} \tag{16}$$

and

$$\begin{aligned} 0 \;=\; & [1 + (\sigma - 1)(1-n)] \left[p x_{F,zz} + (1-p) x_{F,nz} - x_{F,zz} \right] \\ & + (\sigma - 1) n \left[p x_{H,zz} + (1-p) x_{H,nz} - x_{H,zz} \right] \\ & + p\pi^*_{F,zz} + (1-p) \pi^*_{F,nz} + \underline{r}. \end{aligned} \tag{17}$$

For the state nz, we have

$$\pi_{H,nz} = \gamma_H x_{H,nz} + \gamma_{H,F} (1-n) x_{F,nz} + \beta q \pi_{H,nz}, \tag{18}$$

$$\pi^*_{F,nz} = \gamma_{H,F} n x_{H,nz} + \gamma_F x_{F,nz} + \beta q \pi^*_{F,nz}, \tag{19}$$

$$i_{H,nz} = [1 + (\sigma - 1) n] (q x_{H,nz} - x_{H,nz}) + (\sigma - 1)(1-n)(q x_{F,nz} - x_{F,nz}) + q\pi_{H,nz} + r, \tag{20}$$

and

$$0 = [1 + (\sigma - 1)(1-n)] (q x_{F,nz} - x_{F,nz}) + (\sigma - 1) n (q x_{H,nz} - x_{H,nz}) + q\pi^*_{F,nz} + \underline{r}. \tag{21}$$

Now, we have eight unknowns, $\pi_{H,zz}$, $\pi^*_{F,zz}$, $x_{H,zz}$, $x_{F,zz}$, $\pi_{H,nz}$, $\pi^*_{F,nz}$, $x_{H,nz}$, and $x_{F,nz}$ for the eight equations above. Here, $i_{H,nz}$ is the policy variable that is chosen by the home central bank.

4 Optimal Monetary Policy under Discretion

Under discretion, the home central bank chooses $i_{H,nz}$ to maximize social welfare taking expectations as given for the state nz. The policy interest rate set by the home country central bank at the state nz, $i_{H,nz}$, is chosen to minimize the contemporaneous loss:

$$L_{nz}^{D} = \frac{\gamma_H n_H}{\theta} x_{H,nz}^2 + \frac{2\gamma_{H,F} n_H n_F}{\theta} x_{H,nz} x_{F,nz} + \frac{\gamma_F n_F}{\theta} x_{F,nz}^2 + n_H \pi_{H,nz}^2 + n_F \pi_{F,nz}^{*2}, \quad (22)$$

subject to the following five constraints:

$$\pi_{H,nz} = \gamma_H x_{H,nz} + \gamma_{H,F} (1-n) x_{F,nz},$$

$$\pi_{F,nz}^{*} = \gamma_{H,F} n x_{H,nz} + \gamma_F x_{F,nz},$$

$$i_{H,nz} = -[1 + (\sigma - 1) n] x_{H,nz} - (\sigma - 1)(1-n) x_{F,nz} + r,$$

$$0 = -[1 + (\sigma - 1)(1-n)] x_{F,nz} - (\sigma - 1) n x_{H,nz} + \underline{r},$$

and

$$i_{H,nz} \geq 0.$$

Assuming that the nonnegativity constraint on $i_{H,nz}$ does not bind, because there are four constraints with four unknown variables, $\pi_{H,nz}$, $\pi_{F,nz}^{*}$, $x_{H,nz}$, and $x_{F,nz}$, the optimal discretionary policy is obtained by choosing $i_{H,nz}$ to minimize the loss function in equation (22). When the nonnegativity constraint binds, $i_{H,nz}$ is set to zero. Because expectations are taken as given by the two central banks at the state nz, the probability p does not directly affect the optimal monetary policy. However, $i_{H,nz}$ indirectly depends on q, since the economic structure at the state nz is affected by the future expectation associated with q.

Simulation results are shown in Tables 2, 3, 4, and 5. Tables 2 and 3 demonstrate how policy interest rate, output, and inflation change with the expected duration of the adverse shock in the foreign country, namely, q, for the state zz and the state nz, respectively. Here, we fix the other parameters including $p = 0.25$. Similarly, Tables 4 and 5 demonstrate how the variables change with the expected duration of the adverse shock in the home country, namely, p, for the state zz and the state nz, respectively, keeping the other parameters including $q = 0.25$.

Under discretion, the optimal monetary policy $i_{H,nz}$ is characterized by the size of the international spillover, that is captured by the parameter σ.[12] We first describe this using Table 3. When $\sigma = 1$, the output and inflation in the home country are perfectly stabilized at the state nz by setting $i_{H,nz} = \frac{1-\beta}{\beta}$, regardless of the value of q. Because the output gap and inflation in the foreign country does not affect those of the home country in this case, the expected duration of the foreign adverse shock does not affect the home variables. For the foreign country, on the other hand, output gap and inflation vary with q, because longer q implies that the foreign adverse shock stays longer. When q increases from 0.0 to 0.75, the output gap and inflation in the foreign country decrease monotonically. On the other hand, they take positive values when $q = 0.90$. In this case, under our restricted solution, the longer expected adverse shock together with the zero interest rate induces too much easing in terms of the real interest rate since an elasticity of the inflation to output gap increases as q increases.[13] When $\sigma \neq 1$, there is international spillover and the home central bank sets $i_{H,nz}$, depending on σ, because of the two reasons. First, the two central banks focus on the monetary policy coordination, and the home country sets the policy interest rate to maximize the global welfare rather than the home country welfare. Second, there is the interaction between the two countries through the economic structure as we discussed above. Consequently, the size of $i_{H,nz}$ is characterized by σ.[14] To illustrate this, we rewrite the IS equation for the foreign country at the state nz as follows:

[12] The key parameter of interdependence across countries may change according to the model specification. In this paper, we employ the standard new open macroeconomy model of Clarida, Gali, and Gertler (2002), Corsetti and Pesenti (2001), or Corsetti and Pesenti (2005) in which many of the properties of international spillover are studied. In this model, the interdependence is well captured by the inverse of the intertemporal elasticity of substitution parameter σ. However, when we assume a different utility following Greenwood, Hercowitz, and Huffman (1988), for example, the other parameters may play a key role affecting the interdependence effects across countries.

[13] As we see below, we have the similar situation for the case where p takes a large value for both under discretion and under commitment under a given zero interest rate.

[14] More precisely, under the cooperative policy, the two central banks maximize the welfare given by equation (11). Under the non-cooperative policy, each central bank maximizes its own welfare. Admittedly the two optimal policy interest rates can differ since objectives are different. See Clarida, Gali, and Gertler (2002) for the comparison of the two policies in the case where two countries are not in the liquidity trap.

$$i_{F,nz} = [1 + (\sigma - 1)(1 - n)](qx_{F,nz} - x_{F,nz}) + (\sigma - 1) n (qx_{H,nz} - x_{H,nz}) + q\pi^*_{F,nz} + \underline{r}.$$

We first consider the case in which the two countries are insular ($\sigma = 1$). When the zero lower bound constraint is binding, this IS equation is reduced to

$$0 = (q - 1) x_{F,nz} + q\pi^*_{F,nz} + \underline{r}.$$

In this case, as we discussed in Section 2.6, there is no spillover effect across countries. Thus, the home central bank does not have any international spillover affecting the dynamics of the output gap $x_{F,nz}$ and inflation $\pi^*_{F,nz}$ in the above equation.

We now turn to the case in which there is an interdependence between the two countries ($\sigma \neq 1$) and the zero lower bound is binding:

$$0 = [1 + (\sigma - 1)(1 - n)](qx_{F,nz} - x_{F,nz}) + (\sigma - 1) n (qx_{H,nz} - x_{H,nz}) + q\pi^*_{F,nz} + \underline{r}.$$

It is obvious from the third term on the right-hand side of the above equation that if $\sigma > (<)\ 1$, the deflationary pressure to the foreign output gap and inflation is mitigated by decreasing (increasing) the home output gap $x_{H,nz}$, to net out the negative natural rate of interest in the foreign country. In particular, while the foreign output and inflation are negative, the home central bank sets its policy rate higher (lower) than the case of $\sigma = 1$ when $\sigma > (<)\ 1$, so that the home output declines (rises) compared with the case of $\sigma = 1$. By setting the appropriate policy interest rate, the negative output gap of the foreign country is reduced even in the liquidity trap, and the welfare loss of the two countries associated with the adverse shocks is reduced.[15] This result is consistent with the existing literature that discusses the spillover of monetary policy across countries. For instance, Corsetti and Pesenti (2001), using a framework similar to ours, report that a foreign monetary expansion has a negative (positive) impact on home output when $\sigma >$ $(<)\ 1$, because the two goods are substitutes (complements) and the marginal utility of

[15] Table 3 shows that when q is sufficiently large, the adverse shock in the foreign country causes the positive output and inflation, rather than negative output and inflation in the foreign country. In this case, the home central bank sets $i_{H,nz}$ lower (higher) than the case of $\sigma = 1$ for $\sigma > (<)\ 1$, so as to mitigate the inflationary pressure in the foreign country.

home goods decreases (increases) with the consumption of foreign goods. Based on their arguments, therefore, to increase the output gap in the home country, foreign output needs to be lower (higher) for $\sigma > (<) 1$.

Table 5 shows the case for p. The role of international spillover is clearly observed in how the home policy interest rate is set depending on σ. In contrast to q, the expectation about the adverse shock in the home country p, does not affect the economy at the state nz, since the state nz has realized already. This result contrasts sharply with that under the quasi-optimal commitment policy as shown in the later section. This is because the central banks take the values of future variables as given under discretionary policy.

As shown in Table 2 and 4, the variables at the state nz affect those at the state zz through the agents' expectations. When $\sigma = 1$, the home variables at the state zz are independent from the variations in q, and dependent on the variations in p, because longer duration of the adverse state in the foreign country does not influence the home country. The foreign variables, on the other hand, vary with both p and q, because both duration parameters affect the expected duration that the foreign adverse shock prevails. When $\sigma \neq 1$, the home country is affected by q, because of the presence of the spillover effect from the foreign country.

To summarize the results under discretion, the nature of the optimal cooperative monetary policy is characterized by international spillover. When two countries are not insular, a country can mitigate the deflationary pressure of the other country by boosting or contracting its own economy.

5 Quasi-optimal Policy under Commitment

We next discuss our commitment policy. We call the commitment monetary policy under our setting as quasi-optimal commitment policy, since we restrict the state dynamics as explained above. Under quasi-optimal commitment policy, the home central bank under cooperation chooses $i_{H,nz}$ to minimize the present discounted value of the social loss in equation (11) expressed as

$$\begin{aligned} L^{C}\left(s^{t}\right) &= L\left(s^{t}\right)+\beta\sum_{s_{t+1}}\mu\left(s_{t+1}|s^{t}\right)L\left(s_{t+1}|s^{t}\right)+\beta^{2}\sum_{s_{t+2}}\mu\left(s_{t+2}|s_{t+1}\right)L\left(s_{t+2}|s_{t+1}\right)\ldots. \\ &= L\left(s^{t}\right)+\sum_{k=1}^{\infty}\sum_{s_{t+k}}\beta^{k}\mu\left(s_{t+1}|s^{t}\right)L\left(s_{t+1}|s^{t}\right), \end{aligned} \tag{23}$$

subject to equations (14) to (21), and the zero lower bound constraint

$$i_{H,nz}\geq 0.$$

The analytical form of this loss function (23) is shown in the Appendix.

Simulation results are shown in Tables 6, 7, 8, and 9. Tables 6 and 7 demonstrate how policy interest rate, output, and inflation change with the expected duration of the adverse shock in the home country, namely, p, for the state zz and the state nz, respectively. Here, we fix the other parameters including $q=0.25$. Similarly, Tables 8 and 9 demonstrate how the variables change with the expected duration of the adverse shock in the foreign country, namely, q, for the state zz and the state nz, respectively, keeping the other parameters including $p=0.25$.

Under commitment, the optimal monetary policy $i_{H,nz}$ is characterized by the history dependency as well as the international spillover. As Table 7 shows, in contrast to discretion displayed in Table 3, the policy interest rate is affected by p even though the state nz has realized already. This is a pure effect of the monetary policy commitment, since the central banks take the values of future variables into consideration under commitment. The home country's output gap and inflation increase with p, since greater policy stimulus is needed to offset the contractionary impact of the longer expected duration of the adverse shock in the home country. In addition to this history dependency, there is the international spillover effect. When $\sigma>(<)\ 1$, the home policy interest rate $i_{H,nz}$ is set higher (lower) than the case of $\sigma=1$, being consistent with the discussion in the previous section.

Next we discuss the relationship between $i_{H,nz}$ and q, shown in Table 9. Lower q implies a shorter expected duration of the state nz, during which the home central bank is able to maintain its accommodative monetary policy. When $\sigma=1$, there is no foreign effect. Thus the home central bank sets lower value for $i_{H,nz}$ for a smaller q, only to mitigate the effect of the adverse shock in the home country at the state zz. As q increases, the home policy interest rate becomes less accommodative, because the accommodative period becomes longer in the good state for the home country. When $\sigma \neq 1$, there is international spillover effect as well as history dependency, and the relationship between the expected duration and the policy interest rate becomes less evident.

Similarly to the results under discretion, the variables at the state nz affect those at the state zz through the agents' expectations. Under commitment, however, even when $\sigma = 1$, the home variables at the state zz vary with q as well as with p, reflecting the fact that setting of the policy interest rate is history dependent under commitment policy.

In summary, the monetary policy cooperation under commitment is characterized by the history dependence existing studies have found in a closed economy, which is contrary to the results under discretion.[16] However, similar to the outcomes under discretion, the level of these low optimal policy rates is determined by the size of σ. An additional finding in the two-country model is that there are two ways to mitigate the effect of adverse shocks. One is the international spillover channel through which one of the central banks affects the output and inflation in the other country through the interdependence of the two countries. The other is the intertemporal (or history dependent) channel through which both central banks commit to lower future policy interest rates to mitigate the adverse shocks.

6 Welfare analysis

Table 10, 11, 12, and 13 report the expected welfare loss given by (23) under discretion policy and quasi-optimal commitment policy, for various sizes of p and q. The loss, in general, tends to be small when the expected duration of the adverse shocks are small. The comparisons among the tables illustrate the gains from the commitment. Clearly, the welfare loss is smaller under the quasi-optimal commitment policy than that under the discretion for every combinations of p and q as implied by Woodford (2003).

[16] As shown in Adam and Billi (2007), if an economy falls into a liquidity trap again after being in the steady-state, optimal discretionary policy may have history dependency.

7 Conclusion

How should monetary policy cooperation be designed when more than one country simultaneously face zero lower bounds on nominal interest rates? To answer this question, we provided a tractable framework within a two-country dynamic general equilibrium model under a Markov equilibrium. Analysis of the nature of optimal policy cooperation in such a situation provides the key feature of the policy under optimal discretionary and commitment policies. Under discretion, optimal monetary policy cooperation is characterized by no history dependency and international spillover in which the parameter of IES plays an important role. On the other hand, under commitment, monetary policy is characterized by history dependence. It is recommended that the country commits to low future nominal interest rates. Quantitatively, the size of σ, the inverse of IES, affects the optimal level of interest rates.

Yet, in practice, making credible commitments to future policy is a difficult task in open economies. Central banks need to effectively inform citizens not only in the home country but also in foreign countries about the nature of the commitments. At the same time, agents across the globe would fully understand the statements made by the central banks, regardless of whether these are written in their own language or not. Thus, because of these potential obstacles in implementing cooperative commitment policy in open economies, it is equally important for central banks to understand the paths of the policy interest rates under optimal discretion policy.

There are several possible extensions to this study. First, we should investigate the nature of policy cooperation for a global liquidity trap with a less-restrictive model framework. In this paper, we assume that central banks maintain policy interest rates within a state to obtain analytical solutions and clear policy implications in a tractable manner. Although we believe that there should not be qualitatively significant differences, it would be worth analyzing policy cooperation without this restriction.[17] Second, to suggest real monetary policy implementations, it is important to incorporate more empirical realism

[17] Our accompanying paper, Fujiwara, Nakajima, Sudo, and Teranishi (2009), provides the solution of the optimal monetary policy using a model with a general setting.

through estimation and a richer model structure. Third, it would be intriguing to gauge the gains from cooperation for a global liquidity trap by comparing the welfare loss under cooperation with that under noncooperation. Here, it would also be interesting to think of a situation where one country deviates from cooperation. We will address these issues in future research.

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Appendix: Loss Function under Commitment

By substituting equations (14) to (21) into equation (23) and using the formula for the geometric series, we can obtain the loss function that the central banks under commitment aim to minimize:

$$
\begin{aligned}
L^C\left(s^t\right) &= L_t\left(s^t\right)+\beta \mathrm{E}_t L_{t+1}\left(s^{t+1}\right)+\beta^2 \mathrm{E}_t L_{t+2}\left(s^{t+2}\right)+\ldots.. \\
&= \frac{\gamma_H n_H}{\theta} x_{H,zz}^2+\frac{2\gamma_{H,F} n_H n_F}{\theta} x_{H,zz} x_{F,zz}+\frac{\gamma_F n_F}{\theta} x_{F,zz}^2+n_H \pi_{H,zz}^2+n_F \pi_{F,zz}^{*2} \\
&\quad +\beta p\left[\frac{\gamma_H n_H}{\theta} x_{H,zz}^2+\frac{2\gamma_{H,F} n_H n_F}{\theta} x_{H,zz} x_{F,zz}+\frac{\gamma_F n_F}{\theta} x_{F,zz}^2+n_H \pi_{H,zz}^2+n_F \pi_{F,zz}^{*2}\right] \\
&\quad +\beta(1-p)\left[\frac{\gamma_H n_H}{\theta} x_{H,nz}^2+\frac{2\gamma_{H,F} n_H n_F}{\theta} x_{H,nz} x_{F,nz}+\frac{\gamma_F n_F}{\theta} x_{F,nz}^2+n_H \pi_{H,nz}^2+n_F \pi_{F,nz}^{*2}\right] \\
&\quad +\beta^2 p^2\left[\frac{\gamma_H n_H}{\theta} x_{H,zz}^2+\frac{2\gamma_{H,F} n_H n_F}{\theta} x_{H,zz} x_{F,zz}+\frac{\gamma_F n_F}{\theta} x_{F,zz}^2+n_H \pi_{H,zz}^2+n_F \pi_{F,zz}^{*2}\right] \\
&\quad +\beta^2\left[(1-p) q+p(1-p)\right] \\
&\quad \times\left[\frac{\gamma_H n_H}{\theta} x_{H,nz}^2+\frac{2\gamma_{H,F} n_H n_F}{\theta} x_{H,nz} x_{F,nz}+\frac{\gamma_F n_F}{\theta} x_{F,nz}^2+n_H \pi_{H,nz}^2+n_F \pi_{F,nz}^{*2}\right] \\
&\quad \ldots+\beta^k p^k\left[\frac{\gamma_H n_H}{\theta} x_{H,zz}^2+\frac{2\gamma_{H,F} n_H n_F}{\theta} x_{H,zz} x_{F,zz}+\frac{\gamma_F n_F}{\theta} x_{F,zz}^2+n_H \pi_{H,zz}^2+n_F \pi_{F,zz}^{*2}\right] \\
&\quad +\beta^k\left[(1-p)\sum_{j=0}^{k-1} q^{(k-1)-j} p^j\right] \\
&\quad \times\left[\frac{\gamma_H n_H}{\theta} x_{H,nz}^2+\frac{2\gamma_{H,F} n_H n_F}{\theta} x_{H,nz} x_{F,nz}+\frac{\gamma_F n_F}{\theta} x_{F,nz}^2+n_H \pi_{H,nz}^2+n_F \pi_{F,nz}^{*2}\right] \ldots.
\end{aligned}
$$

Then we have for $p \neq q$,

$$
\begin{aligned}
L^C\left(s^t\right) &= \frac{1}{1-\beta p}\left[\frac{\gamma_H n_H}{\theta} x_{H,zz}^2+\frac{2\gamma_{H,F} n_H n_F}{\theta} x_{H,zz} x_{F,zz}+\frac{\gamma_F n_F}{\theta} x_{F,zz}^2+n_H \pi_{H,zz}^2+n_F \pi_{F,zz}^{*2}\right] \\
&\quad +\frac{(1-p)}{q-p}\left[\frac{1}{1-\beta q}-\frac{1}{1-\beta p}\right] \\
&\quad \times\left[\frac{\gamma_H n_H}{\theta} x_{H,nz}^2+\frac{2\gamma_{H,F} n_H n_F}{\theta} x_{H,nz} x_{F,nz}+\frac{\gamma_F n_F}{\theta} x_{F,nz}^2+n_H \pi_{H,nz}^2+n_F \pi_{F,nz}^{*2}\right],
\end{aligned}
$$

and for $p = q$,

$$
L^C\left(s^t\right)=\frac{1}{1-\beta p}\left[\frac{\gamma_H n_H}{\theta} x_{H,zz}^2+\frac{2\gamma_{H,F} n_H n_F}{\theta} x_{H,zz} x_{F,zz}+\frac{\gamma_F n_F}{\theta} x_{F,zz}^2+n_H \pi_{H,zz}^2+n_F \pi_{F,zz}^{*2}\right].
$$

Table 1: Transition probability.

	zz	nz	nn
zz	p	$1-p$	0
nz	0	q	$1-q$
nn	0	0	1

Table 2: Policy interest rate, output gap and inflation under discretion at state zz for various size of q. Variables are presented in terms of percentage points at annual rates.

		$q = 0.0$	$q = 0.5$	$q = 0.75$	$q = 0.90$
$\sigma = 1$	$\pi_{H,zz}$	-0.4	-0.4	-0.4	-0.4
	$x_{H,zz}$	-1.4	-1.4	-1.4	-1.4
	$i_{H,zz}$	0.0	0.0	0.0	0.0
	$\pi_{F,zz}$	-1.0	-2.2	-14.7	6.6
	$x_{F,zz}$	-2.5	-4.0	-16.5	3.4
	$i_{F,zz}$	0.0	0.0	0.0	0.0
$\sigma = 0.5$	$\pi_{H,zz}$	-0.4	-0.1	8.7	-0.3
	$x_{H,zz}$	-2.7	-2.7	4.6	-0.9
	$i_{H,zz}$	0.0	0.0	0.0	0.0
	$\pi_{F,zz}$	-1.1	-2.5	-17.5	6.3
	$x_{F,zz}$	-4.2	-6.4	-23.2	3.6
	$i_{F,zz}$	0.0	0.0	0.0	0.0
$\sigma = 5.988$	$\pi_{H,zz}$	-0.5	-0.7	-3.4	-0.7
	$x_{H,zz}$	-0.2	0.2	1.2	-2.4
	$i_{H,zz}$	0.0	0.0	0.0	0.0
	$\pi_{F,zz}$	-0.7	-1.7	-8.5	7.0
	$x_{F,zz}$	-0.6	-1.2	-4.2	2.9
	$i_{F,zz}$	0.0	0.0	0.0	0.0

Table 3: Policy interest rate, output gap and inflation under discretion at state nz for various size of q. Variables are presented in terms of percentage points at annual rates.

		$q = 0.0$	$q = 0.5$	$q = 0.75$	$q = 0.90$
$\sigma = 1$	$\pi_{H,nz}$	0.0	0.0	0.0	0.0
	$x_{H,nz}$	0.0	0.0	0.0	0.0
	$i_{H,nz}$	4.0	4.0	4.0	4.0
	$\pi_{F,nz}$	-0.2	-1.0	-10.0	5.7
	$x_{F,nz}$	-1.0	-2.2	-11.4	2.8
	$i_{F,nz}$	0.0	0.0	0.0	0.0
$\sigma = 0.5$	$\pi_{H,nz}$	0.1	0.3	6.5	0.1
	$x_{H,nz}$	0.1	0.1	5.9	0.8
	$i_{H,nz}$	2.4	2.5	0.7	4.2
	$\pi_{F,nz}$	-0.2	-1.1	-11.7	5.5
	$x_{F,nz}$	-1.3	-3.0	-15.1	3.4
	$i_{F,nz}$	0.0	0.0	0.0	0.0
$\sigma = 5.988$	$\pi_{H,nz}$	-0.1	-0.3	-2.2	-0.3
	$x_{H,nz}$	0.1	0.3	1.0	-1.2
	$i_{H,nz}$	6.4	5.9	6.6	3.7
	$\pi_{F,nz}$	-0.2	-0.8	-5.8	6.0
	$x_{F,nz}$	-0.3	-0.9	-3.1	1.8
	$i_{F,nz}$	0.0	0.0	0.0	0.0

Table 4: Policy interest rate, output gap and inflation under discretion at state zz for various size of p. Variables are presented in terms of percentage points at annual rates.

		$p = 0.0$	$p = 0.5$	$p = 0.75$	$p = 0.90$
	$\pi_{H,zz}$	-0.2	-1.0	-9.9	5.7
$\sigma = 1$	$x_{H,zz}$	-1.0	-2.2	-11.4	2.8
	$i_{H,zz}$	0.0	0.0	0.0	0.0
	$\pi_{F,zz}$	-1.0	-2.2	-14.7	6.6
	$x_{F,zz}$	-2.5	-4.0	-16.5	3.4
	$i_{F,zz}$	0.0	0.0	0.0	0.0
	$\pi_{H,zz}$	-0.1	-1.1	-33.7	4.9
$\sigma = 0.5$	$x_{H,zz}$	-1.9	-4.6	-61.0	3.8
	$i_{H,zz}$	0.0	0.0	0.0	0.0
	$\pi_{F,zz}$	-1.1	-2.7	-40.3	6.2
	$x_{F,zz}$	-3.9	-7.1	-68.0	4.7
	$i_{F,zz}$	0.0	0.0	0.0	0.0
	$\pi_{H,zz}$	-0.4	-1.1	-5.5	6.9
$\sigma = 5.988$	$x_{H,zz}$	-0.0	-0.2	-0.5	0.7
	$i_{H,zz}$	0.0	0.0	0.0	0.0
	$\pi_{F,zz}$	-0.7	-1.6	-7.7	7.3
	$x_{F,zz}$	-0.7	-1.0	-2.8	1.0
	$i_{F,zz}$	0.0	0.0	0.0	0.0

Table 5: Policy interest rate, output gap and inflation under discretion at state nz for various size of p. Variables are presented in terms of percentage points at annual rates.

		$p = 0.0$	$p = 0.5$	$p = 0.75$	$p = 0.90$
	$\pi_{H,nz}$	0.0	0.0	0.0	0.0
$\sigma = 1$	$x_{H,nz}$	0.0	0.0	0.0	0.0
	$i_{H,nz}$	4.0	4.0	4.0	4.0
	$\pi_{F,nz}$	-0.4	-0.4	-0.4	-0.4
	$x_{F,nz}$	-1.4	-1.4	-1.4	-1.4
	$i_{F,nz}$	0.0	0.0	0.0	0.0
	$\pi_{H,nz}$	0.1	0.1	0.1	0.1
$\sigma = 0.5$	$x_{H,nz}$	0.1	0.1	0.1	0.1
	$i_{H,nz}$	2.5	2.5	2.5	2.5
	$\pi_{F,nz}$	-0.4	-0.4	-0.4	-0.4
	$x_{F,nz}$	-1.8	-1.8	-1.8	-1.8
	$i_{F,nz}$	0.0	0.0	0.0	0.0
	$\pi_{H,nz}$	-0.3	-0.3	-0.3	-0.3
$\sigma = 5.988$	$x_{H,nz}$	-0.5	-0.5	-0.5	-0.5
	$i_{H,nz}$	6.2	6.2	6.2	6.2
	$\pi_{F,nz}$	-0.3	-0.3	-0.3	-0.3
	$x_{F,nz}$	-0.5	-0.5	-0.5	-0.5
	$i_{F,nz}$	0.0	0.0	0.0	0.0

Table 6: Policy interest rate, output gap and inflation under commitment at state zz for various size of p. Variables are presented in terms of percentage points at annual rates.

		$p = 0.0$	$p = 0.5$	$p = 0.75$	$p = 0.90$
	$\pi_{H,zz}$	-0.0	-0.0	-5.1	4.7
$\sigma = 1$	$x_{H,zz}$	-0.6	-0.8	-6.3	2.1
	$i_{H,zz}$	0.0	0.0	0.0	0.0
	$\pi_{F,zz}$	-1.0	-2.2	-14.7	6.6
	$x_{F,zz}$	-2.5	-4.0	-16.4	3.4
	$i_{F,zz}$	0.0	0.0	0.0	0.0
	$\pi_{H,zz}$	0.2	-0.2	-27.4	4.4
$\sigma = 0.5$	$x_{H,zz}$	-1.1	-2.9	-51.1	3.3
	$i_{H,zz}$	0.0	0.0	0.0	0.0
	$\pi_{F,zz}$	-1.1	-2.5	-36.9	6.3
	$x_{F,zz}$	-3.7	-6.4	-61.3	4.6
	$i_{F,zz}$	0.0	0.0	0.0	0.0
	$\pi_{H,zz}$	-0.3	-0.5	-3.3	5.4
$\sigma = 5.988$	$x_{H,zz}$	-0.1	-0.4	-1.2	0.1
	$i_{H,zz}$	0.0	0.0	0.0	0.0
	$\pi_{F,zz}$	-0.8	-1.7	-8.5	7.2
	$x_{F,zz}$	-0.8	-1.4	-4.3	1.3
	$i_{F,zz}$	0.0	0.0	0.0	0.0

Table 7: Policy interest rate, output gap and inflation under commitment at state nz for various size of p. Variables are presented in terms of percentage points at annual rates.

		$p = 0.0$	$p = 0.5$	$p = 0.75$	$p = 0.90$
$\sigma = 1$	$\pi_{H,nz}$	0.1	0.3	0.4	0.4
	$x_{H,nz}$	0.4	1.1	1.4	1.4
	$i_{H,nz}$	3.0	0.8	0.0	0.0
	$\pi_{F,nz}$	-0.4	-0.4	-0.4	-0.4
	$x_{F,nz}$	-1.4	-1.4	-1.4	-1.4
	$i_{F,nz}$	0.0	0.0	0.0	0.0
$\sigma = 0.5$	$\pi_{H,nz}$	0.3	0.4	0.4	0.4
	$x_{H,nz}$	0.8	1.4	1.4	1.4
	$i_{H,nz}$	1.0	0.0	0.0	0.0
	$\pi_{F,nz}$	-0.4	-0.4	-0.4	-0.4
	$x_{F,nz}$	-1.5	-1.4	-1.4	-1.4
	$i_{F,nz}$	0.0	0.0	0.0	0.0
$\sigma = 5.988$	$\pi_{H,nz}$	-0.1	0.1	0.1	0.4
	$x_{H,nz}$	0.3	0.6	0.6	1.4
	$i_{H,nz}$	5.4	4.0	3.7	0.0
	$\pi_{F,nz}$	-0.3	-0.4	-0.4	-0.4
	$x_{F,nz}$	-0.6	-0.8	-0.8	-1.4
	$i_{F,nz}$	0.0	0.0	0.0	0.0

Table 8: Policy interest rate, output gap and inflation under commitment at state zz for various size of q. Variables are presented in terms of percentage points at annual rates.

		$q = 0.0$	$q = 0.5$	$q = 0.75$	$q = 0.90$
	$\pi_{H,zz}$	-0.0	-0.1	-0.2	-0.3
$\sigma = 1$	$x_{H,zz}$	-0.6	-0.9	-1.1	-1.3
	$i_{H,zz}$	0.0	0.0	0.0	0.0
	$\pi_{F,zz}$	-1.0	-2.2	-14.7	6.6
	$x_{F,zz}$	-2.5	-4.0	-16.5	3.4
	$i_{F,zz}$	0.0	0.0	0.0	0.0
	$\pi_{H,zz}$	0.0	0.4	9.2	-0.2
$\sigma = 0.5$	$x_{H,zz}$	-1.6	-1.7	5.3	-0.8
	$i_{H,zz}$	0.0	0.0	0.0	0.0
	$\pi_{F,zz}$	-1.1	-2.4	-17.2	6.3
	$x_{F,zz}$	-3.9	-6.0	-22.7	3.6
	$i_{F,zz}$	0.0	0.0	0.0	0.0
	$\pi_{H,zz}$	-0.3	-0.5	-3.4	-0.6
$\sigma = 5.988$	$x_{H,zz}$	-0.2	0.4	1.2	-2.3
	$i_{H,zz}$	0.0	0.0	0.0	0.0
	$\pi_{F,zz}$	-0.8	-1.7	-8.5	7.0
	$x_{F,zz}$	-0.8	-1.4	-4.2	2.8
	$i_{F,zz}$	0.0	0.0	0.0	0.0

Table 9: Policy interest rate, output gap and inflation under commitment at state nz for various size of q. Variables are presented in terms of percentage points at annual rates.

		$q = 0.0$	$q = 0.5$	$q = 0.75$	$q = 0.90$
	$\pi_{H,nz}$	0.2	0.2	0.1	0.1
$\sigma = 1$	$x_{H,nz}$	0.7	0.4	0.2	0.0
	$i_{H,nz}$	1.2	3.3	4.0	4.1
	$\pi_{F,nz}$	-0.2	-1.0	-9.9	5.7
	$x_{F,nz}$	-1.0	-2.2	-11.4	2.8
	$i_{F,nz}$	0.0	0.0	0.0	0.0
	$\pi_{H,nz}$	0.2	0.6	6.8	0.2
$\sigma = 0.5$	$x_{H,nz}$	1.0	1.0	6.5	0.8
	$i_{H,nz}$	0.0	1.5	0.6	4.3
	$\pi_{F,nz}$	-0.2	-1.1	-11.6	5.5
	$x_{F,nz}$	-1.0	-2.7	-14.7	3.4
	$i_{F,nz}$	0.0	0.0	0.0	0.0
	$\pi_{H,nz}$	0.0	-0.1	-2.2	-0.2
$\sigma = 5.988$	$x_{H,nz}$	0.4	0.5	1.0	-1.2
	$i_{H,nz}$	4.5	5.4	6.6	3.7
	$\pi_{F,nz}$	-0.2	-0.8	-5.8	6.0
	$x_{F,nz}$	-0.5	-1.0	-3.1	1.8
	$i_{F,nz}$	0.0	0.0	0.0	0.0

Table 10: Expected social welfare loss at the state zz for various size of p under discretion (numbers are multiplied by 10^7).

	$p = 0.0$	$p = 0.5$	$p = 0.75$	$p = 0.90$
$\sigma = 1$	70	516	$43,549$	$22,413$
$\sigma = 0.5$	107	875	$411,564$	$18,990$
$\sigma = 5.988$	34	260	$11,316$	$29,073$

Table 11: Expected social welfare loss at the state zz for various size of q under discretion (numbers are multiplied by 10^7).

	$q = 0.0$	$q = 0.5$	$q = 0.75$	$q = 0.90$
$\sigma = 1$	87	382	$23,753$	$11,247$
$\sigma = 0.5$	146	556	$43,070$	$10,548$
$\sigma = 5.988$	40	204	$8,434$	$12,375$

Table 12: Expected social welfare loss at the state zz for various size of p under commitment (numbers are multiplied by 10^7).

	$p = 0.0$	$p = 0.5$	$p = 0.75$	$p = 0.90$
$\sigma = 1$	68	434	33,572	19,505
$\sigma = 0.5$	99	677	314,317	17,420
$\sigma = 5.988$	33	233	10,581	23,507

Table 13: Expected social welfare loss at the state zz for various size of q under commitment (numbers are multiplied by 10^7).

	$q = 0.0$	$q = 0.5$	$q = 0.75$	$q = 0.90$
$\sigma = 1$	76	374	23,748	11,245
$\sigma = 0.5$	117	530	43,033	10,547
$\sigma = 5.988$	36	202	8,434	12,373

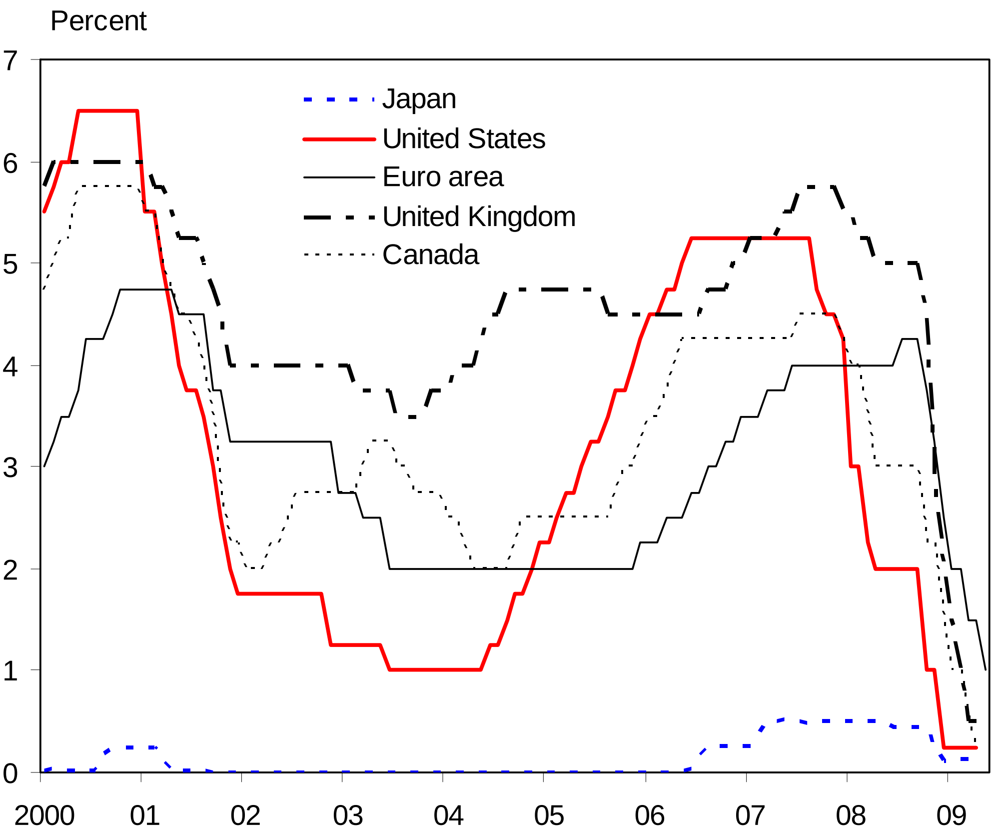

Figure 1: Policy interest rates.

Note: All data are from central banks: the call rate for Japan, the federal fund rate for United States, the main refinancing operations fixed rate for Euro area, the bank rate for United Kingdom, and the overnight rate target for Canada.