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Chained Credit Contracts and Financial Accelerators

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Abstract

Based on the financial accelerator model of Bernanke *et al.* (1999), we develop a dynamic general equilibrium model for a chain of credit contracts in which financial intermediaries (hereafter FIs) as well as entrepreneurs are subject to credit constraints. Financial intermediation takes place through chained-credit contracts, lending from the market to FIs, and from FIs to entrepreneurs. Calibrated to U.S. data, our model shows that the chained credit contracts enhance the financial accelerator effect, depending on the net worth distribution across sectors: (1) our model reinforces the effects of the net worth shock and the technology shock, compared with a model that omits the FIs' credit friction *à la* Bernanke *et al.* (1999); (2) the sectoral shock to FIs has a greater impact than the sectoral shock to entrepreneurs; and (3) the redistribution of net worth from entrepreneurs to FIs reduces the amplification of the technology shock. The key features of the results arise from the asymmetry of the two borrowing sectors: smaller net worth and larger bankruptcy costs of FIs relative to those of entrepreneurs.

Keywords: Chain of Credit Contracts; Net Worth of Financial Intermediaries; Cross-sectional Net Worth Distribution; Financial Accelerator effect

JEL classification: E22, E44

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1 Introduction

A large number of empirical studies suggest that declines of financial intermediaries' (hereafter FIs) net worth generates a macroeconomic downturn (Peek and Rosengren, 1997; 2000; Calomiris and Mason, 2003; Ashcraft, 2005). Because FIs tighten their loans to firms as their credit conditions deteriorate, and the firms in the economy cannot conduct their projects without financing them, the shock to the FI is propagated and amplified to the real economy. The recent financial crisis starting in 2007 confirms the importance of this channel. Declines in asset prices have damaged the balance sheets of the major FIs drastically. Consequently, the interbank market has failed to function and the spread between riskless rates and interbank lending rates has widened. This made it more difficult for FIs to borrow funds from the interbank market, and in turn, for firms to borrow funds from the FIs. Aggregate investment has dropped, and the real economy in many countries has fallen into a deep recession.

From a policy perspective, the net worth (or capital) of the borrowing sectors, especially that of the FI sector, has become an important policy target in many of the governmental initiatives conducted during the financial crisis. For example, the U.S. government has injected capital to distressed large financial institutions.

Despite the importance of FIs' net worth, studies using dynamic stochastic general equilibrium (DSGE) models have focused more on the non-financial firms' net worth rather than the FIs' net worth.[1] In Bernanke, Gertler, and Gilchrist (1999) (hereafter BGG), one of the workhorse models, for example, they consider the case where only the entrepreneurs are credit constrained. There, the net worth of the entrepreneurial sector affects the agency problem of the credit contract, and helps to propagate and amplify the adverse shock hitting the economy (financial accelerator effect). However, the BGG model ignores the fact that FIs are credit constrained, creating another source of the agency problem. In their setting, FIs are competitive agents that have zero net worth, so that they play no role in the financial accelerator effect.

In this paper, we extend the BGG model by incorporating credit-constrained FIs. In this economy, financial intermediation takes place through chained credit contracts. FIs make loans to the entrepreneurs, but the loans are financed by the borrowings from the market, in addition to the FIs' own net worth. The entrepreneur is also credit constrained and finances his/her investment project using his/her own net worth and borrowings from the FIs. Agency problems exist in both of the two borrowing contracts. Consequently, the external finance premium for entrepreneurs depends on FIs' net worth and entrepreneurial net worth.

Using the static version of the model, we first study how the net worth of FIs and entrepreneurs affect the two credit contracts, and how these two credit contracts together affect the aggregate investment decisions. Because the credit contracts are chained, the agency problems in the two credit contracts work complementarily. The supply of funds

[1] Also see the discussions in Allen (2001) and Gorton and Winton (2003).

to entrepreneurs is squeezed if either of the two contracts is severely constrained.

Next, we study the implication on the model dynamics, based on calibration to the U.S. economy. To illustrate the relationship between the financial accelerator effect and the net worth of FIs and entrepreneurs, we conduct three experiments. We first compare our model with a model that abstracts from the FIs' credit friction *à la* BGG. Because credit contracts are chained and they work complementarily, our model, in general, propagates and amplifies the adverse shocks more than a model where only entrepreneurial credit frictions exist. Second, we compare how a shock to the FI sector and a shock to the entrepreneurial sector are propagated to the aggregate economy. We find that a sectoral shock to FIs is propagated more than a shock to the entrepreneurs. This reflects the fact that the agency problem in the FI sector and the entrepreneurial sector are asymmetric. In the United States, FIs have smaller net worth than the entrepreneurs, and the cost associated with FIs' bankruptcy is higher than that of entrepreneurs, resulting in the former facing a more severe agency problem. Consequently, a shock to the FI sector has a larger impact on aggregate variables. Lastly, we examine how the "financial accelerator effect" associated with the aggregate shock can change, when we artificially change the net worth distribution. Taking the other parameters pertaining to the credit markets as given, we find that the financial accelerator effect can be reduced, if appropriate amount of net worth is distributed from entrepreneurs to FIs.

Our model is constructed with reference to two lines of literature. The first strand focuses on the credit friction associated with the entrepreneurs (BGG; Kiyotaki and Moore, 1997; Christiano, Motto, and Rostagno, 2004 [hereafter CMR]). The second strand considers the credit friction of the FIs (Bernanke and Blinder 1988). Extending the idea of Bernanke and Blinder (1988), Goodfriend and McCallum (2007) investigate the role of a banking sector that produces loans and deposits according to a production function with inputs of monitoring effort and collateral while they assume a constant ratio of base money to deposits. Van den Heuvel (2008) analyzes the welfare effects of regulatory requirements for FIs' capital. Gerali *et al.* (2008) and Dib (2009) discuss monopolistically competitive banks in deposit and loan markets. Gertler and Karadi (2009) construct a model in which depositors can force FIs into bankruptcy but cannot recover all of the FIs' assets.

As we explained above, we incorporate the two types of frictions into the model. In this sense, our model is similar to the work of Chen (2001), Aikman and Paustian (2006), and Meh and Moran (2004), who use quantitative extensions of the model of Holmstrom and Tirole (1997). However, there are three notable differences between their models and ours, with respect to the role of net worth.[2] First, their models are built upon

[2]Furthermore, in Holmstrom and Tirole (1997), it is entrepreneurs who optimize contracts between FIs and investors. Entrepreneurs maximize their expected profits subject to the zero profit conditions of other participants of the credit market. However because it is assumed that all projects are perfectly correlated ex post, FIs can earn positive profits if a project succeeds. In our model, FIs are monopolistic and it is the FIs who optimize contracts between FIs and investors. Projects are not correlated, so

the moral hazard problems of the FIs and entrepreneurs. Net worth helps mitigate the two moral hazard problems, so that the incentive-compatible conditions that prohibit shirking are satisfied. Our model is built upon two costly state verification problems, where net worth helps reduce the cost of external finance, thereby increasing aggregate investment. Second, in their models, the sum of the FIs' net worth and entrepreneurial net worth is most important to aggregate investment, because both net worths work similarly in making it easier for the borrowers to borrow external funds from the market. Thus, the role of each net worth is not emphasized. In our model, net worth distribution across sectors also matters for the investment, because two net worths work differently in the chain of credit contracts. Third, our model stresses the role of net worth in affecting the borrowing rates of the credit contracts. Here, net worth affects investments through market price movements. Consequently, the borrowing rates, net worth of the two sectors, and aggregate investment decisions are discussed in a unified framework. In their models, net worth mitigates the moral hazard problems, and affects investment directly by changing the incentive compatibility conditions.

The rest of this paper is organized as follows. Section 2 presents our model in which both FIs and entrepreneurs are credit constrained. In Section 3, we calibrate the model to the U.S. economy, and show the model's quantitative response to sectoral shocks and aggregate shocks. Section 4 concludes.

2 The Model Economy

This section describes the structure of our model and the optimization problems that the economy's agents solve. The economy consists of a credit market and goods market, and seven types of agents; a household, investors, FIs, entrepreneurs, capital goods producers, final goods producers and government.

The participants in the credit market are investors, FIs and entrepreneurs. Investors are subject to perfect competition, earning zero profit. They collect deposits from the household in a competitive market, and invest what they collect as loans to FIs.[3] FIs and entrepreneurs face credit constraints, but earn positive profits, accumulating net worth. FIs are monopolistic lenders to entrepreneurs.[4] They own net worth but not enough to finance their loans to entrepreneurs. Therefore, they engage in credit contracts with

lenders can be exempt from idiosyncratic uncertainty, and without aggregate uncertainty, lenders' ex post profits are the same as ex ante expected profits.

[3]The deposit rate that the household receives from investors equals the risk-free rate. Investors may be interpreted as being the financial institutions that act as fund suppliers to FIs in the financial market.

[4]We assume that the bankruptcy cost associated with investor–enptrepreneur credit contracts is high enough so that there are no direct credit contracts between them. The role of FIs in our model is consistent with recent views about FIs proposed in studies such as those of Allen (2001), Gorton and Winton (2003), and Gorton (2008). In these studies, FIs' economic activity is regarded as overwhelmingly important and broadly described as the 'shadow banking system.'

investors in order to borrow the rest of the funds. Entrepreneurs are the final borrowers of funds in the economy. They own net worth, but not enough to finance their projects. They thus engage in credit contracts with FIs in which they borrow the rest of the required funds from FIs. These two contracts are chained so that the entrepreneurs cannot finance their projects if either of the credit contracts does not hold.

There are agency problems associated with asymmetric information in the credit contracts between FIs and entrepreneurs (hereafter FE contracts) and the credit contracts between investors and FIs (hereafter IF contracts). This makes the borrowing rates dependent on the borrowers' net worth. The contents of the two credit contracts are chosen by monopolistic FIs, so that FIs maximize their profits, thereby ensuring the participation constraints of entrepreneurs and investors.

The structure of our model is based on BGG. However, in BGG, FIs are not credit constrained. FIs and investors are treated as the same institution that faces perfect competition and earns zero profit. There is only one credit contract between FIs and entrepreneurs. Because FIs face perfect competition, in the credit contract, it is entrepreneurs who maximize their profits, ensuring the participation constraint of FIs. FIs' credit conditions do not influence the content of the credit contract.

Our goods markets consist of input markets and output markets for final goods, and capital goods markets. These markets are competitive, and prices of all goods are flexible. Final goods producers have Cobb–Douglas production technology that converts capital and labor into final goods. Capital is supplied by entrepreneurs. Entrepreneurs purchase capital goods from capital goods producers using the funds they borrowed from the credit market, and sell capital goods to the final goods producers. Labor inputs are supplied by the household, FIs and entrepreneurs. Once produced, final goods are allocated to consumption and investment in the competitive final goods market.

2.1 Credit Contracts and Net Worth

The Environment

There is a continuum number of investors, FIs and entrepreneurs. They sign two types of credit contracts. There are three kinds of interest rates, $R(s^t)$, $R^F(s^t)$ and $R^E(s^t)$, that are relevant for the credit contracts, where s^t is state of the economy at t. $R(s^t)$ is the risk-free rate of return in the economy, $R^F(s^t)$ is the ex post return on the loans to entrepreneurs, and $R^E(s^t)$ is the ex post aggregate return to capital. At period t, investors collect deposits from a household for the risk-free rate in the competitive market and lend these deposits to a continuum of FIs. Investors' returns on the loans to FIs are equalized to their opportunity cost given by the risk-free rate. FIs monopolistically supply loans to a continuum of entrepreneurs. Each FI, say a type i FI, makes loan contracts with a specific group of entrepreneurs, say group j_i entrepreneurs, that are attached to the FI.[5] By lending to a continuum of group j_i entrepreneurs, a type i FI diversifies the loan risk associated with a specific entrepreneur and obtains a return of $R^F(s^t)$. Entrepreneurs are final borrowers in the economy. They invest their loans in the purchase of capital goods and receive the return to capital $R^E(s^t)$.

We begin with the FE contract. At the beginning of each period, each type i FI offers a loan contract to group j_i entrepreneurs. Each entrepreneur in group j_i owns net worth $N^E_{j_i}(s^t)$ and purchases capital of $Q(s^t)K_{t,j_i}(s^t)$, where $Q(s^t)$ is the price paid per unit of capital and $K_{t,j_i}(s^t)$ is the quantity of capital purchased by a group j_i entrepreneur. Following BGG, we assume that entrepreneurs are subject to an idiosyncratic productivity shock $\omega^E_{j_i}(s^{t+1})$ so that the net return to capital is $\omega^E_{j_i}(s^{t+1})R^E(s^{t+1})$. The FE contract specifies: (1) the amount of debt that a group j_i entrepreneur borrows from a type i FI, $Q(s^t)K_{j_i}(s^t)-N^E_{j_i}(s^t)$, (2) a cut-off value of idiosyncratic shock $\omega^E_{j_i}(s^{t+1})$, which we denote by $\overline{\omega}^E_{j_i}(s^{t+1}|s^t)$, such that entrepreneurs repay their debt for $\omega^E_{j_i}(s^{t+1}) \geq \overline{\omega}^E_{j_i}(s^{t+1}|s^t)$ and they declare the default for $\omega^E_{j_i}(s^{t+1}) < \overline{\omega}^E_{j_i}(s^{t+1}|s^t)$, and (3) a loan rate that group j_i entrepreneurs repay when they do not default, $Z^E_{j_i}(s^{t+1}|s^t)$. Ex post, non-default entrepreneur j_i receives $\left(\omega^E_{j_i}(s^{t+1}) - \overline{\omega}^E_{j_i}(s^{t+1}|s^t)\right)R^E(s^{t+1})Q(s^t)K_{j_i}(s^t)$ and the default entrepreneur receives nothing from the contract. The relationship between cut-off value $\overline{\omega}^E_{j_i}(s^{t+1}|s^t)$ and non default entrepreneurs' loan rate $Z^E_{j_i}(s^{t+1}|s^t)$ is given by:

$$\overline{\omega}^E_{j_i}\left(s^{t+1}|s^t\right)R^E\left(s^{t+1}|s^t\right)Q\left(s^t\right)K_{j_i}\left(s^t\right) = Z^E_{j_i}\left(s^{t+1}|s^t\right)\left(Q\left(s^t\right)K_{j_i}\left(s^t\right) - N^E_{j_i}\left(s^t\right)\right). \quad (1)$$

There is a participation constraint for entrepreneurs in the FE contract. Instead of participating in the FE contract, group j_i entrepreneurs can purchase capital goods using their own net worth $N^E_{j_i}(s^t)$,without participating in loan contracts with FIs. In this alternative case, the ex post return to their investments equals $\omega^E_{j_i}(s^{t+1})R^E(s^{t+1})N^E_{j_i}(s^t)$. Therefore, an FE contract between an FI and entrepreneurs is agreed only when the following inequality is expected to hold

$$\begin{aligned}
&\overbrace{\left[1-\Gamma^E_t\left(\overline{\omega}^E_{j_i}\left(s^{t+1}|s^t\right)\right)\right]}^{\text{share of entrepreneurial earnings paid to entrepreneur } j_i} \qquad R^E\left(s^{t+1}|s^t\right)Q\left(s^t\right)K_{ji}\left(s^t\right) \\
&\geq \quad R^E\left(s^{t+1}|s^t\right)N^E_{j_i}\left(s^t\right) \\
&\qquad \text{for } \forall j_i, s^{t+1}|s^t, \qquad (2)
\end{aligned}$$

where

[5] We assume that the bankruptcy cost associated with a credit contract between an FI other than a type i FI and group j_i entrepreneurs is high enough. Therefore, group j_i entrepreneurs can borrow funds only from a certain monopolistic FI. See Klein (1971) and Monti (1972) and Freixas and Rochet (2008) for a further discussion about monopolistic FIs.

$$\Gamma_t^E\left(\overline{\omega}_{j_i}^E\left(s^{t+1}|s^t\right)\right) \equiv \overbrace{G_t^E\left(\overline{\omega}_{j_i}^E\left(s^{t+1}|s^t\right)\right)}^{\text{expected productivity of defaulted entrepreneurs}} + \overline{\omega}_{j_i}^E\left(s^{t+1}|s^t\right) \overbrace{\int_{\overline{\omega}_{j_i}^E\left(s^{t+1}|s^t\right)}^{\infty} dF_t^E\left(\omega^E\right)}^{\text{portion of non-defaulted entrepreneurs}},$$

$$G_t^E\left(\overline{\omega}_{j_i}^E\left(s^{t+1}|s^t\right)\right) \equiv \int_0^{\overline{\omega}_{j_i}^E\left(s^{t+1}|s^t\right)} \omega^E dF_t^E\left(\omega^E\right).$$

Note that $1-\Gamma_t^E$ is the expected share of profits from purchasing capital goods that goes to the borrowers in the FE contract. The left-hand side of the inequality (2) shows the expected return from the FE contract for group j_i entrepreneurs, and the right-hand side of the inequality (2) shows the expected return from investing the entrepreneurial net worth $N_i^E\left(s^t\right)$. In this paper, we focus on the case where equation (2) holds with equality.[6]

The inequality (2) also gives the expression for the expected earnings of a type i FI for each FE contract

$$\overbrace{\Phi_{j_i,t}^E\left(\overline{\omega}_{j_i}^E\left(s^{t+1}|s^t\right)\right)}^{\text{share of entrepreneurial earnings paid to FI } i} R^E\left(s^{t+1}|s^t\right) Q\left(s^t\right) K_{ji}\left(s^t\right),$$

where

$$\Phi_{j_i,t}^E\left(\overline{\omega}_{j_i}^E\left(s^{t+1}|s^t\right)\right) \equiv \Gamma_t^E\left(\overline{\omega}_{j_i}^E\left(s^{t+1}|s^t\right)\right) - \mu^E G_t^E\left(\overline{\omega}_{i,t+1}^F\right).$$

Note that $\mu^E \omega_{j_i}^E\left(s^{t+1}\right) R^E\left(s^{t+1}\right) Q\left(s^t\right) K_{ji}\left(s^t\right)$, with $0<\mu^E<1$, is the ex post bankruptcy cost that FIs pay whenever group j_i entrepreneurs declare the default. Because each type i FI lends a continuum number of entrepreneurs in group j_i, the loan risk of the FI is perfectly diversified. For convenience, we define the expected return on the loans to entrepreneurs, $R^F\left(s^{t+1}|s^t\right)$ as

$$\int_{j_i} \overbrace{\left[\Gamma_t^E\left(\overline{\omega}_{j_i}^E\left(s^{t+1}|s^t\right)\right) - \mu^E G_t^E\left(\overline{\omega}_{j_i}^E\left(s^{t+1}|s^t\right)\right)\right]}^{\text{share of entrepreneurial earnings paid to FI } i} R^E\left(s^{t+1}|s^t\right) Q\left(s^t\right) K_{ji}\left(s^t\right) dj_i$$

[6] For some range of parameters, this participation constraint can hold in strict inequality. In this case, FIs choose the optimal level of cut-off value $\overline{\omega}_{j_i}^E\left(s^{t+1}|s^t\right)$ regardless of the entrepreneurial net worth. Interestingly, in this case, the external finance premium comes to depend only on the sum of the net worth of FIs and entrepreneurs. The distribution of net worth between the two borrowing sectors has no effect. According to parameters calibrated to the U.S. data, however, this participation constraint holds in equality. Therefore, in the following analysis, we focus on the case where the equation holds with equality.

$$\equiv R_t^F\left(s^{t+1}|s^t\right)\left(Q\left(s^t\right)K_i\left(s^t\right)-N_i^E\left(s^t\right)\right)\text{ for }\forall s^{t+1}|s^t, \tag{3}$$

where

$$\begin{aligned} K_i\left(s^t\right) &\equiv \int_{j_i} K_{j_i}\left(s^t\right)dj_i, \\ N_i^E\left(s^t\right) &\equiv \int_{j_i} N_{j_i}^E\left(s^t\right)dj_i. \end{aligned}$$

We further define variable $\Phi_{i,t}^F\left(\overline{\omega}_i^F\left(s^{t+1}|s^t\right)\right)$ as follows

$$\Phi_{i,t}^F\left(\overline{\omega}_i^F\left(s^{t+1}|s^t\right)\right)\equiv\Gamma_t^F\left(\overline{\omega}_i^F\left(s^{t+1}|s^t\right)\right)-\mu^F G_t^F\left(\overline{\omega}_i^F\left(s^{t+1}|s^t\right)\right).$$

The left-hand side of equation (3) is the gross profit that a specific type i FI receives from a continuum number of FE contracts with group j_i entrepreneurs.

We next turn to the IF contract. A type i FI splits this gross profit from the FE contract with investors according to another credit contract, the IF contract. The IF contract has the same costly state verification structure as the FE contract, but FIs now need to act as the borrowers rather than lenders. In the IF contract, investors provide loans to a continuum number of FIs. Each type i FI owns the net worth $N_i^F\left(s^t\right)$ and invests in the loans to group j_i entrepreneurs an amount of $Q\left(s^t\right)K_i\left(s^t\right)-N_i^E\left(s^t\right)$. It then borrows the rest $Q\left(s^t\right)K_i\left(s^t\right)-N_i^F\left(s^t\right)-N_i^E\left(s^t\right)$ from investors, and repays the loan using its profit from the FE contracts. We assume that each type i FI is subject to idiosyncratic productivity shock $\omega_i^F\left(s^{t+1}\right)$ [7] and its ex post gross return on the loans to entrepreneurs is given by $\omega_i^F\left(s^{t+1}\right)R^F\left(s^{t+1}\right)$. Here, the IF contract specifies: (1) the amount of debt that a type i FI borrows from investors, $Q\left(s^t\right)K_i\left(s^t\right)-N_i^E\left(s^t\right)-N_i^F\left(s^t\right)$, (2) a cut-off value of idiosyncratic shock $\omega_i^F\left(s^{t+1}\right)$, which we denote by $\overline{\omega}_i^F\left(s^{t+1}|s^t\right)$, such that FIs repay their debt for $\omega_i^F\left(s^{t+1}\right)\geq\overline{\omega}_i^F\left(s^{t+1}|s^t\right)$ and declare the default for $\omega_i^F\left(s^{t+1}\right)<\overline{\omega}_i^F\left(s^{t+1}|s^t\right)$, and (3) the return rate of the loan when the type i FI does not default, $Z_i^F\left(s^{t+1}|s^t\right)$. Here, ex post, non-default FI i receives $\left(\omega_i^F\left(s^{t+1}\right)-\overline{\omega}_i^F\left(s^{t+1}|s^t\right)\right)R^F\left(s^{t+1}\right)Q\left(s^t\right)K_i\left(s^t\right)$ and default FI receives nothing from the contract. The relationship between cut-off value $\overline{\omega}_i^F\left(s^{t+1}|s^t\right)$ and non default FIs' loan rate $Z_i^F\left(s^{t+1}|s^t\right)$ is given by:

$$\begin{aligned} &\overline{\omega}_i^F\left(s^{t+1}|s^t\right)R^F\left(s^{t+1}|s^t\right)\left(Q\left(s^t\right)K_i\left(s^t\right)-N_i^E\left(s^t\right)\right) \\ =\; &Z^F\left(s^{t+1}|s^t\right)\left(Q\left(s^t\right)K_i\left(s^t\right)-N_i^F\left(s^t\right)-N_i^E\left(s^t\right)\right). \end{aligned} \tag{4}$$

[7] The FI's idiosyncratic productivity shock ω_i^F is associated with the shock in bankruptcy costs, technology of financing short-term assets and liabilities, or the quality of borrowers in the FE contract, which differs across FIs. We assume that two variables $\omega_{j_i}^E$ and ω_i^F are unit mean, lognormal random variables distributed independently over time and across entrepreneurs and FIs. We express the density function of these variables as $f_t^E\left(\omega_i^E\right)$ and $f_t^F\left(\omega_i^F\right)$, and their cumulative distribution functions as $F_t^E\left(\omega_i^E\right)$ and $F_t^F\left(\omega_i^F\right)$. Following CMR, we also assume that the standard deviations of $\log\left(\omega_t^E\right)$ and $\log\left(\omega_t^F\right)$, denoted by σ_t^E and σ_t^F, respectively, are stochastic processes.

Similar to the FE contract, there is a participation constraint for the investors in the IF contract. Given the risk-free rate of return in the economy $R\left(s^t\right)$, investors' profit from the investment in the loans to FIs must equal the opportunity cost of lending. That is

$$\overbrace{\left[\Gamma^F\left(\overline{\omega}_i^F\left(s^{t+1}|s^t\right)\right)-\mu^F G_t^F\left(\overline{\omega}_i^F\left(s^{t+1}|s^t\right)\right)\right]}^{\text{share of FIs' earnings paid to investors}} R^F\left(s^{t+1}|s^t\right)\left(Q\left(s^t\right)K_i\left(s^t\right)-N_i^E\left(s^t\right)\right)$$

$$\geq R\left(s^t\right)\left(Q\left(s^t\right)K_i\left(s^t\right)-N_i^F\left(s^t\right)-N_i^E\left(s^t\right)\right), \tag{5}$$

where

$$\begin{aligned}\Gamma_t^F\left(\overline{\omega}_i^F\left(s^{t+1}|s^t\right)\right) \equiv & \overbrace{G_t^F\left(\overline{\omega_i}^F\left(s^{t+1}|s^t\right)\right)}^{\text{expected productivity of defaulted FIs}} \\ & +\overline{\omega}_i^F\left(s^{t+1}|s^t\right)\overbrace{\int_{\overline{\omega}_i^F\left(s^{t+1}|s^t\right)}^{\infty} dF_t^F\left(\omega^F\right)}^{\text{portion of non-defaulted FIs}},\end{aligned}$$

$$G_t^F\left(\overline{\omega}_i^F\left(s^{t+1}|s^t\right)\right) \equiv \int_0^{\overline{\omega}_i^F\left(s^{t+1}|s^t\right)} \omega^F dF_t^F\left(\omega^F\right),$$

and $0<\mu^F<1$ is the bankruptcy cost of FIs. Expected net profit for a type i FI is expressed as

$$\sum_{s^{t+1}} \Pi\left(s^{t+1}|s^t\right) \overbrace{\left[1-\Gamma_t^F\left(\overline{\omega}_i^F\left(s^{t+1}|s^t\right)\right)\right]}^{\text{share of FIs earnings paid to FIs}} R^F\left(s^{t+1}|s^t\right)\left(Q_t\left(s^t\right)K_i\left(s^t\right)-N_i^E\left(s^t\right)\right), \tag{6}$$

where $\Pi\left(s^{t+1}|s^t\right)$ is the probability weight for state s^{t+1}, depending on the information set available at period t.

Optimal Credit Contract

Type i FI maximizes their expected profit (6) by optimally choosing the variables $\overline{\omega}_i^F$, K_i, $\overline{\omega}_{j_i}^E$, K_{j_i}, subject to the investors' participation constraint (5) and entrepreneurial participation constraint (2). Combining the first-order conditions yields the following equation:

$$0 = \sum_{s^{t+1}|s^t} \Pi\left(s^{t+1}|s^t\right)\left\{\left(1-\Gamma_t^F\left(\overline{\omega}_i^F\left(s^{t+1}|s^t\right)\right)\right)\Phi_{i,t}^E\left(s^{t+1}|s^t\right)R^E\left(s^{t+1}|s^t\right)\right.$$

$$+\frac{\Gamma_t'^F\left(\overline{\omega}_i^F\left(s^{t+1}|s^t\right)\right)}{\Phi_{i,t}'^F\left(s^{t+1}|s^t\right)}\Phi_{i,t}^F\left(s^{t+1}|s^t\right)\Phi_{i,t}^E\left(s^{t+1}|s^t\right)R_{t+1}^E\left(s^{t+1}|s^t\right)$$

$$-\frac{\Gamma_t'^F\left(\overline{\omega}_i^F\left(s^{t+1}|s^t\right)\right)}{\Phi_{i,t}'^F\left(s^{t+1}|s^t\right)}R(s_t)$$

$$+\frac{\left\{1-\Gamma_t^F\left(\overline{\omega}_i^F\left(s^{t+1}|s^t\right)\right)\right\}\Phi'^E\left(s^{t+1}|s^t\right)}{\Gamma_t'^E\left(\overline{\omega}_{j_i}^E\left(s^{t+1}|s^t\right)\right)}\left(1-\Gamma_t^E\left(\overline{\omega}_{j_i}^E\left(s^{t+1}|s^t\right)\right)\right)R^E\left(s^{t+1}|s^t\right)$$

$$\left.+\frac{\Gamma_B'\left(\overline{\omega}_i^F\left(s^{t+1}|s^t\right)\right)\Phi^F\left(s^{t+1}|s^t\right)\Phi'^E\left(s^{t+1}|s^t\right)}{\Phi'^F\left(s^{t+1}|s^t\right)\Gamma'^E\left(\overline{\omega}_{j_i}^E\left(s^{t+1}|s^t\right)\right)}\left(1-\Gamma_t^E\left(\overline{\omega}_{j_i}^E\left(s^{t+1}|s^t\right)\right)\right)R^E\left(s^{t+1}|s^t\right)\right\}$$

$$\text{for } \forall j_i. \tag{7}$$

From the first-order condition (7) and the two participation constraints, (5) and (2), we can derive the following relation for FIs' optimal choice of capital $Q(s^t)K(s^t)$, taking external finance premium $E_t\left\{R^E\left(s^{t+1}\right)/R\left(s^{t+1}\right)\right\}$, FIs' own net worth $N^F(s^t)$ and entrepreneurial net worth $N^E(s^t)$ as given

$$\begin{aligned}
\frac{\mathrm{E}_t\left\{R^E\left(s^{t+1}\right)\right\}}{R\left(s^t\right)} &= \overbrace{\Phi_t^F\left(\overline{\omega}_t^F\left(\frac{N^F(s^t)}{Q(s^t)K(s^t)},\frac{N^E(s^t)}{Q(s^t)K(s^t)}\right)\right)^{-1}}^{\text{inverse of share of profit going to the investors in the IF contract}} \\
&\times \overbrace{\Phi_t^E\left(\overline{\omega}_t^E\left(\frac{N^E(s^t)}{Q(s^t)K(s^t)}\right)\right)^{-1}}^{\text{inverse of share of profit going to the FI in the FE contract}} \\
&\times \overbrace{\left(1-\frac{N^F(s^t)}{Q(s^t)K(s^t)}-\frac{N^E(s^t)}{Q(s^t)K(s^t)}\right)}^{\text{ratio of the debt to the size of the capital investment}} \\
&\equiv S_t\left(n^F\left(s^t\right),n^E\left(s^t\right)\right),
\end{aligned} \tag{8}$$

where $n_t^F(s^t)$ and $n_t^E(s^t)$ are the ratio of each of FIs' net worth and entrepreneurial net worth to the total amount of capital. The aggregation of the FIs and the entrepreneurs becomes tractable, because the ratio of net worth to capital is the same across the FIs, and across the entrepreneurs.

This equation is the key to the model, which describes the relationship between the two net worths $N_t^F(s^t)$ and $N_t^E(s^t)$, and the external finance premium $\mathrm{E}_t\left\{R^E\left(s^{t+1}\right)\right\}R\left(s^t\right)^{-1}$.

Because $R^E(s^{t+1})$ corresponds to the return from capital investment, a higher external finance premium implies lower capital investment. It is notable that each net worth as well as the sum of the net worths are important determinants of the external finance premium. The third term on the right-hand side of the equation indicates that the external finance premium decreases with the sum of the net worth held by the two borrowing sectors. This is because if the leverage of the IF and FE contracts is low, the investors do not require high returns from the contract. In addition, the first and the second terms on the right-hand side of the equation make each net worth as well as the sum of the two net worths affect the external finance premium separately. As FIs' net worth and entrepreneurial net worth work differently through the two credit contracts, they influence the external finance premium nonlinearly. Below we display the relationship $S(\cdot)$, using numerical exercises. We first study the relationship between each of the two net worths $N_t^F(s^t)$ and $N_t^E(s^t)$, and the external finance premium. We then investigate how the relative size of the two net worths, or equivalently, the distribution of net worth across the two sectors, is related to the external finance premium.

Cost-of-Funds Curve

Figure 1 displays the cost-of-funds curve in our economy. This curve presents the relationship between the external finance premium, or equivalently, the expected discounted return to capital, and the net worth/capital ratio in each of the sectors, based on the function $S(\cdot)$.[8] The net worth/capital ratio in each sector is depicted on the horizontal axis, and the external finance premium is depicted on the vertical axis. In the left panel of Figure 1, we show the values of the external finance premium for various sizes of the FIs' net worth/capital ratio, maintaining a constant entrepreneurial net worth/capital ratio.

According to the panel, the external finance premium is decreasing in the FIs' net worth/capital ratio. As capital investment increases relative to the FIs' net worth, the expected bankruptcy costs associated with the IF contract rise. This is demonstrated in the top left panel of Figure 2. In contrast, a fall in the FIs' net worth does not affect the expected default cost of the FE contract, because the entrepreneurs' participation constraint is independent from the FIs' net worth. Given higher expected bankruptcy costs, a marginal increase in the capital investment is profitable to FIs only if the expected discounted return to capital is high enough.

Another important feature of this curve is the role of the net worth held by entrepreneurs. As the right panel of Figure 1 indicates, the external finance premium is also decreasing in the entrepreneurial net worth/capital ratio. The bottom right panel of

[8]For the exercises displayed in Figures 1 and 2, we set the model parameters pertaining to the two credit contracts following BGG. Namely, we set the values for parameters μ^E, σ^E and $1-\gamma^E$ equal to the values of the bankruptcy cost, variance of entrepreneurial idiosyncratic productivity and death rate reported in BGG, respectively. We further assume that $\mu^F = \mu^E, \sigma^F = \sigma^F$ and $\gamma^F = \gamma^E$ so that the two credit contracts are symmetric in terms of these parameters.

Figure 2 shows that a rise in entrepreneurial net worth reduces the expected default cost associated with the FE contract, reducing the cost of capital investment. Furthermore, the bottom left panel of Figure 2 suggests that the expected default cost of the IF contract decreases with entrepreneurial net worth. Because the two credit contracts are chained, the decline of the entrepreneurial default probability increases the return of the loan to the FIs, making the FIs' default probability lower, too.

We next discuss the role of the net worth distribution. Unlike BGG, our net worth is distributed across two distinct sectors. Because the two net worths contribute to the two credit contracts differently, the relative size of the each net worth is important to the capital investment. Figure 3 displays the share of the net worth held by the FI sector on the horizontal axis and the external finance premium on the vertical axis. We set the ratio of total net worth to the total amount of capital investment equal to .6, and investigate how the external finance premium changes with an increase in the FIs' share. The solid line in the two panels of Figure 3 represents the cost-of-funds curve, which gives the relationship between the share and the external finance premium when the bankruptcy costs and the variance of idiosyncratic productivity are symmetric across the two contracts.[9]

We find that the cost-of-funds curve is U-shaped with respect to the FIs' share. The required expected discounted return to capital is decreasing in the FI's share, when the share is smaller than 40%, and it is increasing in the share, when the share is above 40%. The curve implies that it is profitable for the FIs to conduct smaller capital investment even when entrepreneurial (FIs') net worth is large, if the net worth held by the FI (entrepreneurs) sector is very small.[10] Consequently, the net worth in a sector that owns a smaller amount of net worth affects the capital size disproportionately.

Figure 4 illustrates this point in more detail. In the region where the FIs' share is small, the expected default cost of the IF contract declines significantly and that of the FE contract increases moderately with the FIs' net worth. On the other hand, in the region where the FIs' share is large, the expected default cost of the IF contract decreases moderately while that of the FE contract increases significantly with the FIs' net worth.

Admittedly, this property is not independent from how the expected default costs of the contracts are related to the net worth/capital ratio of the borrowers. We therefore discuss the cases in which the size of the bankruptcy cost or the distribution of borrowers' idiosyncratic productivity is different across the contracts. First, we study the case in which entrepreneurial bankruptcy cost is higher, $\mu^E = \mu = 2\mu^F$, and vice versa. The lines with black circles in Figures 3 and 4 show the case when $\mu^E = \mu = 2\mu^F$. Here, the

[9]Here, we set $\mu^F = \mu^E = \mu$ and $\sigma^F = \sigma^E = \sigma$, where μ and σ are the bankruptcy cost and the variance used in BGG.

[10]The external finance premium is lowest when the FIs' share is not 50% but around 40%. This indicates asymmetry between the two net worths, reflecting that the two credit contracts are not horizontally but vertically chained. Quantitatively, however, the effect of this asymmetry on the external finance premium is small compared with the effect of the asymmetry coming from bankruptcy costs and variance of idiosyncratic shocks discussed in the next section.

external finance premium is increasing with the FIs' net worth share. The higher the FIs' share is, the lower the FIs' default probability and the higher the entrepreneurial default probability. This is because a rise in the default cost of entrepreneurs in the FE contract dominates a decline in the default cost of the FIs in the IF contract. Consequently, FIs require a higher expected discounted return to capital as net worth is more distributed to the FI sector. The dashed lines with black circles in the upper panel of Figures 3 and 4 show the opposite case. In this case, the external finance premium is decreasing with the FIs' net worth share by a similar but opposite mechanism.

Finally, we discuss the case in which the variance of borrowers' idiosyncratic productivity is different between the IF contract and FE contract. We study the case in which the variance of the FIs' idiosyncratic productivity is higher, $\sigma^F = \sigma = 2\sigma^E$, and the opposite case, $\sigma^E = \sigma = 2\sigma^F$. Figures 5 and 6 present the outcomes of these exercises. Similar to the results for changing bankruptcy costs, asymmetric variances across the two borrowers shift the cost-of-funds curve downwards. However, in contrast to the case of changing bankruptcy costs, the U-shape of the curve is only slightly modified under changes of variances across credit contracts.

Dynamic Behavior of Net Worth

The net worths of FIs and entrepreneurs, $N^F(s^t)$ and $N^E(s^t)$, depend on their earnings from the credit contracts and their labor income. In addition to the profits from entrepreneurial projects, both FIs and entrepreneurs inelastically supply a unit of labor to final goods producers and receive labor income $W^F(s^t)$ and $W^E(s^t)$.[11] We assume that each FI and entrepreneur survives to the next period with a constant probability γ^F and γ^E; then, the aggregate net worths of FIs and entrepreneurs are given by

$$N^F\left(s^{t+1}\right) = \gamma^F V^F\left(s^t\right) + W^F\left(s^t\right), \tag{9}$$

$$N^E\left(s^{t+1}\right) = \gamma^E V^E\left(s^t\right) + W^E\left(s^t\right), \tag{10}$$

with:

$$\begin{aligned} V^F\left(s^t\right) &\equiv \left(1-\Gamma_t^F\left(\overline{\omega}^F\left(s^{t+1}\right)\right)\right)\left(\Gamma_t^E\left(\overline{\omega}^E\left(s^{t+1}\right)\right)-\mu^E G_t^E\left(\overline{\omega}^E\left(s^{t+1}\right)\right)\right) \\ &\quad \cdot R^E\left(s^{t+1}\right) Q\left(s^t\right) K\left(s^t\right), \\ V^E\left(s^t\right) &\equiv \left(1-\Gamma_t^E\left(\overline{\omega}^E\left(s^{t+1}\right)\right)\right) R^E\left(s^{t+1}\right) Q\left(s^t\right) K\left(s^t\right). \end{aligned}$$

FIs and entrepreneurs that fail to survive at period t consume $\left(1-\gamma^F\right)V^F\left(s^t\right)$ and $\left(1-\gamma^E\right)V^E\left(s^t\right)$, respectively.

[11] See BGG and CMR for the technical reason for this specification.

2.2 Rest of the Economy

Household

A representative household is infinitely lived, and maximizes the following utility function subject to the budget constraint

$$\max_{C(s^t),H(s^t),D(s^t)} \sum_{l=0} \beta^{t+l} \mathrm{E}_t \left\{ \log C\left(s^{t+l}\right) - \chi \frac{H\left(s^{t+l}\right)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right\}, \tag{11}$$

subject to

$$C\left(s^{t+l}\right) + D\left(s^{t+l}\right) \leq W\left(s^{t+l}\right) H\left(s^{t+l}\right) + R\left(s^{t+l}\right) D\left(s^{t+l}\right) - T\left(s^{t+l}\right),$$

where $C\left(s^t\right)$ is final goods consumption, $H\left(s^t\right)$ is hours worked, $D\left(s^t\right)$ is real deposits held by investors, $W\left(s^t\right)$ is the real wage measured by the final goods, $R\left(s^t\right)$ is the real risk-free return from the deposit $D\left(s^t\right)$ between time t and $t+1$, and $T\left(s^t\right)$ is the lump-sum transfer. $\beta \in (0,1)$, η and χ are the subjective discount factor, the elasticity of leisure, and the utility weight on leisure.

The first-order conditions associated with the household's maximization problem are:

$$\frac{1}{C\left(s^t\right)} = \beta E_t \left\{ \frac{1}{C\left(s^{t+1}\right)} R\left(s^t\right) \right\}, \tag{12}$$

$$W\left(s^t\right) = \chi H\left(s^t\right)^{\frac{1}{\eta}} C\left(s^t\right). \tag{13}$$

Final Goods Producers

Final goods producers are price takers in both input markets and output markets. They hire three types of labor inputs: $H\left(s^t\right)$, $H^F\left(s^t\right)$ and $H^E\left(s^t\right)$, from a household, FIs and entrepreneurs, and pay real wages $W\left(s^t\right)$, $W^F\left(s^t\right)$ and $W^E\left(s^t\right)$ to each type of labor input, respectively. Capital $K\left(s^{t-1}\right)$ is supplied from entrepreneurs with rental price $R^E\left(s^t\right)$. At the end of each period, the capital is sold back to entrepreneurs at price $Q\left(s^{t-1}\right)$. The maximization problem for final goods producers is given by

$$\begin{aligned} \max_{Y(s^t),K(s^{t-1}),H(s^t),H^F(s^t),H^E(s^t)} & Y\left(s^t\right) + Q\left(s^{t-1}\right) K\left(s^{t-1}\right)(1-\delta) \\ & -R^E\left(s^t\right) Q\left(s^{t-1}\right) K\left(s^{t-1}\right) - W\left(s^t\right) H\left(s^t\right) \\ & -W^F\left(s^t\right) H^F\left(s^t\right) - W^E\left(s^t\right) H^E\left(s^t\right), \end{aligned}$$

subject to

$$Y\left(s^{t}\right) = A\exp\left(e_{t}^{A}\left(s^{t}\right)\right)K\left(s^{t-1}\right)^{\alpha}L\left(s^{t}\right)^{1-\alpha},$$
$$L\left(s^{t}\right) \equiv \left(H\left(s^{t}\right)\right)^{1-\Omega_{E}-\Omega_{F}}\left(H^{F}\left(s^{t}\right)\right)^{\Omega_{F}}\left(H^{E}\left(s^{t}\right)\right)^{\Omega_{E}},$$

where $Y\left(s^{t}\right)$ is the final goods produced and $A\exp\left(e^{A}\left(s^{t}\right)\right)$ denotes the level of technology of final goods production. $\delta\in(0,1]$, α, Ω_E and Ω_F are the depreciation rate of capital goods, the capital share, the share of FIs' labor inputs and the share of entrepreneurial labor inputs. The first-order conditions for final goods producers are:

$$\alpha\frac{Y\left(s^{t}\right)}{K\left(s^{t-1}\right)}-R^{E}\left(s^{t}\right)Q\left(s^{t-1}\right)+Q\left(s^{t-1}\right)(1-\delta)=0, \tag{14}$$

$$(1-\alpha)\left(1-\Omega_{F}-\Omega_{E}\right)\frac{Y\left(s^{t}\right)}{H\left(s^{t}\right)} = W\left(s^{t}\right), \tag{15}$$
$$(1-\alpha)\Omega_{F}\frac{Y\left(s^{t}\right)}{H^{F}\left(s^{t}\right)} = W^{F}\left(s^{t}\right), \tag{16}$$
$$(1-\alpha)\Omega_{E}\frac{Y\left(s^{t}\right)}{H^{E}\left(s^{t}\right)} = W^{E}\left(s^{t}\right). \tag{17}$$

Capital Goods Producers

Capital goods producers own technology that converts final goods to capital goods. They sell capital goods in a competitive market with price $Q\left(s^{t-1}\right)$. At each period, the capital goods producers purchase $I\left(s^{t}\right)$ amount of final goods from final goods producers. They also receive $K\left(s^{t-1}\right)(1-\delta)$ of used capital goods from the final goods producers at price $Q\left(s^{t-1}\right)$. They then produce capital goods $K\left(s^{t}\right)$, using technology F_I. The capital goods producers' problem is to maximize the profit function given below

$$\max_{I_t}\sum_{l=0}^{\infty}\Pi\left(s^{t+l}|s^{t}\right)\Lambda_{t+l}\left(s^{t+l}\right) \cdot\left[Q_{t+l}\left(s^{t+l-1}\right)\left(1-F_{I}\left(I_{t+l}\left(s^{t+l}\right),I_{t+l-1}\left(s^{t+l-1}\right)\right)\right)I_{t+l}\left(s^{t+l}\right)-I_{t+l}\left(s^{t+l}\right)\right], \tag{18}$$

where F_I is defined as follows:

$$F_{I}\left(I_{t+l}\left(s^{t+l}\right),I_{t+l-1}\left(s^{t+l-1}\right)\right)\equiv\frac{\kappa}{2}\left(\frac{I_{t+l}\left(s^{t+l}\right)}{I_{t+l-1}\left(s^{t+l-1}\right)}-1\right)^{2}.$$

Note that κ is a parameter that is associated with investment adjustment cost.[12]

[12] Equation (18) does not have a term for used capital K_{t-1} that is sold by entrepreneurs at the end of the last period. This is because, following BGG, we assume that the price of capital that entrepreneurs sell to the capital goods producers at the end of the period, say $\overline{Q}_t$, is close to the price of newly produced capital Q_t around the steady state.

Because capital depreciates at each period, the evolvement of total capital available at period t is given by:

$$K\left(s^{t}\right)=\left(1-F_{I}\left(I\left(s^{t}\right),I\left(s^{t-1}\right)\right)\right)I\left(s^{t}\right)+\left(1-\delta\right)K\left(s^{t-1}\right). \tag{19}$$

Government

The government collects a lump-sum tax from a household $T\left(s^{t}\right)$, and spends $G\left(s^{t}\right)$. A balanced budget is maintained for each period t as:

$$G\left(s^{t}\right)=T\left(s^{t}\right). \tag{20}$$

Resource Constraint

The resource constraint for final goods is written as:

$$\begin{aligned}Y\left(s^{t}\right) = {} & C\left(s^{t}\right)+I\left(s^{t}\right)+G\left(s^{t}\right)+\mu^{E}G_{t}^{E}\left(\overline{\omega}^{E}\left(s^{t}\right)\right)R^{E}\left(s^{t}\right)Q\left(s^{t-1}\right)K\left(s^{t-1}\right)\\ & +\mu^{F}G_{t}^{F}\left(\overline{\omega}^{F}\left(s^{t}\right)\right)R^{F}\left(s^{t}\right)\left(Q\left(s^{t-1}\right)K\left(s^{t-1}\right)-N^{E}\left(s^{t-1}\right).\right.\\ & +\left(1-\gamma^{FI}\right)\left(1-\Gamma_{t}^{F}\left(\overline{\omega}^{F}\left(s^{t}\right)\right)\right)\Gamma_{t}^{E}\left(\overline{\omega}^{E}\left(s^{t}\right)\right)R^{E}\left(s^{t}\right)Q\left(s^{t-1}\right)K\left(s^{t-1}\right)\\ & -\left(1-\gamma^{FI}\right)\left(1-\Gamma_{t}^{F}\left(\overline{\omega}^{F}\left(s^{t}\right)\right)\right)\mu^{E}G_{t}^{E}\left(\overline{\omega}^{E}\left(s^{t}\right)\right)R^{E}\left(s^{t}\right)Q\left(s^{t-1}\right)K\left(s^{t-1}\right)\\ & +\left(1-\gamma^{E}\right)\left(1-\Gamma_{t}^{E}\left(\overline{\omega}^{E}\left(s^{t}\right)\right)\right)R^{E}\left(s^{t}\right)Q\left(s^{t-1}\right)K\left(s^{t-1}\right).\end{aligned} \tag{21}$$

Note that the fourth and the fifth terms on the right-hand side of the equation correspond to the bankruptcy costs spent by FIs and the household, respectively.

Exogenous Variables

The exogenous shocks to the model, that is, the technology shock, the shocks to the standard error of the idiosyncratic productivity of the FIs, and the shocks to the standard error of the idiosyncratic productivity of the entrepreneurs follow the processes

$$e^{A}\left(s^{t}\right)=\rho_{A}e^{A}\left(s^{t-1}\right)+\varepsilon^{A}\left(s^{t}\right), \tag{22}$$

$$\log\left(\frac{\sigma^{F}\left(s^{t}\right)}{\overline{\sigma}^{F}}\right)=\rho_{\sigma_{F}}\log\left(\frac{\sigma^{F}\left(s^{t-1}\right)}{\overline{\sigma}^{F}}\right)+\varepsilon^{\sigma_{F}}\left(s^{t}\right), \tag{23}$$

$$\log\left(\frac{\sigma^{E}\left(s^{t}\right)}{\overline{\sigma}^{E}}\right)=\rho_{\sigma_{E}}\log\left(\frac{\sigma^{E}\left(s^{t-1}\right)}{\overline{\sigma}^{E}}\right)+\varepsilon^{\sigma_{E}}\left(s^{t}\right), \tag{24}$$

where ρ_{A}, $\rho_{\sigma_{E}}$ and $\rho_{\sigma_{F}}\in(0,1)$ are autoregressive roots of the exogenous variables, and $\varepsilon^{A}\left(s^{t}\right),\varepsilon^{\sigma_{E}}\left(s^{t}\right)$ and $\varepsilon^{\sigma_{F}}\left(s^{t}\right)$ are innovations that are mutually independent, serially uncorrelated and normally distributed with mean zero and variances $\sigma_{A}^{2},\sigma_{\sigma^{F}}^{2}$ and $\sigma_{\sigma^{E}}^{2}$, respectively.

2.3 Equilibrium Condition

An equilibrium consists of a set of prices, $\{R(s^t), R^F(s^t), R^E(s^t), W(s^t), W^F(s^t), W^E(s^t), Q(s^t), R^F(s^{t+1}|s^t), R^E(s^{t+1}|s^t), Z^F(s^{t+1}|s^t), Z^E(s^{t+1}|s^t)\}_{t=0}^{\infty}$, and the allocations $\{\{\overline{\omega}_i^F(s^{t+1}|s^t)\}_{i=1}^{\infty}\}_{t=0}^{\infty}$, $\{\{\overline{\omega}_{j_i}^E(s^{t+1}|s^t)\}_{j_i=1}^{\infty}\}_{t=0}^{\infty}$, $\{\{N_i^F(s^t)\}_{i=1}^{\infty}\}_{t=0}^{\infty}$, $\{\{N_{j_i}^E(s^t)\}_{j_i=1}^{\infty}\}_{t=0}^{\infty}$ $\{Y(s^t), C(s^t), D(s^t), I(s^t), K(s^t), H(s^t)\}\}_{t=0}^{\infty}$, for a given government policy $\{G(s^t), T(s^t)\}_{t=0}^{\infty}$, realization of exogenous variables $\{\varepsilon^A(s^t), \varepsilon^{\sigma_E}(s^t), \varepsilon^{\sigma_F}(s^t)\}_{t=0}^{\infty}$ and initial conditions $\{N_{i,-1}^F\}_{i=1}^{\infty}$, $\{N_{j_i,-1}^E\}_{j_i=1}^{\infty}$, $\{K_{-1}\}$ such that for all t, i, j_i and h : (i) the household maximizes its utility given the prices; (ii) the FIs maximize their profits given the prices; (iii) the entrepreneurs maximize their profits given the prices; (iv) final goods producers maximize their profits given the prices; (v) capital goods producers maximizes their profit given the prices; (vi) the government budget constraint holds; (vii) and markets clear.

3 Simulation

To study the quantitative relationships between the external finance premium and net worth of borrowing sectors, we first calibrate the parameters to the U.S. data. Our calibration reveals that the cost-of-funds curve is a decreasing function of the share of the FIs' net worth in the United States. Next, we calculate the steady state of the model, and linearize the equilibrium conditions (7), (9), (10), (12), (13), (14), (15), (16), (17) and (19) around the steady state. We then compute the equilibrium response of the economy to the adverse shocks that are commonly analyzed in the literature. We study five types of adverse shocks: (1) a net worth shock in the FI sector, (2) a net worth shock in the entrepreneurial sector, (3) a shock to the standard error of idiosyncratic productivity in the FI sector, (4) a shock to the standard error of idiosyncratic productivity in the entrepreneurial sector, and (5) a shock to the technology in the final goods sector. (1), (2), (3) and (4) are sectoral shocks that hit each of the participants in the credit market, and (5) is an aggregate shock. For the first two shocks, we introduce an innovation either in equation (9) or (10), following Gilchrist and Leahy (2002). The other shocks are illustrated in the equations (23), (24) and (22), by $\varepsilon^{\sigma_F}(s^t)$, $\varepsilon^{\sigma_E}(s^t)$ and $\varepsilon^A(s^t)$. Hereafter, we call the shock to the standard error of borrowers' idiosyncratic productivity the "riskiness shock," following the terminology of CMR.[13] The riskiness shock, to our knowledge, was first introduced in the financial accelerator model of CMR. We assume that both FIs and entrepreneurs are subject to this riskiness shock.

The quantitative exercises illustrate the relationship between the financial accelerator effect and the nature of the two credit contracts. In particular, our analysis reveals how the FIs' credit constraint affects the propagation and amplification mechanism. Using

[13] Bernanke (1983) and Bernanke and Blinder (1988) argue that the Great Depression partly stemmed from the increased riskiness of loans.

the model calibrated to the U.S. economy, which we call the "baseline model," we conduct the following three separate exercises.

First, we compare our calibrated model with an alternative model in which only entrepreneurs are credit constrained and FIs are not constrained. We call this model the "BGG model." This model can be considered as a special case of our baseline model in which the credit friction of the IF contract is negligibly small. By comparing the economic responses to adverse shocks under the BGG model and baseline model, we isolate the financial accelerator effect coming from the IF contract.

Second, we compare the macroeconomic consequences of a sectoral shock to the FI sector with those of a sectoral shock to the entrepreneurial sector. The comparison between the two shocks illustrates the relative significance of the two borrowing sectors in the financial accelerator mechanism.

Third, we study the model's response to the aggregate adverse shock. We again compare our baseline model with now alternative models in which the net worth across sectors is allocated differently from that under the baseline model. By comparing the economic responses to the aggregate shock, we show the quantitative role of the net worth distribution in the financial accelerator mechanism.

3.1 Calibration

We choose several parameter values used in BGG for our benchmark model. These include, the quarterly discount factor β, labor supply elasticity η, capital share α, quarterly depreciation rate δ, and steady state share of government expenditure in total output G/Y. We set values for six parameters that are linked to the IF contract and FE contract so that these values are consistent with the following seven conditions. These are (1) the risk spread, $R^E - R$, equals to 200 basis points annually; (2) the ratio of net worth held by FIs to capital, N^F/QK, is .1, which is close to the actual value according to the data;[14] (3) the ratio of net worth held by entrepreneurs to capital, N^E/QK, is .5, the approximate value in the data; (4) the annualized failure rate of FIs is 2%;[15] and (5) the annualized failure rate of entrepreneurs is 2%. Conditions (1), (3), and (5) are the same as those used in BGG. Two more conditions are set so as to be approximately consistent with the U.S. data: (6) the spread between the FIs' loan rate and the FIs' borrowing rate $Z^E - Z^F$ equals 230 basis points annually, which equals the historical average spread between the prime lending rate and the six-month certificates of deposit rate from 1980 to 2006; and (7) the spread between the FIs' borrowing rate and risk-

[14] We calculate the steady state value of N^E/QK based on the Flow of Fund data, released by the Federal Reserve Board. We calculate the historical series of the sum of corporate equities and equity in noncorporate business held by financial sectors divided by total liability and equity of nonfinancial business sector, and set at the steady state value of .1 for N^E/QK, which is the historical average from 1990 to 2005.

[15] Although the FI's failure rate may seem to be lower than the entrepreneur's failure rate, we set them to this value based on the observation of the CDS premium data during the recent crisis periods.

free rate $Z^F - R$ equals 60 basis points annually, which turns out to be approximately the historical average spread between the six-month certificates of deposit rate and the six-month treasury bill rate from 1980 to 2006.

The estimated parameters from these steady state conditions include the lenders' bankruptcy cost in the IF contract μ^F, the lenders' bankruptcy cost in the FE contract μ^E, the standard error of the idiosyncratic productivity shock in the FI sector σ_t^F, the standard error of the idiosyncratic productivity shock in the entrepreneurial sector σ_t^E, the survival rate of FIs γ^F and survival rate of entrepreneurs γ^E. See Appendices B and C for details.[16]

3.2 Cost-of-Funds Curve

Figure 7 displays the quantitative relationships between the external finance premium and net worth distribution, under parameters consistent with the U.S. economy. The U.S. cost-of-funds curve looks like a mixture of the curves shown in the sections above. Two observations are worth noting. First, in the current U.S. economy, the external finance premium decreases with the share of the FIs' net worth, because the share of the FIs' net worth is 0.1/(0.1+0.5)=0.17. Second, the figure is U-shaped and similar to those depicted by the solid line in Figures 3 and 5, but this curve is tilted to the right. As a result, the external finance premium decreases with the FIs' share for a wider range of share values.

This observation stems from the difference in the bankruptcy costs between FIs and entrepreneurs. According to the U.S. data, the spread between the FIs' loan rate and the FIs' borrowing rate, $Z^E - Z^F$, is on average three times larger than the spread between the FIs' borrowing rate and risk-free rate $Z^F - R$. It implies that the bankruptcy cost of the FIs is more expensive than that of entrepreneurs.

3.3 Comparison with BGG Model

We first compare the quantitative implications of our baseline model with those of the "BGG model." Here, our goal is to isolate the financial accelerator effect coming from the FI sector from that coming from the entrepreneurial sector. To make a fair comparison between the two models, we newly construct a "BGG model" that differs from the baseline model only in terms of credit market settings. The FI sector is dropped from this BGG model, and the investors and entrepreneurs make direct credit contracts in the same manner as in the model of BGG.[17] Because there is no agency problem associated

[16] Lenders' bankruptcy cost associated with entrepreneurs, μ^E, is 0.013. This number is much smaller than that in BGG, 0.12. This is because we use the same risk spread (200 basis points) as BGG, while our model incorporates additional credit frictions in the FI sector.

[17] The differences from the original BGG model are that (*i*) we assume that all goods prices are flexible, and (*ii*) we set parameter values equal to our baseline model.

with the FIs, the FIs' net worth plays no role. Thus the external finance premium reflects only the entrepreneurial net worth or entrepreneurial riskiness. Consequently, the financial accelerator effect of the BGG model comes only from the entrepreneurial sector.[18]

Figure 8 presents the results of this exercise. The upper panels display the responses of the endogenous variables to the net worth shocks under the two models. We consider the case where each sector of the two models is subject to an unexpected, once-and-for-all decline in the net worth of .01 units of its steady state value. The line with black circles denotes the response of the baseline model to a .01 units exogenous decline in the FIs' net worth. The dotted and solid lines denote the models' response to a .01 units exogenous decline in the entrepreneurial net worth, under our baseline economy and our BGG model, respectively.

The middle panels display the economic responses to the positive riskiness shocks. We assume that the value of $\sigma^F(s^t)$ $\left(\sigma^E(s^t)\right)$ exogenously jumps up at the initial period by 1%, and gradually returns to its steady state exogenously at the rate of $\rho_{\sigma_F}(s^t)$ $\left(\rho_{\sigma_E}(s^t)\right)$ following equations (23) and (24). The lines with black circles denotes the response of the baseline model to an increase in the FIs' riskiness $\sigma^F(s^t)$. The dotted and solid lines denote the models' response to an increase in entrepreneurial riskiness $\sigma^E(s^t)$, under our baseline economy and under our BGG model.[19]

Lastly, the lower panels display the responses of endogenous variables to the negative technology shock. We consider the case where the productivity of final goods drops at the initial period then gradually returns to its steady state at the rate of ρ_A. The line with black circles denotes the response of the baseline model. The dotted line denotes the model's response under our BGG model.

Comparing the economic responses to the net worth shock, the effect of the shock to the FI sector under the baseline model is the largest, and that to the entrepreneurial sector under the BGG model is the smallest. In response to the negative technology shock, the effect on the external finance premium and investment is larger under the baseline model than under the BGG model. The effect of the FIs' riskiness shock is also larger under the baseline model while that of the entrepreneurial riskiness shock is almost equivalent but slightly smaller under the baseline model than the BGG model.[20]

[18] We set parameter values related to the entrepreneurial sector in our BGG model to the same values used in our baseline model. Thus we set the values of σ^E, μ^E, and n^E the same across the two models. Furthermore, we choose the same steady state return to capital R^E for the two models. We choose to do so because we aim to compare the models' dynamics in a similar economic environment with respect to aggregate investment. Our choice yields the recalibrated values of γ^E and R for the BGG model, which differ from the baseline model.

[19] In this experiment, for comparison, we set the size of the riskiness shock to 1% of its steady state level for FIs and entrepreneurs. An alternative is to give the same size shock to the two sectors. Because the calibrated value of σ^F is lower than that of σ^E, this alternative yields an even larger response of the external finance premium and investment to the FIs' riskiness shock than to the entrepreneurial riskiness shock.

[20] For sensitivity analysis, we also compare our baseline model with the BGG model which is calibrated

To summarize, the existence of the agency problem in the FI sector drastically affects the financial accelerator effect. In response to the technology shock, our model yields larger movements of endogenous variables, suggesting the importance of the FI sector as an amplifier of the exogenous shocks. In our credit market, the IF contract works together with the FE contract, to reinforce the credit market imperfection, making the economic response larger.

In terms of the shocks to the credit market, shocks to the FI sector are more magnified compared with those to the entrepreneurial sector in both models, suggesting the importance of the FI sector as a source of the fluctuations. In addition, our simulation implies that the sectoral shock to the entrepreneurial sector may also be magnified by the presence of the credit-constrained FIs. In response to the net worth shock to the entrepreneurial sector, the model with the IF contract generates a larger response compared with the BGG model.

In the next two sections, we look more closely at our baseline model's response to sectoral shocks (shocks to the credit market) and the technology shock.

3.4 Shocks to the Credit Market

We next discuss the propagation and amplification mechanism of our model, in response to two types of sectoral shocks, net worth shocks and riskiness shocks. We first consider an experiment where the baseline economy is subject to an unexpected, once-and-for-all decline of the FIs' net worth by .01 units of net worth. The solid line in Figure 9 presents the economy's response to the shock under the baseline model. In response to the shock, the entrepreneurial net worth as well as FIs' net worth decreases. As the two net worths decline, the external finance premium rises, reducing the aggregate investment. These endogenous developments of the net worths decrease the FIs' net worth further, which magnifies the financial accelerator effect.

For comparison, we simulate the economy's response to the once-and-for-all decline in the entrepreneurial net worth by .01 units of net worth, using a dashed line in Figure 9. This shock generates qualitatively similar dynamics as those for the FI sector. Its quantitative impacts are, however, relatively moderate. The reason for this asymmetry is the bias of the net worth distribution. Recall in Figure 7 that the effect of a one-unit change of the FIs' net worth on the external finance premium dominates that of the entrepreneurial net worth, when the net worth distribution is biased toward the entrepreneurial sector as it is in the United States.

Similarly, we conduct an experiment where the baseline economy is subjected to an unexpected, 1% increase in the FIs' riskiness $\sigma^F(s^t)$ and entrepreneurial riskiness $\sigma^E(s^t)$. Figure 10 shows that for both shocks, an increase in the riskiness generates a decrease in

in a different manner (not reported). We set the same value of the riskfree rate R across the models, and set different values of the steady state R^E for the two models. The economic responses of this BGG model are qualitatively similar to the BGG model that is calibrated differently.

the capital goods price $Q(s^t)$. As (14) implies, it reduces the return to capital, resulting in a decrease of the capital investment demand. The asymmetry is also observed in the riskiness shock. The quantitative impacts of the FIs' riskiness shock on aggregate variables are, however, larger than those of the entrepreneurial riskiness shock.

To see the source of the asymmetry, we consider another two alternative economies with different initial net worth distributions. In the first economy, we distribute the net worth across sectors equally.[21] In the second economy, we distribute more net worth to the FI sector. Figure 11 demonstrates the responses of investment to the four adverse shocks, under economies with three different net worth distributions. The solid line with black circles depicts the model's response under the baseline net worth distribution. The solid line depicts the case in which net worths are equally distributed, so that $n^F(s^t) = n^E(s^t) = 0.3$. The dotted line depicts the case in which the net worth is distributed more to the FIs' sector, so that $n^F(s^t) = 0.5$ and $n^E(s^t) = 0.1$. The figure indicates that the propagation of the FIs' shock is drastically reduced as more net worth is distributed to the FIs, while that of the entrepreneurial shock increases. Because the agency problems of the FI sector are mitigated, propagation of the FIs' shocks becomes smaller.

3.5 Technology Shock

Lastly, we consider an experiment where the baseline economy is subject to an unexpected temporary decrease in productivity in the final goods sector. The solid line with black circles in Figure 12 presents the economy's response to the shock. As equation (14) implies, this productivity shock decreases the ex post discounted return to capital. Consequently, the expected demand for the capital goods drops, causing the capital goods price $Q(s^t)$ to fall. Through the same mechanism in the previous subsection, the net worths of the two sectors decline, causing a rise in the external finance premium that drives down the aggregate investment.

Similar to the sectoral shocks, the magnification of the financial accelerator effect in response to the productivity shock is affected by the net worth distribution. We compare the baseline model with the alternative two models we studied in the previous section. The solid line in Figure 12 depicts the case in which the initial net worth is equally distributed across sectors. The dotted line in the figure depicts the case in which more net worth is distributed to the FI sector. Clearly, as the FIs have relatively higher net worth, the amplification effect is less enhanced. As Figure 7 indicates, the cost-of-funds curve in the U.S. economy is tilted to the right and the external finance premium

[21] In order to focus on the effect of the net worth distribution, we hold all the technology and distribution parameters pertaining to the credit contracts, two bankruptcy costs μ^F, μ^E and two variances of borrowers' idiosyncratic productivities σ^F, σ^E fixed at the values of the baseline model. We re-calculate the survival rates of the FIs and entrepreneurs γ^F and γ^E for each of the three models so that all of the equilibrium conditions (1), (2), (4), (5), (7), (9), (10) and the model-specific net worth distribution hold at the steady state.

decreases as more net worth is distributed to the FIs, being consistent with the three lines in Figure 12.

4 Conclusion

Empirical evidence suggests that the net worth in the FI sector and the credit frictions associated with the FI sector affect macroeconomic activity in a significant way. Based on the financial accelerator model of Bernanke *et al.* (1999), we developed a dynamic general equilibrium model in which FIs as well as entrepreneurs are subject to credit constraints. In this model, the credit conditions of the two borrowing sectors, especially borrowers' net worths, work as the key determinants of the market borrowing rates, because of the agency problems in the credit market. The two net worths function differently through the chained credit contracts. Therefore the size and cross-sectional distribution of the net worths play an important role in this process. Consequently, the dynamics of aggregate investment depend crucially on the credit conditions of each of the borrowing sectors.

Based on a model calibrated to U.S. data, we studied how that propagation and amplification mechanism of the credit market changes by explicitly incorporating a credit-constrained FI sector into the model. We found the following novel properties. First, the FI sector enhances the financial accelerator effect in response to the aggregate shocks and net worth shocks, compared with an economy that omits the credit friction in the FI sector *à la* BGG. Second, adverse shocks to the FI sector cause larger economic downturns than the adverse shocks to the entrepreneurial sector. Third, amplification of the aggregate shock can be reduced if the net worth distribution is changed from the current distribution. These properties stem from the fact that the cross-sectional net worth distribution is important in determining the financial accelerator effect, and that its current distribution in the United States is unequally biased to the entrepreneurial sector.

Our results have policy implications for governmental initiatives during the financial crisis. For example, confronting the financial crisis starting in 2007, several OECD countries, including the United States and Japan, have injected public funds into financial institutions or entrepreneurs, aimed at stabilizing the financial markets. Other things being equal, our results suggest that capital injection would be more effective in revitalizing aggregate investment, when FIs are targeted rather than entrepreneurs.[22]

[22] Admittedly, however, it is uncertain whether the capital injection policy improves social welfare, without measuring the social welfare under each policy. In Hirakata, Sudo, and Ueda (2009), we investigated the welfare implications of the capital injections to FIs and to entrepreneurs, and for several classes of monetary policy rules, using a version of the current model.

A Analytical Expressions for the Variables Appearing in the Credit Contracts

In this section, we provide the analytical expressions for $G_t^F\left(\overline{\omega}_t^F\right)$, $G_t^E\left(\overline{\omega}_t^E\right)$, $\Gamma_t^F\left(\overline{\omega}_t^F\right)$, $\Gamma_t^E\left(\overline{\omega}_t^E\right)$ and their differentials with respect to their cut-off values. Following BGG and CMR, we assume that both ω_t^F and ω_t^E obey different log-normal distributions, with $E\left(\omega_t^F\right)=1$ and $E\left(\omega_t^E\right)=1$, respectively, and we denote the cdf of the two distributions by $F_t\left(\omega_t^F\right)$ and $F_t\left(\omega_t^E\right)$, and denote the variance of $\log\omega_t^F$ and $\log\omega_t^E$ by $\sigma_{t,F}^2$ and $\sigma_{t,E}^2$.

Variables $G_t^F\left(\overline{\omega}_t^F\right)$ $\left(G_t^E\left(\overline{\omega}_t^E\right)\right)$ are the expected return from the default FIs (the default entrepreneurs) for the IF contract (the FE contract). Using the assumptions about the distribution of ω_t^F and ω_t^E, they are expressed as

$$G_t^F\left(\overline{\omega}_t^F\right)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{\log\overline{\omega}_t^F-0.5\sigma_{t,F}^2}{\sigma_{t,F}}}\exp\left(-\frac{v_F^2}{2}\right)dv_F,$$

$$G_t^E\left(\overline{\omega}_t^E\right)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{\log\overline{\omega}_t^E-0.5\sigma_{t,E}^2}{\sigma_{t,E}}}\exp\left(-\frac{v_E^2}{2}\right)dv_E.$$

Note that $G_t^F\left(\overline{\omega}_t^F\right)$ and $G_t^E\left(\overline{\omega}_t^E\right)$ are functions of the current value of time-varying riskiness σ_t^F and σ_t^E. Differentials of $G_t^F\left(\overline{\omega}_t^F\right)$ and $G_t^E\left(\overline{\omega}_t^E\right)$ with respect to $\overline{\omega}_t^F$ and $\overline{\omega}_t^E$ are given by

$$G_t'^F\left(\overline{\omega}_t^F\right)=\left(\frac{1}{\sqrt{2\pi}}\right)\left(\frac{1}{\overline{\omega}_t^F\sigma_{t,F}}\right)\exp\left(-.5\left(\frac{\log\overline{\omega}_t^F-0.5\sigma_{t,F}^2}{\sigma_{t,F}}\right)^2\right),$$

$$G_t'^E\left(\overline{\omega}_t^E\right)=\left(\frac{1}{\sqrt{2\pi}}\right)\left(\frac{1}{\overline{\omega}_t^E\sigma_{t,E}}\right)\exp\left(-.5\left(\frac{\log\overline{\omega}_t^E-0.5\sigma_{t,E}^2}{\sigma_{t,E}}\right)^2\right).$$

$\Gamma_t^F\left(\overline{\omega}_t^F\right)$ $\left(\Gamma_t^E\left(\overline{\omega}_t^E\right)\right)$ are the net share of profit going to investors (FIs) in the IF contract (FE contract). These are expressed as

$$\Gamma_t^F\left(\overline{\omega}_t^F\right)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{\log\overline{\omega}_t^F-0.5\sigma_{t,F}^2}{\sigma_{t,F}}}\exp\left(-\frac{v_F^2}{2}\right)dv_F+\frac{\overline{\omega}_t^F}{\sqrt{2\pi}}\int_{\frac{\log\overline{\omega}_t^F+0.5\sigma_{t,F}^2}{\sigma_{t,F}}}^{\infty}\exp\left(-\frac{v_F^2}{2}\right)dv_F,$$

$$\Gamma_t^E\left(\overline{\omega}_t^E\right)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{\log\overline{\omega}_t^E-0.5\sigma_{t,E}^2}{\sigma_{t,E}}}\exp\left(-\frac{x^2}{2}\right)dx+\frac{\overline{\omega}_t^E}{\sqrt{2\pi}}\int_{\frac{\log\overline{\omega}_t^E+0.5\sigma_{t,E}^2}{\sigma_{t,E}}}^{\infty}\exp\left(-\frac{v_E^2}{2}\right)dv_E.$$

Similarly, the differentials of $\Gamma_t^F\left(\overline{\omega}_t^F\right)$ and $\Gamma_t^E\left(\overline{\omega}_t^E\right)$ with respect to $\overline{\omega}_t^F$ and $\overline{\omega}_t^E$ are given by

$$
\begin{aligned}
\Gamma_t'^F\left(\overline{\omega}_t^F\right) &= \frac{1}{\sqrt{2\pi}\overline{\omega}_t^F\sigma_{t,F}}\exp\left(-.5\left(\frac{\log\overline{\omega}_t^F-0.5\sigma_{t,F}^2}{\sigma_{t,F}}\right)^2\right)dx \\
&+\frac{1}{\sqrt{2\pi}}\int_{\frac{\log\overline{\omega}_t^F+0.5\sigma_{t,F}^2}{\sigma_{t,F}}}^{\infty}\exp\left(-\frac{v_F^2}{2}\right)dv_F \\
&-\frac{1}{\sqrt{2\pi}\sigma_{t,F}}\exp\left(-\frac{\left(\frac{\log\overline{\omega}_t^F+0.5\sigma_{t,F}^2}{\sigma_{t,F}}\right)^2}{2}\right)dx,
\end{aligned}
$$

$$
\begin{aligned}
\Gamma_t'^E\left(\overline{\omega}_t^E\right) &= \frac{1}{\sqrt{2\pi}\overline{\omega}_t^E\sigma_{t,E}}\exp\left(-.5\left(\frac{\log\overline{\omega}_t^E-0.5\sigma_{t,E}^2}{\sigma_{t,E}}\right)^2\right)dx \\
&+\frac{1}{\sqrt{2\pi}}\int_{\frac{\log\overline{\omega}_t^E+0.5\sigma_{t,E}^2}{\sigma_{t,E}}}^{\infty}\exp\left(-\frac{v_E^2}{2}\right)dv_E \\
&-\frac{1}{\sqrt{2\pi}\sigma_{t,E}}\exp\left(-.5\left(\frac{\log\overline{\omega}_t^E+0.5\sigma_{t,E}^2}{\sigma_{t,E}}\right)^2\right)dx.
\end{aligned}
$$

B Parameterization I

This appendix provides parameterization of the variables associated with the household, wholesalers, capital goods producers, retailers, final goods producers, government and monetary authority. Following earlier studies including BGG and CMR, we choose conventional values for these parameters.

Parameters[23]

Parameter	Value	Description
β	.99	Discount Factor
δ	.025	Depreciation rate
α	.35	Capital share
R	$.99^{-1}$	Risk-free rate
η	3	Elasticity of labor
χ	.3	Utility weight on leisure
κ	2.5	Investment adjustment cost
ρ_a, ρ_{σ_F}, $\rho_{\sigma E}$	.85	Autoregressive parameters of shocks

[23]Figures are quarterly unless otherwise stated.

C Parameterization II

This appendix provides parameterization of the variables that are related to the credit contracts among investors, FIs and entrepreneurs. We choose six parameters so that they are consistent with the equilibrium conditions (1), (2), (4), (5), (7), (9) and (10) evaluated using the steady state values for the risk-free rate R, FIs' loan rate Z^E, FIs' borrowing rate Z^F, entrepreneurial default probability $F\left(\overline{\omega}^E\right)$, FIs' default probability $F\left(\overline{\omega}^F\right)$, entrepreneurial net worth/capital ratio n^E and FIs' net worth/capital ratio n^F shown in the lower table.

Calibrated parameters[24]

Parameter	Value	Description
σ_F	0.107366	S.E. of FIs idiosyncratic productivity at steady state
σ_E	0.312687	S.E. of entrepreneurial idiosyncratic productivity at steady state
μ_F	0.033046	Bankruptcy cost associated with FIs
μ_E	0.013123	Bankruptcy cost associated with entrepreneurs
γ_F	0.963286	Survival rate of FIs
γ_E	0.983840	Survival rate of entrepreneurs

Steady state conditions

Condition	Description
$R = .99^{-1}$	Risk-free rate is the inverse of the subjective discount factor.
$Z^E = Z^F + .023^{.25}$	Premium for FIs' loan rate is $.023^{.25}$.
$Z^F = R + .006^{.25}$	Premium for FIs' borrowing rate is $.006^{.25}$.
$F\left(\overline{\omega}^F\right) = .02$	Default probability in the IF contract is .02.
$F\left(\overline{\omega}^E\right) = .02$	Default probability in the FE contract is .02.
$n^F = .1$	FIs' net worth/capital ratio is set to .1
$n^E = .5$	Entrepreneurial net worth/capital ratio is set to .5.

[24]Figures are quarterly unless otherwise stated.

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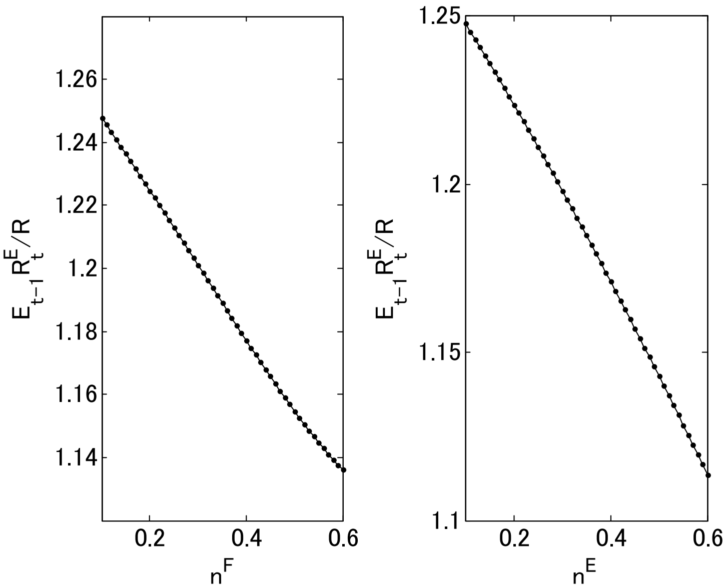

Figure 1. Effect of the net worth in the FI sector (left panel) and in the entrepreneurial sector (right panel). The ratio of net worth to capital in each sector is depicted on the horizontal axis and the external finance premium is depicted on the vertical axis.

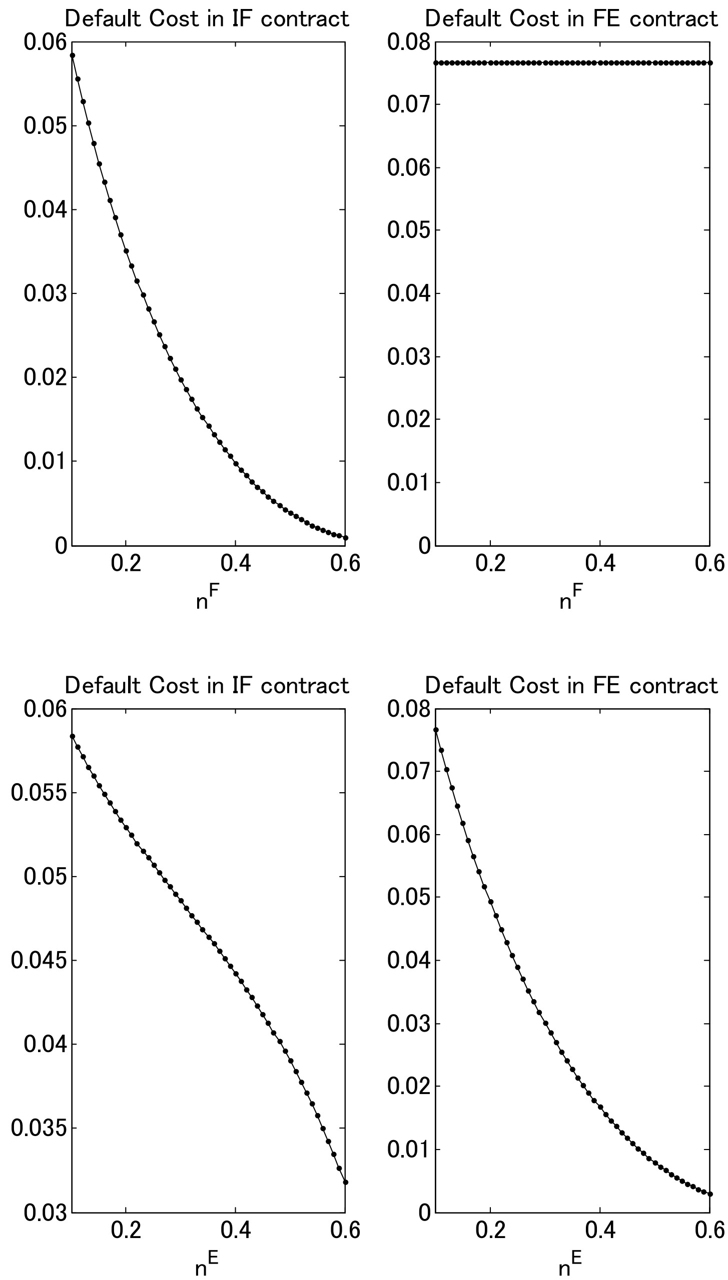

Figure 2. Effect of the net worth in the FI sector (left panel) and in the entrepreneurial sector (right panel). The ratio of net worth to capital in each sector is depicted on the horizontal axis and the expected default rate is depicted on the vertical axis.

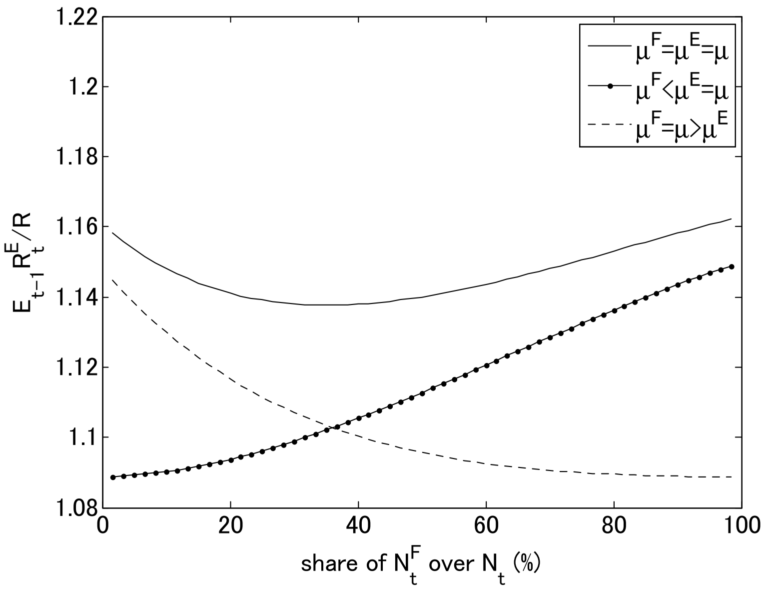

Figure 3. Effect of net worth distribution on the external finance premium. The ratio of FIs' net worth over total net worth is depicted on the horizontal axis and the external finance premium is depicted on the vertical axis. The solid line depicts the case in which bankruptcy costs and variances of idiosyncratic productivities are symmetric in the IF contract and the FE contract. The solid line with black circles (dashed line) depicts the case in which FIs' (entrepreneurial) bankruptcy cost is lower than the entrepreneurial (FIs') bankruptcy cost.

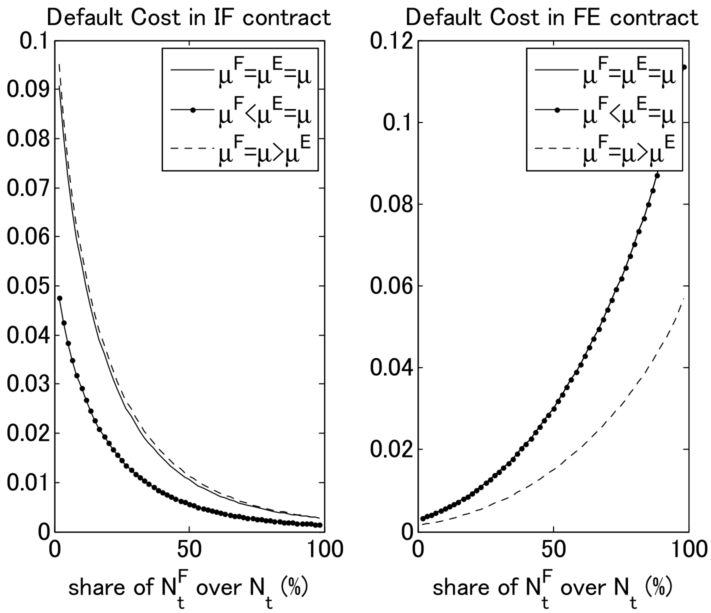

Figure 4. Effect of net worth distribution on the expected default costs. The share of FIs' net worth over total net worth is depicted on the horizontal axis and the expected default cost of the IF contracts (FE contracts) is depicted on the vertical axis in the left (right) panel. The solid line illustrates the case in which bankruptcy costs and variances of idiosyncratic productivities are symmetric in the IF and the FE contracts. The solid line with black circles (dashed line) depicts the case in which the FIs' (entrepreneurial) bankruptcy cost is lower than that of the entrepreneurial (FIs') bankruptcy cost.

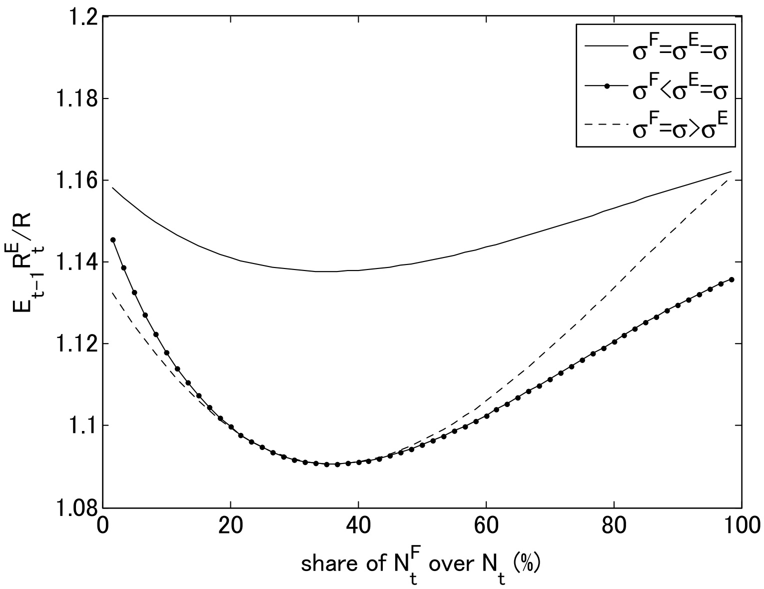

Figure 5. Effect of net worth distribution on the external finance premium. The share of the FIs' net worth over the total net worth is depicted on the horizontal axis and the external finance premium is depicted on the vertical axis. The solid line illustrates the case in which bankruptcy costs and the variances of the idiosyncratic productivities are symmetric in the IF and the FE contracts. The solid line with black circles (dashed line) shows the case in which the variance of the FIs' (entrepreneurial) idiosyncratic productivity is lower than that of the entrepreneurs (FIs).

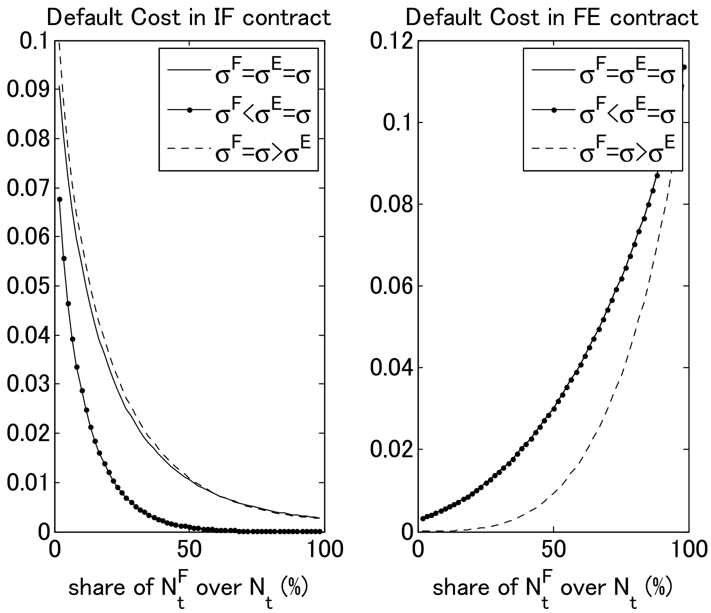

Figure 6. Effect of net worth distribution on the expected default costs. The share of the FIs' net worth over the total net worth is depicted on the horizontal axis and the expected default cost of the IF contract (the FE contract) is depicted on the vertical axis in the left (right) panel. The solid line with black circles (dashed line) illustrates the case in which the variance of the FIs' (entrepreneurial) idiosyncratic productivity is lower than that of the entrepreneurs (FIs).

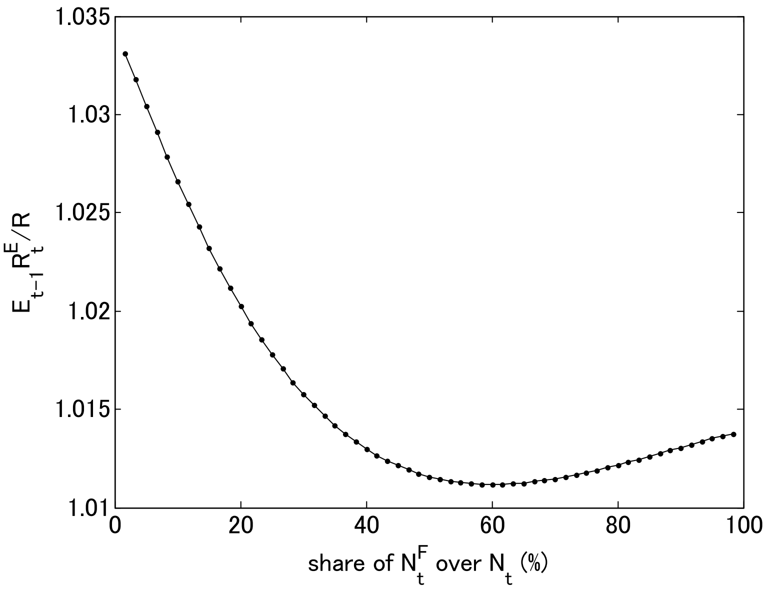

Figure 7. Effect of the net worth distribution on the external finance premium. μ^F, μ^E, σ^F and σ^E are calibrated to the US economy. The y-axis denotes the external finance premium and the x-axis denotes the share of the FIs' net worth over the total net worth.

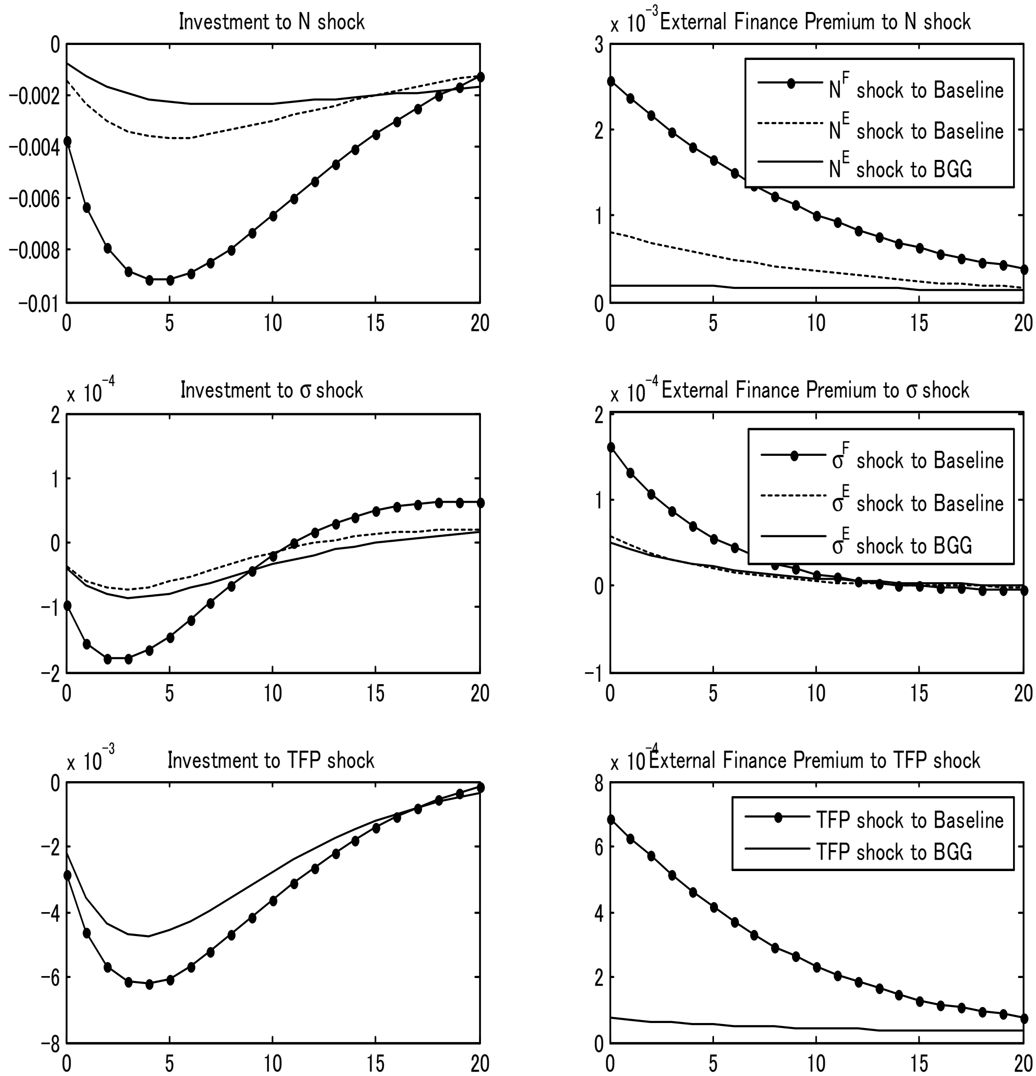

Figure 8. Impulse responses of the endogenous variables to adverse shocks under baseline model and our BGG model. The upper panels display the responses to the shocks to net worth, the middle panels display the responses to the shocks to riskiness, and the lower panels display the responses to the shocks to the productivity of final goods production.

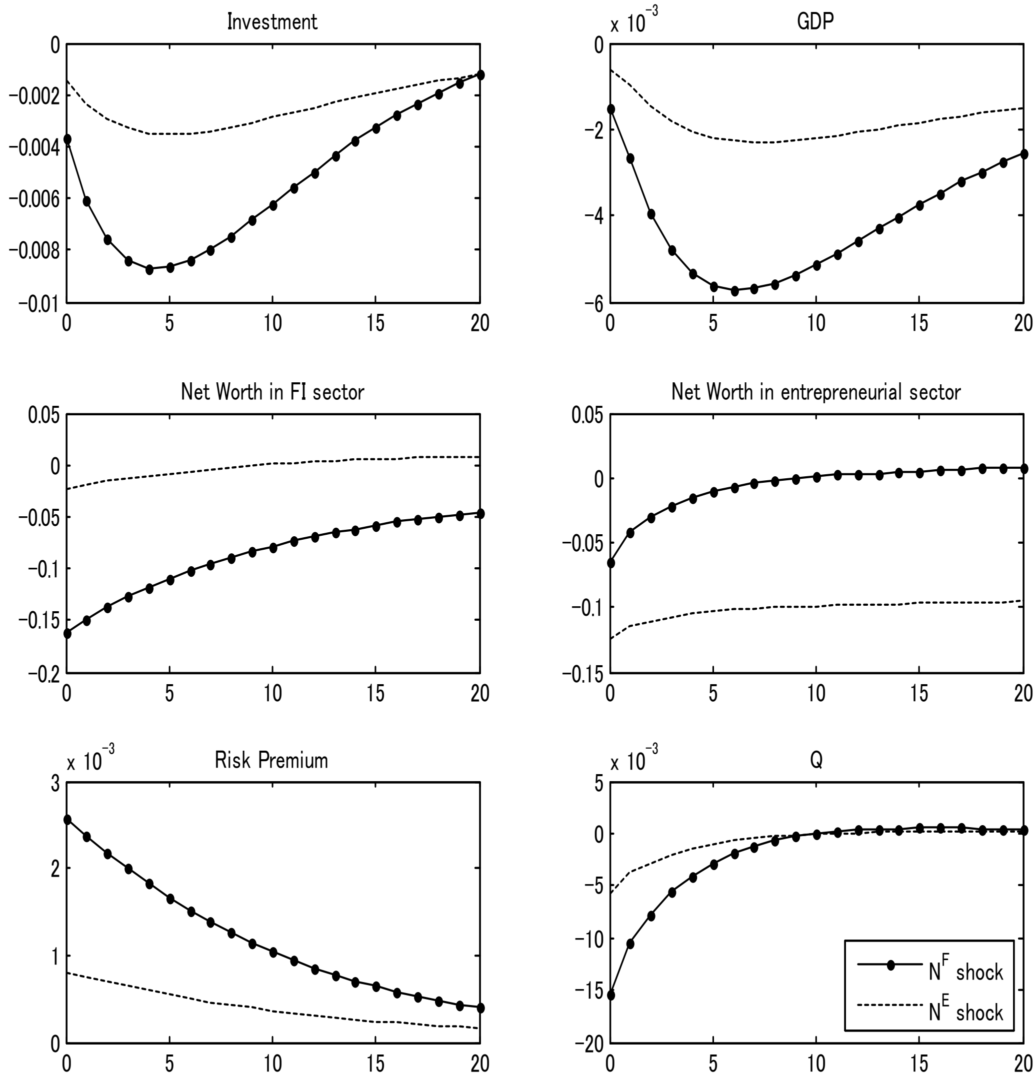

Figure 9. Effect of the net worth shocks. Impulse responses of the endogenous variables to a once-and-for-all decline (N^F shock, N^E shock, respectively) are depicted on the vertical axis.

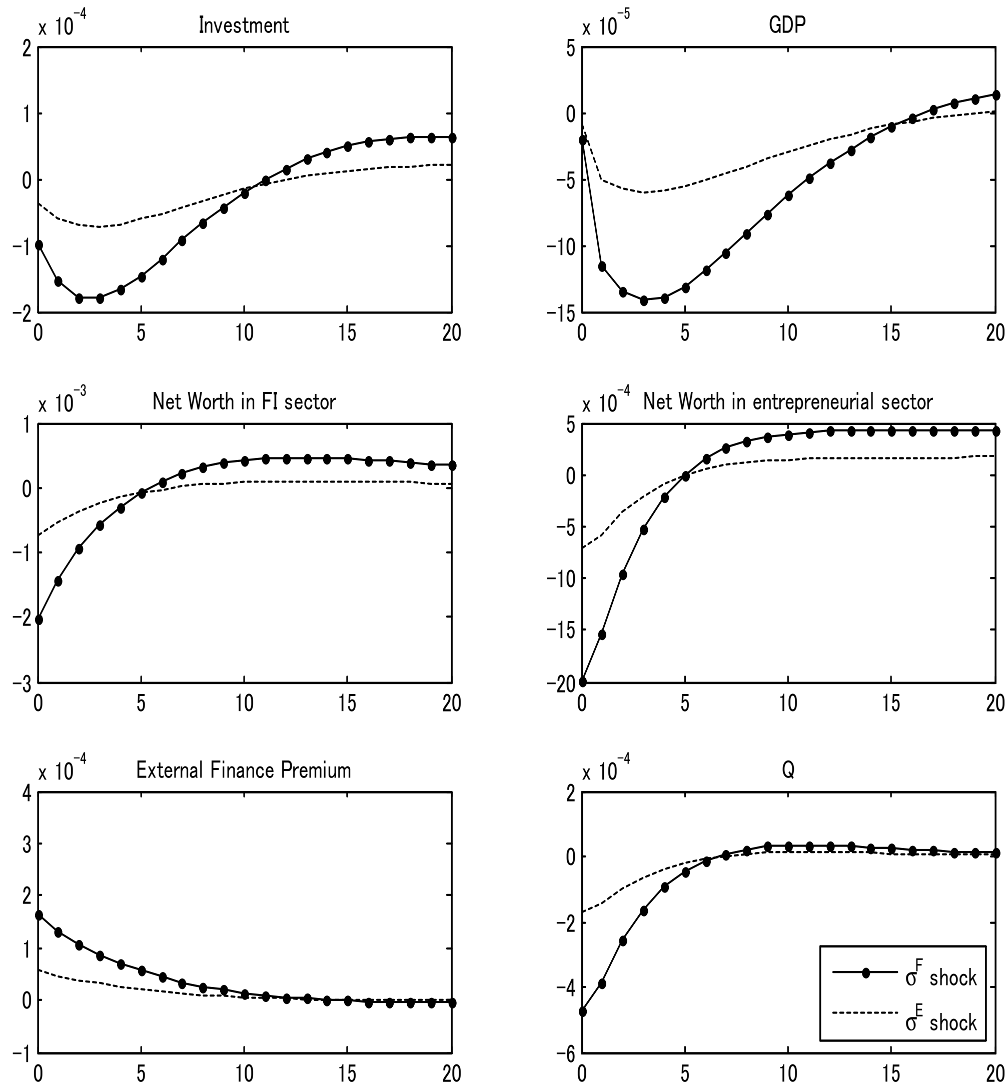

Figure 10. Effect of riskiness shocks. Impulse responses of the endogenous variables to a positive riskiness shock in FIs and entrepreneurs (σ^F shock, σ^E shock, respectively) are depicted on the y-axis.

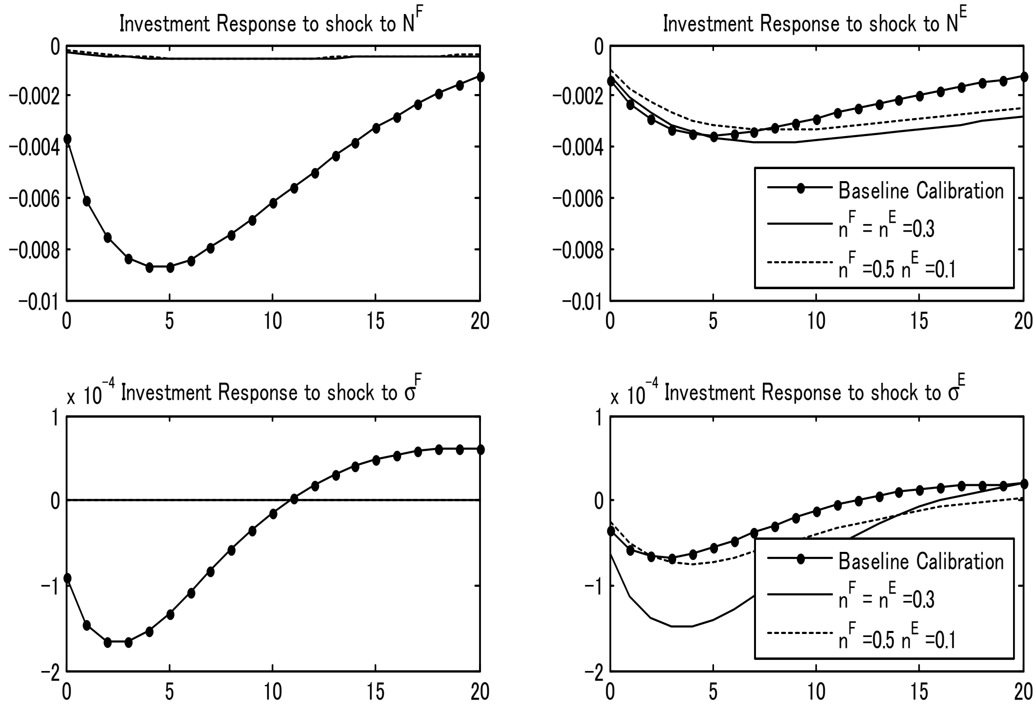

Figure 11. Effect of the net worth distribution on the sectoral shock-propagation mechanism of the credit market. Impulse responses of investment after an unexpected decline in the FIs' net worth (upper left panel), an unexpected decline in the entrepreneurial net worth (upper right panel), an unexpected rise in the FIs' riskiness (lower right panel) and an unexpected rise in the entrepreneurial riskiness (lower right panel) are depicted.

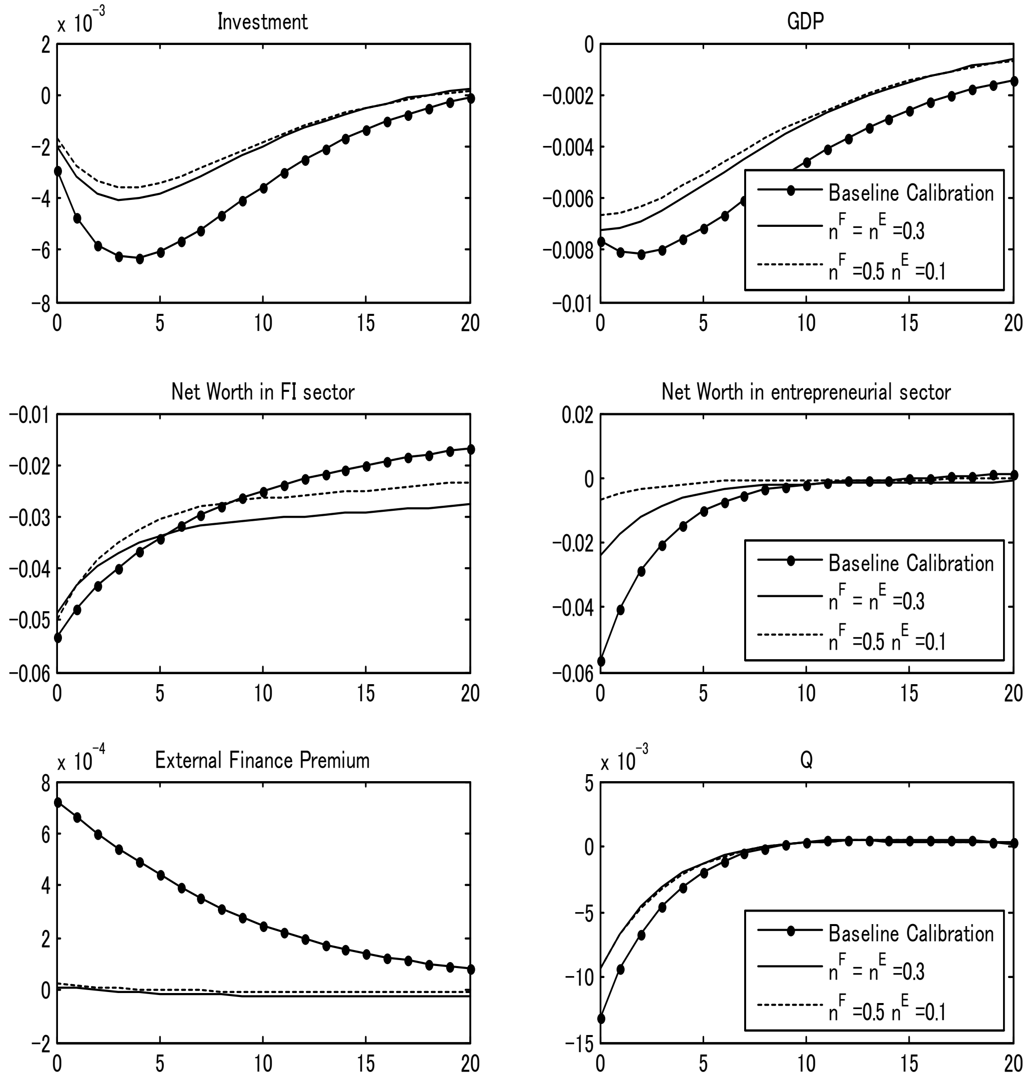

Figure 12. Effect of the net worth distribution on the aggregate amplification mechanism of the credit market. Impulse response of the endogenous variables to a temporary decline in the productivity of the final goods sector are depicted on the vertical axis.