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Abstract

Recent studies argue that the spread-adjusted Taylor rule (STR), which includes a response to the credit spread, replicates monetary policy in the United State. We show (1) STR is a theoretically optimal monetary policy under heterogeneous loan interest rate contracts in both discretionay and commitment monetary policies, (2) however, the optimal response to the credit spread is ambiguous given the financial market structure in theoretically derived STR, and (3) there, a commitment policy is effective in narrowing the credit spread when the central bank hits the zero lower bound constraint of the policy rate.

Keywords: Credit spread; optimal monetary policy
JEL classification: E44, E52

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1 Introduction

Taylor (2008) and Cúrdia and Woodford (2008) have recently discussed whether central banks should respond to the credit spread of interest rates. Taylor (2008) argues that in the U.S. the Federal Reserve Board (FRB) has reacted negatively to the credit spread in the money market during the last few years to stimulate the economy. Taylor (2008) points out that *spread-adjusted Taylor rule* (STR) that additionally includes the credit spread term in the standard *Taylor Rule* (TR) can well explain the easing of monetary policy by the FRB. Similarly, Cúrdia and Woodford (2008) theoretically investigate whether a central bank should react to the credit spread between savers and borrowers. They show that an STR including a negative response term to the credit spread can approximate the optimal monetary policy.

In this paper, we first construct an otherwise New Keynesian model with heterogeneous loan rate contracts following Sudo and Teranishi (2008). We then derive a second-order approximated loss function and the optimal monetary policy rule in this model. Our conclusion is that the STR is a theoretically optimal monetary policy rule though the central bank's optimal response to the credit spread can be positive or negative depending on the financial structure. In brief, we show that a commitment monetary policy is very effective in reducing the credit spread when the central bank hits the zero lower bound on the policy rate.

The paper is organized as follows. Section 2 describes our model with heterogeneous loan contracts and derives a second-order approximated loss function. Section 3 analyzes the optimal monetary policy rule. Section 4 discusses the monetary policy for the credit spread in a liquidity trap. Section 5 concludes.

2 Model

We basically extend the model in Sudo and Teranishi (2008) where there are heterogeneous private banks that provide differentiated loan rate contracts to firms. Our departure from Sudo and Teranishi (2008) is that we assume flexible loan rate contracts where the loan rate perfectly adjusts in every period and assume a monetary transaction cost for the consumer. Unlike Cúrdia and Woodford (2008), we consider that market imperfections induce the credit spread through the cost channel as in Bernanke, Gertler and Gilchrist (1999) and Ravenna and Walsh (2006).

The model is given by the following four equations:[1]

$$x_t = \mathrm{E}_t x_{t+1} - \sigma(\widehat{i}_t - \mathrm{E}_t \pi_{t+1}), \tag{1}$$

$$\pi_t = \kappa x_t + \Theta_1 \widehat{R}_{R,t} + \Theta_2 \widehat{R}_{S,t} + \beta \mathrm{E}_t \pi_{t+1}, \tag{2}$$

$$\widehat{R}_{R,t} = \lambda_R \widehat{i}_t + u_{R,t}, \tag{3}$$

$$\widehat{R}_{S,t} = \lambda_S \widehat{i}_t + u_{S,t}, \tag{4}$$

where x_t is the output gap, π_{t+1} is inflation, $\widehat{R}_{R,t}$ is the loan interest rate for an R type loan, $\widehat{R}_{S,t}$ is the loan interest rate for an S type loan, and $\widehat{i}_t$ is the policy interest rate. We define each variable as the log deviation from its steady state where the price is flexible and the loan rates are constant (the log-linearized version of variable h_t is expressed by $\widehat{h}_t = \ln(h_t/\overline{h})$, where $\overline{h}$ is the steady state value of h_t). σ, κ, Θ_1, Θ_2, β, λ_L, and λ_S are positive parameters. $u_{R,t}$ and $u_{S,t}$ are exogenous shocks. E_t denotes the expectation at time t.

The first equation is the IS curve expressing the relation between the output gap and the real interest rate. The second equation is the augmented Phillips curve expressing the relation between inflation and the output gap, which additionally includes loan interest rates since firm needs the external finance. The last two equations are the loan rate curves under flexible loan rate contracts. We assume that half of the firms' business units are financed by loan type R and the other half are financed by loan type S. The distinction between loan type R and S is due to different property between two types of loans, which induces different shock and different ratio of external finance in these loans.

A new property in this paper that stands it apart from the standard New Keynesian model is the heterogeneous cost channel expressed by the last three equations.

[1]See Appendix A for details.

In particular, the impacts of loan rates on the economy from monetary policy are squeezed by Θ_1, Θ_2, λ_R, and λ_S. Our aim is to investigate the effects of this heterogeneous cost channel on the monetary policy. These parameters are given by:

$$\Theta_1 \equiv \frac{1}{2}\chi\frac{\gamma^R(\varepsilon-1)+1-\varepsilon\gamma^R\left(\beta^{-1}-1\right)}{(\varepsilon-1)+1-\varepsilon\gamma^R\left(\beta^{-1}-1\right)},$$

$$\Theta_2 \equiv \frac{1}{2}\chi\frac{\gamma^S(\varepsilon-1)+1-\varepsilon\gamma^S\left(\beta^{-1}-1\right)}{(\varepsilon-1)+1-\varepsilon\gamma^S\left(\beta^{-1}-1\right)},$$

$$\lambda_R = \frac{\varepsilon}{\varepsilon-1}\frac{\gamma^R(\varepsilon-1)+\gamma^R\left(\beta^{-1}-1\right)(\varepsilon-1)}{\gamma^R(\varepsilon-1)+1+\varepsilon\gamma^R\left(\beta^{-1}-1\right)},$$

$$\lambda_S = \frac{\varepsilon}{\varepsilon-1}\frac{\gamma^S(\varepsilon-1)+\gamma^S\left(\beta^{-1}-1\right)(\varepsilon-1)}{\gamma^S(\varepsilon-1)+1+\varepsilon\gamma^S\left(\beta^{-1}-1\right)},$$

where γ^R and γ^S are the external finance ratios in loan R and loan S in production, respectively, ε defines the labor-type difference, χ is related to a price stickiness, and β is the discount factor. When a firm finance all production costs by external loans, γ^R and γ^S are ones. Note that the ratio of γ^R and γ^S expresses the market share of the two types of loans as half of the firms' business units are financed by loan type R or S. Moreover, any difference in γ^R and γ^S infers a heterogeneous cost channel effect on the economy.[2] As the external finance ratios increases, the cost channel effect captured by λ_R, λ_S, Θ_1, or Θ_2 monotonically increases.

3 Approximated welfare function

The consumer's discounted welfare at time 0 is given by:[3]

$$\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t UT_t = \mathrm{E}_0\left\{\sum_{t=0}^{\infty}\beta^t\left[U(C_t,m_t)-\int_0^n V(l_t(h))dh-\int_n^1 V(l_t(\overline{h}))d\overline{h}\right]\right\},$$

where UT_t is the one-period utility, $U(C_t, m_t)$ is a strictly increasing and concave function of consumption C_t and real money holding m_t, where the two elements are separable, and $V(l_t(h))$ is an increasing and convex function of labor supply $l_t(h)$.

[2] As extensions, we can assume different ε and introduce heterogeneous stickiness for different loans, as in Sudo and Teranishi (2008), to make the heterogeneous cost channel. However, the implications for monetary policy do not change.

[3] See Appendix B for a detailed derivation.

Under the monetary transaction cost, we have a second-order approximated loss function L_t to UT_t as follows:

$$\begin{aligned} & UT_t \simeq -\Lambda L_t \\ = \quad & -\Lambda \left(\lambda_x (x_t - x^*)^2 + \lambda_\pi \pi_t^2 + \lambda_i (\widehat{i}_t - i^*)^2 + \lambda_{RS} \left(\Theta_1 \widehat{R}_{R,t} - \Theta_2 \widehat{R}_{S,t} \right)^2 \right), \end{aligned}$$

where Λ, λ_x, x^*, λ_π, λ_i, i^*, and λ_{RS} are positive parameters. Note that the welfare loss is evaluated in terms of the deviation from the steady state where the price is flexible and the loan rates are constant.

4 Optimal monetary policy rule

The central bank minimizes the discounted sum of the future loss subject to the four constraints, (1), (2), (3), and (4).

4.1 Optimal discretionary policy

Under a discretionary policy following Woodford (2003), the optimal monetary policy rule is given by:[4]

$$\widehat{i}_t = i^* + \rho_x (x_t - x^*) + \rho_\pi \pi_t + \rho_R \left(\Theta_1 \widehat{R}_{R,t} - \Theta_2 \widehat{R}_{S,t} \right), \tag{5}$$

where

$$\begin{aligned} \rho_x &\equiv \frac{\lambda_x \sigma}{\lambda_i} > 0, \\ \rho_\pi &\equiv \frac{\lambda_\pi}{\lambda_i} \left(\kappa\sigma - \Theta_1 \lambda_R - \Theta_2 \lambda_S \right) \lesseqgtr 0, \\ \rho_R &\equiv -\frac{\lambda_{RS}}{\lambda_i} \left(\Theta_1 \lambda_R - \Theta_2 \lambda_S \right) \lesseqgtr 0. \end{aligned}$$

We emphasize three points. First, the optimal discretionary rule holds the term of the credit spread. This simply means that the STR is the optimal monetary

[4] For regular parameters, as in Woodford (2003), the following rule satisfies determinacy given equations (1), (2), (3), and (4).

policy in a model including explicitly heterogeneous bank loans. When there is no loan contract, the STR is reduced to the regular TR as:

$$\widehat{i}_t = i^* + \rho_x(x_t - x^*) + \rho_\pi \pi_t.$$

Second, rather than a pure credit spread, the credit spread that can be adjusted by the effects to the economy as $\left(\Theta_1 \widehat{R}_{R,t} - \Theta_2 \widehat{R}_{S,t}\right)$ is the central bank's policy purpose. Thus, the central bank more sensitively changes the policy rate to the loan rate that has a larger effect on the economy.

Third, the sign of ρ_R depends on the sign of $(\Theta_1\lambda_R - \Theta_2\lambda_S)$. Thus, the optimal monetary policy response to the positive credit spread of $\left(\Theta_1 \widehat{R}_{R,t} - \Theta_2 \widehat{R}_{S,t}\right)$ can be negative or positive by the parameters.[5] If $\Theta_1\lambda_R > \Theta_2\lambda_S$ $(\Theta_1\lambda_R < \Theta_2\lambda_S)$, the last term in the rule of equation (5) demands a negative (positive) response to the credit spread.[6][7] Because $\Theta_1\lambda_R$ $(\Theta_2\lambda_S)$ monotonically increases as γ^R (γ^S) increases, the optimal monetary policy response to the credit spread depends on the financial market structure, *i.e.,* cost channel structure.

In the context of the spread from the policy rate to the market rate, the same argument holds. Now we assume $u_{R,t} > 0$ and $u_{S,t} = 0$, so the loan rate of type S is a proxy for the policy (safe) rate and the loan rate of type R is a proxy for the risky rate. The optimal response of the policy rate to the spread on the risky rate from the policy rate is ambiguous according to the cost channel structure.

4.2 Optimal commitment policy

Under the *timeless perspective commitment policy* in Woodford (2003), the optimal monetary policy rule is given as:

$$(1 - \eta_1 L)(1 - \eta_2 L)(\widehat{i}_t - i^*) = (1 - \eta_1 L)(1 - \eta_2 L)\rho_R \left(\Theta_1 \widehat{R}_{L,t} - \Theta_2 \widehat{R}_{S,t}\right)^2 + f_t,$$

[5]The reason for ambiguity of ρ_π is that the policy rate increases inflation due to external finance being a production cost. However, for the regular parameters, as in Woodford (2003), ρ_π is positive.

[6]$\Theta_1\lambda_R = \frac{\gamma^R}{1-\gamma^R\left(\beta^{-1}-1\right)}$ and $\Theta_2\lambda_S = \frac{\gamma^S}{1-\gamma^S\left(\beta^{-1}-1\right)}$.

[7]Eventually, the central bank changes the policy rate to adjust the more elastic loan rate to the policy rate, as captured by λ_R or λ_S, towards the less elastic loan rate to narrow the credit spread.

where

$$
\begin{aligned}
f_t &= (1-\eta_1 L)(1-\eta_2 L)\left(\Theta_1\lambda_R+\Theta_2\lambda_S\right)\lambda_x\lambda_i^{-1}\kappa^{-1}(x_t-x^*)^2 \\
&\quad +(\eta_3+\eta_4 L)\left(\lambda_\pi\lambda_i^{-1}\kappa\pi_t+\lambda_x\lambda_i^{-1}\triangle x_t\right),
\end{aligned}
$$

and $\eta_1+\eta_2 = 1+\beta^{-1}+\kappa\sigma\beta^{-1}$, $\eta_1\eta_2 = \beta^{-1}$, $\eta_3 = \sigma-\kappa^{-1}\left(\Theta_1\lambda_R+\Theta_2\lambda_S\right)$, and $\eta_4 = \kappa^{-1}\beta^{-1}\left(\Theta_1\lambda_R+\Theta_2\lambda_S\right)$.

We can see the same properties in the credit spread term, although the rule is more complicated.

5 Discussion

In some developed countries, such as Japan, the U.S., and the euro area, and other European countries, we can see that their central banks have hit the zero lower bound on nominal interest rates because of the U.S. subprime mortgage crisis from fall 2007. However, their aim is still to reduce the credit spread between the policy rate and other (risky) market rates.

As shown in the preceding section, the central bank should narrow the credit spread between the policy (safe) rate and risky rate. In this section, we investigate the effect of the zero lower bound constraint on the optimal monetary policy for the credit spread. Note that we assume the situation where the central bank lowers the policy rate to reduce the credit spread on the risky rate from the policy rate.

Under a flexible loan rate, both the discretionary and commitment policies can no longer decrease the credit spread when the policy rate hits the zero lower bound constraint. However, when we introduce the sticky loan rate setting, where the loan rate can not perfectly adjust in every period, to the risky loan rate curve given by equation (3), the two monetary policies make different implications. Borrowing from Sudo and Teranishi (2008), the risky loan rate curve under a sticky loan rate setting is given by:

$$
\widehat{R}_{R,t} = \lambda_1^R \mathrm{E}_t \widehat{R}_{R,t+1} + \lambda_2^R \widehat{R}_{R,t-1} + \lambda_3^R \widehat{i}_t,
$$

where λ_1^R, λ_2^R, and λ_3^R are positive parameters. In this case, the commitment policy, *i.e.,* the future promise on the low policy rate path, can reduce the risky loan rate, implying a narrowing of the credit spread between the policy rate $\widehat{i}_t$ and the risky rate $\widehat{R}_{R,t}$. This holds even under the zero lower bound constraint since the sticky loan rate curves include future expectation $\mathrm{E}_t \widehat{R}_{R,t+1}$.[8]

This implies that a commitment monetary policy is effective in reducing the current credit spread when the central banks have exhausted scope for cutting the policy rate particularly in the case where the loan rate is not perfectly flexible. We, however, note that just reducing the current credit spread to zero is not optimal response for the central banks since the future promise on the low policy rate makes a negative effect on the future credit spread at the sametime.

6 Concluding remarks

We theoretically show that the STR is the optimal monetary policy rule. This supports Taylor's suggestion that the FRB has reacted negatively to the credit spread during the last few years. However, we show that the optimal monetary policy response to the credit spread shock is ambiguous given the financial market structure. A commitment monetary policy is effective in reducing the credit spread when the central bank hits the zero lower bound constraint of the policy rate.

[8]See Appendix B for the approximated loss function under this setting.

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[2] Cúrdia, V. and M. Woodford. "Credit Friction and Optimal Monetary Policy." *Mimeo*, 2008.

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Appendix (not for publication)

A Derivation of the model

Following Sudo and Teranishi (2008), we introduce heterogeneous nominal loan interest rate contracts between private banks and firms into a standard New Keynesian framework model based on Woodford (2003). The model consists of four agents: a representative consumer, firms, a central bank, and private banks.

A.1 Cost minimization

In this model, we have two cost minimization problems. The first determines the optimal allocation of differentiated goods for the consumer. The second determines the optimal allocation of labor services, given the loan rates and wages for the firm's president.

For the consumer, we assume that the consumer's utility from consumption is increasing and concave in the consumption index, which is defined as a Dixit-Stiglitz aggregator as in Dixit and Stiglitz (1977)[9], of bundles of differentiated goods $f \in [0, 1]$ produced by the firm's project groups as:

$$C_t \equiv \left[\int_0^1 c_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}},$$

where C_t is aggregate consumption, $c_t(f)$ is a particular differentiated good along a continuum produced by the firm's project group f, and $\theta > 1$ is the elasticity of substitution across goods produced by the project groups. For the consumption aggregator, the appropriate consumption-based price index is given by:

$$P_t \equiv \left[\int_0^1 p_t(f)^{1-\theta} df \right]^{\frac{1}{1-\theta}},$$

[9] Dixit, A. and J. Stiglitz. "Monopolistic Competition and Optimum Product Diversity." *American Economic Review*, Vol. 67, 1977, pp. 297–308.

where P_t is the aggregate price and $p_t(f)$ is the price of a particular differentiated good $c_t(f)$. As in other applications of the Dixit-Stiglitz aggregator, the consumer's allocation across differentiated goods in each time period must solve a cost minimization problem. This means that the relative expenditures on a particular good are decided according to:

$$c_t(f) = C_t \left[\frac{p_t(f)}{P_t} \right]^{-\theta}. \tag{6}$$

An advantage of this consumption distribution rule is to imply that the consumer's total expenditure on consumption goods is given by $P_t C_t$. We use this demand function for differentiated goods in the firm sector.

Firms optimally hire differentiated labor as price takers. This optimal labor allocation is carried out through a two-step cost minimization problems. Firm f hires all types of labor. Here, each firm has to use two types of loan, loans R and loans S. To replicate this situation, we assume that to finance a labor cost for labor type $h \in [0, n)$, the firm has to use loan R, and to finance the cost for labor type $\overline{h} \in [n, 1]$, it has to use loan S. We can think of this setting as a firm uses loan R for some project characterized by labor type h, but uses loan S for some project characterized by labor type $\overline{h}$. When hiring a labor from $h \in [0, n)$, a portion of the labor cost associated with labor type h, which we denote as γ^R, is financed by borrowing from the bank h. Then, the first-step cost minimization problem for the allocation of differentiated labor from $h \in [0, n)$ is given by:

$$\min_{l_t(h,f)} \int_0^n \left[1 + \gamma^R r_t\left(h\right)\right] w_t\left(h\right) l_t(h,f) dh,$$

subject to the aggregate domestic labor supply to firm f:

$$L_t\left(f\right) \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\epsilon^R}} \int_0^n l_t\left(h,f\right)^{\frac{\epsilon^R - 1}{\epsilon^R}} dh \right]^{\frac{\epsilon^R}{\epsilon^R - 1}},$$

where $r_t(h)$ is the loan R interest rate applied to the employment of a particular labor type h, $l_t(h,f)$ is the differentiated labor input with respect to h that is supplied to firm f, and ϵ^R is a preference parameter on differentiated labor. The bank h

that provides loan R has some monopoly power over setting the loan interest rate. Thus, we assume monopolistic competition in the loan market where the transactions between banks and firms take place. The relative demand for differentiated labor is given as follows:

$$l_t(h,f) = \frac{1}{n} L_t \left\{ \frac{\left[1+\gamma^R r_t(h)\right] w_t(h)}{\Omega_t} \right\}^{-\epsilon^R}, \tag{7}$$

where

$$\Omega_t \equiv \left\{ \frac{1}{n} \int_0^n \left\{ \left[1+\gamma^R r_t(h)\right] w_t(h) \right\}^{1-\epsilon^R} dh \right\}^{\frac{1}{1-\epsilon^R}}. \tag{8}$$

As a result, we can derive:

$$\int_0^n \left[1+\gamma^R r_t(h)\right] w_t(h)\, l_t(h,f)\, dh = \Omega_t L_t(f).$$

Through a similar cost minimization problem, we can derive the relative demand for each type of differentiated labor from $\overline{h} \in [n,1]$ as:

$$l_t\left(\overline{h},f\right) = \frac{1}{1-n} \overline{L}_t \left\{ \frac{\left[1+\gamma^S r_t^*\left(\overline{h}\right)\right] w_t\left(\overline{h}\right)}{\overline{\Omega}_t} \right\}^{-\epsilon^S}, \tag{9}$$

where

$$\overline{\Omega}_t \equiv \left\{ \frac{1}{1-n} \int_n^1 \left\{ \left[1+\gamma^S r_t^*\left(\overline{h}\right)\right] w_t\left(\overline{h}\right) \right\}^{1-\epsilon^S} d\overline{h} \right\}^{\frac{1}{1-\epsilon^S}}, \tag{10}$$

and $r_t^*\left(\overline{h}\right)$ is the loan S interest rate, γ^S is a portion of the labor cost financed by bank $\overline{h}$, and ϵ^S is a preference parameter for differentiated labors. In this model, we set $\epsilon^R = \epsilon^S = \epsilon$. Then,

$$\int_n^1 \left[1+\gamma^S r_t^*\left(\overline{h}\right)\right] w_t\left(\overline{h}\right) l_t\left(\overline{h},f\right) dh = \overline{\Omega}_t \overline{L}_t(f).$$

According to the above optimality conditions, firms optimally choose the allocation of differentiated workers between the two groups. Because firms have a production function that hires n workers from $h \in [0,n)$ and $(1-n)$ workers from $\overline{h} \in [n,1]$, the second-step cost minimization problem describing the allocation of differentiated labor between these groups is given by:

$$\min_{L_t,\overline{L}_t} \Omega_t L_t(f) + \overline{\Omega}_t \overline{L}_t(f),$$

subject to the labor index:

$$\widetilde{L}_t(f) \equiv \frac{[L_t(f)]^n \left[\overline{L}_t(f)\right]^{1-n}}{n^n (1-n)^{1-n}}. \tag{11}$$

Then, the relative demand functions for each differentiated type of labor are derived as follows:

$$L_t(f) = n\widetilde{L}_t(f) \left(\frac{\Omega_t}{\widetilde{\Omega}_t}\right)^{-1}, \tag{12}$$

$$\overline{L}_t(f) = (1-n)\widetilde{L}_t(f) \left(\frac{\overline{\Omega}_t}{\widetilde{\Omega}_t}\right)^{-1}, \tag{13}$$

and

$$\widetilde{\Omega}_t \equiv \Omega_t^n \overline{\Omega}_t^{1-n}.$$

Therefore, we can obtain the following equations:

$$\Omega_t L_t(f) + \overline{\Omega}_t \overline{L}_t(f) = \widetilde{\Omega}_t \widetilde{L}_t(f),$$

$$l_t(h,f) = \left\{\frac{\left[1+\gamma^R r_t(h)\right] w_t(h)}{\Omega_t}\right\}^{-\epsilon} \left(\frac{\Omega_t}{\widetilde{\Omega}_t}\right)^{-1} \widetilde{L}_t(f),$$

and

$$l_t\left(\overline{h},f\right) = \left\{\frac{\left[1+\gamma^S r_t\left(\overline{h}\right)\right] w_t\left(\overline{h}\right)}{\overline{\Omega}_t}\right\}^{-\epsilon} \left(\frac{\overline{\Omega}_t}{\widetilde{\Omega}_t}\right)^{-1} \widetilde{L}_t(f),$$

from equations (7), (9), (12), and (13). We can now clearly see that the demand for each differentiated worker depends on wages and loan interest rates, given the total demand for labor.

Finally, from the assumption that the firms finance part of the labor costs by loans, we can derive:

$$\begin{aligned} q_t(h,f) &= \gamma^R w_t(h)\, l_t(h,f) \\ &= \gamma^R w_t(h) \left\{\frac{\left[1+\gamma^R r_t(h)\right] w_t(h)}{\Omega_t}\right\}^{-\epsilon} \left(\frac{\Omega_t}{\widetilde{\Omega}_t}\right)^{-1} \widetilde{L}_t(f), \end{aligned}$$

and

$$\begin{aligned} q_t\left(\overline{h},f\right) &= \gamma^S w_t\left(\overline{h}\right) l_t\left(\overline{h},f\right) \\ &= \gamma^S w_t\left(\overline{h}\right) \left\{\frac{\left[1+\gamma^S r_t^*\left(\overline{h}\right)\right] w_t\left(\overline{h}\right)}{\overline{\Omega}_t}\right\}^{-\epsilon} \left(\frac{\overline{\Omega}_t}{\widetilde{\Omega}_t}\right)^{-1} \widetilde{L}_t(f). \end{aligned}$$

These conditions demonstrate that the demands for each differentiated loan also depend on the wages and loan interest rates, given the total labor demand.

For aggregate labor demand conditions, we obtain the following expression:

$$\widetilde{L}_t = \int_0^1 \widetilde{L}_t(f)\, df.$$

A.2 Consumer

We consider a representative consumer who derives utility from consumption and disutility from the supply of work. The consumer maximizes the following utility function:

$$UT_t = \mathrm{E}_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_t, m_t) - \int_0^n V(l_T(h))dh - \int_n^1 V(l_T(\overline{h}))d\overline{h} \right] \right\},$$

where E_t is an expectation conditional on the state of nature at date t. The function U is increasing and concave in the consumption index, as shown in the preceding subsection, and real money holding. The budget constraint of the consumer is given by:

$$\begin{aligned} P_t C_t + \mathrm{E}_t\left[X_{t,t+1} B_{t+1}\right] + D_t \leq\ & B_t + (1+i_{t-1})D_{t-1} + \int_0^n w_t(h) l_t(h) dh \\ & + \int_n^1 w_t(\overline{h}) l_t(\overline{h}) d\overline{h} + \Pi_t^B + \Pi_t^F, \end{aligned} \tag{14}$$

where B_t is a risky asset, D_t is the amount of bank deposits, i_t is the nominal deposit rate set by the central bank from t to $t+1$, $w_t(h)$ is the nominal wage for labor supply, $l_t(h)$, to the firm's business unit of type h, $\Pi_t^B = \int_0^1 \Pi_{t-1}^B(h)dh$ is the nominal dividend stemming from the ownership of banks, $\Pi_t^F = \int_0^1 \Pi_{t-1}^F(f)df$ is the nominal dividend from the ownership of firms, and $X_{t,t+1}$ is the stochastic discount factor. We assume a complete financial market for risky assets. Thus, we can hold a unique discount factor and can characterize the relationship between the deposit rate and the stochastic discount factor:

$$\frac{1}{1+i_t} = \mathrm{E}_t \left[X_{t,t+1} \right]. \tag{15}$$

Given the optimal allocation of consumption expenditure across the differentiated goods, the consumer must choose the total amount of consumption, the optimal amount of risky assets to hold, and an optimal amount to deposit in each period. The necessary and sufficient conditions are given by:

$$U_C(C_t, \nu_t) = \beta(1+i_t)\mathrm{E}_t \left[U_C(C_{t+1}, \nu_{t+1}) \frac{P_t}{P_{t+1}} \right], \tag{16}$$

$$\frac{U_C(C_t, \nu_t)}{U_C(C_{t+1}, \nu_{t+1})} = \frac{\beta}{X_{t,t+1}} \frac{P_t}{P_{t+1}}.$$

Together with equation (15), we find that the condition given by equation (16) expresses the intertemporal optimal allocation on aggregate consumption. Assuming that the market clears such that the supply of each differentiated good equals its demand, $c_t(f) = y_t(f)$ and $C_t = Y_t$, we finally obtain the standard New Keynesian IS curve by log-linearizing equation (16):

$$x_t = E_t x_{t+1} - \sigma(\widehat{i}_t - E_t \pi_{t+1} - \widehat{r}_t^n),$$

where we call x_t the output gap as defined in the next section, π_{t+1} is inflation, and $\widehat{r}_t^n$ is the natural rate of interest. $\widehat{r}_t^n$ is an exogenous shock. Each variable is defined as the log deviation from its steady state where the price is flexible and the loan rates are constant (except x_t and π_t. Also, the log-linearized version of variable m_t is expressed by $\widehat{m}_t = \ln(m_t/\overline{m})$, where $\overline{m}$ is the steady state value of m_t.). We define $\sigma \equiv -\frac{U_Y}{U_{YY}\overline{Y}} > 0$.

In this model, the consumer provides differentiated types of labor to the firm and so holds the power to decide the wage of each type of labor as in Erceg, Henderson and Levin (2000).[10] We assume that each project group hires all types of workers in the same proportion. The consumer sets each wage $w_t(h)$ for any h in every period

[10] Erceg, C., D. Henderson, and A. Levin. “Optimal Monetary Policy with Staggered Wage and Price Contracts.” *Journal of Monetary Economics*, Vol. 46, 2000, pp. 281–313.

to maximize its utility subject to the budget constraint given by equation (14) and the demand function of labor given by equation (7).[11] Then we have the following relation:

$$\frac{w_t(h)}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{V_l [l_t(h)]}{U_C(C_t)}, \tag{17}$$

and

$$\frac{w_t(\overline{h})}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{V_l [l_t(\overline{h})]}{U_C(C_t)}. \tag{18}$$

In this paper, we assume that the consumer supplies its labor only to the firm, not the private bank. We use the relations given by equations (17) and (18) on the firm side.

A.3 Firms

There exists a continuum of firms populated over unit mass $[0, 1]$. Each firm plays two roles. First, each firm decides the amount of differentiated labor to be employed from both $h \in [0, n)$ and $\overline{h} \in [n, 1]$, through the two-step cost minimization problem on production cost. Part of the costs of labor must be financed by external loans from banks. For example, to finance the costs of hiring workers from $h \in [0, n)$, the firm must borrow from banks that provide loan R. However, to finance the costs of hiring workers from $\overline{h} \in [n, 1]$, the firm must borrow from banks that provide loan S. Here, we assume that firms must use all types of labor and therefore borrow both loan R and loan S in a fixed proportion.[12] Second, in a monopolistically competitive goods market, where individual demand curves on differentiated consumption goods are offered by consumers, each firm sets a differentiated goods price to maximize profit. Prices are set in a staggered manner, as in a Calvo (1983)[13] - Yun (1992)[14]

[11] We assume a flexible wage setting in the sense that the consumer can change the wage in every period.

[12] The same structure is assumed for employment in Woodford (2003).

[13] Calvo, G. "Staggered Prices in a Utility Maximizing Framework." *Journal of Monetary Economics*, Vol. 12, 1983, pp. 383-398.

[14] Yun, T. "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles." *Journal of Monetary Economics*, Vol. 37, 1996, pp. 345-370.

framework.

As standard in New Keynesian models following the Calvo (1983) - Yun (1992) framework, each firm f resets its price with probability $(1-\alpha)$ and maximizes the present discounted value of profit given by:

$$\mathrm{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} X_{t,T} \left[p_t(f)\, c_{t,T}(f) - \widetilde{\Omega}_T \widetilde{L}_T(f) \right], \tag{19}$$

where we assume the production function as $y_t(f) = F(\widetilde{L}_T(f))$. The production function is increasing and concave. Here, the firm sets $p_t(f)$ under the Calvo (1983) - Yun (1992) framework. The present discounted value of the profit given by equation (19) is further transformed into:

$$\mathrm{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} X_{t,T} \left\{ p_t(f) \left[\frac{p_t(f)}{P_T} \right]^{-\theta} C_T - \widetilde{\Omega}_T \widetilde{L}_T(f) \right\}.$$

We note that price setting is independent of the loan interest rate setting of private banks.

The optimal price setting of $\overline{p}_t(f)$ in the situation where managers can reset their prices with probability $(1-\alpha)$ is given by:

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{U_C(C_T)}{P_T} y_{t,T}(f) \left[\frac{\theta-1}{\theta} \overline{p}_t(f) - \widetilde{\Omega}_T \frac{\partial \widetilde{L}_T(f)}{\partial y_{t,T}(f)} \right] = 0, \tag{20}$$

where we substitute equation (6). By further substituting equations (17) and (18) into equation (20), equation (20) can now be rewritten as:

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} U_C(C_T)\, y_{t,T}(f) \left[\frac{\theta-1}{\theta} \frac{\overline{p}_t(f)}{P_t} \frac{P_t}{P_T} - \frac{\epsilon}{\epsilon-1} Z_{t,T}(f) \right] = 0, \tag{21}$$

where

$$\begin{aligned} Z_{t,T}(f) &= \left\{ \left(\frac{1}{n}\right) \int_0^n \left[1+\gamma^R r_t(h)\right]^{1-\epsilon} \left\{ \frac{V_l\left[l_T(h)\right]}{U_C(C_t)} \frac{\partial \widetilde{L}_{t,T}(f)}{\partial y_{t,T}(f)} \right\}^{1-\epsilon} dh \right\}^{\frac{n}{1-\epsilon}} \\ &\quad \times \left\{ \left(\frac{1}{1-n}\right) \int_n^1 \left[1+\gamma^S r_t\left(\overline{h}\right)\right]^{1-\epsilon} \left\{ \frac{V_l\left[l_T\left(\overline{h}\right)\right]}{U_C(C_t)} \frac{\partial \widetilde{L}_{t,T}(f)}{\partial y_{t,T}(f)} \right\}^{1-\epsilon} d\overline{h} \right\}^{\frac{1-n}{1-\epsilon}}. \end{aligned}$$

By log-linearizing equation (21), we derive:

$$\frac{1}{1-\alpha\beta}\widehat{\widetilde{p}}_t(f) = \mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\sum_{\tau=t+1}^{T} \pi_{H,\tau} + \Theta_1^* \widehat{R}_{R,T} + \Theta_2^* \widehat{R}_{S,T} + \widehat{mc}_{t,T}(f) \right], \quad (22)$$

where $\Theta_1^* \equiv n \frac{\gamma^R(1+\overline{R}_R)}{1+\gamma^R \overline{R}_R}$ and $\Theta_2^* \equiv (1-n) \frac{\gamma^S(1+\overline{R}_S)}{1+\gamma^S \overline{R}_S}$ are positive parameters, and we define the real marginal cost as:

$$\widehat{mc}_{t,T}(f) \equiv \int_0^n \widehat{mc}_{t,T}(h,f)\, dh + \int_n^1 \widehat{mc}_{t,T}\left(\overline{h},f\right) d\overline{h},$$

where

$$mc_{t,T}(h,f) \equiv \frac{V_l\left[l_T(h)\right]}{U_Y(C_T)} \frac{\partial \widetilde{L}_{t,T}(f)}{\partial y_{t,T}(f)},$$

and

$$mc_{t,T}\left(\overline{h},f\right) \equiv \frac{V_l\left[l_T\left(\overline{h}\right)\right]}{U_Y(C_T)} \frac{\partial \widetilde{L}_{t,T}(f)}{\partial y_{t,T}(f)}.$$

We also define:

$$R_{R,t} \equiv \frac{1}{n}\int_0^n r_t(h)\, dh, \quad (23)$$

$$R_{S,t} \equiv \frac{1}{1-n}\int_n^1 r_t\left(\overline{h}\right) d\overline{h}, \quad (24)$$

$$\widetilde{p}_t(f) \equiv \frac{\overline{p}_t(f)}{P_t} \text{ and } \pi_t \equiv \frac{P_t}{P_{t-1}}.$$

Then, equation (22) can be transformed into:

$$\frac{1}{1-\alpha\beta}\widehat{\widetilde{p}}_t(f) = \mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[(1+\omega_p\sigma)^{-1} \left(\widehat{mc}_T + \Theta_1^* \widehat{R}_{R,T} + \Theta_2^* \widehat{R}_{S,T} \right) + \sum_{\tau=t+1}^{T} \pi_\tau \right], \quad (25)$$

where we make use of the relationship:

$$\widehat{mc}_{t,T}(f) = \widehat{mc}_T - \omega_p \theta \left[\widehat{\widetilde{p}}_t(f) - \sum_{\tau=t+1}^{T} \pi_\tau \right],$$

where ω_p is the elasticity of $\frac{\partial \widetilde{L}_{t,T}(f)}{\partial y_{t,T}(f)}$ with respect to y. We further denote the average real marginal cost as:

$$\widehat{mc}_T \equiv \int_0^n \widehat{mc}_T(h)\, dh + \int_n^1 \widehat{mc}_T\left(\overline{h}\right) d\overline{h},$$

where

$$mc_T(h) \equiv \frac{V_l[l_T(h)]}{U_Y(C_T)} \frac{\partial \widetilde{L}_T}{\partial Y_{H,T}},$$

and

$$mc_T(\overline{h}) \equiv \frac{V_l[l_T(\overline{h})]}{U_Y(C_T)} \frac{\partial \widetilde{L}_T}{\partial Y_{H,T}}.$$

The point is that the unit marginal cost is the same for all firms in the situation where each firm uses all types of labor and loans in the same proportion. Thus, all firms set the same price if they have the opportunity to reset their prices at time t.

In the Calvo (1983) - Yun (1992) setting, the evolution of the aggregate price index P is described by the following law of motion:

$$\int_0^1 p_t(f)^{1-\theta} df = \alpha \int_0^1 p_{t-1}(f)^{1-\theta} df + (1-\alpha) \int_0^1 \overline{p}_t(f)^{1-\theta} df,$$

$$\Longrightarrow P_t^{1-\theta} = \alpha P_{t-1}^{1-\theta} + (1-\alpha)(\overline{p}_t)^{1-\theta}, \tag{26}$$

where

$$P_t^{1-\theta} \equiv \int_0^1 p_t(f)^{1-\theta} df \text{ and } \overline{p}_t^{1-\theta} \equiv \int_0^1 \overline{p}_t(f)^{1-\theta} df.$$

The current aggregate price is given by the weighted average of the changed and unchanged prices. Because the chances of resetting prices are randomly assigned to each firm with equal probability, an aggregate price change at time t should be evaluated by the average price change for all firms. By log-linearizing equation (26), together with equation (25), we derive the following New Keynesian Phillips curve:

$$\pi_t = \chi\left(\widehat{mc}_t + \Theta_1^* \widehat{R}_{R,t} + \Theta_2^* \widehat{R}_{S,t}\right) + \beta \mathrm{E}_t \pi_{t+1},$$

where the slope coefficient $\chi \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\omega_p\theta)}$ is a positive parameter. This is quite similar to the standard New Keynesian Phillips curve, though it contains loan interest rates as cost components.

Here, according to the discussion in Woodford (2003), we define the natural rate of output Y_t^n from equation (21) as:

$$
\begin{aligned}
\frac{\theta-1}{\theta} &= \left[\frac{\epsilon}{\epsilon-1}\right]\left[1+\gamma^R\overline{R}_R\right]^n\left[1+\gamma^S\overline{R}_S\right]^{1-n} \\
&\times\left\{\left(\frac{1}{n}\right)\int_0^n\left\{\frac{V_l\left[l_t^n(h)\right]}{U_C(C_t)}\frac{\partial\widetilde{L}_t^n(f)}{\partial Y_t^n(f)}\right\}^{1-\epsilon}dh\right\}^{\frac{n}{1-\epsilon}} \\
&\times\left\{\left(\frac{1}{1-n}\right)\int_n^1\left\{\frac{V_l\left[l_t^n(\overline{h})\right]}{U_C(Y_t^n)}\frac{\partial\widetilde{L}_t^n(f)}{\partial Y_t^n(f)}\right\}^{1-\epsilon}d\overline{h}\right\}^{\frac{1-n}{1-\epsilon}},
\end{aligned}
$$

where, under the natural rate of output, we assume a flexible price setting, $p_t^*(f) = P_t$, and assume no impact of monetary policy, $r_t(h) = r_t(\overline{h}) = \overline{R}$, and so hold $y_t(f) = Y_t^n$. $l_t^n(h)$, $l_t^n(\overline{h})$, $\widetilde{L}_t^n(f)$, and $\widetilde{L}_t^n(f)$ are the amount of labor under Y_t^n, respectively. Then, we have:

$$
\widehat{mc}_t = (\omega+\sigma^{-1})(\widehat{Y}_t - \widehat{Y}_t^n),
$$

where $\widehat{Y}_t \equiv \ln(Y_t/\overline{Y})$, and $\widehat{Y}_t^n \equiv \ln(Y_t^n/\overline{Y})$, and ω is the sum of the elasticity of the marginal disutility of work with respect to the output increase and the elasticity of $\frac{1}{F'(F^{-1}(y))}$ with respect to output increase.[15] By defining $x_t \equiv \widehat{Y}_t - \widehat{Y}_t^n$, we finally have:

$$
\begin{aligned}
\pi_t &= \kappa x_t + \chi\left(\Theta_1^*\widehat{R}_{R,t} + \Theta_2^*\widehat{R}_{S,t}\right) + \beta \mathrm{E}_t\pi_{t+1} \\
&= \kappa x_t + \Theta_1\widehat{R}_{R,t} + \Theta_2\widehat{R}_{S,t} + \beta \mathrm{E}_t\pi_{t+1},
\end{aligned}
$$

where $\kappa \equiv \chi(\omega+\sigma^{-1})$, $\Theta_1 = \chi\Theta_1^*$, and $\Theta_2 = \chi\Theta_2^*$.

A.4 Private banks

There exists a continuum of private banks populated over $[0,1]$. There are two types of banks: banks that provide loans R populated over $[0,n)$ and banks that provide loans S populated over $[n,1]$. In this model, unlike Sudo and Teranishi (2008), this

[15] $\omega \equiv \omega_p + \omega_w$, where ω_w is the elasticity of the marginal disutility of work with respect to the output increase in $\frac{V_l(l_t(h),\nu_t)}{U_Y(Y_t,\nu_t)}$. Woodford (Ch. 3, 2003) provides a more detailed derivation.

heterogeneity is not explained by the heterogeneous stickiness of the loan interest rate. Thus, we assume that the distinction between loans R and loans S lies in the difference in the monopolistic power of private banks. This varies according to the ratios of external finance as it induces different markup distortion.[16] Each private bank plays two roles: (1) to collect deposits from consumers; and (2) under the monopolistically competitive loan market, to set differentiated nominal loan interest rates according to their individual loan demand curves, given the amount of deposits. We assume that each bank sets the differentiated nominal loan interest rate according to the types of labor force.

A bank that provides loan R only lends to firms when they hire labor from $h \in [0, n)$. However, a bank that provides loan S only lends to firms when they hire labor from $\overline{h} \in [n, 1]$. First, we describe the optimization problem of a bank that provides loan R. Under the segmented environment stemming from the differences in labor supply, private banks can set different loan interest rates depending on the types of labor. Consequently, the private bank holds some monopoly power over the loan interest rate to firms. Therefore, the bank h chooses the loan interest rate $r_t(h)$ that maximizes profit:

$$q_{t,T}(h, f)\{[1 + r_t(h)] - (1 + i_T)\}.$$

The optimal loan condition is now given by:

$$q_{t,T}(h)\left\{\left[1 + \gamma^R r_t(h)\right] - \epsilon\gamma^R\{[1 + r_t(h)] - (1 + i_T)\}\right\} = 0. \tag{27}$$

By log-linearizing equations (27), we can determine the relationship between the loan and deposit interest rate as follows:

$$\widehat{R}_{R,t} = \lambda_R \widehat{i}_t,$$

where $\lambda_R \equiv \frac{\epsilon}{\epsilon-1}\frac{1+\overline{i}}{1+\overline{R}_R}$ is a positive parameter.[17] This equation describes the loan interest rate (supply) curve by the banks that provide loans R.

[16] This distinction can also be explained by idiosyncratic shocks that two types of loans face.

[17] Here we have $\left[1 + \gamma^R\overline{R}_R\right] - \epsilon\gamma^R\left[1 + \overline{R}_R\right] - \left(1 + \overline{i}\right) = 0$ in relationship between $\overline{R}_R$ and $\overline{i}$.

Similarly, from the optimization problem of bank $\overline{h}$ that provide loan S, we can obtain the relationship between the loan and deposit interest rates as follows:

$$\widehat{R}_{S,t} = \lambda_S \widehat{i}_t,$$

where $\lambda_S \equiv \frac{\epsilon}{\epsilon-1}\frac{1+\bar{i}}{1+\overline{R}_S}$ is a positive parameter.

The market clearing loan conditions are expressed as:

$$q_{t,T}(h) = \int_0^1 q_{t,T}(h,f)df \text{ and } q_{t,T}\left(\overline{h}\right) = \int_0^1 q_{t,T}\left(\overline{h},f\right)df,$$

$$\int_0^n q_{t,T}(h)dh = nD_T \text{ and } \int_n^1 q_{t,T}\left(\overline{h}\right)d\overline{h} = (1-n)\,D_T.$$

B Derivation of the loss function

B.1 Under a flexible loan rate

In derivation of approximated welfare function, we basically follow the way of Woodford (2003). Note that the welfare approximation is calculated in terms of the deviation from the steady state where the price is flexible and the loan rates are constant.

Under the situation where the supply of goods matches the demand for goods at every level, $Y_t = C_t$ and $y_t(f) = c_t(f)$ for any f, the welfare criterion of the consumer is given by

$$\mathrm{E}_0\left\{\sum_{t=0}^{\infty}\beta^t UT_t\right\},$$

where

$$UT_t = U(C_t, m_t) - \int_0^n V(l_t(h))dh - \int_n^1 V(l_t(\overline{h}))d\overline{h}, \tag{28}$$

and

$$Y_t \equiv \left[\int_0^1 y_t(f)^{\frac{\theta-1}{\theta}}df\right]^{\frac{\theta}{\theta-1}}.$$

We log-linearize equation (28) step by step to derive an approximated welfare function. Firstly, we log-linearize the first term of equation (28). In this log-linearization, we follow the derivation in Woodford (Ch. 6 and Appendix E, 2003).

$$\begin{aligned} U(Y_t, m_t; \nu_t) &= \overline{Y}U_c \left[\begin{array}{c} \widehat{Y}_t + \frac{1}{2}(1-\sigma^{-1})\widehat{Y}_t^2 + \sigma^{-1} g_t \widehat{Y}_t + s_m \eta_y \widehat{Y}_t \\ -s_m \eta_i \widehat{i}_t + \frac{1}{2}\chi \eta_y \widehat{Y}_t^2 + \chi \epsilon_t^m \widehat{Y}_t - \frac{1}{2}\overline{\upsilon}^{-1}\eta_i (\widehat{i}_t)^2 \end{array} \right] + t.i.p + Order(\| \xi \|^3) \\ &= \overline{Y}U_c \left[\begin{array}{c} \widehat{Y}_t + \frac{1}{2}(1-\sigma^{-1})\widehat{Y}_t^2 + \sigma^{-1} g_t \widehat{Y}_t + s_m \eta_y \widehat{Y}_t \\ +\frac{1}{2}\chi \eta_y \widehat{Y}_t^2 + \chi \epsilon_t^m \widehat{Y}_t - \frac{1}{2}\overline{\upsilon}^{-1}\eta_i (\widehat{i}_t - i^*)^2 \end{array} \right] + t.i.p + Order(\| \xi \|^3), \end{aligned} \tag{29}$$

where $\overline{U} \equiv U(\overline{Y}; 0)$, ν_t is an exogenous shock, $t.i.p$ is the term that is independent of monetary policy, $Order(\| \xi \|^3)$ expresses order terms higher than the second-order approximation, $\sigma^{-1} \equiv -\frac{\overline{Y}U_{cc}}{U_c} > 0$, $g_t \equiv -\frac{U_{c\nu}\nu_t}{\overline{Y}U_{cc}}$, $s_m \equiv \frac{\overline{m}u_m}{\overline{c}u_c} > 0$, $\eta_y \equiv -\frac{\overline{Y}u_{cm}}{\overline{m}u_{mm}} > 0$, $\eta_i \equiv -\frac{u_c}{\overline{m}u_{mm}} > 0$, $\chi \equiv \frac{\overline{m}u_{cm}}{u_c} > 0$, $\epsilon_t^m \equiv \left(\chi - \frac{\overline{m}u_{mm}}{u_m}\right)^{-1} \left[\frac{u_{m\nu}}{u_m}\nu_t - \sigma^{-1}g_t\right]$, $\overline{\upsilon} \equiv \frac{\overline{Y}}{\overline{m}}$, $i^* = \ln \frac{\overline{i}-\overline{i}^m}{1+\overline{i}}$, $\overline{i}$ is the steady state value of the policy interest rate, and $\overline{i}^m$ is the steady state value of the interest rate on the real money holding. Here, we assume the interest rate on the real money is constant.

Secondly, we log-linearize the second term of equation (28) in a similar manner.

$$\begin{aligned} \frac{1}{n}\int_0^n V(l_t(h);\nu_t)dh &= V_l\overline{L}(E_h\widehat{l}_t(h) + \frac{1}{2}E_h(\widehat{l}_t(h))^2) + \frac{1}{2}V_{ll}\overline{L}^2 E_h(\widehat{l}_t(h))^2 + V_{l\nu}\overline{L}\nu_t E_h\widehat{l}_t(h) \\ &\quad + t.i.p + Order(\| \xi \|^3) \\ &= \overline{L}V_l\left[\widehat{L}_t + \frac{1}{2}(1+\nu)\widehat{L}_t^2 - \nu\widetilde{\nu}_t\widehat{L}_t + \frac{1}{2}(\nu + \frac{1}{\epsilon})var_h\widehat{l}_t(h)\right] \\ &\quad + t.i.p + Order(\| \xi \|^3), \end{aligned} \tag{30}$$

where $\widetilde{\nu}_t \equiv -\frac{V_{l\nu}\nu_t}{\overline{L}V_{ll}}$, $\nu \equiv \frac{\overline{L}V_{ll}}{V_l}$, $\phi_h \equiv \frac{\overline{Y}}{\overline{L}f_L}$, $\omega_p \equiv \frac{ff_{LL}}{(f_L)^2}$, $q_t \equiv (1+\omega^{-1})a_t + \omega^{-1}\nu\widetilde{\nu}_t$, $a_t \equiv \ln A_t$, $var_h\widehat{l}_t(h)$ is the variance of $\widehat{l}_t(h)$ across all types of labor, and $var_f\widehat{p}_t(f)$ is the variance of $\widehat{p}_t(f)$ across all differentiated good prices. Here the definition of the labor sub-aggregator is given by:

$$L_t \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n l_t\left(h\right)^{\frac{\epsilon-1}{\epsilon}} dh\right]^{\frac{\epsilon}{\epsilon-1}},$$

and so we have $\widehat{L}_t = E_h \widehat{l}_t(h) + \frac{1}{2}\frac{\epsilon-1}{\epsilon} var_h \widehat{l}_t(h) + Order(\parallel \xi \parallel^3)$ in the second order approximation. We use this relation in the second line.

Thirdly, we log-linearize the third term of equation (28) by a similar manner.

$$\begin{aligned}\frac{1}{1-n}\int_n^1 V(l_t(\overline{h});\nu_t)d\overline{h} &= V_l\overline{L}(E_{\overline{h}}\widehat{l}_t(\overline{h}) + \frac{1}{2}E_{\overline{h}}(\widehat{l}_t(\overline{h}))^2) + \frac{1}{2}V_{ll}\overline{L}^2 E_{\overline{h}}(\widehat{l}_t(\overline{h}))^2 + V_{l\nu}\overline{L}\nu_t E_{\overline{h}}\widehat{l}_t(\overline{h}) \\ +t.i.p + Order(&\parallel \xi \parallel^3) \\ &= \overline{L}V_l\left[\widehat{\overline{L}}_t + \frac{1}{2}(1+\nu)\widehat{\overline{L}}_t^2 - \nu\widetilde{\nu}_t\widehat{\overline{L}}_t + \frac{1}{2}(\nu+\frac{1}{\epsilon})var_{\overline{h}}\widehat{l}_t(\overline{h})\right] \\ +t.i.p + Order(&\parallel \xi \parallel^3). \end{aligned} \tag{31}$$

Here the definition of the labor sub-aggregator is given by

$$\overline{L}_t \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}}\int_n^1 l_t\left(\overline{h}\right)^{\frac{\epsilon-1}{\epsilon}} d\overline{h}\right]^{\frac{\epsilon}{\epsilon-1}},$$

and so we have $\widehat{\overline{L}}_t = E_{\overline{h}}\widehat{l}_t(\overline{h}) + \frac{1}{2}\frac{\epsilon-1}{\epsilon}var_{\overline{h}}\widehat{l}_t(\overline{h}) + Order(\parallel \xi \parallel^3)$ in the second order approximation. We use this relation in the second line.

Then, from equations (30) and (31), we have:

$$\begin{aligned} &\int_0^n V(l_t(h);\nu_t)dh + \int_n^1 V(l_t(\overline{h});\nu_t)d\overline{h} \\ &= \overline{L}V_l\left[\begin{array}{c} n\widehat{L}_t + \frac{n}{2}(1+\nu)\widehat{L}_t^2 - n\nu\widetilde{\nu}_t\widehat{L}_t + \frac{n}{2}(\nu+\frac{1}{\epsilon})var_h\widehat{l}_t(h) \\ +(1-n)\widehat{\overline{L}}_t + \frac{1-n}{2}(1+\nu)\widehat{\overline{L}}_t^2 - (1-n)\nu\widetilde{\nu}_t\widehat{\overline{L}}_t + \frac{1-n}{2}(\nu+\frac{1}{\epsilon})var_{\overline{h}}\widehat{l}_t(\overline{h}) \end{array}\right] \\ &+t.i.p + Order(\parallel \xi \parallel^3) \\ &= \overline{L}V_l\left[\begin{array}{c} \widehat{\widehat{L}}_t + \frac{1+\nu}{2}\widehat{\widehat{L}}_t^2 - \nu\widetilde{\nu}_t\phi_h^{-1}\widehat{\widehat{L}}_t + n(1-n)\frac{1+\nu}{2}\left(\widehat{L}_t - \widehat{\overline{L}}_t\right)^2 \\ +\frac{n}{2}(\nu+\frac{1}{\epsilon})var_h\widehat{l}_t(h) + \frac{1-n}{2}(\nu+\frac{1}{\epsilon})var_{\overline{h}}\widehat{l}_t(\overline{h}) \end{array}\right] \\ &+t.i.p + Order(\parallel \xi \parallel^3), \end{aligned} \tag{32}$$

where we use:

$$\widehat{\widehat{L}}_t = n\widehat{L}_t + (1-n)\widehat{\overline{L}}_t,$$

from equation (11). Then, we employ the condition that the demand for labor is equal to the supply of labor as:

$$\widetilde{L}_t = \int_0^1 \widetilde{L}_t(f)df = \int_0^1 f^{-1}(\frac{y_t(f)}{A_t})df,$$

where the production function is given by $y_t(f) = A_t f(L_t(f))$, where $f(\cdot)$ is an increasing and concave function. By taking the second order approximation, we have:

$$\widehat{\overline{L}}_t = \phi_h(\widehat{Y}_t - a_t) + \frac{1}{2}(1+\omega_p - \phi_h)\phi_h(\widehat{Y}_t - a_t)^2 + \frac{1}{2}(1+\omega_p\theta)\theta var_f \widehat{p}_t(f) + Order(\| \xi \|^3),$$

where we log-linearize the demand function for differentiated goods to derive the relation $var_f \ln y_t(f) = \theta^2 var_f \ln p_t(f)$, which can be derived from the consumer's cost minimization problem under the Dixit-Stiglitz aggregator, as:

$$y_t(f) = Y_t \left[\frac{p_t(f)}{P_t}\right]^{-\theta},$$

where the aggregate price index is given by $P_t \equiv \left[\int_0^1 p_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}$. Also, we use the relation of $\phi_h \nu = \omega_w$ and $\omega = \omega_p + \omega_w$, where ω_w is the elasticity of the real wage under a flexible-wage labor supply with respect to aggregate output. We can transform equation (32) as:

$$\begin{aligned}
&\int_0^n V(l_t(h);\nu_t)dh + \int_n^1 V(l_t(\overline{h});\nu_t)d\overline{h} \\
&= \phi_h \overline{L} V_l \left[\begin{array}{c} \widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega q_t \widehat{Y}_t + n(1-n)\frac{1+\nu}{2}\left(\widehat{L}_t - \widehat{\overline{L}}_t\right)^2 \\ +\frac{1}{2}(1+\omega_p\theta)\theta var_f \ln p_t(f) + \frac{n}{2}\phi_h^{-1}(\nu + \frac{1}{\epsilon})var_h \ln l_t(h) \\ +\frac{1-n}{2}\phi_h^{-1}(\nu+\frac{1}{\epsilon})var_{\overline{h}} \ln l_t(\overline{h}) \end{array}\right] \\
&+t.i.p + Order(\| \xi \|^3) \\
&= \phi_h \overline{L} V_l \left[\begin{array}{c} \widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega q_t \widehat{Y}_t \\ +n(1-n)\frac{1+\nu}{2}\left(\frac{1}{1+\nu}\right)^2\left(\Theta\widehat{R}_{R,t} - \Theta^*\widehat{R}_{S,t}\right)^2 + \frac{1}{2}(1+\omega_p\theta)\theta var_f \ln p_t(f) \\ +\frac{n}{2}\phi_h^{-1}(\nu+\frac{1}{\epsilon})var_h \ln l_t(h) + \frac{1-n}{2}\phi_h^{-1}(\nu+\frac{1}{\epsilon})var_{\overline{h}} \ln l_t(\overline{h}) \end{array}\right] \\
&+t.i.p + Order(\| \xi \|^3).
\end{aligned}$$

From the second line to the third line, we use following transformations:

$$\begin{aligned}
\widehat{L}_t - \widehat{\overline{L}}_t &= \widehat{\overline{\Omega}}_t - \widehat{\Omega}_t \\
&= \left(\Theta\widehat{R}_{R,t} - \Theta^*\widehat{R}_{S,t}\right) + \frac{1}{1-n}\int_n^1 w_t(\overline{h})d\overline{h} - \frac{1}{n}\int_0^n w_t(h)dh \\
&= \left(\Theta\widehat{R}_{R,t} - \Theta^*\widehat{R}_{S,t}\right) - \nu\left(\widehat{L}_t - \widehat{\overline{L}}_t\right),
\end{aligned}$$

where $\Theta \equiv \frac{\gamma^R(1+\overline{R}_R)}{1+\gamma^R\overline{R}_R}$ and $\Theta^* \equiv \frac{\gamma^S(1+\overline{R}_S)}{1+\gamma^S\overline{R}_S}$. There we use log-linear relations from equation (12), equation (13), equation (17), and equation (18) and the definitions from equation (7), equation (8), equation (9), equation (10), equation (23), and equation (24).

Furthermore, we can replace $\phi_h \overline{L} V_l$ with $(1-\Phi)\overline{Y}U_c$ as:

$$\begin{aligned} & \int_0^n V(l_t(h);\nu_t)dh + \int_n^1 V(l_t(\overline{h});\nu_t) \\ = \; & \overline{Y}U_c \begin{bmatrix} (1-\Phi)\widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega q_t \widehat{Y}_t + \frac{1}{2}(1+\omega_p\theta)\theta var_f \ln p_t(f) \\ +\frac{n}{2}\phi_h^{-1}(\nu+\frac{1}{\epsilon})var_h \ln l_t(h) + \frac{1-n}{2}\phi_h^{-1}(\nu+\frac{1}{\epsilon})var_{\overline{h}} \ln l_t(\overline{h}) \\ +n(1-n)\frac{1+\nu}{2}\left(\frac{1}{1+\nu}\right)^2\left(\Theta\widehat{R}_{R,t} - \Theta^*\widehat{R}_{S,t}\right)^2 \end{bmatrix} \end{aligned}$$

$$+t.i.p + Order(\quad \| \quad \xi \|^3), \tag{33}$$

Here, we use the assumption that distortion of the output level Φ is induced by firm's price markup through:

$$\left\{\left(\frac{1}{n}\right)\int_0^n \left\{\frac{V_l\left[l_T\left(h\right)\right]}{U_C\left(C_t\right)}\frac{\partial \widetilde{L}_{t,T}\left(f\right)}{\partial y_{t,T}\left(f\right)}\right\}^{1-\epsilon} dh\right\}^{\frac{n}{1-\epsilon}} \times \left\{\left(\frac{1}{1-n}\right)\int_n^1 \left\{\frac{V_l\left[l_T\left(\overline{h}\right)\right]}{U_C\left(C_t\right)}\frac{\partial \widetilde{L}_{t,T}\left(f\right)}{\partial y_{t,T}\left(f\right)}\right\}^{1-\epsilon} d\overline{h}\right\}^{\frac{1-n}{1-\epsilon}}, \tag{34}$$

where a flexible price and no role of monetary policy is of order one, as in Woodford (2003).[18] Thus, in terms of the natural rate of output, we actually assume that the real marginal cost function of firm $Z(\cdot)$ in order to supply a good f is given by:

$$\begin{aligned} Z_t(f) \; = \; & Z(y_t(f), Y_t, r_t; \nu_t) = \left\{\left(\frac{1}{n}\right)\int_0^n \left[1+\gamma^R r_t\left(h\right)\right]^{1-\epsilon}\left\{\frac{V_l\left[l_T\left(h\right)\right]}{U_C\left(C_t,\nu_t\right)}\frac{\partial \widetilde{L}_{t,T}\left(f\right)}{\partial y_{t,T}\left(f\right)}\right\}^{1-\epsilon} dh\right\}^{\frac{n}{1-\epsilon}} \\ & \times \left\{\left(\frac{1}{1-n}\right)\int_n^1 \left[1+\gamma^S r_t\left(\overline{h}\right)\right]^{1-\epsilon}\left\{\frac{V_l\left[l_T\left(\overline{h}\right)\right]}{U_C\left(C_t,\nu_t\right)}\frac{\partial \widetilde{L}_{t,T}\left(f\right)}{\partial y_{t,T}\left(f\right)}\right\}^{1-\epsilon} d\overline{h}\right\}^{\frac{1-n}{1-\epsilon}}, \end{aligned}$$

then the natural rate of output $Y_t^n = Y^n(\nu_t)$ is given by

$$Z(Y_t^n, Y_t^n, \overline{R}; \nu_t) = \frac{\theta-1}{\theta} = \left[\frac{\epsilon}{\epsilon-1}\right]\left[1+\gamma^R\overline{R}_R\right]^n\left[1+\gamma^S\overline{R}_S\right]^{1-n}(1-\Phi) \tag{35}$$

[18] We assume that monetary policy has no impact on the level of the natural rate of output.

where the parameter Φ is of order one and expresses the distortion of the output level.[19] Then, we can combine equations (29) and (33):

$$
\begin{aligned}
UT_t &\simeq \overline{Y}U_c \left[\begin{array}{c} \left(\Phi + s_m\eta_y\right)\widehat{Y}_t + (\sigma^{-1}g_t + \omega q_t + \chi\epsilon_t^m)\widehat{Y}_t - \frac{1}{2}\epsilon_{mc}\widehat{Y}_t^2 \\ -\frac{1}{2}\eta_\pi var_f \ln p_t(f) - \frac{n}{2}\eta_l var_h \ln l_t(h) - \frac{1-n}{2}\eta_s var_{\overline{h}} \ln l_t(\overline{h}) \\ +n(1-n)\frac{1+\nu}{2}\left(\frac{1}{1+\nu}\right)^2\left(\Theta\widehat{R}_{R,t} - \Theta^*\widehat{R}_{S,t}\right)^2 \\ -\frac{1}{2}\overline{\upsilon}^{-1}\eta_i(\widehat{i}_t - i^*)^2 \end{array} \right] \\
&+t.i.p + Order(\parallel \xi \parallel^3) \\
&= -\frac{1}{2}\overline{Y}U_c \left[\begin{array}{c} \epsilon_{mc}(x_t - x^*)^2 + \eta_\pi var_f \ln p_t(f) \\ +n\eta_l var_h \ln l_t(h) + (1-n)\eta_l var_h \ln l_t(\overline{h}) \\ +n(1-n)\frac{1+\nu}{2}\left(\frac{1}{1+\nu}\right)^2\left(\Theta\widehat{R}_{R,t} - \Theta^*\widehat{R}_{S,t}\right)^2 \\ +\overline{\upsilon}^{-1}\eta_i(\widehat{i}_t - i^*)^2 \end{array} \right] \\
&+t.i.p + Order(\parallel \xi \parallel^3),
\end{aligned}
$$

where $\epsilon_{mc} \equiv \sigma^{-1} + \omega - \chi\eta_y$, $\eta_\pi \equiv \theta(1 + \omega_p\theta)$, $\eta_l \equiv \phi_h^{-1}(\nu + \epsilon^{-1})$, $\eta_s \equiv \phi_{\overline{h}}^{-1}(\nu + \epsilon^{-1})$, $x_t \equiv \widehat{Y}_t - \widehat{Y}_t^n$, and $x^* \equiv \ln(Y^*/\overline{Y})$. Here Y_t^* is a solution in equation (35) when $\Phi = 0$, called the efficient level of output as defined in Woodford (2003). In the second line, we use the log-linearization of equation (35) as:

$$
\widehat{Y}_t^n \equiv \ln(Y_t^n/\overline{Y}) = \frac{\sigma^{-1}g_t + \omega q_t}{\sigma^{-1} + \omega},
$$

and the relation as:

$$
\ln(Y_t^n/Y_t^*) = -(\sigma^{-1} + \omega)\Phi + Order(\parallel \xi \parallel^2),
$$

which is given by the relation between the efficient level of output and the natural rate of output in terms of one by equation (34). Note that the same is true for x^* when Φ is positive and $Order(\parallel \xi \parallel)$. This expresses that the percentage difference

[19] By assuming a proper proportional tax on sales τ as

$$
Z(Y_t^n, Y_t^n, \overline{R}; \nu_t) = \frac{\theta - 1}{\theta}(1 - \tau) = \left[\frac{\epsilon}{\epsilon - 1}\right]\left[1 + \gamma^R\overline{R}_R\right]^n\left[1 + \gamma^S\overline{R}_S\right]^{1-n}(1 - \Phi),
$$

we can induce $\Phi = 0$ as in Rotemberg and Woodford (1997).

between Y_t^n and Y_t^* is independent from shocks in the first-order approximation. Here, we have:

$$var_h \ln l_t(h) = 0,$$

$$var_h \ln l_t(\overline{h}) = 0,$$

since we assume a flexible loan setting. Then, equation (35) is transformed into:

$$\begin{aligned} UT_t \simeq & -\frac{1}{2}\overline{Y}U_c \left[\begin{array}{c} \epsilon_{mc}(x_t - x^*)^2 + \eta_\pi var_f \ln p_t(f) \\ +n(1-n)\frac{1+\nu}{2}\left(\frac{1}{1+\nu}\right)^2 \left(\Theta \widehat{R}_{R,t} - \Theta^* \widehat{R}_{S,t}\right)^2 \\ +\overline{\upsilon}^{-1}\eta_i(\widehat{i}_t - i^*)^2 \end{array} \right] \\ & +t.i.p + Order(\quad \| \quad \xi \|^3). \end{aligned} \tag{36}$$

The remaining work needed to derive the approximated welfare function is to evaluate $var_h \ln p_t(f)$ and $var_h \ln(1 + r_t(h))$ in equation (36). Following Woodford (2003), we define:

$$\overline{P}_t \equiv E_f \ln p_t(f),$$

$$\triangle_t \equiv var_f \ln p_t(f).$$

Then, we can make the following relations:

$$\begin{aligned} \overline{P}_t - \overline{P}_{t-1} &= E_f \left[\ln p_t(f) - \overline{P}_{t-1}\right] \\ &= \alpha E_f \left[\ln p_{t-1}(f) - \overline{P}_{t-1}\right] + (1-\alpha)E_f \left[\ln p_t^*(f) - \overline{P}_{t-1}\right] \\ &= (1-\alpha)E_f \left[\ln p_t^*(f) - \overline{P}_{t-1}\right], \end{aligned} \tag{37}$$

and we also have:

$$\begin{aligned}
\triangle_t &= var_f\left[\ln p_t(f) - \overline{P}_{t-1}\right] \\
&= E_f\left\{\left[\ln p_t(f) - \overline{P}_{t-1}\right]^2\right\} - (E_f \ln p_t(f) - \overline{P}_{t-1})^2 \\
&= \alpha E_f\left\{\left[\ln p_{t-1}(f) - \overline{P}_{t-1}\right]^2\right\} + (1-\alpha)E_f\left\{\left[\ln p_t^*(f) - \overline{P}_{t-1}\right]^2\right\} - (\overline{P}_t - \overline{P}_{t-1})^2 \\
&= \alpha\triangle_{t-1} + (1-\alpha)E_f\left\{\left[\ln p_t^*(f) - \overline{P}_{t-1}\right]^2\right\} - (\overline{P}_t - \overline{P}_{t-1})^2 \\
&= \alpha\triangle_{t-1} + (1-\alpha)(var_f(\ln p_t^*(f) - \overline{P}_{t-1}) + \left\{E_f\left[\ln p_t^*(f) - \overline{P}_{t-1}\right]\right\}^2) - (\overline{P}_t - \overline{P}_{t-1})^2 \\
&= \alpha\triangle_{t-1} + \frac{\alpha}{1-\alpha}(\overline{P}_t - \overline{P}_{t-1}), \qquad (38)
\end{aligned}$$

where we use equation (37) and $p_t^*(f)$ is the optimal price setting by the agent f following the Calvo (1983) - Yun (1992) framework. We note that all project groups re-set the same price at time t when they are selected to change prices, because the unit marginal cost of production is the same for all project groups. In addition, we have the following relation that relates $\overline{P}_t$ with P_t:

$$\overline{P}_t = \ln P_t + Order(\parallel \xi \parallel^2),$$

where $Order(\parallel \xi \parallel^2)$ is in order terms higher than the first-order approximation. Here we make use of the definition of the price aggregator $P_t \equiv \left[\int_0^1 p_t(f)^{1-\theta}df\right]^{\frac{1}{1-\theta}}$. Equation (38) can then be transformed as:

$$\triangle_t = \alpha\triangle_{t-1} + \frac{\alpha}{1-\alpha}\pi_t, \qquad (39)$$

where $\pi_t \equiv \ln\frac{P_t}{P_{t-1}}$. From equation (39), we have:

$$\triangle_t = \alpha^{t+1}\triangle_{-1} + \sum_{s=0}^{t}\alpha^{t-s}\left(\frac{\alpha}{1-\alpha}\right)\pi_s^2,$$

and so,

$$\sum_{t=0}^{\infty}\beta^t\triangle_t = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2 + t.i.p + Order(\parallel \xi \parallel^3).$$

Then, equation (36) can finally be transformed as:

$$\sum_{t=0}^{\infty}\beta^t UT_t \simeq -\Lambda\sum_{t=0}^{\infty}\beta^t \begin{pmatrix} \lambda_\pi \pi_t^2 + \lambda_x (x_t - x^*)^2 + \lambda_i (\widehat{i}_t - i^*)^2 \\ +\lambda_{RS}^* \left(\Theta \widehat{R}_{R,t} - \Theta^* \widehat{R}_{S,t}\right)^2 \end{pmatrix},$$

where $\Lambda \equiv \frac{1}{2}\overline{Y}u_c$, $\lambda_\pi \equiv \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\theta(1+\omega_p\theta)$, $\lambda_x \equiv (\sigma^{-1}+\omega)$, $\lambda_{RS}^* \equiv n(1-n)\frac{1}{(1+\nu)}$, and $\lambda_i \equiv \overline{v}^{-1}\eta_i$. When $n = \frac{1}{2}$, we have:

$$\sum_{t=0}^{\infty}\beta^t UT_t \simeq -\Lambda\sum_{t=0}^{\infty}\beta^t \begin{pmatrix} \lambda_\pi \pi_t^2 + \lambda_x (x_t - x^*)^2 + \lambda_i (\widehat{i}_t - i^*)^2 \\ +\lambda_{RS} \left(\Theta_1 \widehat{R}_{R,t} - \Theta_2 \widehat{R}_{S,t}\right)^2 \end{pmatrix},$$

where $\lambda_{RS} = \frac{1}{(1+\nu)\chi^2}$.

B.2 Under a sticky loan rate

When one of the loan rate curves, in particular the risky rate curve, is determined in a sticky loan rate setting, the approximated loss function is given by:

$$\sum_{t=0}^{\infty}\beta^t UT_t \simeq -\Lambda\sum_{t=0}^{\infty}\beta^t \begin{pmatrix} \lambda_\pi \pi_t^2 + \lambda_x (x_t - x^*)^2 + \lambda_i (\widehat{i}_t - i^*)^2 \\ +\lambda_R (\widehat{R}_{R,t} - \widehat{R}_{R,t-1})^2 + \lambda_{RS} \left(\Theta_1 \widehat{R}_{R,t} - \Theta_2 \widehat{R}_{S,t}\right)^2 \end{pmatrix},$$

where $\lambda_R \equiv \frac{1}{2}\phi_h^{-1}\frac{\gamma^R\left(1+\overline{R}_R\right)}{1+\gamma^R\overline{R}_R}\frac{\epsilon}{(1+\nu\epsilon)}\frac{\varphi^R}{(1-\varphi^R)(1-\varphi^R\beta)}$. φ^R defines the probability of re-setting the loan rate, implying a sticky loan rate, following Calvo (1983) - Yun (1996).