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Abstract

The purpose of this paper is to study whether trade frictions in durable goods markets help account for the patterns of household consumption expenditures observed in the Consumer Expenditure Survey (CEX), namely that the response of durable goods expenditures to income shocks is 78 percent larger than that of nondurable goods and the variance of the idiosyncratic part of log durable goods expenditures is four times as high as that of log nondurable goods expenditures. To do so, I develop a model with a continuum of households that purchase durable as well as nondurable goods. The key assumption is that durable goods cannot be rented or sold after purchase. By comparing stationary distributions of the model with and without trade frictions, I find that trade frictions are crucial in accounting for the expenditure patterns observed in the data.

Keywords: Durable Goods; Irreversibility; Consumption Insurance
JEL classification: E21

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1 Introduction

Because perfect risk sharing has been statistically rejected in much of the literature, economists have been trying to understand the causes of imperfect risk sharing and the extent to which households can insure their income risk.[1] Although this recent work has produced many insights, most of this work abstracts from all frictions in goods markets to simplify the analysis. By contrast, I quantitatively examine the effect of trade frictions in durable goods markets on household consumption smoothing in this paper.

Using data from the *Consumer Expenditure Survey* (CEX), I document that the elasticity of 'small' durable goods expenditures, which excludes expenditures on vehicles and houses, with respect to idiosyncratic income is 78 percent higher than that of nondurable goods expenditures.[2] Furthermore, the cross-sectional variance of the idiosyncratic part of log expenditures on small durable goods is four times as high as that of log nondurable goods expenditures. I show that taking into account trade frictions in small durable goods markets, modeled as irreversibility constraints, is crucial in accounting for these patterns of household consumption expenditures.

To examine the empirical relevance of trade frictions in durable goods markets, this paper compares the stationary distributions of three models to the data. The three models differ in the degree of trade frictions in financial and durable goods markets. The first model is a variant of the Huggett (1993) model with durable goods. I assume that households cannot resell their durable goods, or that trade frictions are large enough to close all secondary markets. I call this model the *Incomplete Markets with Durable Goods* (IMD) model. The second model has no friction in secondary

[1] See, among others, Nelson (1994), Attanasio and Davis (1996), Hayashi et al. (1996), Dynarski and Gruber (1997), and Gervais and Klein (2006) for tests of perfect risk sharing. Krueger and Perri (2006) document that consumption inequality increased less than income inequality over the 1980–2003 period in the U.S. The authors note that a higher income risk endogenously relaxes financial constraints households face in the model with limited commitment, which can, at least qualitatively, account for the empirical evidence. Blundell et al. (2008) argue that changes in the persistence of income shocks can explain the evolution of consumption inequality. Incorporating changes in the U.S. wage structure in detail, Heathcote et al. (2008) show that their overlapping-generations model with exogenous borrowing constraints and endogenous labor supply can replicate the evolution of the cross-sectional distributions of wages, hours worked, earnings, and consumption.

[2] Small durable goods include apparel, household furnishings and equipment, medical equipment, televisions, radios, sound equipment, sporting goods, games, toys, film and photographic equipment, and books, which account for about 16% of total expenditure in the CEX.

durable goods trades, as opposed to the IMD model. Because this model does not have trade frictions in durable goods markets, it can be rewritten as a model with rental markets. For this reason, I call this model the *Incomplete Markets with Durable Goods Rental* (IMDR) model. The third model differs from the IMDR model in that it allows durable goods to be used as collateral for borrowing. With the addition of collateralized borrowing opportunities, the third model features the same equilibrium consumption allocation as that in a model with exogenous borrowing constraints and two nondurable goods. I call this model the *Standard Incomplete Markets* (SIM) model.

Before examining the quantitative implications of the three models, I study the qualitative properties of household expenditure allocations in these models. In the SIM model, regardless of whether or not collateral constraints bind, households keep the ratio of nondurable and durable goods consumption constant. In the IMDR model, when households who are currently liquidity constrained receive an adverse income shock, they reduce their expenditures on durable goods to maintain a constant level of nondurable goods.[3] Hence, the response of durable goods expenditures to income changes tends to be larger in the IMDR model than in the SIM model. In the IMD model, irreversibility of durable goods expenditures dampens the response of durable goods expenditures to income changes for the following two reasons. First, households cannot sell durable goods even if they would like to do so ex post. Second, when households receive positive income shocks, they do not increase their durable expenditures as much as they would without the irreversibility constraints, because they cannot sell durable goods in the future. Therefore, the IMD model tends to feature less adjustment in durable goods expenditures than the SIM and IMDR models.

The next step is to study whether the trade frictions are quantitatively important in accounting for the empirical facts described above. To answer this question, I calibrate the above three models and compare the stationary distributions of the models to the CEX data. Results show that the models without trade frictions in durable goods markets (the IMDR and SIM models) generate an elasticity of durable goods expenditures with respect to income and the variance of log durable goods expenditures more than 10 times as high as those from the CEX data. Although

[3] Browning and Crossley (2004) find empirical evidence for this behavior, using data on food and clothing expenditures from the Canadian Out of Employment Panel Survey.

the IMD model still overstates the elasticity of durable goods expenditures, it closely matches the empirical estimates of the variance of log durable and nondurable goods expenditures, generating the variance of log durable goods expenditures that is only 9.5% percent higher than the estimate from the CEX data.

I conclude this section by reviewing related articles. Browning and Crossley (2004) examine the effects of cuts in the unemployment insurance benefits on household food, clothing, and total expenditures using data from the Canadian Out of Employment Panel Survey. The authors find that the effect of marginal dollars of benefit on clothing expenditures is twice as large in absolute terms (dollars) as the effect on food expenditures, despite the fact that the budget share of food expenditures is much larger than that of clothing expenditures in their sample. Using the data from the CEX, I also find that the response of small durable goods expenditures to income changes is larger than that of nondurable goods expenditures. In contrast to Browning and Crossley (2004), however, I compare steady states of the models with and without trade frictions in durable goods markets with the empirical evidence to quantitatively examine the implications of trade frictions households face. Luengo-Prado (2006) examines a model with durable and nondurable goods that nests the IMDR and SIM models as special cases. The author mainly considers houses and vehicles as durable goods and thus models collateral constraints, down payment requirements, and adjustment costs. The author finds that the down payment requirements and adjustment costs help account for both excess smoothness and excess sensitivity observed in the US aggregate data. Unlike Luengo-Prado (2006), I focus my attention on small durable goods, excluding houses and vehicles, and examine the empirical relevance of trade frictions modeled by irreversibility constraints on durable goods expenditures.

The rest of the paper is organized as follows. Section 2 presents empirical results. Section 3 describes the environment. Section 4 compares the three models in terms of the first order conditions of the households' problem. Section 5 explains the benchmark parameterization and reports the main findings. Section 6 concludes. The equivalence between models with no friction in durable goods markets and models with rental markets for durable goods is formally established in the Appendix.

2 Empirical Evidence

In this section, I first describe the Consumer Expenditure Survey (CEX). Then I present empirical evidence from the CEX data on the elasticity of nondurable and durable goods expenditures with respect to the idiosyncratic part of household income as well as the cross-sectional variance of the logarithm of the two types of expenditures.

2.1 The Consumer Expenditure Survey

The Consumer Expenditure Survey is a unique survey in the United States that provides detailed information on both expenditure and income at the household level. In the survey, each household is interviewed once every three months over five consecutive quarters. In the second through fifth interviews, households report their expenditures for the last three months to the time of each interview. Income information is collected in the second and fifth interviews and refers to the last 12 months from the time of the interview.

I use the CEX data for 1980 to 2001. My sample consists of households whose head is at least 21 years of age but no older than 65. To ensure data quality for income and expenditure, I reject observations if a household has been classified as an incomplete income reporter, has missed interviews, has not reported characteristics necessary for my analysis, such as gender, race, and education, or reported inconsistent characteristics across interviews, has reported zero food expenditure or only food expenditures, has reported non-positive earnings or income, has reported household head's wage less than half the minimum wage, or has reported negative medical expenditures. Table 10 in the Appendix reports the step-by-step sample selection.

2.1.1 Expenditure Variables

The category of nondurable goods includes food at home, food away from home, alcoholic beverages, household operations, rental of house furnishings, utilities, apparel services, rental of educational books, fees and admissions, vehicle-related expenses such as rental, licenses, insurance, and gasoline and motor oil, public transportation, medical services, prescription drugs, personal care products and services, newspapers and magazines, and tobacco and smoking supplies. Expenditures in this category account for about 50% of total expenditures. The category of durable goods includes apparel, household furnishings and equipment, medical equipments, televisions, radios, sound equipment, sporting goods, games, toys, film and photographic equipment, and books, which account for about 16% of total expenditures in the CEX data. I deflate expenditure data by detailed CPI data with the base years of 1982–1984 and by an adult equivalence scale taken to be the square root of family size.

Housing expenses, such as mortgage interest payments and property taxes and expenses of vehicle purchases are not included in the above categories because houses and vehicles are very different from the durable goods listed above, especially in lumpiness and development of secondary markets. Because the computational burden would become overwhelming if these goods were additionally incorporated, I focus my attention on consumption smoothing with regards to other goods in this paper. To treat home owners and renters symmetrically, the rent paid by renters is also excluded from nondurable goods. Tuition, health insurance, and life insurance are not included because they are not expenditures for consumption.

2.1.2 Income Variable

My income measure is after-tax income plus transfers. This income concept is defined as the sum of labor earnings and transfers, minus total taxes paid, social security contributions, and retirement contributions. Labor earnings consist of wages and salaries and a fraction (0.864) of self-employment income.[4] Transfers consist of private and public transfers, such as alimony payments received, social security benefits, unemployment compensation, public assistance, and welfare payments. I deflate income data by the CPI for all items for the relevant 12 months and by the square root of the family size.

[4]The fraction of self-employment income that is considered labor income is taken from Díaz-Giménez et al. (1997).

2.2 Empirical Results

To characterize how idiosyncratic variation in income translates into idiosyncratic variation in nondurable and durable goods expenditures at the household level, I present two statistics using the CEX data. The first statistic is the income elasticity of both types of expenditures. The second is the variance of logarithms. To extract the idiosyncratic component of the expenditure and income variables, I first regress the logarithm of a given variable on a constant, the quadratic of the household head's age, household size, the number of earners in the household, dummy variables for region of residence, household head's gender, and household head's and spouse's (if present) race, education, and occupation. Let $c_{i,t}$, $x_{i,t}$ and $y_{i,t}$ denote nondurable goods expenditures, durable goods expenditures, and income of household i in period t, respectively. Then the regression equations are as follows:[5]

$$\begin{aligned} \ln c_{i,t} &= \beta^c_{0,t} + z'_{i,t}\beta^c_{1,t} + u^c_{i,t} \\ \ln x_{i,t} &= \beta^x_{0,t} + z'_{i,t}\beta^x_{1,t} + u^x_{i,t} \\ \ln y_{i,t} &= \beta^y_{0,t} + z'_{i,t}\beta^y_{1,t} + u^y_{i,t}, \end{aligned}$$

where $z_{i,t}$ represents a vector of the observable characteristics of household i in period t and $u^c_{i,t}$, $u^x_{i,t}$, and $u^y_{i,t}$ are residuals. I take the residuals as a measure of the idiosyncratic component of the given variable.

To compute the elasticity of nondurable and durable expenditures with respect to the idiosyncratic component of household income, I run the following regressions:

$$\begin{aligned} \triangle u^c_{i,t} &= \alpha^c \triangle u^y_{i,t} + \epsilon_{i,t}, \\ \triangle u^x_{i,t} &= \alpha^x \triangle u^y_{i,t} + \epsilon_{i,t}, \end{aligned}$$

where $\triangle$ is a difference operator: for example, $\triangle u^c_{i,t} = u^c_{i,t} - u^c_{i,t-1}$. For this exercise, given the survey structure of the CEX described above, I use annual income data collected at the second and fifth interview which overlap for three months. As for expenditure, I use quarterly expenditure data collected at the second and fifth interviews. Regression coefficients α^c and α^x represent the income elasticity of nondurable and durable goods expenditures, respectively.

[5] I run the cross-sectional regressions separately year by year, allowing the coefficients to vary over time.

Table 1 presents the estimation results. The estimate of α^c is 0.0711, while that of α^x is 0.1268. Note that α^x is 78% higher than α^c. The OLS estimates of α^c and α^x are, however, likely to be biased toward zero because income growth is likely to be measured with error.[6] To address this issue, I include (classical) measurement error in the simulation analysis presented in Section 5.2 to evaluate the model's ability to replicate the empirical evidence. Note that α^c is significantly greater than zero even with the possible downward bias, which strongly suggests that households are not fully insured against idiosyncratic income risk.

The last three columns in Table 1 report the cross-sectional variance of the idiosyncratic component of log income, nondurable and durable goods expenditures. For this exercise, I use income data reported in the fifth interview and annual nondurable and durable goods expenditures, summing up quarterly expenditures reported in the second through fifth interviews. Note that the annual expenditure data refer to exactly the same period as the income reported in the fifth interview for each observation. First, one notices that the variance of the idiosyncratic part of log nondurable goods expenditures is about half the variance of the idiosyncratic part of log income. Second, the variance of the idiosyncratic part of log durable goods expenditures is four times as high as that of log nondurable goods expenditures. Furthermore note that the variance of log durable goods expenditures is twice as high as that of log income.

3 The Model

3.1 The Environment

There is a continuum of households with identical preferences. There are two types of goods, one is nondurable and the other is durable. Let x_t denote expenditures on

[6] In the literature, several studies address the issue of measurement error in estimating the response of consumption to income changes using the CEX data. Dynarski and Gruber (1997) use an instrumental variable (IV) approach with an alternative income variable available in the CEX as an instrument. Gervais and Klein (2006) note that measurement error arises in the CEX, because two income reports overlap for three months due to the survey design. The authors argue that the instrumental variable used in Dynarski and Gruber (1997) is invalid for this type of measurement error and the IV estimator tends to overstate the consumption response. The authors propose a projection-based estimation and show that their estimate lies between the OLS and IV estimates.

Table 1: Patterns of Nondurable and Durable Goods Expenditures

α^c	α^x	$Var(u^y_{i,t})$	$Var(u^c_{i,t})$	$Var(u^x_{i,t})$
0.0711	0.1268	0.2558	0.1286	0.5190
(0.0046)	(0.0117)	(0.0047)	(0.0017)	(0.0090)

Notes. Columns labelled α_c and α_x report the OLS estimates of the elasticity of nondurable and durable goods expenditures with respect to income. $Var(u^y_{i,t})$, $Var(u^c_{i,t})$, and $Var(u^x_{i,t})$ are the average of the cross-sectional variance of the idiosyncratic component of the given variable over the 1980–2001 period. Standard errors are in parentheses. Standard errors of the variances are computed by bootstrap with 100 repetitions. (Source: CEX)

durable goods in period t and k_t the stock of durable goods in the beginning of period t. Households make durable goods expenditures in the beginning of each period. Then, the stock of durable goods evolves as follows: $k_{t+1} = (1-\delta)(k_t + x_t)$, where δ is the depreciation rate. Let d_t denote the consumption services of durable goods in period t. The consumption services of durable goods in period t are measured by the sum of stocks plus expenditures: that is, $d_t = k_t + x_t$. Let c_t denote expenditures on nondurable goods in period t. The consumption services of nondurable goods are simply measured by c_t. Households obtain utility from c_t and d_t. Household preferences are represented by the following utility function over $\{c_t, d_t\}_t$:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t, d_t)\right],$$

where β is the discount factor and E_0 is the expectation operator conditional on information available in period 0. The period utility function is as follows:

$$u(c,d) = \left(\frac{1}{1-\sigma}\right)\left(\xi^{\frac{1}{\theta}}c^{\frac{\theta-1}{\theta}} + (1-\xi)^{\frac{1}{\theta}}d^{\frac{\theta-1}{\theta}}\right)^{\frac{(1-\sigma)\theta}{\theta-1}}, \tag{1}$$

where σ is the degree of relative risk aversion, θ is the elasticity of substitution between nondurable and durable goods and ξ governs the ratio of nondurable goods to consumption services of durable goods. Each household faces a stochastic endowment process $\{y_t\}_t$. The endowment process is a stationary Markov chain with finite support Y, transition probabilities $\pi(y'|y)$, and unique invariant measure Π.

3.2 Market Structures

This section describes the market structure of the three models whose quantitative properties I study in the following section. In all three models, the only financial assets that households can trade are risk-free bonds denoted a_t. Let r denote the interest rate of the risk-free bond.

In the first model, households cannot sell their durable goods. That is, $x_t \geq 0$. In addition, households face borrowing constraints of the form $a_{t+1} \geq -\overline{a}$, where $\overline{a}$ is exogenously given. The household problem, in recursive form, is as follows:

$$V(s) = \max_{c,x,a'} \left\{ u(c,d) + \beta \sum_{y'} \pi(y'|y) V(s') \right\}$$

subject to

$$c + x + a' \leq y + (1+r)a, \tag{2}$$

$$a' \geq -\overline{a}, \tag{3}$$

$$x \geq 0, \tag{4}$$

$$c \geq 0, \tag{5}$$

$$d = k + x \geq 0, \tag{6}$$

where variables with a prime denote the value of the corresponding variable in the next period and $s = (y, a, k)$. This is the IMD model.

The second model differs from the IMD model in that this model has no frictions in durable goods markets. Households can buy and sell durable goods without any additional restrictions. Thus, in this model, households do not face constraint (4). The household problem is otherwise identical to the above. This is the IMDR model. The reason for this name is that this model can be represented by a model with rental markets for durable goods, as shown in the Appendix.

The third model has a different financial market structure from the above. In addition to non-collateralized borrowing with the limit $\overline{a}$, households are now allowed to borrow using durable goods as collateral. Thus, instead of the constraint (3), households face the following constraint:

$$a' \geq -\overline{a} - \frac{k'}{1+r}. \tag{7}$$

Households can buy and sell durable goods with no trade frictions as in the IMDR model. As shown in the Appendix, the household problem in this model can be rewritten as that of two nondurable goods (c and d) and exogenous borrowing constraints of the form in (3) in terms of net wealth defined by $a + k/(1 + r)$.[7] Because this model may be a choice if one abstracts from trade frictions in durable goods markets, it is of interest to compare this model with the other two models, although collateralized borrowing may be unlikely given the type of durable goods considered in this paper.[8] Because the third model can be represented by a model with two nondurable goods and simple exogenous borrowing constraints, I call it the SIM model. Table 2 summarizes the differences of the three market structures considered in this paper.

To study expenditure allocations of nondurable and small durable goods in tractable settings, I abstract from many markets, in particular (frictional) markets for houses and vehicles. Therefore, I consider an open-economy version of the above models with the risk-free interest rate exogenously fixed and compare the stationary distribution of the models with the CEX data in terms of the statistics presented in Section 2.2.

Table 2: Three Market Structures

Model	Irreversibility Constraint	Collateral Borrowing
IMD	✓	×
IMDR	×	×
SIM	×	✓

Notes. The check mark ✓ means that the corresponding model has the constraint, while × means that the corresponding model does not have the constraint.

4 Qualitative Analysis

In this section, I qualitatively examine the effects of borrowing and irreversibility constraints on optimal expenditure allocations over nondurable and durable goods.

[7] Luengo-Prado (2006) also notes this reformulation of the household problem with the change of variables.

[8] Note that households can borrow money from pawnshops in exchange for small durable goods such as jewelry.

I start with the SIM model where households do not face irreversibility constraints in durable goods markets but face collateral constraints (7) in financial markets. Let y^t denote a history up to period t. In this model, for all t and y^t, the first order condition reads

$$u_2(c_t(y^t), d_t(y^t)) = \gamma u_1(c_t(y^t), d_t(y^t))$$

where $\gamma = (r+\delta)/(1+r)$, which is typically referred to as the user cost of the durable good in the literature. Under the utility function specified in Section 3.1, this becomes $c/d = \gamma^\theta(\xi/(1-\xi))$. It is important to note that this equation holds regardless of whether or not the borrowing constraint binds.

In the SIM model, households can borrow up to the present value of the stock of durable goods, $k_{t+1}/(1+r)$, plus the exogenously determined borrowing constraint, $\overline{a}$. The presence of collateral borrowing opportunities allows households to separate the intertemporal allocation of resources and the intratemporal allocation of consumption services over nondurable and durable goods. When the collateral constraint binds, therefore, households adjust consumption of nondurable and durable goods proportionately, keeping the ratio of nondurable and durable goods consumption constant.

Next, consider the IMDR model, which corresponds to the above SIM model except that households can no longer use durable goods as collateral. The optimal intratemporal allocation rule must satisfy the following first order condition:

$$u_2(c_t(y^t), d_t(y^t)) = \gamma u_1(c_t(y^t), d_t(y^t)) + \left(\frac{1-\delta}{1+r}\right)\mu_t(y^t)$$

where μ is the Lagrange multiplier associated with the borrowing constraint. Unlike the SIM model, the borrowing constraints affect intratemporal allocations over c and d. When the borrowing constraint binds, $c/d > \gamma^\theta(\xi/(1-\xi))$. In the current environment, households face a binding borrowing constraint when their income is sufficiently low, given their asset holdings. Thus, the first order condition implies that when households receive adverse income shocks and cannot borrow any more, households cut back their expenditures on durable goods more than those on nondurable goods or even sell their durable goods to smooth nondurable consumption over time. Since this type of behavior is absent in the SIM model, the IMDR model tends to feature a larger response of durable goods expenditures to income changes than the SIM model.

Lastly, consider the IMD model where households face irreversibility constraints on durable goods expenditures in addition to borrowing constraints in financial markets. The first order condition becomes:

$$\begin{aligned} u_2(c_t(y^t), d_t(y^t)) &= \gamma u_1(c_t(y^t), d_t(y^t)) + \left(\frac{1-\delta}{1+r}\right) \mu_t(y^t) \\ &- \eta_t(y^t) + (1-\delta)\beta \sum_{y_{t+1} \in Y} \pi(y_{t+1}|y_t)\eta_{t+1}((y^t, y_{t+1})), \end{aligned}$$

where η is the Lagrange multiplier associated with the irreversibility constraints. In addition to the borrowing constraints, the irreversibility constraints affect the optimal intratemporal allocation rule. First note that the coefficient on η_t, the Lagrange multiplier associated with the irreversibility constraint in the current period, is negative. It is not surprising because households cannot sell their durable goods and consequently hold a relatively high level of durable goods stock when they receive large adverse income shocks, compared with the case without the irreversibility constraints.

On the other hand, if the irreversibility constraints bind in some state tomorrow and no other constraints bind, it holds that $c/d > \gamma^\theta(\xi/(1-\xi))$. Because neither the borrowing nor irreversibility constraints bind in the current period, households tend to have high income. Then the first order condition means that in the presence of irreversibility constraints, households do not increase durable goods expenditures as much as they would do without the irreversibility constraints.

The above analysis shows that the irreversibility constraints affect optimal expenditure allocations over nondurable and durable goods in two different ways. First, there is a direct effect: namely, that households cannot sell durable goods even if they want to do so. Second, there is also an indirect (precautionary) effect. When households receive positive income shocks, households do not increase durable expenditures as much as they would without the irreversibility constraints because they cannot sell durable goods in the future. To summarize, the IMD model features less adjustment in durable goods expenditures than the SIM and IMDR models.

5 Quantitative Analysis

In this section, I quantitatively examine whether the irreversibility constraints (trade frictions in durable goods markets) can help account for the empirical evidence on the patterns of nondurable and durable goods expenditures presented in Table 1. To do so, I first set parameter values in the IMD, IMDR, and SIM models so that the stationary distribution of each model captures some important aspects of the US economy over the 1980–2001 period. Second, I estimate the elasticity of nondurable and durable goods expenditures with respect to income and the variance of log nondurable and durable goods expenditures using simulated data.[9] Then I compare estimates from the simulated data with those from the CEX data.

5.1 Benchmark Parameterization

The model period is set to one year. I set all the parameter values except for the value of the discount factor β externally based on empirical evidence or the literature. Then for each model, I calibrate β so that the stationary distribution of the given model features a target ratio of average wealth to average income. As noted in Storesletten et al. (2004a), it is important that households have a realistic amount of wealth in the stationary distribution because it determines the amount of self-insurance, in addition to the adjustment of durable goods expenditures. For the benchmark, I choose the value of 2.6 for the wealth (including financial wealth and housing wealth) to income ratio, computed by Krueger and Perri (2006) using the CEX data for 1980–1981. In the rest of this section, I explain the benchmark values of the other parameters.

5.1.1 Preference and Technology

I set the degree of relative risk aversion σ to 2 and the elasticity of substitution between nondurable and durable goods θ to 1.[10] I set the risk-free rate, r, to 4

[9] I simulate a sample of 5000 agents for 300 periods starting from a degenerate distribution. To insulate the results from the effect of the initial conditions, I use the simulated data for the last 50 periods to compute the statistics. Then I repeat the simulation five times and report the averages of the given statistics over the five repetitions.

[10] There is no conclusive evidence on θ in the literature. See Fernández-Villaverde and Krueger (2002) for a discussion on this issue.

percent, which is the after-tax real return on physical capital found in McGrattan and Prescott (2003). The exogenous borrowing limit, $\overline{a}$, is set equal to 1, which corresponds to the average annual income.

I select the depreciation rate δ based on data from *Fixed Reproducible Tangible Wealth in the United States, 1925–1994*, Bils and Klenow (1998) and the CEX. More precisely, I compute δ by by dividing a constant, called a declining balance rate, by the weighted average of the expected life of component durable goods with expenditure shares as weights. I set the declining balance rate to 1.65, the value used by the Bureau of Economic Analysis for consumer durables that roughly correspond to the small durable goods considered in this paper. Table 11 shows the expected life and expenditure shares of durable goods. Data on the expected life of each durable good are from Table 2 of Bils and Klenow (1998) and Table C of *Fixed Reproducible Tangible Wealth in the United States, 1925-1994*. Data on expenditure share of each durable good are from the CEX. This exercise sets $\delta = 0.1839$.

For the benchmark calibration, I choose the share parameter ξ using the unconstrained intratemporal expenditure allocation rule over nondurable and durable goods. The first order condition is given by:

$$(1-\xi)c(s) = \left(\frac{r+\delta}{1+r}\right)^{\theta} \xi d(s).$$

Note that $d = k + x$. Thus:

$$E[c] = \left(\frac{r+\delta}{1+r}\right)\left(\frac{\xi}{1-\xi}\right)^{\theta}(E[k]+E[x]). \tag{8}$$

Meanwhile, the law of motion of stocks of durable goods and stationarity imply that $E[k] = ((1-\delta)/\delta)E[x]$. Substituting this into Equation (8) yields:

$$\frac{E[c]}{E[x]} = \frac{1}{\delta}\left(\frac{r+\delta}{1+r}\right)^{\theta}\left(\frac{\xi}{1-\xi}\right).$$

Solving this equation for ξ yields:

$$\xi = \frac{F_2}{F_1+F_2}.$$

where $F_1 = (1/\delta)((r+\delta)/(1+r))^{\theta}$ and $F_2 = E[c]/E[x]$. I use the sample average of deflated expenditures on nondurable and durable goods in the CEX as a proxy for $E[c]$ and $E[x]$. Given the benchmark values of r, δ, and θ, the formula yields $\xi = 0.7205$.

Table 3: Benchmark Parameter Values (Preference and Technology)

σ	Degree of relative risk aversion	2
θ	Substitution elasticity between nondurable and durable	1
r	Risk free rate	0.04
δ	Depreciation rate	0.1839
ξ	Weight on nondurable goods	0.7205

5.1.2 Income Process

I model the idiosyncratic part of the logarithm of household income as follows:

$$\begin{aligned} \ln y_{it} &= z_{it} + \varepsilon_{it} + \nu_{it}, \text{ with } \varepsilon_{it} \sim N(0, \sigma_\varepsilon^2), \text{ and } \nu_{it} \sim N(0, \sigma_\nu^2), \\ z_{it} &= \rho z_{it-1} + \eta_{it}, \text{ with } \eta_{it} \sim N(0, \sigma_\eta^2), \end{aligned}$$

where $\ln y_{it}$ represents the idiosyncratic part of the logarithm of the observed income of household i at t, z_{it} represents the persistent component of the observed idiosyncratic income, ε_{it} represents the transitory component, ν_{it} represents (classical) measurement error, and η_{it} represents an innovation to the persistent component. It is important to include the measurement error component, because income data in the CEX are likely to contain measurement error that biases the estimate of income risk as well as the expenditure elasticity with respect to income. I assume that ε_{it}, ν_{it}, and η_{it} are i.i.d., independent of each other, and serially uncorrelated.

I set autocorrelation ρ to 0.95, which is an intermediate value of the point estimates presented in Storesletten et al. (2004b).[11] I select the variance of the measurement error σ_ν^2, using validation studies on annual earnings for the Current Population Survey (CPS) and the Panel Study of Income Dynamics (PSID). Bound and Krueger (1994) document that measurement error explains 28 percent of the overall variance of the rate of growth of earnings in the CPS, while Bound et al. (1994) document that the corresponding value for the PSID data is 22 percent. Hence I set the benchmark level to an intermediate value of 25 percent, which implies $\sigma_\nu^2 = 0.0259$ for my CEX sample.[12] Given the values of ρ and σ_ν^2, I estimate σ_ε^2 and σ_z^2 by solving the following

[11] Storesletten et al. (2004b) estimate an earnings process with persistent and transitory components using data from the Panel Study of Income Dynamics for 1968-1993.

[12] Meghir and Pistaferri (2004) use the same value, 25 percent, for their estimation of an earnings

moment conditions with the empirical moments computed in the CEX data:

$$\begin{aligned}\overline{Cov(\ln y_{it}, \ln y_{it-1})} &= \rho\sigma_z^2 \\ \overline{Var(\ln y_{it})} &= \sigma_z^2 + \sigma_\varepsilon^2 + \sigma_\nu^2,\end{aligned}$$

where $\overline{Cov(\ln y_{it}, \ln y_{it-1})}$ and $\overline{Var(\ln y_{it})}$ are the average of $Cov(\ln y_{it}, \ln y_{it-1})$ and $Var(\ln y_{it})$ over the 1980–2001 period. The resulting point estimates are $\sigma_z^2 = 0.1733$ and $\sigma_\epsilon^2 = 0.0565$.

For the numerical analysis, I discretize the stochastic process described above. The transitory and measurement error components take two values with equal probability, with $\varepsilon_{it} \in \{-\sigma_\varepsilon, \sigma_\varepsilon\}$ and $\nu_{it} \in \{-\sigma_\nu, \sigma_\nu\}$. I discretize the AR(1) process for the persistent component to a seven-state Markov chain by the Tauchen and Hussey (1991) procedure, matching the empirical estimate of σ_z^2.

Table 4: Benchmark Parameter Values (Income Process)

ρ	Autocorrelation	0.95
σ_z^2	Variance of persistent component	0.1733
σ_ϵ^2	Variance of transitory component	0.0565
σ_ν^2	Variance of measurement error	0.0259

5.2 Quantitative Results

Table 5 reports the benchmark result.[13] The top panel contains estimates of the expenditure/consumption elasticity with respect to idiosyncratic income from the CEX data and those from the stationary distributions of the three models considered in this paper. With the benchmark parameter values, all three models slightly understates the elasticity of nondurable goods expenditures (α^c) and largely overstates that of durable goods expenditures (α^x) relative to the empirical evidence from the CEX

process with measurement error using the PSID data, referring to the same validation studies. However, the survey design of the CEX described in Section 2.1 may add some noise to the variance of income growth that is not present in other surveys. To partially address this issue, I conduct sensitivity analysis with alternative levels of the variance of the measurement error in Section 5.3.4.

[13] Since the CEX does not provide information on sales, I set durable goods expenditure x_{it} to zero if it takes on a negative value in simulated data to make the CEX and simulated data comparable.

Table 5: Comparison of Models and Data

	α^c	α^x	$\triangle\alpha^{xc}$	α^d
CEX	0.0711	0.1268	0.0557	n.a.
IMD	0.0624	0.6196	0.5572	0.0718
IMDR	0.0613	1.7219	1.6606	0.0784
SIM	0.0658	1.3281	1.2623	0.0660
	$Var(\ln c_{it})$	$Var(\ln x_{it})$	$\triangle Var^{xc}$	β
CEX	0.1286	0.5190	0.3904	n.a.
IMD	0.1283	0.5683	0.4400	0.9444
IMDR	0.1280	8.1109	7.9829	0.9445
SIM	0.1375	6.6394	6.5019	0.9456

Notes. α^c, α^x, and α^d represent the OLS estimate of the elasticity of nondurable goods expenditure with respect to income, that of durable goods expenditure, and that of durable goods consumption, respectively. The regression equation is described in Section 2.2. $\triangle\alpha^{xc} = \alpha^x - \alpha^c$. $\triangle Var^{xc} = Var(\ln x_{it}) - Var(\ln c_{it})$. For the CEX, columns $Var(\ln c_{it})$ and $Var(\ln x_{it})$ report $Var(u^c_{it})$ and $Var(u^x_{it})$, respectively. The discount factor β is calibrated to the wealth–income ratio of 2.6, given the other parameter values.

data. The elasticity of nondurable goods expenditures in the IMD model is 0.0624, whiich is 12 percent lower than the estimate from the CEX data. On the other hand, the elasticity of durable goods expenditures is estimated as 0.6196 in the IMD model, about five times higher than the empirical counterpart. Consequently, the difference between the elasticity of nondurable goods expenditures and that of durable goods expenditures is 10 times as high in the IMD model as that in the CEX data. However, compared with the IMDR and SIM models, one notices that the irreversibility constraints on durable goods expenditures in the IMD model substantially reduce the responsiveness of durable goods expenditures, bringing the model closer to the empirical evidence. The IMDR model generates an elasticity of durable goods expenditures with respect to idiosyncratic income 2.8 times higher than that in the IMD model and 13.6 times higher than the estimate from the CEX data. Note that, confirming the observation in the qualitative analysis, the IMDR model features higher α^x than the SIM model, because of the binding borrowing constraints.

With a larger adjustment of durable goods expenditures to income changes in the IMDR model, nondurable goods expenditure is slightly less responsive to income in the IMDR model than in the IMD model, while the consumption of durable goods is more responsive to income in the IMDR model than in the IMD model. The SIM model generates an elasticity of nondurable goods of 0.0658, which is slightly higher than those in the IMD and IMDR models.

The bottom panel of Table 5 reports the cross-sectional variance of nondurable and durable goods expenditures, $Var(\ln c_{it})$ and $Var(\ln x_{it})$, as well as the calibrated value of the discount factor β. In terms of cross-sectional variance, the IMD model matches the empirical evidence quite well, generating $Var(\ln x_{it}) - Var(\ln c_{it})$, only 12.7 percent higher than the CEX data. This observation seems inconsistent with the large response of durable goods expenditures to income changes in the IMD model relative to the CEX data. It may suggest that either α^x is substantially more biased downward than α^c because of the use of quarterly expenditures instead of annual expenditures in the estimation using the CEX data[14], or the data on annual durable goods expenditures contain larger measurement errors than annual nondurable goods expenditures. However, the IMDR and SIM models still largely overstate the variance of durable goods expenditures relative to the empirical evidence, indicating that the irreversibility constraints are quantitatively important. The calibrated subjective discount factor is around 0.1 percent higher in the SIM model than in the IMD and IMDR models because of the additional collateral borrowing opportunities available to households in the SIM model.

5.3 Sensitivity Analysis

To better understand the quantitative results, I conduct sensitivity analyses with respect to the elasticity of substitution between nondurable and durable goods θ, the rate of depreciation of durable goods δ, the exogenous borrowing limit $\overline{a}$, and the variance of measurement error σ_ν^2. In each case, I recalibrate the subjective discount factor β to the wealth to income ratio of 2.6.

[14]Because I use quarterly expenditures to measure the growth of annual expenditures in the CEX data, the OLS estimates of the expenditure elasticity with respect to income are likely to be biased. In particular, it is likely that my measure does not capture the growth of annual durable goods expenditures as well as that of annual nondurable goods expenditures because purchases of durable goods tend to occur infrequently. It may result in a larger downward bias in the OLS estimate of the elasticity of durable goods expenditures with respect to income than that of nondurable goods expenditures.

Table 6: Change in Substitution Elasticity: θ

	$\theta = 0.5$				$\theta = 1.5$			
Model	α^c	α^x	α^d	β	α^c	α^x	α^d	β
IMD	0.0639	0.5682	0.0683	0.9443	0.0618	0.6417	0.0733	0.9445
IMDR	0.0634	1.3245	0.0723	0.9444	0.0601	1.9326	0.0821	0.9446
SIM	0.0657	1.2947	0.0658	0.9455	0.0657	1.3037	0.0659	0.9455

Notes. α^c, α^x, and α^d represent the OLS estimate of the elasticity of nondurable goods expenditure with respect to income, that of durable goods expenditure, and that of durable goods consumption, respectively. The regression equation is described in Section 2.2. The discount factor β is calibrated to the wealth–income ratio of 2.6.

5.3.1 Alternative Elasticity of Substitution

With the utility function specified as (1), nondurable and durable goods become complements when θ is less than one, while they become substitutes when θ is greater than one.[15] Table 6 presents the expenditure/consumption elasticity with respect to idiosyncratic income (α^c, α^x, and α^d) and the calibrated value of the subjective discount factor β for $\theta = 0.5$ and $\theta = 1.5$. In addition to the discount factor, I also recalibrate ξ according to the benchmark calibration rule for this exercise. When $\theta = 0.5$, the elasticity of durable goods expenditure with respect to income (α^x) decreases in all the models, compared with the benchmark levels. However, note that α^x is still much larger than the empirical counterpart. As a result of the smaller adjustment of durable goods expenditures, the expenditures on nondurable goods become more responsive to income changes, while the consumption of durable goods become less responsive. It is intuitive, because households like to smooth out the ratio of nondurable and durable goods consumption more as these two goods become complements. The opposite is true when nondurable and durable goods are substitutes. Therefore, when $\theta = 1.5$, households adjust durable goods to income changes to a larger extent, compared with the benchmark case.

Table 7: Change in Depreciation Rate: δ

	$\delta = 0.1$				$\delta = 0.3$			
Model	α^c	α^x	α^d	β	α^c	α^x	α^d	β
IMD	0.0589	0.8037	0.0498	0.9468	0.0673	0.3590	0.0784	0.9430
IMDR	0.0512	4.4803	0.0629	0.9469	0.0658	0.4154	0.0787	0.9428
SIM	0.0561	3.4866	0.0561	0.9485	0.0686	0.3653	0.0692	0.9435

Notes. α^c, α^x, and α^d represent the OLS estimate of the elasticity of nondurable goods expenditure with respect to income, that of durable goods expenditure, and that of durable goods consumption, respectively. The regression equation is described in Section 2.2. The discount factor β is calibrated to the wealth–income ratio of 2.6.

5.3.2 Alternative Depreciation Rate

To examine the role of adjustment of durable goods expenditures for household consumption smoothing, it is helpful to estimate the expenditure/consumption elasticities with respect to income with alternative depreciation rates of durable goods. If the depreciation rate is low, households can cut back their expenditures on durable goods to a larger extent when they receive adverse income shocks, because the consumption services of durable goods last longer. Table 7 reports the expenditure/consumption elasticity with respect to income for $\delta = 0.1$ and $\delta = 0.3$. Note that, as in the sensitivity analysis with the substitution elasticity, I recalibrate ξ according to the benchmark calibration rule, as I change the depreciation rate.

The quantitative effects of the change in depreciation rates on expenditure allocations are substantial. When $\delta = 0.1$, which is lower than the benchmark level of 0.1839, the household's precautionary saving motive decreases, and thus the discount factor is calibrated to be 0.25 to 0.3 percent higher than the benchmark level. In the IMD model, the response of durable goods expenditures to income changes increases by 29.7 percent, while it increased by around 160 percent in the IMDR and SIM models. As a result of the larger adjustment of durable goods expenditures, the response of nondurable goods expenditures to income changes (α^c) becomes 5 percent lower than the benchmark level in the IMD model. In the IMDR and SIM models, the reduction of the response is larger, around 15 percent, because households do not face trade frictions in durable goods markets in these models.

[15] The utility function becomes Leontief as $\theta \to 0$, and it becomes linear as $\theta \to \infty$.

Table 8: Change in Borrowing Limit: $\overline{a}$

	$\overline{a} = 0.2$				$\overline{a} = 2.0$			
Model	α^c	α^x	α^d	β	α^c	α^x	α^d	β
IMD	0.0679	0.6866	0.0784	0.9417	0.0592	0.5487	0.0646	0.9467
IMDR	0.0618	1.7990	0.0827	0.9418	0.0550	1.3902	0.0669	0.9466
SIM	0.0700	1.4774	0.0698	0.9432	0.0592	1.1046	0.0590	0.9473

Notes. α^c, α^x, and α^d represent the OLS estimate of the elasticity of nondurable goods expenditure with respect to income, that of durable goods expenditure, and that of durable goods consumption, respectively. The regression equation is described in Section 2.2. The discount factor β is calibrated to the wealth–income ratio of 2.6.

When $\delta = 0.3$, α^c and α^d increases, α^x decreases, and β decreases below the benchmark. Quantitatively, one notices that the difference between the IMD and IMDR models (that is, the difference between the case with and without irreversibility constraints on durable goods expenditures) falls substantially, with α^x in the IMDR model only 15.7 percent higher than that in the IMD model.

5.3.3 Alternative Borrowing Limit

Table 8 reports results with alternative borrowing limits. Calibrating the exogenous borrowing limit to the proportion of agents with negative or zero wealth, Heathcote et al. (2008) find that the borrowing limit is 20 percent of mean annual individual after-tax earnings in their model, which corresponds to $\overline{a} = 0.2$. For an upper bound, I examine the case with $\overline{a} = 2$. With the tighter borrowing limit of $\overline{a} = 0.2$, the responses of nondurable and durable goods consumption to income changes (α^c and α^d) increase. Furthermore, the response of durable goods expenditure to income changes (α^x) increases in this case. Note that α^c in the IMD model is 9.9 percent higher than that in the IMDR model in this case, which is larger than the benchmark value of 1.8 percent. This observation indicates that trade frictions in durable goods markets matter more for household consumption smoothing when households face tighter constraints in financial markets. Because a household's precautionary saving motive is high in this case, the calibrated discount factor is about 0.3 percent lower than the benchmark level in each model.

When $\overline{a} = 2$, as expected, α^c, α^x, and α^d are all lower than the benchmark

Table 9: Change in Variance of Measurement Error: σ_ν^2 ($\kappa = \sigma_\nu^2/(\sigma_\varepsilon^2 + \sigma_\nu^2)$)

	$\kappa = 0$				$\kappa = 0.5$			
Model	α^c	α^x	α^d	β	α^c	α^x	α^d	β
IMD	0.1062	0.9698	0.1135	0.9438	0.0574	0.5392	0.0638	0.9447
IMDR	0.0965	2.2610	0.1193	0.9441	0.0547	1.3880	0.0673	0.9447
SIM	0.1034	1.8447	0.1042	0.9451	0.0591	1.1746	0.0594	0.9457

Notes. α^c, α^x, and α^d represent the OLS estimate of the elasticity of nondurable goods expenditure with respect to income, that of durable goods expenditure, and that of durable goods consumption, respectively. The regression equation is described in Section 2.2. The discount factor β is calibrated to the wealth–income ratio of 2.6.

values, while β is higher. Note that with large borrowing opportunities in financial markets, the quantitative impact of the irreversibility constraints on durable goods expenditures is smaller than that in the benchmark.

5.3.4 Alternative Variance of Measurement Error

In this section, I report the results for alternative levels of the variance of measurement error. For this exercise, I keep the total variance of the transitory component of the observed idiosyncratic income, $\sigma_\epsilon^2 + \sigma_\nu^2$, constant and change the fraction of the variance of measurement error out of the total variance of the transitory component, $\sigma_\nu^2/(\sigma_\varepsilon^2 + \sigma_\nu^2)$. Let κ denote the fraction, that is, $\kappa = \sigma_\nu^2/(\sigma_\varepsilon^2 + \sigma_\nu^2)$. With $\sigma_\nu^2 = 0.0259$, κ is 0.3143 in the benchmark. Table 9 reports results with $\kappa = 0$ (no measurement error) and $\kappa = 0.5$. First note that households face lower overall income risk when κ is high, which results in a higher discount factor β for $\kappa = 0.5$ compared with the case with $\kappa = 0$. Expenditure/consumption responses to income changes (α^c, α^x, and α^d) fall by half when κ changes from 0 to 0.5, because of the downward bias caused by the measurement error. However, one can confirm that the benchmark results in terms of a comparison between the IMD, IMDR, and SIM models are robust to the variation in the variance of measurement error considered in this section.

6 Conclusion

In this paper, I examined whether trade frictions in small durable goods markets, excluding houses and vehicles, help account for the patterns of household consumption expenditures observed in the CEX. As trade frictions, I considered an extreme case in which households cannot sell durable goods after purchase (or, expenditures on durable goods are irreversible). Empirical evidence from the CEX data shows that the elasticity of durable goods expenditures with respect to idiosyncratic income is 78 percent higher than that of nondurable goods expenditures, and the cross-sectional variance of the idiosyncratic part of log durable goods expenditures is four times as high as that of log nondurable goods expenditures. Both results are consistent with households adjusting durable goods expenditures to insulate their consumption of nondurable goods from idiosyncratic income risk.

Calibrating the stationary distributions of the models with and without trade frictions in durable goods markets, I found that irreversibility constraints on durable goods expenditures are quantitatively important accounting for the empirical evidence. The comparison between the case with and without the trade frictions in durable goods markets is robust to small variations in key model parameters including the elasticity of substitution between nondurable and durable goods, the borrowing limit, the depreciation rate of durable goods, and the variance of measurement error in idiosyncratic income, with relatively higher sensitivity with respect to the depreciation rate. The sensitivity analysis also showed that households adjust durable good purchases to income shocks to a larger extent when borrowing constraints become tighter or the consumption services of durable goods last longer. Hence the irreversibility constraints on durable goods expenditures have a larger effect on household consumption smoothing in those cases.

Finally, I discuss the limitations of this paper. First, no direct evidence on secondary transactions of small durable goods has been presented in this paper. It would be interesting to look into availability of data on this issue. Second, it is also a limitation of this paper that no other possible explanations such as adjustment costs are examined. To address these limitations, more research is required.

Appendix

Rental Markets for Durable Goods

This appendix formally establishes that a model with no frictions in durable goods trades (IMDR and SIM models) can be represented by a model with rental markets for durable goods. In particular, it is shown that the SIM model corresponds to the rental markets model with simple exogenous borrowing constraints in bond holdings. Because the particular rental markets model may be a choice if one abstracts from trade frictions in durable goods, it is of interest to compare the SIM model with the IMD model in addition to the IMDR model.

The environment of the rental markets model is the same as the one described in Section 3.1, except for the presence of a financial intermediary. Rental markets for durable goods are now introduced. It is assumed that households do not own durable goods but only rent them and the financial intermediary accumulates durable goods and rents them out to households. In addition to trading durable goods, the financial intermediary sells and buys risk-free bonds. Then the no arbitrage condition is as follows:

$$\gamma = \frac{r+\delta}{1+r}. \tag{9}$$

Let z denote the amount of risk-free bonds that households hold. Let $s = (y, z)$ denote the state of a household. Households have a stochastic endowment process as before. With their endowments, they purchase nondurable goods, rent durable goods, and trade risk-free bonds. Given the risk-free interest rate r and the rental price of durable goods γ, which is equal to $(r + \delta)/(1 + r)$ as described above, households solve the following problem:

$$V(s) \quad = \quad \max_{c,d,z'} \{u(c, d) + \beta \sum_{y'} \pi(y'|y)V(s')\},$$

subject to

$$c + \gamma d + z' \leq y + (1+r)z, \tag{10}$$

$$z' - \left(\frac{1-\delta}{1+r}\right) d \geq -\bar{a}, \tag{11}$$

$$c \geq 0, \tag{12}$$

$$d \geq 0, \tag{13}$$

$$\tag{14}$$

where

$$u(c,d) = \left(\frac{1}{1-\sigma}\right)\left(\xi^{\frac{1}{\theta}} c^{\frac{\theta-1}{\theta}} + (1-\xi)^{\frac{1}{\theta}} d^{\frac{\theta-1}{\theta}}\right)^{\frac{(1-\sigma)\theta}{\theta-1}},$$

Constraint (10) is the budget constraint and constraint (11) is the borrowing constraint, which makes this model equivalent to the IMDR model as will become clear in Proposition 1.

Proposition 1 *Given a risk-free interest rate r, optimal policies c and d for the household problem in the IMDR model coincide with those in the model with rental markets.*[16]

Proof. Note that without irreversibility constraints, households do not need to know the composition of their wealth. That is, $c(y,a,k) = c(y,\tilde{a},\tilde{k})$ and $d(y,a,k) = d(y,\tilde{a},\tilde{k})$ if $(1+r)a+k = (1+r)\tilde{a}+\tilde{k}$. Thus, one can write $c(y,a,k) = c(y,(1+r)a+k)$ and $d(y,a,k) = d(y,(1+r)a+k)$.

Substituting (6) into (2) yields:

$$\begin{aligned} c + d + a' &= y + (1+r)a + k, \\ \Leftrightarrow c + \left(\frac{r+\delta}{1+r}\right) d + \left(\frac{1-\delta}{1+r}\right) d + a' &= y + (1+r)a + k, \end{aligned}$$

Now define $z' = a' + ((1-\delta)/(1+r))d = a' + (1/(1+r))k'$. Of course, $z = a + (1/(1+r))k$. Then:

$$c + \left(\frac{r+\delta}{1+r}\right) d + z' = y + (1+r)z.$$

[16]This proposition immediately implies that the stationary equilibrium of the IMDR model, defined in a standard way, and that of the corresponding rental-market model share the same optimal policies c and d and the same equilibrium interest rate r.

Because $\gamma = (r+\delta)/(1+r)$ in the model with rental markets, this is exactly the same as (10). The definition of z' and (3) in the IMDR model imply (11) in the model with rental markets. Then the household problems in these two models share the same constraint set. Because household preferences are the same in both models, it implies that, under the same interest rate r, optimal policies c and d are the same. $\square$

Corollary 1 *Replace constraint (11) with $z' \geq -\bar{a}$. Then the household problem in the model with rental markets corresponds to that in the SIM model.*

Proposition 1 shows that the IMDR model itself is not represented by a rental markets model with simple borrowing constraints. To establish the correspondence, the rental-markets model needs to have constraint (11), which requires a 'down payment' for consumption services of durable goods. On the other hand, if the rental-markets model has simple borrowing constraints, then the corresponding model, which is the SIM model, must have collateralized borrowing constraints.

Table 10: Sample Selection

	Observations deleted	Remaining observations
Original data set (1980–2001)		167919
Aged less than 21 or more than 64	74511	93408
Incomplete income respondents	20621	72787
Zero food expenditure	256	72531
Only food expenditure	41	72490
Missing interviews	25745	46745
Inconsistent characteristics	3027	43718
Missing main characteristics	3752	39966
Change in marital status	464	39502
Non-positive, missing annual income	732	38770
Non-positive, missing labor earnings	3136	35634
Non-positive weeks worked	53	35131
Head's wage less than half min wage	2763	32368
Negative medical care expenditures	2299	30069

Main characteristics include region of residence and head's and spouse's (if present) sex, race, education, and occupation.

Table 11: Expected Service Life and Expenditure Share

	Expected life (in years)	Expenditure share
Men's hosiery	1.7	0.0022
Men's suits and coats	4.1	0.0243
Men's shirts and nightwear	2.7	0.0168
Men's underwear	2.2	0.0032
Men's pants	2.7	0.0221
Men's other apparel	2.68	0.0078
Boys' hosiery	1.7	0.0008
Boys' suits and coats	4.1	0.0042
Boys' shirts and nightwear	2.7	0.0055
Boys' underwear	2.2	0.0013
Boys' pants	2.7	0.0094
Boys' other apparel	2.68	0.0025
Women's hosiery	1	0.0086
Women's shirts and blouses	2.3	0.0167
Women's dresses and suits	4	0.0257
Women's coats	4.3	0.0210
Women's underwear	1.8	0.0108
Women's pants	2.7	0.0324
Women's other apparel	3.02	0.0112
Girls' hosiery	1	0.0010
Girls' shirts and blouses	2.3	0.0063
Girls' dresses and suits	4	0.0039
Girls' coats	4.3	0.0023
Girls' underwear	1.8	0.0022
Girls' other apparel	2.68	0.0032
Infants' apparel	1.675	0.0200
Footwear	2.5	0.0462
Jewelry	5.5	0.0212
Watches	15.5	0.0060
Apparel related products	10	0.0035
Carpet and rugs	11.1	0.0120
Textile house furnishings	4.2	0.1157

	Expected life (in years)	Expenditure share
Furniture	8.1	0.0050
Mattresses and springs	15	0.0162
Blinds and shades	10.9	0.0264
Luggage	17.5	0.0503
Glassware	10	0.0111
China	17.5	0.0089
Cookware	17.5	0.0196
Silverware	27.5	0.0826
lawn mowers	7.5	0.0084
Stoves and ovens	14.1	0.0027
Refrigerators and freezers	15	0.0020
Washers and dryers	11	0.0013
Portable heaters	11.3	0.0028
Vacuums cleaners	9.5	0.0037
Lamps	16.7	0.0012
Clocks	15.5	0.0091
Telephones	7.1	0.0084
Computers	9	0.0096
Other household appliances	10	0.0086
TV and sound equipment	11.9	0.0051
Musical instruments	13	0.0035
CD and tapes	5	0.0027
Sporting goods and equipment	10	0.0013
Games and toys	5	0.0042
Film and photographic equipment	6.7	0.0204
Maintenance and repairs (vehicles)	3	0.0173
Eyeglasses and contacts	10	0.1664
Other medical equipments	6	0.0140
Books	11	0.0018
Educational books	11	0.0156

Notes: Expected life of men's and women's other apparel is the average of that of hosiery, suits, shirts, underwear, and pants. Expected life of boys' and girls' apparel is set to that of men's and women's apparel, respectively. Expected life of women's pants is set to that of men's pants. Expected life of infants' apparel is the average of that of hosiery and underwear. Expected life of apparel related product is set to that of other household appliances. (Source: Table 2 of Bils and Klenow (1998) and the CEX.)

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