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Abstract

We examine the expectational stability (E-stability) of the rational expectations equilibrium (REE) in a simple New Keynesian model in which private agents engage in adaptive learning by referring to the central bank's forecast. In this environment, to satisfy the E-stability condition, the central bank must respond more strongly to the expected inflation rate than the so-called Taylor principle suggests. On the other hand, the central bank's strong reaction to the expected inflation rate raises the possibility of indeterminacy of the REE. In considering these problems, a robust policy is to respond to the current inflation rate to a certain degree.

Keywords: Adaptive Learning; E-stability; New Keynesian Model; Monetary Policy; Taylor principle

JEL classification: E52, D84

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1 Introduction

Since the development of adaptive learning in macroeconomics (Evans and Honkapohja [2001]), many studies have investigated the expectational stability (E-stability) conditions of the rational expectations equilibrium (REE) in various macroeconomic models. One of the important applications to monetary economics is Bullard and Mitra [2002]. They examine the E-stability condition in a simple class of the New Keynesian model, which consists of an IS equation, a New Keynesian Phillips curve (NKPC), and a Taylor-type monetary policy rule.[1] Their results indicate that the so-called *Taylor principle*, which requires the central bank to adjust the nominal interest rate by more than one-for-one with the inflation rate, corresponds to the E-stability condition under some versions of Taylor-type monetary policy rules, including a forward-looking rule that incorporates the expectations for the future inflation rate and output gap, which are assumed to be homogeneous between the central bank and private agents.[2]

Honkapohja and Mitra [2005] extend the analysis of Bullard and Mitra [2002] to introduce heterogeneous expectations between the central bank and private agents. They show that, even if the central bank and private agents initially have different expectations, the correspondence between the E-stability condition and the Taylor principle holds, as long as the learning algorithms used by these two agents are asymptotically identical. However, they further show that, if the difference of learning algorithms remains even in the long run, the Taylor principle does not generally correspond to the E-stability condition. Therefore, their analysis points out that the heterogeneity between the central bank and private agents is a key issue for determining the E-stability condition in a simple New Keynesian model.

However, we should note that the environments of these previous studies are still quite simple because the studies assume that the central bank and private agents independently (or simultaneously) engage in adaptive learning. In other words, the previous studies assume that there is no interaction in the learning process of the central bank and private agents. Of course, as Honkapohja and Mitra [2005] noted, this assumption is introduced as a natural benchmark.[3] However, the validity of this assumption is empirically arguable when we take account of possible interactions between the central bank and private agents. In this respect, Fujiwara [2005] provides empirical evidence that, in Japan's survey data, the central bank's forecast influences the forecast of private agents (not vice versa). Therefore, his results indicate that

[1] Evans and Honkapohja [2003a] review the studies of adaptive learning in New Keynesian models.

[2] The issue of stability under learning when the central bank introduces an interest-rate rule is originally raised by Howitt [1992] in an IS-LM model with a New Classical Phillips curve.

[3] Honkapohja and Mitra [2005] stated that, "We will focus on the situation in which both the private sector and the central bank use their own forecasts in their decision-making and the forecasts are not available to the other agents. Consequently, the forecasts have no strategic role. This case can be seen as a natural benchmark."

the central bank is the *leader* and private agents are the *followers* of expectation formations.[4]

In this study, we examine the E-stability of the REE in a simple New Keynesian model in which the central bank is the leader and private agents are the followers of expectation formations. This means that private agents engage in adaptive learning by referring to the central bank's forecast.[5] This kind of leader-follower relationship of adaptive learning has already been introduced by Granato, Guse, and Wong [2007] in the traditional "cobweb" model. However, their analysis investigates the heterogeneous expectations among private agents. In contrast, the distinctive feature of our study is that it investigates the heterogeneous expectations between the policymaker (namely, the central bank) and private agents.

Since we assume that private agents refer to the central bank's forecast, our study introduces the heterogeneity concerning the perceived law of motion (PLM) used by the central bank and private agents. However, as for the learning algorithm, we assume that both the central bank and private agents use the recursive least squares (RLS) with decreasing gain, which is the most standard algorithm in the literature.[6] In these respects, the environment of our study contrasts sharply with that of Honkapohja and Mitra [2005], which assumes that the PLMs are homogeneous and that the learning algorithms are heterogeneous.

The rest of this paper is organized as follows. In Section 2, we present our simple version of the New Keynesian model. We show that, if the central bank and private agents are independently learning, the E-stability condition corresponds to the Taylor principle, as reported by Bullard and Mitra [2002]. In Section 3, we examine the E-stability condition when private agents engage in adaptive learning by referring to the central bank's forecast. In Section 4, we investigate the relationship between the E-stability and the determinacy (uniqueness) of the REE. In Section 5, we examine the E-stability in the reverse situation in which the central bank engages in adaptive learning by referring to the forecast of private agents. In Section 6, we conclude our analysis.

[4] Fujiwara [2005] suggests, "In the learning context, it would be better to suppose that the central bank is a leader rather than a follower when analyzing monetary policy in Japan, since the results in this paper indicate that professional forecasters tend to learn from the central bank rather than to influence it (p. 261)."

[5] In this study, we restrict our attention to a Taylor-type simple monetary policy rule. Thus, we do not examine the E-stability under optimal monetary policy. The property of optimal monetary policy in the presence of adaptive learning is examined in Evans and Honkapohja [2003b, 2006].

[6] An alternative algorithm is RLS with constant gain, which is typically used to describe a situation in which agents take account of the possibility of structural changes (as is explained by Evans and Honkapohja [2001]). Honkapohja and Mitra [2005] introduce heterogeneous constant gains between the central bank and private agents. They show that, if the difference of constant gains remains in the long run, then it matters for the E-stability condition.

2 Framework

2.1 Model

We use a simple version of the New Keynesian model. Our model is simpler than that of Bullard and Mitra [2002] or Honkapohja and Mitra [2005], since we introduce a static version of the IS equation, in which the current output gap does not depend on the expectation for the future output gap. We choose this formulation because (i) we can analytically derive the E-stability condition and (ii) we can numerically obtain essentially the same results in an extended model which introduces a dynamic version of the IS equation.[7] Therefore, we consider that the current version of our model is useful to investigate the essence of our problem.

The static IS equation and the NKPC are given as follows:

$$x_t = -\sigma(r_t - r_t^n - E_t^P \pi_{t+1}), \tag{1}$$

$$\pi_t = \beta E_t^P \pi_{t+1} + \kappa x_t, \tag{2}$$

where x_t is the output gap, π_t is the inflation rate, r_t is the nominal interest rate, and r_t^n is the natural rate of real interest. Each variable is defined as the deviation from its steady state.[8] In particular, r_t is the deviation of the nominal interest rate from its steady-state level, which is consistent with zero inflation and steady-state output growth. E_t^P denotes private agents' subjective (possibly nonrational) expectation. σ, β, and κ are the structural parameters which satisfy $\sigma > 0$, $1 \geq \beta > 0$, and $\kappa > 0$.

The process of natural rate of real interest is given by

$$r_t^n = \rho r_{t-1}^n + \varepsilon_t, \tag{3}$$

where ρ satisfies $1 > \rho > 0$ and ε_t follows i.i.d. with zero mean.

The central bank introduces a forward-looking monetary policy rule:

$$r_t = \phi E_t^{CB} \pi_{t+1}, \tag{4}$$

[7] By using the model of Bullard and Mitra [2002], which introduces a dynamic version of the IS equation, we numerically obtain the result that the Taylor principle is not a sufficient condition for the E-stability of the REE if private agents engage in by referring to the central bank's forecast, which is the main finding of this study.

[8] Preston [2006] examines the E-stability of the REE in a New Keynesian model, in which private agents make current decisions about spending and pricing based on the long-horizon forecasts. He shows that the Taylor principle is not a sufficient condition for the E-stability under a forward-looking version of the Taylor rule. This result differs from Bullard and Mitra [2002]. However, Evans, Honkapohja, and Mitra [2003] show that the approaches of Preston [2006] and Bullard and Mitra [2002] are not inconsistent, because if the law of iterated expectations at an individual and aggregate levels holds, then the IS equation and the NKPC with long-horizon forecasts correspond to those with one-period-ahead forecasts. Since this study assumes that the law of iterated expectations at an individual and aggregate levels holds, we introduce the NKPC with a one-period-ahead forecast.

where ϕ is the responsiveness to the expected inflation rate. E_t^{CB} denotes the central bank's subjective expectation. In this study, we investigate mainly the situation in which the central bank and private agents have heterogeneous expectations (at least in the short run). Therefore, the central bank's expectations (E_t^{CB}) and private agents' expectations (E_t^P) are potentially different.

The model can be reduced to the model of inflation dynamics:

$$\pi_t = A + BE_t^P \pi_{t+1} + CE_t^{CB} \pi_{t+1} + Dr_t^n, \tag{5}$$

where $A = 0$, $B = \kappa\sigma + \beta$, $C = -\kappa\sigma\phi$, and $D = \kappa\sigma$.

2.2 E-stability When the Central Bank and Private Agents Are Independently Learning

Next, we present the E-stability condition of the REE when the central bank and private agents independently engage in adaptive learning. Following Bullard and Mitra [2002], we introduce a simplifying assumption that the central bank and private agents have an identical PLM such as

$$\pi_t = \widetilde{a} + \widetilde{b} r_t^n, \tag{6}$$

where $\widetilde{a}$ and $\widetilde{b}$ are coefficients, which are updated in every period. Since the functional form of (6) corresponds to the minimal state variables (MSV) solution of the system (5), we call the learning process of (6) "MSV learning."[9]

Based on PLM, the one-period-ahead expectation is calculated as follows:

$$E_t^{CB} \pi_{t+1} = E_t^P \pi_{t+1} = \widetilde{a} + \rho \widetilde{b} r_t^n. \tag{7}$$

By substituting (7) into (5), we derive the actual law of motion (ALM) as follows:

$$\pi_t = A + (B + C)\widetilde{a} + (\rho(B + C)\widetilde{b} + D)r_t^n. \tag{8}$$

From (6) and (8), the mapping functions (T-maps) from PLM to ALM are as follows:

$$\begin{aligned} T_a(\widetilde{a}) &= A + (B + C)\widetilde{a}, & (9)\\ T_b(\widetilde{b}) &= \rho(B + C)\widetilde{b} + D. & (10) \end{aligned}$$

The REE with the MSV form (MSV solution) is obtained as the fixed point of T-maps. The parameters of the MSV solution ($\overline{a}$ and $\overline{b}$) are computed as follows:

$$\overline{a} = (1 - (B + C))^{-1}A, \ \overline{b} = (1 - \rho(B + C))^{-1}D.$$

[9] See McCallum [1983] for the details of MSV solution.

Note that the combination of $\overline{a}$ and $\overline{b}$ is unique. It means that, if we restrict attention to the MSV form, the solution is unique, regardless of the values of structural parameters. For the moment (except for Section 4), we focus on the MSV solution.

Next, we derive the E-stability condition of the REE. In this study, we assume that both the central bank and private agents use RLS with decreasing gain. Then, the E-stability of the REE is defined as the local asymptotic stability of the ordinary differential equations (ODEs) associated with the T-maps ((9) and (10)):

$$
\begin{aligned}
h_a(\widetilde{a}) &\equiv \frac{da}{d\tau} = T_a(\widetilde{a}) - \widetilde{a} = A + (B + C - 1)\widetilde{a}, && (11) \\
h_b(\widetilde{b}) &\equiv \frac{db}{d\tau} = T_a(\widetilde{b}) - \widetilde{b} = (\rho(B + C) - 1)\widetilde{b} + D, && (12)
\end{aligned}
$$

where τ is "notional" or "artificial" time.

From these ODEs, the E-stability condition is derived as two inequalities:

$$
\begin{aligned}
Dh_a(\widetilde{a}) &= B + C - 1 < 0, && (13) \\
Dh_b(\widetilde{b}) &= \rho(B + C) - 1 < 0. && (14)
\end{aligned}
$$

Since $1 > \rho > 0$, (14) holds if (13) holds. Therefore, the necessary and sufficient condition for the E-stability of the REE is (13). (13) is rewritten as follows:

$$
\phi > 1 + \frac{\beta - 1}{\kappa\sigma}. \tag{15}
$$

Since β is usually very close to unity (such as 0.99), (15) indicates that the E-stability corresponds to the Taylor principle, which requires the central bank to adjust the nominal interest rate by more than one-for-one with the inflation rate. This is one of the main findings of Bullard and Mitra [2002].

3 E-stability When Private Agents Are Learning from the Central Bank's Forecast

In this section, we examine the E-stability condition when private agents engage in adaptive learning by referring to the central bank's forecast. It means that the central bank is the leader and private agents are the followers of expectation formations.

3.1 PLM and ALM

As in the previous section, we assume that the central bank engages in MSV learning. Then, the central bank's PLM is the same as in the previous section:

$$
\pi_t = \widetilde{a} + \widetilde{b} r_t^n. \tag{16}
$$

At the beginning of period t, the central bank updates the parameters of $\widetilde{a}$ and $\widetilde{b}$ by using the data of period $t-1$ (y_{t-1} and r_{t-1}^n). Then, the central bank observes the realization of the natural rate of real interest at period t (r_t^n). By using the newest estimates of $\widetilde{a}$ and $\widetilde{b}$, the central bank calculates the forward-looking expectations as follows:

$$E_t^{CB}\pi_{t+1} = \widetilde{a} + \rho\widetilde{b}r_t^n. \tag{17}$$

After calculating (17), the central bank announces this forecast to private agents.

The expectation formation of private agents is the core part of this analysis. In this respect, we assume that private agents can observe the central bank's forecast $E_t^{CB}\pi_{t+1}$ when they form the expectation $E_t^P\pi_{t+1}$. However, we also assume that private agents do not know how the central bank calculates the forecast $E_t^{CB}\pi_{t+1}$. Namely, private agents do not know the model used by the central bank. This is a usual assumption of adaptive learning, in which agents do not use structural knowledge to form their expectations.

We assume that private agents determine how to utilize the central bank's forecast in forming their expectations by evaluating the historical performance of the central bank's forecast. Specifically, we assume that private agents estimate the following PLM:

$$\pi_t = \widetilde{c} + \widetilde{d}E_{t-1}^{CB}\pi_t. \tag{18}$$

By estimating (18) with RLS, private agents assess the historical performance of the central bank's forecast.[10] If the forecast has historically performed well, the constant term $\widetilde{c}$ approximates zero, and the slope $\widetilde{d}$ should be close to unity. In contrast, if the central bank's forecast has performed poorly, $\widetilde{c}$ approximates the sample average of π_t, and $\widetilde{d}$ should be close to zero.[11]

Private agents update the parameters of $\widetilde{c}$ and $\widetilde{d}$ by using the data of period $t-1$ (π_{t-1} and $E_{t-2}^{CB}\pi_{t-1}$). Since private agents are the followers, they can use the central bank's forecast $E_t^{CB}\pi_{t+1}$ in forming their expectations at period t ($E_t^P\pi_{t+1}$). To calculate $E_t^P\pi_{t+1}$, private agents use their evaluation of the performance of the central bank's forecast as follows:

$$E_t^P\pi_{t+1} = \widetilde{c} + \widetilde{d}E_t^{CB}\pi_{t+1}. \tag{19}$$

(19) indicates that the forecast of private agents is influenced by the central bank's forecast. In addition, the influence of the central bank's forecast on the forecast of

[10] As is seen in the next subsection, the use of RLS in estimating (18) is consistent with the REE.

[11] This means that private agents do not automatically follow the central bank's forecast (if such is the case, the coefficients are fixed as $\widetilde{c} = 0$ and $\widetilde{d} = 1$, which corresponds to the case of homogeneous learning). This is a natural assumption, because private agents do not have any reason to follow the central bank's forecast if the historical performance of central bank forecast is very poor. The empirical analysis of Fujiwara [2005] supports this idea, because his results show that private agents do not perfectly equalize their forecast as the central bank's forecast even after observing the forecast.

private agents is determined by the estimated parameter $\widetilde{d}$. Therefore, (19) illustrates a situation in which private agents refer to the central bank's forecast, depending on its historical performance.

By inserting both agents' expectations ((17) and (19)) into the system of (5), we derive ALM for π_t as follows:

$$\pi_t = A + B(\widetilde{c} + \widetilde{d}\widetilde{a}) + C\widetilde{a} + (\rho(B\widetilde{d} + C)\widetilde{b} + D)r_t^n. \tag{20}$$

3.2 Equilibrium

Next, we derive the T-maps from PLM to ALM. From (16) and (20), the T-maps about parameters of $\widetilde{a}$ and $\widetilde{b}$ are given as follows:

$$\begin{aligned} T_a(\widetilde{a}) &= A + B(\widetilde{c} + \widetilde{d}\widetilde{a}) + C\widetilde{a}, && (21) \\ T_b(\widetilde{b}) &= \rho(B\widetilde{d} + C)\widetilde{b} + D. && (22) \end{aligned}$$

Since private agents' PLM (18) is not the MSV form, we must derive the T-maps from the relevant orthogonality conditions.[12] From (17) and (18), private agents' "projected" ALM is defined as follows:

$$\pi_t = T_c + T_d(\widetilde{a} + \rho\widetilde{b}r_{t-1}^n). \tag{23}$$

The corresponding orthogonality conditions are given by

$$E\left[1 \cdot \left(\pi_t - T_c - T_d(\widetilde{a} + \rho\widetilde{b}r_{t-1}^n)\right)\right] = 0, \tag{24}$$

$$E\left[(\widetilde{a} + \rho\widetilde{b}r_{t-1}^n)\left(\pi_t - T_c - T_d(\widetilde{a} + \rho\widetilde{b}r_{t-1}^n)\right)\right] = 0. \tag{25}$$

From (20), (24), and (25), we derive the T-maps about $\widetilde{c}$ and $\widetilde{d}$ as follows:

$$\begin{aligned} T_c(\widetilde{c}) &= A + B\widetilde{c} + (1-\rho)(B\widetilde{d} + C)\widetilde{a} - \widetilde{b}^{-1}D\widetilde{a}, && (26) \\ T_d(\widetilde{d}) &= \rho B\widetilde{d} + \rho C + \widetilde{b}^{-1}D. && (27) \end{aligned}$$

The equilibrium is derived as the fixed points of the T-maps ((21), (22), (26), and (27)). The coefficients at the equilibrium are given as follows:

$$\overline{a} = (1-(B+C))^{-1}A, \overline{b} = (1-\rho(B+C))^{-1}D, \overline{c} = 0, \overline{d} = 1.$$

Note that, at the equilibrium, (19) becomes as follows:

$$E_t^P\pi_{t+1} = E_t^{CB}\pi_{t+1} = (1-(B+C))^{-1}A + \rho(1-\rho(B+C))^{-1}Dr_t^n. \tag{28}$$

[12] See Branch [2004] for the derivation of T-maps using orthogonality conditions.

Therefore, at the equilibrium, expectations become homogeneous between the central bank and private agents. Furthermore, these expectations are the same as the expectation at the MSV solution in Section 2. Therefore, the expectation of (28) is the rational expectation and this equilibrium is the REE. This means that the economic dynamics at equilibrium are exactly the same in the two cases: (i) the case in which the central bank and private agents are independently learning and (ii) the case in which private agents engage in adaptive learning by referring to the central bank's forecast. However, in considering the transition dynamics under adaptive learning, the E-stability conditions of the REE can differ between these two cases. In the next subsection, we examine this issue.

3.3 E-stability

Next, we examine the E-stability condition. The E-stability of the equilibrium is the local asymptotic stability of ODEs associated with the T-maps of (21), (22), (26), and (27). Although these T-maps are interdependent, $T_b(\widetilde{b})$ and $T_d(\widetilde{d})$ only depend on $\widetilde{b}$ and $\widetilde{d}$. Therefore, we can examine the stability of $\widetilde{b}$ and $\widetilde{d}$, independently of the stability of $\widetilde{a}$ and $\widetilde{c}$.

To examine the stability of $\widetilde{b}$ and $\widetilde{d}$, we define the ODEs associated with the T-maps of $\widetilde{b}$ and $\widetilde{d}$ ((22) and (27)) as follows:

$$h\begin{pmatrix} \widetilde{b} \\ \widetilde{d} \end{pmatrix} \equiv \begin{pmatrix} T_b(\widetilde{b}) - \widetilde{b} \\ T_d(\widetilde{d}) - \widetilde{d} \end{pmatrix} = \begin{pmatrix} \rho(B\widetilde{d} + C)\widetilde{b} + D - \widetilde{b} \\ \rho B\widetilde{d} + \rho C + \widetilde{b}^{-1}D - \widetilde{d} \end{pmatrix}. \tag{29}$$

Given the convergence of $\widetilde{b}$ and $\widetilde{d}$, we can examine the stability of $\widetilde{a}$ and $\widetilde{c}$ by using the following ODEs:

$$h\begin{pmatrix} \widetilde{a} \\ \widetilde{c} \end{pmatrix} \equiv \begin{pmatrix} T_a(\widetilde{a}) - \widetilde{a} \\ T_c(\widetilde{c}) - \widetilde{c} \end{pmatrix} = \begin{pmatrix} A + B(\widetilde{c} + \widetilde{d}\widetilde{a}) + C\widetilde{a} - \widetilde{a} \\ A + B\widetilde{c} + (1-\rho)(B\widetilde{d} + C)\widetilde{a} - \widetilde{b}^{-1}D\widetilde{a} - \widetilde{c} \end{pmatrix}. \tag{30}$$

We derive the E-stability condition as the necessary and sufficient condition for the ODEs of (29) and (30) to be locally asymptotically stable around the REE. The result is given by the following proposition.

Proposition 1 *Suppose that the central bank engages in MSV learning and all private agents engage in adaptive learning by referring to the central bank's forecast. Then, the REE of (5) is E-stable if and only if (31) and (32) hold.*

$$\phi > 2 + \frac{2(\beta - 1)}{\kappa\sigma}, \tag{31}$$

$$\phi > 1 + \frac{\beta - 1}{\kappa\sigma}. \tag{32}$$

Proof. *See Appendix A.* ■

Proposition 1 leads to the following corollary.

Corollary 1 *Suppose that the central bank engages in MSV learning and all private agents engage in adaptive learning by referring to the central bank's forecast. Then, if $\kappa\sigma > 1 - \beta$, the REE of (5) is E-stable if and only if (31) holds.*

Since β is close to unity, $\kappa\sigma > 1 - \beta$ holds for a wide range of parameter sets (κ and σ). Then, the Taylor principle, which is expressed as (32), is not a sufficient condition for the E-stability, because the E-stability condition corresponds to (31). This means that, to satisfy the E-stability condition, the central bank must adjust the nominal interest rate by more than double the rise of central bank's expected inflation rate.

Thus, the E-stability condition in this situation is quite different from the condition in the benchmark case analyzed in Section 2. Although the equilibrium dynamics of these two cases are exactly the same, the E-stability condition is severer in the environment of this section. This means that, if private agents engage in adaptive learning by referring to the central bank's forecast, the central bank must respond to the expected inflation rate more strongly than the Taylor principle suggests.

3.4 Intuition for the Result

As is shown in the previous subsection, if private agents engage in adaptive learning by referring to the central bank's forecast, the central bank has to respond more strongly to the expected inflation rate than the Taylor principle suggests.

The basic intuition for the result is given by the fact that, in the situation of this section, private agents' forecast errors, which are defined as the deviations of private agents' expectations from rational expectations, are magnified, compared to the central bank's forecast errors. The reason is twofold. Firstly, private agents can make estimation errors concerning the parameters $\widetilde{c}$ and $\widetilde{d}$, which are introduced in their PLM (18). These estimation errors are the first source of private agents' forecast errors. Secondly, as in (19), the central bank's forecast errors influence the forecast of private agents. Since the parameter $\widetilde{d}$ is almost unity around the equilibrium, the central bank's forecast errors bring about almost the same amount of forecast errors as those of private agents. This is the second source of private agents' forecast errors. Summing up these two sources, the total forecast errors of private agents exceed the central bank's forecast errors.

Since the central bank introduces its own forecast in the monetary policy rule (4), the central bank responds to its own forecast errors. However, this policy response is insufficient to offset the forecast errors of private agents. Because private agents have larger forecast errors than the central bank, the central bank must respond very strongly to its own forecast, in order to ensure the convergence of economy to the REE. This is why the E-stability is not attained solely by the Taylor principle.

3.5 E-stability When Part of Private Agents Are Learning from the Central Bank's Forecast

So far, we have assumed that all private agents refer to the central bank's forecast in adaptive learning. However, this could be regarded as an extreme case.[13] In this subsection, therefore, we consider a more realistic environment in which some private agents engage in adaptive learning by referring to the central bank's forecast.

Suppose that a proportion μ of private agents engage in adaptive learning by referring to the central bank's forecast ($1 \geq \mu \geq 0$). The remaining $1-\mu$ of private agents engage in MSV learning. Denote $E_t^{P1}\pi_{t+1}$ as the forecast of the former private agents and $E_t^{P2}\pi_{t+1}$ as the forecast of the latter private agents. Note that the forecast made by the latter is just the same as the central bank's forecast. Therefore, the aggregate forecast of private agents ($E_t^P\pi_{t+1}$) can be expressed as follows:[14]

$$\begin{aligned} E_t^P\pi_{t+1} &= \mu E_t^{P1}\pi_{t+1} + (1-\mu)E_t^{P2}\pi_{t+1} \\ &= \mu E_t^{P1}\pi_{t+1} + (1-\mu)E_t^{CB}\pi_{t+1}. \end{aligned} \tag{33}$$

By substituting (33) into (5), we obtain the following ALM:

$$\pi_t = A + \widehat{B}E_t^{P1}\pi_{t+1} + \widehat{C}E_t^{CB}\pi_{t+1} + Dr_t^n, \tag{34}$$

where $\widehat{B} = \mu B$ and $\widehat{C} = (1-\mu)B + C$.

(34) has the same form as (5). Therefore, in order to examine the E-stability of the REE, we can follow the same steps of the subsections 3.2 and 3.3, by replacing the matrices of B and C with $\widehat{B}$ and $\widehat{C}$. Then, the result for the E-stability of the REE is given by the following proposition.[15]

Proposition 2 *Suppose that the central bank and a proportion $1-\mu$ of private agents engage in MSV learning. In addition, suppose that a proportion μ of private agents engage in adaptive learning by referring to the central bank's forecast. Then, the REE*

[13] In this respect, Kohn [2005] judges that private agents do not rely perfectly on the central bank's expectation. He remarks that "in the United States, we have some indirect evidence that crowding out of private views has not increased even as the Federal Reserve has become more talkative. Market interest rates have continued to respond substantially to surprises in economic data."

[14] Guse [2005] incorporates a convex combination of heterogeneous forecasts into a simple macroeconomic model with multiple equilibria. Branch and McGough [2006] present the underlying assumptions for the validity of a convex combination of heterogeneous forecasts. These include (i) the identical expectations at steady state, (ii) some linearity properties of expectations, and (iii) the law of iterated expectations at both an individual and aggregate level. We assume that all of these assumptions are satisfied.

[15] We can easily find that the equilibrium of (34) is just the same as the REE of (5).

of (5) is E-stable if and only if (35) and (36) hold.

$$\phi > 1+\mu+\frac{(1+\mu)\beta-2}{\kappa\sigma}, \tag{35}$$

$$\phi > 1+\frac{\beta-1}{\kappa\sigma}. \tag{36}$$

Proof. *See Appendix B.* ■

Proposition 2 leads to the following corollary.

Corollary 2 *Suppose that the central bank and a proportion $1-\mu$ of private agents engage in MSV learning. In addition, suppose that a proportion μ of private agents engage in adaptive learning by referring to the central bank's forecast. Then, if $\mu \geq (\kappa\sigma+\beta)^{-1}$, the REE of (5) is E-stable if and only if (35) holds. If $\mu < (\kappa\sigma+\beta)^{-1}$, the REE of (5) is E-stable if and only if (36) holds.*

Thus, if μ is relatively low, then the Taylor principle is the necessary and sufficient condition for the E-stability.[16] However, if μ is relatively high, to ensure the convergence to the REE, the central bank must respond more strongly to the expected inflation rate than the Taylor principle suggests.

4 Determinacy and E-stability

In the previous sections, we have examined the E-stability condition of the REE. However, in the standard analysis, the condition for determinacy (uniqueness) of the REE is also regarded as the minimum criterion which should be satisfied in monetary policy rules. In this regard, Bernanke and Woodford [1997] point out that the issue of determinacy is especially relevant when the central bank introduces a forward-looking monetary policy rule, such as (4). The reason why the determinacy condition has not been examined in the previous sections is that we have restricted our attention to the MSV solution, which is unique in our model. However, if we broaden our scope to introduce the solution forms other than the MSV form (i.e., sunspot equilibria), we must examine the condition for determinacy of the REE.[17] In particular, we must investigate the relationship between the determinacy condition and the E-stability condition. In this section, we examine this issue.

[16] Note that, for a wide range of parameter sets, the value of $(\kappa\sigma+\beta)^{-1}$ is between 0 to 1, since β is almost unity.

[17] Honkapohja and Mitra [2004] examine the existence of learnable sunspot equilibria in a simple New Keynesian model with a forward-looking Taylor rule. In contrast to our study, they introduce a benchmark assumption that the central bank and private agents engage in independently adaptive learning. They show that learnable sunspot equilibria can exist even if the policy rule satisfies the Taylor principle.

4.1 Determinacy of the REE

The determinacy condition is presented by Blanchard and Kahn [1980]. Since the system is reduced as the univariate model of (5), the derivation of determinacy condition is easy. In the REE, the system of (5) is rewritten as follows:

$$\pi_t = A + (B + C)E_t\pi_{t+1} + Dr_t^n. \tag{37}$$

Blanchard and Kahn [1980] show the determinacy condition of (37) as $|B + C| < 1$. This leads to the following proposition.

Proposition 3 *The economy of (5) has a unique REE if and only if the following condition holds:*

$$1 + \frac{\beta + 1}{\kappa\sigma} > \phi > 1 + \frac{\beta - 1}{\kappa\sigma}. \tag{38}$$

Thus, the determinacy condition sets the upper bound of ϕ. This result means that the central bank should not respond to the expected inflation rate very strongly, because such a strong response causes the emergence of sunspot equilibria. This is the issue raised by Bernanke and Woodford [1997].

4.2 Relationship between Determinacy and E-stability

Next, we examine the relationship between the determinacy condition and the E-stability condition. Specifically, we investigate a situation in which all private agents engage in adaptive learning by referring to the central bank's forecast.[18] In this case, the E-stability condition is given by Proposition 1 and Corollary 1. By combining these with Proposition 3, we obtain the following proposition.

Proposition 4 *Suppose that the central bank engages in MSV learning and all private agents engage in adaptive learning by referring to the central bank's forecast. Then, the following statements hold.*

(i) If $1 > \beta + \kappa\sigma$, the necessary and sufficient condition for the REE of (5) to be E-stable and determinate is given by

$$1 + \frac{\beta + 1}{\kappa\sigma} > \phi > 1 + \frac{\beta - 1}{\kappa\sigma}. \tag{39}$$

(ii) If $3 \geq \beta + \kappa\sigma \geq 1$, the necessary and sufficient condition for the REE of (5) to be E-stable and determinate is given by

$$1 + \frac{\beta + 1}{\kappa\sigma} > \phi > 2 + \frac{2(\beta - 1)}{\kappa\sigma}. \tag{40}$$

[18] The extension to the situation in which some private agents engage in adaptive learning by referring to the central bank's forecast is straightforward.

(iii) If $\beta + \kappa\sigma > 3$, the REE of (5) cannot be both E-stable and determinate for any value of ϕ.

The condition (39) is the same as the determinacy condition (38). This means that, in the case (i), the determinacy condition is a sufficient condition for the E-stability of the REE. However, this is a relatively special case, because $\beta + \kappa\sigma$ is usually more than unity (since β is close to unity).

Therefore, for a wide range of the parameter sets, determinacy is not a sufficient condition for the E-stability of the REE. This is an important finding in the literature, because McCallum [2007] points out that, if the current-period information is available in the process of adaptive learning, determinacy becomes a sufficient condition for the E-stability of the REE, in a broad class of linear models. In contrast to the argument of McCallum [2007], Proposition 4 indicates that the determinacy is not necessarily a sufficient condition for the E-stability, even though both the central bank and private agents calculate the expectations ($E_t^{CB}\pi_{t+1}$ and $E_t^P\pi_{t+1}$) by using the information at period t. This result suggests that, in the presence of the leader-follower relationship in adaptive learning, the determinacy does not automatically guarantee the E-stability of the REE.

Since $\beta + \kappa\sigma$ is usually greater than unity, the cases of (ii) and (iii) deserve our attention. In the case (ii), the region of E-stable and determinate REE is narrow. This means that the central bank's choice of the value ϕ is highly restrictive. The environment of the case (iii) is even more severe, because the central bank cannot simultaneously satisfy the conditions of determinacy and E-stability. In the case (iii), we obtain the following proposition.

Proposition 5 *Suppose that the central bank engages in MSV learning and all private agents engage in adaptive learning by referring to the central bank's forecast. Then, under the condition of $\beta + \kappa\sigma \geq 3$, the following statements hold.*

(i) The REE of (5) is E-stable and indeterminate if

$$\phi > 2 + \frac{2(\beta - 1)}{\kappa\sigma}. \tag{41}$$

(ii) The REE of (5) is E-unstable and determinate if

$$1 + \frac{\beta + 1}{\kappa\sigma} > \phi > 1 + \frac{\beta - 1}{\kappa\sigma}. \tag{42}$$

(iii) The REE of (5) is E-unstable and indeterminate if

$$2 + \frac{2(\beta - 1)}{\kappa\sigma} \geq \phi \geq 1 + \frac{\beta + 1}{\kappa\sigma} \quad \text{or} \quad 1 + \frac{\beta - 1}{\kappa\sigma} > \phi. \tag{43}$$

Thus, if $\beta + \kappa\sigma > 3$, the central bank must choose either determinacy or E-stability. If the monetary policy rule satisfies (41), then the E-stable sunspot equilibria emerge.

This is the situation investigated by Honkapohja and Mitra [2004]. In this case, the central bank's strong reaction to the expected inflation rate guarantees the E-stability. However, the endogenous fluctuations can occur, because multiple REE satisfy the E-stability. Honkapohja and Mitra [2004] recommend that the monetary policy rule should rule out this possibility. However, if the central bank avoids the emergence of E-stable sunspot equilibria, the REE must be E-unstable. In this sense, the central bank faces a serious trade-off.

In sum, the results indicate that, if private agents engage in adaptive learning by referring to the central bank's forecast, the central bank's policymaking must be more restrictive than in the benchmark case in which both the central bank and private agents independently engage in adaptive learning. This means that, if the central bank is the leader of expectation formation, a forward-looking monetary policy rule has more serious problems than those pointed out in Bernanke and Woodford [1997].

4.3 A Remedy

As in the previous subsection, we find that a forward-looking policy rule has serious problems when private agents engage in adaptive learning by referring to the central bank's forecast. A possible remedy for this problem is that the central bank additionally introduces the contemporaneous data of the inflation rate into a policy rule. Suppose that the central bank introduces the following monetary policy rule:

$$r_t = \phi E_t^{CB} \pi_{t+1} + \gamma \pi_t, \tag{44}$$

where γ is the responsiveness to the contemporaneous data of the inflation rate. Then, the reduced model has the same form of (5). However, the coefficients are replaced by $A = 0$, $B = \frac{\kappa\sigma+\beta}{1+\kappa\sigma\gamma}$, $C = \frac{-\kappa\sigma\phi}{1+\kappa\sigma\gamma}$, and $D = \frac{\kappa\sigma}{1+\kappa\sigma\gamma}$.

As in Section 4.1, the determinacy condition is obtained as $|B + C| < 1$. This leads to the following proposition.

Proposition 6 *The economy of (1), (2), (3), and (44) has a unique REE if and only if the following condition holds:*

$$1 + \gamma + \frac{\beta + 1}{\kappa\sigma} > \phi > 1 - \gamma + \frac{\beta - 1}{\kappa\sigma}. \tag{45}$$

Thus, the central bank can relax the determinacy condition by increasing the value of γ. This is a natural consequence because previous studies (including Bullard and Mitra [2002]) have shown that the rule with contemporaneous data is more robust for determinacy than the rule with forward-looking expectations. By responding to the contemporaneous data of the inflation rate, the central bank can reduce the sensitivity of the economic system to forward-looking expectations. This is why determinacy is more easily satisfied under rule (44) than (4).

Next, we examine the E-stability condition under rule (44). Suppose that all private agents engage in adaptive learning by referring to the central bank's forecast. Then, we can derive the E-stability condition, following the same steps in Section 3. The result is given by the following proposition.

Proposition 7 *Suppose that the central bank engages in MSV learning and all private agents engage in adaptive learning by referring to the central bank's forecast. Then, the REE of (1), (2), (3), and (44) is E-stable if and only if (46) and (47) hold.*

$$\phi + 2\gamma \quad > \quad 2 + \frac{2(\beta - 1)}{\kappa\sigma}, \tag{46}$$

$$\phi + \gamma \quad > \quad 1 + \frac{\beta - 1}{\kappa\sigma}. \tag{47}$$

Thus, the E-stability condition is relaxed by introducing the coefficient γ. By increasing the value of γ, the central bank can easily attain the E-stability of the REE. The reason for this result is explained by the fact that, in the NKPC (3), the contemporaneous inflation rate is determined by private agents' expected inflation rate. Because of this property, the central bank can respond to the forecast errors of private agents, by responding to the contemporaneous data of the inflation rate.

Therefore, the central bank can simultaneously relax the conditions of determinacy and E-stability, by responding to the contemporaneous movements of the inflation rate. This result suggests that it is dangerous for the central bank to introduce a purely forward-looking monetary policy rule. A more robust policy strategy is to respond to the contemporaneous movements of the inflation rate to a certain degree.

5 E-stability in the Reverse Situation

In this study, we have examined the situation in which private agents engage in adaptive learning by referring to the central bank's forecast. Readers may be interested in the E-stability condition in the reverse situation in which the central bank engages in adaptive learning by referring to the forecast of private agents.

The derivation of the E-stability condition is just the same as in Section 3. Following similar steps, we obtain the following proposition.

Proposition 8 *Suppose that all private agents engage in MSV learning and the central bank engages in adaptive learning by referring to the aggregate forecast of private agents. Then, the REE of (5) is E-stable if and only if (15) holds.*

Proof. *See Appendix C.* ■

Therefore, in this reverse situation, the E-stability condition corresponds to the Taylor principle. Intuitively, this result can be interpreted as follows. In this situation, the central bank's forecast errors exceed the forecast errors of private agents (following the exactly opposite logic of Section 3.4). Therefore, in order to offset private agents' forecast errors, the central bank's reaction to its own forecast need not to be as large as the Taylor principle suggests (i.e. ϕ can be smaller than unity). However, to offset the central bank's own forecast errors, the Taylor principle is still required. This is why the E-stability condition is given by the Taylor principle.

Therefore, if private agents are the leaders and the central bank is the follower, the E-stability condition is just the same as in the benchmark case, which is investigated in Section 2. In this environment, the central bank can guarantee both the E-stability and determinacy of the REE by satisfying the Taylor principle. This implies that the central bank can more easily ensure macroeconomic stability in a case in which the central bank is the follower, rather than the leader of expectation formation.

6 Conclusion

In this study, we have examined the E-stability of the REE in a simple New Keynesian model in which private agents engage in adaptive learning by referring to the central bank's forecast. In contrast to a situation in which both the central bank and private agents independently (or simultaneously) engage in adaptive learning (such as the case of Bullard and Mitra [2002]), the E-stability is not attained solely by the so-called Taylor principle. To ensure the convergence to the REE, the central bank must respond more strongly to the expected inflation rate than the Taylor principle suggests. On the other hand, the central bank's strong reaction to the expected inflation rate raises the possibility of indeterminacy of the REE, as pointed out in Bernanke and Woodford [1997]. In considering these problems, a robust policy strategy is to respond to the contemporaneous data of the inflation rate to a certain degree.

Appendix A: Proof of Proposition 1

The local asymptotic stability of $\widetilde{b}$ and $\widetilde{d}$ is satisfied if and only if all the eigenvalues of the Jacobian of (29) at the REE, which is expressed in (A1), have negative real parts:

$$Dh\begin{pmatrix}\overline{b}\\ \overline{d}\end{pmatrix}|_{\widetilde{b}=\overline{b},\widetilde{d}=\overline{d}}=\begin{pmatrix}\rho(B+C)-1 & \rho B(1-\rho(B+C))^{-1}D\\ -(1-\rho(B+C))^2D^{-1} & \rho B-1\end{pmatrix}. \quad (A1)$$

The characteristic polynomial of (A1) is given as follows:

$$\lambda^2+(2-2\rho B-\rho C)\lambda+1-\rho B-\rho C=0. \quad (A2)$$

All the eigenvalues of (A1) have negative real parts if and only if $(2-2\rho B-\rho C)>0$ and $1-\rho B-\rho C>0$. From the definition of B and C, it corresponds to the following conditions:

$$\phi > 2+\frac{2(\beta-\rho^{-1})}{\kappa\sigma}, \quad (A3)$$
$$\phi > 1+\frac{\beta-\rho^{-1}}{\kappa\sigma}. \quad (A4)$$

Next, we examine the local asymptotic stability of $\widetilde{a}$ and $\widetilde{c}$. The Jacobian of (30) is derived as follows:

$$Dh\begin{pmatrix}\widetilde{a}\\ \widetilde{c}\end{pmatrix}|_{\widetilde{b}=\overline{b},\widetilde{d}=\overline{d}}=\begin{pmatrix}B+C-1 & B\\ B+C-1 & B-1\end{pmatrix}. \quad (A5)$$

The characteristic polynomial of (A5) is as follows:

$$\lambda^2+(2-2B-C)\lambda+1-B-C=0. \quad (A6)$$

Therefore, the local asymptotic stability of (A5) at REE is satisfied if and only if $2-2B-C>0$ and $1-B-C>0$. These correspond to the following conditions:

$$\phi > 2+\frac{2(\beta-1)}{\kappa\sigma}, \quad (A7)$$
$$\phi > 1+\frac{\beta-1}{\kappa\sigma}. \quad (A8)$$

Note that, since $1>\rho>0$, (A3) holds if (A7) holds. Similarly, (A4) holds if (A8) holds. Therefore, the E-stability condition corresponds to (A7) and (A8).

Appendix B: Proof of Proposition 2

If the central bank and a proportion $1-\mu$ of private agents engage in MSV learning and a proportion μ of private agents engage in adaptive learning by referring to the central bank's forecast, the relevant characteristic polynomials are given as follows:

$$\begin{aligned} \lambda^2 + (2 - 2\rho\widehat{B} - \rho\widehat{C})\lambda + 1 - \rho\widehat{B} - \rho\widehat{C} &= 0, && \text{(B1)} \\ \lambda^2 + (2 - 2\widehat{B} - \widehat{C})\lambda + 1 - \widehat{B} - \widehat{C} &= 0. && \text{(B2)} \end{aligned}$$

Then, the E-stability condition corresponds to that in which all of $2 - \rho\widehat{B} - 2\rho\widehat{C}$, $1 - \rho\widehat{B} - \rho\widehat{C}$, $2 - \widehat{B} - 2\widehat{C}$, and $1 - \widehat{B} - \widehat{C}$ are strictly positive. These are equivalent to the following conditions:

$$\begin{aligned} \phi &> 1 + \mu + \frac{(1+\mu)\beta - 2\rho^{-1}}{\kappa\sigma}, && \text{(B3)} \\ \phi &> 1 + \frac{\beta - \rho^{-1}}{\kappa\sigma}. && \text{(B4)} \\ \phi &> 1 + \mu + \frac{(1+\mu)\beta - 2}{\kappa\sigma}, && \text{(B5)} \\ \phi &> 1 + \frac{\beta - 1}{\kappa\sigma}. && \text{(B6)} \end{aligned}$$

Since $1 > \rho > 0$, (B3) holds if (B5) holds. Similarly, (B4) holds if (B6) holds. Therefore, the E-stability condition corresponds to (B5) and (B6).

Appendix C: Proof of Proposition 8

If all private agents engage in MSV learning and the central bank engages in adaptive learning by referring to the aggregate forecast of private agents, the relevant characteristic polynomials are given as follows:

$$\lambda^2 + (2 - \rho B - 2\rho C)\lambda + 1 - \rho B - \rho C = 0, \tag{C1}$$

$$\lambda^2 + (2 - B - 2C)\lambda + 1 - B - C = 0. \tag{C2}$$

Then, the E-stability condition corresponds to that in which all of $2 - \rho B - 2\rho C$, $1 - \rho B - \rho C$, $2 - B - 2C$, and $1 - B - C$ are strictly positive. These are equivalent to the following conditions:

$$\phi > \frac{1}{2} + \frac{\beta - 2\rho^{-1}}{2\kappa\sigma}, \tag{C3}$$

$$\phi > 1 + \frac{\beta - \rho^{-1}}{\kappa\sigma}, \tag{C4}$$

$$\phi > \frac{1}{2} + \frac{\beta - 2}{2\kappa\sigma}, \tag{C5}$$

$$\phi > 1 + \frac{\beta - 1}{\kappa\sigma}. \tag{C6}$$

Since $1 > \rho > 0$, (C3) holds if (C5) holds. Similarly, (C4) holds if (C6) holds. Furthermore, (C5) holds if (C6) holds. Therefore, the E-stability condition is given by (C6).

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