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Abstract

In this paper, we first construct a dynamic new Keynesian model that incorporates life-cycle behavior *a la* Gertler (1999), in order to study whether structural shocks to the economy have asymmetric effects on heterogeneous agents, namely workers and retirees. We also examine whether considerations of life-cycle and demographic structure alter the dynamic properties of the monetary business cycle model, specifically the degree of amplification in impulse responses. According to our simulation results, shocks indeed have asymmetric impacts on different households and the demographic structure does alter the size of responses against shocks by changing the degree of the trade-off between substitution and income effects.

Keywords: Heterogenous Agents; Life-Cycle; New Keynesian Model
JEL classification: E32, E50, J14

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1 Introduction

Societal aging is one of the biggest economic issues facing many developed countries. In Japan, in particular, society is aging so rapidly that not only is the working population (those older than 15 but younger than 65) already shrinking, but the total population is also expected to start decreasing by 2007. This movement suggests that the central bank should have an even greater interest in how monetary policy affects heterogenous agents, namely workers and retirees, differently and how the consideration of this demographic structure may alter the amplification of the variables against structural shocks.[1] Seminal research by Woodford (2003) depicts the various forms of the dynamic New Keynesian model corresponding to different economic conditions and has had a significant influence on central banks' views of monetary business cycle. However, to date very little research has paid attention to monetary business cycle model with heterogenous agents, particularly within a life-cycle setting.[2]

In this paper, we first set up a dynamic stochastic general equilibrium model with nominal rigidities and capital adjustment costs that incorporates life-cycle behavior *a la* Gertler (1999). Then, we show whether the structural shocks to the economy have asymmetric effects on heterogeneous agents and whether the considerations of the life-cycle and demographic structure alter the dynamic properties of the solution, under different settings of life-cycle behavior.[3] Of course, as mentioned in Bean (2004), it is true that "the glacial nature of demographic change appears to suggest that the implications for monetary policy should be modest." We, however, believe that it becomes more important for central banks to acknowledge the asymmetric effects on heterogeneous agents within a life-cycle economy with a stationary population, since societal aging in many developing country necessitates the consideration of the distributional consequences of monetary policy.[4] Furthermore, central banks must always understand the monetary transmission mechanism as well as macroeconomic responses to structural shocks in detail. Therefore, we focus on the impulse

[1] For example, "The Coming Demographic Transition: Will We Treat Future Generations Fairly," the chairman, Ben Bernanke, discusses the societal ageing and comments that "the broader perspective shows clearly that adequate preparation for the coming demographic transition may well involve significant adjustments in our patterns of consumption, work effort and saving," although the remarks are mainly on the sharing the burden of population ageing.

[2] As a large-scale dynamic general equilibrium model used for central bank projections and policy simulations, Bank of Finland construnts a model with lifecycle behaviour as examined in this paper (see Kilponen, Ripatti and Vilmunen, 2004).

[3] This aim is similar to those in recent literature on real rigidities that try to explain realistic inflation persistence with reasonable calibration, such as Sveeen and Weinke (2005), Altig, Christiano, Eichenbaum and Linde (2005), and Levin, Lopez-Salido and Yun (2006). Yet, the demographic structure does not alter the persistence but the volatility of the endogenous variables against structural shocks.

[4] Williamson (2005) analyzes monetary policy and resulting distribution in an island economy. Furthermore, Doepke and Schneider (2005) shows that young borrowers benefit more from inflation than retirees, and inflation can be welfare-enhancing since it acts like a tax on foreign share holders. Their analyses are, however, not based on the canonical dynamic new Keynesian model, heavily used among central banks.

responses of this life-cycle economy. We know that the impact of societal aging on general equilibrium has two separate aspects, and it is important to distinguish between the two. First, the "transition" toward the aging society can be most naturally considered in terms of a macro shock, which will affect monetary policy decisions. Second, the impulse responses derived from a dynamic new Keynesian model in a "stationary population" may be quite different for an elderly society than for a young society. For the purposes of the current paper, we focus on the second of these two aspects.

Bean (2004) summarizes the previous findings in this field, pointing out their implications for a central bank: (1) demographic developments represent a macroeconomic shock, which may lead to abrupt movements in asset prices and sharp movements in saving behavior; (2) the natural rate of interest falls both along the transition path and in the steady state; (3) the natural rate of unemployment may also be affected through the matching mechanism;[5] (4) the wealth channel is likely to become a more important transmission channel of monetary policy than intertemporal substitution; (5) the Phillips curve is flatter due to immigration and the increased participation of retired workers whose supply of labor is considered to be relatively elastic; (6) the constituency for keeping inflation low will be larger thanks to higher average wealth accumulation; and (7) societal aging may induce diversification and risk-shifting with a securitized market rather than bank-intermediated finance, which has implications for financial stability. Although not all the topics raised by Bean (2004) can be covered in this paper, we formally verify (4) using the dynamic general equilibrium model with sticky price and life-cycle behavior. Yet, as societal aging deepens, although income effect becomes stronger for retirees, that of workers becomes weaker. Hence, in aggregate, a tightening monetary policy shock still has negative impacts on the aggregate demand. Even though consumption of retirees increase, worker's consumption or the investment need to be lowered since there is no expansion in the production frontier. At the same time, we will show that (2) is not a general result due to the endogenized labor participation by retirees. Furthermore, we find that since retirees do not work as much as workers, they benefit less from improved technology, a point not by raised by Bean (2004). Therefore, the monetary response to a positive technology shock can be smaller in a greyer society. Anecdotal evidence on these points has abounded, but there have been very few studies that have tackled this problem in a theoretically consistent dynamic general equilibrium framework,[6] which is the workhorse model for modern monetary policy analysis. Our main conclusions are as follows. Since retired people rely more on interest income than on wages from their labor supply, shocks indeed have different impacts on different households. Furthermore, the demographic structure does alter the size of the response to shocks by changing the degree of the trade-off between substitution and income effects. Therefore, societal aging and life-cycle considerations, have

[5] We would like to consider incorporating matching mechanism similar to Merz (1995) and Andolfatto (1996) in our future research.

[6] Miles (2002) is one exception, but not based on the canonical dynamic new Keynesian model.

important implications for monetary policy.

Another interesting point is to compare optimal monetary policy in this life-cycle economy to the one obtained in a standard model with homogeneous agents. Stochastic welfare analysis, however, raises difficult issues such as what discount rate should be used by firms, how we should distribute initial profit stemming at the time when the shock hits the economy among different agents, and which welfare measures are appropriate with the setting of the model in this paper. As for last point, it is not trivial to compute aggregate welfare with heterogenous agents. In other words, it is difficult to weigh the welfare of each heterogenous agent in order to obtain aggregate welfare measure. Similarly, to determine the weights when distributing the initial profit between workers and retirees is not a trivial task either.[7] The first is an especially large concern with incomplete markets and heterogenous agents, because marginal utility growth rates are no longer equalized across agents and therefore the choice of the appropriate discount factor is no longer obvious.[8] Thus, we restrict our analysis to the case of perfect foresight with one-time unanticipated shocks that may be auto-correlated as examined in Ghironi (2006), Kilponen and Ripatti (2006), and Kilponen, Kinnunen, and Ripatti (2006). In a perfect-foresight setting, all assets including the one-period bond must yield the same return. As a result some very difficult (and as yet unsolved) problems regarding asset valuation can be side stepped.

This paper is put together as follows. In section two, we describe the model employed in this analysis. Then, section three discusses the nature of our stationary population under different demographic structures. We show that the natural rate of interest is quite different among the different demographic setups. In section four, we show the impulse responses and show that shocks indeed have different impacts on different households and the demographic structure does alter the size of responses against shocks by changing the degree of the trade-off between substitution and income effects. Finally, section five concludes.

2 Model

The model examined here is based on Gertler (1999). We add a sticky price mechanism with endogenous capital whose accumulation is subject to an investment adjustment cost. This type of the model has been referred to as the "canonical model" by Edge (2003). We have six agents in this model, namely

[7] When we solve the dynamic general equilibrium model with solved-out consumption function as in this paper, we need to determine the initial profit stemming from an unexpected change in the rate of return at the period when the shock hits the economy. This will not be any problem when agents are homogeneous, but will become an issue when heterogeneous agents are considered. Hence, as you will see later, the impulse responses of non-aggregated variables (consumption, labor supply and welfare of workers and retirees) at the initial period is explained by the allocation rule rather than economic dynamics.

[8] One solution is to use a discount factor that is a weighted average of the marginal utility of wealth of each group of agents with the group weights given by the relative share holdings as examined in Heathcote and Perri (2004) for foreign and domestic shareholders.

firms, capital producers, financial intermediaries, households, a government, and the central bank. As explained above, we limit our analysis to the cases with perfect foresight to avoid aggregation issues stemming from heterogeneous agents.

2.1 Firms

Firms face a cost minimization problem via price setting subject to a Rotemberg (1982) - type adjustment cost.

2.1.1 Marginal Cost

Marginal cost, where there exist two inputs, namely labor L and capital K, is computed as in Christiano Eichenbaum and Evans (2005). By denoting real wages by $\frac{W}{P}$ and real cost of capital by r^K, each firm j minimizes total costs:

$$\frac{W_t}{P_t} L_{j,t} + r_t^K K_{j,t}$$

subject to the standard Cobb-Douglas production technology with capital share being α,

$$Y_{j,t} = \left[Z_t \exp\left(z_t\right) L_{j,t}\right]^{1-\alpha} K_{j,t}^{\alpha}, \tag{1}$$

where Y is output, Z is deterministic technology growth, and z is a temporary technology shock. The Lagrangian multiplier of this optimization problem is the real marginal cost φ. This is assumed to be symmetric across monopolistically competitive firms:

$$\varphi_t = \left[\frac{W_t}{(1-\alpha) Z_t \exp\left(z_t\right) P_t}\right]^{1-\alpha} \left(\frac{r_t^K}{\alpha}\right)^{\alpha},$$

where π is the rate of inflation. Similarly, real wages and the cost of capital are also defined as follows:

$$\frac{W_t}{P_t} = (1-\alpha)\varphi_t \left[Z_t \exp\left(z_t\right)\right]^{1-\alpha} L_{j,t}^{-\alpha} K_{j,t}^{\alpha}, \tag{2}$$

$$r_t^K = \alpha\varphi_t \left[Z_t \exp\left(z_t\right)\right]^{1-\alpha} L_{j,t}^{1-\alpha} K_{j,t}^{\alpha-1}. \tag{3}$$

2.1.2 Price Setting

Under monopolistic competition and a Rotemberg-type adjustment cost ϕ, each firm sets prices in order to maximize its real dividend D:

$$D_{j,t} = (1+\tau) \frac{P_{j,t,}}{P_t} Y_{j,t} - \varphi_t Y_{j,t} - \frac{\phi}{2}\left(\frac{P_{j,t}}{P_{j,t-1}} - 1\right)^2 Y_t, \tag{4}$$

subject to a downward sloping demand curve with elasticity of substitution κ:

$$Y_{j,t} = \left(\frac{P_{j,t,}}{P_t}\right)^{-\kappa} Y_t.$$

τ denotes a tax subsidy which is levied on households in lump-sum manner[9] and is assumed to eliminate the distortion in the stationary population stemming from monopolistic competition. Therefore,

$$\tau = \frac{1}{\kappa - 1}.$$

In a symmetric equilibrium where $P_{j,t} = P_t$ and the target level of inflation is zero, the first order necessary condition implies

$$\kappa\left(\varphi_t - 1\right) - \phi\left(\pi_t - 1\right)\pi_t + \frac{m_{0,t+1}}{m_{0,t}}\phi\left(\pi_{t+1} - 1\right)\pi_{t+1} = 0, \tag{5}$$

where gross inflation rate is defined by

$$\pi_t = \frac{P_t}{P_{t-1}},$$

m denotes the stochastic discount factor that is used to discount profits over time. Formally, it is determined as the weighted marginal utilities of the share holders as shown in Heathcote and Perri (2004) and Carceles-Poveda and Coen-Pirani (2005):

$$m_{0,t} = \beta^t \left[w^w \frac{\partial V_t^w}{\partial C_t^w} + (1 - w^w) \frac{\partial V_t^r}{\partial C_t^r} \right],$$

where β is the subjective discount factor, w^w is the weight on the workers' marginal utility, which is, for example, assumed to be given by the relative share holdings in Heathcote and Perri (2004), V and C denote recursive utility and consumption respectively. The superscript w describes a variable that applies to workers, while r stands for retirees. We do not face the usual complication in aggregation stemming from heterogeneous agents, since we limit our analysis to the linearized model with the one-period bond around stationary population under perfect foresight. The link between this pricing kernel and the risk-free rate is given in the subsection below.

2.2 Capital Producers

Capital producers, who maximize profits under a perfectly competitive market, enter the current period t with K_t units of capital. This is the amount of capital they can rent to final goods producers in the current period and receive a (nominal) factor payment of $P_t r_t^K K_t$. Furthermore, these capital-producing firms have borrowed from financial intermediary in this period. The nominal value of these funds is A_t, since A_t is total nominal funds invested by the financial intermediary in period t. The firm pays a gross nominal interest rate R_t^K on these funds. The firm also undertakes new investment, $P_t I_t$, and borrows

[9] In this model, since households are heterogeneous, the size of the lump sum tax to each agent is determined by the population ratio between workers and retirees. As a result, the same lump sum tax is levied on each agent.

the amount A_{t+1} from households. The firms' period real profits Π_t^K are given by:

$$\Pi_t^K = \frac{A_{t+1}}{P_t} + r_t^K K_t - I_t - R_t^K \frac{A_t}{P_t}. \quad (6)$$

Firms maximize these profits using the appropriate asset pricing kernel, subject to the production technology of capital used by firms:

$$K_{t+1} = (1-\delta) K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \quad (7)$$

where $S(\cdot)$ is the adjustment cost function used in Christiano, Eichenbaum and Evans (2005):[10]

$$S\left(\frac{I_t}{I_{t-1}}\right) = [(1+z)(1+n)]^2 S'' \left[\frac{\left(\frac{I_t}{I_{t-1}}\right)^2}{2[(1+z)(1+n)]^2} - \frac{I_t}{I_{t-1}(1+z)(1+n)} + \frac{1}{2}\right],$$

where S'' determines the size of this adjustment cost, and is the second derivative of this adjustment cost function with respect to I_t/I_{t-1}. From first order necessary conditions, we can obtain the equation for rental cost of capital:

$$Q_t = \frac{\pi_{t+1}}{R_{t+1}^K}\left[Q_{t+1}(1-\delta) + r_{t+1}^K\right], \quad (8)$$

where Q is the Lagrange multiplier on the capital formation, whose dynamics are expressed as:

$$Q_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + \frac{\pi_{t+1}}{R_{t+1}^K} Q_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 = 1. \quad (9)$$

Here we have used the definition of the pricing kernel obtained from optimal condition as:

$$\frac{m_{0,t+1}}{m_{0,t}} = \frac{\pi_{t+1}}{R_{t+1}^K}. \quad (10)$$

2.3 Financial Intermediaries

Our model also includes financial intermediaries that give funds to the capital producers and holds equity of the final goods producers and the capital producers. Households can put their money into this financial intermediary and receive a return that is composed of the returns on equities and the funds that are given to the capital producers. Financial intermediaries maximize their real

[10] As shown in Dupor (2001), Carlstrom and Fuerst (2005) and Woodford (2003), realistic empirical properties of this sort of the model, especially in responses to a policy shock, are only obtained with a capital adjustment cost.

profits:

$$
\begin{aligned}
\Pi_t^{FI} &= R_t^K \frac{A_t}{P_t} + \left(\overline{P}_t^F + D_t\right) x_t^F + \left(\overline{P}_t^K + \Pi_t^K\right) x_t^K \\
&\quad -\overline{P}_t^F x_{t+1}^F - \overline{P}_t^K x_{t+1}^K - \frac{A_{t+1}}{P_t} + \frac{\left(FA_{t+1}^w + FA_{t+1}^r\right)}{P_t} - R_t \frac{FA_t^w + FA_t^r}{P_t},
\end{aligned}
$$

where x is the share of the equity, and $\overline{P}$ is the price of such assets, where superscript F denotes those of final goods firms while K denotes those of capital producers. From the first order necessary conditions, we obtain the arbitrage conditions:[11]

$$\frac{m_{0,t}}{m_{0,t+1}} = \frac{E_{t+1}^F + D_{t+1}}{E_t^F}, \tag{11}$$

$$\frac{m_{0,t}}{m_{0,t+1}} = \frac{E_{t+1}^K + \Pi_{t+1}^K}{E_t^K}, \tag{12}$$

$$\frac{m_{0,t}}{m_{0,t+1}} = \frac{R_{t+1}^K}{\pi_{t+1}},$$

and

$$\frac{m_{0,t}}{m_{0,t+1}} = \frac{R_{t+1}}{\pi_{t+1}}, \tag{13}$$

where we define E as the real amount of equity holdings.[12] Since we only conduct perfect foresight simulations in this paper, all assets produce the same rate of return. Therefore, portfolio choice is irrelevant as is obvious from the condition below:

$$\frac{P_{t+1}^F + D_{t+1}}{P_t^F} = \frac{P_{t+1}^K + \Pi_{t+1}^K}{P_t^K} = \frac{R_{t+1}^K}{\pi_{t+1}} = \frac{R_{t+1}}{\pi_{t+1}}.$$

2.4 Government

The government simply collects a lump-sum tax to subsidize firms in order to eliminate the distortion caused by monopolistic competition. We assume that each agent faces the same level of the lump sum tax T. Therefore, the budget constraint which the government faces is simply:

$$\frac{1}{\kappa - 1} Y_t = \left(N_t + N_t^r\right) T_t, \tag{14}$$

where N denotes the population of workers while N^r is that of retirees.

[11] Since these arbitrage conditions hold only in the absence of the unexpected shocks, these do not apply on the initial date when the shock hits the economy. Therefore, we distribute the initial profits from financial intermediaries between different agents according to their amounts of financial assets holdings in period zero, namely in the stationary population.

[12] This equals to the relative price of equities since the share is unity in aggregate.

2.5 Households

In the life-cycle economy assumed in this model, there are two types of households: retirees and workers.

2.5.1 Retiree

Retirees, denoted by superscript r, who were born at j and become retired at k, are assumed to maximize their recursive utility[13] from consumption and leisure $1-L$:[14]

$$V_t^{rjk} = \left\{ \left[\left(C_t^{rjk} \right)^v \left(1 - L_t^{rjk} \right)^{1-v} \right]^\rho + \beta\gamma \left(V_{t+1}^{rjk} \right)^\rho \right\}^{\frac{1}{\rho}},$$

where ρ determines the intertemporal elasticity of substitution and ν defines the marginal rate of transformation between consumption and leisure. Since rate of survival of retirees, specifically his probability of surviving until the next period, is assumed to be γ, future welfare is discounted by common subjective discount factor multiplied with γ. This optimization problem is subject to their intertemporal budget constraint:

$$\frac{FA_{t+1}^{rjk}}{P_t} = \left(\frac{R_t}{\gamma} \right) \frac{FA_t^{rjk}}{P_t} + \frac{W_t}{P_t} \xi L_t^{rjk} - C_t^{rjk} - T_t,$$

where FA is the amount of their financial asset holding and $\xi \in [0,1]$ is the relative marginal product of labor of the retirees to workers. It is natural to assume that retirees receive less compensation than workers.[15] The real rate of return is now $\frac{R_t}{\gamma}$ since bequests are distributed among retirees by the life insurance company in a perfectly competitive market.[16] From the first order necessary conditions, we can derive the relationship between consumption and labor supply:

$$L_t^{rjk} = 1 - \frac{1-v}{v} \frac{P_t}{\xi W_t} C_t^{rjk},$$

and the consumption Euler equation:

$$C_{t+1}^{rjk} = \left[\beta R_{t+1} \left(\frac{1}{\pi_{t+1}} \right)^{1-\rho+v\rho} \left(\frac{W_t}{W_{t+1}} \right)^{(1-v)\rho} \right]^{\frac{1}{1-\rho}} C_t^{rjk}. \tag{15}$$

[13] The functional form is quite similar to the one in Lucas and Stokey (1984), and Epstein and Zin (1989).

[14] A Cobb-Douglas utility function satisfies the balanced growth restriction.

[15] There are other interpretations for ξ. It can be considered to reflect the incentive and legal structure, pension system, the relative labor income tax rate, or the custom taken by firms to reduce the total personnel expenses.

[16] In this paper, insurance is perfect within generations, but transitional risk from a worker to a retiree is uninsurable. For details, see Blanchard (1985) and Yaari (1965).

From these equations, we derive the consumption function, in the form of total wealth multiplied by the marginal propensity to consume out of wealth $\epsilon\theta$:

$$C_t^{rjk} = \epsilon_t\theta_t \left[\left(\frac{R_t}{\gamma} \right) \frac{A_t^{rjk}}{P_t} + H_t^{rjk} + PT_t^{rjk} \right]. \tag{16}$$

By iterating the budget constraint forward, human wealth H can be expressed in a recursive manner:

$$H_t^{rjk} = \frac{W_t}{P_t} \xi L_t^{rjk} + \frac{\gamma \pi_{t+1}}{R_{t+1}} H_{t+1}^{rjk}.$$

Similarly, the present discounted value of lump sum taxes PT, is expressed as:

$$PT_t^{rjk} = -T_t + \frac{\gamma \pi_{t+1}}{R_{t+1}} PT_{t+1}^{rjk}$$

We can then derive the dynamic equation for the marginal propensity to consume:

$$\epsilon_t\theta_t = 1 - \frac{\epsilon_t\theta_t}{\epsilon_{t+1}\theta_{t+1}} \gamma R_{t+1}^{\frac{\rho}{1-\rho}} \beta^{\frac{1}{1-\rho}} \left(\frac{1}{\pi_{t+1}} \right)^{\frac{v\rho}{1-\rho}} \left(\frac{W_t}{W_{t+1}} \right)^{\frac{(1-v)\rho}{1-\rho}}. \tag{17}$$

Furthermore, we can find a value function that satisfies the above conditions:

$$V_t^{rjk} = (\epsilon_t\theta_t)^{-\frac{1}{\rho}} C_t^{rjk} \left(\frac{1-v}{v} \frac{P_t}{\xi W_t} \right)^{1-v}. \tag{18}$$

2.5.2 Workers

Workers, denoted by superscript w, who were born at j, maximize their recursive utility:

$$V_t^{wj} = \left\{ \left[\left(C_t^{wj} \right)^v \left(1 - L_t^{wj} \right)^{1-v} \right]^\rho + \beta \left[\omega V_{t+1}^{wj} + (1-\omega) \left(V_{t+1}^{rj} \right) \right]^\rho \right\}^{\frac{1}{\rho}}$$

subject to

$$\frac{FA_{t+1}^{wj}}{P_t} = R_t \frac{FA_t^{wj}}{P_t} + \frac{W_t}{P_t} L_t^{wj} - C_t^{wj} - T_t,$$

where ω is the probability that the current worker will remain a worker in the next period. From the first order necessary conditions, we can derive the relationship between consumption and labor supply:

$$L_t^{wj} = 1 - \frac{1-v}{v} \frac{P_t}{W_t} C_t^{wj}.$$

and the consumption Euler equation:

$$\begin{aligned}
&\left[\left(C_t^{wj}\right)^v\left(1-L_t^{wj}\right)^{1-v}\right]^{\rho-1}\left(1-L_t^{wj}\right)^{1-v}v\left(C_t^{wj}\right)^{v-1} \\
= \;& \beta\left[\omega V_{t+1}^{wj}+(1-\omega)\left(V_{t+1}^{rj}\right)\right]^{\rho-1} \\
& v\frac{R_{t+1}}{P_{t+1}}\left(\begin{array}{c}\omega P_t\left(V_{t+1}^{wj}\right)^{1-\rho}\left(C_{t+1}^{wj}\right)^{v\rho-1}\left(1-L_{t+1}^{wj}\right)^{(1-v)\rho} \\ +(1-\omega)P_t\left(V_{t+1}^{rj}\right)^{1-\rho}\left(C_{t+1}^{rj}\right)^{v\rho-1}\left(1-L_{t+1}^{rj}\right)^{(1-v)\rho}\end{array}\right).
\end{aligned}$$

Then, we guess that the value function takes the form:

$$V_t^{wj}=(\theta_t)^{-\frac{1}{\rho}}C_t^{wj}\left(\frac{1-v}{v}\frac{P_t}{W_t}\right)^{1-v}, \tag{19}$$

where θ is the marginal propensity to consume for workers. This leads to the Euler condition:

$$\begin{aligned}
&\omega C_{t+1}^{wj}+(1-\omega)(\epsilon_{t+1})^{-\frac{1}{\rho}}C_{t+1}^{rj(t+1)}\left(\frac{1}{\xi}\right)^{1-v} \qquad (20)\\
= \;&\left[\beta\frac{R_{t+1}}{\pi_{t+1}}\left(\omega+(1-\omega)(\epsilon_{t+1})^{-\frac{1-\rho}{\rho}}\left(\frac{1}{\xi}\right)^{1-v}\right)\left(\frac{W_{t+1}}{W_t\pi_{t+1}}\right)^{(v-1)\rho}\right]^{\frac{1}{1-\rho}}C_t^{wj}.
\end{aligned}$$

As with the case for retirees, we are looking to identify the marginal propensity to consume out of wealth in the consumption demand:

$$C_t^{wj}=\theta_t\left(R_t\frac{A_t^{wj}}{P_t}+H_t^{wj}+PT_t^{wj}\right). \tag{21}$$

By using the consumption Euler equation and the consumption function, we can derive a dynamic equation as follows

$$\begin{aligned}
&\left\{1-\beta^{\frac{1}{1-\rho}}\left(\frac{P_tR_{t+1}}{P_{t+1}}\Omega_{t+1}\right)^{\frac{\rho}{1-\rho}}\left(\frac{P_t}{P_{t+1}}\frac{W_{t+1}}{W_t}\right)^{\frac{(v-1)\rho}{1-\rho}}\frac{\theta_t}{\theta_{t+1}}-\theta_t\right\}R_t\frac{A_t^{wj}}{P_t} \\
= \;&\left\{-1+\theta_t+\beta^{\frac{1}{1-\rho}}\left(\frac{P_tR_{t+1}}{P_{t+1}}\Omega_{t+1}\right)^{\frac{\rho}{1-\rho}}\left(\frac{P_t}{P_{t+1}}\frac{W_{t+1}}{W_t}\right)^{\frac{(v-1)\rho}{1-\rho}}\frac{\theta_t}{\theta_{t+1}}\right\}H_t^{wj} \\
&\left(H_t^{wj}+F_t^{wj}\right)-\left(\frac{W_t}{P_t}L_t^{wj}+D_t^{wj}\right)-\frac{P_{t+1}}{P_tR_{t+1}}\frac{\omega}{\Omega_{t+1}}H_{t+1}^{wj} \\
&-\frac{P_{t+1}}{P_tR_{t+1}}\frac{1}{\Omega_{t+1}}(1-\omega)(\epsilon_{t+1})^{-\frac{1}{\rho}}\left(\frac{1}{\xi}\right)^{1-v}\epsilon_{t+1}H_{t+1}^{rj},
\end{aligned}$$

where we define

$$\Omega_{t+1}=\omega+(1-\omega)(\epsilon_{t+1})^{-\frac{1-\rho}{\rho}}\left(\frac{1}{\xi}\right)^{1-v}. \tag{22}$$

This equation holds if

$$\theta_t = 1 - \beta^{\frac{1}{1-\rho}} \left(\frac{1}{\pi_{t+1}}\right)^{\frac{\rho v}{1-\rho}} (R_{t+1})^{\frac{\rho}{1-\rho}} \Omega_{t+1}^{\frac{\rho}{1-\rho}} \left(\frac{W_{t+1}}{W_t}\right)^{\frac{(v-1)\rho}{1-\rho}} \frac{\theta_t}{\theta_{t+1}}, \tag{23}$$

$$H_t^{wj} = \frac{W_t}{P_t} L_t^{wj} + \omega \frac{P_{t+1}}{P_t} \frac{H_{t+1}^{wj}}{R_{t+1}\Omega_{t+1}} + (1-\omega)(\epsilon_{t+1})^{\frac{\rho-1}{\rho}} \left(\frac{1}{\xi}\right)^{1-v} \frac{P_{t+1}}{P_t} \frac{H_{t+1}^{rj}}{R_{t+1}\Omega_{t+1}},$$

and

$$PT_t^{wj} = -T_t + \omega \frac{P_{t+1}}{P_t} \frac{PT_{t+1}^{wj}}{R_{t+1}\Omega_{t+1}} + (1-\omega)(\epsilon_{t+1})^{\frac{\rho-1}{\rho}} \left(\frac{1}{\xi}\right)^{1-v} \frac{P_{t+1}}{P_t} \frac{PT_{t+1}^{rj}}{R_{t+1}\Omega_{t+1}},$$

If these equations are satisfied, the surmised value function has a solution.

2.5.3 No Life-Cycle Benchmark

As a test-case for our model, we first examine the case where $\omega = 1$. With this assumption, workers live forever, and so we delete the life-cycle aspects of the model. Since this assumption does not affect the behavior in other sectors, we show just the households' choice problem. The equilibrium condition for households are obtained by maximizing the recursive utility as below:

$$V_t = \left\{ \left[(C_t)^v (1-L_t)^{1-v} \right]^\rho + \beta (V_{t+1})^\rho \right\}^{\frac{1}{\rho}},$$

subject to the standard budget constraint.

$$\frac{FA_{t+1}}{P_t} = R_t \frac{FA_t}{P_t} + \frac{W_t}{P_t} L_t - C_t - T_t.$$

2.6 Monetary Policy

Monetary policy follows a standard Taylor type instrument rule as in Taylor (1993):

$$R_{t+1} = R_{SS} + 1.5\pi_t + \frac{0.5}{4}\left(\frac{Y_t}{Y_t^F} - 1\right) + e_t, \tag{24}$$

where R_{SS} is the short-term nominal interest rate in stationary population, Y^F is the output in the flexible price equilibrium, computed simultaneously by assuming the absence of nominal rigidities. Since all variables are in quarterly terms, the coefficient on the output gap is divided by four. As is obvious from the setting of a Rotemberg-type adjustment cost and this equation, the target level of inflation is set to zero.

2.7 Aggregation

In this subsection, we first summarize the population growth in this model. Then, we transform equations that define individual behavior into aggregate form.

2.7.1 Population Growth

The dynamics of the population of workers are expressed as follows:

$$\begin{aligned} N_{t+1} &= (1-\omega+n)\,N_t+\omega N_t \\ &= (1+n)\,N_t, \end{aligned}$$

where n is the growth rate of workers, while that of retirees is:

$$N^r_{t+1}=(1-\omega)\,N_t+\gamma N^r_t.$$

Hence, assuming a stationary population, the ratio of the number of retirees to that of workers remains constant:

$$\frac{N^r}{N}=\frac{1-\omega}{1+n-\gamma}=\Gamma, \tag{25}$$

which means that both the working and retired populations grow at the same rate n.

2.7.2 Aggregation

If we assume the existence of a non-profit life insurance company that distributes wealth among retirees, the marginal propensity to consume is equated across all retirees. Therefore, subscripts j and k in most equations above can be removed. Below, we show the aggregate equilibrium conditions. The law of motion of assets held by retirees in aggregate is defined by:

$$\begin{aligned} \frac{FA^r_{t+1}}{P_t} &= R_t\frac{FA^r_t}{P_t}+\frac{W_t}{P_t}\xi L^r_t-C^r_t-\frac{\Gamma}{1+\Gamma}\frac{1}{\kappa-1}Y_t \\ &\quad +(1-\omega)\left(R_t\frac{FA^w_t}{P_t}+\frac{W_t}{P_t}L^w_t-C^w_t-\frac{1}{1+\Gamma}\frac{1}{\kappa-1}Y_t\right), \end{aligned} \tag{26}$$

where we express the expenses stemming from the lump sum tax by using equations (14) and (25). Next, we aggregate individual labor supply. This simply involves multiplying by the population of each category:

$$L^r_t=\Gamma N_t-\frac{1-v}{v}\frac{P_t}{\xi W_t}C^r_t, \tag{27}$$

and

$$L^w_t=N_t-\frac{1-v}{v}\frac{P_t}{W_t}C^w_t. \tag{28}$$

Furthermore, because the population growth rate in each category is $(1+n)$, the discount rate when computing financial and human wealth also changes. Therefore,

$$H^r_t=\frac{W_t}{P_t}\xi L^r_t+\frac{\gamma\pi_{t+1}}{(1+n)\,R_{t+1}}H^r_{t+1}, \tag{29}$$

$$H_t^w = \frac{W_t}{P_t} L_t^w + \omega \frac{H_{t+1}^w \pi_{t+1}}{(1+n) R_{t+1} \Omega_{t+1}} + (1-\omega) (\epsilon_{t+1})^{\frac{\rho-1}{\rho}} \left(\frac{1}{\xi}\right)^{1-v} \frac{H_{t+1}^r \pi_{t+1}}{(1+n) R_{t+1} \Omega_{t+1}}. \tag{30}$$

Similarly, we can rewrite the present discounted values for the lump sum tax:

$$PT_t^r = -\frac{\Gamma}{1+\Gamma} \frac{1}{\kappa - 1} Y_t + \frac{\gamma \pi_{t+1}}{(1+n) R_{t+1}} PT_{t+1}^r, \tag{31}$$

$$PT_t^w = -\frac{1}{1+\Gamma} \frac{1}{\kappa - 1} Y_t + \omega \frac{PT_{t+1}^w \pi_{t+1}}{(1+n) R_{t+1} \Omega_{t+1}} + (1-\omega) (\epsilon_{t+1})^{\frac{\rho-1}{\rho}} \left(\frac{1}{\xi}\right)^{1-v} \frac{PT_{t+1}^r \pi_{t+1}}{(1+n) R_{t+1} \Omega_{t+1}}. \tag{32}$$

2.7.3 Market Clearing

Finally, applying the market clearing conditions, the financial market equilibrium is

$$\frac{FA_{t+1}^w + FA_{t+1}^r}{P_t} = E_{t+1}^F + E_{t+1}^K + \frac{A_{t+1}}{P_t}. \tag{33}$$

The labor market clearing condition is

$$L_t = L_t^w + \xi L_t^r, \tag{34}$$

and the goods market clearing condition, namely the resource constraint, is expressed as:

$$Y_t = C_t^r + C_t^w + I_t. \tag{35}$$

2.7.4 System of Equations

The system of equations consists of structural equations:[17] (1), (2), (3), (4), (5), (6), (7), (8), (9), (10), (11), (12), (13), (16), (17), (18), (19), (21), (22), (23), (24), (26), (27), (28), (29), (30), (31), (32), (33), (34), (35), and their counterparts in a flexible equilibrium, that are computed by excluding nominal rigidity and therefore equation (24). Since we assume both deterministic technology and population growth, endogenous variables are de-trended: $C, D, F, H, I, K, PT, Y, \Pi^K$ are de-trended by ZN; A and FA is de-trended by ZNP; W is de-trended by ZP; L is de-trended by N; and V is by $Z^v N$.

3 Properties under Stationary Population

Parameters in the baseline life-cycle economy are depicted in Table 1. Since the model is solved with quarterly frequency, parameters are on quarterly terms.

[17] Some are derived before assuming the symmetry of agents and aggregation.

Table 1
Parameter values

Parameter	Value	Description and Definitions
θ	5	$\theta / (\theta - 1)$ is markup
σ	.25	Intertemporal elasticity of substitution
ϕ	60	Price adjustment cost
β	$1.04^{-.25}$	Subjective discount factor
ω	$1.023^{-.25}$	Probability of remaining as worker
α	0.667	Labor share
δ	$1.1^{2.5} - 1$	Capital depreciation rate
ρ	-3	$(\sigma - 1) / \sigma$
ν	.4	Utility weight on consumption
ξ	.6	Labor productivity of retirees
γ	$1.1^{-.25}$	Probability of remaining as retirees
S''	2.48	Second derivative of adjustment cost
Z	$1.01^{0.25} - 1$	Technology growth rate
n	$1.01^{0.25} - 1$	Population growth rate
Γ	0.21	Stationary population ratio of retirees over workers
ρ_z	0.8	Technology shock persistence

In our baseline life-cycle economy, we assume that people work from age 21 to 65, which is defined by ω, while people remain in retirement from age 66 to 75, defined by γ, based on Auerbach and Kotlikoff (1987). Other parameters which define life-cycle behavior are exactly the same as those in Gertler (1999) except for newly added parameters, namely θ, ϕ, and S". Elasticity of substitution among goods θ is set at the conventional value as calibrated in Smets and Wouters (2003), or Levin, Onatski, Williams and Williams (2005). The size of the Rotemberg-type adjustment cost ϕ is also set at the conventional value so that on average one-forth of firms change prices in each period in a linearly observational equivalent specification *a la* Calvo (1983). The size of capital adjustment cost, S" is taken from the estimated value in Christiano Eichenbaum and Evans (2005).

3.1 Values under Stationary Population

In this subsection, we study the long-run effects of different demographic structures. For this purpose, three versions of a stationary population are computed: (i) the base line life-cycle economy is a younger economy (low γ), (ii) an older economy (high γ), and (iii) no life-cycle economy. Concerning demographic differences, individuals' life expectancy becomes longer in (ii) older economy. In this economy, individuals retire at the age of 65 but are supposed to live until 85. To achieve this, the parameter that determines the probability of remaining as retirees, γ, is altered from $1.1^{-.25}$ to $1.05^{-.25}$. As a result of this alteration, the percentage of workers to retirees Γ changes from 21% to 39%. In (iii) no life-cycle economy, all households are symmetric, since they live infinitely long. An interpretation of this setting is that households can completely insure their risk of transition from workers and retirees, and are so altruistic that they consider the welfare of their descendents in their utility functions. Table 2 shows the values of major endogenous variables in (i) younger economy, (ii) older economy and (iii) no life-cycle economy.

Table 2
Values under Stationary Population

	c/y	k/y	R	a^r/k	θ	$\epsilon\theta$	l^w	l^r
(i)	.70	10.3	.01	.16	.017	.03	.41	.05
(ii)	.70	10.3	.01	.22	.014	.02	.41	.12
(iii)	.76	8.2	.02	–	.013*	–	.37*	–

* shows the variables of the representative agent in (iii).

First, we contrast the results in a model with heterogenous agents in incomplete market setup with homogenous agents in complete market. In the former, workers need to prepare for the future period in which they become retirees because their labor productivity is lower than that of workers. Their incentive to save tends to become higher in this economy. These movements should result in higher capital-output ratio, and therefore, real interest rates should be lower in (ii) older economy so that we can derive the usual results of a life-cycle model, namely dynamic inefficiency due to too much saving.[18]

As it turns out, comparing different life-cycle economies is somewhat counter-intuitive. With the same reasoning as above, it seems natural that the capital-output ratio is higher and real interest rates are lower in (ii) older economy than (i) younger economy. The results in Table 2, however, show the opposite albeit slightly. The capital output ratio is 10.2684 in (i) and 10.2677 in (ii) while real interest rates are 0.008478 in (i) and 0.008480 in (ii). Yet, this is not a puzzle at all, if we understand the labor supply of the retirees. The longer life expectancy in retirees induces two different effects, namely the tension between worker's marginal disutility for further saving and the retiree's marginal disutility for further reduction in leisure. As explained above, workers try to save more to prepare for longer retirement period. On the other hand, retirees need to work more even though their labor productivity is lower to maintain an optimized level of consumption. If the former dominates, real interest rates fall even in the economy where retirees have longer life expectancy. In this example, the latter dominates the former. Labor supply of retirees rises so much that workers accumulate less capital in (ii). This result that younger people save less in a economy where retiree's life expectancy is longer does, however, not hold generally. Under some parameter settings, real interest rates indeed become

[18] Comparison of marginal propsensity to consume and labor supply between life-cycle economies and no life-cycle economy is not trivial since we need to compute average marginal propensity to consume and labor supply of workers and retirees in life-cycle economies.

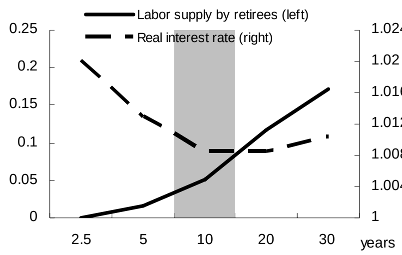

Figure 1: Long-run effects of average life expectancy

lower in a greyer society, which is consistent with the conventional wisdom as the second point raised by Bean (2004) in introduction. Thus, it is of great importance for a central bank to take demographic structure into account, since the natural rate of interest should be quite different across different demographic settings. We will check the sensitivity of our results in long-run properties model in the next subsection. Concerning the marginal propensity to consume, those of both workers and retirees decrease in (ii) because life expectancy increases.

3.2 Long-Run Properties under Alternative Demographic Structure

We next explore how the above results change when we assume alternative demographic structures by altering fundamental structural parameters, γ, and ξ. Figure 1 shows the transition of the retiree's labor supply and real interest rates by altering γ from $1.2^{-0.25}$ to $1.025^{-0.25}$. Accordingly, the horizontal axis shows the average retirement years. Furthermore, values in shaded region coincide with those in (i) younger economy. As explained in the above subsection, the labor supply of retirees increases, as life expectancy grows. The relationship between the worker's saving and life expectancy is, however, not monotonic. While the average length of retirement is below 10 years, real interest rates fall, as consistent with conventional views. On the other hand, the length becomes longer than 10 years, real interest rates rise due to much higher labor participation by retirees.

How does the relative labor productivity of retirees to workers change due to movements in the stationary population? To answer this question, we compute the stationary population for ξ being 0.4 to 1.0 as shown in Figure 2. As the labor productivity becomes higher, retirees tend to work longer since they can receive more compensation per unit of labor supply. Furthermore, this implies that workers become less worried about becoming retirees and less productive. Hence, they consume more and save less. This results in higher real interest rates as the relative productivity of workers increases. Another intriguing result from this exercise is that retiree's labor supply is much lower than that of

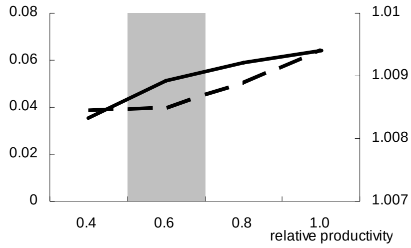

Figure 2: Long-run effects of labor productivity of retirees

workers, even though there is no difference in labor productivity between workers and retirees, namely the economy is at the far right in the horizontal axis. In this situation, the only intrinsic difference between workers and retirees is the subjective discount factor. The fact that retirees have a lower subjective discount factor makes the marginal propensity to consume higher for retirees than for workers. Therefore, even if the labor productivity is the same across workers and retirees, retirees choose to consume more and to have additional leisure than increase labor supply.

We will check how above different demographic structures may alter the dynamic properties, namely the degree of amplification of impulse responses in the next section.

4 Dynamic Properties

In this section, we answer the two major questions raised in this paper, namely whether the structural shocks to the economy have asymmetric effects on workers and retirees and whether the considerations of life-cycle and demographic structure alter the dynamic properties of the solution. For this aim, we first analyze the impulse responses to a level technology shock. Then, we investigate the transmission mechanism of a monetary policy shock. The reason why we choose these two shocks is that a technology shock can possibly have asymmetric effects on workers and retirees through the difference in labor productivity, while a positive shock on nominal interest rates may enhance the welfare of retirees since they rely more on financial assets.

4.1 Technology Shock

In this subsection, we study the impulse responses to a positive technology shock. First, we show impulse responses in (iii) no life-cycle economy as a benchmark.[19] Then, we compare these benchmark responses with those ob-

[19] This is because the model in this paper differs from the standard model in utility specification. There have been very few researches on the new Keynesian model with a recursive

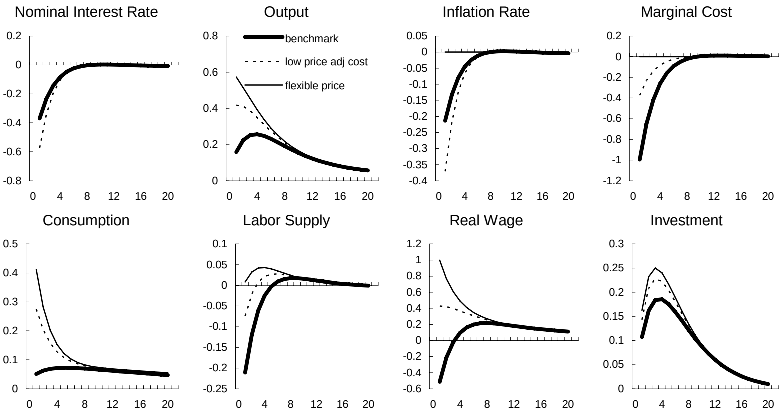

Figure 3: Responses to a technology shock in no life-cycle economy

tained in (i) younger economy and (ii) older economy. Finally, we demonstrate the shifts in impulse responses as the life expectancy of retirees becomes longer and their relative productivity becomes higher.

4.1.1 Benchmark: No Life-Cycle Economy

The thick black lines in Figure 3 show the impulse responses to a shock to the level of technology in (iii) no life-cycle economy. Although the model is based on exotic preference, responses are similar to those obtained in standard dynamic new Keynesian model with capital. A shock to the level of technology naturally increases the level of output, consumption and investment. Since the investment adjustment cost is embedded in the model, the response of investment is hump shaped. At the same time, however, deflation occurs, and therefore the nominal interest rate falls. Unlike the technology shock usually employed in a dynamic new Keynesian model without capital, explained in Walsh (2003) and Woodford (2003), the shock assumed in this model is a level shock. Since the shock decays gradually, it is recognized as a negative growth shock in the Euler equations of a log linearized system, where the percentage deviation of current consumption from its stationary population value receives positive effects from growth shock of technology. Therefore, even though GDP increases, the inflation rate and nominal interest rates are decreased.

Furthermore, as emphasized in Gali (1999) and Gali and Rabanal (2004), a shock to the level of technology reduces the labor supply due to the sticky price mechanism embedded in this model. Since monetary policy does not fully offset the distortion caused by varying the markup, the ability for a full increase in output reflecting the shock which expands the production frontier is limited. Therefore, a technology shock reduces labor supply in this model. The

utility.

response of the real wage seems somewhat puzzling. The real wage decreases after a positive technology shock. This, however, is not puzzling at all. In a baseline case, after a level technology shock hits the economy, the existence of the Rotemberg-type price adjustment cost makes final goods' producers increase the markup and therefore, lower the marginal cost. The effect of the lowered marginal cost dominates the increase in technology itself. Therefore, as the labor demand curve shifts in, the real wage is reduced right after a shock hits the economy. Thin dotted lines show the responses when the Rotemberg-type price adjustment cost ϕ is reduced by one-fifth of that in the baseline case.[20] In this case, lowered price adjustment cost makes the markup less variable. Hence, a positive technology shock raises the real wage immediately. As consistent with theory, both labor supply and the real wage increase when the price is flexible as depicted by thin solid lines. Whether real wage decreases after the positive technology shock depends also on the investment adjustment cost. When monetary policy cannot fully accommodate shocks to technology, firms react by reducing their inputs mainly via labor demand since they face an investment adjustment cost. Thus, the leftward shift of the labor demand becomes more significant when the investment adjustment cost is higher.[21]

4.1.2 Societal Aging Effects

Here, we compare the responses in two life-cycle economies so that we can understand how the societal aging influences the time-properties of the economy. Then, we further investigate how shocks can have asymmetric effects on workers and retirees, respectively.

Figure 4 shows the responses to a technology shock in life-cycle economies. The thin lines demonstrate those in (i) younger economy and dotted lines show those in (ii) older economy. Overall, responses in life-cycle economies are similar to those obtained in the economy with homogeneous agents. There are, however, a few intriguing differences. First, a level technology shock has quite different effects on workers and retirees and hence, there could exist a welfare trade-off between workers and retirees when stabilizing the economy. Second, consumption in retirees shows much more of an increase in (ii) older economy. As a result, responses in (ii) older economy become more similar to those in (iii) no life-cycle economy.

On the first point, the responses of workers are both quantitatively and qualitatively quite different from those of retirees. For example, both responses of consumption and labor supply are much smaller especially in (i) younger economy, while consumption of workers increase right after a positive technology shock,[22] that of retirees is almost unchanged except for the initial period due

[20] As a result, 50% of firms can change prices in each period.

[21] This implies that introduction of habit persistence or labor adjustment cost will alter responses as shown in Vigfusson (2004). Furthermore, if sticky wage *a la* Erceg, Henderson and Levin (2000) is included, we will not see an immediate decrease in real wage.

[22] Initial decrease is due to the decrease in labor supply and real wage that are explained in the previous subsection.

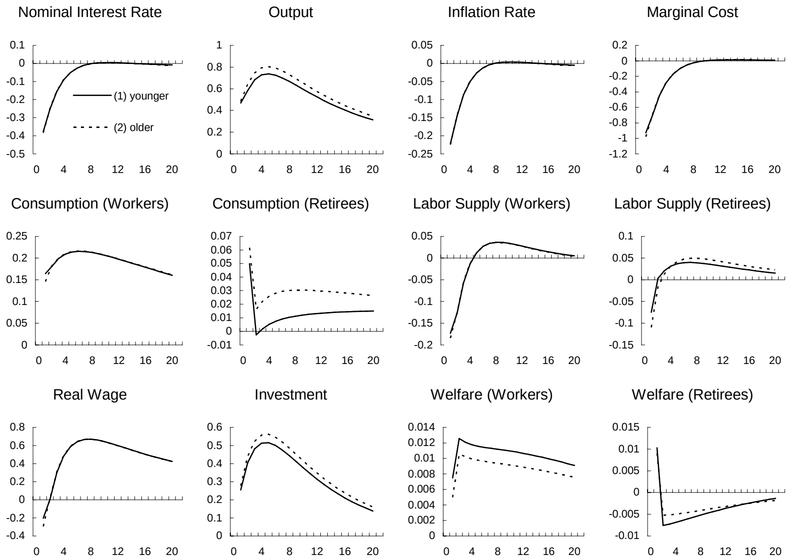

Figure 4: Responses to a technology shock in life-cycle economies

to the theoretically consistent distribution of initial profits.[23] Since retirees are less productive than workers, their labor supply is much smaller than those of workers, as shown in the previous section. Therefore, the gain from higher technology affects workers more than retirees. Consequently, the welfare, namely the recursive utility, of workers rises after a positive technology shock, but that of retirees is lowered since they need to work more even though their productivity is low.

Concerning the second point, comparing the responses between in (i) younger economy and in (ii) older economy leads us to notice that differences between workers and retirees become smaller in (ii) older life-cycle economy. As shown in previous section, in (i) older economy, retirees need to increase their labor supply to maintain optimal level of consumption. Thus, as life expectancy grows, retirees work more and therefore, retirees can enjoy the benefit of higher technology as much as workers can. Consequently, responses in (ii) older life-cycle economy becomes more similar to those in (iii) no life-cycle economy.

[23] As has been mentioned, how to distribute the initial profit at the time when the shock hits the economy determines the responses at the first period. In this simulation, a positive technology shock increases the initial profit, part of which is distributed to retirees as well. This results in an initial increase of retiree's consumption. As you will see later, the opposite happens with the monetary policy shock.

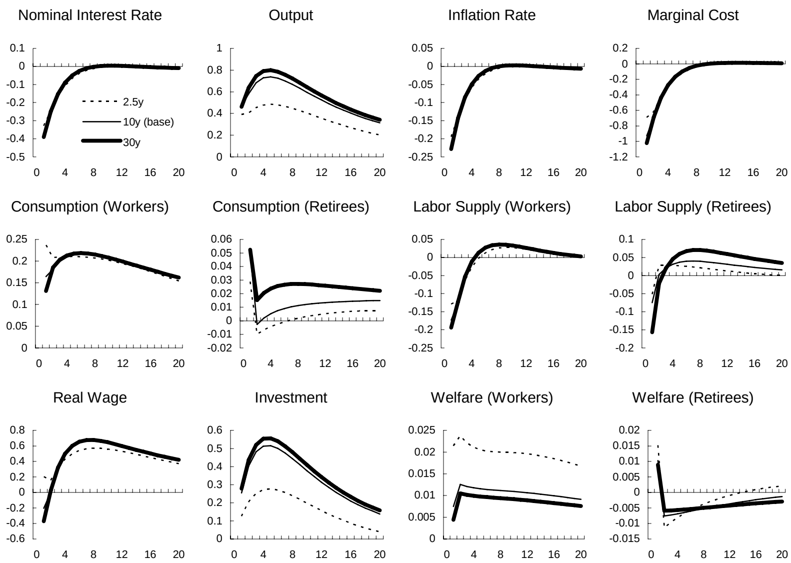

Figure 5: Responses to a technology shock as retiree's life expectancy becomes longer

4.1.3 Sensitivity Analysis

Here, we show how different demographic setups affect the responses to a technology shock.

Life Expectancy of Retirees Figure 5 shows the shifts in impulse responses to a level technology shock, as the life expectancy of retirees becomes longer. The thicker the line becomes, the longer the life expectancy of the retirees become. There, 2.5y means that average life-expectancy of retirees is 2.5 years. With shorter life expectancy, retirees rely more on income from returns on financial assets. Therefore, retirees do not alter consumption as well as leisure significantly. Reflecting these developments in retirees, workers can receive most of the benefits from higher technology. Since they do not have to prepare seriously for the life after retirement, they save (invest) less and therefore can enjoy huge welfare gain. On the other hand, the responses of aggregate output and the inflation rate become much larger as the retirees' labor supply in stationary population increase due to longer life expectancy as shown in Figure 1.

Productivity of Retirees Figure 6 shows the responses as the relative labor productivity of retirees to workers changes. This time, the thicker the line becomes, the more productive the retirees become. Since there are no significant shifts in values in the stationary population as seen in Figure 2, different settings

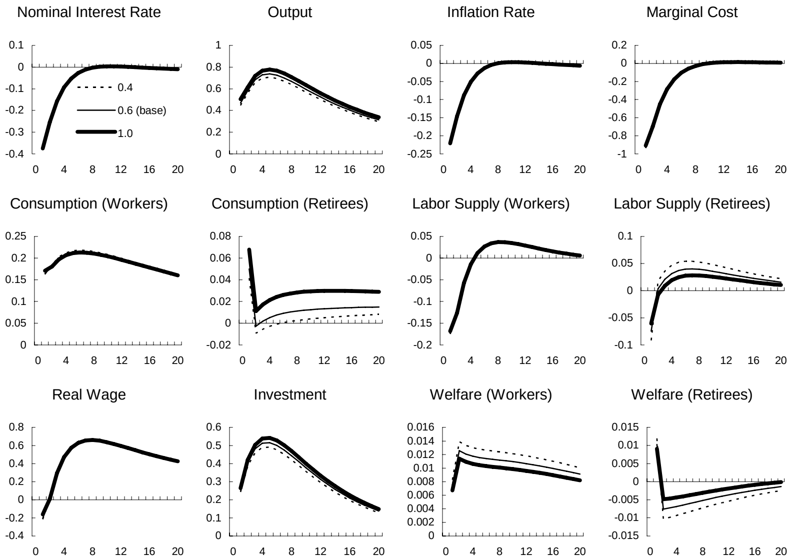

Figure 6: Responses to a technology shock as retiree's labor productivity becomes higher

only result in minor differences for most variables. Yet, as the retirees become more productive, the difference between workers and retirees becomes smaller.

4.2 Monetary Policy Shock

In this subsection, we study the impulse responses to a monetary policy shock. First, we show the impulse responses in (iii) no life-cycle economy as a benchmark. Then, we compare this benchmark responses with those obtained in (i) younger economy and (ii) older economy. Finally, we demonstrate shifts in impulse responses as the life expectancy of retirees becomes longer and their relative productivity becomes higher.

4.2.1 Benchmark: No Life-Cycle Economy

The impulse responses to a monetary policy shock in (iii) no life-cycle economy are shown in Figure 7. In this paper, we assume that the monetary policy shock is not serially correlated. A tightening shock reduces investment, consumption, and output via higher real interest rates. The decrease in output necessitates the reduction on inputs. Therefore, the real wage and eventually the marginal cost also fall. Consequently, we see a deflation after a positive shock to nominal interest rates.

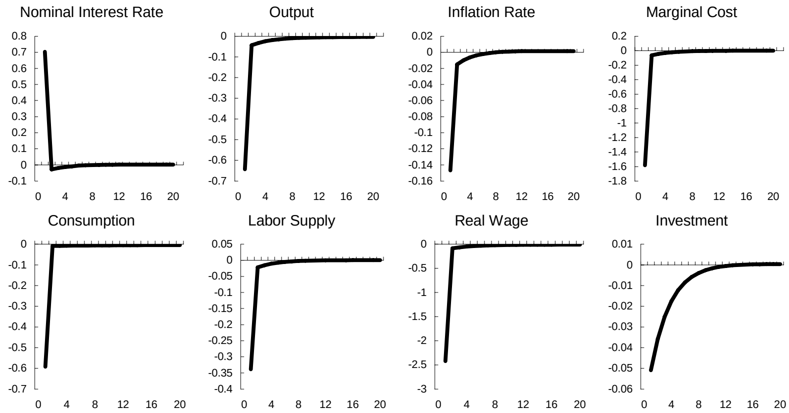

Figure 7: Responses to a monetary policy shock in no life-cycle economy

4.2.2 Societal Aging Effects

Figure 8 below compares the responses to a monetary policy shock in two life-cycle economies. As in Figure 4, the thin lines demonstrate those of (i) younger economy and dotted lines show those of (ii) older economy. Similarly to the case with a positive technology shock, responses of retirees are quite different from those of workers. Since retirees rely more on financial assets accumulated when they were workers, they mostly increase their consumption and leisure after a positive monetary tightening shock. In other words, income effects from increase in nominal interest rates dominates substitution effects for retirees. As a result, the welfare of workers falls significantly while that of retirees mostly increase. This implies that a central bank may face a severe policy trade-off: If the central bank cares more about retirees, or if monetary policy is determined mainly through opinions of older people because of their bargaining power in politics over younger people, there may be a bias towards higher nominal interest rates.[24]

Again, in (ii) older economy, differences between workers and retirees become smaller. Due to higher labor supply to maintain optimal level of consumption by retirees, they become more similar to workers. As a result, a tightening monetary policy shock has much larger negative effects on the macroeconomy as a whole. Below, we will see how the effectiveness of a surprise in monetary policy can change as the demographic structure is altered.

[24] A stochastic welfare analysis is thus very interesting. The aggregation problem for the appropriate pricing kernel makes stochastic analysis not very trivial. This is left for our future research.

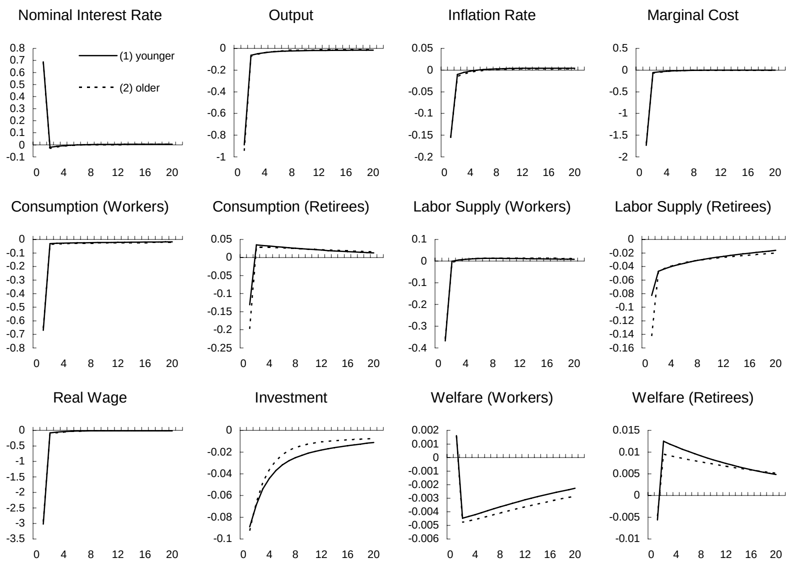

Figure 8: Responses to a monetary policy shock in life-cycle economies

4.2.3 Sensitivity Analysis

We here demonstrate how the life expectancy and productivity of retirees may change the responses to a monetary policy shock.

Life Expectancy of Retirees Figure 10 demonstrates the responses to a monetary policy shock when the life expectancy of retirees becomes longer. As shown in Figure 1, labor supply of retirees is the smallest when the average life expectancy of retirees is only 2.5 years. In this case, since retirees hardly rely on their labor income but instead on the returns from financial assets, the consumption of retirees mostly increases while labor supply is even reduced. Yet, as societal aging deepens, although income effect becomes stronger for retirees, that of workers becomes weaker. Hence, in aggregate, a tightening monetary policy shock still has negative impacts on the aggregate demand. Even though consumption of retirees increase, worker's consumption or the investment need to be lowered since there is no expansion of the production frontier.

Productivity of Retirees Figure 9 below demonstrates the responses with changing labor productivity of retirees. With lower relative productivity of retirees, their labor supply in stationary population is very small as show in Figure 2. Therefore, consumption of retirees mostly increases due to an increase in nominal interest rates. A positive shock on nominal interest rates increases the income from financial assets holdings for retirees, so they reduce

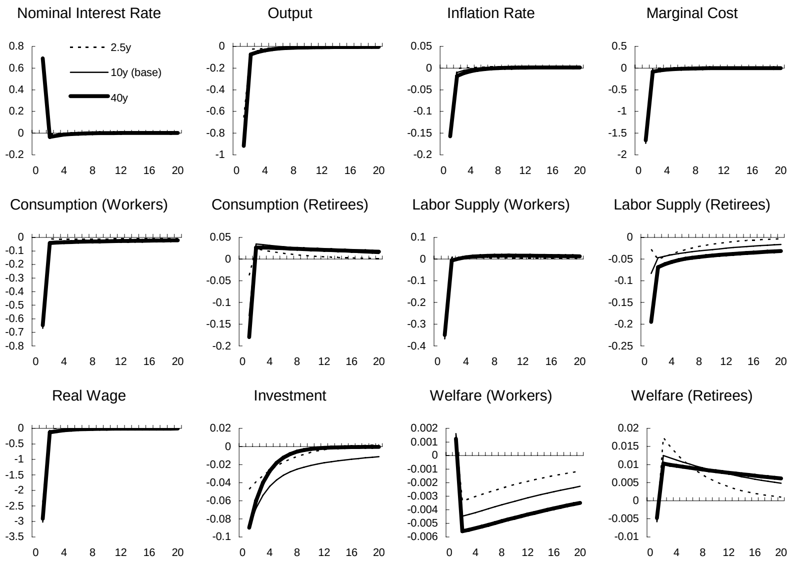

Figure 9: Responses to a monetary policy shock as retiree's life expectancy becomes longer

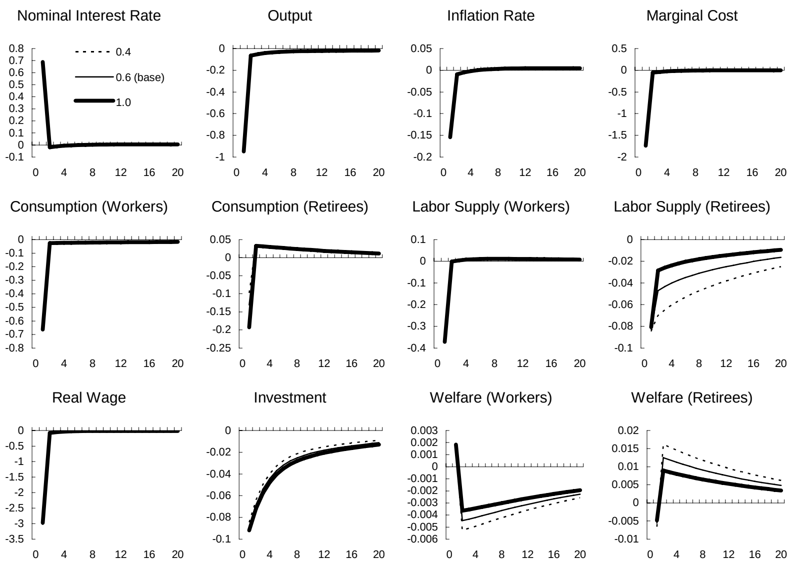

Figure 10: Responses to a monetary policy shock as retiree's labor productivity becomes higher

labor supply. As a result of higher consumption and lower labor supply, retiree's welfare improves the most when the labor supply of retirees is the lowest.

5 Conclusion

In this paper, we construct a dynamic new Keynesian life-cycle model based on Gertler (1999). We show that it is of great importance for a central bank to consider the demographics for sound monetary policy. First, the natural rate of interest differs as the demographics change. Second, the structural shocks to the economy have asymmetric effects on heterogeneous agents, namely workers and retirees. Differences in the labor productivity between workers and retirees make a positive technology shock enhance the worker's but decrease the retiree's welfare. Furthermore, due to the higher reliance on the financial asset, retirees become better off with a positive shock on nominal interest rates. This, especially, implies that a central bank may face a severe policy trade-off. If the central bank cares more about retirees, or if monetary policy is determined mainly through opinions of older people because of their bargaining power in politics over younger people, there may be a bias towards higher nominal interest rates. Yet, even though societal aging deepens, in aggregate, a tightening monetary policy shock still has negative impacts on the aggregate demand since worker's consumption or the investment need to be lowered while consumption of retirees increase. Finally, the demographic structure changes the dynamic properties of the solution, namely the degree of amplification in the impulse responses. The less retirees work under stationary population, the less volatile macroeconomic variables are against shocks. Under such circumstances, a positive technology shock does not increase retiree's labor supply and so the negative effects from a positive monetary policy shock are alleviated by an increase in retiree's consumption.

In particular, the latter two points are very relevant to the recent economy developments in Japan, where the total population is expected to start decreasing and baby boomers are to retire *en masse* around 2007. Depending on whether retirees continue to work or not, macroeconomic responses to shocks could be dramatically changed in foreseeable future. Not only would the effectiveness of monetary policy via surprise could change but also a positive monetary policy shock may even temporarily increase output and inflation.

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