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Masashi Saito

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## Changes in Perceptions of the Future: Macroeconomic Implications

Masashi Saito\*

### Abstract

We develop a real business cycle model that exhibits both the persistent and forecastable movements in consumption, hours, output, and investment which are broadly consistent with U.S. data. In the model, agents solve a signal extraction problem to learn about infrequent shifts in the drift of technology growth that are obscured by transitory shocks. We estimate a Markov regime-switching model of U.S. technology growth to calibrate the shock process. Real-time inferences about the drift of technology growth exhibit similar characteristics over time to the Index of Consumer Sentiment. Learning about the drift of technology growth provides an internal propagation mechanism. The mechanism works through the effects of revisions in the expectations about future technology on the decisions of forward-looking agents.

**Keywords:** Technology growth, regime switching, filtering, nonlinear impulse response, propagation

**JEL classification:** E13, E32, O41

\*Economist, Bank of Japan, Institute for Monetary and Economic Studies (E-mail: masashi.saitou@boj.or.jp)

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# 1 Introduction

This paper analyzes the macroeconomic implications of learning about the transitory and persistent components of technology growth. We develop a real business cycle model in which two types of shocks affect technology growth. One, a shock that induces infrequent shifts in the drift of technology growth, has a persistent impact on technology growth. The other is a shock that occurs frequently and has only a transitory impact on technology growth. We consider an environment of imperfect information in which agents do not observe the two shocks separately and solve a signal extraction problem, and analyze the economic dynamics when agents make decisions based on their inference. To grasp the effects of learning on macroeconomic activity, we also consider a model in which agents have full information and observe both shocks to technology growth.

We find that a model with both transitory and persistent shocks to technology growth explains several features of the U.S. data which a model with random-walk technology does not satisfactorily explain, including the persistence in the growth rates of hours, output, and investment, and the properties of the forecastable movements in macroeconomic variables. The process of learning under imperfect information is an essential element to replicate the persistence in the growth rates in macroeconomic variables. The model with full information, however, performs nearly as well as the model with imperfect information in replicating the properties of the forecastable movements.

A number of empirical studies report the presence of persistent movements in technology growth. French (2001), Roberts (2001), and Kahn and Rich (2003) report that U.S. long-term productivity growth slowed in the 1970s and then accelerated in the 1990s. Cagetti, Hansen, Sargent, and Williams (2002) and an empirical analysis in this paper find persistent movements in technology growth at the business cycle frequency: average technology growth is higher during expansion than during contraction over U.S. business cycles. Both findings indicate that technology growth has a persistent component.

In practice, it is difficult to distinguish between the persistent movements in technology growth and the transitory movements in real time. To illustrate this difficulty, Edge, Laubach, and Williams (2004) document that U.S. long-term productivity growth increased around the mid-1990s, but the corresponding estimates made by economists and professional forecasters changed little until 1999. Because of this, we

consider a case in which agents do not know whether an observed movement in technology growth is transitory or persistent and solve a signal extraction problem.

Our specification of the technology shock process is a special case of the stochastic process considered in Hamilton (1989). In particular, we assume that the drift of technology growth follows a two-state Markov regime-switching process and the transitory component of technology growth follows an *i.i.d.* process with normal distribution. We construct a quarterly U.S. technology series and estimate the parameters for the technology shock process. When we calibrate the model based on these estimates, historical evolution of the real-time inference about the drift of technology growth is similar to the movements in the Index of Consumer Sentiment over U.S. business cycles. For this reason, our model may be interpreted as one that incorporates endogenous fluctuations in consumer sentiment into an otherwise standard real business cycle model.

Changes in the inference about the drift of technology growth affect the nature of economic dynamics. This can be seen, for example, if we compare the effects of a positive transitory shock to technology growth under two information structures. Under full information, agents immediately understand that the increase in technology growth is transitory. In this case, the peak impacts on investment and hours worked occur immediately after the shock, as in a standard real business cycle model. In contrast, under imperfect information, agents do not observe the realizations of the two shocks to technology growth. Observing an increase in technology growth, the rational inference is that it is partly due to a transitory shock and partly due to a persistent shift in the drift. To the extent that agents believe the increase in technology growth is persistent, they have an incentive to reduce hours worked and investment upon impact with a plan to increase them in the future when the technology is more efficient. In this case, the peak impacts on hours, output, and investment occur several quarters after the shock.

We find that the model which includes both transitory and persistent shocks to technology growth explains two properties of the data that a standard model with random-walk technology does not satisfactorily explain. The first property of the data we consider in this paper is the positive autocorrelations in the growth rates of hours, output, and investment. A model with random-walk technology does not replicate this property of the data (Cogley and Nason, 1995). We find that the model with both transitory and persistent shocks to technology growth generates positive autocorrelations in the growth rates of hours, output, and investment, consistent with

the data. A key finding is that the process of learning about the drift of technology growth provides an internal propagation mechanism. The mechanism explains why the autocorrelation of output growth is greater than that of technology growth, a feature of the U.S. data. Contrarily, the model with full information predicts that output growth is no more persistent than technology growth.<sup>1</sup>

The second property of the data we consider in this paper is related to the forecastable movements in consumption, hours, output, and investment. In the U.S. data, about 50% of output movements are forecastable. In addition, consumption, hours, output, and investment are all expected to move in the same direction. In contrast, a standard real business cycle model produces few forecastable movements in output. Moreover, the standard model predicts that the forecastable movements in consumption, hours, and output are not all in the same direction (Rotemberg and Woodford, 1996). We find that the model with both transitory and persistent shocks to technology growth explains the properties of the forecastable movements as found in the data better than a model with random-walk technology. The model with full information performs nearly as well as the model with imperfect information on this dimension.

Several studies have considered the effects of a persistent shock to technology growth on aggregate activity within the framework of a real business cycle model (Campbell, 1994; Danthine, Donaldson, and Johnsen, 1998; Aguiar and Gopinath, 2004; Lindé, 2004; and Ireland and Schuh, 2006). These studies focus on the case of full information in which agents distinguish between transitory and persistent shocks to technology growth. In this paper, we show that the model with imperfect information replicates the autocorrelation property of the data better than the model with full information.

With motivations similar to those in our study, Pakko (2002), Tambalotti (2003), and Edge, Laubach, and Williams (2004) incorporate a signal extraction problem about the transitory and persistent components of technology growth into a dynamic general equilibrium model. In these studies, the persistent component of technology growth follows a linear autoregressive process.<sup>2</sup> We, on the other hand, assume that the per-

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<sup>1</sup>Several modifications to a standard real business cycle model are considered to enrich the internal propagation mechanism of the model. These include adjustment costs and gestation lags (Cogley and Nason, 1995), search frictions in the labor market (Andolfatto, 1996), agency costs in the financial market (Carlstrom and Fuerst, 1997), putty-clay technology (Gilchrist and Williams, 2000), indeterminacy (Schmitt-Grohé, 2000; Benhabib and Wen, 2004), and rational inattention (Luo and Young, 2005).

<sup>2</sup>Kydland and Prescott (1982) also consider a real business cycle model in which both transitory and persistent shocks affect technology, but neither of these shocks has a persistent impact on technology

sistent component of technology growth follows a two-state Markov regime-switching process. We will demonstrate that a model with a nonlinear shock structure like ours has the potential to explain nonlinearity and asymmetry in business cycle fluctuations as found in the data, including short and sharp contractions and long and gradual expansions. Cagetti, Hansen, Sargent, and Williams (2002) and David (1997) use a nonlinear technology shock process similar to ours when they study the asset pricing implications of a stochastic growth model. None of these studies, however, analyze the model's implications for the autocorrelations in the macroeconomic variables and the properties of the forecastable component of macroeconomic variables as we do in this paper.<sup>3</sup>

The rest of the paper is structured as follows. Section 2 presents the model and describes the calibration of the parameters. Section 3 presents the model dynamics under both full and imperfect information. Section 4 presents the model's predictions for both the autocorrelation in the growth rates of macroeconomic variables and the properties of the forecastable movements in consumption, hours, output, and investment. Section 5 concludes.

## 2 Model

This section presents the model, the optimality conditions, and the calibration of parameters.

### 2.1 Structure of the Economy

The model is a real business cycle model (King, Plosser, and Rebelo, 1988). The model abstracts from any nominal or real frictions to isolate the effect of our modification to both the technology shock process and the information structure.

We denote per capita variables with a lowercase letter and aggregate variables with 

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growth in their model. As we shall see, the presence of a persistent shock to technology growth is key to improving the model's predictions for both the persistence and the forecastable movements in macroeconomic variables.

<sup>3</sup>A similar nonlinear shock structure is applied to money growth (Andolfatto and Gomme, 2003), dividend growth (Moore and Schaller, 1996; Veronesi, 1999; Lettau, Ludvigson, and Wachter, 2005), the target inflation rate (Schorfheide, 2005), and the debt-output ratio (Davig, 2004) within the framework of a dynamic general equilibrium model.

a capital letter. The preferences of an agent are given by

$$U(c_t, n_t) = \ln c_t - n_t,$$

where  $c_t$  is the per capita consumption in period  $t$ , and  $n_t$  is the labor input per population in period  $t$ . Given the additive separability, the logarithmic specification for the utility from consumption is required to ensure the existence of balanced growth (King, Plosser, and Rebelo, 2002). The linearity of the disutility from hours worked can be motivated along the line of the indivisible labor model.

A representative household with  $H_t$  members in period  $t$  maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t H_t U(c_t, n_t).$$

By normalizing the number of households at one, we interpret  $H_t$  as the total population in period  $t$ . Assuming that the population grows at a constant rate  $g^p$ , we have  $H_t = H_{t-1} \exp(g^p) = H_0 (\exp(g^p))^t$ . Normalizing  $H_0 = 1$ , the objective function of the household can be written as  $E_0 \sum_{t=0}^{\infty} (\beta \exp(g^p))^t U(c_t, n_t)$ .

The production function is given by

$$Y_t = (A_t N_t)^\alpha K_t^{1-\alpha},$$

where  $Y_t = y_t H_t$  is the aggregate output,  $A_t$  is technology,  $N_t = n_t H_t$  is the aggregate labor input,  $K_t = k_t H_t$  is the aggregate capital stock, and  $\alpha$  is a parameter that governs the share of labor in total income.

The aggregate capital stock evolves according to a capital accumulation equation:

$$K_{t+1} = (1 - \delta)K_t + I_t,$$

where  $\delta$  is the constant depreciation rate of capital, and  $I_t = i_t H_t$  is the aggregate investment.<sup>4</sup>

In each period, the sum of the aggregate consumption and the aggregate investment

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<sup>4</sup>We also considered a model with capital adjustment costs. Since the results in this paper are not sensitive to the size of capital adjustment costs, we report the results for the case of no capital adjustment costs.



cannot exceed the aggregate output:

$$C_t + I_t \leq Y_t.$$

## 2.2 Technology Shock Process

The technology process is a special case of the stochastic process considered in Hamilton (1989).

Let  $\tilde{A}_t \equiv (\ln A_t - \ln A_{t-1})$  denote the growth rate of technology. We assume that technology growth has both transitory and persistent components:

$$\tilde{A}_t = g_t + \varepsilon_t.$$

The persistent component,  $\{g_t\}_{t=0,1,2,\dots}$ , follows a two-state, first-order, Markov regime-switching process defined by

$$\begin{aligned} \text{prob} [g_{t+1} = g^H | g_t = g^H] &= p \\ \text{prob} [g_{t+1} = g^L | g_t = g^H] &= 1 - p \\ \text{prob} [g_{t+1} = g^L | g_t = g^L] &= q \\ \text{prob} [g_{t+1} = g^H | g_t = g^L] &= 1 - q, \end{aligned} \tag{1}$$

where  $g^H$  and  $g^L$  are the two values that the persistent component can take, and  $p$  and  $q$  are the transition probabilities that govern the average durations of the two regimes. When the parameters  $p$  and  $q$  are between 0.5 and 1, a change in  $g_t$  has a persistent impact on technology growth. This component introduces a positive autocorrelation in technology growth.

The transitory component,  $\{\varepsilon_t\}_{t=0,1,2,\dots}$ , follows an *i.i.d.* process with the distribution given by  $N(0, \sigma_\varepsilon^2)$  and is independent of  $g_s$  for all  $s$ . A change in  $\varepsilon_t$  has a transitory impact on technology growth (or, equivalently, a permanent impact on the level of technology).<sup>5</sup>

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<sup>5</sup>We assume that the standard deviation of the transitory component is constant. Allowing for infrequent shifts in the volatility of shocks is an important avenue for future research. Lettau, Ludvigson, and Wachter (2005) and Sill (2005) find that a decline in the volatility of shocks which occurred in the United States in the mid-1980s explains a decline in the equity premium during the same period.

## 2.3 Information Structure

We consider two information structures. Under full information, agents observe both  $g_t$  and  $\varepsilon_t$  in each period. Under imperfect information, agents observe the growth rate of technology,  $\tilde{A}_t$ , or the sum of the transitory and persistent components,  $(g_t + \varepsilon_t)$ , but not the two components separately.<sup>6</sup>

## 2.4 Filtering under Imperfect Information

Let  $p[g_t = g^H | \Omega_t]$  denote the probability that agents assign to the possibility of currently being in regime  $g^H$  conditional on the observations of current and past technology growth rates,  $\Omega_t \equiv (\tilde{A}_t, \tilde{A}_{t-1}, \tilde{A}_{t-2}, \dots)$ . Thus,  $p[g_t = g^H | \Omega_t]$  represents agents' belief about the current state of the drift of technology growth.<sup>7</sup>

In each period, agents update their beliefs based on current and past observations of technology growth as well as their knowledge of both the shock structure and the parameter values  $(g^H, g^L, p, q, \sigma_\varepsilon^2)$ .<sup>8</sup> Agents enter period  $t$  with a belief formed in the previous period,  $p[g_{t-1} = g^H | \Omega_{t-1}]$ . Upon observing current technology growth,  $\tilde{A}_t$ , they update their beliefs using Bayes' rule. The belief follows a recursion:

$$\begin{aligned}
 & p[g_t = g^H | \Omega_t] \\
 & \quad f(\tilde{A}_t | g_t = g^H, \Omega_{t-1}) \times \sum_{i=g^H, g^L} \{p[g_t = g^H | g_{t-1} = i] \times p[g_{t-1} = i | \Omega_{t-1}]\} \\
 = & \frac{\quad}{\sum_{j=g^H, g^L} \left[ f(\tilde{A}_t | g_t = j, \Omega_{t-1}) \times \sum_{i=g^H, g^L} \{p[g_t = j | g_{t-1} = i] \times p[g_{t-1} = i | \Omega_{t-1}]\} \right]}, \quad (2)
 \end{aligned}$$

where the transition probabilities  $p[g_t = j | g_{t-1} = i]$  for  $j, i = g^H, g^L$  are defined by (1). Given that  $\varepsilon_t \sim i.i.d.N(0, \sigma_\varepsilon^2)$  independent of  $g_s$  for all  $s$ , the conditional probability density functions are given by

$$f(\tilde{A}_t | g_t = j, \Omega_{t-1}) = \frac{1}{\sqrt{2\pi\sigma_\varepsilon}} \exp \left[ -\frac{1}{2\sigma_\varepsilon^2} (\tilde{A}_t - j)^2 \right]$$

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<sup>6</sup>Equivalently, agents observe both  $\tilde{A}_t$  and  $g_t$  under full information and only  $\tilde{A}_t$  under imperfect information.

<sup>7</sup>The case of full information can be considered a special case in which the belief takes only one of the two values, 0 or 1.

<sup>8</sup>We maintain the assumption of rational expectations. Bullard and Duffy (2004) consider a stochastic growth model in which agents learn about a structural break in productivity growth using a recursive adaptive learning algorithm.

for  $j = g^H, g^L$ .<sup>9</sup>

Given the belief, the inference about the persistent component of technology growth is

$$E[g_t|\Omega_t] \equiv g^H \times p[g_t = g^H|\Omega_t] + g^L \times \{1 - p[g_t = g^H|\Omega_t]\},$$

which is a linear transformation of the belief. Also, the inference about the transitory component of technology growth is

$$E[\varepsilon_t|\Omega_t] = \tilde{A}_t - E[g_t|\Omega_t].$$

The state of the economy is characterized by  $(K_t, g_t, \tilde{A}_t)$  under full information and  $(K_t, p[g_t = g^H|\Omega_t], \tilde{A}_t)$  under imperfect information.<sup>10</sup>

## 2.5 Normalizations

Along a balanced growth path with growth rate of technology  $g$ , the per capita consumption, output, investment, and capital stock all grow at rate  $g$ . We normalize all the per capita variables (except hours worked per capita, which is stationary) by the level of technology so that the normalized variables are stationary. We write the normalized variables as

$$\tilde{c}_t \equiv \frac{c_t}{A_t}, \tilde{i}_t \equiv \frac{i_t}{A_t}, \tilde{k}_t \equiv \frac{k_t}{A_t}, \tilde{y}_t \equiv \frac{y_t}{A_t}.$$

In terms of the normalized variables, the objective function,<sup>11</sup> the production func-

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<sup>9</sup>The numerator on the right-hand side (RHS) of equation (2) is the likelihood that the observed technology growth in the current period,  $\tilde{A}_t$ , is generated under regime  $g^H$ . The denominator of the RHS is the sum of the likelihood that  $\tilde{A}_t$  is generated under regime  $g^H$  and the likelihood that  $\tilde{A}_t$  is generated under regime  $g^L$ . The numerator can be decomposed into two components: (i) the likelihood that  $\tilde{A}_t$  is generated under  $g_t = g^H$  and  $g_{t-1} = g^H$ ; and (ii) the likelihood that  $\tilde{A}_t$  is generated under  $g_t = g^H$  and  $g_{t-1} = g^L$ . Similarly, the denominator can be decomposed into four components.

<sup>10</sup>Alternative representations of the state are possible. For instance, one could replace one of the states,  $p[g_t = g^H|\Omega_t]$ , with  $E[g_t|\Omega_t]$  under imperfect information.

<sup>11</sup>The objective function can be written as  $E_0 \sum_{t=0}^{\infty} (\beta \exp(g^p))^t U(\tilde{c}_t A_t, n_t) = E_0 \sum_{t=0}^{\infty} (\beta \exp(g^p))^t (\ln \tilde{c}_t - n_t) + E_0 \sum_{t=0}^{\infty} (\beta \exp(g^p))^t \ln A_t = E_0 \sum_{t=0}^{\infty} (\beta \exp(g^p))^t U(\tilde{c}_t, n_t) + E_0 \sum_{t=0}^{\infty} (\beta \exp(g^p))^t \ln A_t$ . Since the last term does not affect the preference ordering, we ignore this term in the optimization problem.

tion, the capital accumulation equation, and the resource constraint are given by

$$E_0 \sum_{t=0}^{\infty} (\beta \exp(g^p))^t U(\tilde{c}_t, n_t), \quad (3)$$

$$\tilde{y}_t = n_t^\alpha \tilde{k}_t^{1-\alpha}, \quad (4)$$

$$\tilde{k}_{t+1} = \frac{1}{\exp(g_{t+1} + \varepsilon_{t+1}) \exp(g^p)} \left\{ (1 - \delta) \tilde{k}_t + \tilde{i}_t \right\}, \quad (5)$$

$$\tilde{c}_t + \tilde{i}_t \leq \tilde{y}_t. \quad (6)$$

The state of the transformed economy is characterized by  $(\tilde{k}_t, g_t)$  under full information and  $(\tilde{k}_t, p[g_t = g^H | \Omega_t])$  under imperfect information. There is a nonstochastic steady state in which the normalized per capita variables are constant.

## 2.6 Optimality Conditions

Under both full and imperfect information, the optimality conditions include

$$\alpha \frac{\tilde{y}_t}{n_t} \frac{1}{\tilde{c}_t} = 1 \quad (7)$$

and

$$\frac{1}{\tilde{c}_t} = \beta E_t \left[ \frac{1}{\exp(g_{t+1} + \varepsilon_{t+1})} \frac{1}{\tilde{c}_{t+1}} \left\{ 1 - \delta + (1 - \alpha) \left( \frac{n_{t+1}}{\tilde{k}_{t+1}} \right)^\alpha \right\} \right]. \quad (8)$$

Equation (7) is the intratemporal optimality condition. This implies that a high level of consumption to output ratio is accompanied by a low level of hours worked. Equation (8) is the Euler equation. Equations (4), (5), (6), (7), and (8) jointly characterize the sequence of optimal allocations  $\left\{ \tilde{c}_t, n_t, \tilde{i}_t, \tilde{y}_t, \tilde{k}_{t+1} \right\}_{t=0,1,2,\dots}$  given an initial condition for  $\tilde{k}_0$  and the sequence of shocks  $\{g_t, \varepsilon_t\}_{t=0,1,2,\dots}$ .

Note that the expectation over  $g_{t+1}$  on the RHS of the Euler equation (8) is taken conditional on different information across full and imperfect information: under full information it is conditional on the realization of  $g_t$ , while under imperfect information it is conditional on the belief  $p[g_t = g^H | \Omega_t]$ . We now use this fact to rewrite the Euler equation under both full and imperfect information.

### 2.6.1 Full Information Case

When the current regime of the drift of technology growth is  $g^H$ , agents expect that  $g_{t+1}$  is  $g^H$  with probability  $p$  and  $g^L$  with the remaining probability  $(1 - p)$ . In this case, the Euler equation is

$$\begin{aligned} \frac{1}{\tilde{c}_t^H} = & \beta E \left[ \frac{1}{\exp(g^H + \varepsilon_{t+1})} \frac{1}{\tilde{c}_{t+1}^{H,H}} \left\{ 1 - \delta + (1 - \alpha) \left( \frac{n_{t+1}^{H,H}}{\tilde{k}_{t+1}^{H,H}} \right)^\alpha \right\} \right] \times p \\ & + \beta E \left[ \frac{1}{\exp(g^L + \varepsilon_{t+1})} \frac{1}{\tilde{c}_{t+1}^{L,H}} \left\{ 1 - \delta + (1 - \alpha) \left( \frac{n_{t+1}^{L,H}}{\tilde{k}_{t+1}^{L,H}} \right)^\alpha \right\} \right] \times (1 - p), \end{aligned} \quad (9)$$

where the unconditional expectation operator  $E$  is taken over  $\varepsilon_{t+1}$ .<sup>12</sup> Equation (9) holds for any current state  $\tilde{k}_t$ .

Instead, when the current regime of the drift is  $g^L$ , we can write the Euler equation as

$$\begin{aligned} \frac{1}{\tilde{c}_t^L} = & \beta E \left[ \frac{1}{\exp(g^L + \varepsilon_{t+1})} \frac{1}{\tilde{c}_{t+1}^{L,L}} \left\{ 1 - \delta + (1 - \alpha) \left( \frac{n_{t+1}^{L,L}}{\tilde{k}_{t+1}^{L,L}} \right)^\alpha \right\} \right] \times q \\ & + \beta E \left[ \frac{1}{\exp(g^H + \varepsilon_{t+1})} \frac{1}{\tilde{c}_{t+1}^{H,L}} \left\{ 1 - \delta + (1 - \alpha) \left( \frac{n_{t+1}^{H,L}}{\tilde{k}_{t+1}^{H,L}} \right)^\alpha \right\} \right] \times (1 - q). \end{aligned} \quad (10)$$

The Appendix provides the definitions of the variables in equations (9) and (10). We use equations (9) and (10) jointly to solve for a decision rule under full information.

### 2.6.2 Imperfect Information Case

The belief about the drift of technology growth is a state variable under imperfect information. When the current belief is  $p[g_t = g^H | \Omega_t]$ , agents expect that  $g_{t+1}$  is  $g^H$  with probability  $p \times p[g_t = g^H | \Omega_t] + (1 - q) \times (1 - p[g_t = g^H | \Omega_t])$  and  $g^L$  with the remaining probability  $(1 - p) \times p[g_t = g^H | \Omega_t] + q \times (1 - p[g_t = g^H | \Omega_t])$ . Thus, for any

<sup>12</sup>The conditional expectation operator in the original Euler equation (8) is replaced by an unconditional expectation operator because  $\varepsilon_{t+1}$  is *i.i.d.* and no information in period  $t$  helps predict it.

current state  $(\tilde{k}_t, p[g_t = g^H | \Omega_t])$ , we can write the Euler equation as

$$\begin{aligned} \frac{1}{\tilde{c}_t} &= \beta E \left[ \frac{1}{\exp(g^H + \varepsilon_{t+1})} \frac{1}{\tilde{c}_{t+1}^H} \left\{ 1 - \delta + (1 - \alpha) \left( \frac{n_{t+1}^H}{\tilde{k}_{t+1}^H} \right)^\alpha \right\} \right] \\ &\times \{ p \times p[g_t = g^H | \Omega_t] + (1 - q) \times (1 - p[g_t = g^H | \Omega_t]) \} \\ &+ \beta E \left[ \frac{1}{\exp(g^L + \varepsilon_{t+1})} \frac{1}{\tilde{c}_{t+1}^L} \left\{ 1 - \delta + (1 - \alpha) \left( \frac{n_{t+1}^L}{\tilde{k}_{t+1}^L} \right)^\alpha \right\} \right] \\ &\times \{ (1 - p) \times p[g_t = g^H | \Omega_t] + q \times (1 - p[g_t = g^H | \Omega_t]) \}, \end{aligned} \quad (11)$$

where the unconditional expectation operator  $E$  is taken over  $\varepsilon_{t+1}$  and the belief follows the recursion (2). The Appendix provides the definitions of the variables in equation (11).

Under both full and imperfect information, we solve for a decision rule using the projection method (Judd, 1992).

## 2.7 Calibration

A period in the model is a quarter. We set the labor income share, the discount factor, and the depreciation rate at the standard values in the literature:  $\alpha = 2/3$ ,  $\beta = 0.984$ , and  $\delta = 0.02$ . We set the quarterly population growth rate at  $g^p = 0.0037$ , which is the average population growth rate in the United States between 1959:I and 2002:I.

We estimate the technology shock parameters  $(p, q, g^H, g^L, \sigma_\varepsilon)$  using a quarterly U.S. technology series.<sup>13</sup> Table 1 reports the estimated shock parameters. These estimates imply that the two regimes of the drift of technology growth correspond in their average durations to the expansion and contraction in the business cycle.<sup>14</sup> Cagetti, Hansen, Sargent, and Williams (2002) interpret similarly the regime-switching drift of technology growth in their model.<sup>15</sup>

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<sup>13</sup>We construct a quarterly technology series using the data on hours, output, and capital, as described in the Appendix. Our construction of the technology series is similar to that in Beaudry and Portier (2006). We use the original code for Hamilton (1989) to estimate the technology shock parameters.

<sup>14</sup>The average duration of regime  $g^H$  is  $1/(1 - p) = 20.43$  quarters, while the average duration of regime  $g^L$  is  $1/(1 - q) = 3.27$  quarters. According to the National Bureau of Economic Research (NBER), the average durations of expansion and contraction during 1945:I-2001:IV are 19 quarters and 3.3 quarters, respectively.

<sup>15</sup>In theory, changes in the drift of technology growth could represent shifts in the long-term tech-

## 2.8 Historical Evolution of Belief and Consumer Sentiment

The upper panel of Figure 1 shows how agents update their beliefs about the drift of technology growth as they observe the U.S. technology in real time. We assume that agents know the shock parameters listed in Table 1 as of the starting date for our historical simulation, 1959:I. Agents use equation (2) when they update their beliefs. We set the initial belief in 1959:I at the unconditional belief given by  $p[g_t = g^H] = (1 - q)/(2 - p - q)$ .

As shown in Figure 1, most of the time agents strongly believe that the regime of the drift of technology growth is  $g^H$ . At certain times, which correspond roughly to the contraction periods in the U.S. business cycle, agents substantially revise their beliefs downward. The belief also displays small downward movements even at periods during which we know retrospectively that the U.S. economy was not in contraction. The last observation implies that agents find it difficult to distinguish between transitory and persistent changes in technology growth in real time.

The lower panel of Figure 1 plots the Index of Consumer Sentiment in the United States, constructed by the University of Michigan, Surveys of Consumers.<sup>16</sup> Comparing the upper and lower panels of Figure 1, we see that the belief of agents about the drift of technology growth implied by the model exhibits movement similar to the Index of Consumer Sentiment, especially at the business cycle frequency.<sup>17</sup> Thus, the model offers a simple explanation for changes in consumer sentiment.

## 3 Model Dynamics

In this section, we present the response of the model economy to transitory and persistent shocks to technology growth. In each case, we present the model dynamics under both full and imperfect information to illustrate the effects of gradual learning about the drift of technology growth. At the end of this section, we consider two experiments

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nology growth such as a slowdown in U.S. technology growth in the 1970s followed by a resurgence starting in the 1990s. Edge, Laubach, and Williams (2004) calibrate their model based on this interpretation.

<sup>16</sup>The index is constructed based on five survey questions. Three of them are concerned with the current economic conditions. The remaining two concern the perceptions of future economic conditions. Ludvigson (2004) describes the index and analyzes the relationship between the index and consumer spending.

<sup>17</sup>The inference about the transitory component of technology growth has little persistence, and thus is not very similar to the Index of Consumer Sentiment.

which show that the impulse response of the economy features nonlinearity and asymmetry when the shock process is nonlinear and agents have imperfect information, as in our model.

### 3.1 Transitory Shock to Technology Growth

Figure 2 presents the responses of the belief and macroeconomic variables to a 1% transitory shock to technology growth, holding the drift of technology growth at  $g^L$ .<sup>18</sup> Under full information, agents immediately understand that the increase in technology growth is transitory. In this case, the response of the economy is the same as in a standard real business cycle model with random-walk technology. In particular, growth rates of consumption, hours, output, and investment all increase upon impact and then decrease, tracking the movement of technology growth.

Under imperfect information, agents observe an increase in technology growth, but do not observe the two underlying shocks. They solve a signal extraction problem to infer the relative contribution of the transitory and persistent shocks. Initially, they attribute some fraction of the increase in technology growth to a persistent shock and the remainder to a transitory shock: the belief about the drift of technology growth is revised upward. Because of the belief revision, the wealth effect of the shock is larger under imperfect information than under full information. As a result, the increase in consumption is greater under imperfect information. Both the large wealth effect and the incentive for intertemporal substitution induce agents to reduce hours worked and investment upon impact. When agents expect that the production technology will be more efficient in the future, they reduce hours and investment upon impact and plan to increase them in the future.<sup>19</sup>

From Figure 2, we see that output growth is more persistent under imperfect information than under full information. Under imperfect information, a transitory shock to technology growth generates a persistent movement in output growth. This is because agents increase hours worked and investment after technology growth returns to its initial rate. Under full information, output growth is no more persistent than

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<sup>18</sup>In section 3.3, we show that the response of the economy would differ if we fix the drift of technology growth at  $g^H$ .

<sup>19</sup>Note that there is another kind of intertemporal substitution effect due to the fact that technology at the impact period is more efficient than in the pre-shock period. This effect alone works to increase hours and investment upon impact. The response shown in the figure represents the net of all the effects including those described in the text.



technology growth. These contrasting results between full information and imperfect information suggest that the process of learning provides a model with an internal propagation mechanism. We will return to this issue in the next section.

A number of empirical studies based on a structural vector autoregression (VAR) report that hours worked decline in response to a permanent increase in the level of technology.<sup>20</sup> Sticky price models provide one explanation for the negative conditional correlation between technology and hours (Galí, 1999). The impulse response under imperfect information shown in Figure 2 suggests that a real business cycle model which incorporates gradual learning about the drift of technology growth provides an alternative explanation.

### 3.2 Permanent Shock to Technology Growth

Next, we consider the effects of a permanent increase in technology growth. Figure 3 presents the responses of the belief and macroeconomic variables to a permanent shift in the drift of technology growth from  $g^L$  to  $g^H$ , holding the level of transitory shock at zero.<sup>21</sup> We assume that the regime of the drift has been  $g^L$  for many consecutive periods before a switch in regimes occurs in period 3.

Under full information, agents immediately understand that the increase in technology growth is permanent. They perceive a large wealth effect and greatly increase consumption. Since they expect a further improvement in technology in the future, they decrease hours worked and investment upon impact and plan to increase them in the future.<sup>22</sup>

Cochrane (1994) reports a related finding. He finds that a mere anticipation of future improvement in technology—good news about future technology without any change in current technology—leads to immediate increases in both consumption and leisure in a standard real business cycle model. Cochrane calls the increases in consumption and leisure a *binge* and *vacation* (Cochrane, 1994, p. 351). This description

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<sup>20</sup>Galí and Rabanal (2004) review this literature.

<sup>21</sup>When the parameters  $p$  and  $q$  are below one as in our estimates of these parameters, a switch in regimes is persistent, but not permanent. Here, we consider a permanent switch in regimes by setting  $p = q = 1$  for simplicity. We consider the effects of a persistent switch in regimes in the next subsection.

<sup>22</sup>As noted earlier, another kind of intertemporal substitution effect arises from the fact that technology at the impact period of the regime switch is more efficient than in the pre-shock period. This effect alone works to increase hours and investment upon impact.

is consistent with the short-run effects of a persistent increase in technology growth on consumption and leisure in our model.<sup>23</sup>

Under imperfect information, agents initially attribute some part of the observed increase in technology growth to a transitory shock. The initial response of the economy in this case contains some features of the efficient response of the economy to a transitory shock to technology growth under full information. In particular, the initial decreases in hours and investment and the initial increase in consumption under imperfect information are not as large as under full information.

### 3.3 Nonlinear Impulse Response

We conduct two experiments to illustrate that the model with imperfect information generates a nonlinear and asymmetric impulse response of the economy.<sup>24</sup> The first experiment shows that the response of the economy to a given shock differs depending on the initial belief of agents, which in turn depends on the history of shocks. The second experiment shows that a downward shift in the drift of technology growth produces a short and sharp decrease in output growth, while an upward shift in the drift of technology growth produces a long and gradual increase in output growth, similar to the cyclical pattern of output over U.S. business cycles.<sup>25</sup>

The decision rule of the economy is approximately linear in the belief, one of the state variables under imperfect information. Therefore, most of the nonlinearity and asymmetry in the impulse response of the economy we report here can be attributed to the nature of belief adjustment.

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<sup>23</sup>Barro and King (1984) and Cochrane (1994) note that it is difficult for a standard real business cycle model to generate a positive comovement in consumption, hours, and investment in response to a “news shock”—a shock that conveys news about the future technology but leaves the current technology unchanged. Our model is similar to the news shock models because both a persistent shock to technology growth under full information and a transitory shock to technology growth under imperfect information affect the economy similarly as a news shock. A difference is that, in our model, good news about the future is revealed to agents through an observation of current technology improvement. Several modifications to a real business cycle model have been considered to generate a positive comovement among macroeconomic variables following a news shock. Beaudry and Portier (2004) consider a multi-sector model, Den Haan and Kaltenbrunner (2004) introduce labor market matching, and Jaimovich and Rebelo (2006) introduce investment adjustment costs and modify the utility function.

<sup>24</sup>See Beaudry and Koop (1993) and Potter (1995) for empirical evidence for nonlinearity in the impulse response.

<sup>25</sup>The model produces another kind of nonlinearity in the impulse response: the size of shocks has nontrivial implications for the response of the economic activity.

### 3.3.1 Dependence on the Initial Belief

Consider two economies, one in which the technology growth rate has been  $g^H$  and the other in which the technology growth rate has been  $g^L$ , both for many consecutive periods.<sup>26</sup> Given the history of technology growth, the belief of agents is initially high in the former economy and low in the latter, as shown in Figure 4.

Given these different initial conditions for the belief, suppose that both economies experience a 1% positive transitory shock to technology growth in period 3. Assume that agents have imperfect information. Upon impact, agents with a low initial belief drastically revise their beliefs upward, perceive a large wealth effect from the observed increase in technology growth, and greatly increase consumption. Since they believe that technology will become more efficient in the future, they decrease hours worked and investment upon impact and plan to increase them in the future.

On the other hand, when the belief is initially high, the same shock has a relatively small impact on the belief. In this case, the response of the economy resembles the efficient response to a transitory shock to technology growth under full information: the growth rates of hours, output, and investment increase upon impact but then immediately return to their initial rates. The growth rates of consumption, hours, output, and investment all exhibit movements similar to technology growth.

### 3.3.2 Asymmetry over the Direction of the Regime Switch

Next, we show that the economy's response to a switch in regimes of the drift of technology growth in one direction differs from the response to a switch in regimes in the opposite direction.

The drift of technology growth switches between two regimes in a stochastic manner, with the *average* durations of the two regimes governed by the transition probabilities  $p$  and  $q$ . In particular, since  $p$  is larger than  $q$ , the average duration of regime  $g^H$  is longer than that of regime  $g^L$ .

Figure 5 presents the average response of the economy to a switch in regimes. The impulse response shown in this figure is the mean from 1,000 replications of a particular regime-switch experiment. For simplicity, we fix the level of transitory shock at zero

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<sup>26</sup>Specifically, we assume that the drift of technology growth has been  $g^H$  and the transitory shock has been zero in the former, and that the drift of technology growth has been  $g^L$  and the transitory shock has been zero in the latter.

throughout.

The top left panel of Figure 5 shows that, after an initial switch in regimes from  $g^L$  to  $g^H$ , technology growth tends to remain high because  $p$  is close to one. In contrast, the top right panel of Figure 5 shows that, after an initial switch in regimes in the opposite direction, technology growth rebounds relatively quickly because  $q$  is relatively low. In both cases, technology growth is expected to converge to the unconditional mean given by  $g^H \times (1 - q)/(2 - p - q) + g^L \times (1 - p)/(2 - p - q) = 0.0043$ .

The second row of Figure 5 presents the response of the belief under imperfect information. The impact response of the belief is larger in the case of a switch in regimes from  $g^L$  to  $g^H$  compared to the other case, although the magnitude of the initial change in technology growth is the same across the two cases. The reason is that, when the level of belief is initially high (as in the experiment of a switch in regimes from  $g^H$  to  $g^L$ ), the large value of the transition probability  $p$  implies that agents expect the regime  $g^H$  is likely to continue with a high probability. In this case, when agents observe a decrease in technology growth, they attribute a large fraction of the decrease to a transitory shock, and thus do not adjust their beliefs about the drift of technology growth as much.

On the other hand, when the belief is initially low (as in the experiment of a switch in regimes from  $g^L$  to  $g^H$ ), the small value of the transition probability  $q$  implies that agents expect the next regime switch will occur soon in the future. In this case, when agents observe an increase in technology growth, they greatly revise their beliefs upward. In both cases, the belief is expected to converge to a value around 0.95.

The third and fourth rows of Figure 5 present the response of economic activity under imperfect information. In the long run, consumption, output, and investment all grow at rate  $g^H \times (1 - q)/(2 - p - q) + g^L \times (1 - p)/(2 - p - q)$  and the growth rate of hours returns to zero in both experiments. The short-run effects differ greatly depending on the direction of the regime switch. The asymmetry in the belief adjustments across the two experiments is key to understanding this difference. Following a switch in regimes from  $g^L$  to  $g^H$ , agents greatly revise their beliefs upward and believe strongly that the technology growth will be persistently high in the future. Hours and investment decline and consumption increases greatly upon impact. The effect of improvement in technology on output is partially offset by a decline in hours. Thus, output growth responds only modestly upon impact. Hours and investment start increasing several periods after the shock. The peak impact on output growth occurs several periods

after the shock.

In contrast, in the case of a switch in regimes from  $g^H$  to  $g^L$ , the peak effect on economic activity occurs at the impact period. The reason is that the belief adjustment is small in this case, and therefore agents make economic decisions as if the observed decrease in technology growth is purely transitory. Hours, output, and investment immediately fall, and the movements in the growth rates of these variables closely track the movement of technology growth.

### 3.3.3 Sharp Contractions and Gradual Expansions

In the U.S. business cycle, contractions are short and sharp and expansions are long and gradual (Beaudry and Koop, 1993; Potter, 1995). To the extent that expansions and contractions in the aggregate activity are related to shifts in the drift of technology growth, our model with imperfect information replicates this cyclical pattern (see the response of output growth in Figure 5). A key finding here is that the asymmetry in the belief adjustments helps a model with imperfect information to produce sharp contractions and gradual expansions. A full information model with asymmetric transition probabilities in the drift of technology growth produces short contractions and long expansions, but not sharp contractions and gradual expansions.<sup>27</sup>

## 4 Quantitative Implications

In this section, we compare the quantitative predictions of the model to the data. In particular, we consider the model's predictions for both the unconditional moments and the forecastable movements in macroeconomic variables.

Throughout this section, we compare the imperfect information model with both transitory and persistent shocks to technology growth to two other models. One is the full information model with both transitory and persistent shocks to technology growth. The other is a model in which the log of technology is a random walk with constant drift:

$$\tilde{A}_t = \mu + \hat{\varepsilon}_t,$$

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<sup>27</sup>Nieuwerburgh and Veldkamp (2006) present a model that produces sharp contractions and gradual expansions. In their model, the speed of learning about the current state of technology varies over the phase of business cycles. Specifically, agents recognize a technological regress in periods of high economic activity faster than a technological progress in periods of low economic activity.

where  $\tilde{A}_t \equiv (\ln A_t - \ln A_{t-1})$  and  $\hat{\varepsilon}_t \sim i.i.d.N(0, \sigma_{\hat{\varepsilon}}^2)$ . We consider this single-shock model because it is typically used as a benchmark in the literature when the quantitative properties of real business cycle models are analyzed.<sup>28</sup>

We calibrate the technology shock parameters in the single-shock model based on the U.S. technology series, the same series as that used to estimate technology shock parameters in the model with both transitory and persistent shocks to technology growth.  $\mu = 0.00427$  is the average quarterly growth rate of technology between 1959:I and 2002:I and  $\sigma_{\hat{\varepsilon}} = 0.0134$  is the standard deviation of technology growth during the same period. The remaining parameters  $(\alpha, \beta, \delta, g^p)$  are set equal to those in the model with both transitory and persistent shocks to technology growth.

## 4.1 Unconditional Moments

Table 2 reports the unconditional moments of the growth rates of macroeconomic variables both in the U.S. quarterly data and in the artificial data generated by three models. The moments for the models are the mean from 1,000 replications of stochastic simulation, with each simulation having 173 periods.<sup>29</sup>

In terms of standard deviations and contemporaneous correlations, the predictions of the three models are similar to each other, except that hours and investment are less volatile and consumption is more volatile in the model with two shocks and imperfect information than in the two other models. This is because a transitory shock to technology growth, which occurs more often than persistent shocks, has a large wealth effect under imperfect information because agents tend to confuse it with a persistent shock.<sup>30</sup>

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<sup>28</sup>See, for instance, Cogley and Nason (1995) and Rotemberg and Woodford (1996). One may also wish to consider a model in which technology growth is subject to a persistent shock only:  $\tilde{A}_t = \bar{\mu} + \rho\tilde{A}_{t-1} + \bar{\varepsilon}_t$  where  $\bar{\varepsilon}_t \sim i.i.d.N(0, \sigma_{\bar{\varepsilon}}^2)$ . This model generates a negative contemporaneous correlation between consumption and investment, as we can see from the experiment of a permanent shock to technology growth under full information. Since this negative correlation is not consistent with the data, we do not consider this model in this paper.

<sup>29</sup>To be precise, we run a simulation of 373 periods and discard the first 200 periods to reduce the effects of initial condition.

<sup>30</sup>Danthine, Donaldson, and Johnsen (1998) report a related finding. They consider a full information model in which both transitory and persistent shocks affect technology growth. They find that, when the transitory and persistent shocks have a positive contemporaneous correlation, the full information model generates a large volatility in consumption and small volatilities in hours and investment. The mechanism behind their result is similar to ours in the case of imperfect information: in our model, whenever agents with imperfect information observe a movement in technology growth, they initially believe that both transitory and persistent shocks are partially responsible.

## 4.2 Persistence and Propagation

We find that the model with two shocks and imperfect information generates positive autocorrelations in the growth rates of hours, output, and investment, a feature of the U.S. data. We show that this is not only because technology growth is positively autocorrelated in that model, but also because the process of learning about the drift of technology growth provides a model with an internal propagation mechanism.

Figure 6 presents the autocorrelation function for the growth rate of per capita output both for the data and for three models. The model with two shocks and imperfect information generates a positive autocorrelation in output growth for the first several lags, the same as we find in the data. The model with two shocks and full information also generates positive autocorrelation in output growth, but we will see shortly that this is simply because technology growth in that model has a positive autocorrelation. The autocorrelation in output growth is higher in the model with imperfect information than in the model with full information, although these two models share the same shock process. The single-shock model with random-walk technology generates a flat autocorrelation function for output growth, implying that output growth is close to white noise. This is because technology growth is not autocorrelated in that model and the model has a weak internal propagation mechanism (Cogley and Nason, 1995).<sup>31</sup>

A further inspection of the data reveals that the autocorrelation in output growth is not only positive but also higher than the autocorrelation in technology growth for the first several lags in the data, as shown in the top left panel of Figure 7. Figure 7 also shows that only the model with two shocks and imperfect information replicates this feature of the data. Thus, the process of learning about the transitory and persistent components of technology growth provides a model with an internal propagation mechanism.

To understand how this propagation mechanism works, recall the impulse response to a transitory shock to technology growth (Figure 2). Under imperfect information, agents attribute part of the increase in technology growth to a persistent shock, which calls for a decrease in hours. This decrease in hours partially offsets the direct effect

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<sup>31</sup>Cogley and Nason (1995) report that incorporating capital adjustment costs into a model has little effect on the autocorrelation in output growth. The reason is that capital adjustment costs affect the flow of investment, but the size of investment is relatively small compared to the size of capital stock. It is the changes in the capital stock that directly influence output dynamics. Cogley and Nason (1995) also report that employment decision lags and labor adjustment costs help a model to generate positive autocorrelations in output growth.

of the improvement in technology on output, weakening the initial response of output. Agents start increasing hours and investment after technology growth returns to its initial rate. The peak impact on output occurs several quarters after the shock. This is why output growth is more persistent than technology growth in the model with imperfect information.

Figure 8 confirms this logic by showing that growth in hours and in investment also has a positive autocorrelation in the imperfect information model.

The upper panel of Figure 8 presents the autocorrelation function for the growth rate of hours per capita implied by the data and three models. Only the imperfect information model with two shocks generates a positive autocorrelation in growth in hours, consistent with the data. The full information model with two shocks generates a slightly negative autocorrelation in growth in hours for the first several lags. This is not surprising if we recall the impulse response analysis: hours initially fall and subsequently rise following a persistent shock to technology growth, generating a negative autocorrelation in growth in hours. The model with random-walk technology generates a flat autocorrelation function for growth in hours since technology growth has no autocorrelation and the model has a weak internal propagation mechanism.

The lower panel of Figure 8 shows that the model with imperfect information replicates the positive autocorrelation in investment growth, while autocorrelation in investment growth is approximately zero in the two other models.

### 4.3 Forecastable Movements

Rotemberg and Woodford (1996) show that a standard real business cycle model with random-walk technology makes predictions for the forecastable movements in macroeconomic variables that differ greatly from the data. In particular, a standard model implies that a very small part of output movements is forecastable, and that consumption and hours are expected to move in opposite directions. Contrarily, in the U.S. data, about half of the output movements are forecastable, and consumption, hours, output, and investment are all expected to move in the same direction.

In this subsection, we show that the model with both transitory and persistent shocks to technology growth improves upon the standard model in replicating the properties of the forecastable movements as found in the data. The improvement over the standard model is obtained for both full and imperfect information.



For any horizon (in quarters)  $k = 1, 2, \dots$ , let

$$\widehat{\Delta y_t^k} \equiv E_t \log(y_{t+k}) - \log(y_t)$$

denote the percentage change in output per capita from period  $t$  to period  $t + k$  that is forecasted in period  $t$ . Following Rotemberg and Woodford (1996), we assume that these forecasts are obtained by estimating a three-variable VAR that consists of output growth, the log of consumption to output ratio, and the log of hours per capita, with two lags of each variable.<sup>32</sup> Also, let

$$\Delta y_t^k \equiv \log(y_{t+k}) - \log(y_t)$$

denote the percentage change in output from period  $t$  to period  $t + k$ . This includes both forecastable and unforecastable changes in output. Similar notation is used for per capita consumption ( $c$ ), per capita hours worked ( $n$ ), and per capita investment ( $i$ ).

We obtain the statistics for the forecastable movements implied by the single-shock model in the same way as in Rotemberg and Woodford (1996). Specifically, we first solve a log-linearized version of the model and write the solution in a VAR form. We then use the VAR system to compute statistics for the forecastable changes.

We cannot write the solution to the model with both transitory and persistent shocks to technology growth in a VAR form since the solution to the model is a non-linear function of the state variables. Our strategy for computing the statistics for the forecastable movements in this case is as follows. We first simulate the model to generate data on consumption and output. We then use the simulated data to estimate a bivariate VAR that consists of output growth and the log of consumption to output ratio, with two lags of each variable. We do not estimate a three-variable VAR because the model implies that one of the equations in the three-variable VAR system holds without errors: the optimality condition (7) implies that the log of hours worked is a linear function of the log of consumption to output ratio. After estimating a bivariate VAR, we recover the structure of the three-variable VAR system by augmenting

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<sup>32</sup>The only difference from the specification of Rotemberg and Woodford (1996) is that we use hours per capita instead of linearly detrended total hours as a measure of hours worked. We do so because the former corresponds more closely to the hours worked in the model. We find that this difference has little effect on the results that we report here.

the bivariate VAR system with the log-linearized version of the optimality condition (7). We then use the resulting three-variable VAR system to compute statistics for the forecastable movements. The statistics we report below are the mean from 1,000 replications, with each simulation having 173 periods.

#### 4.3.1 Size of Forecastable Movements

We begin our analysis by considering the properties of the forecastable movements implied by a model with random-walk technology. Figure 9 presents the impulse response to a 1% transitory shock to technology growth in this model.<sup>33</sup> Since this is the only kind of shock present in this particular model, we can understand the properties of the forecastable movements from Figure 9 alone. As indicated by Figure 9, output is expected to change little after the impact period. Almost all the changes in output, which occur at the impact period, are not forecastable in this model. In other words, output is close to a random walk.<sup>34</sup> In contrast, about 50% of the output changes are forecastable in the data, as seen in Table 3.

According to Table 3, the model with both transitory and persistent shocks to technology growth implies that a sizeable fraction of output movements is forecastable, for both full and imperfect information. This is so for several reasons. First, the presence of a persistent shock to technology growth enables a model to generate a large future deviation in output from its level at the impact period. This is true under both full and imperfect information. Second, the size of unforecastable changes in output is small in the model with both transitory and persistent shocks, again under both full and imperfect information. Under full information, hours worked initially move in the opposite direction from technology following a persistent shock to technology growth. This decreases the size of unforecastable changes in output. The same mechanism is present under imperfect information, although this mechanism is weaker than in the case of full information. Third, under imperfect information, the size of unforecastable changes in output is small following a transitory shock to technology growth. This is because agents initially attribute part of an observed movement in technology growth

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<sup>33</sup>As Rotemberg and Woodford (1996, section IV) argue, the main argument here does not depend on the specifications of preferences and technology.

<sup>34</sup>Watson (1993) shows that the spectrum of output growth predicted by the model with random-walk technology does not have any noticeable peak at business cycle frequencies unlike the spectrum of output growth estimated from the data. This is another way to demonstrate the lack of forecastable movements in output in the model with random-walk technology.

to a persistent shock. This weakens the initial response of hours worked, and decreases the size of unforecastable changes in output in the imperfect information model.

Table 4 compares the magnitudes of the forecastable changes in consumption, hours, output, and investment. The single-shock model with random-walk technology implies that the size of forecastable changes in consumption is larger than the size of forecastable changes in output. We see the opposite pattern in the data.

The model with both transitory and persistent shocks to technology growth brings the model's predictions closer to the data, especially for short horizons. This result holds for both full and imperfect information. As previously noted, the size of forecastable changes in output is relatively large in the model with both transitory and persistent shocks to technology growth. The size of forecastable changes in consumption is relatively small in this model for the following reasons. Under full information, a persistent shock to technology growth has a large wealth effect. This renders the size of unforecastable changes in consumption large. Everything else equal, this implies that the size of forecastable changes in consumption is small. The mechanism is also present under imperfect information. In addition, under imperfect information, a transitory shock to technology growth has a large wealth effect. This also increases the size of unforecastable changes in consumption and decreases the size of forecastable changes in consumption.

### 4.3.2 Correlations among Forecastable Movements

We now consider the models' predictions for the correlations among the forecastable movements in consumption, hours, output, and investment. As Rotemberg and Woodford (1996) point out, predictions of the model with random-walk technology are not consistent with the data in this respect. In particular, the model predicts that consumption is expected to move in the opposite direction from hours and investment, as seen from Figure 9. In contrast, consumption, hours, output, and investment are all expected to move in the same direction in the data, as reported in Table 5.

We can attribute the failure of the standard model on this dimension to the fact that the peak impacts on hours and investment occur immediately after a shock to technology. The imperfect information model with both transitory and persistent shocks to technology growth mitigates this problem by making the impact responses of hours and investment weak. Table 5 shows that the correlations among the forecastable move-

ments in consumption, hours, output, and investment are all positive in the imperfect information model, consistent with the data.

The correlations among the forecastable changes implied by the imperfect information model are not as strong as in the data, however. This is because the response of the macroeconomic variables to a particular shock varies depending on several factors, including the size of the shock, the direction of the shock, and the history of shocks.

The full information model with both transitory and persistent shocks to technology growth improves slightly upon the single-shock model. In particular, Table 5 shows that the correlation between the forecastable changes in consumption and output turns slightly positive for a short forecast horizon. This improvement over the single-shock model is brought about by the presence of a persistent shock to technology growth. Following a persistent shock to technology growth, consumption, hours, output, and investment are all expected to move in the direction of technology, especially for a short horizon. However, due to the presence of transitory shocks to technology growth, the overall correlations among the forecastable movements remain weak.

Finally, Table 6 reports the regression coefficients of the expected changes in consumption, hours, and investment on the expected changes in output. In the model with random-walk technology, the regression coefficient of the forecastable changes in consumption on the forecastable changes in output is negative, inconsistent with the data. Also, since forecastable changes in output are so small in this particular model, the regression coefficients are too large (in absolute values) to be consistent with the data. The model with both transitory and persistent shocks to technology growth improves upon the model with random-walk technology in replicating the regression coefficients among expected changes as found in the data.

## 5 Conclusion

This paper is based on the idea that it is difficult to distinguish between transitory and persistent movements in technology growth in real time. To obtain the macroeconomic implications, we have developed a real business cycle model in which both transitory and persistent shocks affect technology growth. We have then analyzed the quantitative properties of the model when agents do not observe the two shocks and solve a signal extraction problem to infer their realizations.

We have estimated a two-state regime-switching model of U.S. technology growth to calibrate the technology shock parameters in the model. In our calibration, the real-time inference about the persistent component of technology growth implied by the model has characteristics similar to the Index of Consumer Sentiment over U.S. business cycles.

We have found that the model with imperfect information replicates a feature of the data in which the autocorrelations in the growth rates of hours, output, and investment are higher than the autocorrelation in technology growth. In contrast, when agents observe both transitory and persistent shocks to technology growth, the model predicts that the autocorrelations in the growth rates of macroeconomic variables are no greater than the autocorrelation in technology growth. Thus, the process of learning about the drift of technology growth provides a model with an internal propagation mechanism.

We have also found that the model with both transitory and persistent shocks to technology growth improves upon a model with random-walk technology in replicating the properties of the forecastable movements in consumption, hours, output, and investment as found in the data. On this dimension, the model with full information performs nearly as well as the model with imperfect information.

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# Appendix

## I. Definitions of the Variables in Equations (9) and (10)

This section provides the definitions of the variables in the Euler equations under full information (9) and (10).

For  $j = H, L$ ,  $\tilde{c}_t^j$  is the consumption policy function when the current state is  $(\tilde{k}_t, g_t = g^j)$ :

$$\tilde{c}_t^j \equiv \tilde{c}(\tilde{k}_t, g_t = g^j) = \exp\left(\sum_{i=1}^n a_i^j \times \varphi_i(\tilde{k}_t)\right),$$

where  $\varphi_i(\cdot)$  for  $i = 1, 2, \dots, n$  are the basis functions. We use Chebyshev polynomials as the basis.

Using  $\tilde{c}_t^j$  obtained in (A-1), hours in regime  $g^j$  can be written as a function of  $(\tilde{k}_t, g_t = g^j)$ . For  $j = H, L$ :

$$n_t^j \equiv n(\tilde{k}_t, g_t = g^j) = \left(\alpha \frac{\tilde{k}_t^{1-\alpha}}{\tilde{c}_t^j}\right)^{\frac{1}{1-\alpha}}. \quad (\text{A-2})$$

Using (A-1) and (A-2), we can write investment in regime  $g^j$  as a function of  $(\tilde{k}_t, g_t = g^j)$ . For  $j = H, L$ :

$$\tilde{i}_t^j \equiv \tilde{i}(\tilde{k}_t, g_t = g^j) = \tilde{y}_t^j - \tilde{c}_t^j = (n_t^j)^\alpha \tilde{k}_t^{1-\alpha} - \tilde{c}_t^j. \quad (\text{A-3})$$

Using  $\tilde{i}_t^j$  obtained in (A-3), we can write  $\tilde{k}_{t+1}^{s,j}$  as a function of  $(\tilde{k}_t, g_t = g^j, g_{t+1} = g^s, \varepsilon_{t+1})$ . For  $j = H, L$  and  $s = H, L$ :

$$\tilde{k}_{t+1}^{s,j} = \frac{1}{\exp(g^s + \varepsilon_{t+1}) \exp(g^j)} \left[ (1 - \delta) \tilde{k}_t + \tilde{i}_t^j \right]. \quad (\text{A-4})$$

$\tilde{c}_{t+1}^{s,j}$  is the consumption policy function in period  $t+1$  when the state is  $(\tilde{k}_{t+1}^{s,j}, g_{t+1} = g^s)$ . For  $j = H, L$  and  $s = H, L$ :

$$\tilde{c}_{t+1}^{s,j} \equiv \tilde{c}(\tilde{k}_{t+1}^{s,j}, g_{t+1} = g^s) = \exp\left(\sum_{i=1}^n a_i^s \times \varphi_i(\tilde{k}_{t+1}^{s,j})\right). \quad (\text{A-5})$$

Using  $\tilde{k}_{t+1}^{s,j}$  and  $\tilde{c}_{t+1}^{s,j} \equiv \tilde{c}(\tilde{k}_{t+1}^{s,j}, g_{t+1} = g^s)$  obtained in (A-4) and (A-5),  $n_{t+1}^{s,j}$  for

$j = H, L$  and  $s = H, L$  can be written as

$$n_{t+1}^{s,j} \equiv n\left(\tilde{k}_{t+1}^{s,j}, g_{t+1} = g^s\right) = \left(\alpha \frac{\left(\tilde{k}_{t+1}^{s,j}\right)^{1-\alpha}}{\tilde{c}_{t+1}^{s,j}}\right)^{\frac{1}{1-\alpha}}. \quad (\text{A-6})$$

The investment in period  $t + 1$  is, for  $j = H, L$  and  $s = H, L$ :

$$\begin{aligned} \tilde{i}_{t+1}^{s,j} &\equiv \tilde{i}\left(\tilde{k}_{t+1}^{s,j}, g_{t+1} = g^s\right) \\ &= \tilde{y}_{t+1}^{s,j} - \tilde{c}_{t+1}^{s,j} = \left(\alpha \frac{\left(\tilde{k}_{t+1}^{s,j}\right)^{1-\alpha}}{\tilde{c}_{t+1}^{s,j}}\right)^{\frac{\alpha}{1-\alpha}} \left(\tilde{k}_{t+1}^{s,j}\right)^{1-\alpha} - \tilde{c}_{t+1}^{s,j}. \end{aligned} \quad (\text{A-7})$$

## II. Definitions of the Variables in Equation (11)

This section provides the definitions of the variables in the Euler equation under imperfect information (11).

$\tilde{c}_t$  is the consumption policy function when the current state is  $(\tilde{k}_t, p[g_t = g^H | \Omega_t])$ :

$$\tilde{c}_t \equiv \tilde{c}(\tilde{k}_t, p[g_t = g^H | \Omega_t]) = \exp\left(\sum_{i=1}^n a_i \times \varphi_i(\tilde{k}_t, p[g_t = g^H | \Omega_t])\right), \quad (\text{B-1})$$

where  $\varphi_i(\cdot)$  for  $i = 1, 2, \dots, n$  are the basis functions.

Using  $\tilde{c}_t \equiv \tilde{c}(\tilde{k}_t, p[g_t = g^H | \Omega_t])$  obtained in (B-1), hours can be written in terms of  $(\tilde{k}_t, p[g_t = g^H | \Omega_t])$ :

$$n_t \equiv n(\tilde{k}_t, p[g_t = g^H | \Omega_t]) = \left(\alpha \frac{\tilde{k}_t^{1-\alpha}}{\tilde{c}_t}\right)^{\frac{1}{1-\alpha}}. \quad (\text{B-2})$$

Using  $\tilde{c}_t$  and  $n_t$  obtained in (B-1) and (B-2), the investment can be written in terms of  $(\tilde{k}_t, p[g_t = g^H | \Omega_t])$ :

$$\tilde{i}_t \equiv \tilde{i}(\tilde{k}_t, p[g_t = g^H | \Omega_t]) = \tilde{y}_t - \tilde{c}_t = n_t^\alpha \tilde{k}_t^{1-\alpha} - \tilde{c}_t. \quad (\text{B-3})$$

Using  $\tilde{c}_t$ ,  $n_t$ , and  $\tilde{i}_t$  obtained in (B-1), (B-2), and (B-3),  $\tilde{k}_{t+1}^s$  for  $s = g^H, g^L$  can be written in terms of  $(\tilde{k}_t, p[g_t = g^H | \Omega_t], g_{t+1} = g^s, \varepsilon_{t+1})$ :

$$\tilde{k}_{t+1}^s = \frac{1}{\exp(g^s + \varepsilon_{t+1}) \exp(g^p)} \left[ (1 - \delta) \tilde{k}_t + \tilde{i}_t \right]. \quad (\text{B-4})$$

The consumption policy function in period  $t + 1$  in state  $(\tilde{k}_{t+1}^H, p_1[g_{t+1} = g^H | \Omega_{t+1}])$  is

$$\tilde{c}_{t+1}^H \equiv \tilde{c}(\tilde{k}_{t+1}^H, p_1[g_{t+1} = g^H | \Omega_{t+1}]) = \exp \left( \sum_{i=1}^n a_i \times \varphi_i(\tilde{k}_{t+1}^H, p_1[g_{t+1} = g^H | \Omega_{t+1}]) \right), \quad (\text{B-5})$$

where  $\Omega_{t+1} \equiv (\tilde{A}_{t+1}, \tilde{A}_t, \tilde{A}_{t-1}, \dots)$ , and  $p_1[g_{t+1} = g^H | \Omega_{t+1}]$  is defined as

$$\begin{aligned} & p_1[g_{t+1} = g^H | \Omega_{t+1}] \\ & f_1(\tilde{A}_{t+1} | g_{t+1} = g^H, \Omega_t) \times \sum_{i=g^H, g^L} \{p[g_{t+1} = g^H | g_t = i] \times p[g_t = i | \Omega_t]\} \\ \equiv & \frac{\sum_{i=g^H, g^L} \left[ f_1(\tilde{A}_{t+1} | g_{t+1} = i, \Omega_t) \times \sum_{i=g^H, g^L} \{p[g_{t+1} = i | g_t = i] \times p[g_t = i | \Omega_t]\} \right]}{\sum_{j=g^H, g^L} \left[ f_1(\tilde{A}_{t+1} | g_{t+1} = j, \Omega_t) \times \sum_{i=g^H, g^L} \{p[g_{t+1} = j | g_t = i] \times p[g_t = i | \Omega_t]\} \right]}, \quad (\text{B-6}) \end{aligned}$$

where  $f_1(\tilde{A}_{t+1} | g_{t+1} = g^H, \Omega_t)$  and  $f_1(\tilde{A}_{t+1} | g_{t+1} = g^L, \Omega_t)$  are defined as

$$\begin{aligned} f_1(\tilde{A}_{t+1} | g_{t+1} = g^H, \Omega_t) & \equiv \frac{1}{\sqrt{2\pi}\sigma_\varepsilon} \exp \left[ -\frac{1}{2\sigma_\varepsilon^2} (g^H + \varepsilon_{t+1} - g^H)^2 \right] \\ & = \frac{1}{\sqrt{2\pi}\sigma_\varepsilon} \exp \left[ -\frac{1}{2\sigma_\varepsilon^2} \varepsilon_{t+1}^2 \right] \end{aligned}$$

and

$$f_1(\tilde{A}_{t+1} | g_{t+1} = g^L, \Omega_t) \equiv \exp \left[ -\frac{1}{2\sigma_\varepsilon^2} (g^H + \varepsilon_{t+1} - g^L)^2 \right].$$

The consumption policy function in period  $t + 1$  in state  $(\tilde{k}_{t+1}^L, p_2[g_{t+1} = g^H | \Omega_{t+1}])$  is

$$\begin{aligned} \tilde{c}_{t+1}^L & \equiv \tilde{c}(\tilde{k}_{t+1}^L, p_2[g_{t+1} = g^H | \Omega_{t+1}]) \\ & = \exp \left( \sum_{i=1}^n a_i \times \varphi_i(\tilde{k}_{t+1}^L, p_2[g_{t+1} = g^H | \Omega_{t+1}]) \right), \quad (\text{B-7}) \end{aligned}$$

where  $p_2[g_{t+1} = g^H | \Omega_{t+1}]$  is defined as

$$\begin{aligned} & p_2[g_{t+1} = g^H | \Omega_{t+1}] \\ & f_2(\tilde{A}_{t+1} | g_{t+1} = g^H, \Omega_t) \times \sum_{i=g^H, g^L} \{p[g_{t+1} = g^H | g_t = i] \times p[g_t = i | \Omega_t]\} \\ \equiv & \frac{\sum_{i=g^H, g^L} \left[ f_2(\tilde{A}_{t+1} | g_{t+1} = i, \Omega_t) \times \sum_{i=g^H, g^L} \{p[g_{t+1} = i | g_t = i] \times p[g_t = i | \Omega_t]\} \right]}{\sum_{j=g^H, g^L} \left[ f_2(\tilde{A}_{t+1} | g_{t+1} = j, \Omega_t) \times \sum_{i=g^H, g^L} \{p[g_{t+1} = j | g_t = i] \times p[g_t = i | \Omega_t]\} \right]}, \quad (\text{B-8}) \end{aligned}$$

where  $f_2(\tilde{A}_{t+1}|g_{t+1} = g^H, \Omega_t)$  and  $f_2(\tilde{A}_{t+1}|g_{t+1} = g^L, \Omega_t)$  are defined as

$$f_2(\tilde{A}_{t+1}|g_{t+1} = g^H, \Omega_t) \equiv \frac{1}{\sqrt{2\pi\sigma_\varepsilon}} \exp \left[ -\frac{1}{2\sigma_\varepsilon^2} (g^L + \varepsilon_{t+1} - g^H)^2 \right]$$

and

$$\begin{aligned} f_2(\tilde{A}_{t+1}|g_{t+1} = g^L, \Omega_t) &\equiv \frac{1}{\sqrt{2\pi\sigma_\varepsilon}} \exp \left[ -\frac{1}{2\sigma_\varepsilon^2} (g^L + \varepsilon_{t+1} - g^L)^2 \right] \\ &= \frac{1}{\sqrt{2\pi\sigma_\varepsilon}} \exp \left[ -\frac{1}{2\sigma_\varepsilon^2} \varepsilon_{t+1}^2 \right]. \end{aligned}$$

Using  $\tilde{k}_{t+1}^H$  and  $\tilde{c}_{t+1}^H$ , hours in state  $(\tilde{k}_{t+1}^H, p_1[g_{t+1} = g^H|\Omega_{t+1}])$  is

$$n_{t+1}^H \equiv n(\tilde{k}_{t+1}^H, p_1[g_{t+1} = g^H|\Omega_{t+1}]) = \left( \alpha \frac{(\tilde{k}_{t+1}^H)^{1-\alpha}}{\tilde{c}_{t+1}^H} \right)^{\frac{1}{1-\alpha}}. \quad (\text{B-9})$$

Similarly,

$$n_{t+1}^L \equiv n(\tilde{k}_{t+1}^L, p_2[g_{t+1} = g^H|\Omega_{t+1}]) = \left( \alpha \frac{(\tilde{k}_{t+1}^L)^{1-\alpha}}{\tilde{c}_{t+1}^L} \right)^{\frac{1}{1-\alpha}}. \quad (\text{B-10})$$

The investment in state  $(\tilde{k}_{t+1}^H, p_1[g_{t+1} = g^H|\Omega_{t+1}])$  is

$$\begin{aligned} \tilde{i}_{t+1}^H &\equiv \tilde{i}(\tilde{k}_{t+1}^H, p_1[g_{t+1} = g^H|\Omega_{t+1}]) \\ &= \tilde{y}_{t+1}^H - \tilde{c}_{t+1}^H = \left( \alpha \frac{(\tilde{k}_{t+1}^H)^{1-\alpha}}{\tilde{c}_{t+1}^H} \right)^{\frac{\alpha}{1-\alpha}} (\tilde{k}_{t+1}^H)^{1-\alpha} - \tilde{c}_{t+1}^H. \end{aligned} \quad (\text{B-11})$$

Similarly,

$$\begin{aligned} \tilde{i}_{t+1}^L &\equiv \tilde{i}(\tilde{k}_{t+1}^L, p_2[g_{t+1} = g^H|\Omega_{t+1}]) \\ &= \tilde{y}_{t+1}^L - \tilde{c}_{t+1}^L = \left( \alpha \frac{(\tilde{k}_{t+1}^L)^{1-\alpha}}{\tilde{c}_{t+1}^L} \right)^{\frac{\alpha}{1-\alpha}} (\tilde{k}_{t+1}^L)^{1-\alpha} - \tilde{c}_{t+1}^L. \end{aligned} \quad (\text{B-12})$$

### III. Data Description

All the data except capital are quarterly, for 1959:I–2002:I.

Population ( $H$ ) is the civilian noninstitutional population of 16 years and over, obtained from the Bureau of Labor Statistics (BLS) (series ID: LNU00000000Q).

Output ( $Y$ ) is the quarterly nonfarm business sector real output index, obtained from the BLS (series ID: PRS85006043).

Labor ( $N$ ) is the quarterly index of total hours worked by all persons engaged in the nonfarm business sector, obtained from the BLS (series ID: PRS85006033).

Capital ( $K$ ) is based on the annual private nonfarm business sector capital service index, obtained from the BLS (series ID: MPU750025). To construct a quarterly series, we interpolate the annual series under the assumption that the quarterly growth rate of capital is constant within each year. We assume that the capital service reported for the year 1959, for example, is available for use between 1959:I and 1959:IV.

Consumption ( $C$ ) is the sum of two items: (i) the personal consumption expenditures on nondurable goods, and (ii) the personal consumption expenditures on services. Both of these are obtained from National Income and Product Accounts (NIPA) table 1.15, and are converted to real terms with the appropriate deflators reported in NIPA table 1.19.

Investment ( $I$ ) is the sum of two items: (i) the personal consumption expenditures on durable goods in NIPA table 1.15, deflated by the appropriate deflator in NIPA table 1.19, and (ii) the real gross private domestic investment (the sum of fixed investment on equipment and structures and the change in inventories) reported in NIPA table 1.16.

Per capita variables are defined as  $y \equiv Y/H$ ,  $n \equiv N/H$ ,  $k \equiv K/H$ ,  $c \equiv C/H$ ,  $i \equiv I/H$ .

Technology ( $A$ ) is constructed using the production function  $Y = (AN)^\alpha K^{1-\alpha}$  with a labor share parameter  $\alpha = 2/3$ . The technology series is constructed using the expression  $A = [Y/(N^\alpha K^{1-\alpha})]^{1/\alpha}$  given the data on output ( $Y$ ), capital stock ( $K$ ), and hours of all persons ( $N$ ). Beaudry and Portier (2006) construct a technology similarly.

The Index of Consumer Sentiment is obtained from the University of Michigan, Surveys of Consumers. The quarterly index is available from 1960:I. We use the biannual data for the year 1959.

Table 1: Estimated Parameter Values for the Technology Shock Process

Parameter	Description	Estimate
$p$	Transition probability: $prob [g_{t+1} = g^H   g_t = g^H]$	0.9510 (0.0413)
$q$	Transition probability: $prob [g_{t+1} = g^L   g_t = g^L]$	0.6941 (0.1658)
$g^H$	Average technology growth rate in state $g^H$	0.006533 (0.001418)
$g^L$	Average technology growth rate in state $g^L$	-0.009533 (0.004406)
$\sigma_\varepsilon^2$	Variance of transitory shock to technology growth	0.014769 (0.002160)



Table 2: Unconditional Moments (First Difference of log Series)

Variable	Std. dev.	Relative std. dev.	Autocorrelations		Corr. with $\Delta \log(y)$
			1	2	
<i>Data</i>					
$\Delta \log(y)$	1.207	1	0.260	0.198	1
$\Delta \log(c)$	0.473	0.392	0.276	0.207	0.481
$\Delta \log(i)$	3.702	3.067	0.166	0.111	0.879
$\Delta \log(k)$	0.288	0.239	0.790	0.721	0.224
$\Delta \log(n)$	0.828	0.686	0.567	0.342	0.716
<i>Model with two shocks and imperfect information</i>					
$\Delta \log(y)$	1.131	1	0.202	0.127	1
$\Delta \log(c)$	0.800	0.708	0.076	0.078	0.934
$\Delta \log(i)$	2.628	0.320	0.276	0.152	0.922
$\Delta \log(k)$	0.282	0.249	0.961	0.905	0.062
$\Delta \log(n)$	0.481	0.425	0.252	0.133	0.800
<i>Model with two shocks and full information</i>					
$\Delta \log(y)$	1.351	1	0.089	0.055	1
$\Delta \log(c)$	0.785	0.581	0.077	0.079	0.673
$\Delta \log(i)$	4.609	3.408	-0.027	-0.027	0.903
$\Delta \log(k)$	0.305	0.225	0.903	0.814	-0.051
$\Delta \log(n)$	1.007	0.745	-0.081	-0.059	0.821
<i>Model with random-walk technology</i>					
$\Delta \log(y)$	1.332	1	-0.002	-0.005	1
$\Delta \log(c)$	0.679	0.510	0.087	0.076	0.984
$\Delta \log(i)$	3.598	2.700	-0.029	-0.030	0.992
$\Delta \log(k)$	0.240	0.180	0.904	0.815	-0.040
$\Delta \log(n)$	0.675	0.507	-0.034	-0.034	0.984

Table 3: Forecasted Changes in Output as a Fraction of Total Changes

Horizon ( $k$ )	1	4	8	12
<i>Data</i>				
$\frac{std(\widehat{\Delta y_t^k})}{std(\Delta y_t^k)}$	0.586	0.510	0.505	0.547
<i>Model with two shocks and imperfect information</i>				
$\frac{std(\widehat{\Delta y_t^k})}{std(\Delta y_t^k)}$	0.316	0.238	0.206	0.198
<i>Model with two shocks and full information</i>				
$\frac{std(\widehat{\Delta y_t^k})}{std(\Delta y_t^k)}$	0.342	0.268	0.247	0.230
<i>Model with random-walk technology</i>				
$\frac{std(\widehat{\Delta y_t^k})}{std(\Delta y_t^k)}$	0.003	0.005	0.006	0.006

Table 4: Standard Deviations of Forecasted Changes

Horizon ( $k$ )	$std\left(\widehat{\Delta y_t^k}\right)$	$std\left(\widehat{\Delta c_t^k}\right)$	$std\left(\widehat{\Delta i_t^k}\right)$	$std\left(\widehat{\Delta n_t^k}\right)$
<i>Data</i>				
1	0.007	0.002	0.022	0.006
4	0.016	0.005	0.049	0.015
8	0.023	0.006	0.073	0.021
12	0.029	0.007	0.094	0.026
<i>Model with two shocks and imperfect information</i>				
1	0.004	0.002	0.012	0.003
4	0.007	0.005	0.021	0.006
8	0.009	0.008	0.028	0.008
12	0.010	0.011	0.034	0.011
<i>Model with two shocks and full information</i>				
1	0.005	0.002	0.017	0.005
4	0.008	0.005	0.034	0.009
8	0.011	0.007	0.049	0.014
12	0.012	0.008	0.057	0.017
<i>Model with random-walk technology</i>				
1	$3e-5$	0.001	0.005	0.001
4	$1e-4$	0.004	0.020	0.004
8	$2e-4$	0.008	0.034	0.008
12	$3e-4$	0.010	0.046	0.010

Table 5: Correlations among Forecasted Changes

Horizon ( $k$ )	$\text{corr}(\widehat{\Delta c_t^k}, \widehat{\Delta y_t^k})$	$\text{corr}(\widehat{\Delta i_t^k}, \widehat{\Delta y_t^k})$	$\text{corr}(\widehat{\Delta n_t^k}, \widehat{\Delta y_t^k})$
<i>Data</i>			
1	0.864	0.989	0.880
4	0.911	0.994	0.881
8	0.869	0.993	0.838
12	0.850	0.993	0.827
<i>Model with two shocks and imperfect information</i>			
1	0.495	0.921	0.857
4	0.464	0.771	0.598
8	0.361	0.584	0.370
12	0.301	0.485	0.288
<i>Model with two shocks and full information</i>			
1	0.191	0.955	0.920
4	-0.075	0.850	0.789
8	-0.225	0.797	0.751
12	-0.277	0.779	0.743
<i>Model with random-walk technology</i>			
Any $k$	-1.000	1.000	1.000

Table 6: Regression Coefficients among Forecasted Changes

Horizon ( $k$ )	$\widehat{\Delta c_t^k}$ on $\widehat{\Delta y_t^k}$	$\widehat{\Delta i_t^k}$ on $\widehat{\Delta y_t^k}$	$\widehat{\Delta n_t^k}$ on $\widehat{\Delta y_t^k}$
<i>Data</i>			
1	0.276	3.060	0.747
4	0.266	3.089	0.863
8	0.229	3.196	0.779
12	0.212	3.241	0.752
<i>Model with two shocks and imperfect information</i>			
1	0.236	3.176	0.764
4	0.398	2.713	0.602
8	0.481	2.478	0.519
12	0.506	2.407	0.494
<i>Model with two shocks and full information</i>			
1	0.070	3.648	0.931
4	-0.014	3.887	1.014
8	-0.096	4.119	1.096
12	-0.129	4.213	1.129
<i>Model with random-walk technology</i>			
Any $k$	-35.050	161.025	36.050

Figure 1: Belief and the Index of Consumer Sentiment

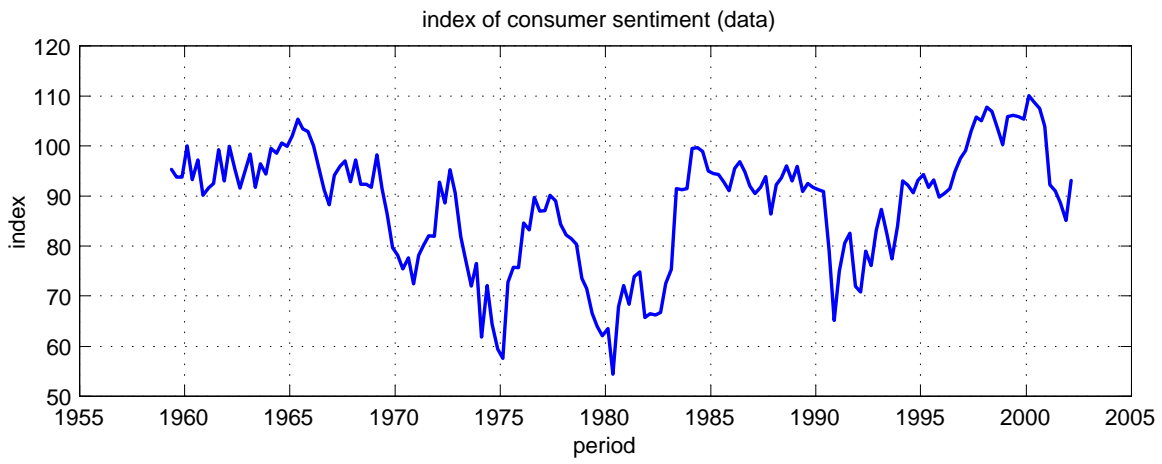
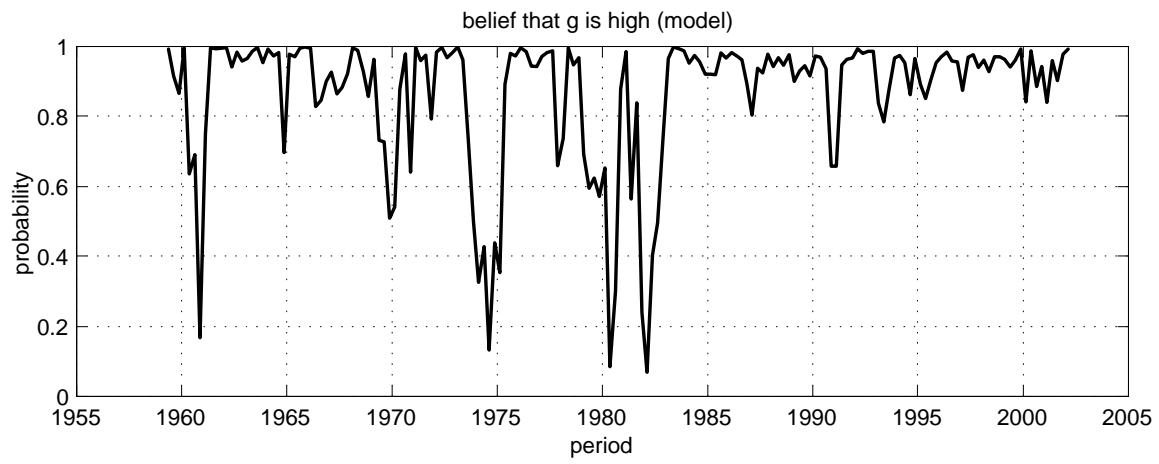


Figure 2: 1% Transitory Shock to Technology Growth Conditional on Regime  $g^L$

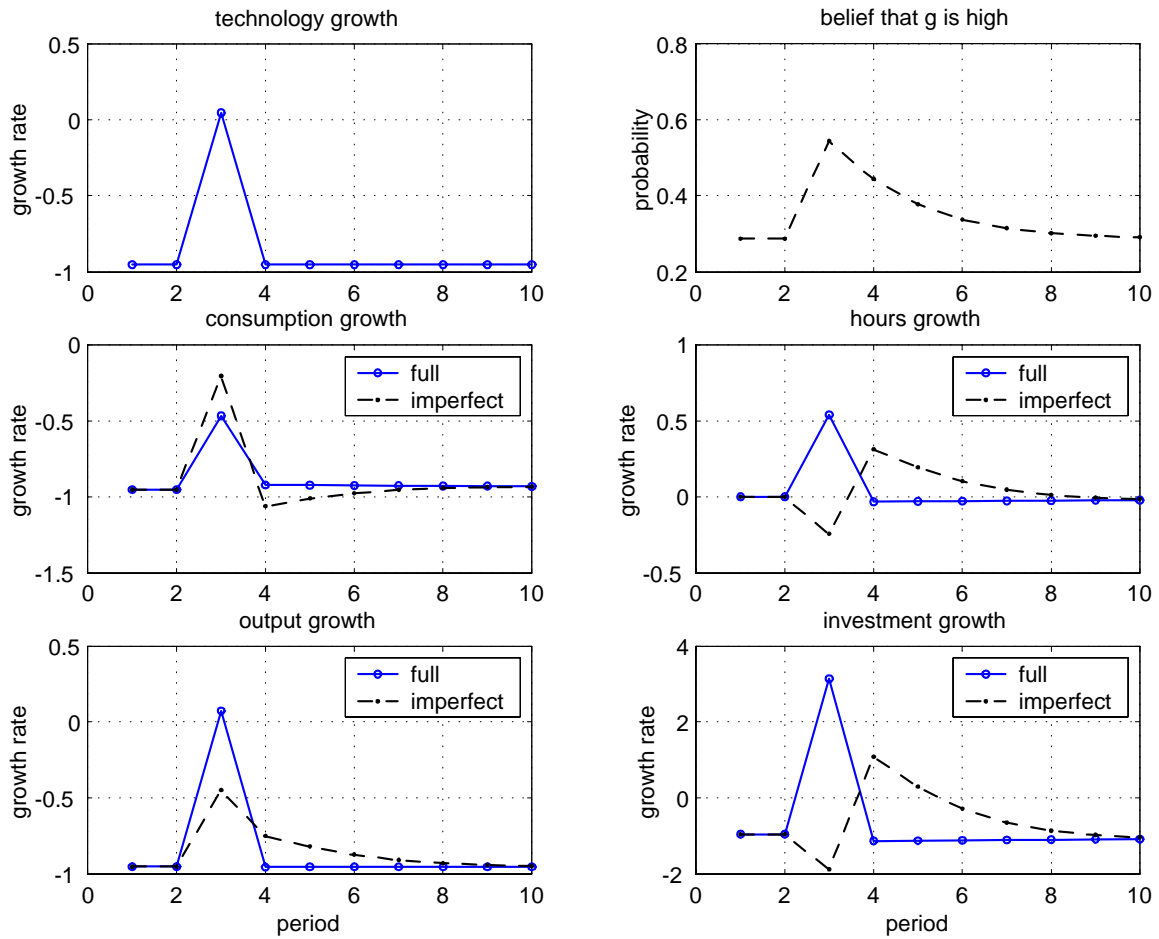


Figure 3: Positive Permanent Shock to Technology Growth (Switch from  $g^L$  to  $g^H$ )

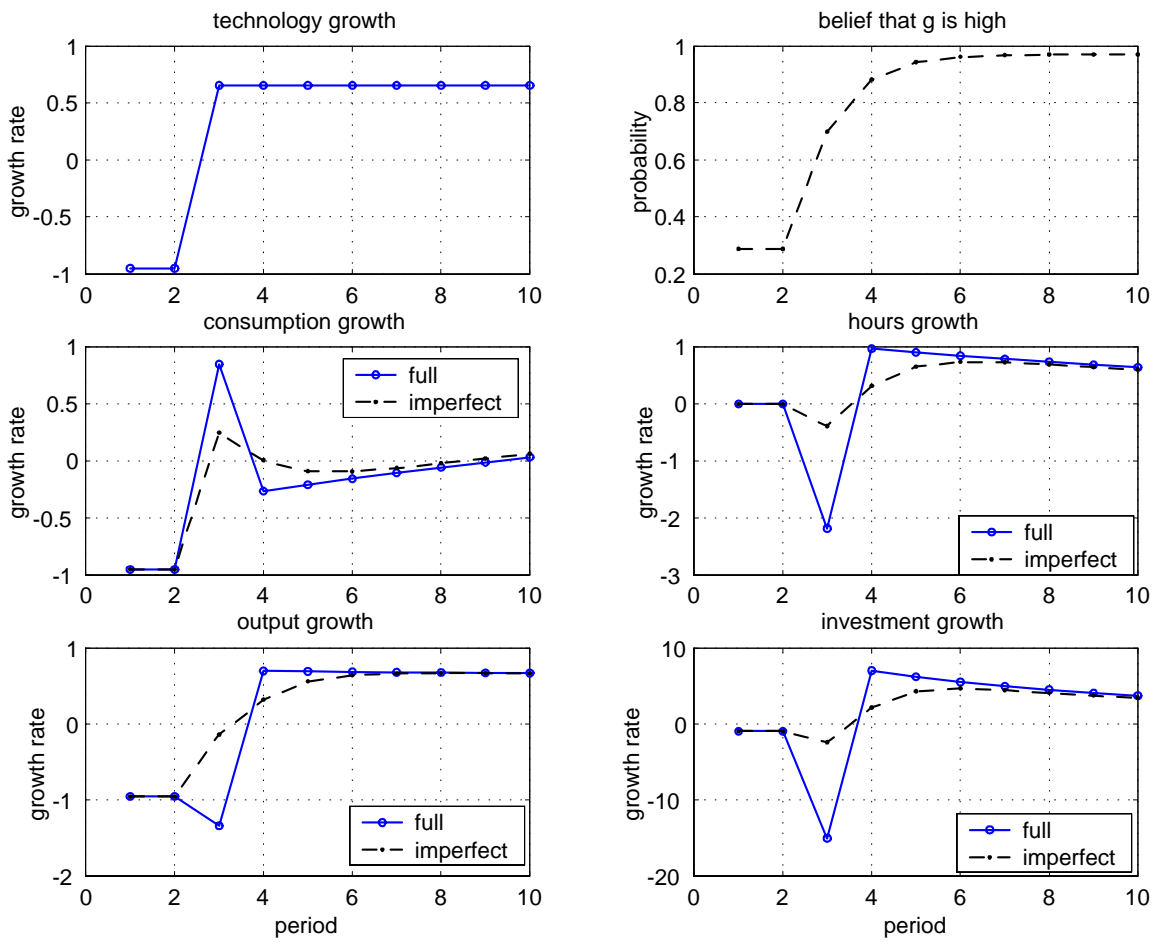




Figure 4: 1% Transitory Shock to Technology Growth Conditional on Regime  $g^H$  (Solid Lines) and Conditional on Regime  $g^L$  (Dashed Lines)

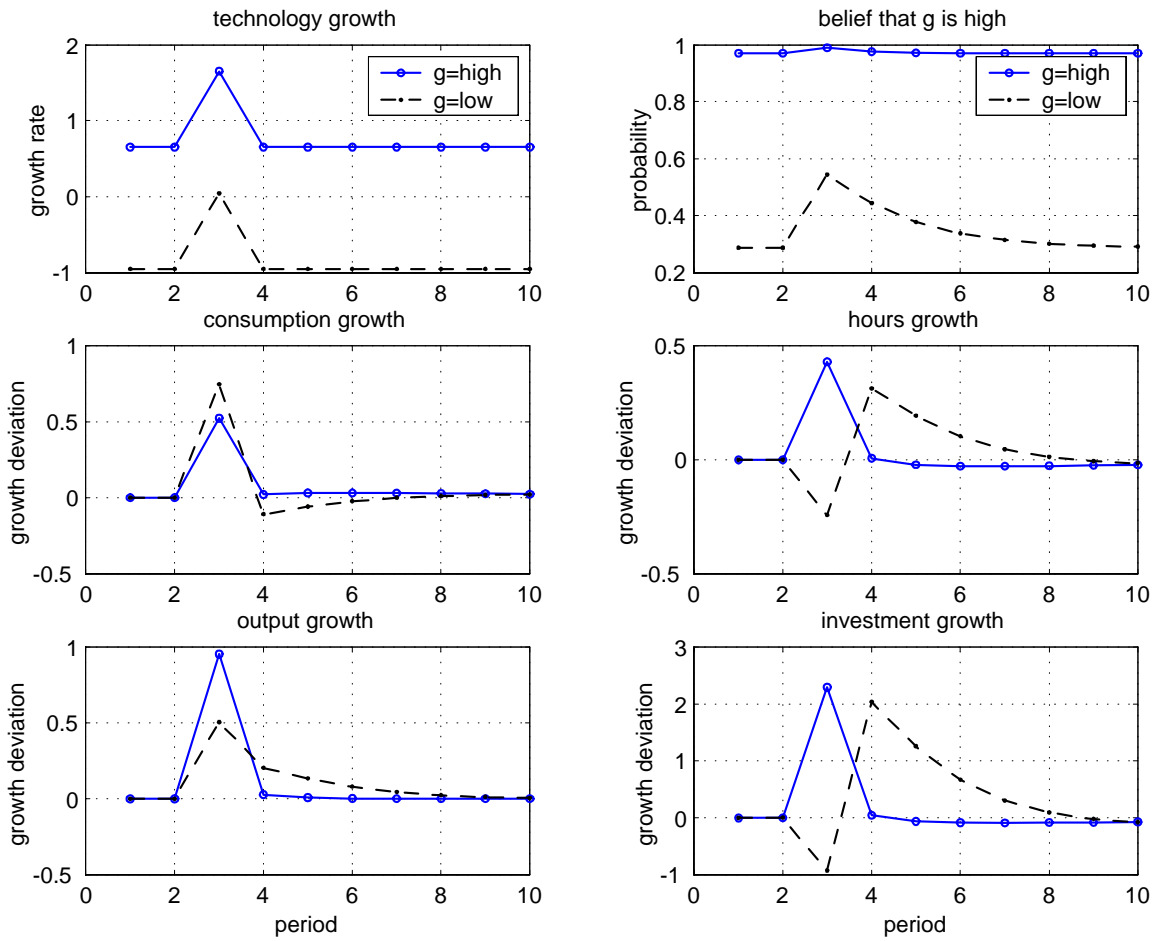


Figure 5: Switch from  $g^L$  to  $g^H$  (Left Panels) and Switch from  $g^H$  to  $g^L$  (Right Panels)

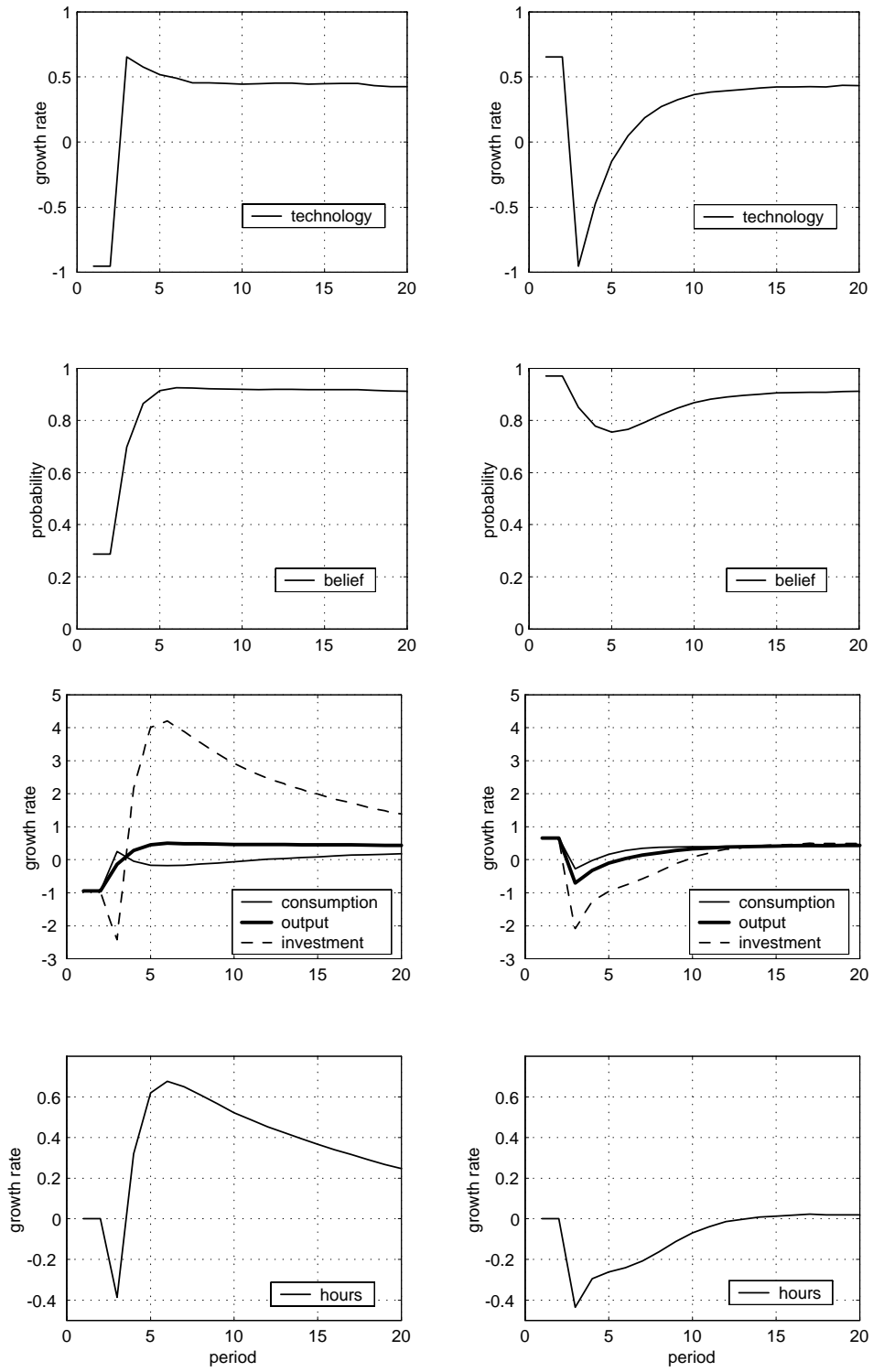


Figure 6: Autocorrelation Function for Output Growth

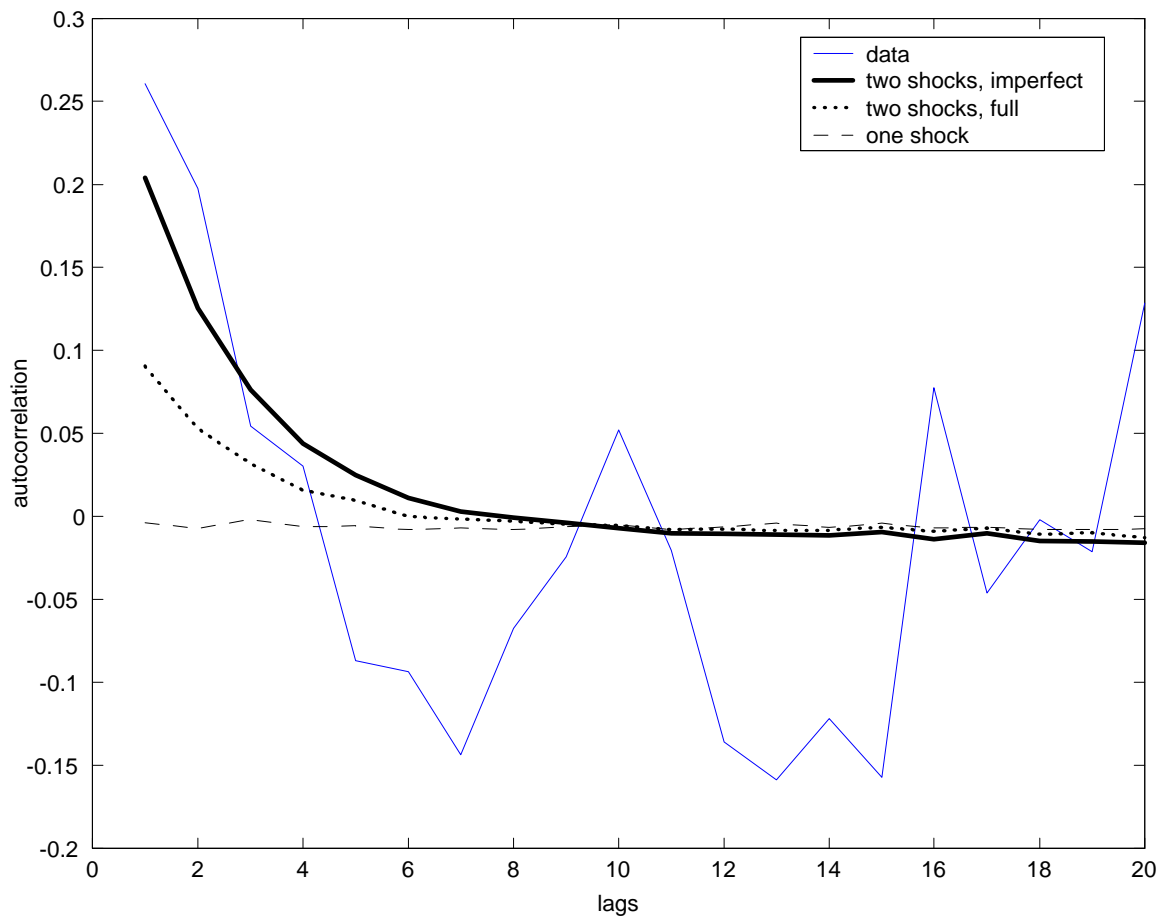


Figure 7: Autocorrelation Functions for Output Growth and Technology Growth

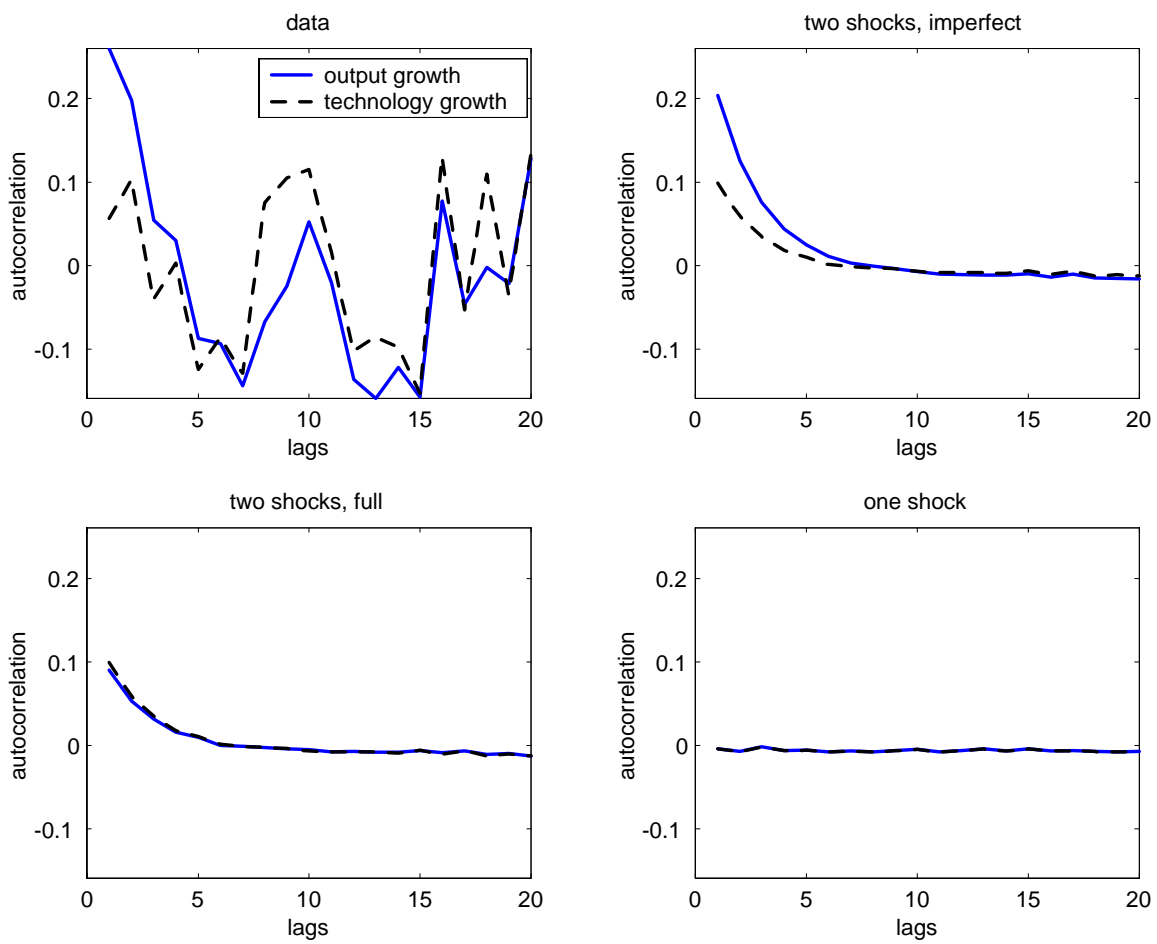


Figure 8: Autocorrelation Function for Hours Growth (Upper Panel) and Investment Growth (Lower Panel)

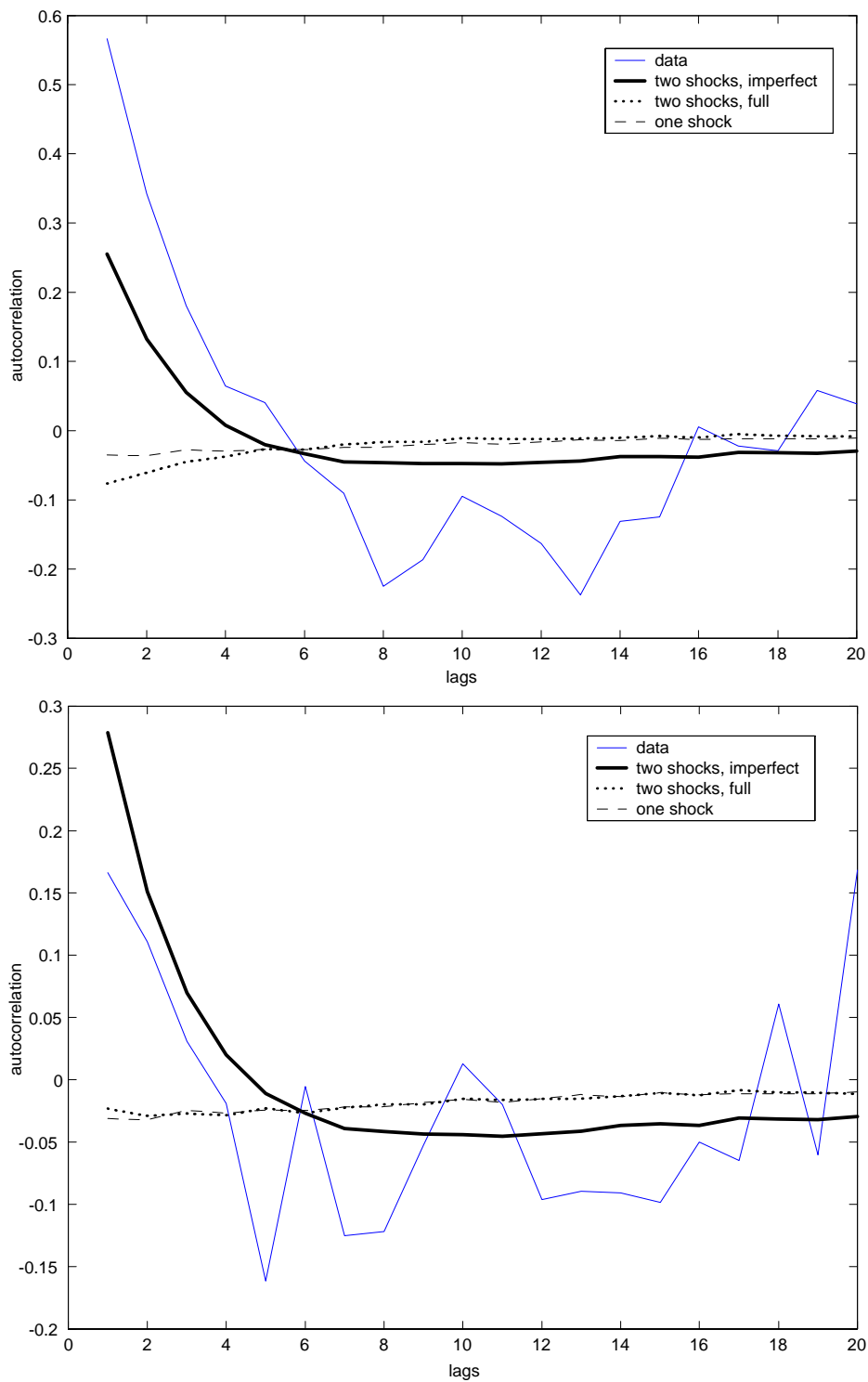


Figure 9: 1% Transitory Shock to Technology Growth in the Model with Random-Walk Technology

