

IMES DISCUSSION PAPER SERIES

**Imperfect Common Knowledge, Staggered Price Setting,
and the Effects of Monetary Policy**

Ichiro Fukunaga

Discussion Paper No. 2006-E-5

IMES

INSTITUTE FOR MONETARY AND ECONOMIC STUDIES

BANK OF JAPAN

C.P.O BOX 203 TOKYO

100-8630 JAPAN

You can download this and other papers at the IMES Web site:

<http://www.imes.boj.or.jp>

Do not reprint or reproduce without permission.

NOTE: IMES Discussion Paper Series is circulated in order to stimulate discussion and comments. Views expressed in Discussion Paper Series are those of authors and do not necessarily reflect those of the Bank of Japan or the Institute for Monetary and Economic Studies.

Imperfect Common Knowledge, Staggered Price Setting, and the Effects of Monetary Policy

Ichiro Fukunaga*

Abstract

This paper studies the consequences of a lack of common knowledge in the transmission of monetary policy by integrating the Woodford (2003a) imperfect common knowledge model with Taylor-Calvo staggered price-setting models. The average price set by monopolistically competitive firms who can only observe the state of the economy through noisy private signals depends on their higher-order expectations about not only the current state but also about the states in the future periods in which prices are to be fixed. This integrated model provides a plausible explanation for the observed effects of monetary policy: it shows analytically how price adjustments are delayed and how the response of output to monetary disturbances is amplified. I also consider a more general information structure in which a noisy public signal, in addition to the private signals, is introduced.

Keywords: Imperfect common knowledge; Higher-order expectations; Public and private information; Staggered price setting

JEL classification: D82, E30

*Economist, Institute for Monetary and Economic Studies, Bank of Japan (E-mail: ichirou.fukunaga@boj.or.jp)

This paper is a revised version of Chapter 1 of my Ph.D. thesis submitted to the London School of Economics and Political Science (LSE) in April 2005. I am deeply grateful to my supervisor, Nobuhiro Kiyotaki, for his valuable suggestions and unstinting support and encouragement. I also thank Wouter denHaan, Neil Rankin, Andrei Sarychev, Hyun Song Shin, and seminar participants at LSE and the Institute for Monetary and Economic Studies (IMES) for helpful discussions. All remaining errors are my own. The views expressed in this paper are those of the author and do not necessarily reflect the official views of the Bank of Japan or IMES.

1 Introduction

Modern macroeconomic theory provides two main explanations for why monetary policy has real effects in the short run: imperfect information about the policy shocks and short-run rigidity in price or wage adjustment. The imperfect information approach was originally developed by Phelps (1970) and Lucas (1972) in the era in which the traditional output-inflation relationship collapsed. However, their arguments were criticized for their practical irrelevance: the Phelps-Lucas models imply that the real effects of monetary policy only last while the precise public information about aggregate disturbances is unavailable, which seems to contradict the observed persistence of business fluctuations despite the availability of macroeconomic data with little delay.¹ To analyze the persistent real effects of monetary policy, many current macroeconomic models of business fluctuations assume short-run rigidity in price or wage adjustment, typically by incorporating staggered price setting as in Taylor (1980) or Calvo (1983).

Some authors have recently reconsidered the imperfect information approach. They have developed monetary business cycle models that can generate persistent real effects of monetary policy and can also overcome a major problem in the Taylor-Calvo staggered price-setting models, namely their inability to explain the observed inflation inertia. Mankiw and Reis (2002) consider sticky information rather than sticky prices, which means parts of current prices are chosen on the basis of old information. Woodford (2003a) considers imperfect common knowledge about nominal disturbances in an environment among monopolistically competitive suppliers whose optimal pricing strategy depends not only on their own estimates of the aggregate disturbances but also on their expectations of the average estimates by other suppliers. These models can explain in particular the stylized fact identified in many empirical studies including Christiano, Eichenbaum, and Evans (2005) that monetary policy shocks initially affect real variables and then have delayed and gradual effects on inflation.

However, these models still leave the original problem in the Phelps-Lucas models unsolved. The source of persistence of the real effects of monetary policy in the Mankiw-Reis model is the outdated information that influences current price setting. In their model some suppliers set their prices based on very old information because the probability of obtaining new information in

¹Lucas (1975) developed a monetary business cycle model that can generate persistent real effects of monetary policy by introducing capital accumulation as well as information lags. Such persistence can also be generated by introducing inventories into monetary business cycle models based on imperfect information without assuming rigidity in price or wage adjustment, as shown in Fukunaga (2005).

each period is constant and identical for all suppliers however recent their last updates. In the Woodford model, suppliers never obtain, nor pay attention to, precise information about aggregate demand and even about the actual quantities they sold at their chosen prices. They choose their prices solely on the basis of the history of their subjective observations that contain idiosyncratic perception errors. In both models, there would be no persistent real effects of monetary policy if the true state of the economy were revealed to all suppliers with a delay of only one period. These models do not explain why price setters fail to use widely and readily available macroeconomic data.²

In this paper, I develop a model that integrates Woodford's imperfect common knowledge model with Taylor-Calvo staggered price-setting models in order to overcome the problems in each of them and explain plausibly the observed effects of monetary policy. The model is based on the standard monopolistic competition framework as in Blanchard and Kiyotaki (1987). Following Woodford, I assume that price setters can only observe the state of the economy through noisy private signals so that the overall price level depends on a weighted sum of price setters' "higher-order expectations," that is, what others expect about what others expect ... about aggregate demand.³ Meanwhile, I drop Woodford's unrealistic assumption that price setters never pay attention to widely available data by assuming that the true state of the economy is revealed to all price setters with a delay of one period. Given staggered price setting, however, the model can generate persistent real effects of monetary policy. The average price chosen in each period depends on higher-order expectations about not only the current state of the economy but also about the states in the future periods in which prices are to be fixed. Although these dynamic and staggered higher-order expectations are complicated, the model can be solved analytically by virtue of the assumption that the true current state becomes common knowledge in the subsequent period.

The main results of the model are as follows. The noisier are the private signals, the more sluggish is the initial response of prices to a monetary disturbance. The response that operates through dynamic and staggered higher-order expectations is, in most cases, more sluggish than the one that

²Some recent studies attempt to explain how agents rationally choose to be inattentive under informational constraints: Sims (2003) considers limited capacity for processing information, while Reis (2005) considers costs of acquiring, absorbing, and processing information and provides a micro-foundation to the assumption of Mankiw and Reis (2002).

³Keynes (1936) described the role of higher-order expectations in an asset-pricing context by introducing the famous metaphor of financial markets as "beauty contests." Recently, higher-order beliefs have been extensively studied in the theoretical literature on "global games" (Morris and Shin, 2003) and applied to various fields.

operates through static and simultaneous higher-order expectations. Following this initial response, price adjustments are delayed and inflation may peak later than in the corresponding full-information staggered price-setting model. The response of output is amplified by the lack of common knowledge and continues to exceed the response in the full-information model. Even a small amount of noise in the private signals may significantly delay the adjustment of prices and amplify the response of output. The model nests the full-information staggered price-setting model as one limit case. As another limit case, it also nests the predetermined-prices model, in which all firms either have no information about the current aggregate disturbances or are simply assumed to set their prices one period in advance.⁴ The case of imperfect common knowledge is between these two limit cases, and explains endogenously how price adjustments are delayed.

I obtain these results analytically in the baseline model in which a lack of common knowledge is incorporated into a simple two-period staggered price-setting model, and then in more general price-setting models that allow for multiple-period staggered price setting including the one analogized with Calvo-type price setting. The latter overcomes a major problem with Calvo-type price setting, namely that the price level jumps in the period of disturbance and inflation responds earlier than does output, which is inconsistent with the stylized fact mentioned above.

I extend my baseline model by introducing a noisy public signal in addition to the private signals and study the consequences of a more general information structure following Hellwig (2002) and Amato and Shin (2003). These authors emphasize the separation of information into public and private signals and criticize the Woodford model for focusing only on private signals and for lacking considerations of problems involving informational interaction between decision makers. Whereas Amato and Shin assume that price setters never obtain precise information as in the Woodford model, I retain the assumption that the true state of the economy is revealed to all price setters with a delay of one period. I show that provision of the public signal alleviates the sluggishness in the initial response of prices to some extent, but the results in the baseline model, including the delayed response of inflation and the amplified response of output, are robust.

The public signal in the extended model may represent preliminary data that is to be revised or noisy information provided by the media, the government, and so on. When it is interpreted as a communication tool of the

⁴The latter limit case is analyzed in Section 3.1 of Chapter 3 in Woodford (2003b) to demonstrate a simple and direct way of generating delayed effects of nominal disturbances on inflation.

monetary authority, the model has interesting implications for the conduct of monetary policy involving, for example, commitment and transparency. As Morris and Shin (2002) argue, in an economy in which decision makers' information sets are heterogeneous, public information has disproportionately large effects on their decisions. While provision of the public signal alleviates the sluggishness in price adjustments to monetary disturbances, it exposes firms to an additional disturbance, namely noise in the public signal, and could destabilize the economy. In this extended model, I show that a *negative* informational disturbance, that is, downwardly biased information about current aggregate demand, generates delayed inflation and positive response of output as does a positive monetary disturbance. In addition, a small improvement in the precision of a relatively noisy public signal amplifies, rather than reduces, the responsiveness to informational disturbances and increases output volatility. Although I conduct neither practical policy analysis nor rigorous welfare analysis in this paper, the introduction of the noisy public signal is important for studying various consequences of a lack of common knowledge as well as for checking the robustness of the results obtained from the baseline model.

Recent studies on imperfect common knowledge have obtained important results for the analysis of monetary policy and social welfare. Adam (2006) determines the optimal monetary policy in an economy in which firms have imperfect common knowledge about real demand and supply shocks. Kawamoto (2004) examines the role of monetary policy in accommodating technology improvements when the central bank as well as private agents have imperfect common knowledge about the state of technology. Amato and Shin (2003) consider a targeting rule in an economy in which firms can access both public and private signals about the natural rate of interest. Lately, there have been remarkable developments in the welfare analysis of the precision of public signals, or central-bank transparency. Svensson (2005) and Woodford (2005) cast doubt on the practical relevance of the main result of Morris and Shin (2002) that improving precision of public information could lower welfare. In response, Morris, Shin, and Tong (2006) and Morris and Shin (2006a) develop their arguments. Hellwig (2005) extends the Hellwig (2002) model of nominal adjustment and finds welfare-improving effects of public information in the form of reduced price dispersion. Angeletos and Pavan (2005) provides a general analytical framework that relates the inefficiency of business cycles to the social value of information. These studies, however, are typically based on static models, or assume an unrealistic lack of awareness or attentiveness as does Woodford (2003a).

Meanwhile, the attempt to integrate imperfect common knowledge with

staggered price setting has never been made until very recently.⁵ Nimark (2005) considers firms' private information about their own marginal costs that includes an idiosyncratic component, and derives a Phillips curve based on Calvo-type price setting. Morris and Shin (2006b) consider higher-order expectations iterated in a forward-looking manner, which can be applied to the purely forward-looking New Keynesian Phillips curve based on Calvo-type price setting. These approaches, however, are not sufficiently tractable for obtaining various analytical results.

The remainder of the paper is organized as follows. In Section 2, I describe the baseline model and present the main results on the effects of monetary disturbances. In Section 3, I extend the baseline model by introducing a noisy public signal, in addition to private signals, and examine the effects of informational disturbances as well as monetary disturbances. In Section 4, I consider more general price setting, including the one analogized with Calvo-type price setting. Section 5 concludes.

2 The Baseline Model

In this section, I incorporate a lack of common knowledge into a simple two-period staggered price-setting model and then analytically examine the effects of monetary disturbances.

2.1 Set-up

Consider an economy in which a continuum of monopolistically competitive firms indexed by $i \in [0, 1]$ produce individual-specific goods and set their own prices. Goods are perishable and capital is not required as a factor of production. Following Woodford (2003a) and Mankiw and Reis (2002), I begin with the static optimal price-setting condition of firm i .⁶

$$p_t^*(i) = E_t^i p_t + \xi E_t^i y_t, \quad 0 < \xi < 1 \quad (1)$$

⁵It has been argued, however, that imperfect information and nominal rigidities are closely related to each other as plausible explanations for the real effects of monetary policy. Ball and Cecchetti (1988) develop a model in which monopolistically competitive firms gain information by observing the prices set by others and then the staggered price setting arises endogenously as the equilibrium outcome under certain conditions. Kiley (2000) develops a model that has costs of nominal price adjustment as well as costs of acquiring information in order to estimate the degree of price stickiness.

⁶This condition can be derived from a standard monopolistic competition model such as Blanchard and Kiyotaki (1987). Woodford (2003a) provides a simple explanation of the background of equation (1).

All variables are expressed in terms of log deviations from the full-information symmetric equilibrium. $p_t^*(i)$ is firm i 's desired price in period t and would be the actual price if firms could set their prices flexibly. p_t is the overall price index and y_t is the output gap. The parameter ξ is assumed to be less than unity so that firms' price-setting decisions are strategic complements. The higher the elasticity of substitution among the differentiated goods or the lower the elasticity of marginal cost with respect to output, the smaller is ξ and the greater is the degree of strategic complementarity.

Firms cannot precisely observe aggregate variables such as p_t and y_t in the current period, t . Moreover, their information sets are heterogeneous, which is the main feature of this model. Accordingly the expectations operator conditional on i 's information set at period t , E_t^i , is applied to p_t and y_t in the above equation. In the next subsection, I explain the details of the private information set and the signal extraction problem.

Next, I introduce the two-period staggered price setting as in Taylor (1980). In period t , half of the firms in the economy set their prices for the current period, t , and the next period, $t + 1$. Since they must set the same price for both periods, prices are not just pre-determined but fixed. The price chosen by firm i who sets its price in t is given by⁷

$$\begin{aligned} x_t(i) &= \frac{1}{2} (p_t^*(i) + E_t^i p_{t+1}^*(i)) \\ &= \frac{1}{2} (E_t^i p_t + \xi E_t^i y_t + E_t^i p_{t+1} + \xi E_t^i y_{t+1}). \end{aligned} \tag{2}$$

In period $t + 1$, the remaining half of the firms set their prices for periods $t + 1$ and $t + 2$. In period $t + 2$, the firms who set their prices in period t then re-set their prices for periods $t + 2$ and $t + 3$, and so on. The overall price index is given by

$$p_t = \frac{1}{2} (x_t + x_{t-1}), \tag{3}$$

where x_t is the average price chosen by the firms who set their prices in t , that is, $x_t \equiv 2 \int_0^{0.5} x_t(i) di$ when $t = \dots, -2, 0, 2, \dots$, and $x_t \equiv 2 \int_{0.5}^1 x_t(i) di$ when $t = \dots, -1, 1, \dots$.

I specify the demand side of the economy by introducing an exogenous stochastic process for aggregate nominal spending as follows.

$$m_t - m_{t-1} = \rho (m_{t-1} - m_{t-2}) + \sigma \epsilon_t, \quad \epsilon_t \sim N(0, 1) \tag{4}$$

⁷For simplicity, I assume that the discount rate applied to the firm's profits in the next period is negligible.

where

$$m_t = p_t + y_t \tag{5}$$

and ϵ_t is Gaussian white noise. One may interpret m_t as “money” that households must hold for their spending. Following Mankiw and Reis (2002), I treat the above process as a plausible stochastic process for representing the actual money supply (M2) in the U.S.⁸ Alternatively, m_t can be interpreted more broadly as a generic variable affecting aggregate demand. This simple specification for aggregate demand, however it is interpreted, allows me to concentrate on examining the consequences of alternative specifications for price-setting behavior.

2.2 Signal Extraction

Here I specify firms’ information sets. As in Lucas (1972) and Woodford (2003a), each individual firm estimates the current state of the economy by using their private information. In period t , firm i has access to a noisy private signal about current aggregate demand, m_t , which is represented as follows.

$$z_t(i) = m_t + \sigma_u u_t(i), \quad u_t(i) \sim N(0, 1) \tag{6}$$

where $u_t(i)$ is Gaussian white noise, which is distributed independently of both ϵ_t and $u_t(j)$ for all $j \neq i$. Unlike Woodford, I assume that the true value of m_t becomes common knowledge among all firms with a delay of only one period, in period $t + 1$. Therefore, the information set of firm i comprises the private signal, $z_t(i)$, and the history of realized aggregate nominal spending, $\{m_{t-s}\}_{s=1}^{\infty}$. The result of firms’ signal extraction for estimating m_t is given by

$$\begin{aligned} E_t^i m_t &\equiv E[m_t \mid z_t(i), m_{t-1}, m_{t-2}, \dots] \\ &= b z_t(i) + (1 - b) \{m_{t-1} + \rho(m_{t-1} - m_{t-2})\} \end{aligned} \tag{7}$$

where

$$b \equiv \frac{\sigma^2}{\sigma^2 + \sigma_u^2}$$

represents firms’ reliance on their private signals. Given the variance of aggregate nominal spending, this reliance is greater, the higher is the precision of the signals (the smaller is σ_u).

⁸Woodford (2003a) specifies almost the same stochastic process as (4), except that he adds a drift term that represents the long-run average growth rate of aggregate nominal spending.

2.3 Higher-Order Expectations

Unlike the Lucas model, this model considers an environment among monopolistically competitive firms whose pricing strategies depend on the other firms' strategies. The prices chosen by the firms depend not only on their own estimates of current aggregate demand but also on their expectations of the average estimate among the other firms, their expectations of the average estimate of that average estimate, and so on.

Averaging (7) over i , I have

$$\begin{aligned}\overline{E}_t m_t &= b m_t + (1 - b) \{m_{t-1} + \rho(m_{t-1} - m_{t-2})\} \\ &= b \sigma \epsilon_t + \{m_{t-1} + \rho(m_{t-1} - m_{t-2})\},\end{aligned}\tag{8}$$

where \overline{E}_t is the average expectations operator. The second line implies that the average estimate is not equal to the true value of m_t defined by (4) despite the assumption that the mean of the private signals is equal to the true value. The average estimate is closer to the true value when the private signals are more precise and reliable. When $\sigma_u = 0$, all firms can access homogeneous precise signals and the average expectations operator no longer needs to be defined.

The average expectations operator, defined for heterogeneous information sets, does not satisfy the law of iterative expectations. Firm i 's expectation of the average estimate (8) can be calculated as follows.

$$\begin{aligned}E_t^i [\overline{E}_t m_t] &= b [b z_t(i) + (1 - b) \{m_{t-1} + \rho(m_{t-1} - m_{t-2})\}] \\ &\quad + (1 - b) \{m_{t-1} + \rho(m_{t-1} - m_{t-2})\}\end{aligned}$$

Averaging again over i , I have

$$\begin{aligned}\overline{E}_t [\overline{E}_t m_t] &= b^2 m_t + (1 - b^2) \{m_{t-1} + \rho(m_{t-1} - m_{t-2})\} \\ &= b^2 \sigma \epsilon_t + \{m_{t-1} + \rho(m_{t-1} - m_{t-2})\},\end{aligned}$$

which differs from (8). Therefore, I need to define the j -th order average expectations as follows.

$$\begin{aligned}\overline{E}_t^{(0)} m_t &\equiv m_t \\ \overline{E}_t^{(j+1)} m_t &\equiv \overline{E}_t [\overline{E}_t^{(j)} m_t]\end{aligned}$$

The higher-order average expectations can be calculated as

$$\begin{aligned}E_t^i [\overline{E}_t^{(j)} m_t] &= b^{j+1} z_t(i) + (1 - b^{j+1}) \{m_{t-1} + \rho(m_{t-1} - m_{t-2})\} \\ \overline{E}_t^{(j+1)} m_t &= b^{j+1} m_t + (1 - b^{j+1}) \{m_{t-1} + \rho(m_{t-1} - m_{t-2})\} \\ &= b^{j+1} \sigma \epsilon_t + \{m_{t-1} + \rho(m_{t-1} - m_{t-2})\}.\end{aligned}\tag{9}$$

Since b is less than 1, the infinite-order average expectation converges to the expectations that are conditional only on common knowledge about the history of realized aggregate nominal spending.

2.4 Solving the Model

I seek to find a rational expectations equilibrium, which is defined as a set of $\{p_t, y_t\}$ that satisfies the model equations (1), (2), (3), and (5) given the exogenous process for aggregate nominal spending (4) and the information structure described in the preceding subsections. The key endogenous variable in the model is the re-set price, x_t . Combining equations (1) through (5) yields

$$\begin{aligned}
x_t(i) &= \frac{1}{2} (E_t^i p_t + \xi E_t^i y_t + E_t^i p_{t+1} + \xi E_t^i y_{t+1}) \\
&= \frac{1}{2} \{ \xi E_t^i m_t + (1 - \xi) E_t^i p_t + \xi E_t^i m_{t+1} + (1 - \xi) E_t^i p_{t+1} \} \\
&= \frac{1}{2} \{ \xi E_t^i m_t + \xi E_t^i m_{t+1} + (1 - \xi) E_t^i x_t + \frac{1 - \xi}{2} E_t^i x_{t+1} + \frac{1 - \xi}{2} x_{t-1} \} \\
&= \frac{1}{2} \{ \xi (2 + \rho) E_t^i m_t - \xi \rho m_{t-1} \\
&\quad + (1 - \xi) E_t^i x_t + \frac{1 - \xi}{2} E_t^i x_{t+1} + \frac{1 - \xi}{2} x_{t-1} \}.
\end{aligned}$$

The price chosen by firm i who sets its price in period t depends on its estimate of current aggregate demand, m_t , its estimate of the average price among the firms who set their prices in the same period, x_t , and its estimate of the future average price chosen by the other group of firms, x_{t+1} . The price also depends on the past realized value of aggregate nominal spending, m_{t-1} , and the past average price chosen by the other group of firms, x_{t-1} , which are known in period t and therefore the expectations operators need not be added to these terms.

Averaging $x_t(i)$ over the group of firms who set their prices in t , I have

$$\begin{aligned}
x_t &= \frac{1}{2} \{ \xi (2 + \rho) \bar{E}_t m_t - \xi \rho m_{t-1} + \\
&\quad (1 - \xi) \bar{E}_t x_t + \frac{1 - \xi}{2} \bar{E}_t x_{t+1} + \frac{1 - \xi}{2} x_{t-1} \}, \tag{10}
\end{aligned}$$

where the average expectations operator is defined as $\bar{E}_t(\cdot) \equiv 2 \int_0^{0.5} E_t^i(\cdot) di$ when $t = \dots, -2, 0, 2, \dots$, and $\bar{E}_t(\cdot) \equiv 2 \int_{0.5}^1 E_t^i(\cdot) di$ when $t = \dots, -1, 1, \dots$.

Apart from the average expectations operator, the above equation can be regarded as a second-order difference equation for x_t , similar to the ordinary

two-period staggered price-setting model with full homogenous information sets. I suppose that all firms in both groups believe that the solution of the difference equation takes the following form.

$$x_t = \lambda x_{t-1} + C_1 m_{t-1} + C_2 m_{t-2} + C_3 \sigma \epsilon_t, \quad (11)$$

where λ , C_1 , C_2 , and C_3 are undetermined coefficients. By substituting this solution form into (10), I eliminate the term of x_{t+1} .

$$\begin{aligned} x_t &= \frac{1}{2} \left\{ \xi (2 + \rho) \bar{E}_t m_t - \xi \rho m_{t-1} + (1 - \xi) \bar{E}_t x_t \right. \\ &\quad \left. + \frac{1 - \xi}{2} (\lambda \bar{E}_t x_t + C_1 \bar{E}_t m_t + C_2 m_{t-1}) + \frac{1 - \xi}{2} x_{t-1} \right\} \\ &= \frac{1}{4} \left[\{2\xi(2 + \rho) + (1 - \xi)C_1\} \bar{E}_t m_t + \{(1 - \xi)C_2 - 2\xi\rho\} m_{t-1} \right. \\ &\quad \left. + (2 + \lambda)(1 - \xi) \bar{E}_t x_t + (1 - \xi) x_{t-1} \right] \end{aligned}$$

Note that $E_t^i \epsilon_{t+1} (= \bar{E}_t \epsilon_{t+1}) = 0$ for all i . Then, iterative substitutions for x_t yield higher-order expectations about m_t .

$$\begin{aligned} x_t &= \frac{2\xi(2 + \rho) + (1 - \xi)C_1}{4} \sum_{j=1}^{\infty} \left\{ \frac{(2 + \lambda)(1 - \xi)}{4} \right\}^{j-1} \bar{E}_t^{(j)} m_t \\ &\quad + \frac{(1 - \xi)C_2 - 2\xi\rho}{4 - (2 + \lambda)(1 - \xi)} m_{t-1} + \frac{1 - \xi}{4 - (2 + \lambda)(1 - \xi)} x_{t-1} \end{aligned} \quad (12)$$

This implies that firms consider the weighted sum of higher-order expectations up to the infinite order when choosing their prices. Using (9) to substitute for $\bar{E}_t^{(j)} m_t$, I obtain

$$\begin{aligned} x_t &= \frac{b\{2\xi(2 + \rho) + (1 - \xi)C_1\}}{4 - (2 + \lambda)(1 - \xi)b} \sigma \epsilon_t \\ &\quad + \frac{2\xi(2 + \rho) + (1 - \xi)C_1}{4 - (2 + \lambda)(1 - \xi)} \{m_{t-1} + \rho(m_{t-1} - m_{t-2})\} \\ &\quad + \frac{(1 - \xi)C_2 - 2\xi\rho}{4 - (2 + \lambda)(1 - \xi)} m_{t-1} + \frac{1 - \xi}{4 - (2 + \lambda)(1 - \xi)} x_{t-1}. \end{aligned}$$

By matching this with the solution form (11), the values of the undetermined coefficients, which provide a unique stable solution of the difference equation for x_t , are identified as follows.

$$\begin{aligned}
\lambda &= \frac{1 - \sqrt{\xi}}{1 + \sqrt{\xi}} < 1 \\
C_1 &= \frac{2\sqrt{\xi}}{1 + \sqrt{\xi}} + \frac{2\rho\sqrt{\xi}(1 + \sqrt{\xi} + \rho\sqrt{\xi})}{(1 + \sqrt{\xi})\{1 + \sqrt{\xi} - \rho(1 - \sqrt{\xi})\}} \\
C_2 &= -\frac{2\rho\sqrt{\xi}(1 + \sqrt{\xi} + \rho\sqrt{\xi})}{(1 + \sqrt{\xi})\{1 + \sqrt{\xi} - \rho(1 - \sqrt{\xi})\}} \\
C_3 &= \frac{2\sqrt{\xi}(1 + \sqrt{\xi})(1 + \sqrt{\xi} + \rho\sqrt{\xi})b}{\{1 + \sqrt{\xi} - \rho(1 - \sqrt{\xi})\}\{4 - b(3 - 2\sqrt{\xi} - \xi)\}}
\end{aligned}$$

The set of equilibrium paths $\{p_t, y_t\}$ can be calculated as

$$\begin{aligned}
p_t &= \frac{1}{2}(\lambda x_{t-1} + C_1 m_{t-1} + C_2 m_{t-2} + C_3 \sigma \epsilon_t \\
&\quad + \lambda x_{t-2} + C_1 m_{t-2} + C_2 m_{t-3} + C_3 \sigma \epsilon_{t-1}) \\
&= \lambda p_{t-1} + \frac{1}{2}\{C_1 m_{t-1} + (C_1 + C_2) m_{t-2} + C_2 m_{t-3} + C_3 \sigma (\epsilon_t + \epsilon_{t-1})\} \quad (13)
\end{aligned}$$

$$\begin{aligned}
y_t &= m_t - \lambda p_{t-1} - \frac{1}{2}\{C_1 m_{t-1} + (C_1 + C_2) m_{t-2} + C_2 m_{t-3} + C_3 \sigma (\epsilon_t + \epsilon_{t-1})\} \\
&= \lambda y_{t-1} + \left(1 - \lambda + \rho - \frac{C_1}{2}\right) m_{t-1} - \left(\rho + \frac{C_1 + C_2}{2}\right) m_{t-2} \\
&\quad - \frac{1}{2}\{C_2 m_{t-3} + C_3 \sigma (\epsilon_t + \epsilon_{t-1})\} \quad (14)
\end{aligned}$$

2.5 Impulse Responses

From the solution of the model obtained in the previous subsection, I examine the impulse responses of output and inflation to a monetary disturbance. I compare the responses in the baseline model with those in the full-information two-period staggered price-setting model to study the consequences of a lack of common knowledge. The baseline model nests as a limit case the full-information two-period staggered price-setting model in which all firms can access homogeneous precise information about the realization of the current aggregate disturbances, $\sigma_u = 0$, so that $b = 1$. The other limit case, $\sigma_u = \infty$, so that $b = 0$, implies that all firms have no information about the current aggregate disturbances or are simply assumed to set their prices one period in advance. The case of imperfect common knowledge is between these two limit cases, and explains endogenously how price adjustments are delayed.

The impulse responses of the price level and output to a unit positive innovation in ϵ_0 are calculated as a set of equilibrium paths $\{\hat{p}_t, \hat{y}_t\}$ with $\epsilon_0 = 1$, $\epsilon_t = 0$ for all $t \neq 0$, $p_{-1} = y_{-1} = m_{-1} = m_{-2} = m_{-3} = 0$, and $\lim_{t \rightarrow \infty} y_t = 0$ in (13) and (14). The main analytical results are summarized in the following proposition.

Proposition 1. *i) The impulse response of output is decreasing in firms' reliance on their private signals, b , i.e.,*

$$\frac{\partial \hat{y}_t}{\partial b} < 0, \quad t \geq 0.$$

ii) The impulse response of inflation is initially increasing in b and later decreasing in b , i.e.,

$$\begin{aligned} \frac{\partial(\hat{p}_t - \hat{p}_{t-1})}{\partial b} &> 0, \quad t = 0, 1. \\ \frac{\partial(\hat{p}_t - \hat{p}_{t-1})}{\partial b} &< 0, \quad t \geq 2. \end{aligned}$$

Proof. i) Taking the partial derivative of \hat{y}_t with respect to b sequentially, I have

$$\begin{aligned} \frac{\partial \hat{y}_0}{\partial b} &= -\frac{4\sqrt{\xi}(1+\sqrt{\xi})(1+\sqrt{\xi}+\rho\sqrt{\xi})\sigma}{\{1+\sqrt{\xi}-\rho(1-\sqrt{\xi})\}\{4-b(3-2\sqrt{\xi}-\xi)\}^2} \\ \frac{\partial \hat{y}_1}{\partial b} &= (1+\lambda)\frac{\partial \hat{y}_0}{\partial b} \\ \frac{\partial \hat{y}_t}{\partial b} &= \lambda\frac{\partial \hat{y}_{t-1}}{\partial b}, \quad t \geq 2. \end{aligned}$$

ii) Taking the partial derivative of $(\hat{p}_t - \hat{p}_{t-1})$ with respect to b sequentially, I have

$$\begin{aligned} \frac{\partial \hat{p}_0}{\partial b} &= \frac{4\sqrt{\xi}(1+\sqrt{\xi})(1+\sqrt{\xi}+\rho\sqrt{\xi})\sigma}{\{1+\sqrt{\xi}-\rho(1-\sqrt{\xi})\}\{4-b(3-2\sqrt{\xi}-\xi)\}^2} \\ \frac{\partial(\hat{p}_1 - \hat{p}_0)}{\partial b} &= \lambda\frac{\partial \hat{p}_0}{\partial b} \\ \frac{\partial(\hat{p}_2 - \hat{p}_1)}{\partial b} &= -(1-\lambda^2)\frac{\partial \hat{p}_0}{\partial b} \\ \frac{\partial(\hat{p}_t - \hat{p}_{t-1})}{\partial b} &= \lambda\frac{\partial(\hat{p}_{t-1} - \hat{p}_{t-2})}{\partial b}, \quad t \geq 3. \quad \blacksquare \end{aligned}$$

The noisier are the private signals, that is, the smaller is b , the more sluggish is the initial response of prices. Accordingly, price adjustments are

delayed and inflation may peak later than in the full-information staggered price-setting model. The response of output is amplified by the lack of common knowledge in period 0 and continues to exceed the response in the full-information model even after precise information about the disturbances becomes common knowledge in period 1. While it takes the same period of time for the responses of output and inflation in the full-information and the imperfect-common-knowledge models to die away, the differences between those in these models persist until then.

Moreover, it can be shown that the extent to which a change in the amount of noise affects the sluggishness in the initial response of prices is greater when the private signals are more precise (b is closer to 1), that is, $\partial^2 \hat{p}_0 / \partial b^2 > 0$. This implies that even a small amount of noise in the private signals may significantly delay the adjustment of prices and amplify the response of output.

Sample sets of impulse responses in the models in which $b = 0, 0.25, 0.5, 0.75$, and $b = 1$ (the full-information model) are shown in Figure 1. In the model with $b = 0.5$, the size of the initial response of prices is about a third of that in the full-information model. The response of output is amplified by more than 45 percent in the first four periods, while the response in the model with $b = 0$ is amplified by more than 65 percent.

In Figure 1, I set the parameter value for the strategic complementarity, ξ , to 0.15 following Woodford (2003a), and set the AR(1) coefficient on the process for quarterly aggregate nominal spending, ρ , to 0.5 following Mankiw and Reis (2002). A smaller ξ , that is, a higher degree of strategic complementarity implies a larger λ , which indicates more persistent responses, and also implies a smaller C_3 , which indicates more sluggishness in the initial response of prices. A smaller ρ , that is, a less persistent shock process implies a smaller C_3 but has no implication for λ .

Another interesting comparison is the one between the amplitude of the initial response of prices in the baseline model and that in a static model of imperfect common knowledge without staggered price setting. In the static model, averaging (1) over i yields the average price in the whole economy as follows.

$$\begin{aligned} p &= \bar{E} p + \xi \bar{E} y \\ &= \xi \bar{E} m + (1 - \xi) \bar{E} p \\ &= \xi \sum_{j=1}^{\infty} (1 - \xi)^{j-1} \bar{E}^{(j)} m, \end{aligned}$$

where the average expectations operator is now defined as $\bar{E}(\cdot) \equiv \int_0^1 E^i(\cdot) di$. Substituting (9) with $\epsilon_t = 1$ and $m_{t-1} = m_{t-2} = \rho = 0$, I have

$$\hat{p}^S = \frac{\xi b \sigma}{1 - b(1 - \xi)}.$$

The corresponding result in the baseline model is (13) with $\epsilon_0 = 1$ and $p_{-1} = m_{-1} = m_{-2} = m_{-3} = \epsilon_{-1} = \rho = 0$, that is,

$$\hat{p}_0^D = \frac{\sqrt{\xi}(1 + \sqrt{\xi})b\sigma}{4 - b(3 - 2\sqrt{\xi} - \xi)}.$$

If b and ξ are very small, in which case the private signals are very noisy and the strategic complementarity in firms' price-setting decisions is very strong, \hat{p}_0^D may exceed \hat{p}^S .⁹ However, for most range of the parameter values, \hat{p}_0^D is smaller than \hat{p}^S , which implies that the response that operates through dynamic and staggered higher-order expectations about the future states of the economy as well as the current state is more sluggish than the one that operates through static and simultaneous higher-order expectations.

3 Public Information

In this section, I introduce a noisy public signal in addition to private signals into the baseline model and study the consequences of a more general information structure following Hellwig (2002) and Amato and Shin (2003). The public signal in the extended model may represent preliminary data that is to be revised or noisy information provided by the media, the government, and so on. As Morris and Shin (2002) argue, in an economy in which decision makers' information sets are heterogeneous, public information has disproportionately large effects on their decisions.

3.1 Private and Public Signals

First I re-specify the firms' information set. In period t , firm i has access to not only private signals (6) but also the following public signal, which is not necessarily precise.

$$z_t^P = m_t + \sigma_v v_t, \quad v_t \sim N(0, 1), \quad (15)$$

where v_t is Gaussian white noise distributed independently of both ϵ_t and $u_t(i)$ for all i . Whereas Amato and Shin (2003) assume that price setters never obtain precise information about aggregate disturbances as in the Woodford model, I retain the assumption that the true value of m_t is revealed to all firms with a delay of only one period, in $t + 1$. Therefore, the

⁹The ratio of \hat{p}_0^D to \hat{p}^S is decreasing in b , that is, $\frac{\partial(\hat{p}_0^D/\hat{p}^S)}{\partial b} < 0$.

information set of firm i comprises the private and public signals and the history of realized aggregate nominal spending, in which noisy information, z_t^P , as well as precise information, $\{m_{t-s}\}_{s=1}^\infty$, is common knowledge. Following Hellwig (2002), firms' signal extraction for estimating m_t can be calculated as

$$\begin{aligned} E_t^i m_t &\equiv E[m_t \mid z_t(i), z_t^P, m_{t-1}, m_{t-2}, \dots] \\ &= \alpha \Delta z_t(i) + (1 - \alpha) \Delta z_t^P \\ &\quad + (1 - \Delta) \{m_{t-1} + \rho(m_{t-1} - m_{t-2})\}, \end{aligned} \quad (16)$$

where

$$\alpha \equiv \frac{\sigma_v^2}{\sigma_v^2 + \sigma_u^2}$$

represents firms' reliance on their private signals *relative to* the public signal. Given the precision of the public signal, this *relative* reliance is greater, the higher is the precision of the private signals (the smaller is σ_u). In addition,

$$\Delta \equiv \frac{\sigma^2}{\sigma^2 + \frac{\sigma_u^2 \sigma_v^2}{\sigma_u^2 + \sigma_v^2}} = \frac{\sigma^2}{\sigma^2 + \alpha \sigma_u^2}$$

represents firms' reliance on the private and public signals. Given the variance of aggregate nominal spending, this reliance is greater, the higher is the precision of the *composite* signal.

As in the baseline model, I calculate the higher-order expectations about m_t as follows.

$$\begin{aligned} E_t^i [\overline{E}_t^{(j)} m_t] &= (\alpha \Delta)^{j+1} z_t(i) + \frac{1 - (\alpha \Delta)^{j+1}}{1 - \alpha \Delta} (1 - \alpha) \Delta z_t^P \\ &\quad + \left\{ 1 - (\alpha \Delta)^{j+1} - \frac{1 - (\alpha \Delta)^{j+1}}{1 - \alpha \Delta} (1 - \alpha) \Delta \right\} \\ &\quad \{m_{t-1} + \rho(m_{t-1} - m_{t-2})\} \end{aligned} \quad (17)$$

$$\begin{aligned} \overline{E}_t^{(j+1)} m_t &= (\alpha \Delta)^{j+1} m_t + \frac{1 - (\alpha \Delta)^{j+1}}{1 - \alpha \Delta} (1 - \alpha) \Delta z_t^P \\ &\quad + \left\{ 1 - (\alpha \Delta)^{j+1} - \frac{1 - (\alpha \Delta)^{j+1}}{1 - \alpha \Delta} (1 - \alpha) \Delta \right\} \\ &\quad \{m_{t-1} + \rho(m_{t-1} - m_{t-2})\} \\ &= \left\{ (\alpha \Delta)^{j+1} + \frac{1 - (\alpha \Delta)^{j+1}}{1 - \alpha \Delta} (1 - \alpha) \Delta \right\} \sigma \epsilon_t \\ &\quad + \frac{1 - (\alpha \Delta)^{j+1}}{1 - \alpha \Delta} (1 - \alpha) \Delta \sigma_v v_t \\ &\quad + \{m_{t-1} + \rho(m_{t-1} - m_{t-2})\} \end{aligned} \quad (18)$$

Compared with firm i 's own estimate of m_t , (16), its expectation of the higher-order average expectations, (17), is more responsive to public information including the public signal and the history of realized aggregate nominal spending, and less responsive to private information. The infinite-order average expectation converges to the expectations that are conditional only on common knowledge.

3.2 Effects of Monetary Disturbances

Substituting (18) into (12) in the baseline model, I obtain a unique stable solution of the following difference equation: $x_t = \lambda x_{t-1} + C_1 m_{t-1} + C_2 m_{t-2} + C_3 \sigma \epsilon_t + C_4 \sigma_v v_t$, where λ , C_1 , and C_2 are the same as in the baseline model and

$$C_3 = \frac{2\sqrt{\xi}(1 + \sqrt{\xi} + \rho\sqrt{\xi})}{(1 + \sqrt{\xi})\{1 + \sqrt{\xi} - \rho(1 - \sqrt{\xi})\}} \frac{\{4 - \alpha(3 - 2\sqrt{\xi} - \xi)\}\Delta}{\{4 - \alpha\Delta(3 - 2\sqrt{\xi} - \xi)\}},$$

$$C_4 = \frac{8\sqrt{\xi}(1 + \sqrt{\xi} + \rho\sqrt{\xi})}{(1 + \sqrt{\xi})\{1 + \sqrt{\xi} - \rho(1 - \sqrt{\xi})\}} \frac{(1 - \alpha)\Delta}{\{4 - \alpha\Delta(3 - 2\sqrt{\xi} - \xi)\}}.$$

As before, I examine the impulse responses of output and inflation to a monetary disturbance, that is, a unit positive innovation in ϵ_0 , and compare these responses with those in the baseline model as well as those in the full-information staggered price-setting model. The baseline model without the public signal corresponds to the case of $\sigma_v = \infty$, so that $\alpha = 1$ and $\Delta = b$, and the full-information staggered price-setting model corresponds to the case of $\sigma_u = 0$ and $\sigma_v = \infty$, so that $\alpha = 1$ and $\Delta = 1$. The responses of the price level and output in this extended model are calculated as a set of equilibrium paths $\{\hat{p}_t^P, \hat{y}_t^P\}$ with $\epsilon_0 = 1$, $\epsilon_t = 0$ for all $t \neq 0$, $v_t = 0$ for all t , $p_{-1} = y_{-1} = m_{-1} = m_{-2} = m_{-3} = 0$, and $\lim_{t \rightarrow \infty} y_t = 0$. The main analytical results are summarized in the following proposition.

Proposition 2. *i) The impulse response of output is increasing in firms' relative reliance on their private signals to the public signal, α , and decreasing in their reliance on the private and public signals, Δ , i.e.,*

$$\frac{\partial \hat{y}_t^P}{\partial \alpha} > 0, \quad \frac{\partial \hat{y}_t^P}{\partial \Delta} < 0, \quad t \geq 0.$$

ii) The impulse response of inflation is initially decreasing in α and increasing in Δ , and later increasing in α and decreasing in Δ i.e.,

$$\frac{\partial(\hat{p}_t^P - \hat{p}_{t-1}^P)}{\partial \alpha} < 0, \quad \frac{\partial(\hat{p}_t^P - \hat{p}_{t-1}^P)}{\partial \Delta} > 0, \quad t = 0, 1.$$

$$\frac{\partial(\hat{p}_t^P - \hat{p}_{t-1}^P)}{\partial \alpha} > 0, \quad \frac{\partial(\hat{p}_t^P - \hat{p}_{t-1}^P)}{\partial \Delta} < 0, \quad t \geq 2.$$

Proof. i) Taking the partial derivatives of \hat{y}_0^P with respect to α and Δ , I have

$$\begin{aligned}\frac{\partial \hat{y}_0^P}{\partial \alpha} &= \frac{4\sqrt{\xi}(1+\sqrt{\xi}+\rho\sqrt{\xi})}{(1+\sqrt{\xi})\{1+\sqrt{\xi}-\rho(1-\sqrt{\xi})\}} \frac{(3-2\sqrt{\xi}-\xi)\Delta(1-\Delta)\sigma}{\{4-\alpha\Delta(3-2\sqrt{\xi}-\xi)\}^2} \\ \frac{\partial \hat{y}_0^P}{\partial \Delta} &= -\frac{4\sqrt{\xi}(1+\sqrt{\xi}+\rho\sqrt{\xi})}{(1+\sqrt{\xi})\{1+\sqrt{\xi}-\rho(1-\sqrt{\xi})\}} \frac{\{4-\alpha(3-2\sqrt{\xi}-\xi)\}\sigma}{\{4-\alpha\Delta(3-2\sqrt{\xi}-\xi)\}^2}.\end{aligned}$$

From \hat{y}_1^P onward,

$$\begin{aligned}\frac{\partial \hat{y}_1^P}{\partial \alpha} &= (1+\lambda)\frac{\partial \hat{y}_0^P}{\partial \alpha}, \quad \frac{\partial \hat{y}_1^P}{\partial \Delta} = (1+\lambda)\frac{\partial \hat{y}_0^P}{\partial \Delta}. \\ \frac{\partial \hat{y}_t^P}{\partial \alpha} &= \lambda\frac{\partial \hat{y}_{t-1}^P}{\partial \alpha}, \quad \frac{\partial \hat{y}_t^P}{\partial \Delta} = \lambda\frac{\partial \hat{y}_{t-1}^P}{\partial \Delta}, \quad t \geq 2.\end{aligned}$$

ii) Taking the partial derivatives of \hat{p}_0^P with respect to α and Δ , I have

$$\begin{aligned}\frac{\partial \hat{p}_0^P}{\partial \alpha} &= -\frac{4\sqrt{\xi}(1+\sqrt{\xi}+\rho\sqrt{\xi})}{(1+\sqrt{\xi})\{1+\sqrt{\xi}-\rho(1-\sqrt{\xi})\}} \frac{(3-2\sqrt{\xi}-\xi)\Delta(1-\Delta)\sigma}{\{4-\alpha\Delta(3-2\sqrt{\xi}-\xi)\}^2} \\ \frac{\partial \hat{p}_0^P}{\partial \Delta} &= \frac{4\sqrt{\xi}(1+\sqrt{\xi}+\rho\sqrt{\xi})}{(1+\sqrt{\xi})\{1+\sqrt{\xi}-\rho(1-\sqrt{\xi})\}} \frac{\{4-\alpha(3-2\sqrt{\xi}-\xi)\}\sigma}{\{4-\alpha\Delta(3-2\sqrt{\xi}-\xi)\}^2}.\end{aligned}$$

From \hat{p}_1^P onward,

$$\begin{aligned}\frac{(\partial \hat{p}_1^P - \partial \hat{p}_0^P)}{\partial \alpha} &= \lambda\frac{\partial \hat{p}_0^P}{\partial \alpha}, \quad \frac{\partial \hat{p}_1^P - \partial \hat{p}_0^P}{\partial \Delta} = \lambda\frac{\partial \hat{p}_0^P}{\partial \Delta}. \\ \frac{(\partial \hat{p}_2^P - \partial \hat{p}_1^P)}{\partial \alpha} &= -(1-\lambda^2)\frac{\partial \hat{p}_0^P}{\partial \alpha}, \quad \frac{\partial \hat{p}_2^P - \partial \hat{p}_1^P}{\partial \Delta} = -(1-\lambda^2)\frac{\partial \hat{p}_0^P}{\partial \Delta}. \\ \frac{\partial \hat{p}_t^P - \partial \hat{p}_{t-1}^P}{\partial \alpha} &= \lambda\frac{\partial \hat{p}_{t-1}^P - \partial \hat{p}_{t-2}^P}{\partial \alpha}, \quad \frac{\partial \hat{p}_t^P - \partial \hat{p}_{t-1}^P}{\partial \Delta} = \lambda\frac{\partial \hat{p}_{t-1}^P - \partial \hat{p}_{t-2}^P}{\partial \Delta}, \quad t \geq 3.\end{aligned}$$

■

The greater is firms' relative reliance on the *public* signal compared to the private signals (the smaller is α), the *less* sluggish is the initial response of prices. Provision of the public signal allows firms to gain common knowledge and, hence, calculate the higher-order average expectations more precisely. This alleviates the sluggishness in their initial response of prices to some extent. Compared with the full-information staggered price-setting model, however, the adjustment of prices is still delayed and the response of output is amplified as long as the signals contain noise. The lower is firms' reliance on the private and public signals (the smaller is Δ), the *more* sluggish is the initial response of prices. Either noisier private signals or a noisier public signal makes the initial response of prices more sluggish, that is, $\partial \hat{p}_0^P / \partial \sigma_u < 0$ and $\partial \hat{p}_0^P / \partial \sigma_v < 0$.

Sample sets of impulse responses in the full-information model, the baseline model ($b = 0.5$), and the extended models ($\alpha = 0$ and 0.5 with $\Delta = 0.5$) are shown in Figure 2. The other parameter values, for ξ and ρ , are the same as in Figure 1. In the extended model of $\alpha = 0$, the size of the initial response of prices is half of that in the full-information model. The response of output is amplified by more than 30 percent in the first four periods.

As before, I compare the amplitude of the initial response of prices in the extended model with that in a static model that incorporates the public signal as well as the private signals. The average price in the static model is

$$\hat{p}^{PS} = \frac{\{1 - \alpha(1 - \xi)\} \Delta \sigma}{\{1 - \alpha \Delta(1 - \xi)\}}.$$

The corresponding result in the extended model is

$$\hat{p}_0^{PD} = \frac{\sqrt{\xi}}{(1 + \sqrt{\xi})} \frac{\{4 - \alpha(3 - 2\sqrt{\xi} - \xi)\} \Delta \sigma}{\{4 - \alpha \Delta(3 - 2\sqrt{\xi} - \xi)\}}.$$

If α is very close to 1 and Δ and ξ are very small, \hat{p}_0^{PD} may exceed \hat{p}^{PS} . However, for most range of the parameter values, \hat{p}_0^{PD} is smaller than \hat{p}^{PS} , which implies that the response that operates through dynamic and staggered higher-order expectations is more sluggish than the one that operates through static and simultaneous higher-order expectations.

3.3 Effects of Informational Disturbances

While the public signal reduces uncertainty in firms' higher-order expectations about aggregate nominal spending, the noise in the public signal itself adds to aggregate uncertainty. Firms with heterogeneous information sets might over-react to the noisy public signal to such an extent that the economy could be destabilized. I consider this side effect of public information by using the extended model to examine the impulse responses to an informational disturbance in the public signal.

The responses of the price level and output to a unit positive innovation in v_0 are calculated as a set of equilibrium paths $\{\tilde{p}_t^P, \tilde{y}_t^P\}$ with $v_0 = 1$, $v_t = 0$ for all $t \neq 0$, $m_t = \epsilon_t = 0$ for all t , $p_{-1} = y_{-1} = 0$, and $\lim_{t \rightarrow \infty} y_t = \lim_{t \rightarrow \infty} p_t = 0$. The responses of output and inflation are given by

$$\begin{aligned} \tilde{y}_0^P &= -\frac{4\sqrt{\xi}(1 + \sqrt{\xi} + \rho\sqrt{\xi})}{(1 + \sqrt{\xi})\{1 + \sqrt{\xi} - \rho(1 - \sqrt{\xi})\}} \frac{(1 - \alpha)\Delta\sigma_v}{\{4 - \alpha\Delta(3 - 2\sqrt{\xi} - \xi)\}} \\ \tilde{y}_1^P &= (1 + \lambda)\tilde{y}_0^P \\ \tilde{y}_t^P &= \lambda\tilde{y}_{t-1}^P, \quad t \geq 2 \end{aligned}$$

and

$$\begin{aligned}
\tilde{p}_0^P &= \frac{4\sqrt{\xi}(1+\sqrt{\xi}+\rho\sqrt{\xi})}{(1+\sqrt{\xi})\{1+\sqrt{\xi}-\rho(1-\sqrt{\xi})\}} \frac{(1-\alpha)\Delta\sigma_v}{\{4-\alpha\Delta(3-2\sqrt{\xi}-\xi)\}} \\
\tilde{p}_1^P - \tilde{p}_0^P &= \lambda\tilde{p}_0^P \\
\tilde{p}_2^P - \tilde{p}_1^P &= -(1-\lambda^2)\tilde{p}_0^P \\
\tilde{p}_t^P - \tilde{p}_{t-1}^P &= \lambda(\tilde{p}_{t-1}^P - \tilde{p}_{t-2}^P), \quad t \geq 3.
\end{aligned}$$

Firms raise their prices when they receive a public signal biased upward from the true value of $m_0(=0)$, believing it to be unbiased. Since the exogenous process for m_t is not affected by the informational disturbance, the increase in prices leads to a corresponding decrease in output.

Sample sets of impulse responses of output and inflation to a *negative* informational disturbance are shown in Figure 3.1, in which $(\alpha, \Delta) = (0.5, 0.5)$ and the other parameter values are the same as in Figure 2. Firms react to the downward-biased public signal by reducing their prices, and output increases accordingly. When the output gap begins to shrink, prices start increasing and then inflation peaks later than output. A combination of this pattern of responses to a negative informational disturbance and the pattern of responses to a positive monetary disturbance examined in the previous subsection further delays the response of inflation and further amplifies the response of output.¹⁰ These effects of the informational disturbance in the public signal could offset the effects of alleviating the sluggishness in the initial response of prices to the monetary disturbance.

Improving precision of the public signal does not necessarily reduce the amplitude of those responses to the informational disturbance. While a small σ_v generates small responses of output and inflation, it also implies that firms rely heavily on the public signal, in which case α is small and Δ is large, and therefore, indirectly generates high responsiveness to informational disturbances.¹¹ If the latter indirect effect dominates, improving precision of the public signal induces firms to over-react to the signal and amplifies the responses. The partial derivative of the initial response of prices with respect to the amount of noise in the public signal is

$$\frac{\partial \tilde{p}_0^P}{\partial \sigma_v} = \frac{4\sqrt{\xi}(1+\sqrt{\xi}+\rho\sqrt{\xi})}{(1+\sqrt{\xi})\{1+\sqrt{\xi}-\rho(1-\sqrt{\xi})\}} \frac{\sigma_u^2 \sigma^2 \{4\sigma_u^2(\sigma^2 - \sigma_v^2) - \sigma_v^2 \sigma^2(1+\sqrt{\xi})^2\}}{\{4\sigma_u^2(\sigma^2 + \sigma_v^2) + \sigma_v^2 \sigma^2(1+\sqrt{\xi})^2\}^2}.$$

The sign is ambiguous, depending on the magnitude of σ_v .

¹⁰A negative informational disturbance accompanied by a positive monetary disturbance implies that the public signal (15) does not reflect the current monetary disturbance.

¹¹It can be shown that $\frac{\partial C_4}{\partial \alpha} < 0$ and $\frac{\partial C_4}{\partial \Delta} > 0$, where $\tilde{y}_0^P = -\frac{1}{2}C_4\sigma_v$ and $\tilde{p}_0^P = \frac{1}{2}C_4\sigma_v$.

In Figure 3.2, I plot \tilde{p}_0^P as a function of the amount of noise in the public signal, σ_v . For the amount of noise in the private signals, σ_u , and the variance of aggregate nominal spending, σ^2 , I choose the following three sets of parameter values: $(\sigma_u, \sigma^2) = (1, 0.5), (2, 0.5), (1, 1)$. When $\sigma_v = 1$, these sets correspond to $(\alpha, \Delta) = (0.5, 0.5), (0.2, 0.385), (0.5, 0.667)$, respectively.¹² The relationship between the precision of the public signal and the responsiveness of prices and output to informational disturbances is non-monotonic: improving precision reduces responsiveness when precision is high (σ_v is small) but amplifies it when precision is low (σ_v is large). The “conservative benchmark” case proposed by Svensson (2005), in which the public signal is as precise as the private signals, is contained within the range in which the precision of public signal is relatively low.¹³ A small improvement in the precision of such a relatively noisy public signal could increase output volatility and destabilize the economy.¹⁴

4 General Staggered Price Setting

The preceding models are based on simple two-period staggered price setting. In this section, I consider more general price setting that allows for multiple-period staggered price setting. Of particular interest is Calvo-type price setting, which is widely used in recent New Keynesian macroeconomic models.

4.1 Set-up

Consider an economy in which a proportion θ_1 of monopolistically competitive firms $i \in \Theta_1$ set their prices in each period, θ_2 of firms $i \in \Theta_2$ set their prices every other period, θ_3 of firms $i \in \Theta_3$ do so every three periods, and θ_4 of firms $i \in \Theta_4$ do so every four periods. Within Θ_j , the periods of price setting are staggered among j equally-sized groups. I assume that the maximum fixed-price length is four periods so that $\sum_{j=1}^4 \theta_j = 1$.

¹²The asterisk on the $(\sigma_u, \sigma^2) = (1, 0.5)$ line corresponds to the initial response of inflation (price level) in Figure 3.1.

¹³Svensson (2005) argues that the public signal should be at least as precise as the private signals in the context of central-bank transparency, and points out that improving precision of the public signal is welfare-improving unless the public signal is much noisier than the private signals in the Morris and Shin (2002) model. Meanwhile, Morris and Shin (2006a) argue that it would be difficult to conclude on the relative precision without taking account of its endogenous nature.

¹⁴Hellwig (2005) studies the welfare effects of providing public and private information with taking account of price dispersion as well as output volatility.

While the static optimal price-setting condition in the baseline model (1) is unchanged, the individual price equation (2) is modified as follows.

$$x_t(i) = \begin{cases} p_t^*(i) & \text{for } i \in \Theta_1 \\ \frac{1}{2}(p_t^*(i) + E_t^i p_{t+1}^*(i)) & \text{for } i \in \Theta_{2,t} \\ \frac{1}{3}(p_t^*(i) + E_t^i p_{t+1}^*(i) + E_t^i p_{t+2}^*(i)) & \text{for } i \in \Theta_{3,t} \\ \frac{1}{4}(p_t^*(i) + E_t^i p_{t+1}^*(i) + E_t^i p_{t+2}^*(i) + E_t^i p_{t+3}^*(i)) & \text{for } i \in \Theta_{4,t} \end{cases}$$

where $\Theta_{j,t}$ is $1/j$ of the firms within Θ_j who set their prices in period t . Averaging over those firms $i \in \Theta_1 \cup \Theta_{2,t} \cup \Theta_{3,t} \cup \Theta_{4,t}$ yields

$$x_t = \omega_1 p_t^* + \omega_2 \bar{E}_t p_{t+1}^* + \omega_3 \bar{E}_t p_{t+2}^* + \omega_4 \bar{E}_t p_{t+3}^*, \quad (19)$$

where

$$p_t^* = \bar{E}_t p_t + \xi \bar{E}_t y_t$$

and

$$\begin{aligned} \omega_1 &= \theta_1 + \frac{1}{2}\theta_2 + \frac{1}{3}\theta_3 + \frac{1}{4}\theta_4 \\ \omega_2 &= \frac{1}{2}\theta_2 + \frac{1}{3}\theta_3 + \frac{1}{4}\theta_4 \\ \omega_3 &= \frac{1}{3}\theta_3 + \frac{1}{4}\theta_4 \\ \omega_4 &= \frac{1}{4}\theta_4. \end{aligned}$$

The average expectations operator \bar{E}_t is now defined as the average of E_t^i over $i \in \Theta_1 \cup \Theta_{2,t} \cup \Theta_{3,t} \cup \Theta_{4,t}$.

The overall price index is the weighted sum of prices set in the current and past periods as in the baseline model. Equation (3) can be rewritten as

$$p_t = \omega_1 x_t + \omega_2 x_{t-1} + \omega_3 x_{t-2} + \omega_4 x_{t-3}. \quad (20)$$

The demand side of the economy, represented by (4) and (5), is the same as in the baseline model. The information structure is the same as in Section 3 in which the public signal and the private signals are incorporated.

4.2 Impulse responses

As before, I examine the impulse responses of output and inflation to a monetary disturbance, and compare them with those in the corresponding full-information staggered price-setting model.

Since the analytical results corresponding to Proposition 2 in Section 3 are qualitatively little changed, I relegate them to the Appendix. Here, I examine the sample sets of impulse responses shown in Figures 4, 5 and 6. In Figure 4, for the model analogized with Calvo-type price setting, I set θ_j to 0.5, 0.25, 0.15, and 0.1 for $j = 1$ to 4 respectively. This implies that the constant probability with which each firm gets the opportunity to change its price is approximated by $\omega_1 = 0.7$. The other parameter values are the same as in the extended model in Section 3.

As in the two-period staggered price-setting models in the preceding sections, the size of the initial response of prices when there are only private signals ($\alpha = 1$ and $\Delta = b = 0.5$) is about a third of the size of the initial response in the corresponding full-information staggered price-setting model. When there are both private and public signals ($\alpha = 0$ and $\Delta = 0.5$), the corresponding proportion is a half. The initial response of output is amplified by more than 80 percent when there are only private signals, and is amplified by more than 60 percent when there are both private and public signals. Although these rates of amplification exceed those in the two-period staggered price-setting models in period 0, they monotonically decrease as the output gap shrinks. However, the response of output remains amplified, that is, it persistently exceeds that in the corresponding full-information staggered price-setting model.

As mentioned in the introduction, a major problem with Calvo-type price setting is that the price level jumps in the period of disturbance and inflation responds earlier than does output. The above results show that this problem can be overcome by incorporating a lack of common knowledge into the model.

In Figure 5, I set θ_j to 0 for $j = 1$ to 3 and $\theta_4 = 1$ with other parameter values unchanged, that is, I assume four-period staggered price setting. When there are only private signals, the initial response of prices is about 40 percent of the size of the response in the corresponding full-information staggered price-setting model, and is half its size when there are both private and public signals. Thereafter, price adjustments in both cases are slightly delayed but inflation peaks in the same period as it does in the full-information staggered price-setting model. The initial response of output is large but the rate of amplification is about 15 percent when there are only private signals and is about 12 percent when there are both private and public signals. These amplification rates remain almost constant (or increase) as the output gap shrinks. Since the proportion of prices set in the period of disturbance under imperfect common knowledge, ω_1 , is smaller than that in the preceding Calvo-analogized model, the responses are closer to those in the corresponding full-information staggered price-setting model.

Lastly, to complete the discussion, I examine the responses to an informational disturbance. Sample sets of impulse responses of output and inflation to a *negative* informational disturbance in the Calvo-analogized model and the four-period staggered price-setting model are shown in Figure 6. While the amplitude of the responses is larger in the former model, the persistence is greater and the peaks are later in the latter model.

5 Concluding Remarks

In this paper I have studied the consequences of a lack of common knowledge in the transmission of monetary policy by integrating the Woodford (2003a) imperfect common knowledge model with Taylor-Calvo staggered price-setting models. The average price set by monopolistically competitive firms who can only observe the state of the economy through noisy private signals depends on their higher-order expectations about not only the current state of the economy but also about the states in the future periods in which prices are to be fixed. This integrated model provides a plausible explanation for the observed effects of monetary policy: it shows analytically how price adjustments are delayed and how the response of output to a monetary disturbance is amplified.

These results are robust in the model of Section 3, which incorporates a noisy public signal as well as private signals, and in the models of Section 4, which generalize staggered price setting. While provision of the public signal alleviates the sluggishness in price adjustments to monetary disturbances, it exposes firms to an additional disturbance, namely noise in the public signal, and could destabilize the economy. The Calvo-analogized model in Section 4 overcomes a major problem with Calvo-type price setting that the price level jumps in the period of disturbance and inflation responds earlier than does output.

Based on the models developed in this paper, at least two directions for future research can be pursued. One is policy research. The model of Section 3 could be further extended to obtain richer implications for the central bank's communication strategy. Another direction is empirical research. For example, deep parameters such as price setters' reliance on their private information could be estimated by matching impulse responses obtained from a structural model with those from an estimated VAR model. Although the models in this paper may be too simple for practical use, they are tractable, flexible, and based on plausible assumptions about information structure. I hope these models serve as a useful building block for future research in those directions.

Appendix

Solution of the General Staggered Price-Setting Model

The key equations of the model are (19) and (20).

$$\begin{aligned} x_t &= \omega_1 p_t^* + \omega_2 \bar{E}_t p_{t+1}^* + \omega_3 \bar{E}_t p_{t+2}^* + \omega_4 \bar{E}_t p_{t+3}^* \\ p_t &= \omega_1 x_t + \omega_2 x_{t-1} + \omega_3 x_{t-2} + \omega_4 x_{t-3} \end{aligned}$$

Combining these, I have

$$\begin{aligned} x_t &= \xi (M_1 \bar{E}_t m_t + M_2 m_{t-1}) \\ &\quad + (1 - \xi) (W_1 x_{t-3} + W_2 x_{t-2} + W_3 x_{t-1} \\ &\quad + W \bar{E}_t x_t + W_3 \bar{E}_t x_{t+1} + W_2 \bar{E}_t x_{t+2} + W_1 \bar{E}_t x_{t+3}), \end{aligned}$$

where

$$\begin{aligned} M_1 &\equiv \omega_1 + \omega_2 (1 + \rho) + \omega_3 (1 + \rho + \rho^2) + \omega_4 (1 + \rho + \rho^2 + \rho^3) \\ M_2 &\equiv -\{\omega_2 \rho + \omega_3 (\rho + \rho^2) + \omega_4 (\rho + \rho^2 + \rho^3)\} \\ W_1 &\equiv \omega_1 \omega_4 \\ W_2 &\equiv \omega_1 \omega_3 + \omega_2 \omega_4 \\ W_3 &\equiv \omega_1 \omega_2 + \omega_2 \omega_3 + \omega_3 \omega_4 \\ W &\equiv \omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2. \end{aligned}$$

Suppose that all firms believe that the solution of the difference equation for x_t takes the following form.

$$\begin{aligned} x_t &= \lambda_1 x_{t-1} + \lambda_2 x_{t-2} + \lambda_3 x_{t-3} \\ &\quad + C_1 m_{t-1} + C_2 m_{t-2} + C_3 \sigma \epsilon_t + C_4 \sigma_v v_t. \end{aligned}$$

As before, the undetermined coefficients are identified as follows.

$$\begin{aligned} C_3 &= \frac{M^* (1 - \alpha W^*) \Delta}{(1 - W^*) (1 - \alpha \Delta W^*)} \\ C_4 &= \frac{M^* (1 - \alpha) \Delta}{(1 - W^*) (1 - \alpha \Delta W^*)}, \end{aligned}$$

where

$$\begin{aligned} M^* &\equiv \xi M_1 + (1 - \xi) \{ W_3 C_1 + W_2 C_2 + (W_2 \lambda_1 + W_1 \lambda_2) C_1 + W_1 \lambda_1 C_2 \\ &\quad + W_1 \lambda_1^2 C_1 + (W_2 C_1 + W_1 C_2 + W_1 \lambda_1 C_1 + W_1 C_1) (1 + \rho) + W_1 C_1 \rho \} \\ W^* &\equiv (1 - \xi) \{ W + W_3 \lambda_1 + W_2 \lambda_2 + W_1 \lambda_3 \\ &\quad + (W_2 \lambda_1 + W_1 \lambda_2) \lambda_1 + W_1 \lambda_1 \lambda_2 + W_1 \lambda_1^3 \}, \end{aligned}$$

while $\lambda_1, \lambda_2, \lambda_3, C_1,$ and C_2 are determined independently of α and Δ by solving the following difference equation

$$\left(\frac{1}{1-\xi} - W_1 L^{-3} - W_2 L^{-2} - W_3 L^{-1} - W - W_3 L - W_2 L^2 - W_1 L^3 \right) x_t = \frac{\xi}{1-\xi} (M_1 + M_2 L) m_t,$$

where L is the lag operator defined as $L x_t \equiv x_{t-1}$.

Now I examine the impulse responses of output and inflation to a monetary disturbance, that is, a unit positive innovation in ϵ_0 . The responses are calculated as a set of equilibrium paths $\{\hat{p}_t^P, \hat{y}_t^P\}$, as in Proposition 2. Taking the partial derivatives of \hat{y}_0^P with respect to α and Δ , I have

$$\frac{\partial \hat{y}_0^P}{\partial \alpha} = \frac{\omega_1 M^* W^* \Delta (1-\Delta) \sigma}{(1-W^*)(1-\alpha \Delta W^*)^2}, \quad \frac{\partial \hat{y}_0^P}{\partial \Delta} = -\frac{\omega_1 M^* (1-\alpha W^*) \sigma}{(1-W^*)(1-\alpha \Delta W^*)^2}.$$

As in Proposition 2, the former is positive and the latter is negative, unless $W^* > 1$. Correspondingly, the partial derivatives of \hat{p}_0^P with respect to α and Δ are

$$\frac{\partial \hat{p}_0^P}{\partial \alpha} = -\frac{\omega_1 M^* W^* \Delta (1-\Delta) \sigma}{(1-W^*)(1-\alpha \Delta W^*)^2}, \quad \frac{\partial \hat{p}_0^P}{\partial \Delta} = \frac{\omega_1 M^* (1-\alpha W^*) \sigma}{(1-W^*)(1-\alpha \Delta W^*)^2}.$$

The partial derivatives of the response of output evolve as follows.

$$\begin{aligned} \frac{\partial \hat{y}_t^P}{\partial \alpha} &= \lambda_1 \frac{\partial \hat{y}_{t-1}^P}{\partial \alpha} + \lambda_2 \frac{\partial \hat{y}_{t-2}^P}{\partial \alpha} + \lambda_3 \frac{\partial \hat{y}_{t-3}^P}{\partial \alpha} + \frac{\omega_{t+1}}{\omega_1} \frac{\partial \hat{y}_0^P}{\partial \alpha}, \quad 0 \leq t \leq 3. \\ &= \lambda_1 \frac{\partial \hat{y}_{t-1}^P}{\partial \alpha} + \lambda_2 \frac{\partial \hat{y}_{t-2}^P}{\partial \alpha} + \lambda_3 \frac{\partial \hat{y}_{t-3}^P}{\partial \alpha}, \quad t \geq 4. \end{aligned}$$

The evolution of the partial derivative with respect to Δ is the same. These imply that the response of output remains amplified as long as the roots $\lambda_1, \lambda_2,$ and λ_3 are positive.

The partial derivatives of the response of inflation evolve as follows.

$$\begin{aligned} \frac{(\partial \hat{p}_t^P - \partial \hat{p}_{t-1}^P)}{\partial \alpha} &= \lambda_1 \frac{(\partial \hat{p}_{t-1}^P - \partial \hat{p}_{t-2}^P)}{\partial \alpha} + \lambda_2 \frac{(\partial \hat{p}_{t-2}^P - \partial \hat{p}_{t-3}^P)}{\partial \alpha} \\ &\quad \lambda_3 \frac{(\partial \hat{p}_{t-3}^P - \partial \hat{p}_{t-4}^P)}{\partial \alpha} - \frac{\omega_t - \omega_{t+1}}{\omega_1} \frac{\partial \hat{p}_0^P}{\partial \alpha}, \quad 1 \leq t \leq 3. \\ &= \lambda_1 \frac{(\partial \hat{p}_{t-1}^P - \partial \hat{p}_{t-2}^P)}{\partial \alpha} + \lambda_2 \frac{(\partial \hat{p}_{t-2}^P - \partial \hat{p}_{t-3}^P)}{\partial \alpha} \\ &\quad \lambda_3 \frac{(\partial \hat{p}_{t-3}^P - \partial \hat{p}_{t-4}^P)}{\partial \alpha} - \frac{\omega_t}{\omega_1} \frac{\partial \hat{p}_0^P}{\partial \alpha}, \quad t = 4. \end{aligned}$$

$$\begin{aligned}
&= \lambda_1 \frac{(\partial \hat{p}_{t-1}^P - \partial \hat{p}_{t-2}^P)}{\partial \alpha} + \lambda_2 \frac{(\partial \hat{p}_{t-2}^P - \partial \hat{p}_{t-3}^P)}{\partial \alpha} \\
&\quad \lambda_3 \frac{(\partial \hat{p}_{t-3}^P - \partial \hat{p}_{t-4}^P)}{\partial \alpha}, \quad t \geq 5.
\end{aligned}$$

The evolution of the partial derivative with respect to Δ is the same. These imply that the sign of the partial derivative in period 0 is later reversed, that is, the price adjustments are delayed after the initial sluggish response.

References

- [1] Adam, K. (2006), “Optimal Monetary Policy with Imperfect Common Knowledge,” *Journal of Monetary Economics*, forthcoming.
- [2] Amato, J. D. and Shin, H. S. (2003), “Public and Private Information in Monetary Policy Models,” BIS Working Papers, 138.
- [3] Angeletos, G. M. and Pavan, A. (2005), “Efficiency and Welfare with Complementarities and Asymmetric Information,” NBER Working Paper, 11826.
- [4] Ball, L. and Cecchetti, S. G. (1988), “Imperfect Information and Staggered Price Setting,” *American Economic Review*, 78(5): 999-1018.
- [5] Blanchard, O. and Kiyotaki, N. (1987), “Monopolistic competition and the Effects of Aggregate Demand,” *American Economic Review*, 77(4): 647-666.
- [6] Calvo, G. A. (1983), “Staggered Prices in a Utility-Maximizing Framework,” *Journal of Monetary Economics*, 12(3): 383-398.
- [7] Christiano, L. J., Eichenbaum, M. and Evans, C. L. (2005), “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy,” *Journal of Political Economy*, 113(1): 1-45.
- [8] Fukunaga, I. (2005), “Inventories, Predetermined Prices, and the Effects of Monetary Policy,” Bank of Japan, IMES Discussion Paper, 2005-E-14.
- [9] Hellwig, C. (2002), “Public Announcements, Adjustment Delays and the Business Cycles,” University of California at Los Angeles.
- [10] Hellwig, C. (2005), “Heterogeneous Information and the Welfare Effects of Public Information Disclosures,” University of California at Los Angeles.

- [11] Kawamoto, T. (2004), "Technology Shocks and the Role of Monetary Policy in the Beauty Contest Monetarist Model," Bank of Japan, IMES Discussion Paper, 2004-E-10.
- [12] Keynes, J. M. (1936), *The General Theory of Employment, Interest, and Money*, MacMillan, London.
- [13] Kiley, M. T. (2000), "Endogenous Price Stickiness and Business Cycle Persistence," *Journal of Money, Credit, and Banking*, 32(1): 28-53.
- [14] Lucas, R. Jr. (1972), "Expectations and the Neutrality of Money," *Journal of Economic Theory*, 4(2): 103-124.
- [15] Lucas, R. Jr. (1975), "An Equilibrium Model of the Business Cycle," *Journal of Political Economy*, 83(6): 1113-1144.
- [16] Mankiw, N. G. and Reis, R. (2002), "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Philips Curve," *Quarterly Journal of Economics*, 117(4): 1295-1328.
- [17] Morris, S. and Shin, H. S. (2002), "Social Value of Public Information," *American Economic Review*, 92(5): 1521-1534.
- [18] Morris, S. and Shin, H. S. (2003), "Global Games: Theory and Applications," in Dewatripont, M., Hansen, L. P. and Turnovsky, S. (eds.) *Advances in Economics and Econometrics: Theory and Applications, Eighth World Congress*, Cambridge University Press.
- [19] Morris, S. and Shin, H. S. (2006a), "Central Bank Transparency and the Signal Value of Prices," *Brookings Papers on Economic Activity*, forthcoming.
- [20] Morris, S. and Shin, H. S. (2006b), "Inertia of Forward-Looking Expectations," paper presented at the one hundred eighteenth annual meeting of the American Economic Association.
- [21] Morris, S., Shin, H. S., and Tong, H. (2006), "Response to 'Social Value of Public Information: Morris and Shin (2002) is Actually Pro Transparency, Not Con'," *American Economic Review*, forthcoming.
- [22] Nimark, K. P. (2005), "Calvo Pricing and Imperfect Common Knowledge: A Forward Looking Model of Rational Inflation Inertia," ECB Working Paper Series, 474.

- [23] Phelps, E. S. (1970), "Introduction: The New Microeconomics in Employment and Inflation Theory," in Phelps, E., et al. *Microeconomic Foundations of Employment and Inflation Theory*, Norton, New York.
- [24] Reis, R. (2005), "Inattentive Producers," NBER Working Paper, 11820, and *Review of Economic Studies*, forthcoming.
- [25] Sims, C. A. (2003), "Implications of Rational Inattention," *Journal of Monetary Economics*, 50(3): 665-690.
- [26] Svensson, L. E. O. (2005), "Social Value of Public Information: Morris and Shin (2002) is Actually Pro Transparency, Not Con," NBER Working paper, 11537, and *American Economic Review*, forthcoming.
- [27] Taylor, J. B.(1980), "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy*, 88(1): 1-23.
- [28] Woodford, M. (2003a), "Imperfect Common Knowledge and the Effects of Monetary Policy," in Aghion, P., Frydman, R., Stiglitz, J. and Woodford, M. (eds.) *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, Princeton University Press.
- [29] Woodford, M. (2003b), *Interest and Prices*, Princeton University Press.
- [30] Woodford, M. (2005), "Central Bank Communication and Policy Effectiveness," NBER Working Paper, 11898.

Figure 1: Baseline Model

Figure 1.1: Impulse responses of OUTPUT to a positive monetary disturbance
(responses: log deviations from steady state)

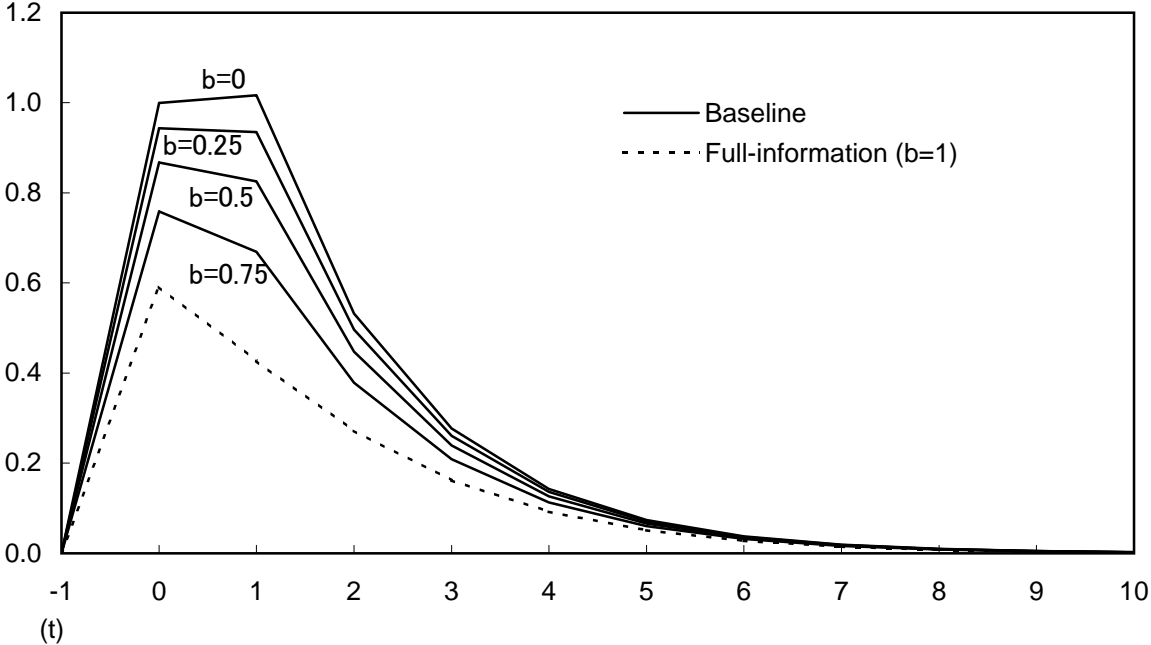


Figure 1.2: Impulse responses of INFLATION to a positive monetary disturbance
(responses: log deviations from steady state)

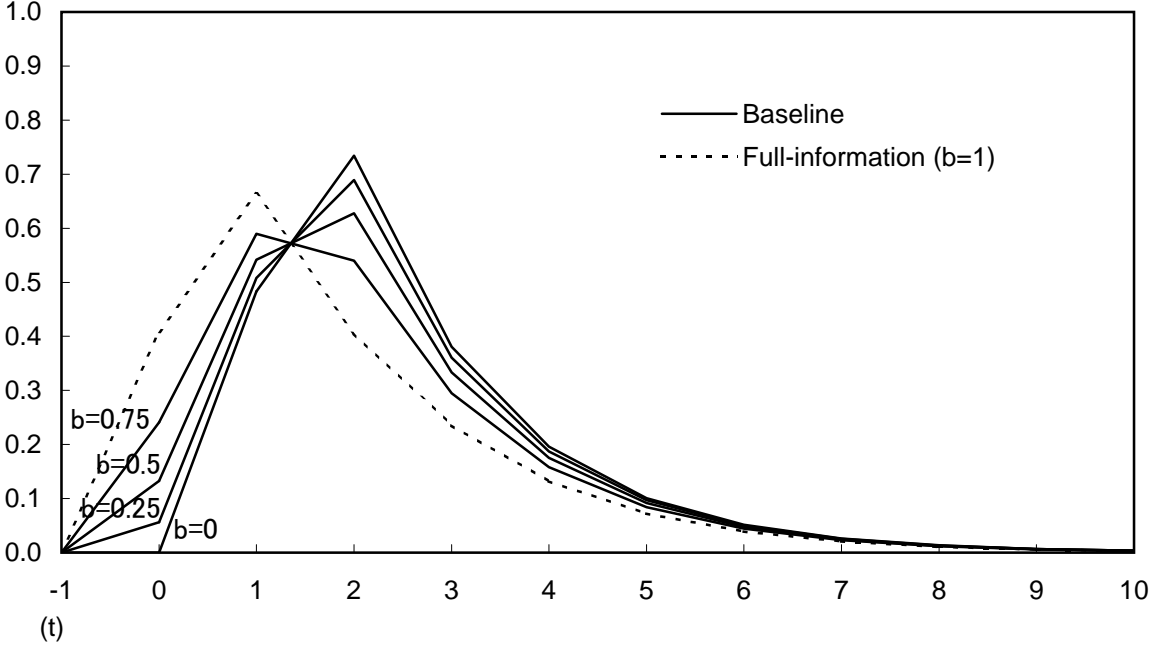


Figure 2: Extended Model (Monetary Disturbance)

Figure 2.1: Impulse responses of OUTPUT to a positive monetary disturbance
(responses: log deviations from steady state)

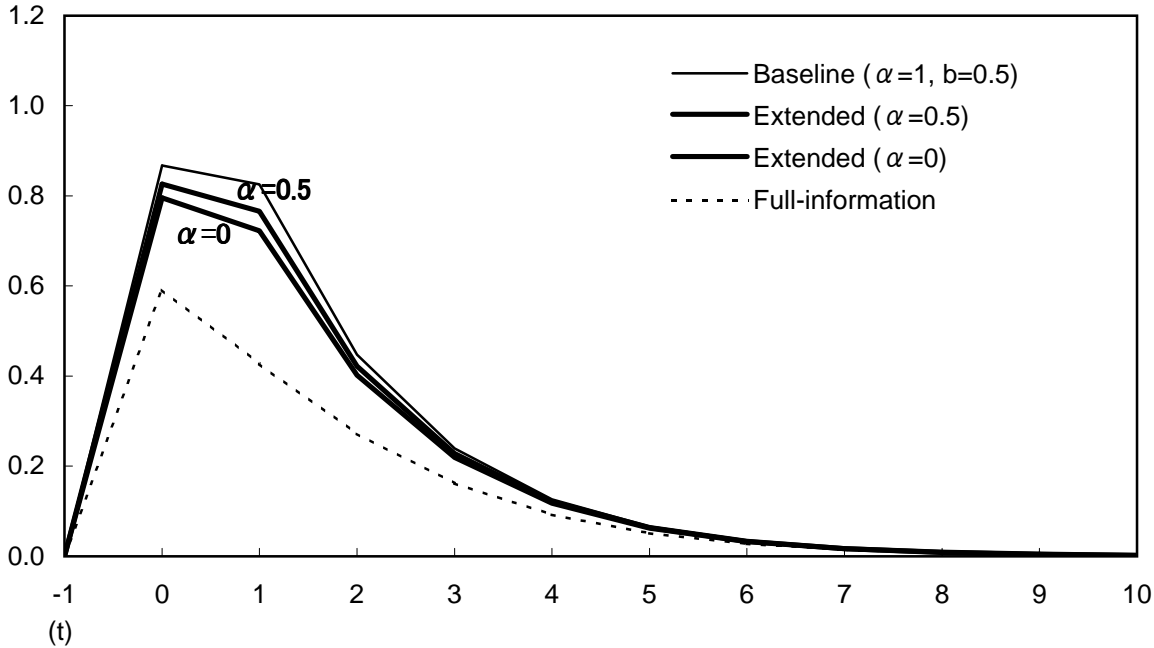


Figure 2.2: Impulse responses of INFLATION to a positive monetary disturbance
(responses: log deviations from steady state)

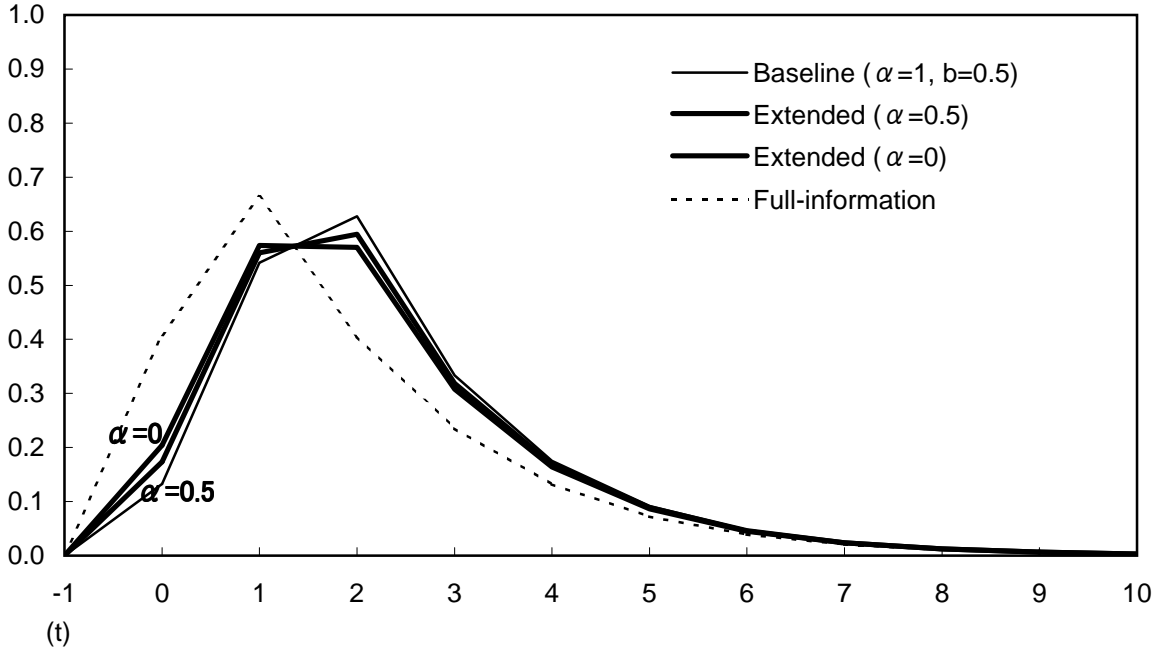


Figure 3: Extended Model (Informational Disturbance)

Figure 3.1: Impulse responses to a NEGATIVE informational disturbance

(responses: log deviations from steady state)

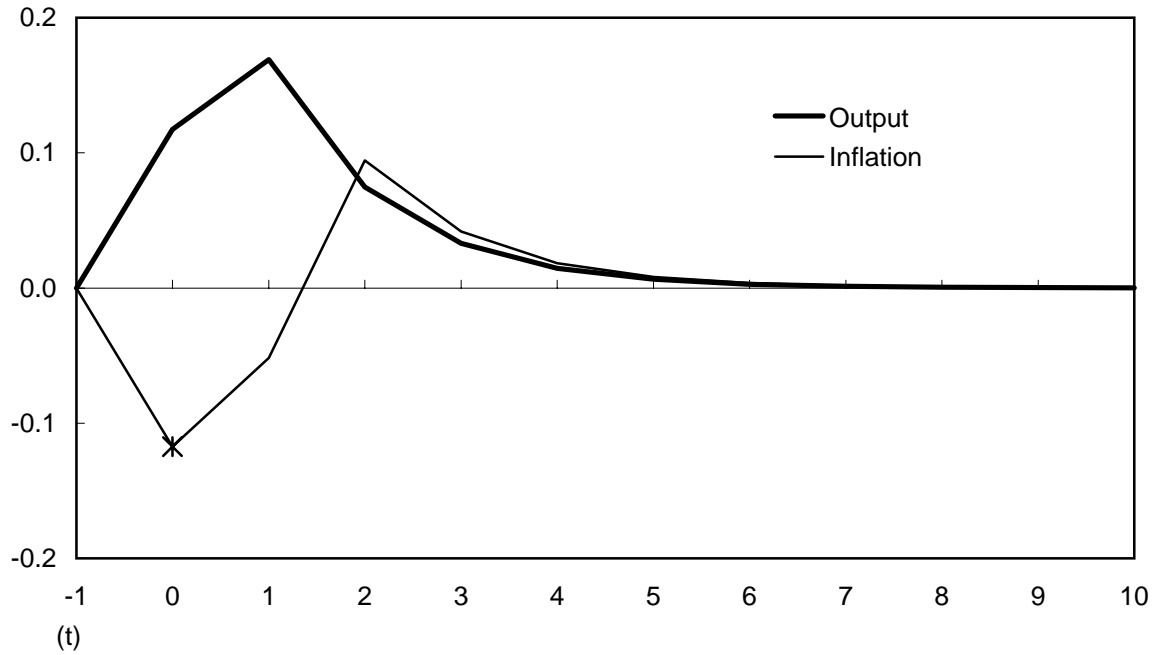
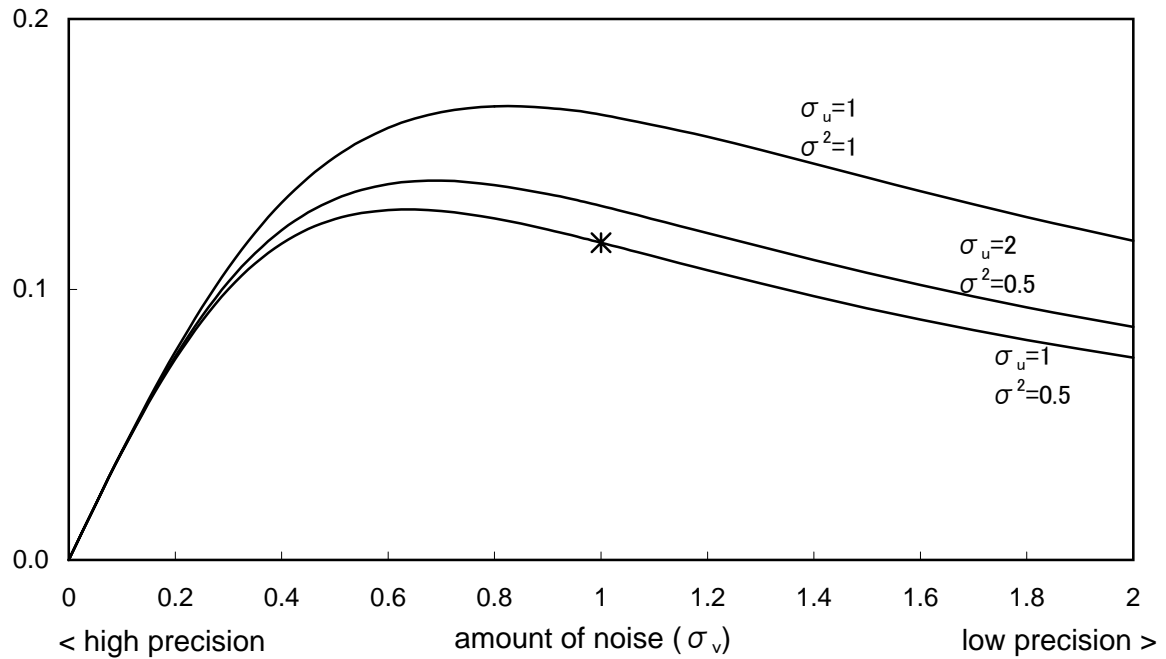


Figure 3.2: Initial response of prices as a function of the precision of public signal

(responses: log deviations from steady state)



Note: The asterisk on the $(\sigma_u, \sigma^2)=(1, 0.5)$ line corresponds to the response of inflation at t=0 in Figure 3.1.

Figure 4: Calvo-type price setting

Figure 4.1: Responses of OUTPUT to a positive monetary disturbance

(responses: log deviations from steady state)

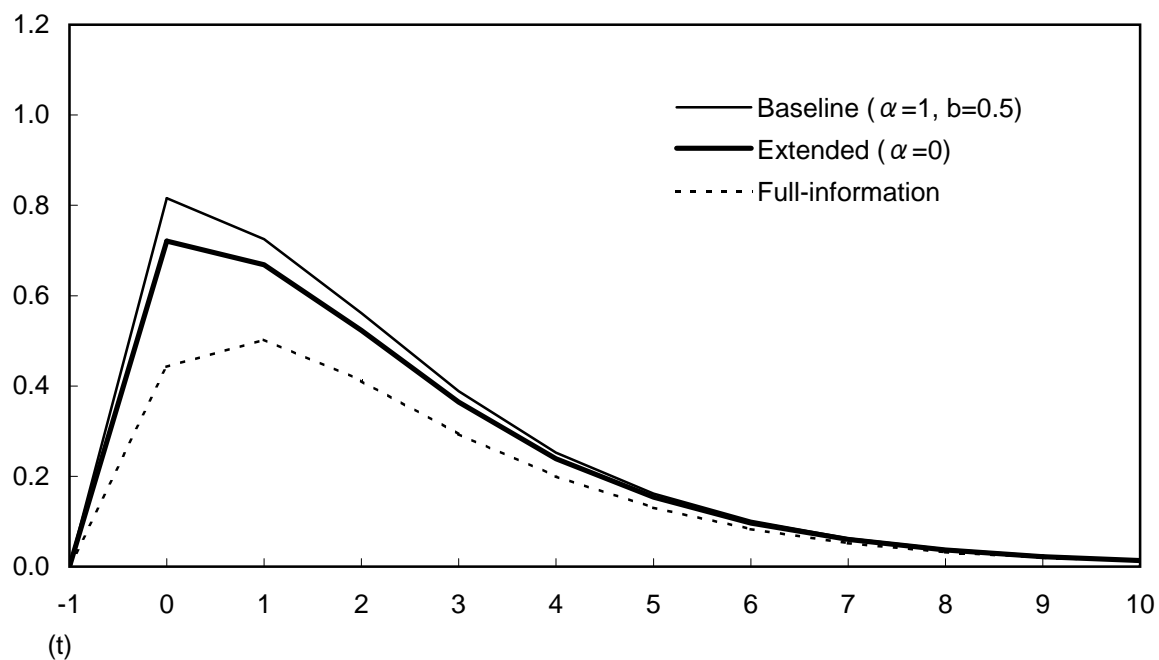


Figure 4.2: Responses of INFLATION to a positive monetary disturbance

(responses: log deviations from steady state)

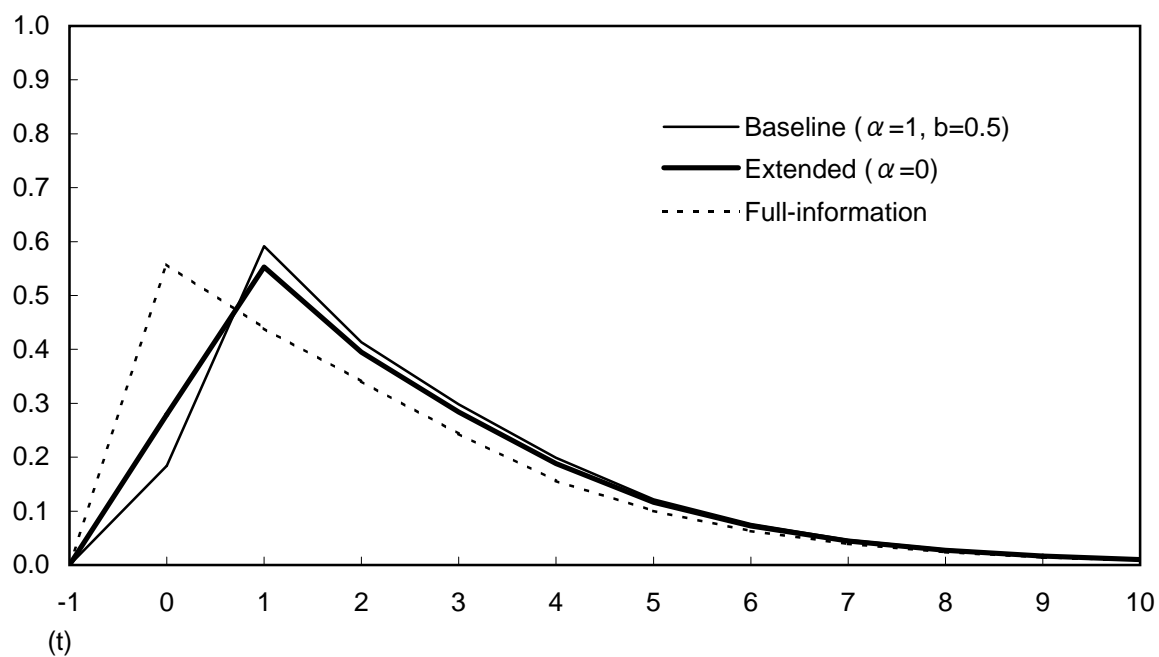


Figure 5: Four-period staggered price setting

Figure 5.1: Responses of OUTPUT to a positive monetary disturbance
(responses: log deviations from steady state)

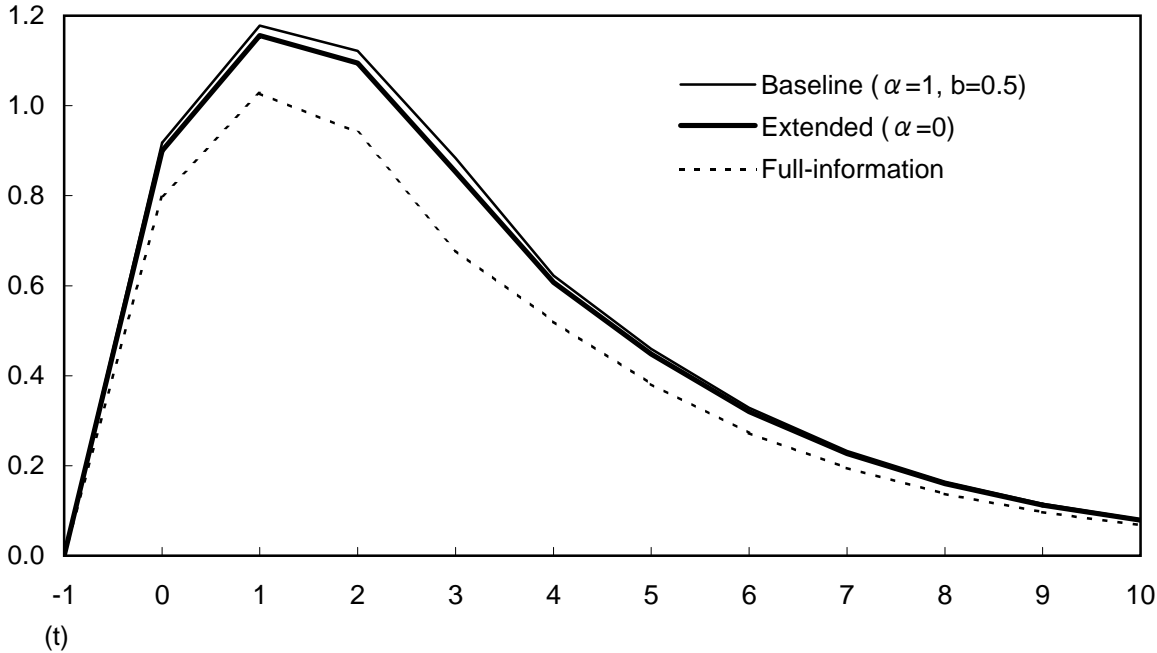


Figure 5.2: Responses of INFLATION to a positive monetary disturbance
(responses: log deviations from steady state)

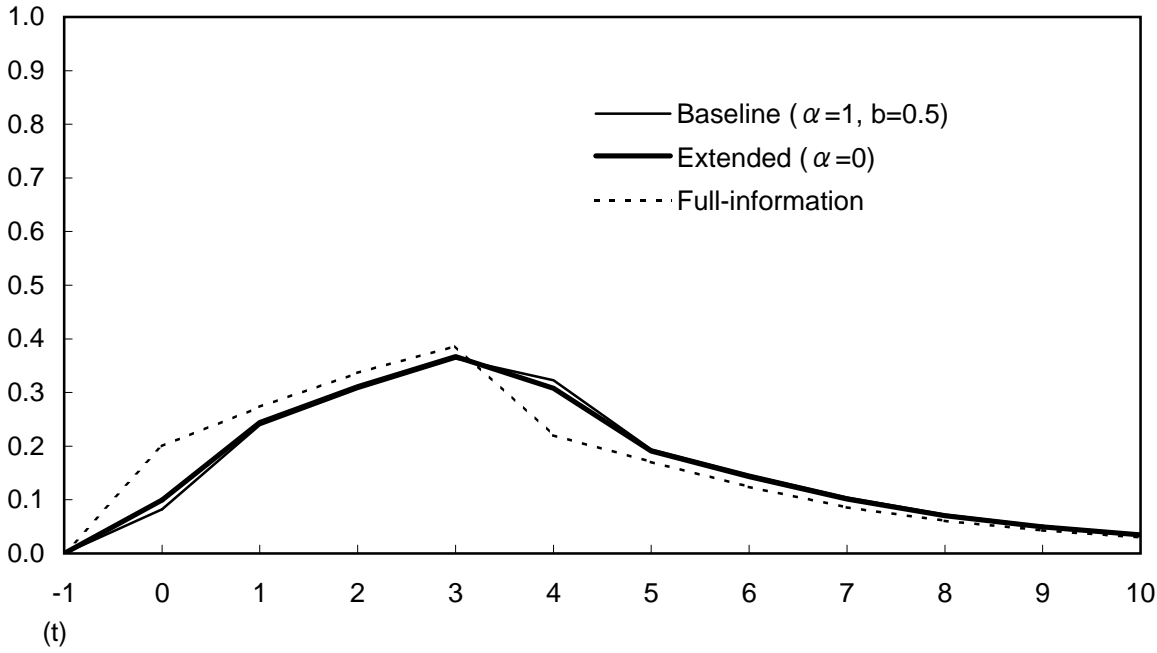


Figure 6: General staggered price setting (Informational Disturbance)

Responses to a NEGATIVE informational disturbance

Figure 6.1: Calvo-type price setting

(responses: log deviations from steady state)

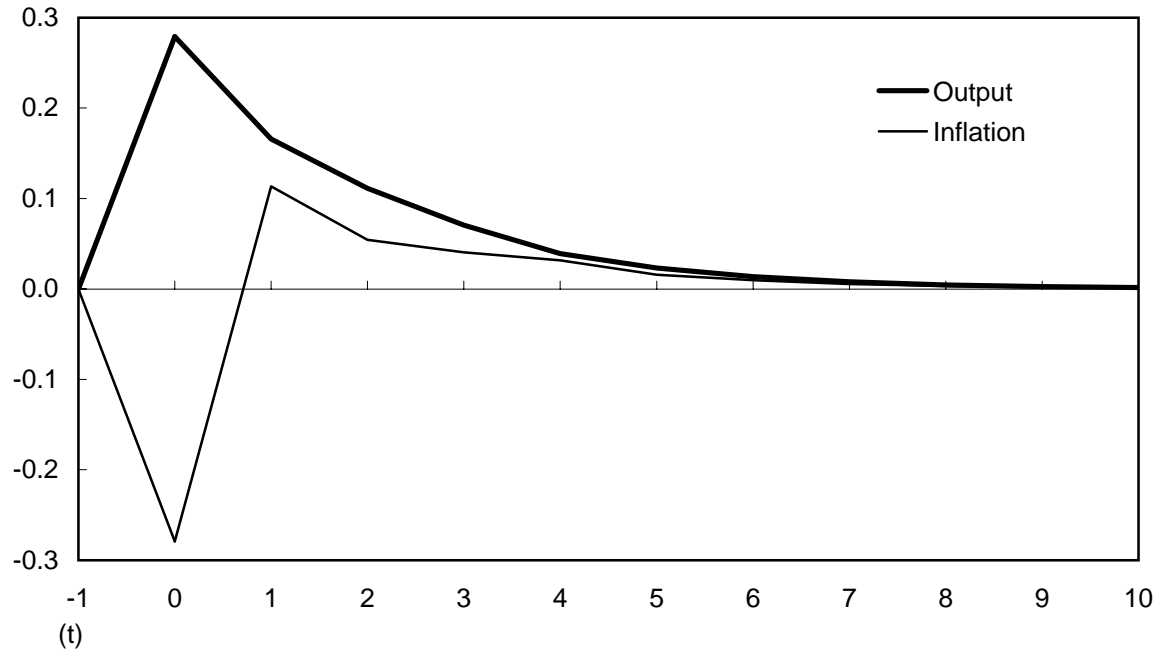


Figure 6.2: Four-period staggered price setting

(responses: log deviations from steady state)

