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a Dynamic Externality

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Abstract

This paper develops a model that can explain the hump-shaped impulse response of inflation to a monetary shock. We utilize a New Keynesian (NK) model with sticky prices and wages, variable capital utilization, habit formation for consumption, and adjustment costs in investment, as in Christiano, Eichenbaum, and Evans (2005). However, instead of assuming a backward-looking indexation, which is often utilized to generate inflation inertia, this paper introduces a dynamic externality into the production function of firms. We show that a dynamic externality can explain the observed hump-shaped behavior of inflation even under *purely forward-looking* nominal rigidities in nominal prices and wages à la Calvo (1983). In addition, we show that in order for inflation to be hump-shaped, sticky wages and variable capital utilization are important as well as a dynamic externality.

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1   Introduction

Sticky prices are one of the most important elements in the New Keynesian (NK) model and the policy analysis based upon it. Under nominal rigidities à la Calvo (1983) or Rotemberg (1982), an expression for inflation can be obtained in a very simple form called the New Keynesian Phillips Curve (NKPC). It has been one of the fundamental equations for the analysis of the monetary policy, as discussed in Clarida, Gali, and Gertler (1999).

The NKPC is theoretically appealing because it can be derived from a rational expectations model with staggered price contracts and gives us intuitive descriptions of the supply side in the economy. However, despite its theoretical appeal, the NKPC has been subject to criticism because of its counterfactual predictions. For example, Fuhrer and Moore (1995) and Fuhrer (1997) point out that NKPC predicts the expected change in inflation must decrease when the output gap is positive. Nelson (1998) concluded that a standard Calvo (1983)-type staggered price setting cannot generate the hump-shaped impulse response function (IRF hereafter) implied by estimated VARs.[1]   Mankiw and Reis (2002) reported similar results: in the sticky price model, monetary policy shocks have their maximum impact on inflation immediately.[2]

In general, the literature has considered two ways of extending the NKPC to generate a hump-shaped IRF for inflation to monetary policy shocks. First, the inclusion of lagged inflation in the equation can yield a hump-shaped IRF for inflation. Fuhrer and Moore (1995) proposed a relative wage contract that allows inflation to be a function of lagged inflation.[3] Gali and

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[1]Delayed responses of inflation to monetary policy shocks can be seen in the VAR literature. Stock and Watson (2001) ran a simple VAR with the inflation rate, the unemployment rate, and the federal funds rate and concluded that the response of inflation to a federal funds rate shock is delayed. Gali (1992) estimated a structural VAR with long-run and short-run restrictions. His IRF of inflation to an M1 shock is hump-shaped and its peak is eight periods after the monetary policy shock.

[2]In some exceptional cases, Taylor (1980)-type nominal rigidities seem to generate a hump-shaped impulse response for price inflation. For example, Erceg (1997) used Taylor-type staggered wages and flexible prices to show that inflation can be hump-shaped in response to a monetary shock, although the reason behind this has not been explored clearly.

[3]Nelson (1998) reported that Fuhrer and Moore (1995)'s expression for inflation is the only one in which the inflation response could be hump-shaped.

Gertler (1999) estimated a hybrid NKPC, which assumes that a fraction of the firms determine price according to a backward-looking rule of thumb. Christiano, Eichenbaum, and Evans (2005, CEE) derived a hybrid NKPC by assuming a (theoretically) nonstandard backward-looking indexation and succeeded in accounting for the observed sluggishness in inflation. However, this extension is often justified on empirical rather than theoretical grounds, because of the good empirical fit of models.

Second, a hump-shaped response of inflation can be generated in response to a monetary policy shock when economic agents have limited information. Erceg and Levin (2003) and Keen (2003) assume imperfect information between the private sector and the central bank. Although Erceg and Levin (2003) did not show a hump-shaped IRF for inflation, Keen (2003) showed that inflation is hump shaped in response to a monetary shock. Dellas (2004) assumed that economic agents observe variables such as the output gap and inflation with error, and then have to solve a signal extraction problem. A number of researchers have even begun to replace the Calvo-type fixed price model with the flexible price model or the predetermined price model involving limited information about the economy.[4]

This paper explores another possible explanation for the hump-shaped response of inflation. In this paper, we do not assume a hybrid NKPC or private agents' limited information about the economy. Instead, this paper develops an NK model with the production function supplemented by organizational capital. We label the effect of organizational capital on productivity a dynamic externality, because we assume organizational capital is accumulated through a production spillover. This paper shows that a dynamic externality can be a powerful mechanism for generating a hump-shaped IRF for inflation if it is combined with sticky wages and variable capital utilization.

The production function with organizational capital is based on the economist's argument that knowledge in organization may be stored and accumulated. A number of researchers have argued that there may be a productivity-enhancing factor in a conventional production function. For example, Rapping (1965) estimated the production function in the shipbuilding industry during World War II, focusing on the effect of organizational (e.g. labor or management) learning stemming from accumulated production experience. Prescott and Visscher (1980) provided examples of productivity-enhancing factors, namely the stock of knowledge on how to assign employees to specific tasks, and how to combine workers in a team efficiently. In their microstudy on the production function, Bahk and Gort (1993) argued that the stock of knowledge is accumulated in an organization as a result of learning-by-doing, and found that it had a significant effect on output. Cooper and

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[4]Examples of this approach include: the sticky information model by Mankiw and Reis (2002) and Burstein (2005); the imperfect common knowledge model by Woodford (2002) and Hellwig (2002); and Adam (2005), who, using a predetermined price model, showed the possibility that an equilibrium-supporting irrational forecasting rule discussed in Roberts (1997) and Ball (2000) can exist under a reasonable range of parameters. These models explain the hump-shaped behavior of inflation.

Johri (2002) considered the role of the stock of knowledge in a business cycle frequency. The stock of knowledge may fluctuate because of learning under some circumstances, such as the reorganization of a production team, hiring and firing workers, and the introduction of new management or supervision. They referred to the stock of knowledge as organizational capital.

To estimate the production function with organizational capital, Bahk and Gort (1993) link a plant's cumulative output since its birth with its organizational capital, which is introduced as a separate input in the production function. This formulation captures only firm-specific knowledge and may not capture industry-wide or economy-wide knowledge, which could be diffused over the industry. Our approach to modeling organizational capital is simple and broad in its meaning: we assume that organizational capital is accumulated through output in the economy. We consider not only that a firm-specific organizational capital may be accumulated through a firm-specific learning-by-doing, but also that a learning-by-doing spillover may affect a firm's organizational capital, because knowledge in an organization may be transferable within the industry or the economy.

In the real business cycle literature, a number of papers have analyzed the effect of organizational capital or learning-by-doing as a propagation mechanism. Cooper and Johri (1997) introduced dynamic complementarities into the standard production function to capture the change in organizational capital. Their analysis suggests that the change in organizational capital induced by an externality can be a propagation mechanism. Cooper and Johri (2002) explicitly modeled organizational capital that is accumulated through organizational learning-by-doing. Chang, Gomes, and Schorfheide (2002) studied the effect of learning-by-doing involving the accumulation of a worker's skill over time through that worker's experience. In an NK framework, on the other hand, the inclusion of organizational capital plays a more important role for hump-shaped inflation than for hump-shaped output, because changes in organizational capital directly affect firms' marginal costs via changes in productivity. In our analysis, the learning-by-doing is assumed to be external rather than internal. However, the dynamics of organizational capital are extremely similar to the model of Cooper and Johri (2002) in that organizational capital accumulates over time according to the level of production activity.

The intuition behind a hump-shaped IRF for inflation is as follows. Expansionary monetary shocks generate two effects on marginal cost. The first effect operates through factor prices. The increased demand for goods raises the demand for inputs, thereby bidding up factor prices and increasing marginal cost. The second effect operates through organizational capital. Owing to production spillovers, the increased organizational capital causes productivity to increase and thus reduces marginal cost. The second effect at least partially offsets the first effect and may actually reduce marginal cost in the short run. The intermediate-run increase in marginal cost can be moderated and delayed if sticky wages and variable capital utilization

slow down the increase in factor prices. Given this marginal cost behavior, forward-looking firms may raise their current price only moderately or even reduce their current price in the short run because they put weight on both the short-run decreases and intermediate-run increases in marginal cost in their determination of their current price. In the future, they will raise their price appreciably because they will no longer be putting any weight on the short-run decreases in marginal cost, which lie in the past. As a result, expansionary monetary shocks have a delayed impact on inflation under a purely forward-looking NKPC.[5]

The rest of the paper is organized as follows. Section 2 presents the specific model used in the simulations. Section 3 shows that the IRF for inflation to a monetary policy shock is hump-shaped and explains the mechanism underlying that shape. The model replicates the stylized facts on the estimated IRF quite well. In addition, the model with a dynamic externality explains the observed IRF for marginal cost, at least qualitatively, better than the model without a dynamic externality. In the section 4, we consider the robustness of hump-shaped inflation to returns to organizational capital. We find that it is quite robust to changes in the returns. Section 5 explains the differences between our model and CEE's, focusing on the behavior of inflation. Section 6 concludes the paper.

2   The Model

In this section, we describe the model economy. Our model is closely related to the Taylor rule version of the CEE model. However, our model assumes a purely forward-looking NKPC implied by static indexation rather than backward-looking indexation. The model consists of a representative goods aggregator, a representative labor aggregator, and a government, as well as monopolistically competitive firms and households. Monopolistically competitive firms rent their capital service from households in a rental capital market and a composite labor service from a labor aggregator. Households have some real frictions: habit formation for consumption, adjustment costs in investment, and variable capital utilization. Nominal rigidities exist in both prices and wages in a purely forward-looking manner. To include sticky prices and wages, we assume that the nominal price and wage adjustments are possible only at some constant hazard rate. This Calvo (1983)-style timing of the nominal rigidity gives us an NKPC for both price and wage inflation.

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[5]Technically, we introduce two endogenous state variables in marginal cost. The first endogenous state variable is real wages given nominal price and wage rigidities. The second endogenous state variable is organizational capital. The combination of the two offsetting endogenous state variables generates not only sluggish behavior of marginal cost but also the short-run decrease and the intermediate-run increase in marginal cost.

2.1 Firms

Following the literature, we introduce an output aggregator with a constant-returns-to-scale technology of the Dixit–Stiglitz form and intermediate good firms under monopolistic competition. An output aggregator produces a final good Y_t for households' consumption and investment in the perfect competitive market. The final good is a transformation of a continuum of differentiated goods, each of which is produced by a single monopolistic firm. Under these assumptions, the demand function for intermediate goods takes the following form:

$$Y_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\epsilon_p} Y_t, \tag{1}$$

where $Y_t(f)$ denotes a differentiated good and $P_t(f)$ is its price. P_t is the aggregate price index. f is the index for intermediate good firms distributed uniformly on [0,1]. $\epsilon_p > 1$ is the elasticity of substitution between the differentiated goods.

We assume a Calvo-type staggered price setting so that each firm is allowed to change its price only with a probability. Instead of deriving it, we simply start with the NKPC that has been derived in the literature from that assumption. Let π_t denote the gross inflation rate $\pi_t = P_t/P_{t-1}$ and $\hat{\pi}_t = \log(\pi_t) - \log(\pi)$, where π is the steady-state value of the gross rate of inflation. Then, the NKPC is given by:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \Psi_p \hat{mc}_t, \tag{2}$$

where Ψ_p and β are parameters satisfying $\Psi_p > 0$ and $0 < \beta < 1$ and $\hat{mc}_t$ is the log-deviation of marginal cost from the steady-state value.

The intermediate good firm faces perfectly competitive factor markets for the *effective* capital input (defined below) $\tilde{K}_t(f)$ and the labor input $L_t(f)$, which it rents in competitive factor markets.[6] For this reason, each intermediate good firm takes the rental price of effective capital, R_t^k, and the aggregate wage index, W_t, as given.

Suppose that the production function for firm f is Cobb–Douglas in effective capital, labor, and organizational capital:

$$Y_t(f) = \tilde{K}_t(f)^\alpha L_t(f)^{1-\alpha} X_t^\phi, \tag{3}$$

where $\alpha \in (0,1)$ and $\phi \geq 0$. X_t is organizational capital. Organizational capital is interpreted as the stock of knowledge embodied in the organization, and we assume it is the same across firms. Obviously, when $\phi = 0$, the effect of organizational capital is negligible and the production function is standard.

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[6]This is not a contradiction to the assumption of monopolistically competitive households in their labor market. The households sell the labor to the labor aggregator in monopolistically competitive markets, but the labor aggregator sells its aggregate labor to the intermediate good firms in a competitive market. For this reason, we may assume the intermediate good firms face a competitive labor market.

The law of motion of organizational capital is given by:

$$\log(X_t) = \gamma \log(X_{t-1}) + \eta \log\left(\frac{Y_t}{Y}\right), \tag{4}$$

where $\gamma \in (0,1)$ captures the persistence of the organizational capital and $\eta > 0$ captures the effect of current aggregate output. Y is the steady-state level of aggregate output. This AR(1) structure allows us to show that output in the past affects the firm's productivity in the current period. The law of motion for organizational capital is similar to Cooper and Johri (2002) in that it considers depreciations of organizational capital over time. However, it differs from theirs in that organizational capital is accumulated through production in the economy Y_t rather than individual production $Y_t(f)$. We employ this assumption because it captures industry-wide or economy-wide learning-by-doing spillovers.[7] Hence, we call this effect of organizational capital accumulated through the aggregate output on a firm's productivity a *dynamic externality.*

Given the production function and the assumption of perfectly competitive factor markets, the real marginal cost function mc_t and the marginal rate of substitution between labor and effective capital from the static cost minimization problem take the form:

$$mc_t = (1-\alpha)^{-(1-\alpha)} \alpha^{-\alpha} w_t^{1-\alpha} (r_t^k)^\alpha X_t^{-\phi}, \tag{5}$$

$$\frac{w_t}{r_t^k} = \frac{1-\alpha}{\alpha} \frac{\tilde{K}_t}{L_t}, \tag{6}$$

where w_t is the real wage rate (i.e., $w_t = W_t/P_t$) and r_t^k is the real rental cost of effective capital (i.e., $r_t^k = R_t^k/P_t$). Note that the index f is dropped because all intermediate good firms face identical factor prices.

2.2   Households

Each household, indexed by $h \in (0,1)$, is assumed to supply a differentiated labor service to firms. We assume a representative labor aggregator that buys households' differentiated labor supply $L_t(h)$ to produce a single composite labor service L_t, which it sells to intermediate good firms. This formation is parallel to the output aggregator. Hence, we obtain the following demand function for the differentiated labor:

$$L_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\epsilon_w} L_t, \tag{7}$$

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[7]In this formulation, the current level of output affects firms' productivity as well as the past level of output. On the other hand, Cooper and Johri (2002) originally employed a specification such that the productivity is affected only by the past level of output like $\log(X_{t+1}) = \gamma \log(X_t) + \eta \log(Y_t/Y)$. However, we find that this slight change of the law of motion for X_t does not greatly change the simulation results in the following sections.

where $\epsilon_w > 1$ is the elasticity of substitution between the differentiated labor and $W_t(h)$ is the nominal wage for differentiated labor.

We set up the household's maximization problem. Following CEE, the expected utility function is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t(h) - bC_{t-1}(h)) - \psi_0(L_t(h))^2 \right\}, \qquad (8)$$

where $\psi_0 > 0$ and $b > 0$. $C_t(h)$ denotes the consumption. Thus, the utility of households is characterized by habit formation. In addition, the disutility of supplying labor is characterized by a quadratic function. While the specifications for consumption and labor are the same as CEE, we abstract the real balance effect from the utility function. Because we specify the monetary policy as an interest rate rule, we can neglect the real balance effect under an additively separable utility function.

Next, let us consider the household's budget constraint. It is given by:

$$W_t(h)L_t(h) + R_t^k \tilde{K}_t(h) + \Gamma_t(h) + T_t(h) \qquad (9)$$
$$= P_t \left[C_t(h) + I_t(h) + a(U_t(h))K_t(h) \right] + B_t(h) - R_{t-1}B_{t-1}(h).$$

On the income side, the household's source of income is labor income $W_t(h)L_t(h)$, returns from effective capital service $R_t^k \tilde{K}_t(h)$, the sum of the profits from the firms in the economy $\Gamma_t(h)$, and a lump-sum transfer from the government to the household $T_t(h)$. Effective capital $\tilde{K}_t(h)$ is defined as the product of the actual capital stock $K_t(h)$ and capital utilization $U_t(h)$:

$$\tilde{K}_t(h) = U_t(h)K_t(h). \qquad (10)$$

On the spending side of the budget constraint, the household purchases the final goods for consumption and investment. In utilizing the actual capital $K_t(h)$, the household loses final goods in the form of the capital utilization costs given by $a(U_t(h))K_t(h)$. The function $a(U_t(h))$ is assumed to be increasing and convex in $U(h)$ (i.e., $a'(\cdot) > 0, a''(\cdot) > 0$.) We assume that the cost is zero when the utilization rate is equal to the steady-state value of one (i.e., a(1)=0). Finally, the household spends its income for financial assets in the form of one-period nominal bonds $(B_t(h) - R_{t-1}B_{t-1}(h))$. We assume a constraint $B_t(h) > -\bar{B}$ for some large positive number $\bar{B}$.

In the formulation of the capital accumulation equation, we employ investment adjustment costs developed in CEE. The actual capital stock evolves according to:

$$K_{t+1}(h) = (1 - \delta)K_t(h) + \left(1 - S\left(\frac{I_t(h)}{I_{t-1}(h)} \right) \right) I_t(h), \qquad (11)$$

where δ denotes the depreciation rate. The second term on the left-hand side characterizes investment adjustment costs. As the growth rate of investment is high, the costs prevent capital accumulation. The function $S(\cdot)$ satisfies $S(1) = S'(1) = 0$ and $s \equiv S''(1) > 0$.

We assume that every household faces the same initial conditions and that the contingent markets are complete. Then, we have the symmetric equilibrium value for control variables except for $W_t(h)$. These assumptions allow us to drop the household index h for $C_t(h)$, $I_t(h)$, $U_t(h)$, $M_t(h)$, $B_t(h)$, and $K_{t+1}(h)$.

In order to make a decision for these variables, the household maximizes its expected utility function (8) subject to (9), (10), and (11). The first-order conditions are as follows:

$$1 = \beta E_t \left\{ \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \frac{R_t}{\pi_{t+1}} \right\}, \tag{12}$$

$$\lambda_t = Q_t \left\{ 1 - S\left(\frac{I_t}{I_{t-1}} \right) - S'\left(\frac{I_t}{I_{t-1}} \right) \left\{ \frac{I_t}{I_{t-1}} \right\} \right\}$$

$$+ \beta E_t \left(Q_{t+1} S'\left(\frac{I_{t+1}}{I_t} \right) \left\{ \frac{I_{t+1}}{I_t} \right\}^2 \right), \tag{13}$$

$$r_t^k = a'(U_t), \tag{14}$$

$$Q_t = \beta E_t \left[(1 - \delta) Q_{t+1} + \lambda_{t+1} \left(r_{t+1}^k U_{t+1} - a(U_{t+1}) \right) \right], \tag{15}$$

where Q_t is the Lagrange multiplier for the capital accumulation equation (11) and λ_t is the marginal utility for current consumption:

$$\lambda_t = \frac{1}{C_t - b C_{t-1}} - \beta b E_t \frac{1}{C_{t+1} - b C_t}. \tag{16}$$

These first-order conditions are quite standard. The equations (12) and (16) imply a consumption Euler equation that equates the marginal utility of consumption today with the discounted marginal utility of consumption tomorrow. The equation (13) is the first-order condition for investment. The left-hand side of (13) is the marginal benefit of increasing an extra unit of investment goods in terms of utility. In period t, the household obtains the benefit of $Q_t(1 - S(I_t/I_{t-1}) - S'(I_t/I_{t-1})I_t/I_{t-1})$ by increasing an extra unit of investment goods. This extra increase in I_t reduces the expected investment adjustment cost by $\beta E_t Q_{t+1} S'(I_{t+1}/I_t)(I_{t+1}/I_t)^2$ in period $t + 1$. Hence, in optimum, the household equates the marginal utility of investment with the marginal utility of consumption by allocating its resources across consumption and investment. The equation (14) is the marginal condition for variable capital utilization. Utilizing more of the capital stock gives the household an additional income of r_t^k per unit of capital stock but requires it to pay the marginal cost of $a'(U_t)$ per unit of capital stock. Equation (15) determines the shadow value of capital in terms of utility.

2.2.1 Wage Setting

We turn to wage setting behavior. We assume that the nominal wage contracts are analogous to the price setting behavior. In each period, the household is allowed to reoptimize its nominal wages with a probability. Under

our preference assumptions, Calvo-type staggered wage setting gives us the following wage NKPC to a first-order approximation:

$$\hat{\pi}_t^w = \beta E_t \hat{\pi}_{t+1}^w + \Psi_w \left[\hat{L}_t - \hat{\lambda}_t - \hat{w}_t \right], \quad \Psi_w > 0 \qquad (17)$$

where $\hat{\pi}_t^w$ is the log-deviation of wage inflation from the steady-state value. That is, $\hat{\pi}_t^w = \log(\pi_t^w) - \log(\pi^w)$, where $\pi_t^w = W_t/W_{t-1}$ and π^w is the gross rate of wage inflation in the steady state. Similarly, $\hat{L}_t$, $\hat{\lambda}_t$, and $\hat{w}_t$ are the log-deviations from the steady state of the labor supply, the marginal utility of consumption, and the real wage, respectively. Finally, Ψ_w is a parameter. The first two terms inside the brackets are the log-deviation of the marginal rate of substitution between labor and consumption from the steady state. Thus, the difference between the marginal rate of substitution and the real wage affects the wage inflation rate.

2.3   Closing the Model

To close the model, we specify some model identities and the monetary policy rule. First, the government budget is balanced every period (i.e., $\int_0^1 T_t(h)dh = 0$, for all t). As we assume that the government does not hold one-period nominal bonds at any period, its total lump-sum transfer is set equal to zero.

Second, the market clearing condition is given by:

$$Y_t = C_t + I_t + a(U_t)K_t. \qquad (18)$$

Third, we have a model identity for the real wage rate:

$$w_t = \frac{\pi_t^w w_{t-1}}{\pi_t}. \qquad (19)$$

Finally, the monetary policy rule is represented as a variant of the Taylor (1993) rule with partial adjustment:

$$\hat{R}_t = \rho \hat{R}_{t-1} + (1-\rho)(a_\pi E_t \hat{\pi}_{t+1} + a_y \hat{Y}_t) + e_t, \quad e_t \sim iid, \qquad (20)$$

where $\hat{R}_t$ is the log-deviation of the nominal interest rate from the steady state. Similarly, $\hat{Y}_t$ is the log-deviation of output from the steady state. The parameters satisfy $0 < \rho < 1$, $a_\pi > 0$, and $a_y > 0$, which represent the adjustment parameter of the federal funds rate, the responsiveness to inflation, and the responsiveness to the output gap, respectively.

2.4   Model Solution and Parameters

2.4.1   Model Solution

The log-linearized model is used to analyze the solution to the model. As some of the equations such as (2), (17) and (20) are already log-linearized, we

take log-linearizations of other Euler equations and several model identities around the steady state. There are 13 equations to be log-linearized: (3)-(6), (10)-(16), (18), and (19). As a result, we obtain 16 log-linearized equations consisting of 16 unknowns that are log-linearized around the steady-state value, $\hat{Y}_t$, $\hat{\pi}_t$, $\hat{\pi}_t^w$, $\hat{r}_t^k$, $\hat{mc}_t$, $\hat{L}_t$, $\hat{U}_t$, $\hat{\tilde{K}}_t$, $\hat{I}_t$, $\hat{Q}_t$, $\hat{\lambda}_t$, $\hat{C}_{t-1}$, $\hat{X}_{t-1}$, $\hat{K}_t$, $\hat{w}_{t-1}$, and $\hat{R}_{t-1}$.

It should be noted that the log-linearized model does not require any specific functional form on the variable capital utilization cost function $a(U_t)$ in (14), (15), and (18). Instead, it requires the elasticity of the variable capital utilization cost with respect to U_t evaluated at the steady-state value (i.e., $a''(1)/a'(1)$). We assume that the elasticity $\mu_a \equiv a''(1)/a'(1)$ is a constant.[8]

Similarly, the log-linearized model does not require any specific functional form of $S(\cdot)$ in (11) but a constant adjustment cost parameter s. Because the steady state of the model is independent of the functional form of $S(\cdot)$, only the calibrated value of s is sufficient to investigate the dynamics of the model.

The last five variables in the list of the variables are the state variables in the model. That is, the lagged consumption, lagged organizational capital, the capital stock, the lagged real wages and the nominal interest rate are predetermined or exogenous. Finally, the log-linearized system of equations has a unique equilibrium at the model parameters calibrated below.

2.4.2 Calibration

We have parameters to be specified from outside the model. To compare the model with the Taylor rule version of CEE, we borrow most calibrated values from CEE's calibration and estimates. The discount factor β is $1.03^{-0.25}$, implying that an annualized real interest rate is 3% in the steady state. The habit parameter b is 0.65. The parameter ψ_0 is assigned so that the steady-state value of L is equal to one.[9] The elasticities of demand functions are $\epsilon_p = 6$ and $\epsilon_w = 21$, respectively. These values are consistent with a 20% price mark-up and a 5% wage mark-up in the steady state. The parameters Ψ_p and Ψ_w in two NKPCs are consistent with CEE: $\Psi_p = 0.2698$ and $\Psi_w = 0.0093$.[10]

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[8]When (14) is evaluated at the steady state, we obtain $r^k = a'(1)$ where r^k is the steady-state value of the real rental cost of capital. Using this steady-state relation, the log-linearized version of (14) and (18) is given by $\hat{r}_t^k = \mu_a \hat{U}_t$ and $\hat{Y}_t = \frac{C}{Y}\hat{C}_t + \frac{I}{Y}\hat{I}_t + r^k\frac{K}{Y}\hat{U}_t$, respectively, where any variable without a time subscript is the corresponding steady-state value. Finally, U_t in (15) affects the shadow price of the capital goods only to the second-order terms. Hence, we need only the elasticity of $a(U_t)$ with respect to U_t alone.

[9]In the log-linearized system of the equations, this parameter does not affect the dynamics of the model.

[10]In the literature of the NK model, Ψ_p is a function of β and the probability that firms can reoptimize their nominal price. Letting $1 - \xi_p$ be the probability, the parameter Ψ_p is given by $\frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}$. $\Psi_p = 0.2698$ is consistent with CEE because CEE's estimate of the probability is $\xi_p = 0.60$, which gives $\Psi_p = 0.2698$. Similarly, letting $1 - \xi_w$ be the probability that households can change their nominal wage, Ψ_w is given by $\frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w(1+\epsilon_w)}$. CEE's estimate ξ_w of 0.64 and their calibrated wage mark-up of 5% imply that $\Psi_w =$

We assume that $\delta = 0.025$, which implies a 10% depreciation in a year in the steady state. The adjustment cost parameter s in (11) is set to 2.48. The parameter of variable capital utilization μ_a is set to 0.01.

As for the production side, we need to assign calibrated values for ϕ, γ, η, and α. Cooper and Johri (2002) estimated their production functions with organizational capital, using different sets of data. Among their specifications, the most useful for our purpose is the Increasing Returns to Scale in the Production Function (IRS-PF). IRS-PF assumes constant returns to the effective capital stock and labor and that $\eta = 1 - \gamma$.[11] Their estimates of ϕ then range from 0.26 to 0.35, while the estimates of γ range from 0.50 to 0.55. Considering their estimates, we take $\gamma = \eta = 0.5$ for the dynamics of $\hat{X}_t$ and $\phi = 0.26$ for returns to organizational capital. The total cost share for effective capital is 0.36; i.e., $\alpha = 0.36$.[12]

Finally, the monetary policy parameters ρ, a_π, and a_y are assumed to be 0.8, 1.5, and 0.1, respectively.

Table 1 summarizes the calibrated parameters.

3   Effects of a Monetary Policy Shock

In this section, we report and evaluate the simulation results of the log-linearized model specified in the previous section. In particular, first we use the US data to estimate IRFs in response to the federal funds rate from an unrestricted VAR and discuss the properties of IRFs. Next, we show that an IRF for inflation from the model under $\phi = 0$ (i.e., the standard production function) appears to be inconsistent with the estimated IRFs, whereas the IRF is dramatically improved when $\phi > 0$. Our results suggest that a dynamic externality can be an alternative explanation for the observed hump-shaped behavior of inflation under purely forward-looking nominal rigidities.

3.1   IRF Estimation

Our VAR is an unrestricted nine-variable VAR composed of real gross domestic product, real consumption, an inflation rate calculated from the GDP deflator, real investment, real wages, labor productivity, real unit labor cost, real profits, and the federal funds rate in the US economy with lag four.[13] All variables except the federal funds rate were logged here. We put a special emphasis on the behavior of (real) marginal cost in accounting for inflation. Therefore, a real unit labor cost is included in the VAR. The sample period

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0.0093.

[11]The reason for this restriction in Cooper and Johri (2002) is that η and ϕ are not identified separately because there is no direct measure of organizational capital.

[12]As in Burnside, Eichenbaum, and Rebelo (1995), Cooper and Johri (2002) used electricity consumption to estimate the total cost share for effective capital input. Our calibrated value for α of 0.36 is not substantially different from their estimates, $\alpha = 0.39$.

[13]The data appendix of variables used in the VAR is available on request.

Preference parameters	
β	$1.03^{-0.25}$
b	0.65
Real rigidities	
ϵ_p	6
ϵ_w	21
Nominal rigidities	
Ψ_p	0.2698
Ψ_w	0.0093
Capital accumulation technology	
δ	0.025
s	2.48
μ_a	0.01
Technology in the production function	
α	0.36
ϕ	0.26
γ	0.5
η	0.5
Taylor rule	
ρ	0.8
a_π	1.5
a_y	0.1

Table 1: Calibrated parameters in the model

is 1965:1 – 2003:4. Here, we specify the federal funds rate as our measure of monetary policy.

We follow a procedure similar to CEE's estimation to identify monetary policy shocks. In our recursive VAR, the ordering of the variables is real GDP, real consumption, inflation, real investment, real wages, labor productivity, real unit labor cost, real profits, and the federal funds rate. Our procedure differs from CEE's VAR in that we included the real unit labor cost in the VAR, we excluded the growth rate of M2, and we used inflation instead of the GDP deflator.[14] In addition, our sample periods are a little longer than CEE's. Other characteristics such as the length of lags are essentially the same.

Figure 1 shows the estimated IRFs resulting from a negative one-standard-deviation shock to the federal funds rate. As shown in the figure, the response of inflation shows a hump-shape with the initial decline. The peak for inflation occurs after three years (in the 12th quarter) in response to the shock. Although the standard error bands are rather large, the price puzzle appears in our recursive VAR: the first five responses are negative. In addition, the peak for the output gap occurs after about one and a half years (in the fifth quarter). These results are consistent with the estimation results by CEE based on a nine-variable recursive VAR.

It should be noted that the unit labor cost shows interesting dynamics. In certain circumstances, unit labor cost proxies well for marginal cost.[15] This approximated marginal cost suggests that the IRF for real marginal cost is reduced for several quarters after an interest rate shock and increases only after these quarters. After the reductions, the peak for unit labor cost occurs in the 13th quarter.

3.2 The Difficulty of Generating Hump-shaped Inflation

Figure 2 shows the IRFs for inflation, output, and the marginal cost in response to a negative one standard deviation shock in the federal funds rate under the standard production function ($\phi = 0$). Each panel has the estimated IRF and the 95% confidence intervals based on the VAR.

Some important features are worth emphasizing here. First, we see from the upper left panel in the figure that the model fails to replicate the observed behavior of inflation when $\phi = 0$. As in a traditional NK model, the maximum response of inflation occurs in the period of the monetary shock. Moreover, the responses for the first few periods are outside the confidence intervals.[16] Second, habit formation and investment adjustment costs gener-

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[14]We find that the results are robust when the growth rate of M2 is included and the GDP deflator is used instead of inflation.

[15]In order for this relation to hold, we need to assume Cobb–Douglas technology and free mobility of all inputs.

[16]Following CEE, the instantaneous response of inflation can be set to zero by assuming that firms cannot see a monetary shock contemporaneously and set their price one period
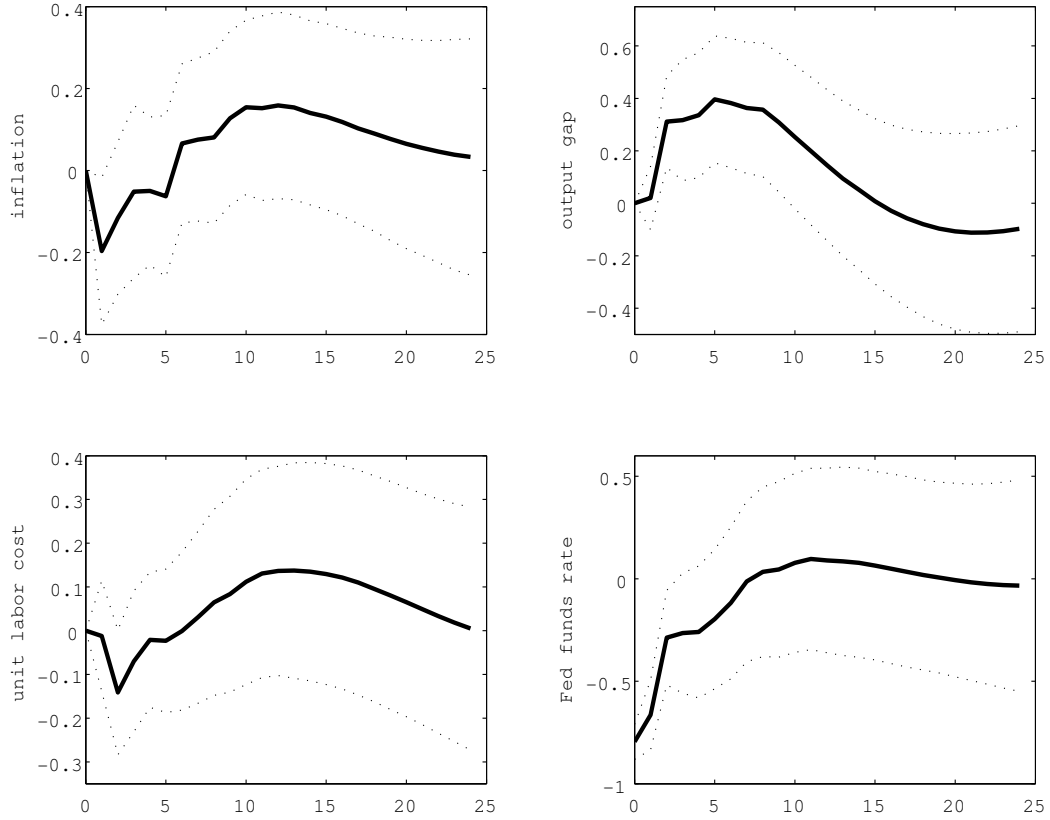
Figure 1: **Estimated IRFs:** IRF for inflation, output, real unit labor cost, and the federal funds rate in response to a negative one standard deviation shock (-0.79) in the federal funds rate. The sample is from 1965:1 to 2003:4. The IRFs are estimated using a nine-variable VAR with lag four. The solid lines are the estimated impulse response functions. The dotted lines are 95% confidence intervals about the IRFs based on bootstrapping. Inflation and the federal funds rate are given in terms of annualized percentage points. All other variables are expressed in percentages.
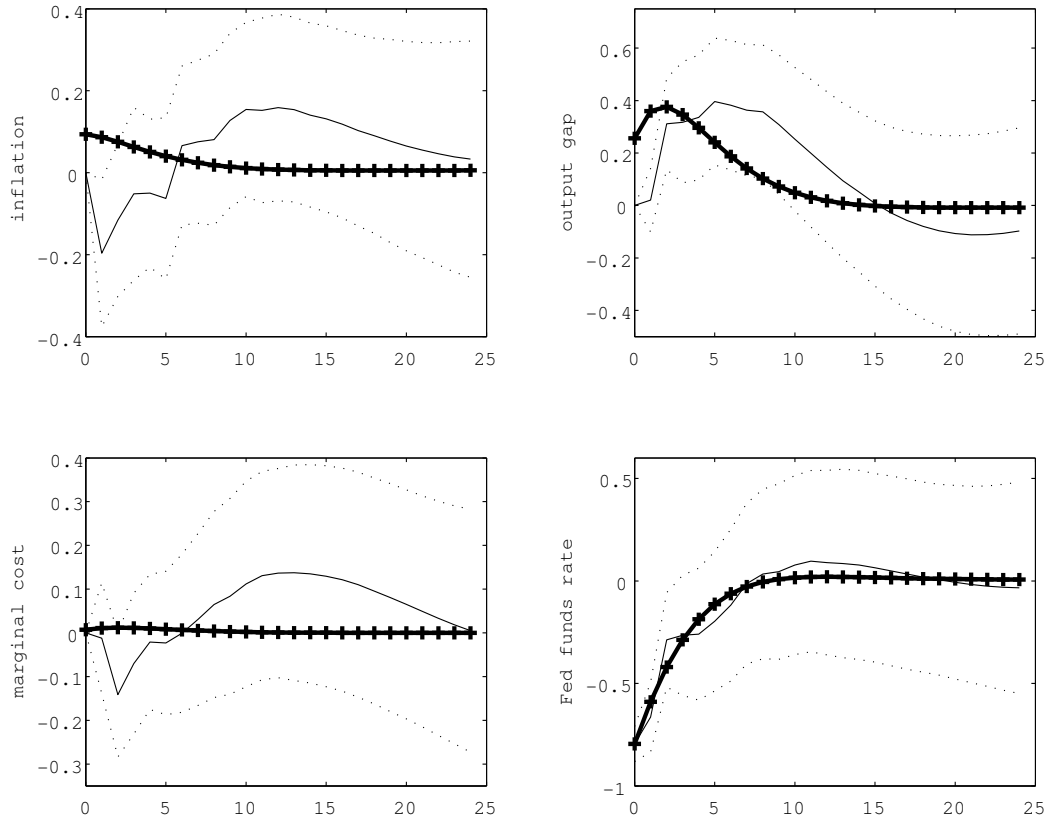
Figure 2: **Without a dynamic externality**: IRF for inflation, output, marginal cost, and the federal funds rate in response to a negative one standard deviation shock in the federal funds rate. The solid lines with (+) are the simulated IRF from the model with $\phi = 0$. The solid lines and dotted lines are the IRF and 95% confidence intervals based on the VAR.

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ate the hump-shaped behavior of the output gap. The IRF for output peaks in the second quarter. The peak is much earlier than the data suggest, but it is at least qualitatively consistent with our estimated IRF. This response of output is reasonably close to CEE's findings for output, considering that we allow consumption and investment to move immediately in response to a monetary policy shock. Finally, turning to the IRF for marginal cost, the response of marginal cost is quite small in its magnitude because of real frictions, such as variable capital utilization and sticky wages.

The simulation results in the figure can be interpreted as being from a stripped-down version of CEE in that this model does not have a backward-looking indexation for prices and wages.[17] Therefore, our finding suggests that sticky wages and variable capital utilization alone may not be able to generate the observed hump-shaped behavior of inflation. In our model, marginal cost is quite inertial because of sticky wages and variable capital utilization. However, inflation shows a counterfactual behavior: a front-loaded response.

In the next subsection, we consider the case for the production function with a dynamic externality. That is, we set ϕ to 0.26, holding the remaining parameters the same. The inclusion of a dynamic externality dramatically improves the model's performance.

3.3   A Dynamic Externality as an Alternative Explanation

We consider the production function with a dynamic externality ($\phi = 0.26$). Figure 3 shows IRFs for inflation, output, and marginal cost implied by our model in response to a negative one standard deviation decrease in the federal funds rate.

The results are strikingly different from the previous figure. First, inflation is strongly hump-shaped, as shown in the upper left panel of the figure. The response of inflation peaks in the ninth quarter after a monetary shock with negative initial responses. Our model even replicates the price puzzle that some researchers have found in the data: inflation decreases for five quarters and then gradually increases. This slow and gradual increase in inflation is qualitatively and quantitatively consistent with our IRF shown in Fig. 1. Second, the output gap (shown in the upper right panel) peaks in the second quarter after the shock. The output response remains very similar to the case of $\phi = 0$, shown in the upper right panel in Fig. 2, except that the responses are slightly larger. Finally, the marginal cost in the lower left panel behaves interestingly. Although its magnitude remains stable and very small

in advance to the shock. However, even when we add this assumption to our model, some initial responses of inflation lie outside the intervals.

[17]Although CEE concluded that the performance of their model for inflation was not substantially affected by removing backward-looking indexation, their conclusion relied on the inclusion of a working capital channel in their model. We discuss this point in section 5.
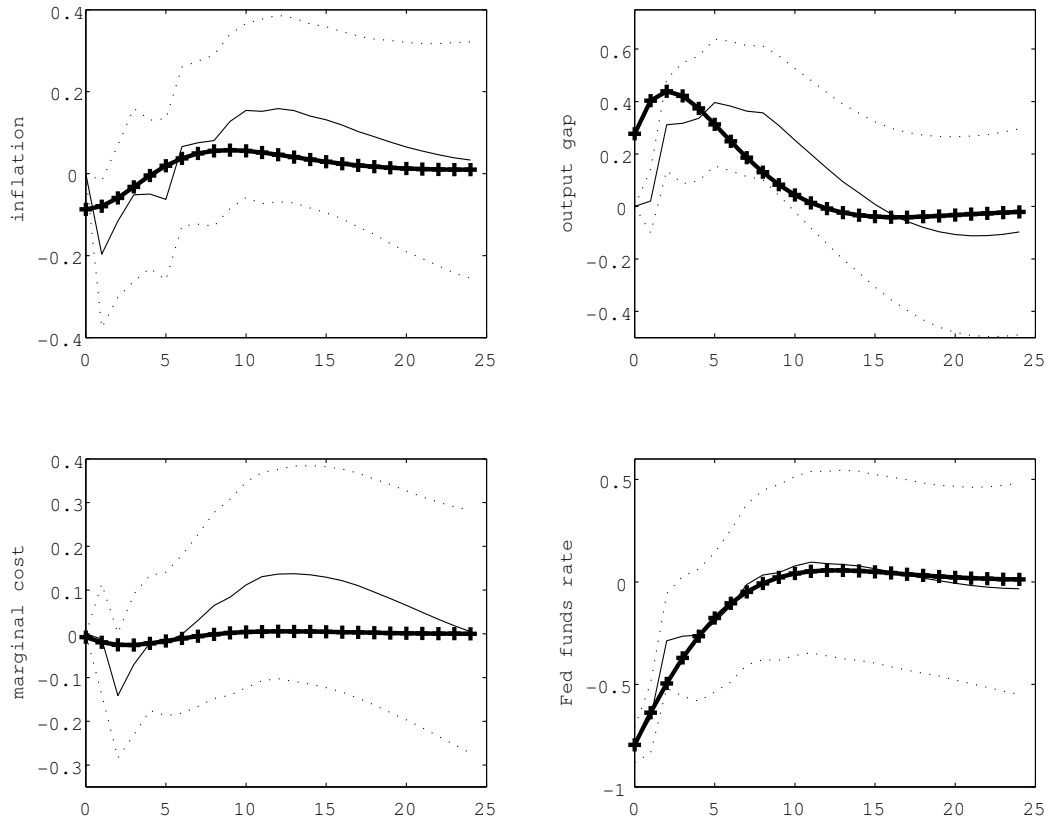
Figure 3: **With a dynamic externality**: IRF for inflation, output, marginal cost, and the federal funds rate in response to a negative one standard deviation shock in the federal funds rate. The solid lines with (+) are the simulated IRFs under $\phi > 0$. The other lines are explained in Fig. 2.

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relative to the data, the log-deviation of marginal cost is negative from the time of the shock through to the eighth quarter after the shock, returning to positive values only in the ninth period. Later, we will see the qualitative properties of the marginal cost in a comparison with the case where there is no externality.

For other variables of interests, the IRFs are shown in Fig. 4 with the estimated IRFs. Overall, these responses are quite similar to the results of CEE, both qualitatively and quantitatively. Owing to the positive effect of the output gap, organizational capital fluctuates procyclically in response to an expansionary monetary shock. This movement helps labor productivity $(\hat{Y}_t - \hat{L}_t)$ in the upper left panel move procyclically. Real profits show a persistent rise that is consistent with our estimate. Because habit formation and investment adjustment costs transform consumption and investment into state variables, their IRFs (the panels in the middle row) show a hump-shaped response. Finally, real wages move modestly procyclically with substantial persistence.[18]

Figure 5 is helpful for seeing the intuition behind the hump-shaped IRF for inflation. The figure magnifies two IRFs for marginal cost based on the two calibrated values of ϕ by changing the scale of the vertical axis. Qualitative differences in the dynamics of marginal cost are now clear. Without a dynamic externality ($\phi = 0$), the response of marginal cost is *uniformly* positive. On the other hand, the marginal cost under $\phi > 0$ takes negative values for the first several periods and then increases gradually. It reaches positive values after about two years and peaks in about three years. Although the magnitude of $\hat{mc}$ is much smaller than indicated by the data, this dynamic pattern of marginal cost under a dynamic externality is qualitatively much more similar to the data than it is under the standard production function.

Why does the marginal cost behave so differently? As is clear in (5), the real marginal cost at each period is decomposed into the effects of factor prices and productivity.

1. The effect of the real wage: marginal cost is higher, the higher is the real wage.

2. The effect of the rental cost of effective capital: marginal cost is higher, the higher is the rental cost of capital.

3. The effect of productivity through a dynamic externality: the higher organizational capital X_t is, the lower is marginal cost.

When the federal funds rate decreases, marginal utility decreases from (12).[19] To induce the decreased marginal utility, consumption and the investment

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[18]The IRFs for these variables shown here are at least as good as the case of the standard production function. For some variables, the simulation results are actually better than the case of $\phi = 0$. For example, under $\phi = 0$, labor productivity and real wages show a hump shape but almost no fluctuations relative to the data. However, including a dynamic externality, we obtain a more pronounced hump shape in these variables.

[19]To see this, we solve the log-linearized equation of (12) forward. Because the marginal
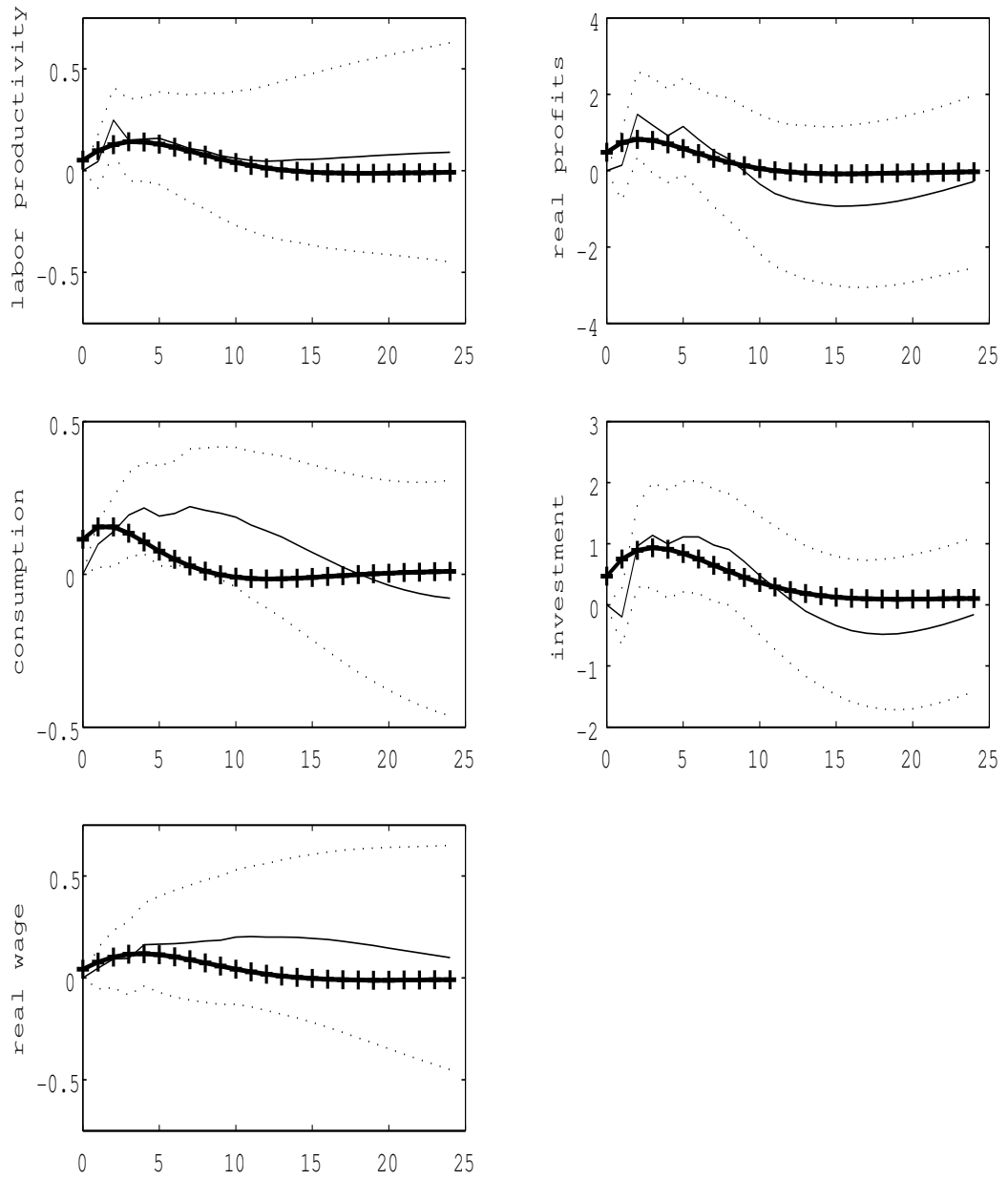
Figure 4: **With a dynamic externality**: IRF for other variables of interest in response to a negative one standard deviation shock in the federal funds rate.
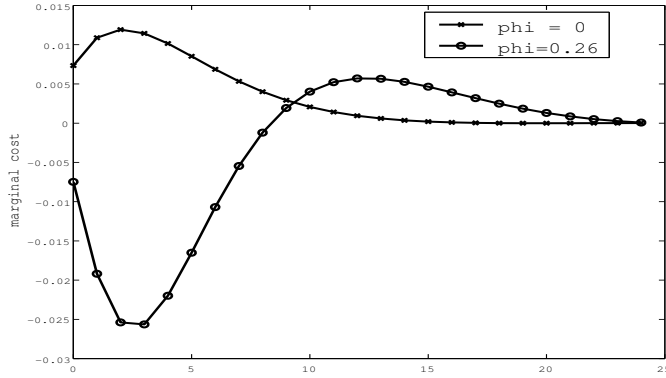
Figure 5: **IRFs for marginal cost based on $\phi = 0$ and 0.26**: IRF for marginal cost to a negative one standard deviation shock in the federal funds rate.

must increase.[20] Thus, the increase in consumption and investment in turn raises the demand for goods. To meet the increased demand for goods, firms need to hire more labor and capital. Thus, the real wage and rental cost of capital are bid upward. The increase in factor prices gives firms the incentive to increase prices. Without a dynamic externality, only the first two effects influence the dynamics of marginal cost. However, under a positive ϕ, organizational capital is accumulated because of the increased consumption and investment, which raises the firm's productivity. The higher productivity gives the firm the incentive to price low. Because the effects of productivity and factor prices are offsetting, the marginal costs may increase or decrease depending on the dynamic structure of these three elements. In the simulation, the marginal cost decreases for several periods after the shock and then increases.

Now, the reason for hump-shaped IRF for inflation is straightforward. As a first approximation, let $\beta = 1$. Then, (2) becomes:

$$E_t \hat{\pi}_{t+1} - \hat{\pi}_t = -\Psi_p \hat{mc}_t. \tag{21}$$

Thus, the expected change in inflation is negatively correlated with marginal

utility positively depends on the sum of short-term real interest rate, the decrease in the real interest rate causes the marginal utility to decrease.

[20]For the increased consumption, log-linearizing (16) yields:

$$\hat{C}_t = \frac{b}{1+\beta b^2}\hat{C}_{t-1} + \beta\frac{b}{1+\beta b^2}E_t\hat{C}_{t+1} - \frac{(1-b)(1-\beta b)}{1+\beta b^2}\hat{\lambda}_t.$$

The decrease in $\hat{\lambda}$ leads to the increased consumption. For the investment, log-linearizing (13) yields:

$$\hat{I}_t = \frac{1}{1+\beta}\hat{I}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{I}_{t+1} + \frac{s^{-1}}{1+\beta}[\hat{Q}_t - \hat{\lambda}_t].$$

Again, the decrease in $\hat{\lambda}_t$ leads to higher investment. In fact, $\hat{Q}_t$ changes and affects the investment, but the change in $\hat{Q}_t$ is small relative to $\hat{\lambda}_t$ in our simulation.

cost. In the simulation in Fig. 2 ($\phi = 0$), the impulse response of the log-deviation of marginal cost to a monetary policy shock is *uniformly* positive. Hence, the expected change of inflation must be uniformly negative in its response. In turn, this uniformly negative response means that inflation decreases over time until it reaches its steady state value of zero. Hence, the IRF for inflation must be front-loaded.

On the other hand, when $\phi = 0.26$, $\hat{mc}_t$ is *negative* until the ninth quarter after a monetary shock. Thus, inflation is expected to increase and actually increases rather than decreasing over time, even though the output gap during these periods is *positive*. After these periods, $\hat{mc}_t$ becomes *positive* until it converges to its steady state level of zero, implying that inflation decreases over time. Inflation increases as long as $\hat{mc}_t$ is negative and decreases as long as $\hat{mc}_t$ is positive.[21] This generates the hump-shaped response of inflation.[22]

The hump-shaped IRF for inflation can be interpreted as follows. After an unexpected monetary shock, a firm observes higher demand for its goods and needs to respond by resetting the price of its goods. When this is possible, the firm will predict that marginal cost will be high in the future, but will remain low for several periods before it begins to increase. As price setting is purely forward-looking, the firm will take into account the low short-run and high intermediate-run marginal costs in determining its price. Therefore, the firm will hesitate to price high while the marginal cost remains low in the short run. However, $\hat{mc}$ is increasing over time, as the externalities weaken and factor prices increase. When it is possible to reset the price again, the forward-looking firm no longer has such a low marginal cost. At this point, the incentive to price low created by the externalities has become small, leading the forward-looking firm to set its price higher. Thus, the inflation response is hump-shaped *not because firms are backward looking, but because they are forward looking.*

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[21]Note that this argument is based on the approximation $\beta \simeq 1$. It is possible that inflation increases while $\hat{mc}$ is positive, because $\beta < 1$. That is, $\beta E_t \pi_{t+1} - \pi_t < 0$ and $E_t \pi_{t+1} - \pi_t > 0$ can occur when $\hat{mc}$ is close enough to zero.

[22]This mechanism implies that one may obtain the observed hump-shaped behavior of inflation under the standard production function if real wages decrease at first and then increase after several periods. The combination of flexible prices and sticky wages may generate this behavior. To explore this possibility, we simulate an extreme case: very low nominal price stickiness and very high nominal wage stickiness. However, in this case, we obtain very weak countercyclical real wages and a procyclical rental cost of effective capital and thus, marginal cost does not show its initial decline. For this reason, inflation in this extreme case shows a front-loaded response. In addition, we consider that the countercyclical movement of real wages is counterfactual at least for the postwar era in the US. In particular, a number of studies suggest that real wages in the US modestly increase in response to expansionary monetary policy shocks. For example, see Christiano, Eichenbaum, and Evans (1997) and Huang, Liu, and Phaneuf (2004).

3.4 The Role of Sticky Wages and Variable Capital Utilization

In this subsection, we discuss the role of sticky wages and variable capital utilization. In the analysis in the previous subsection, we found that the behavior of marginal cost is important: when the log-deviation of the marginal cost takes negative values in the short run and positive values in the intermediate run, inflation can be hump-shaped under purely forward-looking nominal rigidities. It will be shown that sticky wages and variable capital utilization are important for generating such behavior of marginal cost. Without both elements, we lose the short-run decrease in marginal cost and thus inflation is not hump-shaped.

To analyze the effect of sticky wages and variable capital utilization, we use (5) to take the log-linearization of the marginal cost around the steady state:

$$\hat{mc}_t = \hat{P}_t^f - \phi \hat{X}_t,$$

where $\hat{P}_t^f \equiv (1-\alpha)\hat{w}_t + \alpha\hat{r}_t^k$. In other words, $\hat{P}_t^f$ is the weighted average of real wages and the rental cost of effective capital. The second term in the equation $\phi \hat{X}_t$ shows the log-deviation of productivity stemming from the dynamic externality.

Figure 6 makes the effect of sticky wages and capital utilization clear. In each panel of the figure, the IRFs of factor prices $\hat{P}_t^f$ and productivity $\phi \hat{X}_t$ to a monetary shock are shown. The upper left panel of the figure is the benchmark case, whereas the lower right panel of the figure is the case of flexible wages and constant capital utilization. The off-diagonal panels of the figure show the case in which either sticky wages or variable capital utilization is missing in the simulation.

Note that only in the upper left panel does the productivity effect exceed the factor-price effect for the first several periods. Thus, $\hat{mc}$ is first negative and then positive. As shown in the previous subsection, inflation is hump-shaped because of this behavior of $\hat{mc}$. On the other hand, the factor prices in the other panels of the figure are larger than productivity in the log-deviation response, which implies that $\hat{mc}$ takes positive values in response to a monetary shock. Therefore, inflation is never hump-shaped.

We can see that *both* sticky wages and variable capital utilization are important. In the upper right panel of the figure, capital utilization is variable but real wages are flexible. In this case, the factor-price effect overwhelms the productivity effect in its magnitude because real wages are adjusted upward quickly. In the lower left panel, real wages are sticky but capital utilization is constant. As a result, the rental cost of effective capital is adjusted upward so much that factor prices exceed productivity, although the magnitude of factor prices is close to that of productivity. Finally, under flexible wages and constant capital utilization, the effect of factor prices is so strong that the productivity effect is almost negligible in comparison.
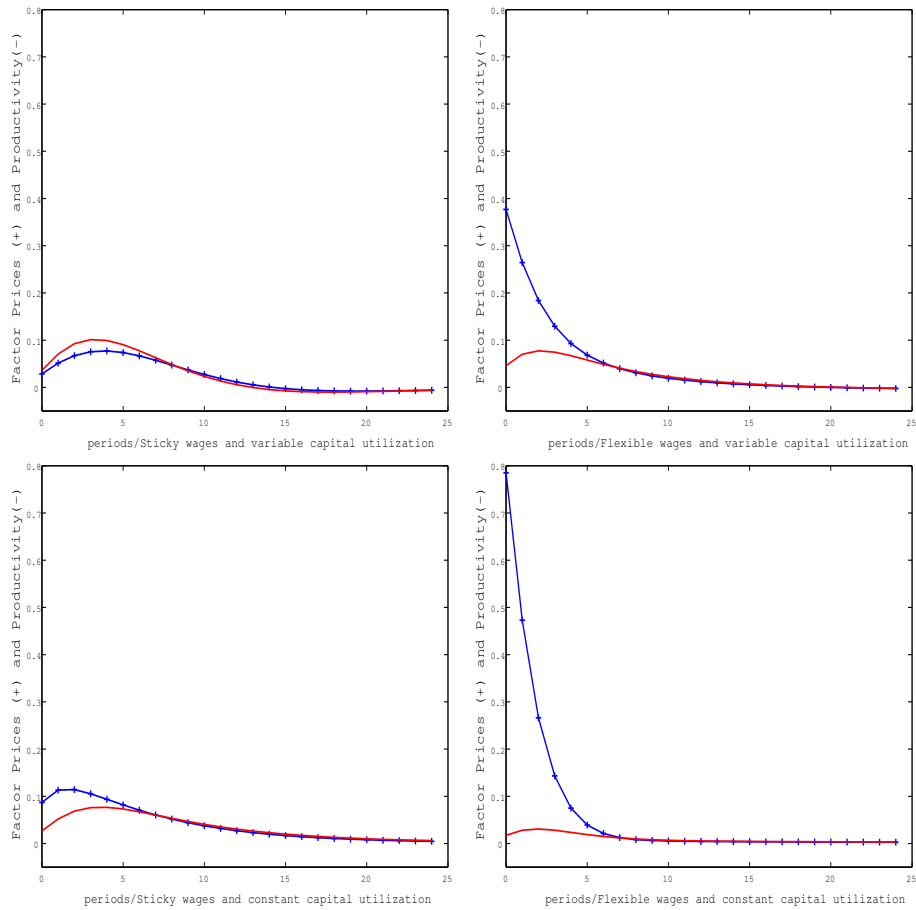
Figure 6: **Factor prices and productivity**: The line with (+) is the log-deviation of factor prices from the steady state value $\hat{P}_t^f$ and the line with (-) is productivity. The factor price minus productivity is the log-deviation of the marginal cost.

4 Sensitivity Analysis Regarding Returns to Organizational Capital

It is natural to ask how robust our externality-driven hump-shaped IRF for inflation is to the extent of the externality. In the literature on a static externality, some authors have argued against external increasing returns to scale. For this reason, Fig. 7 shows the IRFs for inflation and the output gap for smaller values of ϕ. In the figure, the values of ϕ are 0, 0.10, 0.15, 0.20, and 0.26, respectively. The lines in a panel differ in the degree of returns to organizational capital ϕ.

As the top panel in Fig. 7 shows, a pronounced hump shape is obtained even under values of ϕ as small as 0.15. Under this small level of returns, inflation shows a peak response in the seventh quarter.

The bottom panel in Fig. 7 tells us how much the dynamic externality amplifies the effect of monetary shocks on the output gap. Although the hump-shape dynamics of the output gap largely rely on habit formation and investment adjustment costs, the dynamic externality plays the role of an amplification mechanism. The larger the degree of externality, the more pronounced is the hump shape in the output gap. Thus, externalities amplify the effect of habit formation and investment adjustment costs.

The intuition for amplification is simple. The increased demand for goods raises productivity. The price of goods is lower when there are externalities than otherwise because of increased productivity. Because of the lower price of goods, there is a larger increase in the demand for goods.[23] Habit formation and adjustment costs in investment restrict the increase in consumption and investment so that their initial responses are almost the same for all values of ϕ. However, the effect of the monetary shock becomes stronger as ϕ becomes larger. As a result, a hump shape in the output gap is more pronounced when a dynamic externality exists than it is otherwise.

5 Why Is Inflation Hump-shaped in CEE?

In this section, we evaluate the model in CEE and our model. In our discussion, the model with sticky wages and variable capital utilization alone does not show hump-shaped behavior of inflation unless there is a dynamic externality (see Fig. 2 and 3). CEE concluded that sticky wages and variable capital utilization are important for generating quantitatively plausible inflation inertia. In particular, CEE succeeded in generating a hump shape in inflation without a dynamic externality. Moreover, they pointed out that a hump shape is actually obtained even if the assumption of backward-looking indexation is dropped. In this section, we discuss why CEE were able to generate the observed hump-shaped inflation without a dynamic externality.

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[23]More rigorously, this effect of prices occurs indirectly through lower real interest rates. As the parameter ϕ increases, inflation is initially lower. As a result, the decrease in the real interest rate becomes larger, causing higher consumption and investment.
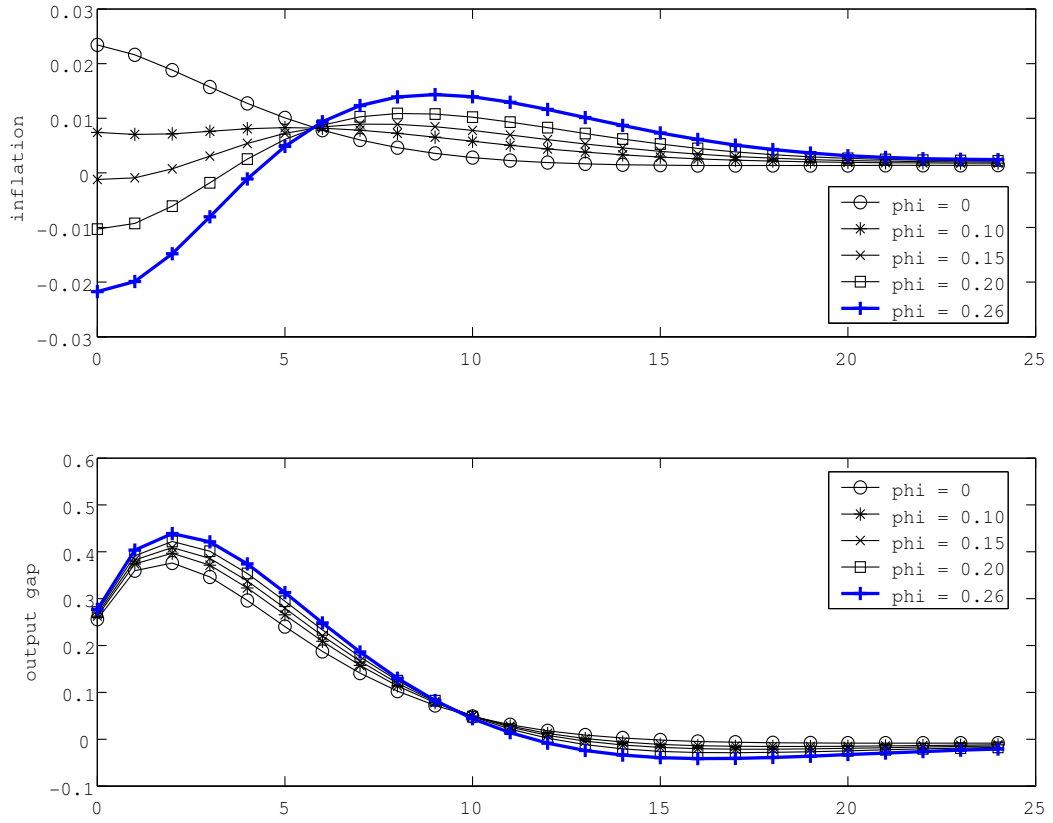
Figure 7: **Robustness to returns to scale**: IRF for inflation and the output gap in response to a negative one standard deviation shock in the federal funds rate based on different values of ϕ.

To discuss this question, it is necessary to disentangle the assumptions made by CEE in the benchmark model. In their model, there are two important assumptions for a hump-shaped IRF for inflation. First, one of the most important assumptions is their use of a hybrid NKPC that incorporates backward-looking indexation in nominal prices. This backward-looking indexation transforms inflation from a jump variable to a state variable because inflation becomes a function of the lagged inflation. In our notations, their hybrid NKPC for prices is given by:

$$\hat{\pi}_t = \frac{1}{1+\beta}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} + \frac{\Psi_p}{1+\beta}\hat{mc}_t.$$

As a result, the first term in the equation results in inflation inertia in CEE's benchmark model.

Second, CEE assumed a working capital channel. In other words, firms must borrow their wage bill from financial intermediaries at the beginning of each period and repay it plus interest at the end of the period. As a result, the real marginal cost is given by:

$$mc_t = (1-\alpha)^{-(1-\alpha)}\alpha^{-\alpha}(R_t w_t)^{1-\alpha}(r_t^k)^{\alpha},$$

where R_t is the gross nominal interest rate. As marginal cost depends on the nominal interest rate, it can decrease appreciably in the short run if the monetary authority reduces the nominal interest rate substantially. As in our model, this reduction in the nominal interest rate generates the hump-shaped behavior of inflation.

A model dropping only one of these assumptions can still generate hump-shaped inflation, as CEE showed in their analysis. When they replaced a backward-looking indexation with a purely forward-looking indexation, they concluded that "inflation continues to be inertial" (p. 32). Indeed, the peak of the IRF occurs in the fourth quarter following a monetary shock, whereas the peak of the IRF with a backward-looking indexation is in the tenth quarter. This relatively quick but hump-shaped response of inflation results from the assumption that working capital creates a short-run decrease in the marginal cost. In addition, CEE concluded that "the role of the working capital assumption is relatively minor" (p. 39) when they dropped it. (The peak response occurs in the fifth quarter.) This is because their hybrid NKPC plays a crucial role in generating a hump shape in inflation.

The simulation results shown in Fig. 2 can be interpreted as the model dropping both the working capital assumption and backward-looking indexation for prices and wages. Because there is neither lagged inflation nor a short-run decrease in marginal cost, the peak response of inflation must occur at the time of the monetary policy shocks.

6 Conclusion

This paper provides an alternative explanation for the observed hump-shaped behavior of inflation in response to monetary policy shocks. In the model, nominal prices are determined in a purely forward-looking manner. Instead of discussing existing ways of generating a hump shape, this paper introduces organizational capital accumulated by production spillovers in the production function, which we call a dynamic externality. In response to expansionary monetary shocks, a dynamic externality gives firms an incentive to price low because firms observe an increase in productivity. This incentive weakens the incentive to price high because of the effect of increased factor prices.

In order for this low-pricing incentive to generate hump-shaped inflation, we require the assumption of sticky wages and variable capital utilization, as in CEE. These assumptions assist in dampening fluctuations in marginal cost. In addition, sticky wages lead to an increase in marginal cost in the intermediate run because real wages adjust upward slowly. We show that these assumptions are important because the effect of productivity induced by a dynamic externality overwhelms the effect of factor prices in the short run. As a consequence, marginal cost can decrease for several initial periods, but it eventually increases. Given the dynamics of marginal cost, forward-looking firms change their prices only moderately for several periods and the response of inflation to a monetary shock can be hump-shaped for a reasonable range of parameters.

We found the delayed response of inflation is quite robust to returns to organizational capital. Inflation is still hump-shaped even when we calibrate returns to organizational capital that are less than the estimates of Cooper and Johri (2002). In our model, a price puzzle can even emerge. Therefore, a dynamic externality could be a possible explanation for the price puzzle found by some researchers.

We compare CEE's mechanism of hump-shaped inflation with ours. To generate plausible inflation inertia under a purely forward-looking NKPC, CEE required the assumption of a working capital channel, whereas our model requires a dynamic externality in the production function. Note that both models predict a short-run decrease in marginal cost. Our finding suggests a dynamic externality may generate a more delayed hump-shaped inflation than does CEE's working capital channel. Indeed, our model generates the peak in inflation in the ninth quarter, whereas CEE's model without a backward-looking indexation generated the peak in the fourth quarter. However, the choice between the two is ultimately an empirical problem.

We must admit that the accumulation equation of organizational capital is only an approximation for simplicity. Although the empirical literature on organizational capital suggests that organizational capital accumulated through output is one way of modeling this, the process of accumulation itself is not micro-founded. Moreover, whether a firm's knowledge is likely to be diffused is a thorny issue. Developing better models to answer these

issues would be an interesting future step in this line of research.

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