IMES DISCUSSION PAPER SERIES

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and the Effects of Monetary Policy

Ichiro Fukunaga

Discussion Paper No. 2005-E-14

IMES

INSTITUTE FOR MONETARY AND ECONOMIC STUDIES

BANK OF JAPAN

C.P.O BOX 203 TOKYO

100-8630 JAPAN

Inventories, Predetermined Prices, and the Effects of Monetary Policy

Ichiro Fukunaga*

Abstract

This paper studies the role of inventories in the propagation of monetary shocks by developing simple dynamic general equilibrium models that assume predetermined prices. Inventories serve as a source of real rigidities, that is, amplify the real effects of monetary policy. I introduce a sales-facilitating motive as well as a production-smoothing motive for holding inventories. Inventories respond procyclically and prices are adjusted gradually to a nominal disturbance only if the sales-facilitating motive is relatively strong; otherwise inventories respond countercyclically and prices are adjusted excessively. I also consider the models that assume that both production and prices are predetermined, in which inventories absorb shocks in an unintended manner. In a case where the decision lag of price setting is longer than that of production, inventories respond countercyclically at first and then move procyclically, which is consistent with the pattern shown in empirical studies.

1 Introduction

There are many mechanisms that cooperate in transmitting monetary policy effects to the economy. This paper studies the role of inventories in the propagation of monetary shocks by developing simple dynamic general equilibrium models.

Changes in inventories are key components in business cycles so that economists closely look at inventory data when they assess the current state of business cycles. Although the share of inventory investment in GDP is less than one percent, reduction in inventories arithmetically accounted for about half of the fall in GDP during postwar U.S. recessions (Ramey and West, 1999). Nonetheless, most existing monetary business-cycle models pay no attention to inventories. For example, the standard dynamic New Keynesian models based on staggered price setting assume that monopolistic suppliers have to produce whatever quantity to meet the demand even in the periods when they cannot set their optimal price. If goods are storable, however, those suppliers may wish to hold inventories in order to smooth production or facilitate sales. Gradual adjustments of the stock of inventories could make the monetary policy effects on production or sales more persistent. This idea was proposed by Blinder and Fischer (1981) in their IS-LM framework, but has not been considered in optimization-based monetary business-cycle models until very recently.

One of the main reasons for the neglect of inventories is probably simply that there is no conventional dynamic general equilibrium model with inventories that can successfully explain stylized facts. According to Khan and Thomas (2004), a core set of empirical regularities in postwar U.S. data that "any useful model of inventories should seek to address" is that: 1) the relative variability of inventory investment is large; 2) the correlation between inventory investment and GDP is positive; 3) the correlation between inventory investment and final sales is positive; 4) the standard deviation of production exceeds that of sales;[1] 5) the correlation between inventory-to-sales ratio and GDP is negative. In addition, Wen (2005) pointed out that inventory investment is procyclical only at relatively low cyclical frequencies such as business-cycle frequencies; it is countercyclical at very high frequencies (2–3 quarters per cycle).[2]

It is not easy for the most popular inventory theory, the production-smoothing theory, to explain the stylized facts that inventory investment is positively correlated with sales and accordingly production is more volatile

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[1]Given the accounting definition of GDP as being equal to final sales plus inventory investment, 3) is sufficient to imply 4).

[2]Similar facts are reported in Kimura and Adachi (1998) for Japanese data.

than sales. The production-smoothing theory simply claims that firms hold inventories in order to smooth production if their short-term production function is concave. It is proved that this theory can be reconciled with the fact of relatively volatile production if cost shocks or highly serially correlated demand shocks are dominant in the economy. It is not clear, however, how the production-smoothing model can generate volatile production in response to *nominal* demand shocks.

Some other theories emphasize sales-related motives for holding inventories.[3] The stockout avoidance theory claims that firms hold inventories in order to avoid losses of opportunity for sales when they cannot adjust production to meet positive demand shocks. Bils and Kahn (2000) further emphasize the relationship between sales and inventories, and assume that sales directly depend on the available inventory stock, that is, the sum of current production and inventory stock at the beginning of a period. They claim that a larger stock of inventories facilitates matching with potential purchasers who arrive with preferences for a specific type of good when the stock is considered as an aggregate of similar goods of different sizes, colors, locations, etc. The relationship between sales and inventories is taken into account by many empirical studies in which a class of the "linear-quadratic models" (Ramey and West, 1999) is typically specified.

One of the main challenges in this paper is to introduce inventories into a monetary dynamic general equilibrium model in a way that is consistent with the above stylized facts of inventories. Monopolistic suppliers in my models have both production-smoothing and sales-facilitating motives for holding inventories. The production-smoothing motive is incorporated by assuming that the marginal disutility of production or labor supply is increasing. The sales-facilitating motive is incorporated by considering a generic cost of sales that can be saved by holding inventories. This motive induces farmers to keep a close relationship between sales and inventories.[4] I introduce a generic cost function of sales and inventories, from which the optimal inventory-to-sales

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[3]Other theories on inventories also include the factor-of-production theory and the (S,s) theory. The former, mainly adopted in the real business-cycle models such as Kydland and Prescott (1982), claims that inventory stock at each stage of production may facilitate shipment, delivery, distribution, and eventually final production and therefore should be treated as a factor of production. The latter, recently incorporated into a dynamic general equilibrium model by Khan and Thomas (2003), emphasizes the role of inventories that saves fixed costs of production or ordering and constructs a model that features the S-s type of decision rule for inventory investment.

[4]The motive for keeping the relationship between sales and inventories amplifies the fluctuations in production, which can be interpreted as a type of the "inventory accelerator" mechanism proposed by Metzler (1941). He also considered "unintended inventories", which I will consider below.

ratio can be explicitly derived in the case of constant return to scale.[5] In my baseline model, which assumes predetermined prices, I will show that inventory investment responds procyclically to a nominal disturbance only if the sales-facilitating motive is relatively strong. Moreover, in an extended model that assumes that production as well as prices are predetermined, inventories initially absorb the shock in an unintended manner so that inventory investment responds countercyclically at first and then moves procyclically, which is consistent with the cyclical pattern pointed out by Wen (2005) and the estimated VAR evidence by Bernanke and Gertler (1995) and Jung and Yun (2005).

Another challenge is to explain plausibly how prices are adjusted to monetary shocks. The only nominal rigidity in my models is the predetermination of prices. The price-setting decisions of the monopolistic suppliers are closely related to their inventory-holding decisions. I will show that prices are adjusted gradually to a nominal disturbance only if inventories move procyclically at business-cycle frequencies, that is, the sales-facilitating motive for holding inventories is relatively strong; otherwise prices are adjusted quickly and even excessively at the early stage of the adjustment. The strong sales-facilitating motive generates not only the procyclical inventories but also the gradual adjustment of prices or "price stickiness" endogenously.

The intuition behind the above inventory-holding and price-setting behaviors is as follows. Suppose that a positive nominal disturbance occurs in the economy, which means that monopolistic suppliers face an unexpected boom in sales because they cannot adjust their prices immediately during the period of disturbance. If the sales-facilitating motive for holding inventories is relatively strong compared with the production-smoothing motive, the suppliers initially produce more than the amount just to meet the demand so that they can hold inventory stock above its normal level to save the cost of sales. Then they start adjusting prices gradually so that sales and inventories can gradually return to their original levels while retaining their relationship. If the production-smoothing motive is relatively strong, on the other hand, the suppliers meet the initial unexpected demand partially by reducing their existing inventory stock because they strongly wish to avoid changing production levels to such an extent. Then they raise prices aggressively, even excessively, to dampen sales so that the reduced inventory stock can gradually recover in parallel with production smoothly returning to its

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[5]This sales-cost function is distinct from the sales function in Bils and Kahn (2000). Chang, Hornstein, and Sarte (2004) consider a specific form of sales-cost function that depends on inventory stock for their quantitative assessment of the employment response to productivity shocks across different degrees of inventory-holding costs and price stickiness.

original level.[6]

By incorporating inventories into a monetary business-cycle model in an appropriate way, the persistence of monetary policy effects is increased, as Blinder and Fischer (1981) argued. I will show that even my baseline inventory model that assumes that all prices must be determined just one period in advance, or equivalently that true information about disturbances is revealed to all price setters just one period later, can generate the real effects of monetary policy lasting several periods. In an extended model that assumes longer-period predetermined prices, the real effects persist even after all prices begin to be adjusted. This implies that the model with inventories does not need to assume that a fraction of prices is determined on the basis of unrealistically old information, as the sticky information model of Mankiw and Reis (2002) does in order to generate persistent real effects. Inventories serve as a source of "real rigidities" (Ball and Romer, 1990); that is, they amplify the effects caused by nominal rigidities.

I will show most of the above results analytically in my sufficiently simplified models where monopolistic "yeoman farmers" produce and directly supply their individual-specific goods and consume all types of those differentiated goods. I examine the effects of nominal disturbances by restricting my attention to stationary fluctuations around the steady state and log-linearizing equilibrium conditions. I extend my models step by step from the baseline model that assumes just one-period predetermined prices to the models that assume both production and prices are predetermined and that the length of the decision lag for price setting is heterogeneous among farmers. Finally, I develop a more realistic model that includes the labor market, capital accumulation, explicit money, and real disturbances and use stochastic simulations to assess quantitatively the cyclical pattern and persistence of aggregate variables. Although not all the results of my simulations quantitatively match the data, they qualitatively well reflect the stylized facts of inventories and support my analytical results.

Some recent studies deal with similar problems to this paper. Boileau and

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[6]Another explanation for the relationship between inventory-holding and price-setting behaviors, interpreting the inventory stock as an asset upon which returns are decreasing, is as follows. The opportunity cost for monopolistic suppliers to deviate inventory stock upwards from its normal level is equal to the expected deflation rate *of their own products* if the nominal interest rate is fixed. Thus they set higher prices for the next period than the current period (the prices are assumed to be predetermined) when they hold inventory stock above its normal level (the case of a strong sales-facilitating motive), which implies gradual increases of prices in response to a positive nominal disturbance. On the other hand, they set lower prices for the next period when they hold inventory stock below its normal level (the case of a weak sales-facilitating motive), which implies gradual decreases after an excessive increase of prices. See footnote 15 in the baseline model below.

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Letendre (2004) consider inventories held for alternative motives under the costly price adjustment as in Rotemberg (1982), and use stochastic simulations to compare the ability of these inventories to generate persistent output and inflation. Hornstein and Sarte (2001) consider inventories held only for the production-smoothing motive under staggered price setting as in Taylor (1980), and show that an aggregation effect intrinsic to price staggering can make aggregate production respond more strongly than aggregate sales to a nominal demand shock. Jung and Yun (2005) consider inventories held for sales-expansion benefits at a given price under staggered price setting as in Calvo (1983), and match the impulse responses to a monetary policy shock from their theoretical model with those from their estimated VAR model.[7]

Besides the differences in focus and strategies, my models differ from theirs in the basic assumptions of inventory-holding and price-setting behaviors. One of the main features of my models is the sales-facilitating motive for holding inventories that is introduced in an original way and shown to be essential for my main results. Another important difference is in the assumption of price setting: my models assume predetermined prices as in Fischer (1977), whose model was a prototype of the sticky information model. My motivations for this assumption are as follows. First, it allows me to obtain many analytical results, which is difficult under the assumptions adopted in the studies mentioned above. Second, the predetermined prices models without inventories cannot generate sticky prices and persistent real effects of monetary policy endogenously, which encourages me to examine how they are created by introducing inventories into the models. Third, I will consider unintended inventories caused by a decision lag of production. It seems reasonable to assume predetermined prices together with predetermined production for examining unintended inventories in response to nominal disturbances. In addition, the predetermined prices or imperfect information models have the following advantages over the more popular staggered price-setting models. As Mankiw and Reis (2002) argued, imperfect information models can explain the delay of price adjustment better than staggered price-setting models. Moreover, I will show that the responses of disaggregate inventories to a nominal disturbance in the predetermined prices models seem more plausible than those in the staggered price-setting models of Hornstein and Sarte (2001) where the level of disaggregate inventory stock

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[7]Although the impulse responses from their theoretical and estimated VAR models are generally well matched, they fail to explain the observed procyclicality in the stock of inventories at business-cycle frequencies. Another important difference between their results and mine is that inventories do not serve as a source of real rigidities: in order to match the estimated inflation persistence, their model requires a higher degree of nominal rigidities than the standard models without inventories.

oscillates as price setters change their prices alternately.

The remainder of this paper is organized as follows. In Section 2, I describe the baseline model and show the main results on the effects of nominal disturbances. In Section 3, I extend the baseline model by assuming that production as well as prices are predetermined. I seek to find a condition under which a model assuming predetermined production can explain more precisely the observed cyclical pattern of inventory-holding behavior that includes unintended inventories. I also consider the case of heterogeneity in the length of the decision lag among price setters. In Section 4, I develop a more realistic quantitative model and report the main results of simulations. Section 5 concludes.

2   The Baseline Model

In this section I develop a basic dynamic general equilibrium model simple enough to obtain analytical results for the properties of the effects of nominal disturbances on aggregate variables including inventories.

The baseline model in this section assumes a minimum nominal rigidity: all prices must be determined one period in advance due to imperfect information or some other constraints. I will consider the cases of longer-period predetermined prices, the predetermination in different periods among price setters, and predetermined production as well as prices in the next section. The models in this and the following sections ignore some basic and realistic factors in the economy such as the labor market, capital accumulation, explicit money, and real disturbances, which I will introduce in the quantitative model in Section 4.

2.1   Set-up

Consider an economy populated by a continuum of infinitely lived "yeoman farmers" indexed by $i \in [0, 1]$ who produce and directly supply their individual-specific goods and consume all types of those differentiated goods. Farmer i who supplies a good of type i seeks to maximize a discounted sum of utilities of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t^i - \frac{y_t(i)^{1+\eta}}{1+\eta} \right\}, \tag{1}$$

where $0 < \beta < 1$ is a discount factor, C_t^i is a constant-elasticity-of-substitution aggregator of farmer i's consumption of each individual good of type j

$$C_t^i \equiv \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{2}$$

6

with $\theta > 1$, and $y_t(i)$ is i's production. I assume η is strictly positive so that the marginal disutility of producing good is increasing and accordingly farmers have incentives to smooth production. The larger η is, the stronger the production-smoothing motive is.

Products are storable in this economy. Sales and production need not match in each period since the gap is adjusted by inventories. The resource constraint for good i is

$$y_t(i) - \Phi(c_t(i), x_t(i)) = c_t(i) + x_t(i) - x_{t-1}(i), \tag{3}$$

where $c_t(i) \equiv \int_0^1 c_t^j(i)dj$ is the sales and $x_t(i)$ is the end-of-period inventory stock of good i. $\Phi(c_t(i), x_t(i))$ is a generic function of sales cost that can be saved by holding inventories. I assume that Φ is a nonnegative increasing function of $c_t(i)$ and a decreasing function of $x_t(i)$ and that the second derivatives satisfy $\Phi_{cc} > 0$, $\Phi_{xx} > 0$ and $\Phi_{cx} < 0$ for the relevant region. The absolute value of Φ_{cx} represents the degree of the sales-facilitating motive for holding inventories: the larger $|\Phi_{cx}|$ is, the stronger the sales-facilitating motive. This motive induces farmers to keep a close relationship between sales and inventories. I also assume $\lim_{x_t(i) \to 0} \Phi_x(c_t(i), x_t(i)) = -\infty$, which implies that zero inventory stock leads to prohibitively high sales cost. Farmers need to hold inventories despite depreciation and some physical storage costs that are included in the generic function.

Financial markets are complete in this economy. Even if the income streams from sales are expected to differ among farmers, they can choose an identical consumption plan under the intertemporal budget constraint for any farmer i from any period t,

$$\sum_{s=t}^{\infty} E_t Q_{t,s} \left[\int_0^1 p_s(j) c_s^i(j) dj \right] \leq W_t + \sum_{s=t}^{\infty} E_t Q_{t,s}[p_s(i) c_s(i)], \tag{4}$$

where $Q_{t,s}$ is a stochastic discount factor, W_t is the beginning-of-period wealth, and $p_t(i)$ is the price of good i. Farmers allocate their consumption across differentiated goods in each period so as to maximize the index (2), taking as given the level of total expenditure $\int_0^1 p_t(j) c_t^i(j) dj$, which implies

$$c_t^i(j) = \left(\frac{p_t(j)}{P_t} \right)^{-\theta} C_t^i, \tag{5}$$

where

$$P_t \equiv \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

is the corresponding price index with which i's optimally allocated total expenditure is equal to $P_t C_t^i$. Taking as given this optimal allocation of

consumption in each period, farmers then choose the optimal path of total consumption. The first-order condition in t is given by

$$\frac{1}{C_t} = \beta \left(1 + i_t\right) E_t \left[\frac{P_t}{P_{t+1}\, C_{t+1}} \right], \tag{6}$$

where i_t is the riskless nominal interest rate that corresponds to $(1+i_t)^{-1} = E_t Q_{t,t+1}$. Here I drop the superscript of C_t^i and use $C_t \equiv \int_0^1 C_t^i di$ instead since all farmers choose the same consumption plan. Similarly, (5) can be rewritten by dropping the superscript i as $c_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\theta} C_t$, which now implies the demand constraint for good j.

At the same time in each period, farmers make a decision about their production and inventory investment. Since they are monopolistic suppliers who set their prices under demand constraints of sales, their decisions on production are partly combined with those on price setting, which I will describe in the next paragraph. Meanwhile, farmers can control their inventory investment independently of their price-setting decisions.[8] The first-order condition for optimal inventory holding is given by

$$y_t(i)^\eta \left\{ 1 + \Phi_x(c_t(i),\, x_t(i)) \right\} = \beta\, E_t y_{t+1}(i)^\eta. \tag{7}$$

They hold inventories so that the marginal cost of increasing inventories in terms of the marginal disutility of production in the current period should be equal to the expected marginal benefit from reducing production in the next period, which satisfies both the production-smoothing and the sales-facilitating motives. This is a key equation in the model.

Farmers choose their prices one period in advance. Using the information set available only in period $t-1$, farmer i sets the price $p_t(i)$ so as to maximize (1) subject to (3), (4), and the demand constraint $c_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} C_t$ that comes from (5). The first-order condition is given by

$$E_{t-1} \left[\frac{p_t(i)}{P_t} \right] = \frac{\theta}{\theta - 1} E_{t-1} \left[C_t\, y_t(i)^\eta \left\{ 1 + \Phi_c(c_t(i),\, x_t(i)) \right\} \right]. \tag{8}$$

Factors in the expectation operator on the right-hand side represent the real marginal cost of supplying good i, which is comprised of the marginal cost of sales in terms of the marginal disutility of production divided by the marginal utility of consumption.

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[8]The first-order condition for optimal inventory holding (7) is unchanged even if farmers are price takers. This does not mean, however, that there is no connection between inventory-holding and price-setting decisions in this model.

Lastly, I introduce an exogenous stochastic process for aggregate nominal spending as follows.

$$\ln \mathcal{M}_t = \ln \mathcal{M}_{t-1} + \epsilon_t, \tag{9}$$

where $\mathcal{M}_t = P_t C_t$ and ϵ_t is white noise. One may interpret $\mathcal{M}_t$ as "money" that farmers must hold for their spending and the above process may be taken as a monetary policy rule specified by a target path for the money supply. Alternatively, one can image a fiscal-monetary policy rule specified by a target path for the aggregate nominal spending that may be achieved by adjusting the nominal interest rate on the government bond.[9] This simple specification for aggregate demand, however it is interpreted, allows me to concentrate on examining the consequences of alternative specifications for aggregate supply such as inventory-holding and price-setting behaviors.

2.2 Equilibrium

In the symmetric equilibrium, every farmer sets the same price and therefore purchases the same amount of each differentiated good, which implies that for any i

$$p_t(i) = P_t$$
$$c_t(i) = C_t.$$

Then equations (3), (7), and (8) are rewritten as

$$Y_t - \Phi(C_t, X_t) = C_t + X_t - X_{t-1} \tag{10}$$
$$Y_t^{\eta}\left\{1 + \Phi_x(C_t, X_t)\right\} = \beta E_t Y_{t+1}^{\eta} \tag{11}$$
$$1 = \frac{\theta}{\theta - 1} E_{t-1}\left[C_t Y_t^{\eta}\left\{1 + \Phi_c(C_t, X_t)\right\}\right] \tag{12}$$

where $Y_t \equiv \int_0^1 y_t(i)di$ and $X_t \equiv \int_0^1 x_t(i)di$.

A rational expectations equilibrium of the economy is defined as a set of $\{C_t, Y_t, X_t, P_t, i_t\}$ that satisfies the resource constraint (10) and the decision rules for consumption (6), inventory holding (11), and price setting (12), given the exogenous process for aggregate nominal spending (9) where $\mathcal{M}_t = P_t C_t$.

Below, I restrict my attention to stationary fluctuations around the steady state. The deterministic steady-state conditions for (10), (6), (11), and (12)

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[9]Under the optimal consumption rule of the private sector (6), this policy is implemented simply by holding the nominal interest rate on $i \equiv \beta^{-1} - 1$.

are as follows.

$$Y - \Phi(C, X) = C \tag{13}$$

$$1 = \beta(1 + i) \tag{14}$$

$$1 + \Phi_x(C, X) = \beta \tag{15}$$

$$1 = \frac{\theta}{\theta - 1} C Y^\eta \{1 + \Phi_c(C, X)\} \tag{16}$$

Around the steady state, $\Phi(C_t, X_t)$, $\Phi_c(C_t, X_t)$, and $\Phi_x(C_t, X_t)$ may be approximated as follows.

$$\Phi(C_t, X_t) \cong \Phi(C, X) + \Phi_C(C_t - C) + \Phi_X(X_t - X)$$

$$\Phi_c(C_t, X_t) \cong \Phi_C + \Phi_{CC}(C_t - C) + \Phi_{CX}(X_t - X)$$

$$\Phi_x(C_t, X_t) \cong \Phi_X + \Phi_{CX}(C_t - C) + \Phi_{XX}(X_t - X)$$

where $\Phi_C \equiv \Phi_c(C, X)$, $\Phi_X \equiv \Phi_x(C, X)$, $\Phi_{CC} \equiv \Phi_{cc}(C, X)$, $\Phi_{CX} \equiv \Phi_{cx}(C, X)$, and $\Phi_{XX} \equiv \Phi_{xx}(C, X)$. Using these and the above steady-state conditions, the whole system consisting of log-linear approximations to the equilibrium conditions can be obtained as follows.

$$\hat{C}_t = E_t \hat{C}_{t+1} - \{\hat{i}_t - (E_t \hat{P}_{t+1} - \hat{P}_t)\} \tag{17}$$

$$\eta \hat{Y}_t = \eta E_t \hat{Y}_{t+1} - \frac{C \Phi_{CX}}{\beta} \hat{C}_t - \frac{X \Phi_{XX}}{\beta} \hat{X}_t \tag{18}$$

$$0 = E_{t-1} \left[\eta \hat{Y}_t + \left(1 + \frac{C \Phi_{CC}}{1 + \Phi_C}\right) \hat{C}_t + \frac{X \Phi_{CX}}{1 + \Phi_C} \hat{X}_t \right] \tag{19}$$

$$\hat{Y}_t = \frac{C}{Y}(1 + \Phi_C) \hat{C}_t + \frac{X}{Y}(\beta \hat{X}_t - \hat{X}_{t-1}) \tag{20}$$

$$\hat{\mathcal{M}}_t = \hat{P}_t + \hat{C}_t \tag{21}$$

$$\hat{\mathcal{M}}_t = \hat{\mathcal{M}}_{t-1} + \epsilon_t \tag{22}$$

where $\hat{\mathcal{M}}_t \equiv \ln \mathcal{M}_t$, $\hat{P}_t \equiv \ln P_t$, $\hat{i}_t \equiv \ln(1 + i_t)/(1 + i)$, and other variables with hat denote log differences (or rate of deviation) from their steady-state values such as $\hat{X}_t \equiv \ln(X_t/X)$.

If the function $\Phi(C_t, X_t)$ is constant returns to scale, the above system can be simplified. I introduce the function $\phi(Z_t) \equiv \Phi(1, X_t/C_t) = \Phi(C_t, X_t)/C_t$ where $Z_t \equiv X_t/C_t$ is the inventory-to-sales ratio. The assumptions about the function Φ imply $\phi'(Z_t) < 0$ and $\phi''(Z_t) > 0$. Here, $\phi''(Z_t)$ rather than $|\Phi_{cx}|$ represents the degree of the sales-facilitating motive. First, the deterministic steady-state conditions (13), (15), and (16) are rewritten as follows.

$$Y = C\{1 + \phi(Z)\} \tag{23}$$

$$1 + \phi'(Z) = \beta \tag{24}$$

$$1 = \frac{\theta}{\theta - 1} C Y^\eta \{1 + \phi(Z) - Z \phi'(Z)\} \tag{25}$$

The optimal steady-state inventory-to-sales ratio Z is given by (24). Around the steady state, the degree of the sales-facilitating motive $\phi''(Z)$ corresponds to the marginal cost of deviating inventory-to-sales ratio from its optimal level Z. Then the equations (18), (19), and (20) in the above log-linearized system are rewritten as follows.

$$\eta \hat{Y}_t = \eta E_t \hat{Y}_{t+1} - \frac{Z \phi''(Z)}{\beta} (\hat{X}_t - \hat{C}_t) \tag{26}$$

$$0 = E_{t-1} \left[\eta \hat{Y}_t + \hat{C}_t - \frac{Z^2 \phi''(Z)}{1 + \phi(Z) - Z \phi'(Z)} (\hat{X}_t - \hat{C}_t) \right] \tag{27}$$

$$\hat{Y}_t = \frac{1 + \phi(Z) - Z \phi'(Z)}{1 + \phi(Z)} \hat{C}_t + \frac{Z}{1 + \phi(Z)} (\beta \hat{X}_t - \hat{X}_{t-1}) \tag{28}$$

In the numerical examples below and the quantitative model in Section 4, I assume for simplicity that the function Φ is constant returns to scale; otherwise I seek to obtain analytical results without specifying Φ.

2.3 Effects of Nominal Disturbances

Using the above log-linearized model, I examine the effects of nominal disturbances on real aggregate variables such as consumption, production, and inventories as well as on prices. The exogenous process for aggregate nominal spending (9) implies that a unit of positive innovation in ϵ_t raises $E_t \hat{\mathcal{M}}_{t+k}$ by one unit for all $k \geq 0$. The disturbance I consider below is such an unexpected permanent increase of one unit in the log of aggregate nominal spending.

Suppose that the economy originally stays at its steady state with $\hat{\mathcal{M}}_{t-1} = \hat{P}_{t-1} = 0$, and then in period t the disturbance, $\epsilon_t = 1$, occurs. Since I assume that prices are predetermined, the initial responses of prices and consumption are

$$\hat{P}_t = 0$$
$$\hat{C}_t = 1.$$

The rational expectations equilibrium that I am interested in is one in which deviations of real variables from their steady-state values are stationary, which requires

$$\lim_{k \to \infty} E_t \hat{P}_{t+k} = 1$$
$$\lim_{k \to \infty} E_t \hat{C}_{t+k} = 0.$$

Also, $E_t \hat{Y}_{t+k}$ and $E_t \hat{X}_{t+k}$ must converge to 0 as $k \to \infty$. In order to obtain

11

such a unique stationary equilibrium, I assume the following conditions of the parameter and the steady-state values.

$$\Phi_C > 0; \ \Phi_X < 0; \ \Phi_{CC} > 0; \ \Phi_{XX} > 0; \ \Phi_{CX} < 0;$$

$$\Phi^* \equiv \Phi_{XX} \left\{ Y \left(1 + \Phi_C + C \, \Phi_{CC} \right) + \eta \, C \left(1 + \Phi_C \right)^2 \right\}$$
$$+ (1 - \beta) \, \eta \, C \left(1 + \Phi_C \right) \Phi_{CX} - C \, Y \, \Phi_{CX}^2 \ > 0 \tag{29}$$

I seek to find a set of sequences $\{ E_t \hat{C}_{t+k}, \ E_t \hat{Y}_{t+k}, \ E_t \hat{X}_{t+k}, \ E_t \hat{P}_{t+k}, \ E_t \hat{\imath}_{t+k} \}$ that satisfies the log-linearized model and is consistent with the above initial and terminal conditions assuming (29). The solution is given in the following proposition.

Proposition 1. *Suppose that the economy around the steady state is approximated by the model (17) through (21) and the aggregate nominal spending evolves according to (22). All variables with hat are zero in period $t - 1$. Prices are determined one period in advance. Then the responses of inventories and consumption to a unit of positive innovation in ϵ_t are given by*

$$E_t \hat{X}_{t+k} = \lambda \, E_t \hat{X}_{t+k-1}$$
$$E_t \hat{C}_{t+k} = \mu \, E_t \hat{X}_{t+k-1}$$

for $k \geq 1$, where $0 < \lambda < 1$ is the smaller of the two real roots of the characteristic polynomial

$$\beta \, \alpha \, \lambda^2 - \left\{ (1 + \beta) \, \alpha - \frac{\Phi^*}{\Phi_{CX}} \right\} \lambda + \alpha = 0$$

where Φ^* is defined in (29) and

$$\alpha \equiv \left\{ \frac{C \left(1 + \Phi_C \right)}{\beta} - \frac{1 + \Phi_C + C \, \Phi_{CC}}{\Phi_{CX}} \right\} \eta \left(1 + \Phi_C \right) \ > 0 \,,$$

and

$$\mu \equiv \frac{(1 - \beta \, \lambda) \, \eta \, X \left(1 + \Phi_C \right) - \lambda \, X \, Y \, \Phi_{CX}}{Y \left(1 + \Phi_C + C \, \Phi_{CC} \right) + \eta \, C \left(1 + \Phi_C \right)^2} \ > 0 \,;$$

while for $k = 0$

$$\hat{X}_t = \frac{C \left(1 + \Phi_C \right) \left\{ -\Phi_{CX} - \eta \, \beta \left(1 + \Phi_C \right) / Y \right\}}{\left(X \, \Phi_{XX} + \eta \, \beta^2 \, X / Y \right) \left(1 + \Phi_C \right) + \mu \, \beta \left(1 + \Phi_C + C \, \Phi_{CC} \right) + \lambda \, \beta \, X \, \Phi_{CX}}$$
$$\hat{C}_t = 1.$$

12

The responses of production, prices, and nominal interest rate are given by

$$E_t \hat{Y}_{t+k} = \frac{C}{Y}(1 + \Phi_C) E_t \hat{C}_{t+k} + \frac{X}{Y}(\beta E_t \hat{X}_{t+k} - E_t \hat{X}_{t+k-1})$$

$$E_t \hat{P}_{t+k} = 1 - E_t \hat{C}_{t+k}$$

$$E_t \hat{i}_{t+k} = 0$$

for $k \geq 0$.

The proof is given in the Appendix.

Here I assume for simplicity that $\Phi_{XX} + \Phi_{XC} > 0$.[10] Then the main properties of the inventory-holding and price-setting behaviors are summarized by the following corollaries.

Corollary 1.1. *i) Inventories respond procyclically, i.e., $\hat{X}_t > 0$ and $E_t \hat{X}_{t+k} \geq 0$ for $k \geq 1$, if $-\Phi_{CX} > \eta \beta (1 + \Phi_C)/Y$. ii) Inventories respond countercyclically, i.e., $\hat{X}_t < 0$ and $E_t \hat{X}_{t+k} \leq 0$ for $k \geq 1$, if $-\Phi_{CX} < \eta \beta (1 + \Phi_C)/Y$. iii) Inventories do not respond, i.e., $E_t \hat{X}_{t+k} = 0$ for $k \geq 0$, if $-\Phi_{CX} = \eta \beta (1 + \Phi_C)/Y$.*

Corollary 1.2. *i) Prices are expected to be adjusted gradually, i.e., $E_t \hat{P}_{t+1} < 1$ and $E_t \hat{P}_{t+k} \leq 1$ for $k \geq 2$, if $-\Phi_{CX} > \eta \beta (1 + \Phi_C)/Y$. ii) Prices are expected to be adjusted excessively, i.e., $E_t \hat{P}_{t+1} > 1$ and $E_t \hat{P}_{t+k} \geq 1$ for $k \geq 2$, if $-\Phi_{CX} < \eta \beta (1 + \Phi_C)/Y$. iii) Prices are expected to be adjusted instantaneously, i.e., $E_t \hat{P}_{t+k} = 1$ for $k \geq 1$, if $-\Phi_{CX} = \eta \beta (1 + \Phi_C)/Y$.*

The deviation from the steady-state level of inventory stock is expected to decay monotonically from its initial response in period t, whether it is positive or negative. The sign of the initial response depends on the difference between $-\Phi_{CX}$ and $\eta \beta (1 + \Phi_C)/Y$, or the coefficient of $\hat{C}_t$ in (18), $-C \Phi_{CX}/\beta$, and $\eta (1 + \Phi_C) C/Y$, as stated in Corollary 1.1.[11] Since $|\Phi_{CX}|$ represents the degree of the sales-facilitating motive for holding inventories and η represents that of the production-smoothing motive, a positive or procyclical response of inventories occurs when the sales-facilitating motive is relatively strong while a negative or countercyclical response occurs when the production-smoothing motive is relatively strong.[12] If neither motive dominates, i.e.,

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[10]This assumption implies $C > X$ if the function Φ is constant returns to scale. Without this assumption, Corollary 1.1 and 1.2 should be modified as follows: inventories respond procyclically and prices are expected to be adjusted gradually if $-\Phi_{CX} > \eta \beta (1 + \Phi_C)/Y$ and also $(X \Phi_{XX} + \eta \beta^2 X/Y)(1 + \Phi_C) + \mu \beta (1 + \Phi_C + C \Phi_{CC}) + \lambda \beta X \Phi_{CX} > 0$.

[11]If the function Φ is constant returns to scale, this condition depends on the difference between $Z \phi''(Z)/\beta$ and $\eta \{ 1 + \phi(Z) - Z \phi'(Z) \} C/Y$.

[12]In my analytical results, I define the procyclical response of inventories as the deviation

$-\Phi_{CX} = \eta\,\beta\,(1 + \Phi_C)/Y$, inventory stock never deviates from its steady-state level.

The inventory dynamics affect the dynamic behaviors of other variables such as consumption, production, and prices. After the adjustment of prices starts in period $t + 1$, the deviations from the steady-state values of consumption and production are expected to decay monotonically, in parallel with that of inventories. This means that the real effects of the nominal disturbance persist even after all farmers get precise information about the disturbance and start adjusting their prices. It takes the same periods for the adjustment of prices to be completed as for the real variables including inventory stock to return to their original levels. If inventories do not respond during the period of disturbance,[13] however, prices will be adjusted instantaneously and the real effects will disappear in the next period, as in the standard predetermined prices or imperfect information models without inventories.

The sign of the initial response of inventories determines how consumption and production are expected to respond and how prices are expected to be adjusted to the disturbance, as stated in Corollary 1.2. A sample set of impulse responses of those variables for the case of a relatively strong sales-facilitating motive is shown in Figure 1.1.[14] Faced with an unexpected boom in sales in period t, farmers produce more than the amount just to meet the demand so that they can hold inventory stock above its normal level to save the cost of sales. After period $t + 1$, they will adjust their prices gradually so that sales and inventories can gradually return to their original levels while retaining their close relationship. Meanwhile, production is sharply reduced from t to $t + 1$ because the farmers have relatively little concern about smoothing production. This movement of production is correlated with that of *changes* in inventory stock, which means inventory *investment* also moves procyclically. Another sample set for the case of relatively weak sales-facilitating motive is shown in Figure 1.2. In this case, since farmers

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of inventory stock upwards from its steady-state level in response to a positive nominal disturbance. In the case of procyclical response of inventories, as I will mention later, inventory *investment* also moves procyclically, that is, its movement is positively correlated with that of production.

[13]This could occur in my model not only when $-\Phi_{CX} = \eta\,\beta\,(1 + \Phi_C)/Y$ but also when $\Phi_{XX} = \infty$, i.e., the cost of deviating inventory stock from its steady-state level is prohibitively high.

[14]In Figure 1.1, I assume the function Φ is constant returns to scale and $\eta = 1.5$, $\beta = 0.99$, $\theta = 10$, $C/Y = 0.99$, $Z \equiv X/C = 2/3$, and $\phi''(Z) = 5$ (which implies $-C\,\Phi_{CX}/\beta = X\,\Phi_{XX}/\beta = Z\,\phi''(Z)/\beta = 3.367$) are chosen for illustrative parameter values. In Figure 1.2, $\phi''(Z) = 1$ (which implies $Z\,\phi''(Z)/\beta = 0.6734$) is chosen with other parameter values unchanged.

strongly wish to avoid significantly changing levels of production, they meet the unexpected demand in period t partially by reducing their existing inventory stock, with relatively little concern about the relationship between sales and inventories. Then in period $t+1$, they raise prices aggressively, even excessively, to dampen sales so that the reduced inventory stock can gradually recover in parallel with production smoothly returning to its original level.[15]

Comparing these two cases, I find that the case of the strong sales-facilitating motive is more plausible: it is consistent with the stylized fact of inventories that inventory investment is procyclical or production is more volatile than sales, and also consistent with much empirical evidence in the literature that prices are adjusted gradually to nominal disturbances. In other words, the sales-facilitating motive rather than the production-smoothing motive is essential for generating price stickiness endogenously and obtaining plausible monetary policy effects in my simple dynamic general equilibrium model with inventories.

3   Unintended Inventories

The baseline model in the previous section assumes that farmers must determine their prices one period in advance whereas they can control their production without any decision lags. In this section I consider the case in which production as well as prices must be predetermined. If production as well as prices cannot be adjusted to unexpected demand within a period, inventories are forced to absorb the shock in an unintended manner, which implies that inventories move in the opposite direction to sales at least until production becomes adjustable. Business economists pay much attention to those countercyclical movements in "unintended inventories" when they try to detect a turning point in business cycles. Indeed, as Wen (2005) pointed out, inventory investment in the data moves countercyclically at very high frequencies while it moves procyclically at business-cycle frequencies. The estimated VAR evidences of the responses of inventory investment/stock to a monetary policy shock in Bernanke and Gertler (1995) and Jung and Yun (2005)

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[15]Equations (17), (18), and (19) imply $E_{t-1}[\hat{i}_t - (\hat{P}_{t+1} - \hat{P}_t)] = E_{t-1}[-\frac{C\,\Phi_{CX}}{\beta}\hat{C}_t - \frac{X\,\Phi_{XX}}{\beta}\hat{X}_t - \frac{C\,\Phi_{CC}}{1+\Phi_C}(\hat{C}_{t+1} - \hat{C}_t) - \frac{X\,\Phi_{CX}}{1+\Phi_C}(\hat{X}_{t+1} - \hat{X}_t)]$. The left-hand side is the marginal opportunity cost of increasing inventories (deviating inventory stock upwards from its steady-state level) and the right-hand side is the marginal benefit (marginal reduction in the sales cost) from holding inventories both directly through the optimal inventory-holding decision and indirectly through the optimal price-setting decision. Since $\hat{i}_t = 0$ for all t, this equation implies that farmers set higher prices for the next period than the current period when they hold inventory stock (or inventory-to-sales ratio) above its steady-state level as stated in footnote 6 in the introduction.

also imply those cyclical movements. I seek to find a condition under which a model assuming predetermined production can explain such a pattern of inventory-holding behavior.

3.1   Identical Lags in Production and Price Setting

First I consider the simplest case in which both production and prices are determined one period in advance, so that the decision lags are identical. In this case, I only have to modify the information set in the inventory-holding decision of the baseline model. The first-order condition (7) is replaced with

$$E_{t-1}[\, y_t(i)^\eta \, \{\, 1 + \Phi_x(c_t(i),\, x_t(i))\, \}\,] = \beta\, E_{t-1} y_{t+1}(i)^\eta.$$

The corresponding log-linearized equation

$$\eta\, E_{t-1}\hat{Y}_t = \eta\, E_{t-1}\hat{Y}_{t+1} - \frac{C\,\Phi_{CX}}{\beta}\, E_{t-1}\hat{C}_t - \frac{X\,\Phi_{XX}}{\beta}\, E_{t-1}\hat{X}_t \tag{30}$$

replaces (18).

Then I examine the effects of a nominal disturbance as before. The results are summarized as follows.

Proposition 2. *Suppose that the economy around the steady state is approximated by the model (17), (19), (20), (21), and (30), and the aggregate nominal spending evolves according to (22). All variables with hat are zero in period $t-1$. Both production and prices are determined one period in advance. Then the responses of inventories and consumption to a unit of positive innovation in ϵ_t are given by*

$$\begin{aligned}
E_t\hat{X}_{t+k} &= \lambda\, E_t\hat{X}_{t+k-1}\\
E_t\hat{C}_{t+k} &= \mu\, E_t\hat{X}_{t+k-1}
\end{aligned}$$

for $k \geq 1$, where λ and μ are given in Proposition 1. For $k = 0$,

$$\begin{aligned}
\hat{X}_t &= -\frac{C\,(1 + \Phi_C)}{\beta\, X}\\
\hat{C}_t &= 1.
\end{aligned}$$

The responses of production, prices, and nominal interest rate are given by the same equations as in Proposition 1.

Corollary 2.1. *Inventories respond countercyclically, i.e., $\hat{X}_t < 0$ and $E_t\hat{X}_{t+k} \leq 0$ for $k \geq 1$.*

Corollary 2.2. *Prices are expected to be adjusted excessively, i.e., $E_t\hat{P}_{t+1} > 1$ and $E_t\hat{P}_{t+k} \geq 1$ for $k \geq 2$.*

The proof of Proposition 2 is given in the Appendix.

The above corollaries imply that the sales-facilitating motive, however strong, does not work as in the baseline model. Farmers who care about the relationship between sales and inventories are expected to raise their prices aggressively in period $t+1$ in order to dampen sales because inventory stock has already declined in an unintended manner in period t. Without a sales-facilitating motive, on the other hand, farmers still wish to dampen sales immediately for their production-smoothing motive. As a result, the response of inventories is countercyclical and the adjustment of prices is excessive for any parameter values within the ranges I assumed.

3.2 Different Lags in Production and Price Setting

Since the case of identical decision lags in production and price setting turned out to be implausible, I next consider the cases of different decision lags. There are two possibilities: a longer decision lag is needed for production than for price setting, or the opposite. First I consider the former. Leaving the assumption of price setting in the baseline model still unchanged, I assume production must be determined *two periods* in advance. Then the only log-linearized equation I have to modify is again (30), which is replaced with

$$\eta \, E_{t-2}\hat{Y}_t = \eta \, E_{t-2}\hat{Y}_{t+1} - \frac{C\,\Phi_{CX}}{\beta}\, E_{t-2}\hat{C}_t - \frac{X\,\Phi_{XX}}{\beta}\, E_{t-2}\hat{X}_t \tag{31}$$

The effects of a nominal disturbance are summarized as follows.

Proposition 3. *Suppose that the economy around the steady state is approximated by the model (17), (19), (20), (21), and (31), and the aggregate nominal spending evolves according to (22). All variables with hat are zero in period $t-1$. Production is determined two periods in advance, while prices are determined one period in advance. Then the responses of inventories and consumption to a unit of positive innovation in ϵ_t are given by*

$$
\begin{aligned}
E_t\hat{X}_{t+k} &= \lambda \, E_t\hat{X}_{t+k-1} \\
E_t\hat{C}_{t+k} &= \mu \, E_t\hat{X}_{t+k-1}
\end{aligned}
$$

for $k \geq 2$, where λ and μ are given in Proposition 1. For $k = 1$,

$$
\begin{aligned}
E_t\hat{X}_{t+1} &= \frac{C\,(1+\Phi_C)\,(1+\Phi_C+C\,\Phi_{CC})}{\beta\,X\,\{\,C\,(1+\Phi_C)\,\Phi_{CX} - \beta\,(1+\Phi_C+C\,\Phi_{CC})\,\}} \;<\; 0 \\
E_t\hat{C}_{t+1} &= -\frac{C\,(1+\Phi_C)\,\Phi_{CX}}{\beta\,\{\,C\,(1+\Phi_C)\,\Phi_{CX} - \beta\,(1+\Phi_C+C\,\Phi_{CC})\,\}} \;<\; 0 .
\end{aligned}
$$

17

For $k = 0$,

$$\hat{X}_t = -\frac{C(1 + \Phi_C)}{\beta X}$$

$$\hat{C}_t = 1.$$

The responses of production, prices, and nominal interest rate are given by the same equations as in Proposition 1.

Corollary 3.1. *Inventories respond countercyclically, i.e., $\hat{X}_t < 0$, $E_t\hat{X}_{t+1} < 0$, and $E_t\hat{X}_{t+k} \leq 0$ for $k \geq 2$.*

Corollary 3.2. *Prices are expected to be adjusted excessively, i.e., $E_t\hat{P}_{t+1} > 1$ and $E_t\hat{P}_{t+k} \geq 1$ for $k \geq 2$.*

The proof of Proposition 3 is given in the Appendix.

As in the case of identical decision lags, farmers are expected to raise their prices aggressively in period $t+1$ regardless of their sales-facilitating motive. Again, the response of inventories is countercyclical and the adjustment of prices is excessive for any parameter values within the ranges I assumed.

Therefore, the only remaining possibility of procyclical inventories and gradual price adjustments explained by a model assuming predetermined production is in the case where a longer decision lag is needed for price setting than for production. I now assume that prices must be determined *two periods* in advance while production must be determined *one period* in advance. (19) in the baseline log-linearized model is replaced with

$$0 = E_{t-2}\left[\eta\hat{Y}_t + \left(1 + \frac{C\,\Phi_{CC}}{1 + \Phi_C}\right)\hat{C}_t + \frac{X\,\Phi_{CX}}{1 + \Phi_C}\hat{X}_t\right] \tag{32}$$

while (18) is replaced with (30) rather than (31). The effects of a nominal disturbance are summarized as follows.

Proposition 4. *Suppose that the economy around the steady state is approximated by the model (17), (20), (21), (30) and (32), and the aggregate nominal spending evolves according to (22). All variables with hat are zero in period $t - 1$. Prices are determined two periods in advance, while production is determined one period in advance. Then the responses of inventories and consumption to a unit of positive innovation in ϵ_t are given by*

$$E_t\hat{X}_{t+k} = \lambda\,E_t\hat{X}_{t+k-1}$$

$$E_t\hat{C}_{t+k} = \mu\,E_t\hat{X}_{t+k-1}$$

for $k \geq 2$, where λ and μ are given in Proposition 1. For $k = 1$,

$$E_t \hat{X}_{t+1} = \frac{C(1 + \Phi_C)\{-\Phi_{CX} - \eta(1 + \beta)(1 + \Phi_C)/Y\}}{(X \Phi_{XX} + \eta \beta^2 X/Y)(1 + \Phi_C) + \mu \beta(1 + \Phi_C + C \Phi_{CC}) + \lambda \beta X \Phi_{CX}}$$

$$E_t \hat{C}_{t+1} = 1.$$

For $k = 0$,

$$\hat{X}_t = -\frac{C(1 + \Phi_C)}{\beta X}$$

$$\hat{C}_t = 1.$$

The responses of production, prices, and nominal interest rate are given by the same equations as in Proposition 1.

Corollary 4.1. *i) Inventories respond countercyclically at first, i.e., $\hat{X}_t < 0$, and then are expected to move procyclically, i.e., $E_t \hat{X}_{t+1} > 0$ and $E_t \hat{X}_{t+k} \geq 0$ for $k \geq 2$, if $-\Phi_{CX} > \eta(1 + \beta)(1 + \Phi_C)/Y$. ii) Inventories respond countercyclically, i.e., $\hat{X}_t < 0$, $E_t \hat{X}_{t+1} < 0$, and $E_t \hat{X}_{t+k} \leq 0$ for $k \geq 2$, if $-\Phi_{CX} < \eta(1 + \beta)(1 + \Phi_C)/Y$. iii) Inventories respond countercyclically at first, i.e., $\hat{X}_t < 0$, and then are expected to return to their original steady-state level instantaneously, i.e., $E_t \hat{X}_{t+k} = 0$ for $k \geq 1$, if $-\Phi_{CX} = \eta(1 + \beta)(1 + \Phi_C)/Y$.*

Corollary 4.2. *i) Prices are expected to be adjusted gradually, i.e., $E_t \hat{P}_{t+1} < 1$ and $E_t \hat{P}_{t+k} \leq 1$ for $k \geq 2$, if $-\Phi_{CX} > \eta(1 + \beta)(1 + \Phi_C)/Y$. ii) Prices are expected to be adjusted excessively, i.e., $E_t \hat{P}_{t+1} > 1$ and $E_t \hat{P}_{t+k} \geq 1$ for $k \geq 2$, if $-\Phi_{CX} < \eta(1 + \beta)(1 + \Phi_C)/Y$. iii) Prices are expected to be adjusted instantaneously, i.e., $E_t \hat{P}_{t+k} = 1$ for $k \geq 1$, if $-\Phi_{CX} = \eta(1 + \beta)(1 + \Phi_C)/Y$.*

The proof of Proposition 4 is given in the Appendix.

In this case, prices still cannot be adjusted in period $t + 1$ and the boom in sales continues from t to $t + 1$. Farmers then try to recover the reduced inventory stock by adjusting production rather than prices, which implies that the growth of production in $t + 1$ is fully explained by the change in inventory investment. If the sales-facilitating motive is relatively strong compared with the production-smoothing motive, i.e., $-\Phi_{CX} > \eta(1 + \beta)(1 + \Phi_C)/Y$, as shown in Figure 2.1,[16] farmers will increase production so much that inventory stock will exceed the original level and rise to the level corresponding

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[16]In Figure 2.1, $\phi''(Z) = 10$ (which implies $-C \Phi_{CX}/\beta = X \Phi_{XX}/\beta = Z \phi''(Z)/\beta = 6.734$), and in Figure 2.2, $\phi''(Z) = 1$ (which implies $\phi''(Z)/\beta = 0.6734$) are chosen for illustrative parameter values with other parameters unchanged from Figure 1.1 and 1.2.

19

to the strong sales in period $t + 1$. After period $t + 2$, they will raise their prices gradually so that sales and inventories can gradually return to their original levels while retaining their close relationship. If the sales-facilitating motive is relatively weak, i.e., $-\Phi_{CX} < \eta (1 + \beta) (1 + \Phi_C)/Y$, as shown in Figure 2.2, farmers will not increase production so much, because of their relatively strong production-smoothing motive, and accordingly the inventory stock in period $t + 1$ will remain below its original level. Then in period $t + 2$, they need to dampen sales by raising prices aggressively for the inventory stock to recover gradually without a large adjustment of production.

Here I find that inventories respond procyclically and prices are adjusted gradually to a nominal disturbance in the case of a longer decision lag for price setting than for production if the sales-facilitating motive is relatively strong, although it needs to be stronger than that in the baseline model. Regardless of whether I consider predetermined production, I need to assume a longer decision lag for price setting than for production for obtaining the procyclical response of inventories and the gradual adjustment of prices. Moreover, the case considering predetermined production captures more precisely the observed pattern of inventory-holding behavior that inventory investment moves countercyclically at very high frequencies and procyclically at business-cycle frequencies.

Is it reasonable to assume that the decision lag for price setting is longer than that for production? One possibility is that there exist some factors that directly cause a substantial decision lag for price setting such as in the formation of long-term contracts or commitments. Another possibility is that the information processing, or "information stickiness" advocated by Mankiw and Reis (2002), for price setting is different from that for production. In the case considering predetermined production, unintended inventories in the period of disturbances have some information about shocks. When farmers look at their unintended inventories, they can use the information in some ways for their decision regarding production or price setting in the subsequent periods. This local information in unintended inventories might be more quickly processed for the decision of production than that of price setting which might require more global information and therefore cause a longer decision lag.

3.3   Heterogeneity in Price Setting

In the end of this section, I provide a more realistic example for the case of a longer decision lag for price setting than for production that includes heterogeneity in the length of the decision lag for price setting among farmers. I assume there are four groups of farmers in the economy. A quarter of farm-

ers have to set their prices one period in advance, another quarter of farmers have to set their prices two periods in advance, a third quarter three periods in advance, and the remaining quarter four periods in advance. Meanwhile, all farmers have to determine their production one period in advance. The equilibrium is no longer symmetric, which means that each group of farmers sets different prices, and accordingly sales, inventories, and production all vary between the groups. The model consists of many equations for each group's decisions, from which I do not seek to derive analytical results, as in the case of symmetric equilibrium.

A sample set of impulse responses of aggregate variables to a nominal disturbance is shown in Figure 3.1. I choose exactly the same parameter values as in Figure 1.1 where the sales-facilitating motive is relatively strong. The real effects persist even after all farmers start adjusting their prices, which implies that this model does not need to assume that some farmers set their prices on the basis of unrealistically old information, as the Mankiw and Reis (2002) sticky information model does in order to generate persistent real effects. The response of production is hump-shaped, which peaks immediately when it becomes adjustable in period $t+1$. The response of inflation peaks later than that of production, as in the sticky information model. On the whole, the results well reflect the main features of the monetary policy effects reported in, for example, Bernanke and Gertler (1995).

The responses of disaggregate variables are shown in Figure 3.2. The four dotted lines in each panel represent the responses of the four groups while the solid line represents the aggregate or average response. The overall price is adjusted gradually as the four groups of farmers start adjusting their prices one after the other from $t+1$ to $t+4$. Sales for goods produced by the farmers who start adjusting their prices earlier decline, while the farmers who cannot adjust their prices face strong demand due to their relatively low prices. Inventories of the farmers who start adjusting their prices earlier respond countercyclically while inventories of those who cannot adjust their prices respond procyclically after production becomes adjustable in $t+1$. Those patterns for disaggregate variables are totally different from those in the staggered price-setting model with inventories that are reported in Hornstein and Sarte (2001). In their model, the level of disaggregate inventory stock oscillates as price setters change their prices alternately, which seems unrealistic. As mentioned in the introduction, this is one advantage of my assumption of predetermined prices over their assumption of staggered price setting.

4   Simulations

The models so far omit various factors for analytical simplicity. In this section I develop a more realistic model that includes the labor market, capital accumulation, explicit money, and real disturbances, and use stochastic simulations to assess quantitatively the cyclical pattern and persistence of aggregate variables.

4.1   Model

Following Chari, Kehoe, and McGrattan (2000, hereafter CKM) and Boileau and Letendre (2004, hereafter BL), I consider a monetary economy in which a large number of identical and infinitely lived agents consume a homogenous consumption-capital good produced from a continuum of intermediate goods indexed by $i \in [0, 1]$. The representative consumer seeks to maximize a discounted sum of utilities of the form

$$E_0 \sum_{t=0}^{\infty} \frac{\beta^t}{1-\sigma} \left[\left(\omega\, C_t^{\frac{\chi-1}{\chi}} + (1-\omega)(M_t/P_t)^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}} (1-N_t)^{\psi} \right]^{1-\sigma}$$

subject to the flow budget constraint

$$P_t\,(C_t + I_t) + M_t + Q_{t,t+1}B_{t+1} = P_t\,(w_t\,N_t + r_t\,K_{t-1}) + M_{t-1} + B_t + T_t + \Pi_t$$

where the capital stock they hold evolves according to

$$I_t - \frac{\nu}{2} \left(\frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1} = K_t - (1-\delta)K_{t-1}.$$

M_t is nominal money balances, N_t is hours worked, w_t is the real wage rate, r_t is the rental rate of capital, T_t is nominal net transfers from the government, and Π_t is the aggregate of profits from the producing and retailing firms described below.

Following BL, I assume there are producing and retailing firms owned by consumers. The competitive retailing firms purchase all types of intermediate goods $s_t(i)$ from the producing firms $i \in [0, 1]$, aggregate them, and sell them to the consumers. They seek to maximize profits

$$P_t\,S_t - \int p_t(i)\,s_t(i)\,di$$

subject to the aggregation technology

$$S_t = \left[\int_0^1 s_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

22

where S_t is the aggregate sales to consumers for their consumption and capital investment. The first-order condition of their demand for intermediate good i is given by

$$s_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} S_t. \qquad (33)$$

The monopolistic producing firm i produces differential intermediate good i and sets the price of their own products. Products are storable. They seek to maximize a discounted sum of profits

$$E_0 \sum_{t=0}^{\infty} Q_{0,t} \left[p_t(i) \, s_t(i) - P_t \{ w_t \, n_t(i) + r_t \, k_{t-1}(i) \} \right]$$

subject to the production technology

$$y_t(i) = A_t \, n_t(i)^\alpha \, k_{t-1}(i)^{1-\alpha},$$

where A_t is the aggregate total factor productivity, the resource constraint for intermediate good i

$$y_t(i) - \Phi(s_t(i), x_t(i)) = s_t(i) + x_t(i) - x_{t-1}(i)$$

where $\Phi(\cdot)$ is the same function of sales cost as in the baseline model, and the demand constraint (33).

Clearing conditions for the final goods, labor, and capital markets are

$$
\begin{aligned}
S_t &= C_t + I_t \\
N_t &= \int n_t(i) \, di \\
K_t &= \int k_t(i) \, di.
\end{aligned}
$$

In order to introduce unintended inventories, I assume that the labor input is determined one period in advance by both the consumers and the producing firms taking the real wage rate as given. Also as in Section 3.3, I assume that a quarter of the producing firms set their prices one period in advance, another quarter two periods in advance, a third quarter three periods in advance, and the remaining quarter four periods in advance.

Lastly, the exogenous shock processes for the productivity and money growth are given by

$$
\begin{aligned}
\ln A_t &= (1 - \rho^A) \ln A + \rho^A \ln A_{t-1} + \epsilon_t^A \\
\ln(M_t/M_{t-1}) &= (1 - \rho^M) \ln(\Delta M) + \rho^M \ln(M_{t-1}/M_{t-2}) + \epsilon_t^M
\end{aligned}
$$

where A is the mean level of the productivity and $\ln(\Delta M)$ is the mean growth rate of money. ϵ_t^A and ϵ_t^M are white noise processes distributed independently of each other and normally with variances $(\sigma^A)^2$ and $(\sigma^M)^2$, respectively. The government is assumed to provide nominal transfers to the consumers in each period so that $T_t = M_t - M_{t-1}$.

4.2 Results

I log linearize the model around the steady state as I did the baseline model, set all parameter and steady-state values, and compute average sample moments such as standard deviations, autocorrelations, and cross correlations over 200 simulations of 200 quarters (50 years).

I choose most parameter values following CKM and BL based on postwar U.S. quarterly data:[17] $\chi = 0.39$, $\theta = 10$, $\beta = 0.99$, $\sigma = 1.5$, and $\alpha = 0.64$.[18] The steady-state share of hours worked is set to 0.3. The steady-state values of capital stock and investment are chosen so that the annualized capital–output ratio is 2.65 and the investment–output ratio is 0.23 in the steady state. ν is chosen so that the standard deviation of investment relative to that of output, which is computed from the Hodrick–Prescott filtered simulated data, is 2.9. For exogenous shock processes, $A = 1$, $\rho^A = 0.979$, $\sigma^A = 0.0072$, $\Delta M = 1$, $\rho^M = 0.69$, and $\sigma^M = 0.006$ are chosen following BL.[19]

I set $S/Y = 0.99$, which implies the steady-state share of inventory investment in GDP is one percent. The steady-state quarterly inventory-to-sales ratio is set to $Z \equiv X/S = 2/3$. Then, if the sales-cost function is constant returns to scale (CRS), ψ and δ are determined by the steady-state equilibrium conditions. With the CRS sales-cost function, I consider a case of a relatively strong sales-facilitating motive, $\phi''(Z) = 2.5$, which implies $S^* \equiv -S\,\Phi_{SX}/\beta = 1.6835$ and $X^* \equiv X\,\Phi_{XX}/\beta = 1.6835$, and a case of a relatively weak sales-facilitating motive, $\phi''(Z) = 1$, which implies $S^* = X^* = 0.6734$. I also consider the cases of non-CRS sales-cost function such that X^* is 100 times larger than S^* leaving S^* unchanged from the above CRS cases, which implies $(S^*, X^*) = (1.6835, 168.35)$ and $(S^*, X^*) = (0.6734, 67.34)$. These extremely high values of X^* or Φ_{XX} represent prohibitively high costs of deviating inventory stock from its steady-state level. To sum up, I consider four models that vary in the values for S^* and X^*: (1.6835, 1.6835), (0.6734, 0.6734), (1.6835, 168.35), and (0.6734, 67.34).

Results are summarized in Table 1. First, compared with the models of prohibitively high inventory-deviating costs (1.6835, 168.35) and (0.6734, 67.34), the models of reasonable costs (1.6835, 1.6835) and (0.6734, 0.6734) generate higher autocorrelations of production and inflation, which implies

---

[17]BL set several parameters to the values used in CKM. While CKM vary some values according to the amount of staggering in price setting and the length of exogenous price stickiness, BK fix those values so that consumers make their decisions quarterly.

[18]While CKM and BL set ω to 0.94, I assume ω is so close to 1 that the effects of real money balances on the marginal utility of consumption and labor supply are negligible.

[19]BL choose the parameter values on the process of productivity following King and Rebelo (1999) and the process of money growth by estimation using the postwar U.S. quarterly data on M2.

that the introduction of inventories allows us to reproduce persistent business-cycle fluctuations. Second, within the models of reasonable inventory-deviating costs, comparing the model of a strong sales-facilitating motive (1.6835, 1.6835) and that of a weak sales-facilitating motive (0.6734, 0.6734), we can see that inventory investment is procyclical, that is, the cross correlation between inventory investment and production is positive and the volatility of sales is smaller than that of production in the strong sales-facilitating model, while inventory investment is countercyclical in the weak sales-facilitating model, as predicted by my analytical results in the preceding sections. As for the first-order autocorrelations, the strong sales-facilitating model generates more persistent inflation due to gradual adjustments of prices in response to money-growth shocks, while the weak sales-facilitating model generates more persistent production due to the relatively strong production-smoothing motive. Meanwhile, within the models of prohibitively high inventory-deviating costs, there is little difference between the model (1.6835, 168.35) and the model (0.6734, 67.34), especially in the procyclicality of inventory investment.

The first row of Table 1 shows the corresponding sample moments in the postwar U.S. data reported in BL. The relative volatility of sales and inventory investment and the cross correlation between inventory investment and production in the data are close to the simulated data generated by the model (1.6835, 1.6835). Meanwhile, the autocorrelations of production and inflation in the data are far larger than those in the simulated data generated by any models, which implies that still longer-period predetermined prices need to be assumed.

Table 1 also shows the results from one of the alternative inventory models of BL. The cross correlation between inventory investment and production is barely positive (0.07), which is the largest value obtained from their models. However, they succeed in reproducing the high autocorrelations of production and inflation.

Although not all the results of my simulation quantitatively match the data, they qualitatively well reflect the stylized facts on inventories[20] and support my analytical results in the preceding sections.

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[20]The simulation results also imply that the relative variability of inventory investment is large (0.11 in the model (1.6835, 1.6835)) compared with the share of inventory investment in GDP (0.01), the cross correlation between inventory investment and sales is positive (0.19), and the cross correlation between inventory-to-sales ratio and production is negative (-0.99), which are consistent with the stylized facts pointed out by Khan and Thomas (2004).

5 Concluding Remarks

In this paper I have studied the role of inventories in the propagation of monetary shocks by developing simple dynamic general equilibrium models. Introducing inventories allows us to generate real effects of monetary policy lasting several periods even in the baseline model that assumes just one-period predetermined prices, which implies that inventories serve as a source of real rigidities. I introduce a sales-facilitating motive as well as a production-smoothing motive for holding inventories. In the baseline model in Section 2, I obtain analytical results that inventories respond procyclically and prices are adjusted gradually to a nominal disturbance only if the sales-facilitating motive is relatively strong; otherwise inventories respond countercyclically and prices are adjusted excessively. In the extended models in Section 3, which assume production as well as prices are predetermined, inventories respond countercyclically at first and then move procyclically only if the sales-facilitating motive is relatively strong and the decision lag of price setting is longer than that of production. In the further extended model in Section 4 that introduces the labor market, capital accumulation, explicit money, and real disturbances, I obtain quantitative results through simulations that support my analytical results in the preceding sections and are consistent with stylized facts on inventories. The above results imply that the sales-facilitating motive rather than the production-smoothing motive is essential for generating price stickiness endogenously and obtaining plausible monetary policy effects in my models.

Based on the models developed in this paper, two directions for future research can be pursued. One is policy research. We may have to consider a monetary policy rule that stabilizes both production and sales in addition to prices in the models with inventories. Another direction is empirical research. Inventory data have been used for identifying various types of shocks in the economy, and could also provide richer information on the monetary transmission mechanism. I hope the models in this paper serve as a useful building block for future research in those directions.

Appendix

Proof of Proposition 1

(19) in the log-linearized model implies

$$E_t \hat{Y}_{t+k} = -\frac{X\,\Phi_{CX}}{\eta\,(1+\Phi_C)}\,E_t \hat{X}_{t+k} - \frac{(1+\Phi_C+C\,\Phi_{CC})}{\eta\,(1+\Phi_C)}\,E_t \hat{C}_{t+k} \tag{34}$$

for $k \geq 1$. Substituting this into (18) and (20), I have

$$
\begin{aligned}
E_t \hat{X}_{t+k} &= -\frac{\beta\,\Phi_{CX}}{(1+\Phi_C)\,\Phi_{XX}-\beta\,\Phi_{CX}}\,E_t\hat{X}_{t+k+1} \\
&\quad +\frac{\beta\,C\,\Phi_{CC}+(\beta-C\,\Phi_{CX})\,(1+\Phi_C)}{(1+\Phi_C)\,\Phi_{XX}-\beta\,\Phi_{CX}}\,E_t\hat{C}_{t+k} \\
&\quad -\frac{\beta\,C\,\Phi_{CC}+\beta\,(1+\Phi_C)}{(1+\Phi_C)\,\Phi_{XX}-\beta\,\Phi_{CX}}\,E_t\hat{C}_{t+k+1} \\
E_t\hat{C}_{t+k} &= -\frac{\beta\,\eta\,X\,(1+\Phi_C)+X\,Y\,\Phi_{CX}}{Y\,(1+\Phi_C+C\,\Phi_{CC})+\eta\,C\,(1+\Phi_C)^2}\,E_t\hat{X}_{t+k} \\
&\quad +\frac{\eta\,X\,(1+\Phi_C)}{Y\,(1+\Phi_C+C\,\Phi_{CC})+\eta\,C\,(1+\Phi_C)^2}\,E_t\hat{X}_{t+k-1} \qquad (35)
\end{aligned}
$$

for $k \geq 1$. Combining these, I obtain the following second-order difference equation for $\tilde{X}_k \equiv E_t\hat{X}_{t+k}$.

$$
A(L)\tilde{X}_{k+1} = 0,
$$

where

$$
\begin{aligned}
A(L) &\equiv \alpha\,L^2 - \left\{(1+\beta)\,\alpha - \frac{\Phi^*}{\Phi_{CX}}\right\}L + \beta\,\alpha \\
\alpha &\equiv \left\{\frac{C\,(1+\Phi_C)}{\beta} - \frac{1+\Phi_C+C\,\Phi_{CC}}{\Phi_{CX}}\right\}\eta\,(1+\Phi_C) \; > 0\,,
\end{aligned}
$$

and L is the lag operator defined as $L\tilde{X}_k \equiv \tilde{X}_{k-1}$. $A(L)$ can be factored as

$$
A(L) = (1 - \lambda_1\,L)(1 - \lambda_2\,L)
$$

where λ_1 and λ_2 are the two roots of the characteristic polynomial

$$
\mathcal{A}(\lambda) \equiv \beta\,\alpha\,\lambda^2 - \left\{(1+\beta)\,\alpha - \frac{\Phi^*}{\Phi_{CX}}\right\}\lambda + \alpha = 0.
$$

On the assumption (29), I have two real roots satisfying $0 < \lambda_1 < 1 < \lambda_2$ because $\mathcal{A}(0) > 0$, $\mathcal{A}(1) < 0$, and $\mathcal{A}(\lambda) > 0$ for large enough λ. To be consistent with the terminal condition $\lim_{k\to\infty}\tilde{X}_k = 0$, the smaller root must be taken in the solution form

$$
\tilde{X}_k = \lambda\,\tilde{X}_{k-1}, \qquad (36)
$$

so that $\lambda \equiv \lambda_1$. Using this, I can rewrite (35) as

$$
E_t\hat{C}_{t+k} = \mu\,E_t\hat{X}_{t+k-1} \qquad (37)
$$

27

where

$$\mu \equiv \frac{(1 - \beta\,\lambda)\,\eta\,X\,(1 + \Phi_C) - \lambda\,X\,Y\,\Phi_{CX}}{Y\,(1 + \Phi_C + C\,\Phi_{CC}) + \eta\,C\,(1 + \Phi_C)^2}.$$

For $k = 0$, (20) with the initial conditions $\hat{C}_t = 1$ and $\hat{X}_{t-1} = 0$ implies

$$\hat{Y}_t = \frac{C}{Y}\,(1 + \Phi_C) + \frac{\beta\,X}{Y}\,\hat{X}_t.$$

Substituting this and (34) for $k = 1$ into (18), I have

$$
\begin{aligned}
\hat{X}_t &= -\frac{\beta\,X\,\Phi_{CX}}{(X\,\Phi_{XX} + \eta\,\beta^2\,X/Y)\,(1 + \Phi_C)}\,E_t\hat{X}_{t+1} \\
&\quad -\frac{\beta\,(1 + \Phi_C + C\,\Phi_{CC})}{(X\,\Phi_{XX} + \eta\,\beta^2\,X/Y)\,(1 + \Phi_C)}\,E_t\hat{C}_{t+1} \\
&\quad +\frac{C\,\{-\Phi_{CX} - \eta\,\beta\,(1 + \Phi_C)/Y\}}{(X\,\Phi_{XX} + \eta\,\beta^2\,X/Y)}.
\end{aligned}
$$

Substituting (36) and (37) into this, I obtain

$$\hat{X}_t = \frac{C\,(1 + \Phi_C)\,\{-\Phi_{CX} - \eta\,\beta\,(1 + \Phi_C)/Y\}}{(X\,\Phi_{XX} + \eta\,\beta^2\,X/Y)\,(1 + \Phi_C) + \mu\,\beta\,(1 + \Phi_C + C\,\Phi_{CC}) + \lambda\,\beta\,X\,\Phi_{CX}}. \tag{38}$$

The full sequence of $E_t\hat{Y}_{t+k}$ for $k \geq 0$ can be obtained by substituting (36), (37), (38) and $\hat{C}_t = 1$ into (20).

$$E_t\hat{Y}_{t+k} = \frac{C}{Y}\,(1 + \Phi_C)\,E_t\hat{C}_{t+k} + \frac{X}{Y}\,(\beta\,E_t\hat{X}_{t+k} - E_t\hat{X}_{t+k-1})$$

Finally, since I consider a disturbance that causes $E_t\hat{\mathcal{M}}_{t+k} = 1$ for all $k \geq 0$, (17) and (21) implies

$$
\begin{aligned}
E_t\hat{P}_{t+k} &= 1 - E_t\hat{C}_{t+k} \tag{39} \\
E_t\hat{\imath}_{t+k} &= 0
\end{aligned}
$$

for all $k \geq 0$.

Proof of Proposition 2

The solutions for $k \geq 1$ are the same as in Proposition 1.

For $k = 0$, since production as well as prices are predetermined, $\hat{Y}_t = 0$, $\hat{P}_t = 0$, and $\hat{C}_t = 1$. Substituting these initial conditions into (20), I obtain

$$\hat{X}_t = -\frac{C\,(1 + \Phi_C)}{\beta\,X}.$$

Proof of Proposition 3

The solutions for $k \geq 2$ and $k = 0$ are the same as in Proposition 2.

For $k = 1$, production still cannot be adjusted, i.e., $E_t \hat{Y}_{t+1} = 0$, while prices become adjustable in $t + 1$. Then (19) implies

$$0 = -X \, \Phi_{CX} \, E_t \hat{X}_{t+1} - (1 + \Phi_C + C \, \Phi_{CC}) \, E_t \hat{C}_{t+1} \tag{40}$$

Meanwhile, substituting the solution for $\hat{X}_t$ into (20), I have

$$0 = C \, (1 + \Phi_C) \, E_t \hat{C}_{t+1} + \beta \, X \, E_t \hat{X}_{t+1} + C \, (1 + \Phi_C)/\beta. \tag{41}$$

Combining (40) and (41), I obtain

$$
\begin{aligned}
E_t \hat{X}_{t+1} &= \frac{C \, (1 + \Phi_C) \, (1 + \Phi_C + C \, \Phi_{CC})}{\beta \, X \, \{ C \, (1 + \Phi_C) \, \Phi_{CX} - \beta \, (1 + \Phi_C + C \, \Phi_{CC}) \}} \\
E_t \hat{C}_{t+1} &= -\frac{C \, (1 + \Phi_C) \, \Phi_{CX}}{\beta \, \{ C \, (1 + \Phi_C) \, \Phi_{CX} - \beta \, (1 + \Phi_C + C \, \Phi_{CC}) \}}.
\end{aligned}
$$

Proof of Proposition 4

The solutions for $k \geq 2$ and $k = 0$ are the same as in Proposition 2.

For $k = 1$, prices still cannot be adjusted, therefore, $E_t \hat{C}_{t+1} = 1$, while prices become adjustable in $t+1$. Substituting this condition and the solution for $\hat{X}_t$ into (20), I have

$$E_t \hat{Y}_{t+1} = \frac{1 + \beta}{\beta} \frac{C}{Y} (1 + \Phi_C) + \frac{\beta \, X}{Y} E_t \hat{X}_{t+1}.$$

Substituting this and (34) for $k = 2$ into (30), I have

$$
\begin{aligned}
E_t \hat{X}_{t+1} &= -\frac{\beta \, X \, \Phi_{CX}}{(X \, \Phi_{XX} + \eta \, \beta^2 \, X/Y) \, (1 + \Phi_C)} E_t \hat{X}_{t+2} \\
&\quad -\frac{\beta \, (1 + \Phi_C + C \, \Phi_{CC})}{(X \, \Phi_{XX} + \eta \, \beta^2 \, X/Y) \, (1 + \Phi_C)} E_t \hat{C}_{t+2} \\
&\quad +\frac{C \, \{-\Phi_{CX} - \eta \, (1 + \beta) \, (1 + \Phi_C)/Y \}}{(X \, \Phi_{XX} + \eta \, \beta^2 \, X/Y)}.
\end{aligned}
$$

Substituting (36) and (37) into this, I obtain

$$E_t \hat{X}_{t+1} = \frac{C \, (1 + \Phi_C) \, \{-\Phi_{CX} - \eta \, (1 + \beta) \, (1 + \Phi_C)/Y \}}{(X \, \Phi_{XX} + \eta \, \beta^2 \, X/Y) \, (1 + \Phi_C) + \mu \, \beta \, (1 + \Phi_C + C \, \Phi_{CC}) + \lambda \, \beta \, X \, \Phi_{CX}}.$$

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Figure 1: Baseline Model
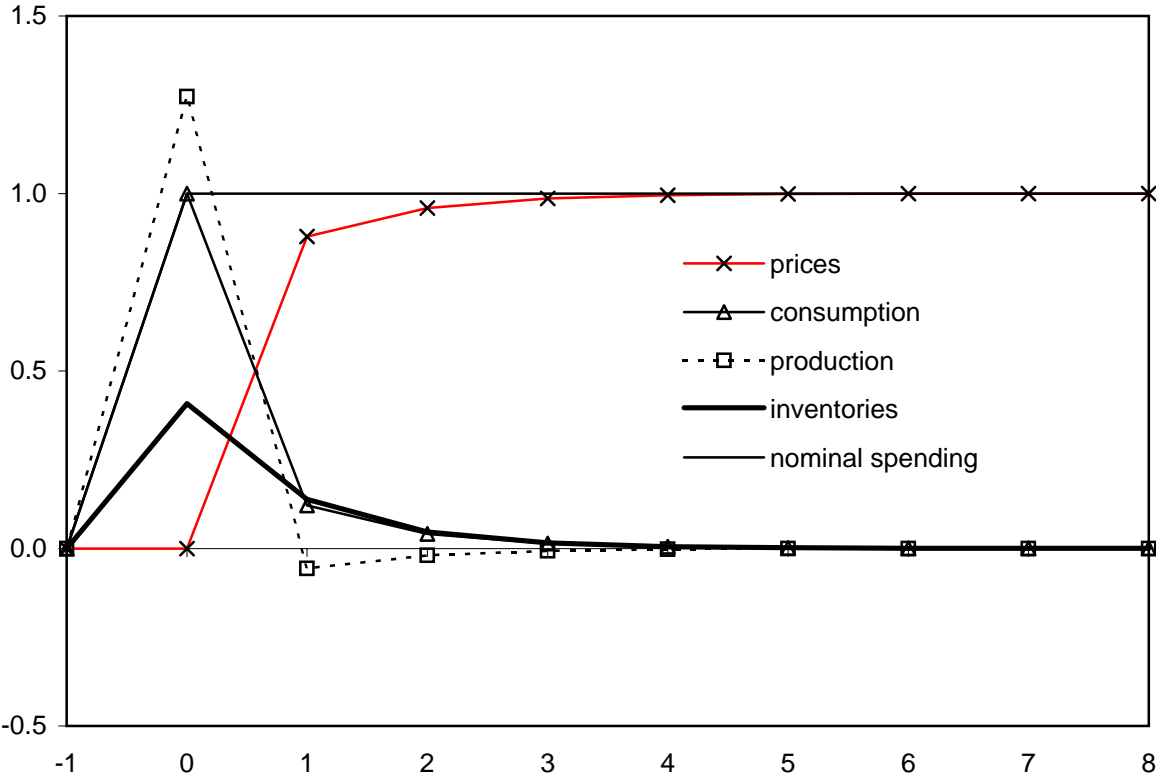
Figure1.1: Strong sales-facilitating motive

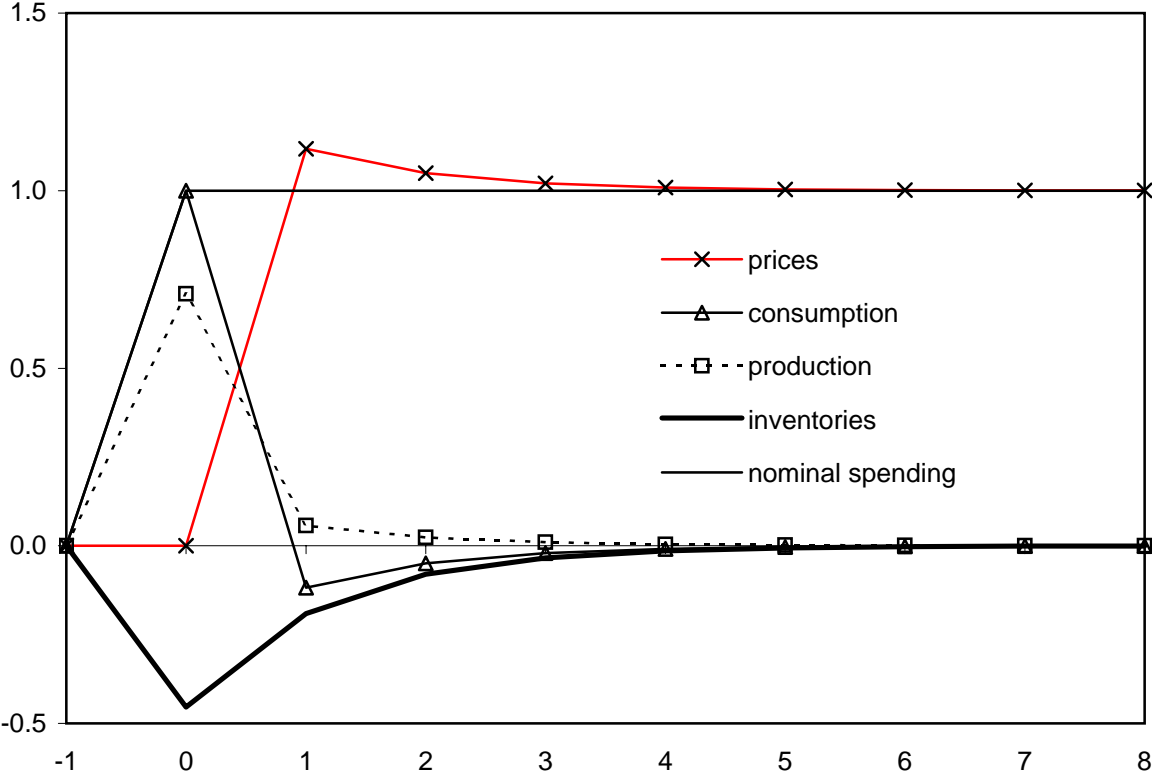### Figure1.2: Weak sales-facilitating motive

Figure 2: Unintended Inventories

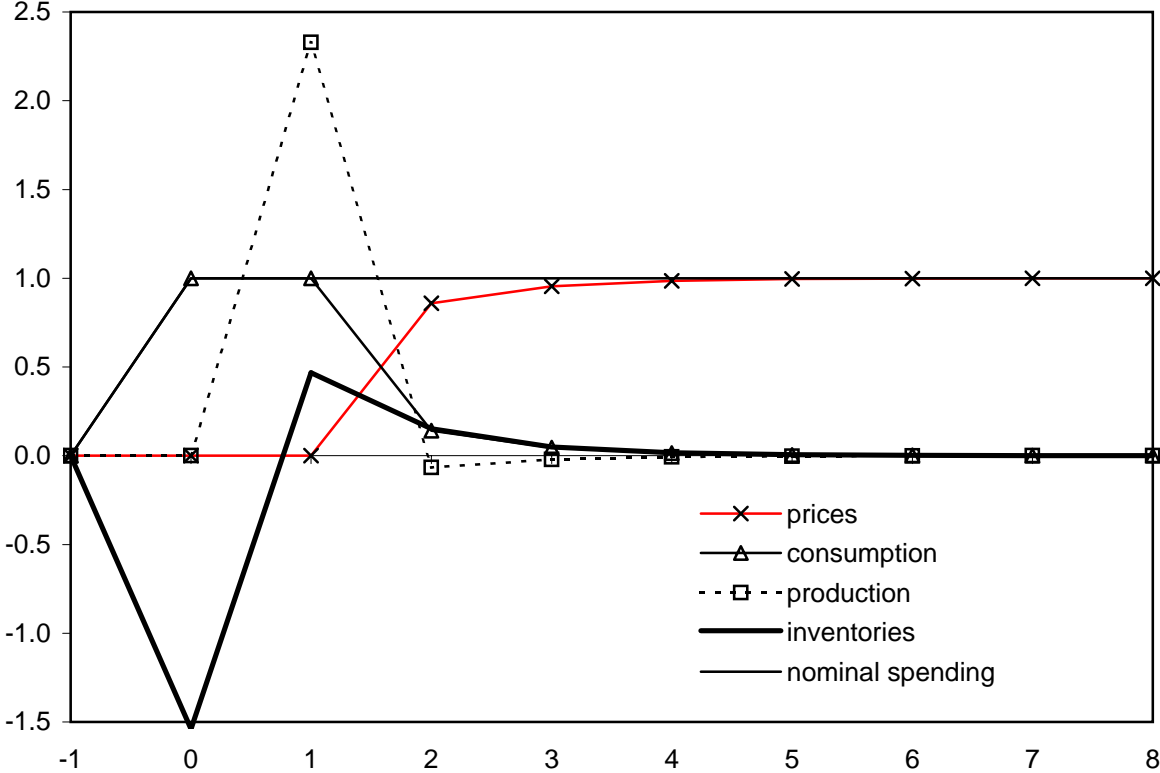## Figure2.1: Strong sales-facilitating motive
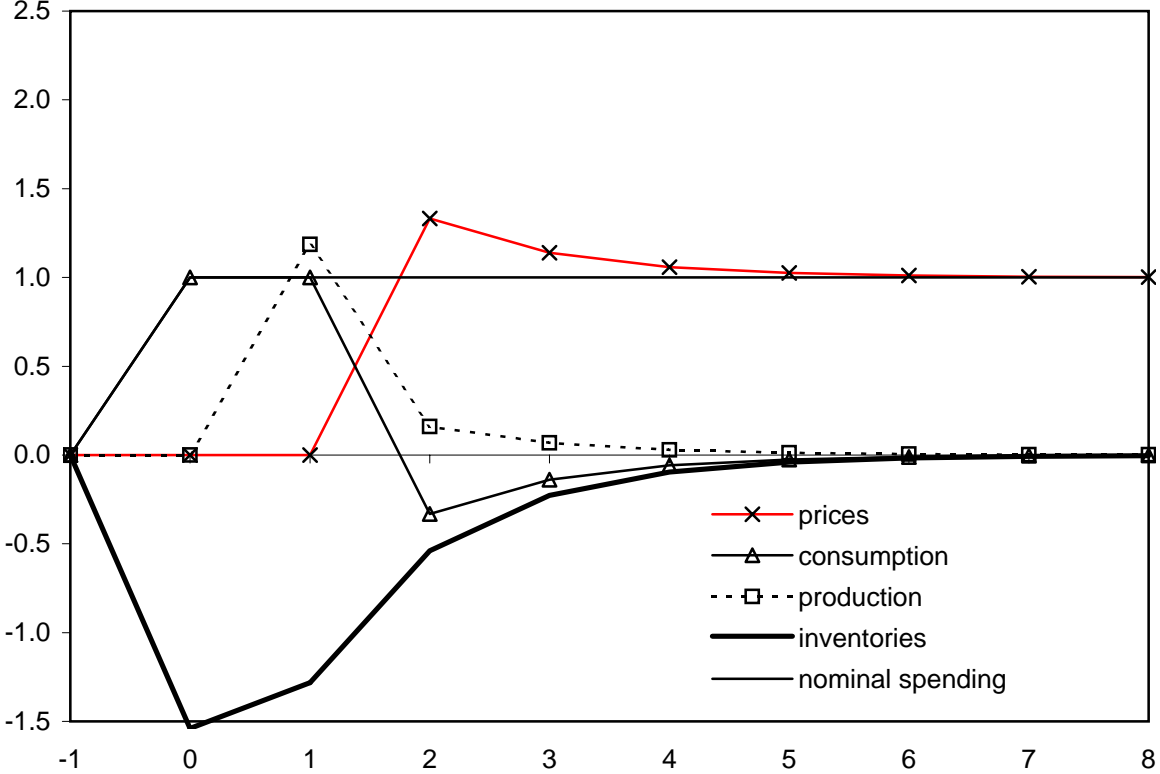

Figure2.2: Weak sales-facilitating motive

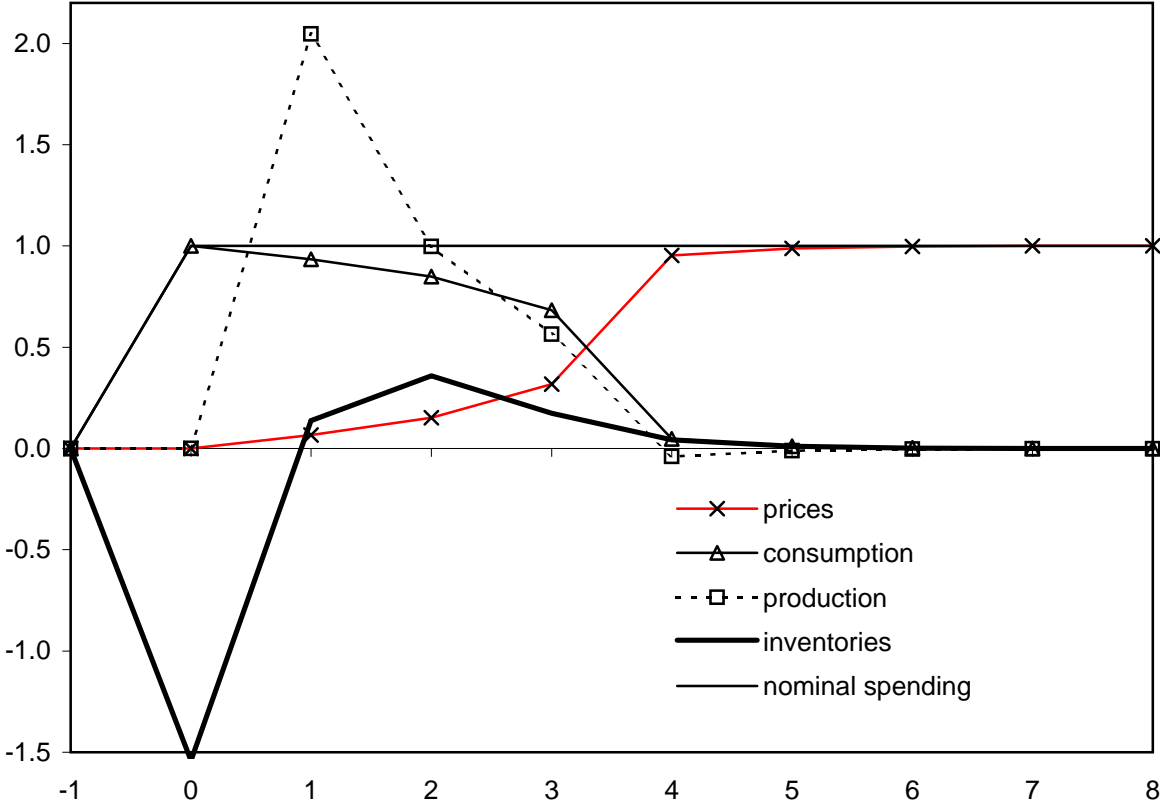# Figure 3: Heterogeneity in Price-Setting

Figure3.1: Aggregate variables

Figure3.2: Disaggregate variables
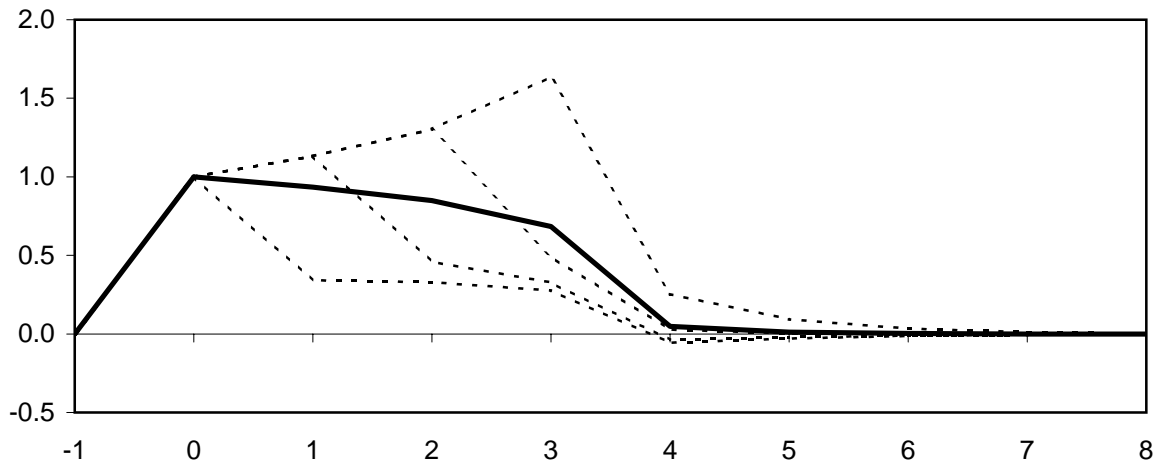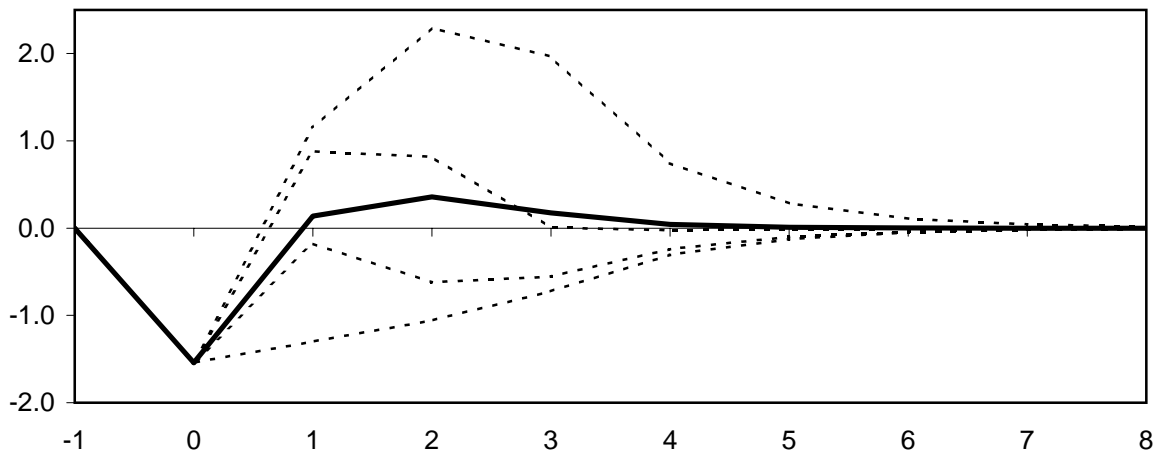
Prices
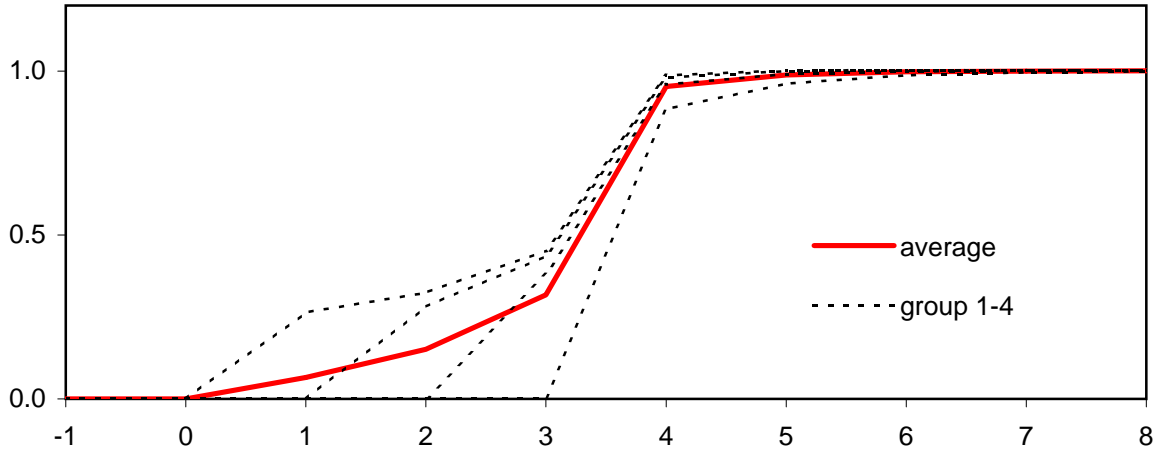
Sales

Inventories

Table 1: Simulations

	Relative Volatility		Cross Correlation	First-Order Autocorrelation	
	S to Y	dX to Y	dX and Y	Y	dP
U.S. data	0.94	0.13	0.50	0.97	0.78
Boileau=Letendre	1.37	1.01	0.07	0.92	0.71
Model					
(S*, X*)					
(1.6835, 1.6835)	0.98	0.11	0.27	0.23	0.20
(0.6734, 0.6734)	1.12	0.65	-0.10	0.46	0.05
(1.6835, 168.35)	0.96	0.07	0.72	0.02	-0.44
(0.6734, 67.34)	0.96	0.08	0.61	0.11	-0.24

Notes:
1. Y is production, S is sales, dX is inventory investment, and dP is inflation.
2. U.S. data are those reported in Boileau and Letendre (2004).
   They calculate the sample moments of quarterly data over 1959:1 to 2000:1
   after removing linear-quadratic trends.
3. The results of simulations by Boileau and Letendre (2004) are those from
   their benchmark shopping-cost model.