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Abstract

This paper provides a theoretical overview of monetary and fiscal policy with the potential to engineer an exit from a deflationary trap, which we define here as sustained deflation in the presence of zero interest rates. We find that the required policy steps are an interest rate hike, a commitment to future currency growth, and a money-financed tax cut. The amount of tax cut required is equal to the increase in the central bank's payments to the treasury resulting from the higher inflation rate (nominal interest rate), while fiscal policymakers must maintain fiscal discipline by stabilizing government debt and the primary balance. There will be a temporary fall in output when prices are sticky, but this is the price that must be paid to conquer deflation. The current commitment to quantitative easing is based on the assumption that the natural interest rate has temporarily declined. If the economy is in a deflationary trap, however, the continuation of zero interest rates reinforces deflationary expectations and may make it perpetually impossible to eliminate deflation. Even under conditions in which the natural rate of interest looks to be positive, if deflation persists, it is probably wise to consider a policy approach that assumes deflationary trap conditions. With this in mind, we believe the conditions required for abandoning the current policy regime should include, in addition to consistently positive growth in the CPI, a consideration of the trend in real GDP.

Key words: Monetary policy, zero bound on interest rates, liquidity trap, deflationary trap, non-Ricardian fiscal policy regime.

JEL classification: E41, E52

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I. Introduction

In February 1999, the Bank of Japan implemented a zero interest rate policy (ZIRP), aimed at supplying the market with ample liquidity while keeping the uncollateralized overnight call rate as low as possible. In August 2000, the BOJ announced the end of ZIRP, but then followed up in March 2001 with its quantitative easing policy, thereby effectively restoring zero interest rates, and this policy remains in effect today. Monetary policy confined to the traditional tools of manipulating interest rates can maximize monetary easing, but Japan's economic recoveries have regularly disappointed, and in recent years, since 1998, the GDP deflator has consistently recorded price declines in year-on-year terms (Figure 1 shows the level of interest rates and inflation since 1990).

Keynes (1936) argued that zero interest rates have the negative aspect of producing a liquidity trap whereby a further increase in the money supply has no expansionary effects, but Bailey (1956) and Friedman (1969) point out a positive aspect, noting that zero interest rates accommodate the optimal supply of money. Previously, the existence of a zero bound on nominal interest rates was often ignored, but since zero interest rates became a fixture in Japan beginning in the 1990s, a considerable body of research has been produced that takes this zero bound into account.

Within the literature dealing with zero interest rates, Auerbach and Obstfeld (2004), Eggertsson (2003), Eggertsson and Woodford (2003), Jung, Teranishi and Watanabe (2004), Krugman (1998), and Woodford (1999) argue that even when the natural rate of interest (the level of interest rates that occurs when prices are flexible) is declining and the nominal interest rate corresponding to the optimal inflation target is negative, the real interest rate remains high since nominal interest rates cannot actually become negative.[1] At the same time, however, they assume that the natural rate of interest would rise in the future, thereby eliminating zero interest rates. In this case, an increase in only the current money supply through open market operations is nothing more than an exchange of government bonds with money, which are perfect substitutes when interest rates are zero, and thus has no impact on the real economy. However, by increasing the money supply at a future date when interest rates are no longer zero, future prices rise, and the effects of this future rise in prices extend to the present. Using a two-period model, Krugman (1998) showed that an increase in future prices raises the current inflation rate, and thus has an expansionary effect on current income via the resulting decline in real interest rates.

Eggertsson and Woodford (2003), Jung, Teranishi and Watanabe (2004) and Woodford (1999) note that, when interest rates are the monetary policymakers' control variable, a commitment to maintain monetary easing even after the natural interest rate recovers to a positive value can make monetary easing effective under zero interest rates by lowering long-term interest rates. In Japan, the pursuit of additional easing effects under zero interest rates by committing to monetary easing in the future has been referred to as the "policy commitment effect," or the "policy duration effect" Essentially, when the BOJ implemented ZIRP in April 1999 it committed to holding interest rates at zero until deflationary concerns were dispelled, while its implementation of quantitative easing came with a commitment to stick with the policy until the CPI (excluding fresh foods) stabilizes at either a zero or positive rate of growth in year-on-year terms. Both Shiratsuka and Fujiki (2001) and Okina and Shiratsuka (2003) examined the effects of this policy commitment and found that the expression of commitment led to a reduction in long-term interest rates, but that monetary policy transmission channels failed to function and thus the effects did not reach the real economy. Whether more proactive monetary policy measures are required has been a major point of contention in recent years.[2]

Nevertheless, with interest rates persisting near zero, it is no easy matter to ascertain when in the future that interest rates will diverge from zero. This is owing to the variety of economic factors that influence the natural interest rate and the extreme difficulty of ascertaining, accurately and in a timely manner, what that rate is. Oda and Muranaga (2003) pointed out the possibility that there may have been periods since 1997 when the natural rate of interest was negative.[3] On the other hand, it is also possible that the natural interest rate did not temporarily decline. For example, Nishimura and Saito (2004) argue that Japan's natural interest rate is not negative.

If Japan's natural rate of interest were currently positive and remained positive moving forward, it would substantially change the debate over monetary policy. First, if nominal interest rates were held to zero under a positive natural interest rate, it would lead to deflation in the long run. Assuming that the real interest rate converges to the natural interest rate over the long term, this can be confirmed from the relationship between interest rates and

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[1] The natural interest rate, positioned by Woodford (2003) as a core concept for monetary policy management, has generated substantial interest recently.
[2] See Svensson (2003) for a range of policy measure proposals suitable under the zero bound on interest rates.
[3] It is important to note here that the policy commitment effect works in forward-looking models where economic agents project the future, but Oda and Muranaga (2003) base their empirical research on a backward-looking model.

the inflation rate shown by the Fisher equation.[4] Second, it is possible to interpret a future increase in the natural interest rate as an economic recovery driven by an increased demand for goods. This would mean that without an increase in the natural interest rate there would be no prospects for a higher level of economic activity stemming from a self-sustained increase in demand in the future. This would be, in other words, an approach toward monetary policy grounded in pessimistic assumptions about the future.

There are essential differences in the framework for analyzing monetary policy between conditions under which zero interest rates were caused by a large, temporary drop in the natural interest rate and conditions characterized by a sustained period of zero interest rates and deflation while natural interest rates are at their normal level. We will distinguish between the two in this paper by referring to the former as a liquidity trap and the latter as a deflationary trap.

One possible reason that the economy falls into a deflationary trap despite the central bank not pursuing a deflation target is that the central bank errs in setting interest rates as a result of the difficulty in quickly recognizing when shocks affect the natural interest rate. Meanwhile, recent research has found that it is possible for the economy to fall into a deflationary trap even if the central bank pursues monetary policies that are consistent with an inflation target. Although Kerr and King (1996) and Leeper (1991) showed that a rational expectations equilibrium could be achieved if monetary policy adheres to the Taylor rule, Benhabib, Schmitt-Grohè and Uribe (2001) noted that this argument ignores the zero bound on nominal interest rates, and showed that when taking account of the zero bound, a deflationary trap was globally stable and that there were an infinite number of paths toward that state. Additionally, Evans and Honkapohja (2003) showed that when economic agents base their decisions on adaptive learning rather than rational expectations, although learning makes it impossible for the deflationary trap itself to occur, there is a possibility that a path leading to an inflation rate that is even lower than under a deflationary trap will be chosen.[5] On the other hand, Bullard and Cho (2002) found that under adaptive learning, the presence of escape dynamics leads to a major decline in both nominal interest rates and inflation, and imply that this could provide an explanation of conditions in Japan.

An explanation based on monetary quantity would be expected to posit that monetary growth would cause inflation, but if zero interest rates become permanent, it would rule out

[4] In the standard use of the Fisher equation, the Fisher effect is posited as a one-for-one increase in nominal interest rates in response to an increase in the inflation rate. Here, we use the Fisher equation to explain the inverse of that relationship.

that future point in time posited by Krugman (1998) when the relationship between the money supply and prices is formed.[6] In Japan today, the monetary base has been growing but prices have been declining, a situation that is impossible to explain based on the quantity theory of money alone. The question of what policies should be pursued to spark inflation under these conditions has been taken up recently in such papers as Benhabib, Schmitt-Grohè and Uribe (2002a), and Eggertsson and Woodford (2003).

Taking cues from progress in this area of research and considering Japan's current situation, this paper addresses the question of what policies should be pursued to get the Japanese economy out of its deflationary trap, assuming it has fallen into such a trap.

This paper will take a two-track approach, searching for conclusions from both the long-run and the short-run perspectives. In section II, we take the long-run view, assuming flexible prices and using a standard monetary growth model to examine the role of monetary policy. To reflect the zero interest rate environment, we use a money-in-utility model, whereby the utility provided by money is the motivation for holding it.[7] When real money balances become sufficiently large and the utility from holding money is saturated, zero interest rates result. We then show that deflation is the long-run consequence of zero interest rates and that, under conditions in which nominal government debt is shrinking, the quantity theory of money does not apply and that deflation remains in place even as the quantity of money increases. The fiscal policy stance plays a critical role here and must be non-Ricardian for inflation to increase in step with monetary growth. We examine exactly what form such a non-Ricardian fiscal policy winds up taking.

If prices are sticky, the possibility that deflation is caused by economic recession must also be considered. In section III, we look at the problem from the short-run perspective. Given that short-run economic fluctuations are the subject of endless disputes in the field of macroeconomics, we have decided against using a specific model and opted instead to

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[5] For a comprehensive explanation of adaptive learning, see Evans and Honkapojha (2001).

[6] The "foolproof way" described by Svensson (2001), in which a weakening of the yen in currency markets is used to spark inflation, also relies on other countries to form the relationship between money and prices. As recognized by Svensson (2003), this approach would not work if interest rates were at zero outside of Japan. Of course, as long as not all countries have zero interest rates, it is theoretically possible to use the exchange rate approach, but given that it is conceivable that other countries would respond with their own monetary easing, if interest rates are initially too low (as they currently are in the US, for example), there is a possibility that both countries would move to zero interest rates and that the foolproof way would fail to work.

[7] Although models where the motivation to hold money is given by the cash-in-advance constraint are common in the literature, for the points we are making here it is possible to make roughly the same arguments as in a money-in-utility model. When citing other papers, we will not make any extra effort to highlight the differences between the cash-in-advance and money-in-utility frameworks.

examine our questions using a variety of models simultaneously. We use 20 dynamic models to examine potential monetary policies for escaping from zero interest rates.

When prices are flexible, an increase in the inflation rate correlates with an increase in interest rates. Under a monetary policy using interest rates as the control variable, interest rates would be increased to eliminate deflation. If prices are slow to adjust, however, when interest rates are the only control variable the increase in the inflation rate can lag the hike in interest rates. In this case, a decline in output caused by the increase in real interest rates can occur. We look at the conditions under which this can occur, and examine its policy implications.

In section IV, we conclude with a summary of our arguments concerning the best policy scenario to escape deflation.

II. Zero interest rates over the long run

A. Budget constraint for consolidated government

When examining the subject of zero interest rates, it is important to look at the interplay, via the budget constraint, between the fiscal authorities (government narrowly defined) and the central bank. This requires taking a closer look at the budget constraint that integrates government narrowly defined (hereinafter just "government") with the central bank, which we call here consolidated government. To do this, we must start by deriving the consolidated government's budget constraint.

We assume no government spending. The government issues new government bonds only by the amount by which interest payments on existing government bonds exceed the total of tax revenues and central bank payments to the treasury. Public debt is short-term nominal debt, and the balance of nominal debt is given as A. The nominal interest rate is *i*, treasury payments from the central bank are *X*, and tax revenues are *T* (all nominal values). Assuming time is continuous, the government's budget equation can be written as follows.

$$\dot{A}_t = i_t A_t - X_t - T_t \qquad (1)$$

The central bank supplies the monetary base through open market operations, which it conducts on a regular basis to ensure the monetary base *M*, a liability on its balance sheet, matches the public debt, which is a central bank asset. We ignore the government's equity in the central bank, and assume the central bank's profits are not retained but immediately transferred to the government via payments to the treasury. Using these assumptions, and representing the government bonds held by the private sector as *B*, the following equalities hold.

$$A_t = B_t + M_t \tag{2}$$

$$X_t = i_t M_t \tag{3}$$

Open market operations cause no change in *A* and are immediately reflected as changes in *B* and *M*. When the level of market operations is small, however, smooth changes in *B* and *M* are possible, and we consider a budget constraint for the consolidated government under such conditions. Based on this, we get

$$\dot{A}_t = \dot{B}_t + \dot{M}_t. \tag{4}$$

Substituting equations (2), (3) and (4) into equation (1), we can express the change in government bonds held by the private sector as

$$\dot{B}_t = i_t B_t - \dot{M}_t - T_t. \tag{5}$$

Equation (5) indicates that government bonds held by the private sector can only increase by the amount that interest payments on those government bonds exceed seniorage and tax revenues.

Representing real values with lower case variables, equation (1) can be transformed into

$$\dot{a}_t = (i_t - \pi_t)a_t - i_t m_t - \tau_t. \tag{6}$$

Here, π is the inflation rate, *m* the real money balance, and τ tax revenue in real terms. The interest payments on those bonds issued by the government that are held by the central bank become treasury payments from the central bank and thus are returned to the government. This is shown by the second term in the RHS in equation (6). Expressing equation (2) in real values we get

$$a_t = b_t + m_t \text{,} \tag{7}$$

which we can use to rewrite the equation expressing the government bonds held by the private sector as

$$\dot{b}_t = (i_t - \pi_t)b_t - (\dot{m}_t + \pi_t m_t) - \tau_t \text{.} \tag{8}$$

Equation (8) says that the fiscal deficit comprises interest payments on government bonds, seniorage and tax revenues. In this equation, *b* is the real value of privately held government bonds. The relationship between the real interest rate *r* and the nominal interest rate *i* is given by the Fisher equation

$$i_t = r_t + \pi_t \text{.} \tag{9}$$

B. Basic model under flexible prices

In this section we assume that prices are flexible and output is always at its natural level. To look at the long-run consequences of the ZIRP, we build an equation that enables analysis of zero interest rate conditions using the standard monetary growth model from Brock (1974, 1975) and Sidrauski (1967).[8] Our paper uses the model from Blanchard and Fischer (1989, Chap. 5), with additions to account for government and zero interest rates. Blanchard and Fischer (1989) provides a detailed exposition of the basic arguments concerning the model's behavior and of related research.

A representative consumer with an infinite time horizon maximizes the following utility function

$$\int_0^\infty u(c_t, m_t)e^{-\bar{r}t}dt \text{.} \tag{10}$$

In the above function, *c* is consumption and $\bar{r}$ is the discount rate. The consumer's budget constraint is given as

[8] Cole and Kocherlakota (1998) examine the possibility of a deflationary trap under a cash-in-advance constraint.

$$c_t + \tau_t + \dot{a}_t = (i_t - \pi_t)a_t - i_t m_t + y_t, \tag{11}$$

where y is income. An additional constraint is the no-Ponzi-game (NPG) condition

$$\lim_{t\to\infty} a_t e^{-rt} \geq 0. \tag{12}$$

The NPG condition could also be expressed as two inequalities, one each for government bonds and money.

$$\lim_{t\to\infty} b_t e^{-rt} \geq 0 \tag{13.a}$$

$$\lim_{t\to\infty} m_t e^{-rt} \geq 0 \tag{13.b}$$

The monetary NPG condition, (13.b), is indisputable because a negative value for *m* is impossible. On the other hand, imposing a constraint on consumer borrowing from the government turns out to be critical, as we will see below. It is possible to satisfy (12) while not satisfying (13.a), which would happen when the consumer borrows from the government and increases the money balance. In this paper, we take the NPG condition as the consumer's budget constraint in continuous time, and thus view it as a constraint that applies only to the consumer's accumulated assets. Therefore, we will use (12). Later in the paper we explain how using (13.a) and (13.b) instead changes the argument.

We further assume that the following equation always holds.

$$\bar{y} = y_t = c_t \tag{14}$$

To solve the representative consumer's optimization problem, the following conditions must be satisfied.

$$u_c(\bar{y}, m_t) = \lambda_t \tag{15.a}$$

$$u_m(\bar{y}, m_t) = \lambda_t i_t \tag{15.b}$$

$$\dot{\lambda}_t = \lambda_t(\bar{r} + \pi_t - i_t) \tag{15.c}$$

Based on NPG (12), we also get the following transversality condition.

$$\lim_{t\to\infty} \lambda_t a_t e^{-\bar{r}t} = 0 \tag{16}$$

If using (13.a) and (13.b) for the NPG condition, our transversality conditions are as follows.

$$\lim_{t\to\infty} \lambda_t b_t e^{-\bar{r}t} = 0 \tag{17.a}$$

$$\lim_{t\to\infty} \lambda_t m_t e^{-\bar{r}t} = 0 \tag{17.b}$$

To simplify matters, we assume that both consumption and real money balances are separable in the instantaneous utility. If so, income would be constant, and therefore the marginal utility of consumption would become constant and the λ of equation (15.a) would become constant over time. Setting $\lambda = 1$ with no loss of generality, equations (15.b) and (15.c) are transformed into the following equations.

$$u_m(m_t) = i_t \tag{18}$$

$$i_t = \bar{r} + \pi_t \tag{19}$$

Equation (19) is the Fischer equation, where $\bar{r}$ represents both the consumer's discount rate and the natural interest rate. Using μ for the growth rate of nominal money, we get

$$\frac{\dot{m}_t}{m_t} = \mu_t - \pi_t\,. \tag{20}$$

Substituting this into equations (18) and (19), we get

$$\dot{m}_t = (\bar{r} + \mu_t)m_t - u_m(m_t)m_t\,. \tag{21}$$

To make it possible to look at the zero interest rate condition, we assume that when real money balances are sufficiently large, utility from holding money is saturated and marginal utility goes to zero. Stated in symbols,

$$u_m(m) = 0 \text{ (when } m \geq \overline{m} \text{).} \tag{22}$$

The point at which utility from holding money is saturated is $\overline{m}$.

C. ZIRP and the quantity theory of money

The policies proposed by Krugman (1988) to stoke inflation through future monetary growth cannot succeed without a commitment that is credible with the private sector. On the other hand, our focus in this section is on the possibility that inflation will not be created under zero interest rates even if there is a credible commitment to future monetary growth. To show this, we assume that the central bank adopts a monetary target that holds nominal money growth fixed over the long run.[9]

Bear in mind that in this case, the dynamic structure of the model changes depending on the relationship between μ and $\overline{r}$.

If $\mu > -\overline{r}$, then economic fluctuations can be represented by the phase diagram shown in Figure 2. In this paper, based on a widely used assumption, we assume the below inequality holds in order to exclude a path whereby real money balances go to zero (a path whereby the rate of inflation exceeds nominal money growth).

$$\lim_{m \to 0} u_m(m)m > 0 \tag{23}$$

This implies a rapid increase in the marginal utility of money when real money balances decline. If this condition is satisfied, $\dot{m}$ becomes negative as m approaches zero, thereby excluding a solution in which m approaches zero.

In an economy with sticky prices, the initial value of m is determined by the initial values of the price level and of the nominal money balance chosen by the central bank, but in an economy with flexible prices, there is no initial condition for the price level and the initial value of m is not based on historical conditions. A major point of contention is whether to exclude the path whereby m becomes infinitely large (a path with an inflation rate below the rate of nominal money growth). On a path where m becomes infinitely large, nominal interest rates will eventually reach zero. Subsequently, zero interest rates will be expected to remain

[9] The central bank makes regular monetary adjustments by manipulating interest rates. If the quantity theory of money holds, targeting a nominal interest rate of $(\overline{r} + \mu)$ can be interpreted as setting a monetary growth target of μ .

in perpetuity, and the inflation rate will remain equal to the natural interest rate times negative one.

$$\pi_t = -\bar{r} \tag{24}$$

Since monetary growth is higher than the inflation rate, real money balances increase. Nevertheless, consumers do not use those funds for consumption, real money balances increase without limit, and the inflation rate remains below the rate of monetary growth. Based on this, the solution can be expressed as follows.

$$u_m(m_t) = 0 \tag{25.a}$$

$$\dot{m}_t / m_t = \mu - \pi > 0 \tag{25.b}$$

$$\pi = -\bar{r} \tag{25.c}$$

The quantity theory of money does not hold in this solution, which is characterized by the persistence of deflation despite an increase in the monetary growth rate. This could degenerate into a fairly serious type of a liquidity trap. The liquidity trap itself is normally associated with a demand function for money that has become perfectly elastic in interest rates. We refer to the condition whereby the quantity theory of money does not apply as a deflationary trap.[10]

Furthermore, since the initial value of m is not given, a solution is obtained irrespective of from what point the path toward infinite growth in m departs, as shown in the upper frame of Figure 2. Accordingly, there is real indeterminacy in the solution, i.e., there exist an infinite number of solutions with different real values.[11]

Excluding the solutions on either side of the deflationary trap and dealing only with the solution at the point where $\dot{m} = 0$, the economy is such that the price level adjusts at the

[10] This terminology is used by Eggertsson and Woodford (2003), Sims (2003), Woodford (2003) and elsewhere. Benhabib, Schimitt-Grohè and Uribe (2002a) use the term liquidity trap. Obstfeld and Rogoff (1983) and Blanchard and Fischer (1989) call this solution "hyperdeflation." Nevertheless, since the absolute value of the deflation rate is the natural rate of interest, deflation is not necessarily going to be that high, and thus hyper-deflation is not the most appropriate term, in our opinion.

[11] The first papers to argue this real indeterminacy include Black (1974), Brock (1975), and Sargent and Wallace (1973). McCallum (2001) calls this solution multiplicity or nonuniqueness.

A different concept is nominal indeterminacy, wherein the real variables within the model are unique, but the nominal variables (nominal money balance and price level) are not. Nominal indeterminacy is discussed in the seminal paper by Patinkin (1949), as well as by McCallum (1981) and Sargent and Wallace (1975).

starting point and m remains stuck at the level where $\dot{m} = 0$. Under this equilibrium, these equations hold:

$$u_m(m_t) = \bar{r} + \pi \qquad (26.a)$$

$$\dot{m}_t / m_t = \mu - \pi = 0. \qquad (26.b)$$

In equilibrium, the monetary growth rate is equal to the inflation rate. Since this seems to be the most plausible behavior, we will call this the normal equilibrium.

The mechanism for excluding a deflationary trap and selecting only the normal equilibrium can be derived from the transversality condition in the consumer's optimization problem. We consider first the transversality condition given by equation (16). Since λ is a constant, equation (16) can be rewritten as follows.

$$\lim_{t\to\infty} a_t e^{-\bar{r}t} = 0 \qquad (27)$$

This is another way of saying that assets held by the consumer are not going to grow at a rate faster than the natural rate of interest $\bar{r}$. This is because, if asset holdings were to grow faster than $\bar{r}$, the consumer would be able to improve utility by consuming a portion of assets.

Since the consumer's assets are the government's liabilities, we will focus on how fiscal authorities manage the government debt.[12] First, consider the case where the nominal growth rate is constant at n. Since the rate of change in prices is $-\bar{r}$ in a deflationary trap, real government debt grows at $n + \bar{r}$ under these conditions. Thus, if government debt is initially positive and $n \geq 0$, it holds that

$$\lim_{t\to\infty} a_t e^{-\bar{r}t} > 0. \qquad (28)$$

This implies that if nominal government debt outstanding is constant or growing, the consumer's transversality condition is not satisfied. Consequently, a deflationary trap cannot be a solution. The economic intuition behind this is as follows. If inflation equal to the rate of monetary growth does not occur, real assets held by consumers will steadily increase. If this

[12] This extreme simplification means that physical capital is ignored in our model. When physical capital reaches a stationary value, marginal increases in consumer assets and government liabilities are equalized, and thus with minor modification our argument is equally applicable to an economy with physical capital.

happens, consumers will tend to divert assets toward consumption, and an increase in prices would be required to balance the supply and demand for goods.

That said, if $n - \pi \geq \mu$, the growth rate of a will be at least as high as the growth rate of m, and thus b will grow at the rate of $n - \pi$. Under normal equilibrium, since the minimum rate of growth in b is $\bar{r}$ when $n \geq \mu + \bar{r}$, equation (17.a) is not satisfied. We follow McCallum (2001) and assume that in this case there would be a loss of faith in government debt that brings government activity to a halt, and thus that this is not a policy stance that the government could take.[13]

Likewise, when $n < 0$ the transversality condition in equation (27) is satisfied, and thus the development of a deflationary trap is not excluded.[14] In this case, all paths that depart from the price level of P or lower corresponding to the level of m when $\dot{m} = 0$ provide a solution, thus leading to real indeterminacy.

The above makes it clear that the presence of a deflationary trap depends upon whether the nominal government debt outstanding is decreasing. The possibility that a deflationary trap could not be excluded based on the transversality condition was pointed out more than a quarter century ago by Brock (1975), who assumed that $a = m$.[15] When the consumer's transversality condition is not satisfied, government debt tends to grow at a rate faster than the natural interest rate. This would be termed a non-Ricardian regime.[16] In other words, to escape from a deflationary trap, fiscal policy must be non-Ricardian. This has been pointed out by Woodford (2003), Benhabib, Schmitt-Grohè and Uribe (2002a), Takeda (2002), Nakajima (2002), and Eggertsson and Woodford (2003), among others. If m grows at a faster rate than $-\bar{r}$ under a non-Ricardian regime (no change in b), m will instantly jump to the point where $\dot{m} = 0$, as previously noted. This means the price level will also jump when nominal money balances do not change.

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[13] The argument that the price level is determined in a way that satisfies the government's transversality condition is apparently equivalent to the fiscal theory of the price level. Such an argument can be found in Woodford (2001, footnote 26).
Given that the fiscal theory of the price level posits a price level whereby prices adjust so that the transversality condition of government debt a is satisfied, even when b is growing at a rate faster than $\bar{r}$, the economy can attain the normal equilibrium even in the region where $n - \pi \geq \mu$. This point highlights the difference between our argument and the fiscal theory of the price level.

[14] Even if real money balances were to grow at a rate above the natural interest rate, the government would offset this by reducing net debt (with the government eventually becoming a net creditor to the private sector). This would mean consumers were borrowing from the government to hold cash.

[15] Brock (1975) did not assume that zero interest rates would develop once the utility from holding cash was saturated, and the basic thrust of his paper was on the indeterminacy of a solution.

[16] This definition of government debt not meeting the transversality condition as a non-Ricardian fiscal policy comes from Benhabib, Schmitt-Grohè, and Uribe (2002a), Woodford (2001) and others. Woodford (1995)

Next, as shown in Figure 3, a different phase diagram can be drawn when $\mu = -\bar{r}$.

$$\dot{m}_t = -u_m(m_t)m_t \tag{29}$$

Based on the above equation, $\dot{m} = 0$ only holds under zero interest rates. This is because the monetary growth rate and the inflation rate are equal, with both equal to the natural interest rate times negative one. Under zero interest rates everything is a solution, and with flexible prices, the price level at the initial point (and along the path beyond that) cannot be uniquely determined.

Finally, when $\mu < -\bar{r}$, in the entire region where $m > 0$, it is the case that $\dot{m} < 0$ and no solution exists, as shown in Figure 4.

The above makes it clear that a variety of solutions can be found depending on the values of μ and n. This relationship is summarized in Figure 5.[17]

Holding growth in nominal debt constant is not the only fiscal policy rule. Another possible rule is for the government to hold the real value of debt *a* constant. This policy leads to a sustainable government budget, which implies the stability of debt as a percentage of GDP.[18] By doing so, the transversality condition (16) is always satisfied, and the path in which *m* becomes infinitely large cannot be excluded.

Another option would be to keep constant *b*, the real value of government bonds held by the private sector. Since this would ensure that *a* and *m* move together, it is equivalent to setting $\mu = n$ when holding nominal growth in government debt constant. In this case, if

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defined a non-Ricardian fiscal policy stance as when public debt held by the private sector does not satisfy the transversality condition.

[17] When *m* becomes infinitely large, the inflation rate is lower than the rate of monetary growth. Ono (1999) and Ono (2001), among others, attempted to explain the long-term slump in output by combining this phenomenon with a version of the Phillips curve,

$$\pi = \mu + \beta[y - \bar{y}].$$

The assumption in these papers was that the marginal utility of money holdings had a positive minimum value, such that *m* would accumulate even when $\mu = 0$. Although these papers emphasized that a lack of saturation of money holdings was a cause of the long-term slump, Shibata (1993) showed that even with saturation, as long as utility did not decline, a sustained slump like that identified by Ono (1992) would occur if $\mu < 0$. Since a perfectly natural assumption is that the minimum marginal utility on cash holdings is zero, the use of a Phillips curve according to the above equation is essential to the occurrence of a sustained slump.

Be aware that, under the standard expectations augmented Phillips curve, with perfect foresight of the inflation rate, output is equal to $\bar{y}$, but because the monetary growth rate is contained in that portion of the above equation corresponding to the expected inflation rate, even with perfect foresight of inflation, output could deviate from $\bar{y}$.

[18] There has been a considerable body of research on the sustainability of government budgets since Hamilton and Flavin (1986). For an examination of the ratio of government debt to GDP, see Corsetti and Roubini (1991).

$\mu \geq 0$, the transversality condition is not satisfied and solutions with *m* growing infinitely large are excluded.

Table 1 summarizes the fiscal policies and monetary targeting rules that would lead to a deflationary trap based on the above arguments. To preclude a deflationary trap when $\mu > -\bar{r}$, the government would have to commit to either holding the nominal growth rate of government debt positive, or keeping the real value of privately held government bonds constant while ensuring positive nominal money growth. In either case, when $n = \mu$, both government debt and money grow at the same rate as inflation and thus their real values are held constant.

It is important to remember, however, that the real values are constant as a result of the previous steps. The appearance of zero interest rates cannot be ruled out if there is merely a commitment to hold the real value of government debt constant. This is because taking such a fiscal policy stance in a deflationary environment would reduce the nominal value of government debt and wind up satisfying the transversality condition. Even under deflation, the transversality condition cannot be ruled out without a commitment to not reduce the nominal value of government debt.

When applying the transversality conditions in (17.a) and (17.b), neither *b* nor *m* are going to grow faster than $\bar{r}$. This implies a negative nominal rate of growth for both *b* and *m*. It follows that the transversality condition would be met and that growth in both nominal government debt and monetary growth would have to be negative for a deflationary trap to appear. Thus, under the transversality conditions in (17.a) and (17.b), the deflationary trap shown in Figure 4, when $\mu \geq 0$, would disappear, and all solutions would be a normal equilibrium when $\mu \geq 0$ in Table 1.

D. Fiscal policies to escape from a deflationary trap

We now consider an economy that is already in a deflationary trap to examine those policies that would enable an escape from that trap. When considering such policies, it is probably necessary to build a model that shows that economic welfare is higher under normal equilibrium than under a deflationary trap. In this paper, however, we avoid complicating the model to do that, and instead consider escape from a deflationary trap as a given policy objective. [19]

[19] Edmond (2002) cited the easing of borrowing constraints on the consumer and improved welfare as reasons why inflation is desirable. When seniorage is returned to the consumer through fixed subsidies, consumers

Under zero interest rates, even when keeping monetary growth at $\mu > 0$, inflation will not occur and escape from a deflationary trap is impossible under a Ricardian fiscal policy regime. Under a non-Ricardian regime, however, it becomes possible to create inflation and escape from a deflationary trap.[20] All that is required of this non-Ricardian fiscal regime is a commitment to not reduce nominal government debt. This commitment cannot be kept under deflation, but under other conditions it is consistent with maintaining prudent fiscal discipline, without any destructive fiscal expansion.[21] Schmitt-Grohè and Uribe (2000), for example, show that a balanced budget would belong to such a non-Ricardian fiscal policy regime.

Next we assume the economy has already fallen into a deflationary trap and consider the policies required to exit that trap as well as their consequences.[22] We can start by looking at the Ricardian regime of holding government debt a constant while in a deflationary trap. In this case, prices would decline at the rate of $-\bar{r}$, as would nominal government debt. The

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facing a borrowing constraint can shift consumption in a desirable direction by receiving the returns from seniorage in their younger years. He showed that, when shifting from deflation to inflation under zero interest rates, there is a possibility that this positive effect would outweigh the negative effect from the opportunity cost of holding money.

Ireland (2001) found that when there is population growth, welfare declines under zero interest rates and improves when zero interest rates are eliminated. In his model, however, the adverse impact on welfare comes from the income transfer from the younger generation to the older generation, and the entire government debt is assumed to be in the form of money. If the government held assets in the same amount as the money balance and the government's net debt was zero, there would be no intergenerational transfer of income and thus no deterioration in welfare.

Without flexible prices, either inflation or deflation would result in erroneous pricing and cause a misallocation of resources. Based on this reason, Woodford (2003) showed that it was desirable to stabilize prices (bring the inflation rate to zero) to prevent this disruption of the pricing mechanism.

Akerlof, Dickens and Perry (1996), on the other hand, emphasized that when prices are downwardly rigid, the long-run Phillips curve ceases to be vertical and that a higher income level can be maintained over the long term by keeping a positive rate of inflation.

[20] Evans and Honkapohja (2003) pointed out that the appropriate fiscal policy approach for exiting a deflationary trap depends on whether the formation of expectations is rational or adaptive. Under rational expectations, a deflationary trap is a stable equilibrium and a sunspot solution emerges when pursuing Ricardian policies. Under adaptive learning, in contrast, such learning is impossible with Ricardian policies, and thus there is no need to give much thought to the formation of a deflationary trap equilibrium. There is a possibility, however, of deflation becoming deeper than in a deflationary trap. Under a non-Ricardian regime, in contrast, learning is possible and it becomes easier for a deflationary trap to form.

The model used by Evans and Honkapohja (2003) does not include a transversality condition on consumer behavior and thus has no mechanism to rule out solutions based on such condition. For this reason, they believe that a Ricardian regime is preferable for exiting a deflationary trap and maintain that monetary policy must be changed before the inflation rate declines to deflationary trap levels.

[21] Within the realm of the fiscal theory of the price level there are proponents of massively expansionary fiscal policy aimed at causing prices to increase, but Kawagoe and Hirose (2003) take a negative view of this, maintaining that such fiscal expansion does nothing more than provoke higher interest rates by raising the risk premium. We take the same position as Kawagoe and Hirose (2003) in this paper.

[22] Another approach is the consideration of a policy rule to avoid a deflationary trap, as in Benhabib, Schmit-Grohè and Uribe (2002). They attempt to avoid falling into a deflationary trap by establishing a policy rule inconsistent with the transversality condition ahead of time, while on a deflationary trap path,. The solution thus achieved is therefore a consistent policy rule. Our approach differs in that we want to analyze the conditions prevailing after having already fallen into a deflationary trap, so we assume an initial policy that is compatible with a deflationary trap and then suppose a subsequent policy change.

government would change policy to exit the deflationary trap, making a commitment to not reduce nominal government debt (a non-Ricardian regime) during the deflationary trap. This would mean a tax cut. Nevertheless, to ensure a solution with a normal equilibrium, fiscal discipline (a Ricardian regime) is required, holding government debt *a* constant under normal equilibrium. Thus the new policy rule requires a non-Ricardian regime under certain circumstances (only when in a deflationary trap).

If economic agents find this commitment to pursue new policies credible, the deflationary trap would not satisfy the transversality condition and therefore not be a solution. Further restricting our focus to a policy of holding the size of the tax cut constant when policy is changed, together with the central bank maintaining a constant rate of monetary growth, the new policy rule we consider here can be expressed with three parameters: the new government debt, the monetary growth rate and the tax cut size. With the government setting the parameters correctly so as to achieve the inflation target under normal equilibrium, as long as transversality is satisfied under normal equilibrium, the economy should quickly reach its new normal equilibrium.

The policy parameters are chosen as follows. The government's budget constraint under a deflationary trap ($\pi_0 = -\bar{r}$) can be written as

$$ra_0 = \tau_0 . \qquad (30)$$

Once out of the deflationary trap, the budget constraint changes to

$$ra_1 - im_1 = \tau_1 . \qquad (31)$$

Here, m_1 is the real money balance that equates to money demand with nominal interest rate *i*. Assuming that the real value *a* of government debt (including government bonds held by the central bank) does not change around the time of the policy change[23], then

$$\tau_0 - \tau_1 = im_1 . \qquad (32)$$

[23] A model that assumes no instantaneous adjustments to nominal government debt also presupposes no instantaneous adjustments to the price level. We consider this a natural constraint that limits the range of policies.

The amount of tax cut required to exit the deflationary trap is given by the new nominal interest rate and money balance attained. The tax cut corresponds to the amount of increase in the BOJ's payments to the national treasury. This would mean no change in the primary balance.

We now estimate how large a money-financed tax cut would be given Japan's current conditions. From equation (32), this can be expressed with two parameters, the nominal interest rate and the monetary base following exit from the deflationary trap. Using the well-known Taylor rule,

$$i = 0.04 + 0.5(\pi - 0.02)\,, \tag{33}$$

we set an inflation rate of 2%, a nominal interest rate of 4%, and a natural interest rate of 2%.[24] Accordingly, in a deflationary trap the nominal interest rate would be zero and the inflation rate –2%, values close to what they currently are in Japan. One way to determine the monetary base corresponding to 4% nominal interest rates is to estimate using a money demand function, but there is another way, and that is to check the historical values of the monetary base when nominal interest rates were at 4%. In keeping with our objective of making it easy to grasp estimation steps, we will use the latter approach. As shown in Figure 1, during the declining phase of interest rates in the early 1990s, short-term interest rates dropped below 4% in February 1992. Figure 6 shows that the monetary base was stable around this time, averaging ¥39 trillion between August 1991 and July 1992. We can therefore say that the monetary base that corresponded to a nominal interest rate of 4% in 1992 was ¥39 trillion. The current monetary base must be thought of in the context of this amount plus the natural growth in GDP. In the post-bubble year of 1992, economic activity was neither exceptionally strong nor exceptionally weak, and GDP was deemed to be at roughly its natural level, so such an approach should not produce any large errors. An estimate of the natural rate of growth in GDP from 1992 until 2002 would vary widely depending on whether the slow growth in the 1990s is blamed on supply-side factors or demand-side factors, and opinions differ widely on this issue. In light of this, we have adopted the middle road in this paper, setting both an upper limit and a lower limit on the natural GDP growth rate. For the upper limit, we use the real GDP growth rate of 3.4% that

[24] The values in equation (33) have been widely used since Taylor (1993). It is normally written with not only the inflation rate but also the output gap as explanatory variables, but we have simplified it here by ignoring the output gap.

was achieved in the 1980s prior to the bubble (1980-86). Assuming no output gap in 1992, this would mean an output gap of 20% in 2002. For the lower limit, we use the actual rate of real GDP growth from 1992 to 2002 of 1.1%. This would be equivalent to assuming an output gap of zero in 2002. Furthermore, the annual change in the GDP deflator from FY1992 until FY2002 was +0.8%. Under these two extremes, the monetary base of ¥39 trillion in 1992 extrapolates to between ¥40.2 and ¥51.5 trillion in 2003.[25] Using this as m in equation (32) and multiplying by i = 4%, the required tax cut comes to ¥1.6-2.1 trillion. With an interest rate hike of 3 percentage points, the tax cut would be ¥1.2-1.5 trillion.

The following is a conceivable exit strategy when the economy is in a deflationary trap. Fiscal policymakers begin with a commitment to prevent negative nominal growth in government debt and coordinate with the central bank a money-financed tax cut (or possibly a "helicopter drop" by the central bank alone). Using the above calculations, the required tax cut would be a maximum of approximately ¥2 trillion. An area of concern with a money-financed tax cut without strict rules in place is the ease with which this could wind up resulting in the central bank underwriting government bonds. Nevertheless, critical to coordination between fiscal policymakers and the central bank is to form a common expectation for the new inflation rate. If the fiscal authorities expect inflation to climb higher, they will have leeway to grow the nominal value of government debt in accord with inflation when holding the real value of government debt constant. Because nominal interest rates increase, however, interest payments on government bonds become higher. When inflation increases by the same amount as nominal interest rates (i.e., real interest rates are constant), the two are perfectly offset. The other factor affecting the government's budget constraint is the change in payments into the national treasury by the BOJ. Since nominal interest rates are higher, these payments – which are government revenues – increase, creating leeway for a tax cut of the same amount. Stated differently, the tax cut is funded by the increase in central bank payments into the treasury resulting from the increase in inflation (nominal interest rates). This is fine as long as the central bank purchases government bonds on the open market to ensure monetary growth at the rate of μ.

When nominal interest rates increase, real money balances must decline, and thus there must be a large, instantaneous increase in prices. Our model assumes flexible prices, so

[25] Our actual calculation for the minimum figure was to multiply ¥39 trillion by the ratio of FY2002 nominal GDP (¥499.1001 trillion) to FY1992 nominal GDP (¥483.6074). For the maximum figure, we took the ratio of the FY2002 GDP deflator (real GDP divided by nominal GDP) to the FY1992 GDP deflator, multiplied that by 11/6 times the ratio of FY1986 real GDP (¥379.8456) to FY1980 real GDP (¥311.9881), and then multiplied that by ¥39 trillion.

such a jump in prices presents no problems within the model. Realistically, however, it is probably best to avoid such a dramatic increase in prices. To do this, the central bank could engage in selling operations to lower nominal money balances and achieve the new real money balance without a jump in prices. Auernheimer (1974) termed such a policy aimed at preventing a jump in prices an “honest government policy.” The above scenario would lead to changes in the main economic variables as shown in Figure 7.

We can now attempt to apply the above argument to the *reductio ad absurdum* described by Bernanke (2000, p. 158). Bernanke says that if monetary growth did not lead to higher prices, the government could reap infinitely large revenues from seniorage with which it would be able to buy goods and assets, but since such a state could not be an equilibrium, monetary growth is going to impact the price level. Thinking in terms of equation (8), if seniorage goes up, either government bonds b held by the private sector are reduced, or taxes can be reduced (demand for goods from the government is ruled out in this model). If the choice is to cut taxes, it would become a non-Ricardian fiscal policy that, as we have shown, would affect prices and produce a result consistent with Bernanke’s argument. If on the other hand there is a reduction in privately held government bonds and the policy satisfies the transversality condition (27), there would be no impact on prices and Bernanke’s argument would not apply.

E. Applicability to the Japanese economy

Is it accurate to characterize Japan's current economic situation as a deflationary trap as defined in section II.C? When the government adopts a fiscal policy stance aimed at reducing nominal debt, it renders the quantity theory of money inapplicable. However, Japan's fiscal deficit is extraordinarily high relative to other industrialized countries, and the amount of increase in nominal debt is also quite large. As depicted in Figured 5, a deflationary trap cannot occur when growth in nominal government debt is positive. Accordingly, it would be possible to argue that the fiscal authorities have adopted policies aimed at avoiding a deflationary trap, and that Japan is not already “trapped.”

On the other hand, it is also possible to interpret Japan's current situation as a deflationary trap by emphasizing the point made in section II.C that characterizes a deflationary trap as sustained deflation despite growth in nominal money balances. In this case, there are a number of ways to reconcile the theory with the current fiscal policy stance.

First, Eggertsson and Woodford (2003) pointed out that a large quantity of Japanese government bonds (JGBs) is held by government agencies other than the central bank, including the postal savings system, postal insurance and public pension funds, and that there is a huge discrepancy between the quantity of government debt held by the private sector and the amount of government bonds that have been issued. Figure 8, which shows the total amount of JGB issuance and the amount of JGBs held privately as a percentage of GDP, shows clearly that private holdings – recently around 40% of GDP – have not shown the same rapid growth as total issuance.

Second, it may be possible to argue that the government has yet to make a clear commitment to escape from the deflationary trap. In fact, the government's commitment to fiscal sustainability could also be interpreted as a commitment to reduce nominal government debt if deflation continues. According to this line of reasoning, what is needed now is a clear statement by the fiscal authorities that it will not reduce nominal debt if deflation continues.

Eggertsson (2003) proposes two ways for the government to clarify its commitment: increase spending (or cut taxes) to inflate the fiscal deficit and engage in open-market operations to purchase real assets and overseas assets. These policies would amount to the government attempting to decrease the real value of the government debt by triggering inflation. If fiscal policymakers and the central bank were to cooperate in an attempt to maximize a common objective function, this "inflation incentive" would be reflected in central bank policy. This would create inflation expectations in the private sector, actually produce inflation, and lead to a rise in output. But if the government and the central bank do not cooperate and the central bank maximizes an independent objective function, inflation expectations would not form. Eggertsson (2003) interprets the lack of inflation despite the large quantity of JGB issuance under zero interest rates as evidence of a lack of cooperation between treasury officials and the central bank.

Third, given that consumers act without perfect foresight on an infinite horizon, it could be argued that the transversality condition on government debt is not included in the model. We expect, however, that such an interpretation would be criticized as arbitrarily assuming bounded rationality of the consumer. A promising area for future research would be to examine the impact that the transversality condition has on price level determination, based on a widely used type of bounded rationality (adaptive learning, for example).

III. Short-run Adjustments

A. The impact from lagged adjustments in the inflation rate

In section II, we assumed flexible prices and output always at its natural level. In this section, because prices are sticky in the short run, we consider situations in which output diverges from its natural level.

This assumption of price rigidity affects the options for escaping from a deflationary trap in the following ways. Using the Fischer equation as a starting point

$$i = r + \pi , \tag{34}$$

when prices are flexible, the real interest rate is equal to the natural interest rate. Assuming these are both constant, the inflation rate and nominal interest rates increase on a one-for-one basis. Accordingly, an increase in nominal interest rates is immediately reflected as an increase in the inflation rate. When the inflation rate does not respond immediately, on the other hand, an increase in the nominal interest rate causes the real interest rate to rise in the short run, only to return to the level of the natural interest rate over time. Since an increase in the real interest rate pushes income lower, income declines in the short run. Because this scenario entails a temporary monetary tightening, we expect actual implementation would meet stiff resistance.

Another scenario for escaping from a deflationary trap would be to pursue a self-sustained increase in demand, which would lower the real interest rate and raise the inflation rate, accompanied by a hike in nominal interest rates. Although raising the nominal interest rate by itself has a contractionary effect, it would probably be supported as a stabilization policy. Because an autonomous increase in demand is, by definition, independent of monetary policy, this scenario is not a policy-driven escape from a deflationary trap, but rather an escape achieved through factors exogenous to monetary policy.

Consequently, it is the first scenario that merits a further analysis. The problem revolves around how much income would decline when attempting an escape from a deflationary trap by changing only monetary policy. We consider this problem in section III.

B. Twenty types of dynamic models

A critical dispute in the field of macroeconomics concerns the theoretical framework for explaining the short-term fluctuations that arise from price rigidities. Although economists are closer to agreement now than they have ever been, there is not yet agreement on a single model.

When there is imperfect understanding of actual economic fluctuations, there is always the risk that selection of an optimal monetary policy under a specific model will bring undesirable results if that model does not present an accurate picture of reality. With this in mind, McCallum (1988) ran an analysis that attempted to assess the consequences of monetary policy rules under multiple conceivable models.[26] In a similar spirit, we consider the short-term fluctuations that would occur under models of differing characteristics when applying the policy scenarios discussed in section II. Therefore, our objective here is not to focus in on those models that provide the closest approximation to reality, but rather to select models that accommodate the widest possible range of characteristics through simple combinations of setting. Of course, not all of the models capable of explaining reality are included. If those models we have neglected exhibit the same dynamic characteristics as the models we included, they should be thought of as effectively analyzed here, albeit represented by simpler models to facilitate analysis.

We consider a dynamic model that represents price adjustments occurring over time. The model is described by three equations, and we consider alternative versions of each.

We consider the following two forms for the demand side.

(A.1) $$\dot{y}_t = \alpha[i_t - \pi_t - \bar{r}], \quad \alpha > 0$$

(A.2) $$y_t - \bar{y} = -\alpha[i_t - \pi_t - \bar{r}], \quad \alpha > 0$$

Derived by Kerr and King (1996) and others, (A.1) is a linear approximation of the Euler equation, where $c = y$. This is also called the expectational IS curve in Kerr and King (1996). (A.2) is the classic IS curve, and implies that when real interest rates are high, income is below its natural level. (A.1) is based on intertemporal utility maximizing behavior, as is the model in section II. This differs from (A.2), which cannot be thought of as a simple extension of section II. When solving forward for (A.1), current income is expressed as a decreasing function of real long-term interest rates, so it is possible to think of the difference between (A.1) and (A.2) as being whether the impact of future short-term interest rates on current demand is taken into account. As will be made clear below, this does not make a critical difference to the thrust of this paper.

We consider three versions of the equation representing the change in prices.

[26] Brock, Daulauf and West (2003) took this approach a step further by considering prior probabilities for different models, and showed a way to evaluate policies under model uncertainty based on decision theory.

(B.1) $\dot{\pi}_t = -\beta[y_t - \bar{y}], \quad \beta > 0$

(B.2) $\dot{\pi}_t = \beta[y_t - \bar{y}], \quad \beta > 0$

(B.3) $\pi_t - \bar{\pi} = \beta[y_t - \bar{y}], \quad \beta > 0$

(B.1) is the price adjustment function from Calvo (1983), and is based on the assumption that firms have an opportunity to adjust prices each period by a given percentage. Since the opportunity to change prices is limited, firms are assumed to set prices based on their future marginal costs. During cyclical fluctuations, assuming that some production factors are fixed and the variable production factors generate diminishing returns, an increase in production would lead to higher marginal costs, and thus the gap in output between the present and the future would have a positive effect on current prices. Solving (B.1) forward makes this relationship clear. The price adjustment mechanism used by Calvo (1983) was reformulated into discrete time by Roberts (1995) as follows.

$$\pi_t = \beta[y_t - \bar{y}] + E_t \pi_{t+1} \tag{35}$$

This is the now widely used equation called the New Keynesian Phillips curve.[27] Here, E_t is the expected value based on information accumulated in period *t*. Since (B.1) and (35) have essentially the same characteristics within our framework, we will refer to (B.1) as the New Keynesian Phillips curve.

(B.2), a formulation dating back to early studies of the Phillips curve, like Gordon (1970) and Solow (1969), that showed the persistence of inflation, implies that when income is high, the inflation rate tends to increase. This is known as the accelerationist Phillips curve (in the Japanese literature, such as Higo and Nakada (2000) and Watanabe (1997), it is called the NAIRU-type Phillips curve). Solving (B.2) backward, we see that the current inflation rate is affected by the past output gap and responds by moving in the same direction. While (B.2) is a backward looking model, (B.1) is forward looking in regards to price adjustments. Although the recent trend has favored the use of forward-looking models, it is recognized that simple forward-looking models do not necessarily match with the empirical data. Furhrer and Moore (1995) consider a hybrid model with both forward-looking and backward-looking elements, and Mankiw and Reis (2002, 2003) attempt to build micro foundations into a backward-looking model.

Equation (B.3) is the classic Phillips curve, which shows that when incomes are high, the inflation rate is high.

The last equation concerns the monetary policy rule, and assumes that the central bank adjusts the nominal interest rate in response to inflation. Since the real interest rate shows up as a variable on the demand side, monetary policy can be considered in two different forms, as follows.

(C.1) $r'(\pi) > 0$

(C.2) $r'(\pi) < 0$

Equation (C.1) implies a policy rule whereby when the inflation rate rises, the nominal interest rate is raised even higher to ensure an increase in the real interest rate. Leeper (1991) calls this an active monetary policy. The Taylor (1993) rule is included in this type of monetary policy. Nevertheless, such a rule comes up against the zero bound on interest rates. That is, when the inflation rate becomes low and interest rates reach zero, it becomes impossible to lower nominal interest rates further, even with further declines in the inflation rate, and thus the real interest rate winds up rising. Accordingly, when constructing a phase diagram we consider $r'(\pi) < 0$ when π is low. Equation (C.2) implies a policy rule whereby even under rising inflation, the increase in the nominal interest rate is moderate enough that the real interest rate declines. Termed a passive monetary policy by Leeper (1991), this includes interest rate targeting and a ZIRP.

In section II, we examined a monetary policy in which nominal money growth was held constant, but in section III, we look at nominal interest rates as a function of inflation. Although the settings in section III are close to actual monetary policy rules, under flexible prices, policies that target nominal interest rates and policies that target nominal money growth are essentially the same outside of a deflationary trap, so the model in section II is not limiting.

The model in section III does not show a and m explicitly. With this in mind, we first consider a Ricardian fiscal regime and analyze the model assuming that the transversality

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[27] As argued in Woodford (2003), the application of micro foundations leads to a coefficient for expected inflation other than one.

condition is always satisfied. Under these conditions, we find that a deflationary trap is a possible equilibrium under zero interest rates. Thus the models in sections II and III share the same challenge, which is finding a policy that will provide an escape from a deflationary trap after falling in.

Substituting (C) into (A) leaves the model with two variables, y and π. There are six conceivable combinations of (A) and (B), but we can exclude the combination of (A.2) and (B.3), since it is not a dynamic model. Adding to this the equations in (C), we now have ten possible combinations. Further considering that π could be either a state variable or a jumping variable gives us the possibility of 20 different types of dynamic models.

These models include some representative models that are widely used in macroeconomics. A combination of the expectational IS curve in (A.1), the New Keynesian Phillips curve in (B.2), and the Taylor rule in (C.1), under the assumption that the inflation rate can jump at the initial point, has recently become a commonly used theoretical framework for examining monetary policy that was used by, e.g., Clarida, Gali and Gertler (1999). Additionally, the combination of the IS curve in (A.2), the accelerationist Phillips curve in (B.2) and the Taylor rule in (C.1), with inflation assumed to be a state variable, is a model seen in newer undergraduate-level textbooks, starting with Taylor (1998).[28]

We conduct our dynamic analysis based on the following assumptions. First, we look at the dynamic structure of the model assuming that the government adopts a Ricardian fiscal regime and that a deflationary trap is a possible solution. Next, we examine how the economy behaves assuming that, after a deflationary trap has developed, policy is changed to exit the trap by adopting a non-Ricardian regime, thereby excluding a deflationary trap from the set of possible solutions. As in section II, under a deflationary trap a non-Ricardian regime could be implemented with a combination of a money-financed tax cut and an interest rate hike implemented at the time of policy change, but since the budget constraints of the consumer and the government are not explicitly written, the tax cut does not show up in the model. We also consider in section III situations where a policy change that is not non-Ricardian in nature is needed.

C. When inflation is a state variable

[28] When emphasizing that (B.1) is a forward-looking model and (B.2) a backward looking model, this model is open to criticism for a lack of theoretical consistency, since the inflation rate is given as an initial condition in (B .1) but can jump in (B.2). Nevertheless, our objective is to examine the robustness of policy scenarios, so we have chosen to use a variety of models rather than narrow our focus to models based on theoretical consistency.

Phase diagrams expressing the ten combinations of (A), (B), and (C) are shown in Figure 9, with the inflation rate on the vertical axis and level of output on the horizontal axis. Each phase diagram is drawn as described below.

In models with (A.1), since at a given inflation rate $\dot{y}_t = \alpha[r(\pi_t) - \bar{r}] = 0$, the curve $\dot{y} = 0$ is represented by a horizontal line. To achieve a normal equilibrium, the monetary policy rule must be set so that such a horizontal line results when inflation is positive. Under active monetary policy rules there is also a need to consider the impact of the zero bound on interest rates. The relationship between interest rates and the inflation rate can be expressed as in Figure 10 (A), with the nominal interest rate on the vertical axis and the inflation rate on the horizontal axis. Under an active monetary policy rule, the nominal interest rate is lowered by more than the decline in the inflation rate, and thus the line connecting *E* and *F* slopes at an angle steeper than 45 degrees. The 45-degree slope connecting *E* to *G* shows the relationship between the nominal interest rate and inflation under natural interest rates, and the intersection of the two lines at *E* is the normal equilibrium. Under an active monetary policy rule, when a decline in inflation brings interest rates to zero as in *F*, further declines in inflation cause the real interest rate to rise, even with the nominal rate held to zero. Figure 10 (B) shows the relationship between the real interest rate and inflation, and indicates that an active monetary policy rule is followed when inflation is higher than the point at *F* where interest rates are zero, but must be passive when the inflation rate is lower than *F* because of the zero bound on interest rates. In models with equation (A.1), $\dot{y} = 0$ when the inflation rate is *E* or *G*, $\dot{y} < 0$ when it lies between *E* and *G*, and $\dot{y} > 0$ when it is either higher than *E* or lower than *G*.

In models with (A.2), substituting (C) into (A.2) yields a solution curve. Under an active monetary policy rule, the real interest rate is reduced by lowering the nominal interest rate by an amount greater than the decline in the inflation rate, and because output increases, the curve is downward sloping. Nevertheless, under zero interest rates, a decline in inflation causes the real interest rate to go up and output to decline. Therefore, as shown in (4) and (5) of Figure 9, the curve is downward sloping under positive interest rates and upward sloping under negative interest rates. This is because an increase in inflation causes an increase in the real interest rate under a passive monetary policy rule. This is shown by the upward slope of the curve.

In models with (B.1) and (B.2), the $\dot{\pi} = 0$ curve appears vertical at $y = \bar{y}$. In models with equation (B.3), the upward sloping Phillips curve becomes a solution curve.

Based on the above arguments, we draw the $\dot{y}=0$ and $\dot{\pi}=0$ curves in each phase diagram to be able to check for the properties of the solution.

When π is a state variable and equilibrium is not a saddle point, there are either an infinite number of solutions or there is no solution. When a unique solution exists with the right choice of monetary policy rule, we assume that such a monetary policy was chosen. If so, of the five combinations of equations (A) and (B), we select the monetary polices that produce a saddle point equilibrium. The results of this are shown in Table 2.

The behavior of the economy can be divided into two types. In the first, there is a positive correlation between inflation and output, and the economy converges on a long-run equilibrium (in response to the Phillips curve relationship). This behavior occurs under either the New Keynesian Phillips curve (B.1) or the classic Phillips curve (B.3), irrespective of the shape of the IS curve, and implies the selection of monetary policy (C.2). In the second, there is a negative correlation between inflation and output, and the economy converges on a long-run equilibrium (in response to the positive interest rate level). This behavior occurs under the accelerationist Phillips curve (B.2), irrespective of the shape of the IS curve, and implies the selection of monetary policy (C.1).

Kerr and King (1996), using a New Keynesian Phillips curve under a Ricardian regime, are able to obtain a unique, rational expectations equilibrium with an active monetary policy, but show that equilibrium is indeterminate with a passive monetary policy. Unlike Kerr and King (1996), we are able to obtain a unique solution under a passive monetary policy rule in models that contain (B.1). This is because we model inflation as a state variable, which gives the model completely different dynamic characteristics. In this paper, although we confine ourselves to a Ricardian regime, we look at monetary policy rules and the problem of indeterminancy using a much wider framework, including treating inflation as a state variable and assuming an accelerationist Phillips curve, than what has been considered in the recent literature. When including the accelerationist Phillips curve (B.3), a unique solution is possible under active monetary policy, but note that this is in agreement with Kerr and King (1996) only as to the conclusion; the Phillips curve and the initial settings for the inflation rate are different, and the dynamic characteristics of the models are completely different.

Be aware of the economy's behavior under monetary policy (C.1) when inflation declines and interest rates reach zero. In the combination (A.1) and (B.2), as the economy approaches equilibrium at zero interest rates and output at its natural level (hereinafter the zero interest rate equilibrium[29]), the dynamic system has complex roots, and whether this is a stable sink or an unstable source depends on the parameters for the IS curve and the Phillips curve. Under certain parameters, the zero interest rate equilibrium is a sink. Given that the absolute value of the unique root in the dynamic system when approaching equilibrium is $|\alpha\beta r'(\pi)|$, it is possible to change the equilibrium to a source by increasing the absolute value of $r'(\pi)$. Since a monetary policy rule becomes passive in the neighborhood of the zero interest rate equilibrium, this amplifies the increase in the real interest rate that accompanies the decline in the inflation rate. By doing this, it is possible to change the equilibrium to a source. Thus, in the phase diagram shown in Figure 11, it is possible to choose a path that converges on a normal equilibrium for a wide range of initial inflation rates.[30]

Under the combination (A.2) and (B.2), when output falls below its natural level under zero interest rates, there is also a possibility of a deflationary spiral, in which both the inflation rate and output decline without bound. The possibility of a deflationary spiral in such a model has been shown by Iwata (2002), Kaizuka (2002), Reifschneider and Williams (2000), and Taylor (2000).

D. When inflation is not a state variable

When the inflation rate adjusts instantaneously, on the other hand, the following can be said. Since in this case neither π nor y is a state variable, unless the equilibrium is a unique source, a unique solution cannot be obtained. In this case, both π and y immediately jump to equilibrium and stay there. When such a solution is possible depending on the monetary policy rule chosen, we assume that the rule enabling the solution is chosen. With either (B.1) or (B.3) in the model, however, a saddle point equilibrium is possible irrespective of the monetary policy chosen. If the equilibrium becomes a saddle point and the initial terms of the two variables are not given, at any point along the saddle point path, all the paths converging

[29] In section II, we defined a deflationary trap as a condition in which zero interest rates and deflation are persistent and the quantity theory of money does not hold. Since we do not specify the rate of monetary growth in section III, however, we cannot exclude the possibility that the quantity theory of money holds under a zero interest rate equilibrium. Although the difference is only slight, to keep the distinction clear in our terminology, when zero interest rates become an equilibrium in the dynamic system in section III, we use the term zero interest rate equilibrium. Since zero interest rate equilibria include a deflationary trap, in section III it is possible to interpret an exit from a deflationary trap as an exit from a zero interest rate equilibrium.

[30] Depending on the parameter settings, a periodic solution is possible. Although they use a model that differs substantially in structure, Benhabib, Schmitt-Grohè and Uribe (2002b) analyze the possibility of a periodic solution. Using an interest rate rule in which the current interest rate is set based on the past rate, they showed that a periodic solution cannot be ruled out if the coefficient for the past rate is less than one, but it can be excluded if it is larger than one.

to the saddle point are a solution. That is, this effectively gives rise to indeterminancy. [31] The combinations including either (B.1) or (B.3) effectively lead to indeterminancy irrespective of the monetary policy chosen. The passive monetary policy (C.2) leads to a saddle point under positive interest rates. In contrast, under an active monetary policy (C.1), a source is possible under positive interest rates, but when interest rates reach zero, monetary policy effectively becomes passive, and thus a saddle point develops just as with (C.2). In this paper, we view the ability to generate a source under positive interest rates as an advantage, and assume that (C.1) is selected.[32] Based on the above argument, Table 3 shows the optimal monetary policy choice for the five combinations of (A) and (B).

The economy's behavior can be divided into two types. In combinations of (A) and (B), the two behaviors are divided in the same way as when the inflation rate does not adjust immediately. That is, the same behavior occurs for both (A.1) and (A.2), and it is the selection of (B) that determines the economy's behavior. With (B.2), monetary policy (C.2) is selected and the economy immediately jumps to a unique normal equilibrium. Accordingly, in this case it can be assumed that the behavior is the same as in a model with flexible prices. When either (B.1) or (B.3) is selected, the economy could go toward either of two equilibria, and the settings chosen here do not determine which. One equilibrium is a source under positive interest rates, and one solution is for the economy to immediately jump to this equilibrium. The other equilibrium is a saddle point under zero interest rates.

E. Exiting from zero interest rates

Next we look at the situation when the economy attains, from among the patterns outlined in Tables 2 and 3, a zero interest rate equilibrium, and consider how to exit that equilibrium.

(1) We start by looking at when a zero interest rate equilibrium develops owing to the interest rate level having fallen too low under a passive monetary policy rule, as shown in Figure 12 (A). In this case, if interest rates are raised as in Figure 12 (B), it becomes possible to reach a normal equilibrium. Figure 13 shows the combination (A.1), (B.2), and (C.2) when inflation is not a state variable, representing with a dotted line the curve $\dot{y}=0$ when the money policy rule is consistent with the zero interest rate equilibrium, and representing with a solid line the same curve when the monetary policy rule is consistent with the normal equilibrium. In this case, when interest rates are raised, the inflation rate rises by exactly the

[31] Calvo (1983) showed that the combination (A.1), (B.1), and (C.2) leads to real indeterminancy.
[32] Kerr and King (1996) showed that there was a unique solution for the combination (A.1), (B.1), and (C.1) when ignoring the possibility of zero interest rates, and our argument coincides with this.

amount of the interest rate hike, and the economy immediately jumps from the zero interest rate equilibrium E_0 to the new equilibrium E_1. Similarly, under the combination (A.2), (B.2), and (C.2), the economy immediately moves to a normal equilibrium. Because this is the same behavior as under flexible prices, there is no need to consider additional elements in the adjustment process.

(2) There are three potential combinations under a passive monetary policy rule when inflation is a state variable: (A.1) and (B.1), (A.1) and (B.3), and (A.2) and (B.1). The saddle path towards equilibrium has the same dynamic characteristics in all three. In this case, as well, it is possible to attain a normal equilibrium if interest rates are raised as in Figure 12 (B). Figure 14 shows how the economy reacts to policy change. We assume first that the economy is at the zero interest rate equilibrium E_0. The economy's dynamics are changed when interest rates are raised, and the new saddle path shifts upwards. Because the inflation rate does not adjust instantaneously, however, output must first be lowered in order to get on the new saddle path. Subsequently, inflation gradually rises, and output returns to its natural level. In other words, if interest rates are raised to increase inflation, since the inflation rate does not adjust immediately, the real interest rate rises temporarily and causes output to decline. This is an additional element to consider in the adjustment process.

(3) Under an active monetary policy with inflation not a state variable, the zero interest rate equilibrium is a saddle point, and an infinite number of solutions converging on this point exist. This is the state described by Benhabib, Schmitt-Grohè and Uribe (2002), and if a fiscal policy stance violates the transversality condition, the economy immediately moves to a normal equilibrium that is a source. Here again, the economy behaves in the same way as under flexible prices when adopting the same policy prescriptions, and thus there is no need to consider additional elements in the adjustment process.

Another theoretically conceivable way out of a deflationary trap is to change the monetary policy rule and raise the interest rate when inflation is low, as shown in Figure 15. In this case, there will no longer be a nominal interest rate that accommodates both deflation and the natural rate of interest, the only equilibrium will be at point E, and the economy will quickly move to that point, which is a normal equilibrium. Furthermore, the elimination of deflation will result in an increase in treasury payments from the central bank, thereby creating room for a tax-cut by the government even when holding government debt constant. Therefore, there is no difference in the variable movements that result between escaping from deflation by pursuing non-Ricardian fiscal policies and by doing so with monetary policy as in Figure 15. Outside of equilibrium, the policy approaches become different.

Under the rule shown by Figure 15, however, there is a large, discontinuous change in nominal interest rates, and at the point of discontinuity the decline in inflation brings an increase in the nominal interest rate. Benhabib, Schmitt-Grohè and Uribe (2001) questioned the plausibility of implementing such a policy. If non-Ricardian fiscal policies are not effective, however, it seems worthwhile to consider a policy of raising interest rates. As argued in section II.E, it is also possible to view Japan's current fiscal stance as non-Ricardian in nature. There is no reason why only one of these policies can be adopted, and it would also be workable to both adopt a non-Ricardian fiscal regime and hike interest rates via a change in rules as shown in Figure 15.

(4) Under an active monetary policy with inflation as a state variable, there are two possible dynamic behaviors. In the first, with the combination of (A.2), (B.2), and (C.1), the zero interest rate equilibrium is a source and thus unstable, but with this combination ruled out, the economy converges on a path toward a normal equilibrium, as shown in Figure 9. If the inflation rate departs only slightly from its level at the zero interest rate equilibrium, the economy will converge on a path toward the normal equilibrium and thus the likelihood that the zero interest rate equilibrium will be achieved is extremely small. This suggests that there is no need to give any special consideration to a policy move aimed at exiting such an equilibrium.

(5) Looking at one more combination under an active monetary policy with inflation as a state variable, (A.2), (B.2), and (C.1), the zero interest rate equilibrium is unstable and, if output is below its equilibrium level, both the inflation rate and output will decline continuously. Under such conditions, an interest rate hike would have the negative effect of lowering income, and the economy would not return to the normal equilibrium. Nevertheless, the long-run result of a downward spiral of output is the same as prior to policy implementation. To return to the normal equilibrium would require somehow increasing demand and raising output above its natural level.

Determining whether such a deflationary spiral is realistic would require a more detailed examination. Such a path would occur under an accelerationist Phillips curve, where the tacit assumption of a constant expected inflation rate plays a crucial role. That is, for an output gap to develop there must be an error in forecasting the inflation rate, and since people continue to expect the same inflation rate as before even with the inflation rate declining, this forecasting error becomes larger. While it is unrealistic to assume perfect foresight wherein forecasting error does not occur, it would be more natural to assume inflation expectations to be modified over the long run. This suggests the need for serious reservations over how realistic the result in (5) is.

In consideration of the above, when prices are not flexible, it is clearly important to be aware of the possibility of inflation not increasing in step with the rise in interest rates. In this case, real interest rates would rise and cause a downward adjustment to output, as described in the second of the five patterns outlined above.

F. Examining the mechanisms that determine the inflation rate

As already touched upon in footnote 28, the 20 dynamic models we examine here include models that are inconsistent with the price determination mechanism normally posited by the Phillips curve. We now look at this in more detail. To make our handling of the state variable more clear, we will describe here a discrete-time model. The behavioral equations that correspond to (A) with (C) substituted in are

(A.1′) $y_{t+1} - y_t = \alpha[r(\pi_t) - \bar{r}]$

(A.2′) $y_t - \bar{y} = -\alpha[r(\pi_t) - \bar{r}]$

while those that correspond to (B) are[33]

(B.1′) $\pi_t = \beta[y_t - \bar{y}] + \pi_{t+1}$

(B.2′) $\pi_t - \pi_{t-1} = \beta[y_{t-1} - \bar{y}]$

(B.3′) $\pi_t - \bar{\pi} = \beta[y_t - \bar{y}].$

Looking at the Phillips curve, the inflation rate is a state variable in equation (B.2′), since it is determined by variables from the prior period, but not in equations (B.1′) and (B.3′), since it does not depend on past variables. Accordingly, there is no problem in interpreting inflation as a state variable in (B.2) and not doing so in (B.1) and (B.3). These correspond to the third, fourth and fifth patterns described in section III.E, all of which are associated with an active monetary policy.

[33] Regarding the timing of the inflation rate used in (A), under a monetary policy that targets the expected inflation rate, the expected inflation rate one period later is used, while under a monetary policy that adjusts the nominal interest rate based on the past inflation rate, the past inflation rate must be used. Nevertheless, these timing differences do not have a significant impact on our argument in this paper, so for simplification, we use a formulation that applies directly to a continuous-time model.

Meanwhile, our approach to the state variable appears to be inconsistent, since in pattern (1) we interpret the inflation rate as a non-state variable in equation (B.2) but in pattern (2) we interpret inflation as a state variable in equations (B.1) and (B.3). In all of these cases, a passive monetary policy is chosen. A strict interpretation of the inconsistency would probably require excluding both patterns (1) and (2) from our model assumptions.

Nevertheless, the Phillips curve considered in this paper adopts a very simple formulation in regards to the time structure of the variables. Empirical research on the Phillips curve normally includes a lag variable for the inflation rate and the unemployment rate (the output gap in our paper), while the actual inflation rate and income probably have a fairly complex lag structure. Accordingly, a wide range of dynamic behaviors can occur. It is probably more important to focus on the fact that the analysis summarized in Tables 2 and 3 provides for a comprehensive set of conclusions, including both a negative correlation and a positive correlation between the inflation rate and output. If dynamic behavior under a Phillips curve with a complex lag structure were to be included in the results summarized in Tables 2 and 3, our analysis could be interpreted as abbreviated models aimed at expressing the various possibilities that could result from the lag structure of the Phillips curve. In that case, we believe it would be inappropriate to exclude some models based on a strict interpretation of their inconsistencies.

In fact, below we will give examples of models that show the same dynamic behavior as in patterns (1) and (2) above using natural assumptions for the state variable by slightly altering the Phillips curve equation.

We look first at (1). The accelerationist Phillips curve is structured such that the inflation rate is determined by its previous levels, and this makes it difficult to consider a model where the inflation rate is not a state variable when using the accelerationist Phillips curve. Nevertheless, when considering a New Keynesian Phillips curve where the output gap has the opposite sign,

$$\pi_t = -\beta[y_t - \bar{y}] + \pi_{t+1}, \tag{36}$$

the inflation rate depends on current and future variables and thus is a jumping variable. The model has the same dynamic behavior as under the accelerationist Phillips curve, while under a passive monetary policy rule the equilibrium is a source and a unique solution is possible. In other words, we get the same result as in (1).

Nevertheless, it is impossible to apply micro foundations to the New Keynesian Phillips curve in this framework, since doing so causes the output gap's impact on the inflation rate to have the wrong sign. Consequently, the argument for ruling out pattern (1) is persuasive. Nevertheless, the result in (1) is the same as when prices are flexible as well as when the inflation rate is a jumping variable, so even if (1) is ruled out, the possibility of ending with the same result does not disappear. Assuming the priority is on the model results, our focus should be on whether to exclude an adjustment mechanism that assumes a positive correlation between inflation and output, as in (2).

Moving to pattern (2), we slightly transform (B.3′) to create an equation whereby the inflation rate is determined by the output gap from the prior period.

$$\pi_t - \bar{\pi} = \beta\left[y_{t-1} - \bar{y}\right] \tag{37}$$

Combining equation (37) with (A.2′), we can express the inflation rate dynamics as

$$\pi_t - \bar{\pi} = -\alpha\beta\left[r(\pi_{t-1}) - \bar{r}\right]. \tag{38}$$

The inflation rate is a state variable in this case, since it is a function of the inflation rate from the prior period. Accordingly, for the dynamic system to be stable and not cause any oscillations, this condition must be satisfied.

$$0 < -\alpha\beta r' < 1 \tag{39}$$

It is clear from the inequality constraint on the LHS of equation (39) that monetary policy must be passive. Also, since the following is a linear approximation of (A.2′),

$$y_t - \bar{y} = -\alpha r'\left[\pi_t - \bar{\pi}\right], \tag{40}$$

it follows that under a passive monetary policy, the path to equilibrium must show a positive correlation between output and the inflation rate. This leads to a dynamic system in which inflation is a state variable and the correlation between output and inflation is positive, the pattern described in (2) above.

The combination of (A.2) and (B.3) is not analyzed in Table 2 because it is static. That said, (A.2) and (B.3) are equivalent to the IS curve and the classic Phillips curve, and many of the arguments developed in undergraduate macroeconomics textbooks are explained through this framework, making it an important model. If it were possible to make a natural dynamic extension in discrete time, we believe this model would be well worth analyzing. Accordingly, if we had begun section III using discrete-time models, we would have had to include this combination within that portion of Table 2 where π and y have a positive correlation. In a discrete-time model, however, it is also possible to include timing for the variables that is different than that described above, in which case the 20 types of models initially envisioned would not be sufficient. By using continuous-time models, we had to oversimplify the complex timing, and the disappearance of the combination of (A.2) and (B.3) before can be viewed as compensation for this.

G. Why raise interest rates?

The policy we are proposing here of raising interest rates under deflationary conditions appear to fly in the face of conventional wisdom in macroeconomics. We will therefore attempt to explain how we reached this conclusion from a different angle.

A discussion of policies aimed at lowering the inflation rate can be found in intermediate macroeconomic textbooks. In the classic explanation, first interest rates are raised, and if the inflation rate does not immediately adjust, the real interest rate increases and output decreases. Eventually, however, the inflation rate declines and output returns to its equilibrium level. This is because raising interest rates at the outset amounts to an active monetary policy rule. Since lowering the inflation rate by 1% causes at least a 1% decline in the nominal interest rate, if the inflation rate declines so does the real interest rate. Accordingly, if the real interest rate started out equal to the natural interest rate, under the new, lower inflation target the real interest rate will wind up lower than the natural interest rate. Therefore, as shown in Figure 16, there is a need to equalize the inflation target with the natural interest rate by adopting a rule of raising interest rates. In a reversal of this policy, the path to follow for raising the inflation rate is to lower interest rates initially, which raises output and then causes the inflation rate to increase.

Of the five patterns described in section III.E, (1) and (2) are passive monetary policy rules. In this case, if the inflation rate is going to be raised with interest rates at their natural level, there is a need to increase the nominal interest rate as in Figure 12. With the active monetary policy rules of patterns (3) to (5), there is a need to distinguish between an interest rate hike implemented by following the rule and an interest rate hike implemented by changing the rule. When monetary policy is proactive, interest rates are raised via a rule change, as in Figure 15. Since a passive monetary policy rule is adopted under zero interest rates, this is another reason why interest rates must be increased to spark inflation under such a scenario.

IV. Conclusion

If the central bank commits to zero interest rates when the natural rate of interest is positive and remains so into the future, there is a risk that deflation will persist. This paper examines ways to escape from such a deflationary trap. In doing so, we take account of the imperfect understanding of the economy's short-term behavior, while also paying attention to differences in the results from each policy option. The required policy measures are a money-financed tax-cut combined with an interest rate hike and a commitment to future monetary growth. The amount of tax-cut required is exactly equal to the increase in the central bank's payments into the national treasury resulting from the increase in inflation (higher nominal interest rates), which allows the fiscal authorities to adhere to a disciplined fiscal policy that maintains both government debt and the primary balance at stable levels. When prices are flexible, the appropriate level of inflation quickly arises in the economy. When prices are sticky, on the other hand, income may decline temporarily, but this is the price that must be paid to eliminate deflation.

We have an incomplete understanding of the interest rate's natural level, and this makes it impossible to provide a definitive answer to the question of whether the persistent deflation now confronting the Japanese economy is a deflationary trap or a liquidity trap that can be explained by a temporary drop in the natural rate of interest. If in a liquidity trap, the proper policy response is to maintain interest rates at zero, but if in a deflationary trap, interest rates must be raised. This is Japan's dilemma: there is no policy that is robust for both a liquidity trap and a deflationary trap.

This becomes a serious problem when devising policies to move interest rates off of zero back to normal levels. The Bank of Japan has declared that it will continue its current quantitative easing policies until the consumer price index (nationwide, excluding fresh food) stabilizes at a zero or positive rate of growth in year-on-year terms. This policy commitment is aimed at reining in long-term interest rates and, with overnight interest rates already at zero, at extracting additional easing benefits. Nevertheless, if in fact the economy is in a deflationary trap, the continuation of zero interest rates feeds into deflationary expectations, creating the possibility that the conditions required to abandon the current policy stance may never be achieved. Accordingly, even within an environment where the natural rate of interest is deemed to be positive (e.g., when we observe rising productivity or improving economic growth), if deflation persists, there may be a need to consider applying policies aimed at escaping from a deflationary trap. In this sense, we believe the conditions required for abandoning the current policy stance should include, in addition to consistently positive growth in the CPI, a reference to the trend in real GDP.

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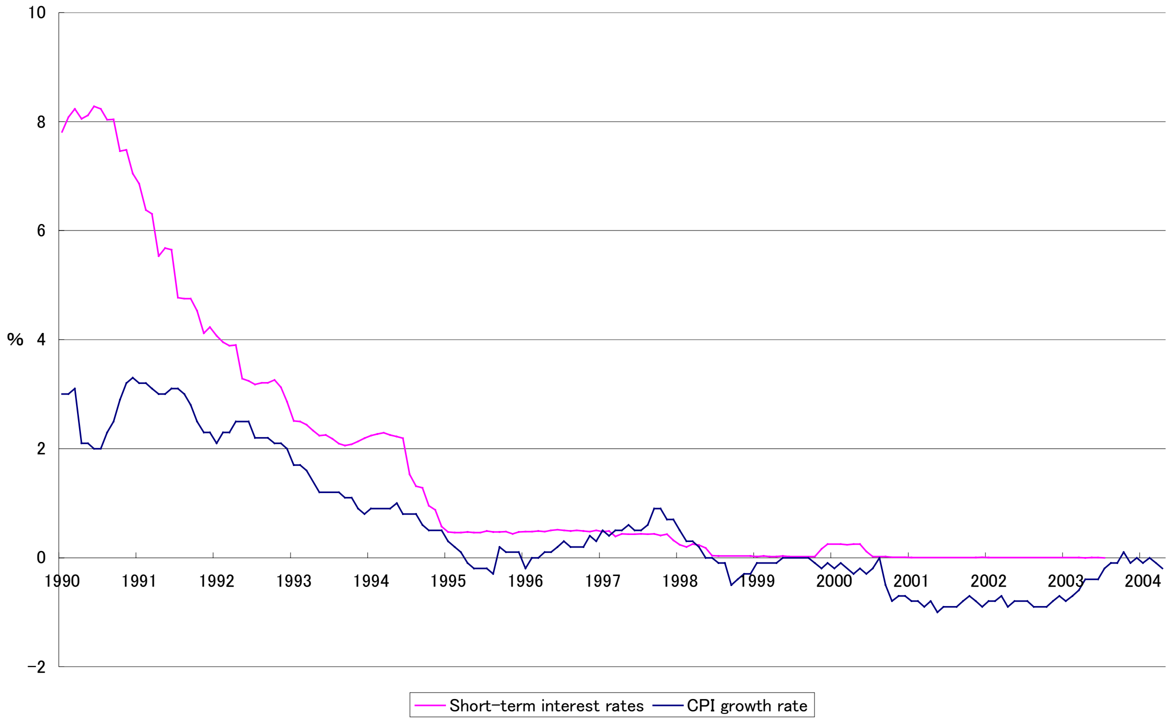
Figure 1 Short-term interest rates and growth in the consumer price index



Note: The short-term interest rate is the (average) uncollateralized overnight call rate. The CPI growth rate is the year-on-year change in the consumer price index (nationwide, excluding fresh foods). For prices following the consumption tax hike in FY1997, we have deducted 1.5% from the actual price to correct for the price increase attributable to the tax hike.

Source: Consumer price index, from the Statistics Bureau, Ministry of Internal Affairs and Communications.

Figure 2 Dynamic path with monetary targeting (when $\mu > -\bar{r}$)

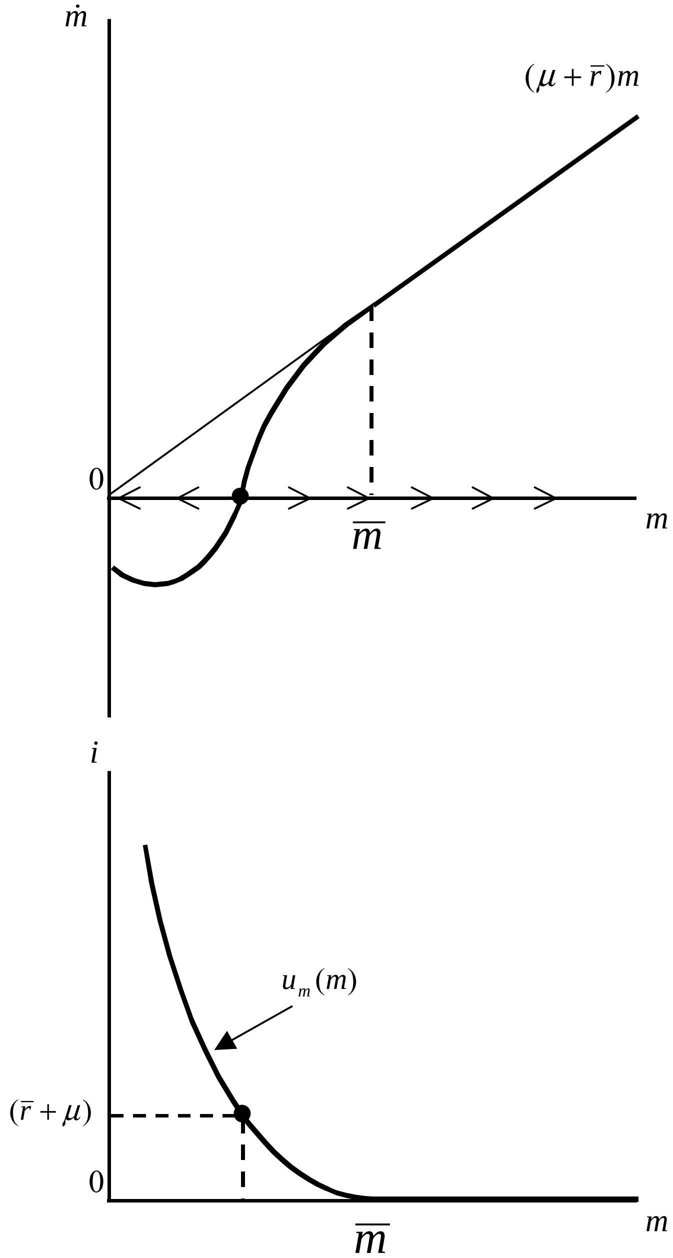


Figure 3 Dynamic path with monetary targeting (when $\mu = -\bar{r}$)

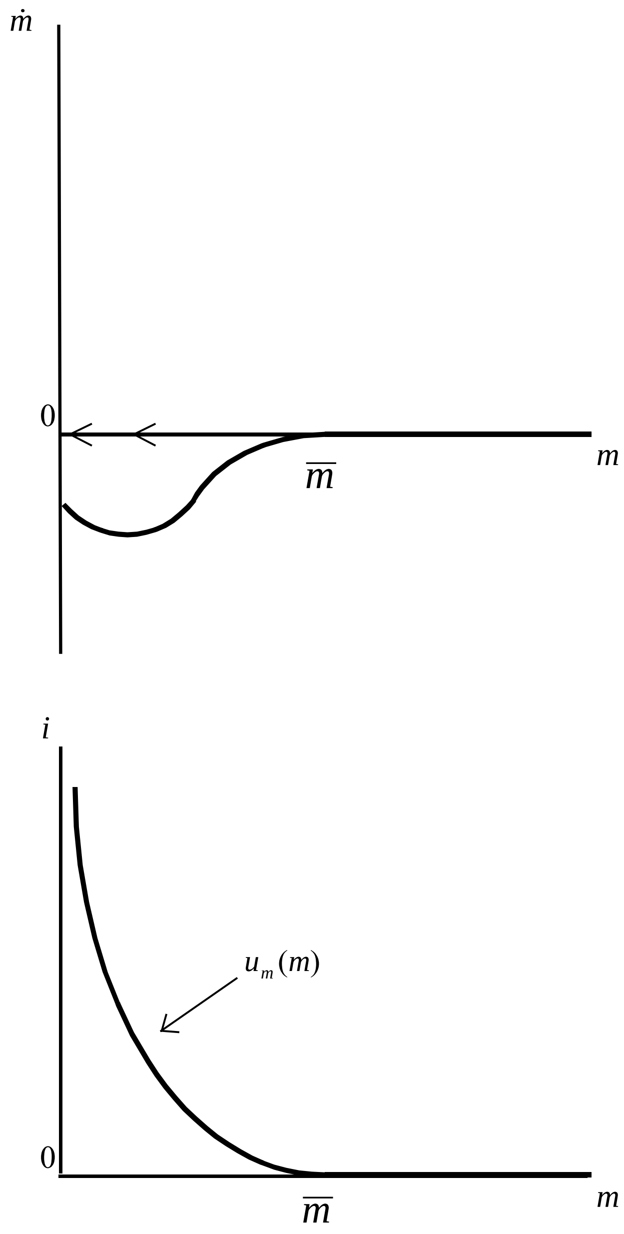


Figure 4 Dynamic path with monetary targeting (when $\mu < -\bar{r}$)

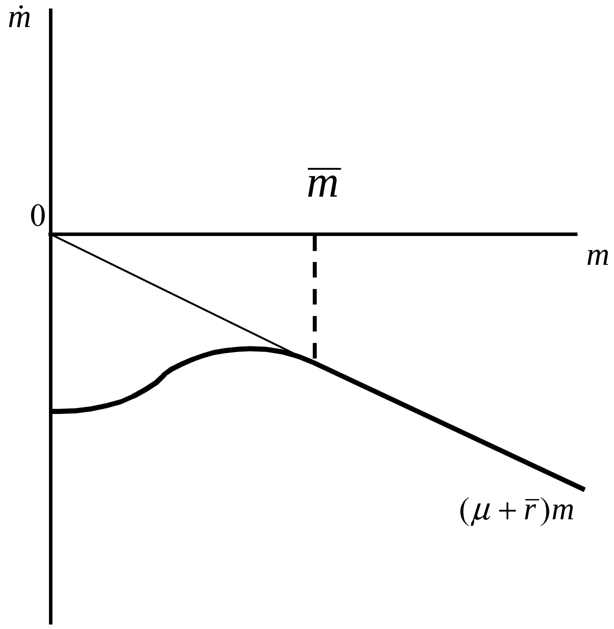


Figure 5 When emergence of a deflationary trap is possible

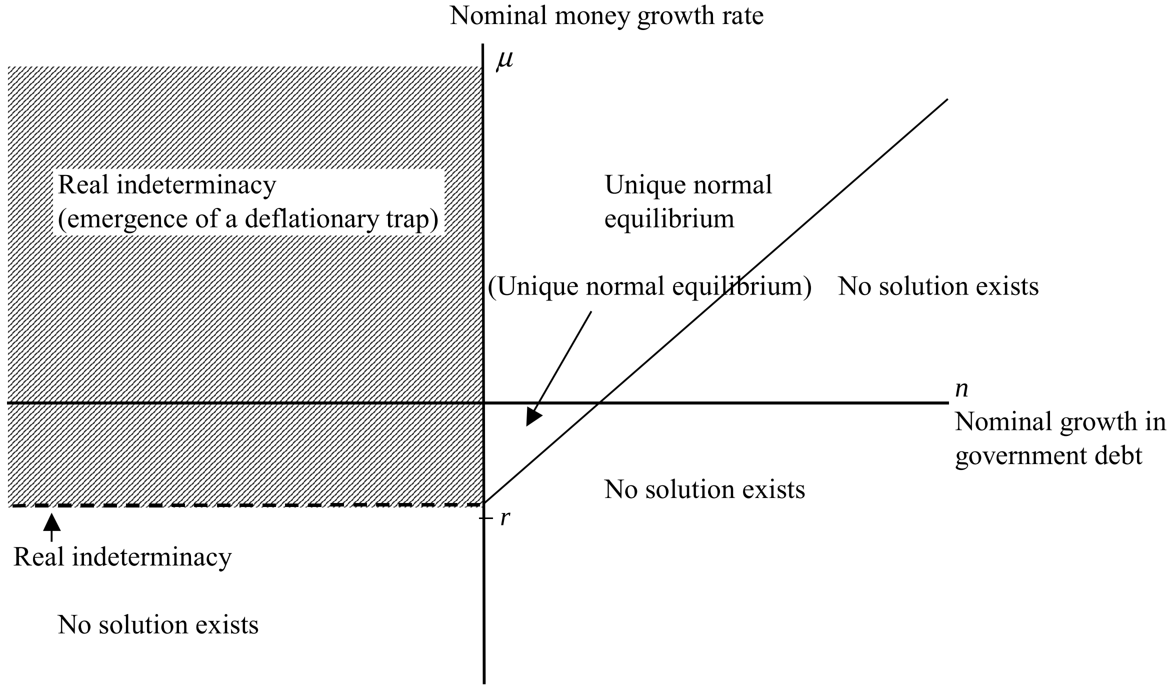


Figure 6 Monetary base over time

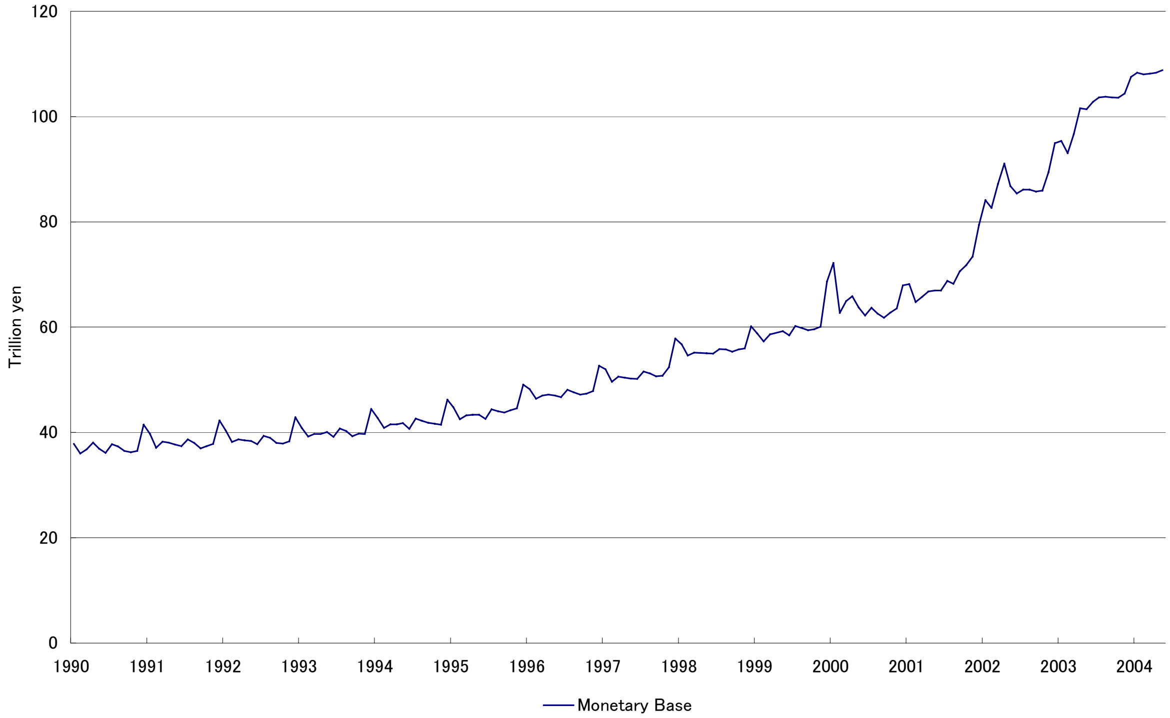


Table 1 Fiscal stance and the presence of a deflationary trap

		Nominal money growth rate (μ) held constant	
		$\mu \geq 0$	$0 > \mu > -\bar{r}$
Real government debt (*a*) held constant		Present	
Government bonds owned by the private sector (*b*) held constant in real terms		Absent	Present
Growth in nominal government debt (*n*) held constant	$n \geq 0$	Absent	
	$n < 0$	Present	

Figure 7 Changes in economic variables when interest rates are raised
(with instantaneous adjustment of the inflation rate)

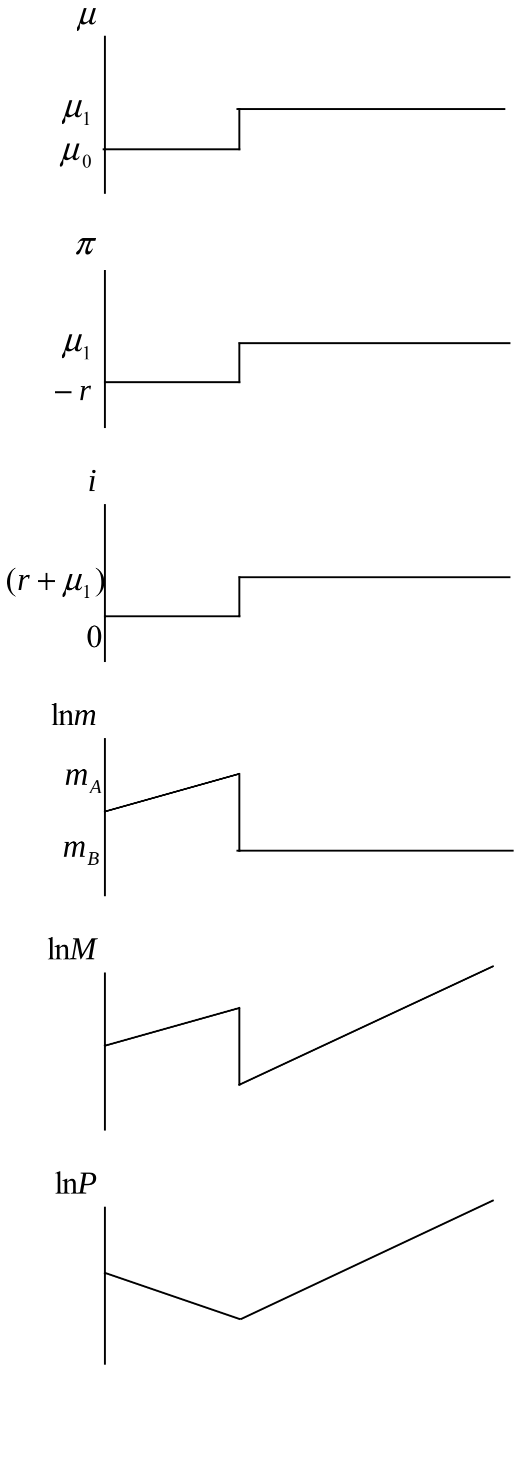


Figure 8 JGBs outstanding (as a percentage of GDP, excluding FILP bonds)

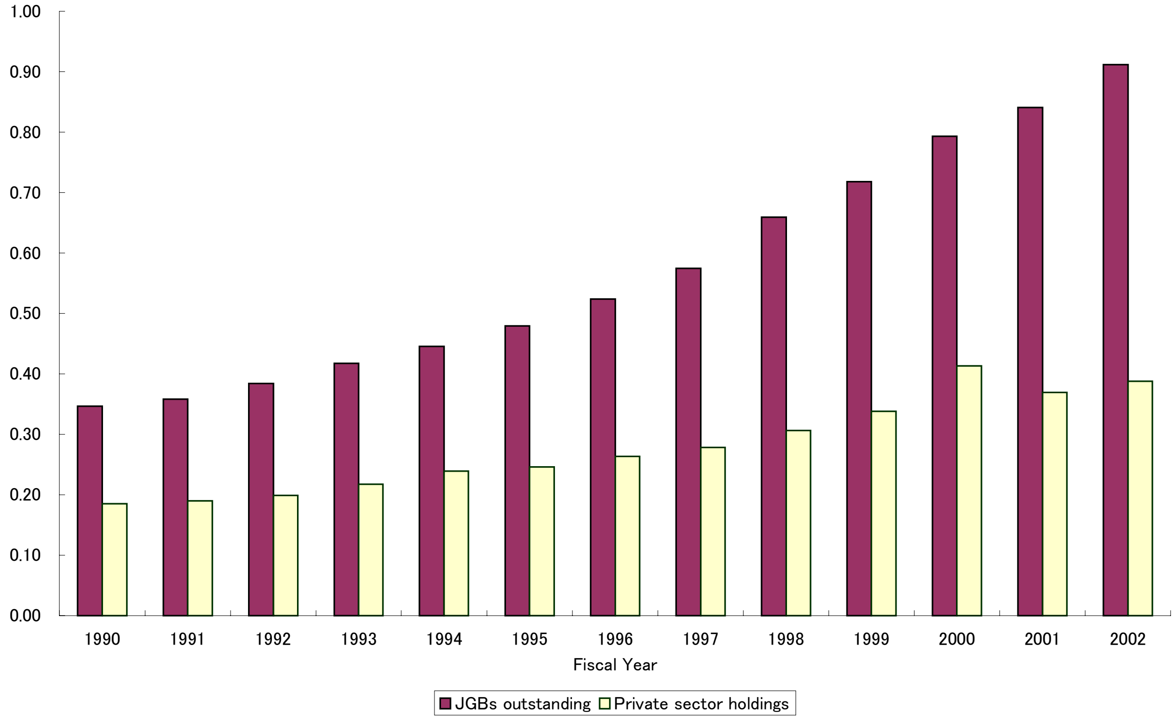
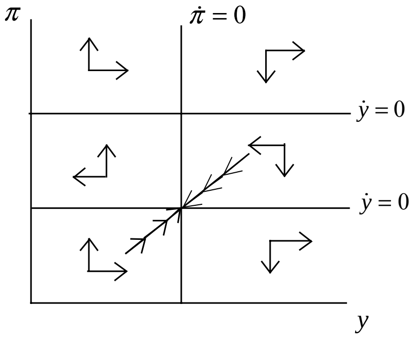


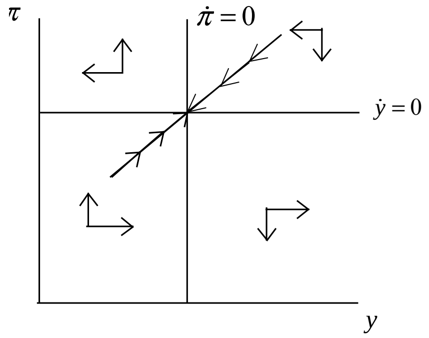
Figure 9 Phase diagram of short-term adjustments

(1)
(A.1) $\dot{y}_t = \alpha[r(\pi_t) - \bar{r}]$
(B.1) $\dot{\pi}_t = -\beta[y_t - \bar{y}]$

(C.1) $r'(\pi) > 0$

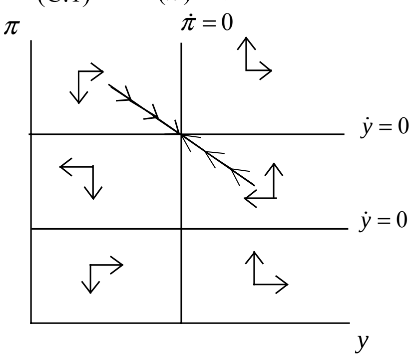


(C.2) $r'(\pi) < 0$

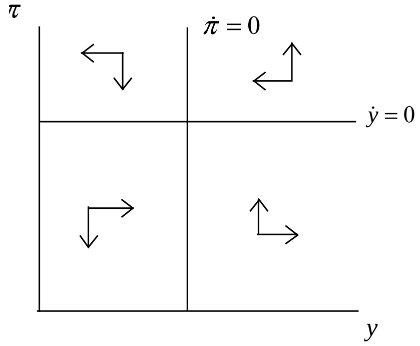


(2)
(A.1) $\dot{y}_t = \alpha[r(\pi_t) - \bar{r}]$
(B.2) $\dot{\pi}_t = \beta[y_t - \bar{y}]$

(C.1) $r'(\pi) > 0$

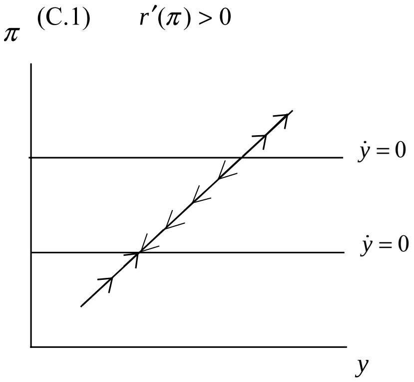


(C.2) $r'(\pi) < 0$



(3)
(A.1) $\dot{y}_t = \alpha[r(\pi_t) - \bar{r}]$
(B.3) $\pi_t - \bar{\pi} = \beta[y_t - \bar{y}]$

(C.1) $r'(\pi) > 0$



(C.2) $r'(\pi) < 0$

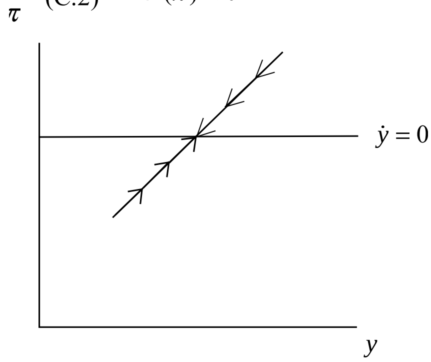
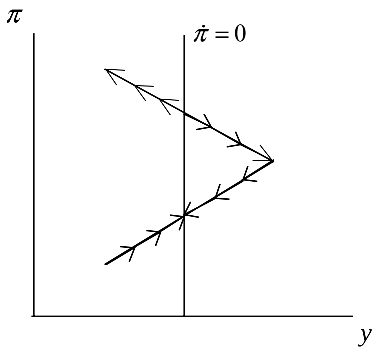


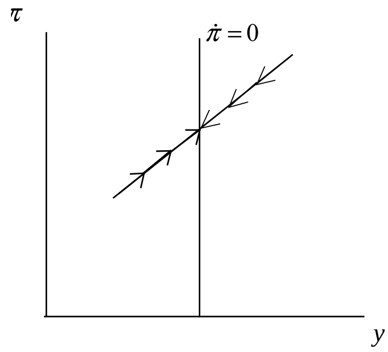
Figure 9 Phase diagram of short-term adjustments (continued)

(4)
(A.2) $y_t - \bar{y} = -\alpha[r(\pi_t) - \bar{r}]$
(B.1) $\dot{\pi}_t = -\beta[y_t - \bar{y}]$

(C.1) $r'(\pi) > 0$

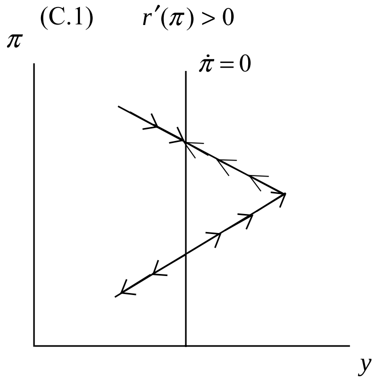


(C.2) $r'(\pi) < 0$



(5)
(A.2) $y_t - \bar{y} = -\alpha[r(\pi_t) - \bar{r}]$
(B.2) $\dot{\pi}_t = \beta[y_t - \bar{y}]$

(C.1) $r'(\pi) > 0$



(C.2) $r'(\pi) < 0$

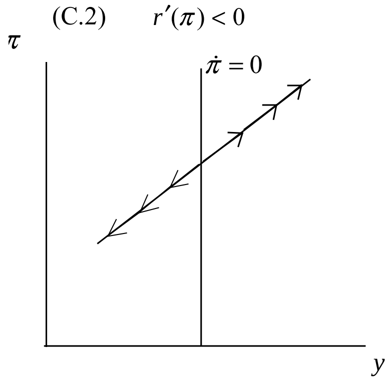
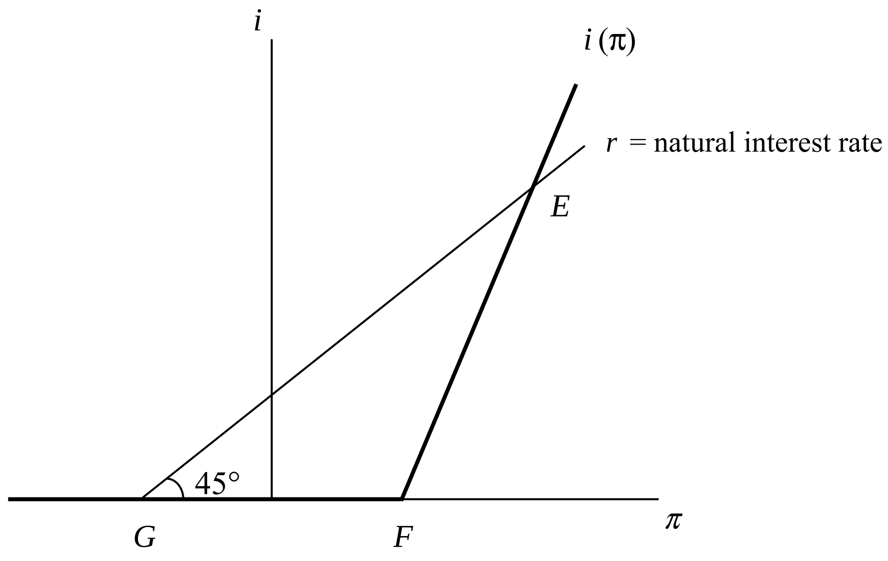


Figure 10 Active monetary policy and the zero bound on interest rates

(A)



(B)

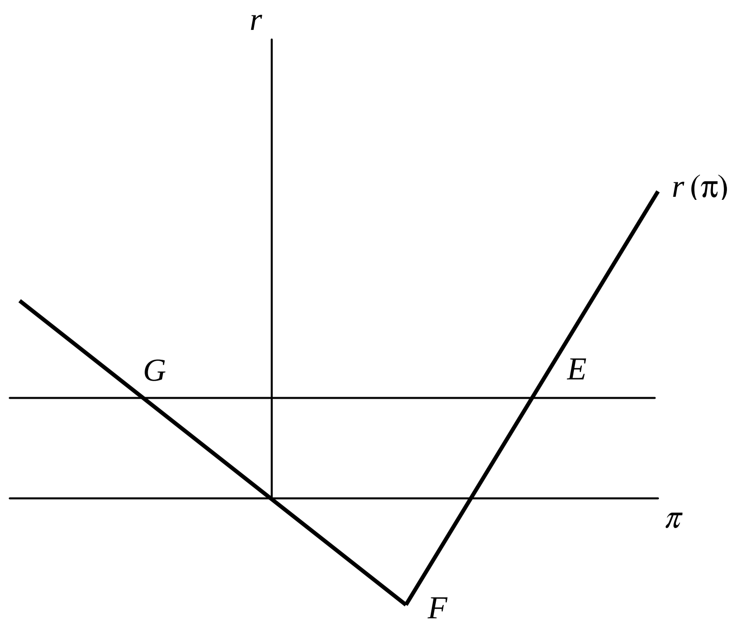


Table 2 The economy’s path and monetary policy (with inflation as a state variable)

<table>
<tr><th></th><th>(A.1)
$\dot{y}_t = \alpha[r(\pi_t) - \bar{r}]$
Expectational IS curve</th><th>(A.2)
$y_t = -\alpha[r(\pi_t) - \bar{r}]$
IS curve</th></tr>
<tr><td>(B.1)
$\dot{\pi}_t = -\beta[y_t - \bar{y}]$
New Keynesian Phillips curve</td><td colspan="2" rowspan="2">(C.2) $r'(\pi) < 0$
Positive correlation between π and y</td></tr>
<tr><td>(B.3)
$\pi_t - \bar{\pi} = \beta[y_t - \bar{y}]$
Phillips curve</td></tr>
<tr><td>(B.2)
$\dot{\pi}_t = \beta[y_t - \bar{y}]$
Accelerationist Phillips curve</td><td colspan="2">(C.1) $r'(\pi) > 0$
Negative correlation between π and y.
Also, a deflationary spiral under zero interest rates.</td></tr>
</table>

Figure 11 Monetary policy choice under the combination (A.2) and (B.2)

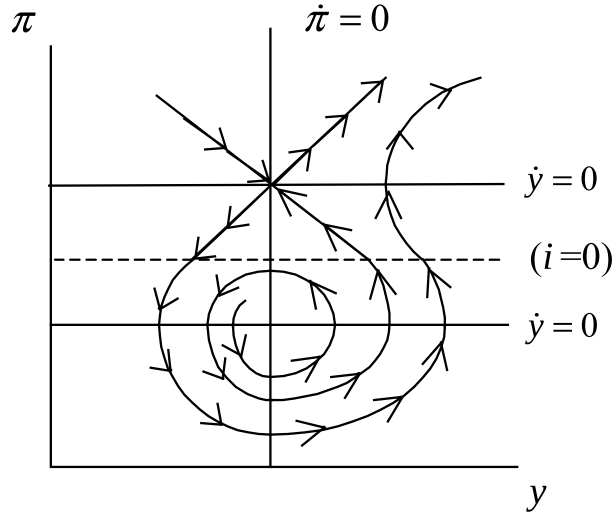


Table 3 The economy's path and monetary policy (with inflation not a state variable)

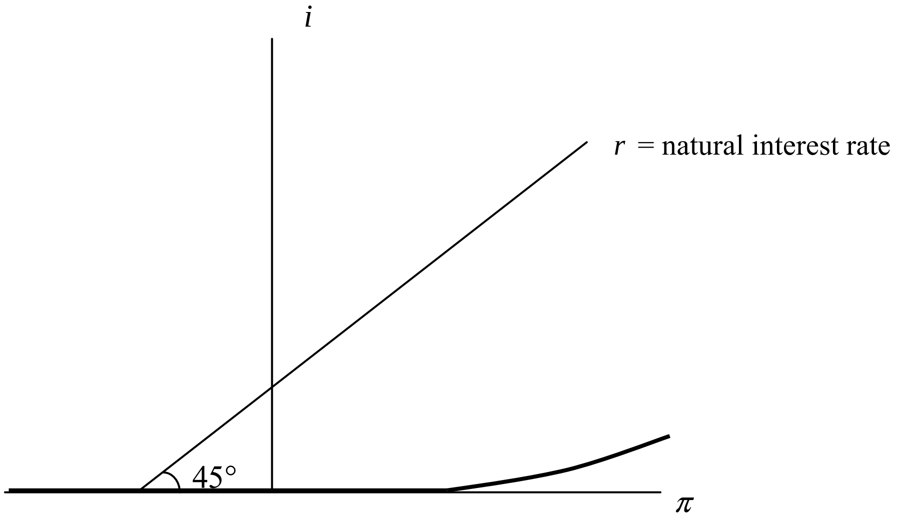
<table>
<tr><td></td><td>(A.1)
$\dot{y}_t = \alpha[r(\pi_t) - \bar{r}]$
Expectational IS curve</td><td>(A.2)
$y_t = -\alpha[r(\pi_t) - \bar{r}]$
IS curve</td></tr>
<tr><td>(B.1)
$\dot{\pi}_t = -\beta[y_t - \bar{y}]$
New Keynesian
Phillips curve</td><td colspan="2" rowspan="2">(C.1) $r'(\pi) > 0$

Instantaneous adjustment to normal equilibrium,
or convergence to zero interest rate equilibrium.</td></tr>
<tr><td>(B.3)
$\pi_t - \bar{\pi} = \beta[y_t - \bar{y}]$
Phillips curve</td></tr>
<tr><td>(B.2)
$\dot{\pi}_t = \beta[y_t - \bar{y}]$
Accelerationist
Phillips curve</td><td colspan="2">(C.2) $r'(\pi) < 0$

Instantaneous adjustment to normal equilibrium</td></tr>
</table>

Figure 12 Escape from deflation under passive monetary policies

(A) Reaching zero interest rate equilibrium



(B) Interest rate hike

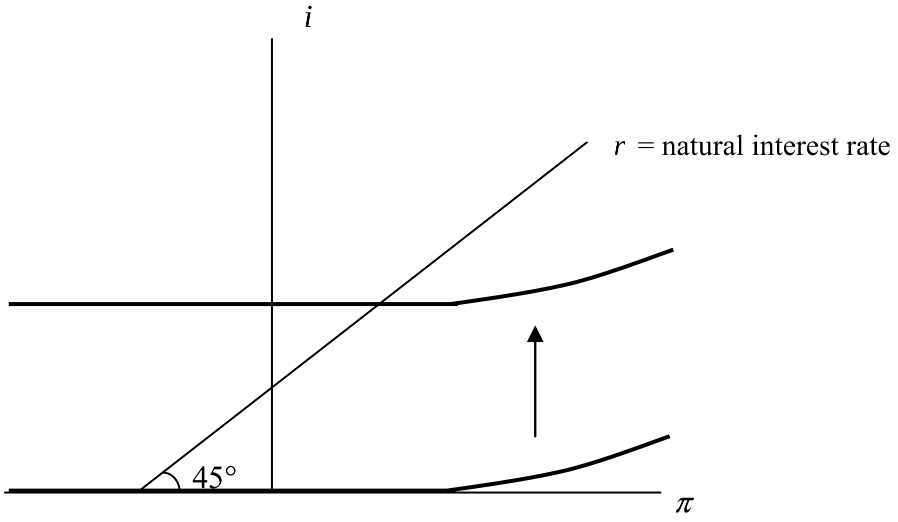


Figure 13 Escape from deflation when the inflation rate adjusts instantaneously

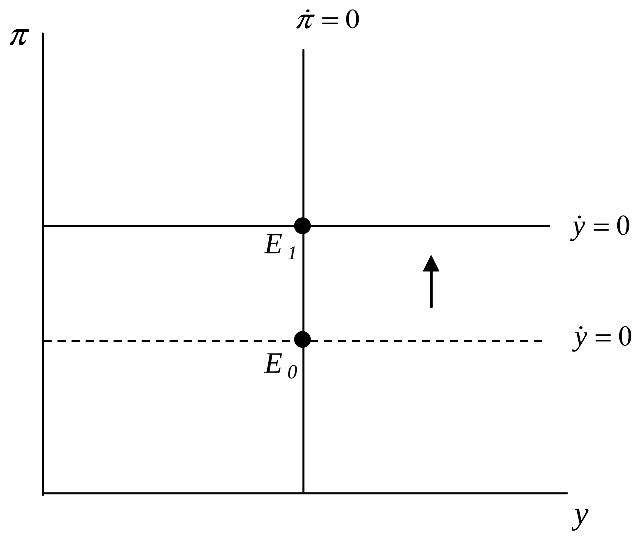


Figure 14 When the inflation rate adjustment lags

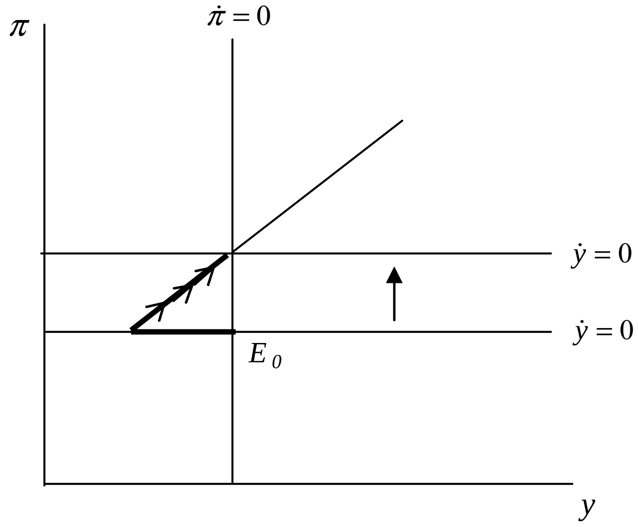


Figure 15 Escape from a deflationary trap by raising interest rates when inflation is low

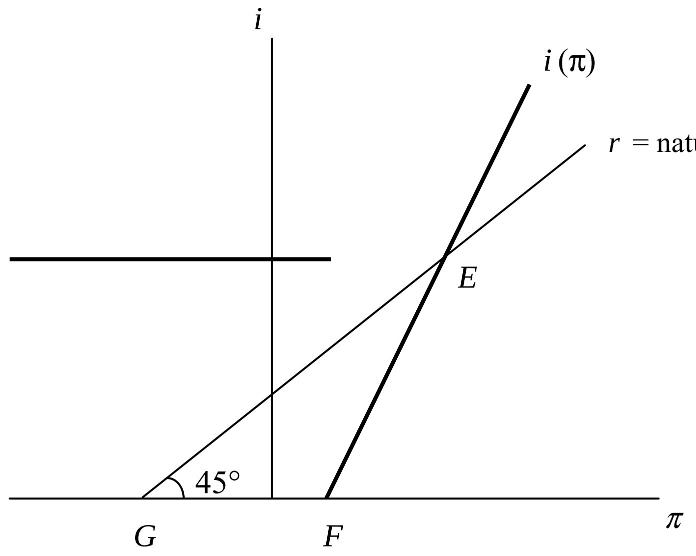


Figure 16 Raising interest rates under an active monetary policy rule