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Abstract

In this paper, we develop a quantitative, general-equilibrium business cycle model with imperfect common knowledge regarding technology shocks. We first show that the model has the ability to explain the short-run contractionary effects of technology improvements that are found by recent empirical studies such as Galí (1999) and Basu, Fernald, and Kimball (2002). In particular, the model predicts that a *positive* technology shock leads to a persistent *decline* in employment and a delayed, sluggish fall in inflation. Then we examine the role of monetary policy in stabilizing macroeconomic fluctuations originating from technology shocks. We show that monetary policy tends to fall short of accommodation of technology improvements when the central bank has only imperfect information on the state of the technology.

Key words: Technology Shocks; Imperfect Common Knowledge; Employment; Inflation; and Monetary Policy

JEL classification: E30, E50

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1. Introduction

Imperfect common knowledge, namely that individual economic agents do not share the same belief regarding exogenous shocks, has a long history in macroeconomics, dating back at least to Pigou (1929). In his explanation of the business cycle, Pigou emphasizes that firms are partially but not completely interconnected, possessing only limited information on what *others* are doing. Pigou then argues that these interconnections tend to promote the mutual generation of forecast errors across sectors, which in turn leads to substantial macroeconomic fluctuations.[1] In the 1970s, Lucas (1972; 1973) develops a macroeconomic model formalizing the idea that if markets are decentralized and participants in each market have only limited information about *other* markets, they misinterpret price signals and respond to monetary shocks they would not have responded to, had they had complete information. Subsequently, Lucas (1975) uses economy-wide *average* beliefs as a state variable, along with money and capital, to explain the persistent real effects of monetary shocks. More recently, Woodford (2003a) introduces monopolistically competitive pricing into an incomplete information model similar to Lucas, and thereby revives Phelps' (1983) insight elegantly that higher-order *average* beliefs play a crucial role in generating the persistent real effects of nominal disturbances.[2]

In this paper, we develop a quantitative, general-equilibrium business cycle model with imperfect common knowledge regarding *technology shocks*, drawing on Phelps (1983) and Woodford (2003a). Specifically, information on aggregate technology is assumed to come in the form of noisy signals to price-setting firms, but the true state of the technology never becomes

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[1] Developing Pigou's idea, Beaudry and Portier (2000) propose a model of business cycles in which recessions and booms arise as the result of difficulties encountered by agents in properly forecasting the economy's future needs in terms of capital.

[2] In the early literature, Townsend (1983) also stresses the importance of higher-order beliefs.

common knowledge: Even though firms may have relatively precise information about the underlying technology shocks, they lack information about *others*' expectations. However, since monopolistic competition makes optimal pricing decisions highly dependent on the prices set by other firms, higher-order average expectations (others' expectations of others' expectations ...) are necessary to forecast the behavior of others. Uncertainty about the higher-order expectations in turn causes firms to set prices that are far less sensitive to *their own* best estimates of the state of the technology.[3]

Relative to Woodford (2003a), we make two contributions. First, in our model business cycle fluctuations are driven by technology shocks leading to stochastic variations in the full-information equilibrium level of output, i.e., the "full employment output." By contrast, Woodford (2003a) abstracts from any kind of real shocks in order to highlight the fact that in contrast with the Lucas model, the existence of both real and nominal disturbances is not necessary to generate the monetary non-neutrality. We show, however, that the model can be more fully developed to allow for technology shocks, which gives us many interesting insights regarding the impulses and propagation mechanism driving business cycles as discussed below. Second, we explicitly model the monetary authority as varying the money supply in the face of noisy information about technology shocks just as with private firms. By doing so, we can explore the interaction between technology shocks and monetary policy, especially the potentially important role of monetary policy in stabilizing macroeconomic fluctuations originating from technology shocks. On the other hand, Woodford (2003a) treats monetary policy

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[3] Because this sort of information structure is reminiscent of a famous "beauty contest" example in Keynes' (1936) *General Theory* and because traditional monetarists stress the limitations of the information available to economic agents, we call such a model a "Beauty Contest Monetarist Model."

as an *exogenous* stochastic process of the money supply unlike his many other studies including Woodford (2003b).

Why do we care about technology shocks in the context of an imperfect information model that is originally designed to capture the monetary non-neutrality? The first compelling reason is that far less research has explored the dynamic effects of technology shocks in the imperfect information literature compared to the sticky price literature.[4] Second, although recent papers on the implementation of monetary policy (e.g., Orphanides, 2003) emphasize the importance of the informational limitations encountered by the central bank in measuring the full employment output, many, if not most, of these papers (implicitly or explicitly) assume that private agents have full information on technology shocks. This is not entirely satisfactory, because if private agents had perfect information on technology it would be relatively easy for the central bank to extract that information and estimate the full employment output quite accurately. This paper represents an effort to fill these gaps.

We first show that our model has the ability to explain the *contractionary* effects of technology improvements ― *positive* technology shocks lead to *declines* in labor input ―, which are found by the recent empirical studies such as Galí (1999) and Basu, Fernald, and Kimball (2002).[5] The intuition behind that result is straightforward. Since the quantity theory governs the

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[4] Cooley and Hansen (1995) is one of the most significant exceptions. Mankiw and Reis (2003) also examine the effects of productivity shocks, using a version of the "sticky information" model they originally propose in Mankiw and Reis (2002). Galí (1999; 2003), Dotsey (1999), Basu, Fernald, and Kimball (2002), Basu and Kimball (2003), and Galí, Lopez-Salido, and Valles (2003) investigate this issue using sticky price models.

[5] Galí (1999) identifies technology shocks using long-run restrictions in a structural VAR, while Basu, Fernald, and Kimball (2002) identify these shocks by estimating Solow-Hall style regressions with proxies for utilization. In both cases, they find significant *negative* correlations of hours worked with the identified technology shock. See also Francis and Ramey (2003) who reexamine Galí's results in depth. Shapiro and Watson (1988) also uncovered this result fifteen years ago, but it has gone unnoticed (See Figures 2 and 5 of their paper).

demand for money in our model, output is proportional to real balances. Assume, for the sake of argument, that the supply of money is fixed. Now suppose a positive technology shock hits the economy. Since each monopolistically competitive firm is unwilling to change its price owing to uncertainty about *others*' perceptions of that shock, the aggregate price level and hence real balances remain (almost) unchanged in the short run. Thus the firm's demand curve and sales do not change — this is an example of the "aggregate demand externality" pointed out by Blanchard and Kiyotaki (1987) —, whereas the firm can produce the same quantity of output using fewer inputs with improved technology. As a result, firms lay off workers and reduce hours despite the favorable technology shock. The important point to note is that our explanation of the contractionary technology improvements does not resort to *exogenous* price stickiness at all, in sharp contrast with the explanations advocated by Galí (1999) and Basu, Fernald, and Kimball (2002): In our explanation, the main reason for incomplete nominal adjustment in the wake of the technology shock is the *absence of common knowledge* regarding the state of the technology, which is *endogenously* generated by our model and seems plausible in the real world.[6]

We find, moreover, that the model predicts a delayed, hump-shaped response of inflation to autocorrelated technology change. Sluggish inflation arises because higher-order average expectations respond only gradually to the underlying technology shock as Woodford (2003a) emphasizes in the context of monetary shocks. This finding is important, because many recent empirical studies have commonly found the sluggishness of inflation (see Furher and Moore, 1995 among others). By contrast, in the standard sticky price model, inflation *jumps down* in response to technology shocks (see Galí, 2003 and Galí, Lopez-Salido, and Valles, 2003). In this

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[6] For instance, think about the debate in the 1990s about whether full employment output was increasing due to advances in information and communication technology. Similarly, in retrospect, the celebrated

respect, we believe that our model provides a more satisfactory explanation for the observed dynamic effects of technology shocks than the sticky price models.[7]

Finally, we demonstrate that monetary policy tends to fall short of accommodation of technology improvements when the central bank has only imperfect information on the state of the technology. The main reason for the central bank to adopt the strategy of the partial accommodation is that its errors in measuring the technology act as "aggregate demand" or "monetary" shocks, which cause price-setting firms to have another uncertainty and thus lead to undesirable fluctuations in the output gap. This result appears consistent with the recent empirical studies alluded above, which indicate that monetary policy in the United States over the postwar period has not responded sufficiently to technology improvements to avoid the contraction of employment. Furthermore, our result accords well with Kiley's (2003) findings that the Federal Reserve has not adjusted nominal income growth (through changes in the growth of the money supply) to changes in productivity growth, and hence the strong negative correlation between inflation and productivity growth has been consistently observed in U.S. Conceptually, the central bank faces two different types of noise in observing the underlying technology: the noise *specific* to the central bank and the noise *common* among the central bank *and* private agents. We also find that an increase in variability of the central-bank-specific noise always lowers the optimal degree of monetary policy responsiveness to movements in technology,

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productivity slowdown of the 1970s does not seem to have been "common knowledge" in real time (see Orphanides, 2003).

[7] To explain the negative technology-hours correlation, Francis and Ramey (2003) propose two models without imperfect nominal adjustment: (i) RBC model with habit formation in consumption and adjustment costs on investment and (ii) RBC model with a Leontief production function. Yet they do *not* explain the apparent negative correlation between inflation and technology, which we consider a fundamental stylized fact of the transmission of technology shocks. Indeed, the negative correlation between inflation and productivity growth in the United States over the postwar period is much *stronger* than the correlation between inflation and unemployment or the correlation between inflation and growth in monetary aggregates (see Kiley, 2003).

whereas the effect of an increase in variability of the common noise is ambiguous, depending on the assumed parameter values.

The paper is structured as follows. The next section presents the assumptions of the model. Section 3 characterizes the rational expectations general equilibrium of the model. Section 4 investigates simulated responses of a calibrated version of the model to technology shocks. Section 5 examines the role of monetary policy in the face of technology shocks. The final section offers concluding thoughts and suggests directions for future research.

2. The Model

This section lays out a simple general-equilibrium business cycle model with imperfect common knowledge regarding technology shocks. The model is a Lucas-Phelps type "island economy" version of the recent dynamic New Keynesian model.[8]

In the model, all agents are rational in the sense that they correctly understand the structure of the economy and make optimal decisions based on their information sets. The economy is composed of a continuum of small islands indexed by $i \in [0,1]$, which can be interpreted as different sectors of the economy. In each island, there exist one household and one monopolistically competitive firm. For simplicity and to focus on the essential aspects of the model, we assume households have full information. By contrast, firms are assumed to have incomplete information, and hence common knowledge of the state of the economy is absent among them. There is no capital (or capital is a predetermined fixed factor) in the model

[8] For some of the many recent references on the dynamic New Keynesian models, see, for example, Kimball (1995), Goodfriend and King (1997), Rotemberg and Woodford (1997; 1999), McCallum and Nelson (1999), King (2000), Chari, Kehoe, and McGrattan (2000), Galí (2003), Woodford (2003b), Christiano, Eichenbaum, and Evans (2003), and Basu and Kimball (2003).

economy as in the standard New Keynesian literature, so the model should be interpreted as describing a *short-run* aggregate dynamics.[9]

2.1 *Households*

The consumer side is fairly standard. The household in island *i* maximizes the discounted value of expected utility, which is given by

$$\max : \mathrm{E}_0 \sum_{t=0}^{\infty} e^{-\rho t} \left[\log\left(C_{it}\right) - \frac{N_{it}^{1+\psi}}{1+\psi} \right], \tag{1}$$

subject to the usual series of budget constraints:

$$P_t C_{it} + M_{it} + \mathrm{E}_t \left[R_{t+1}^{-1} B_{it+1} \right] = W_{it} N_{it} + \int_0^1 \Pi_{it}^j dj + M_{it-1} + B_{it}, \tag{2}$$

and the cash-in-advance constraint:

$$P_t C_{it} \leq M_{it}. \tag{3}$$

C_i is consumption of final goods, P is the price of the final good, N_i is labor supply, W_i is the nominal wage in island *i*, B_i is the total nominal value of financial assets, R^{-1} is the stochastic discount factor, M_i is money balances, and Π_i^j is firm *j*'s economic profit household *i* receives. The parameters ρ and ψ are the subjective discount rate and the inverse of the Frisch labor supply elasticity. For analytical tractability, we adopt the logarithmic utility for consumption, which is standard in the business cycle literature. Note that it is the only utility function for which income and substitution effects of a change in the real wage exactly offset without allowing for non-separabilities between consumption and labor supply.[10]

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[9] New Keynesian models without investment include Rotemberg and Woodford (1997; 1999), McCallum and Nelson (1999), King (2000), Woodford (2003b, chapter 3), and Gali (2003).

[10] See King, Plosser, and Rebelo (1988), Basu and Kimball (2002), and Kimball and Shapiro (2003) for this point.

We assume that (i) there are complete financial markets, and (ii) households have full information about the state of the economy. Since consumers are all *ex ante* identical, these assumptions imply that consumers can insure themselves against idiosyncratic risks in labor income. As a result, they will make the same consumption decisions (the common level of consumption is denoted by C_t). Thus we avoid both keeping track of the distribution of wealth across households and considering how rational expectations equilibria are established in asset markets under differential information. This greatly simplifies the following analysis.[11]

We assume, moreover, that there is no economy-wide, spot labor market. This implies that household *i* supplies his or her labor services N_i only to the firm in the same island *i* (e.g., due to firm specific skill). Workers are thus unable to move freely to other islands to look for higher wages. One consequence of such "labor immobility" or "labor attachment" is that the nominal wage rate W_i will be different across islands.

There is a single final good in the economy, which is an aggregate of individual varieties of goods. Each variety of good is produced by a monopolistically competitive firm as described in the next subsection. Following Kimball (1995), we assume each household *i* solves:

$$\min_{C^j} : \int_0^1 P_j C^j dj,$$

subject to the variety aggregator:

$$1 = \int_0^1 \Upsilon\left(C^j / C\right) dj, \tag{4}$$

for given *C*. Here C^j is the household's purchase of the differentiated good produced by firm *j*, P_j is the associated price, and the function Υ satisfies $\Upsilon(1) = 1$, $\Upsilon'(\xi) > 0$, and $\Upsilon''(\xi) < 0$ for

[11] Similar assumptions can be found in Ball, Mankiw, and Reis (2003).

all $\xi \geq 0$. Note that the variety aggregator (4) is symmetric and constant returns to scale. The first-order condition for this problem is given by

$$P_j = \frac{\Lambda}{C}\Upsilon'\left(C^j / C\right) \quad (5)$$

for some value of the Lagrange multiplier Λ that is constant for all j. This equation in turn can be inverted to the household's demand curve:

$$\frac{C^j}{C} = \Upsilon'^{-1}\left(P_j \frac{C}{\Lambda}\right). \quad (6)$$

Since there is no investment in the model, the market clearing condition implies $C^j = Y_j$ for all j and $C = Y$, where Y_j and Y are the output produced by firm j and the aggregate output. Writing $\xi \equiv C^j / C = Y_j / Y$, the elasticity of demand at various points is given by

$$\varepsilon(\xi) \equiv -\frac{d \log Y_j}{d \log P_j} = -\frac{\Upsilon'(\xi)}{\xi \Upsilon''(\xi)},$$

which in turn determines the *desired* markup of each monopolist:

$$\mu(\xi) = \frac{\varepsilon(\xi)}{\varepsilon(\xi) - 1}. \quad (7)$$

That is, the elasticity of demand $\varepsilon(\cdot)$ and hence the desired markup $\mu(\cdot)$ depend on the firm's market share ξ. As Kimball (1995) shows, this specification allows one to have "convex" (smoothed-off kinked) demand curves that become an important source of "real rigidity" in the sense of Ball and Romer (1990). Convex demand curves make it easier for the firm to lose customers by raising its relative price above one than to gain customers by lowering its relative price below one.[12] This corresponds to the desired markup $\mu(\xi)$ that is sharply increasing in the firm's market share ξ. By contrast, the oft-used Dixit and Stiglitz (1977) CES aggregator restricts one to constant-elasticity demand curves. Log-linearizing (5) around the symmetric steady state, where $Y_j = Y$ and $P_j = P$, we obtain the locally exact demand curve:

$$y_j - y = -\varepsilon^* \left(p_j - p\right) \tag{8}$$

with $\varepsilon^* \equiv \varepsilon\left(Y_j^* / Y^*\right) = \varepsilon(1)$. (We use the notation that lower-case letters represent log-deviations from the steady-state levels of their upper-case counterparts and asterisks represent steady-state values.)

Assuming that there exists a riskless one-period nominal bond with net return i_{t+1}, the standard consumption Euler equation is given by

$$C_t^{-1} = e^{(i_{t+1}-\rho)} \mathrm{E}_t\left[\left(P_t / P_{t+1}\right) C_{t+1}^{-1}\right].$$

Log-linearizing this Euler equation yields

$$c_t = \mathrm{E}_t\left[c_{t+1}\right] - \left(r_{t+1} - \rho\right), \tag{9}$$

where r_{t+1} is the real interest rate: $r_{t+1} = i_{t+1} - \mathrm{E}_t\left[\pi_{t+1}\right]$ and π_{t+1} is the rate of inflation between t and t+1: $\pi_{t+1} = p_{t+1} - p_t$. Turning to the optimal choice of labor supply, we obtain

$$N_{it}^{\psi} = \frac{W_{it}}{P_t} C_t^{-1} = \frac{W_{it}}{P_t} Y_t^{-1}.$$

Log-linearizing this equation gives us

$$n_{it} = \frac{1}{\psi}\left(w_{it} - p_t - y_t\right). \tag{10}$$

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[12] Ball and Romer (1990) suggest that search costs provide a rationale for the convex demand curves.

We assume that the nominal interest rate i_{t+1} is always strictly larger than zero, which ensures that money has a lower return than a riskless bond. This in turn leads to cash in advance constraint (3) holding as an equality as an optimality condition. Thus, the log-linearized "LM" or quantity equation is simply given by

$$y_t = m_t - p_t. \tag{11}$$

We assume that the central bank controls the money supply m_t, which is essentially equivalent to the "nominal GDP targeting" because we abstract from fluctuations in velocity. Note that our specification of monetary policy is much simpler than those in most recent work on policy rules. It is common to assume that the monetary authority controls an interest rate, and to relate the interest rate to output through an Euler equation like equation (9).[13] However, since there is currently no consensus about the right specification for aggregate demand, we follow Ball, Mankiw, and Reis (2003) by adopting the simple quantity equation, which has a long history as a pedagogical tool.

2.2 *Firms*

Firm in island *i* is a monopolist in the production of a single variety of good. The production function is given by

$$Y_{it} = A_t N_{it} - F, \tag{12}$$

where F is an overhead fixed cost of production (paid in units of output), and A is the level of technology. We assume that all firms have the same technology and that technology shocks are the only source of uncertainty. Around the symmetric steady state, the local degree of returns to scale, γ, is

[13] Some researchers call this Euler equation a "new IS" equation.

$$\gamma = \frac{Y^* + F}{Y^*} > 1,$$

and hence the log-linearized production function is given by

$$y_{it} = \gamma\left(a_t + n_{it}\right). \tag{13}$$

Given the demand curve for each product (6), the optimal relative price P_i / P and the optimal relative output Y_i / Y that goes along with it, must satisfy the condition that marginal revenue (*MR*) equals marginal cost (*MC*). Thus the optimal relative price must equal the desired markup (7) times real marginal cost:

$$\mathrm{E}_t^i\left[\frac{P_{it}}{P_t}\right] = \mathrm{E}_t^i\left[\mu\left(Y_{it} / Y_t\right) MC_{it}\right], \tag{14}$$

where $\mathrm{E}_t^i[\cdot] \equiv \mathrm{E}\left[\cdot \middle| \Phi_t(i)\right]$ denotes the expectation operator conditional on firm i's information set as of date *t*, $\Phi_t(i)$.

Following Sims (1998; 2003) and Woodford (2003a), we assume that firms have "limited information processing capacity," i.e., they are unable to pay attention to all of the relevant information in their environment.[14] In particular, each firm is assumed to act on the basis of its own *subjective* perception of the aggregate disturbances, which is modeled as observation of the true value with an idiosyncratic error. Given that technology shocks are the only source of uncertainty in our model, the information received by firm *i* in period *t* is the noisy "private signal":

$$a_t(i) = a_t + v_t(i).$$

[14] Sims (1998; 2003) argues that it is reasonable that agents can only imperfectly filter various data into a set of summary statistics on which to base their decisions, given the vast quantity of information at their disposal.

Here $v_t(i)$ is a Gaussian white noise process with mean zero and constant variances σ_v^2, which is distributed independently across islands. The signal $a_t(i)$ is "private" in the sense that the realization of $a_t(i)$ is not observed by firms in other islands. The complete information set of firm *i* in period *t* consists of the history of these private signals:

$$\Phi_t(i) = \{a_{t-j}(i)\}_{j=0}^{j=t}.$$

We now return to the firm's price-setting behavior. Since real marginal cost is equal to the ratio of the real wage to the marginal product of labor, equation (14) can be written as

$$\mathrm{E}_t^i\left[\frac{P_{it}}{P_t}\right] = \mathrm{E}_t^i\left[\mu(Y_{it}/Y_t)\frac{W_{it}}{P_t}\frac{1}{A_t}\right].$$

Since each worker works at one (and only one) firm, firms face an upward sloping labor supply curve (10), instead of taking the real wage as given as they would if there were an economy-wide labor market. Therefore,

$$\mathrm{E}_t^i\left[\frac{P_{it}}{P_t}\right] = \mathrm{E}_t^i\left[\mu(Y_{it}/Y_t)N_{it}^{\psi}Y_t\frac{1}{A_t}\right].$$

We substitute out N_{it} using the production function (12) to get

$$\mathrm{E}_t^i\left[\frac{P_{it}}{P_t}\right] = \mathrm{E}_t^i\left[\mu(Y_{it}/Y_t)(Y_{it}+F)^{\psi}\,Y_t\frac{1}{A_t^{1+\psi}}\right]. \qquad (15)$$

Note that due to the assumption of labor attachment, labor costs depend on firm output as well as aggregate output. Ball and Romer (1990) and Kimball (1995) show that such a specification leads to a greater degree of real rigidity by making *individual* MC curve (i.e., MC curve internal to the firm) steeper.

We define "full employment output" Y^f to be the level of the aggregate output at which the optimal relative price P_i / P is one under *full information*. If the optimal relative price P_i / P is one, then the desired relative output Y_i / Y is also one. Hence, the full employment output Y^f is the solution to

$$1 = \mu(1)\left(Y_t^f + F\right)^{\psi} Y_t^f \frac{1}{A_t^{1+\psi}}. \tag{16}$$

Clearly, the full employment output Y^f is an implicit function of the technology *A*. Log-linearizing equation (16) yields

$$y_t^f = \chi a_t, \tag{17}$$

where $\chi \equiv (1+\psi)/(1+\psi/\gamma)$. Notice that the full employment output is positively related with the level of aggregate technology.

Following similar steps to Kimball (1995), we now derive the notional "Short-Run Aggregate Supply" (SRAS) curve. First, dividing (15) by (16) and log-linearizing the resulting equation around the steady state, we obtain

$$\mathrm{E}_t^i[p_{it} - p_t] = \left(\frac{\mu'(1)}{\mu(1)} + \frac{\psi}{\gamma}\right)\mathrm{E}_t^i[y_{it} - y_t] + \left(1 + \frac{\psi}{\gamma}\right)\mathrm{E}_t^i\left[y_t - y_t^f\right].$$

Next, using equation (8), we solve for $y_{it} - y_t$ (the logarithmic deviation of the desired relative output) in terms of $y_t - y_t^f$ (the logarithmic deviation of the actual output from the full employment output):

$$\mathrm{E}_t^i[y_{it} - y_t] = -\frac{\Omega}{\omega}\mathrm{E}_t^i\left[y_t - y_t^f\right], \tag{18}$$

where Ω is the elasticity of MC / MR with respect to the proportional increase in *both* aggregate output and firm output:

$$\Omega \equiv 1 + \frac{\psi}{\gamma},$$

and ω is the elasticity of MC / MR with respect to firm output, holding aggregate output fixed:

$$\omega \equiv \frac{1}{\varepsilon^*} + \frac{\mu'(1)}{\mu(1)} + \frac{\psi}{\gamma}.$$

Finally, combining equations (8) and (18) yields the notional SRAS curve:

$$p_{it} = \mathrm{E}_t^i \left[p_t \right] + \alpha \mathrm{E}_t^i \left[y_t - y_t^f \right], \tag{19}$$

where α is the slope of the SRAS curve:

$$\alpha \equiv \frac{\Omega}{\varepsilon^* \omega}.$$

A small value of α can be interpreted as a high degree of “real rigidity” in the sense of Ball and Romer (1990) or a high degree of “strategic complementarity” in the sense of Cooper and John (1988).

We now consider how the aggregate price level p_t is determined in the model. First, substituting the LM curve (11) into the SRAS curve (19) gives us

$$p_{it} = (1-\alpha) \mathrm{E}_t^i \left[p_t \right] + \alpha \mathrm{E}_t^i \left[m_t - y_t^f \right]. \tag{20}$$

Note that price-setting firm i must estimate not only fundamentals $m_t - y_t^f$ but also *others*’ prices p_t conditional on its information set. How can it do that? Following Phelps (1983) and Woodford (2003a), we use the basic insight that each firm correctly understands other firms face exactly the same problem. That is, averaging equation (20) over i gives us an expression for the aggregate price level:

$$p_t = (1-\alpha) \overline{\mathrm{E}}_t \left[p_t \right] + \alpha \overline{\mathrm{E}}_t \left[m_t - y_t^f \right], \tag{21}$$

where $\overline{\mathrm{E}}_t[\cdot] \equiv \int_0^1 \mathrm{E}_t^i[\cdot]\,di$ is the *average* expectation operator. Substituting this expression into (20) yields

$$p_{it} = \alpha \mathrm{E}_t^i\left[m_t - y_t^f\right] + \alpha(1-\alpha)\mathrm{E}_t^i\left[\overline{\mathrm{E}}_t\left[m_t - y_t^f\right]\right] + (1-\alpha)^2 \mathrm{E}_t^i\left[\overline{\mathrm{E}}_t\left[p_t\right]\right].$$

Repeating similar substitutions, we obtain

$$p_{it} = \sum_{k=0}^{\infty} \alpha(1-\alpha)^k \mathrm{E}_t^i\left[\overline{\mathrm{E}}_t^k\left[m_t - y_t^f\right]\right], \tag{22}$$

where $\overline{\mathrm{E}}_t^k[\cdot]$ denotes the *k*th-order average expectation, i.e., $\overline{\mathrm{E}}_t^1[\cdot] = \overline{\mathrm{E}}_t[\cdot]$, $\overline{\mathrm{E}}_t^2[\cdot] = \overline{\mathrm{E}}_t\left[\overline{\mathrm{E}}_t[\cdot]\right]$,…, $\overline{\mathrm{E}}_t^k[\cdot] = \overline{\mathrm{E}}_t\left[\overline{\mathrm{E}}_t^{k-1}[\cdot]\right]$. In other words, $\overline{\mathrm{E}}_t^k[\cdot]$ represents the average expectation of the average expectation of…(repeat *k* times)…of the average expectation. Finally, taking an average of equation (22) once more yields the aggregate price level:

$$p_t = \sum_{k=0}^{\infty} \alpha(1-\alpha)^k \overline{\mathrm{E}}_t^{k+1}\left[m_t - y_t^f\right]. \tag{23}$$

This expression says that the aggregate price level depends on an infinite sum of the higher-order average expectations regarding the money supply adjusted for the full employment output. It is important to note that as the real rigidity parameter α becomes smaller, more weight would be put on the higher-order expectations.

The key to this result is twofold. First, the higher-order expectations matter only when $\alpha \neq 1$. That is, if $\alpha = 1$, then just the *first-order* average expectation, $\overline{\mathrm{E}}_t\left[m_t - y_t^f\right] = \int_0^1 \mathrm{E}_t^i\left[m_t - y_t^f\right]di$, would determine the aggregate price level as in the Lucas model. Moreover, in order to obtain sensible results, we need "strategic complementarity" $(\alpha < 1)$ rather than strategic substitutability $(\alpha > 1)$. As discussed later, we confirm that $\alpha < 1$ under reasonable calibration parameters. Second, the average expectation operator, $\overline{\mathrm{E}}_t[\cdot]$, does *not* satisfy the "law of iterated expectations" due to the absence of common knowledge resulting from differential information on the state of the economy.[15] That is, in our model,

$$\overline{\mathrm{E}}_t\left[\cdot\right] \neq \overline{\mathrm{E}}_t\left[\overline{\mathrm{E}}_t\left[\cdot\right]\right] \quad \text{and} \quad \mathrm{E}_t^i\left[\overline{\mathrm{E}}_t\left[\cdot\right]\right] \neq \mathrm{E}_t^i\left[\cdot\right].$$

In the next section we will discuss this second point using a simple example.

3. General Equilibrium

Given an exogenous stochastic process for the technology, a general equilibrium can be defined as a set of stochastic processes for the endogenous variables that satisfy the individual optimization conditions of households and firms and the aggregate equilibrium conditions described in the previous section. Before plunging into a realistic case, this section begins by considering a simple example in which a technology shock is assumed to be a white noise. We then proceed to the case where the technology shock follows a more realistic stochastic process.

3.1 *A Simple Example*

This subsection illustrates a fairly simple example in order to get some intuition for the nature of the higher-order expectations. We assume that the money supply is constant over time, i.e., $m_t = 0$ for all *t*. In addition, we simply suppose that the technology a_t follows a Gaussian white noise process:

$$a_t = u_t,$$

where $u_t \overset{i.i.d.}{\sim} N\left(0, \sigma_u^2\right)$. Under this assumption, the private signal firm *i* receives is given by

$$a_t\left(i\right) = u_t + v_t\left(i\right).$$

---

[15] The "global game" literature emphasizes this point. See, for example, Morris and Shin (2002).

We first consider the firm i's optimal estimate of a_t conditional on its information set $\Phi_t(i) = \{a_{t-j}(i)\}_{j=0}^{j=t}$, which is obtained by solving a standard "signal extraction" problem:

$$\mathrm{E}_t^i[a_t] = \kappa a_t(i),$$

where κ is a Kalman gain coefficient:

$$\kappa = \frac{\mathrm{Cov}\left[a_t, a_t(i)\right]}{\mathrm{Var}\left[a_t(i)\right]} = \frac{\mathrm{Cov}\left[u_t, \left(u_t + v_t(i)\right)\right]}{\mathrm{Var}\left[u_t + v_t(i)\right]} = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_v^2} = \frac{1}{1 + \sigma_v^2 / \sigma_u^2} < 1.$$

Note that this signal extraction problem is essentially the same as that in the famous Lucas model. Averaging the individual estimate $\mathrm{E}_t^i\left[a_t\right]$ over i in turn gives us the "first-order average expectation":

$$\begin{aligned}
\overline{\mathrm{E}}_t[a_t] &\equiv \int_0^1 \mathrm{E}_t^i[a_t]\,di \\
&= \int_0^1 \kappa a_t(i)\,di \\
&= \kappa \int_0^1 \left[u_t + v_t(i)\right] di \\
&= \kappa u_t,
\end{aligned}$$

where we use the law of large numbers, i.e., $\int_0^1 v_t(i)\,di = 0$.

We now turn to the second-order expectation. Solving a similar signal extraction problem, we obtain the firm i's optimal estimate of the *average* estimate of a_t, $\overline{\mathrm{E}}_t[a_t]$, conditional on its information set:

$$\begin{aligned}
\mathrm{E}_t^i\left[\overline{\mathrm{E}}_t[a_t]\right] &= \frac{\mathrm{Cov}\left[\overline{\mathrm{E}}_t[a_t], a_t(i)\right]}{\mathrm{Var}\left[a_t(i)\right]} a_t(i) \\
&= \kappa \frac{\mathrm{Cov}\left[u_t, \left(u_t + v_t(i)\right)\right]}{\mathrm{Var}\left[u_t + v_t(i)\right]} a_t(i) \\
&= \kappa^2 a_t(i).
\end{aligned}$$

Intuition behind this result is straightforward: Firm i must take account of not only its own mismeasurement but also *others' mismeasurement* when estimating the *average* estimate. This is a consequence of the absence of common knowledge stemming from differential information on the state of the technology. We can easily obtain the "second-order average expectation" by taking an average of the above expression:

$$\begin{aligned}\overline{\mathrm{E}}_t^2\left[a_t\right] &= \int_0^1 \mathrm{E}_t^i\left[\overline{\mathrm{E}}_t\left[a_t\right]\right] di \\ &= \int_0^1 \kappa^2 a_t(i)\, di \\ &= \kappa^2 u_t.\end{aligned}$$

Finally, repeating the same procedure, we can derive the expression for the "*k*th-order expectation" of a_t:

$$\mathrm{E}_t^i\left[\overline{\mathrm{E}}_t^{k-1}\left[a_t\right]\right] = \kappa^k a_t(i) \quad \text{and} \quad \overline{\mathrm{E}}_t^k\left[a_t\right] = \kappa^k u_t. \tag{24}$$

This confirms the nature of the higher-order average expectations: The "law of iterated expectations" does *not* hold since $\kappa < 1$. Note, moreover, that

$$\lim_{k\to\infty} \overline{\mathrm{E}}_t^k\left[a_t\right] \to 0.$$

In words, as k becomes larger, the higher-order expectation approaches to the *unconditional* expectation, i.e., $\mathrm{E}\left[a_t\right] = 0$, which is the best estimate when one has no prior information on the state of the technology.

It is now straightforward to derive analytical solutions for the equilibrium aggregate price, output, and employment in this simple example. Equations (17), (23), and (24) together gives us the equilibrium average price level:

$$
\begin{aligned}
p_t &= -\sum_{k=0}^{\infty} \alpha(1-\alpha)^k \, \overline{\mathrm{E}}_t^{k+1}\left[y_t^f\right] \\
&= -\sum_{k=0}^{\infty} \alpha(1-\alpha)^k \, \chi \overline{\mathrm{E}}_t^{k+1}\left[a_t\right] \\
&= -\sum_{k=0}^{\infty} \alpha(1-\alpha)^k \, \chi \kappa^{k+1} u_t \\
&= -\frac{\alpha\kappa}{1-(1-\alpha)\kappa} \chi u_t.
\end{aligned}
\tag{25}
$$

Using the quantity equation, we obtain the equilibrium output as well:

$$
y_t = -p_t = \frac{\alpha\kappa}{1-(1-\alpha)\kappa} \chi u_t.
$$

Averaging the log-linearized production function (13) over *i*, we can derive the expression for the equilibrium aggregate employment:

$$
\begin{aligned}
n_t &= \frac{1}{\gamma} y_t - a_t \\
&= -\frac{\gamma(1-\kappa)+(\gamma-\chi)\alpha\kappa}{\gamma\left[1-(1-\alpha)\kappa\right]} u_t.
\end{aligned}
$$

Using the relationships $\gamma > \chi > 1$, $\alpha < 1$, and $\kappa < 1$, we can easily prove that the coefficient of u_t in this expression is always negative. In other words, a *positive* technology shock induces a *decline* in labor input.

The story behind that result is as follows. Since the quantity theory governs the demand for money, output is proportional to real balances. Note that the supply of money is assumed to be constant. Now suppose technology improves. Even though all firms experience declines in their marginal cost, the aggregate price level will decline *less than proportionally* to the increase in productivity. This occurs because uncertainty about the higher-order expectations of the technology shock leads firms to set their prices that are far less sensitive to their own (first-order) optimal estimates of that shock — monopolistically competitive firms must repeatedly take into account the possibilities of others' mismeasurement when estimating the average expectations.[16] Thus, real balances and hence output rise less than proportionally to the increase in productivity (this is an example of the aggregate demand externality suggested by Blanchard and Kiyotaki 1987). Firms now need less labor to produce this level of output with improved technology, so they lay off workers and reduce hours.

The contractionary effects of technology improvements are also present in virtually any sticky price model. The important point to note is, however, that our model has produced these effects *without resorting to exogenous price stickiness*. In our model, the lack of common knowledge regarding the state of the technology *endogenously* generates imperfect price adjustment, which in turn leads to declines in employment despite the favorable technology shock. Thus, the result in our model is observationally equivalent to the result obtained by the sticky price model, but the underlying economic logic is quite different in the two models.

3.2 *More Realistic Case*

In the previous subsection we have analyzed the general equilibrium for which a technology shock follows a white noise process. We now take a step toward greater realism. In particular, we assume a plausible stochastic process for the technology and then characterize the rational expectations equilibrium of the model.

For simplicity, we assume $m_t = 0$ for all t as before. We model the growth in technology as following a first-order autoregressive process:

$$\Delta a_t = \rho \Delta a_{t-1} + u_t,$$

[16] As should be clear from the derivation of equation (24), imperfect nominal adjustment is predicted to be greater as α is smaller, which is to say, the greater the degree of "real rigidity" in the sense of Ball and Romer (1990), or the greater the extent of "strategic complementarity" in the sense of Cooper and John (1988).

where $0 \le \rho < 1$ and $u_t \overset{i.i.d.}{\sim} N\left(0, \sigma_u^2\right)$. In the special case where $\rho = 0$, technology follows a random walk. Note that this stochastic process can be equivalently expressed as the "state space" form:

$$X_t = DX_{t-1} + du_t, \tag{26}$$

where

$$X_t \equiv \begin{bmatrix} a_t \\ a_{t-1} \end{bmatrix}, \quad D \equiv \begin{bmatrix} 1+\rho & -\rho \\ 1 & 0 \end{bmatrix}, \quad d \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Since the aggregate price level p_t depends on an *infinite* sum of the higher-order average expectations, it seems that one needs to solve for the law of motion for $\overline{\mathrm{E}}_t^k \left[X_t\right]$ for every *k*. However, such calculations would be very complex in this case unlike the simple white-noise case. Here we adopt another strategy. Specifically, we attempt to solve for the law of motion for a *certain linear combination* of the higher-order expectations directly, instead of specifying the law of motion for each individual expectation.

Following Woodford (2003a), we first conjecture that a firm correctly believes that the "aggregate state" of the economy evolves according to the law of motion given by

$$S_t = GS_{t-1} + gu_t, \tag{27}$$

for a certain matrix *G* and vector *g* that we have yet to specify, where

$$S_t \equiv \begin{bmatrix} X_t \\ H_t \end{bmatrix}$$

and

$$H_t \equiv \sum_{k=0}^{\infty} \alpha (1-\alpha)^k \, \overline{\mathrm{E}}_t^{k+1} \left[X_t\right].$$

Note that p_t equals a linear transformation of the first element of H_t (see equations (17) and (23)). Once the law of motion for p_t is determined, we can readily solve for those for all other endogenous variables as described below. Hence, only the law of motion for a particular combination of the higher-order expectations, i.e., H_t, is actually needed to characterize the rational expectations equilibrium of the whole model. It is in this sense that we have called S_t the aggregate state. Note that given equation (26), the matrix *G* and vector *g* must be of the form:

$$\underset{4\times4}{G} = \begin{bmatrix} \underset{2\times2}{D} & \underset{2\times2}{0} \\ \underset{2\times2}{J} & \underset{2\times2}{L} \end{bmatrix}, \quad g = \begin{bmatrix} \underset{2\times1}{d} \\ \underset{2\times1}{l} \end{bmatrix}.$$

D and *d* are already known, so we need to solve for the parameters of *J*, *L*, and *l* to identify the law of motion.

As in the previous subsection each individual firm solves a signal extraction problem to filter the noise from the observed data. In particular, the firm makes linear-least-squares forecasts of a hidden state vector S_t conditional on its information set. Given the conjectured "state equation" (27), the firm's "observation equation" can be written as

$$a_t(i) = e_1' S_t + v_t(i), \tag{28}$$

where e_j denotes the *j*th unit vector (i.e., a vector the *j*th element of which is one, while all other elements are zeros). This equation just says that each firm observes a_t with an idiosyncratic noise $v_t(i)$. The firm i's optimal estimate of S_t evolves according to a standard Kalman filtering equation:

$$\mathrm{E}_t^i[S_t] = \mathrm{E}_{t-1}^i[S_t] + \kappa\left(a_t(i) - \mathrm{E}_{t-1}^i[a_t(i)]\right), \tag{29}$$

where κ is a 4×1 vector of the Kalman gains.

Given equations (26)-(29), we can *uniquely* identify the matrices *J*, *L*, and vector *l* using the method of undermined coefficients, and thus confirm that the conjectured law of motion for the aggregate state (27) is certainly correct. See Appendix for the details of the solution.

It is now straightforward to find the implied processes for the endogenous variables: p_t, y_t, n_t, $w_t - p_t$, and r_t. Cleary, the aggregate price level is given by

$$\begin{aligned} p_t &= -\chi \sum_{k=0}^{\infty} \alpha (1-\alpha)^k \, \overline{\mathrm{E}}_t^{k+1} [a_t] \\ &= -\chi e_3' S_t. \end{aligned}$$

Next, the quantity equation and the assumption of the constant money supply yield the aggregate output:

$$y_t = -p_t = \chi e_3' S_t.$$

Using the production function, we obtain the aggregate employment level:

$$\begin{aligned} n_t &= \frac{1}{\gamma} y_t - a_t \\ &= \frac{\chi}{\gamma} e_3' S_t - e_1' S_t. \end{aligned}$$

Averaging individual labor supply (10) over *i* gives us the average real wage:

$$\begin{aligned} w_t - p_t &= \psi n_t + y_t \\ &= \left(\frac{\psi}{\gamma} + 1 \right) \chi e_3' S_t - \psi e_1' S_t. \end{aligned}$$

Finally, since there is no investment in our model, the real interest rate is determined by the Euler equation (9):

$$\begin{aligned} r_{t+1} &= \rho + \mathrm{E}_t [y_{t+1}] - y_t \\ &= \rho + \mathrm{E}_t [\chi e_3' S_{t+1}] - \chi e_3' S_t \\ &= \rho + \chi e_3' (G - I_4) S_t, \end{aligned}$$

where we use the assumption that households have full information about the state of the economy.

4. Calibration and Simulation Results

4.1 *Calibration*

This subsection discusses the calibration of our model to compute impulse responses of model variables to technology shocks. The calibration is done so that each model period corresponds to one quarter. The benchmark parameter values are summarized in Table 1.1.

First, recall that we restrict the intertemporal elasticity of substitution for consumption to 1, because consumption and labor supply are assumed to be separable. Following Kimball and Shapiro (2003), we set the Frisch labor supply elasticity $1/\psi$ to 1, a low value relative to most DGE business-cycle models. We assume zero economic profit in the steady state and thus equate the steady-state markup and the steady-state degree of returns to scale. Recent empirical studies, such as Basu and Kimball (1997) and Basu, Fernald, and Kimball (2002), indicate that the plausible degree of imperfect competition and/or the degree of increasing returns is small. Thus, we set $\mu(1) = \gamma = 1.1$, implying that $\varepsilon^* = 11$ and $F/Y^* = 0.1$.[17] In addition, we set the elasticity of the desired markup with respect to a firm's market share $\mu'(1)/\mu(1)$ to 2.5. This value implies that a 1% increase in market share ξ causes a fall in the elasticity of demand from 11 to 9,

[17] However, Barsky, Bergen, Dutta, and Levy (2003) find much higher markups ranging from 1.5 to 2, using retail and wholesale data for a grocery store chain.

so that $\mu(\xi)$ would increase from 11/10=1.1 to 9/8≈1.125. Relative to Kimball's (1995) original parameterization, we assume mildly convex demand curves.[18]

Putting these values together, we obtain

$$\Omega = 1 + \frac{\psi}{\gamma} = 1 + \frac{1}{1.1} \approx 1.9$$

$$\omega = \frac{1}{\varepsilon^*} + \frac{\mu'(1)}{\mu(1)} + \frac{\psi}{\gamma} = \frac{1}{11} + 2.5 + \frac{1}{1.1} \approx 3.5,$$

so that the slope of the SRAS curve α turns out to be

$$\alpha = \frac{\Omega}{\varepsilon^* \omega} = \frac{1.9}{11 \times 3.5} \approx 0.05.$$

Our calibration ensures $\alpha < 1$, and hence firms' price-setting behaviors turn out to be strategic complements in the sense of Cooper and John (1988).

We assume that the growth in technology A follows a first-order autoregressive process: $\Delta a_t = \rho \Delta a_{t-1} + u_t$. To check the sensitivity of our results, we experiment with three different values of ρ: 0, 0.4, 0.8.[19] The relative size of innovation variances σ_v^2 / σ_u^2 is somewhat arbitrarily set to 4 because there is no evidence on the degree of firms' inattentiveness to the underlying technology.

4.2 *Simulation Results*

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[18] Kimball (1995) sets $\mu'(1)/\mu(1) = 4.28$ to obtain a sufficient degree of real rigidity in his model. Chari, Kehoe, and McGrattan (2000) argue that this parameterization implies extraordinarily convex demand curves.

[19] Basu, Fernald, and Kimball (2002) estimate the growth of their identified technology to be approximately an AR(1) process with an autoregressive coefficient of 0.4 at an *annual* frequency. It thus appears plausible to assume that ρ is larger than 0.4 at a *quarterly* frequency. Galí, López-Salido, and Vallés (2003) identify technology shocks as the only source of the unit root in average labor productivity as in Galí (1999), and suggest that ρ=0.7 for the Pre-Volcker period, while ρ=0 for the Volcker-Greenspan period.

This subsection presents impulse responses of the calibrated version of our model to technology shocks. Figures 1.1-1.3 show the dynamic responses of the technology level, employment, output, inflation, the real wage, and the real interest rate to an innovation in technology growth that eventually raises (log) technology by one percentage point. (The innovation is thus of size $u_t = 1 - \rho$.) Figures 1.1, 1.2, and 1.3 correspond to the case of $\rho = 0$, $\rho = 0.4$, and $\rho = 0.8$ respectively. Simulations are done under the assumption of the constant money supply, i.e., we assume that monetary policy does *not* react to the technology shock.

Several interesting results deserve emphasis. First, the model predicts a large, persistent *reduction* in employment in response to a positive technology shock, and hence implies the strong *negative* comovement between technology and labor input. Furthermore, when $\rho > 0$, employment exhibits negative, hump-shaped responses: It reaches a trough 2-3 quarters after the shock. This result accords well with the recent findings by Galí (1999) and Basu, Fernald, and Kimball (2002). Both papers estimate the responses of a number of variables to an identified technology shock. Although the approach to identification is very different in the two papers, the results that emerge are similar: In response to a positive technology shock, employment shows a persistent decline. Hence, conditional on technology as a driving force, the data point to a negative correlation between employment and technology, as well as between employment and output. [20] Our model can easily explain these contractionary effects of technology improvements

[20] Galí (1999), using total hours rather than hours per worker, demonstrates that his results do not depend on the assumption of a unit-root versus trend-stationary hours. By contrast, Christiano, Eichenbaum, and Vigfusson (2003a) argue that a technology improvement leads to a *rise* in hours worked when identified in a model in which *hours per worker* is assumed to be stationary. However, their argument does not explain why Basu, Fernald, and Kimball (2002) find the negative correlation between hours and technology using a quite different identification scheme, as Basu and Kimball (2003) argue. (See Christiano, Eichenbaum, and Vigfusson (2003b) for their reply to such a criticism.) Moreover, Francis and Ramey (2003) criticize Christiano et al. (2003a), showing that the technology shocks identified under the assumption of stationary hours per worker *can* be predicted by the military variables, oil variables,

for the reason discussed in the previous section, although Galí (1999) and Basu, Fernald, and Kimball (2002) advance price stickiness as the main explanation for their findings.

Second, if technology change is positively autocorrelated, the model predicts some of the (dis)inflation inertia. In particular, when $\rho = 0.8$, a technology shock has its maximum impact on inflation after 4 quarters. Inflation responds sluggishly, essentially because it takes a long time for the technology shock to become common knowledge among price-setting firms as Woodford (2003a) emphasizes in the context of monetary shocks.[21] This result is quite consistent with Fuhrer and Moore (1995), who emphasize the persistence of inflation in response to "inflation shocks," which could be interpreted as supply shocks. Our result also matches Galí, Lopez-Salido, and Valles' (2003) finding that a technology shock has its maximum impact on inflation after 4-5 quarters (see Figure 6 in their paper). In the Taylor-Calvo-Rotemberg sticky price models, inflation *jumps down* in response to a technology shock (as well as a monetary shock) *regardless of the persistence of that shock*. Indeed, as Galí (2003, Figure 5.5) shows in the context of a Calvo-type sticky price model, the largest impact on inflation occurs immediately after the technology shock. By contrast, in our model the rate of inflation is by no means a forward-looking variable, and hence there is no need for "front-load" pricing, which is a common feature of the sticky price models. In this respect, we believe that our model provides a more satisfactory explanation for the observed dynamic effects of technology shocks compared to the sticky price models.

5. The Role of Monetary Policy

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and the federal funds rate, whereas those identified under the assumption of unit-root hours per worker *cannot* be.

[21] Note also that due to the imperfect common knowledge, output basically does not respond on impact and then only gradually increases toward the new steady-state level.

Up to this point we have maintained the assumption that monetary policy does not respond to technology shocks. This assumption might seem too conservative, but is far from a bad approximation to reality. Indeed, most of the recent empirical studies mentioned earlier indicate that monetary policy in U.S. over the postwar period has not responded sufficiently to technology shocks to allow actual output to adjust quickly to the new level of the full employment output. Kiley (2003) also provides a variety of evidence supporting the view that the Federal Reserve has not adjusted the nominal income (through changes in the money supply) to changes in productivity growth. These observations thus lead to a natural question: Why does the central bank fail to accommodate technology improvements fully by loosing policy? To answer this question and thereby provide a unified explanation for the observed effects of technology shocks, this section examines the role of monetary policy in stabilizing fluctuations caused by these shocks.

In Subsection 5.1 we define the goal of monetary policy in terms of model-based welfare, which will provide the foundation for the policy analysis in the subsections that follow. Subsection 5.2 derives the optimal policy under the assumption that the central bank has perfect information on the state of the technology. The final subsection considers at length how monetary policy should react to technology shocks when the central bank is not well enough informed about these shocks.

5.1 *The Goal of Monetary Policy*

In setting up the problem facing the monetary authority, we adopt the natural assumption that it seeks to maximize economic welfare as a benevolent policymaker. The model described in Section 2 is plagued with a variety of distortions (static or average markup, “valuable” money, etc). However, as a way to keep things manageable, and to focus on the role of imperfect information in the design of monetary policy, we follow a canonical approach in the literature by maintaining the assumption that all such distortions, with the exception of the presence of imperfect information, have already been corrected by means of appropriate fiscal interventions (e.g., a subsidy that sufficiently offsets the market power).[22] Consequently, the full employment output Y^f coincides with the "efficient" level of output Y^e — the level of output that would be obtained under perfect competition —, i.e., $Y_t^f = Y_t^e = A_t$.[23] Thus, the presence of imperfect information is the *only* source of distortion left for the monetary authority to correct.

We define welfare in a period as the unweighted average level of utility across all households:

$$W_t \equiv \int_0^1 U_{it} di = \log C_t - \int_0^1 \frac{N_{it}^{1+\psi}}{1+\psi} di. \tag{30}$$

Note that in this expression, consumption is identical across households whereas labor supply may vary across households, due to complete contingent claims for consumption but not for labor supply. Following similar steps to Rotemberg and Woodford (1997; 1999), Woodford (2003b, chapter 6), and Ball, Mankiw, and Reis (2003), we can obtain a second-order approximation of the period welfare-losses resulting from deviations from the full employment (efficient) level:

$$\mathrm{Var}\left(y_t - y_t^f\right) + \lambda \mathrm{E}\left[\mathrm{Var}_i\left(p_{it} - p_t\right)\right], \tag{31}$$

where $\lambda = \varepsilon^{*2}\left(\varepsilon^{*-1} + \psi\right)/\left(1+\psi\right)$. The first term is the variability of output around the full employment level. The second term captures the *cross-sectional* variance of relative prices across different firms. Variability at the firm level reduces welfare since it creates inefficient variability in labor supply around the full employment level.

[22] Similar assumptions can be found in, for example, Rotemberg and Woodford (1997; 1999), Woodford (2003b, chapter 6), and Ball, Mankiw, and Reis (2003).

[23] Substituting $\mu(1) = 1$ and $F = 0$ into equation (16) yields the expression in the text.

It is important to note that in our model, inefficient cross-sectional price dispersion arises purely from idiosyncratic measurement errors in firms' observations. As a result, the cross-sectional price variability is unrelated to aggregate variables and thus *independent of monetary policy*. Hence, the loss function in our model the central bank seeks to minimize is given by

$$L_t = \mathrm{Var}\left(y_t - y_t^f \right). \tag{32}$$

In words, the central bank focuses on correcting a distortion stemming from the variance of the output gap.

5.2 *Monetary Policy with Full Information*

Given the loss function (32), the optimal monetary policy clearly requires that

$$y_t = y_t^f \tag{33}$$

for all *t*. The key point to note is that the optimal policy in our model does *not* require zero inflation, contrasting sharply with that in the standard sticky price model. This difference stems from the difference in loss functions in the two models. In the Calvo-type sticky price model, the cross-sectional variability of prices is determined by the variance of inflation around zero rather than around some mean level, so that the optimal monetary policy requires zero inflation (see Woodford, 2003b among others).[24] By contrast, in our model, the cross-sectional price variability is independent of aggregate variables, and hence the monetary authority cannot affect (and do not care about) such variability as discussed earlier.

Equation (33), i.e., the allocation associated with the full employment level, can be exactly replicated with appropriate monetary policy, under the assumption that the state of the

[24] This reflects the fact that even steady inflation causes inefficient relative-price variability under Calvo's assumption of staggered price *levels*.

technology can be observed *accurately* by the central bank. Specifically, such allocation can be implemented in practice using the following monetary targeting rule:

$$m_t = y_t^f = a_t.$$

Substituting this policy rule into equation (23) causes the aggregate price level to be constant over time.[25] It follows from the quantity equation that the optimal allocation (32) always holds.

Thus, with full information on the state of aggregate technology, implementation of appropriate monetary policy is really easy for the central bank: If it just equates the money supply to the level of technology in every period, then the economy continues to stay at the full employment level. There is nothing to prevent the central bank from accommodating the effects of technology shocks perfectly.

5.3 *Monetary Policy with Imperfect Information*

In the previous subsection, we have assumed that the monetary authority can observe technology shocks accurately. In practice, this, of course, is not the case. It is more reasonable to suppose that the central bank has imperfect information regarding the state of the technology just as private agents do so. Indeed, a number of previous authors have emphasized the limitations of the information available to policymakers in real time. For example, in his famous American Economic Association presidential address, Milton Friedman (1968) claims:

> What if the monetary authority chose the 'natural' rate – either of interest or unemployment – as its target? One problem is that it cannot know what the natural rate is. Unfortunately, we have as yet devised no method to estimate accurately and readily

[25] Since the optimal monetary policy in our model does *not* require zero inflation, we can also allow for deterministic trend inflation. In implementing such a policy, the monetary authority would set

the natural rate of either interest or unemployment. And the 'natural' rate will itself change from time to time.

We think Friedman's point is still alive and well (see Orphanides, 2003). From this standpoint, this subsection considers how the central bank should respond to technology shocks when it has access only to the noisy signals on these shocks.

We first assume that the information received by the central bank in period *t* is the noisy signal:

$$a_t(CB) = a_t + \theta_t + z_t. \tag{34}$$

Meanwhile, we modify the signal received by private firm *i* as follows:

$$a_t(i) = a_t + \theta_t + v_t(i). \tag{35}$$

Here a_t is the "true" level of aggregate technology as before, and θ_t, z_t, $v_t(i)$ are Gaussian white noise errors with zero mean and constant variances $(\sigma_\theta^2, \sigma_z^2, \sigma_v^2)$. Equation (34) means that the central bank observes the state of the technology with two kinds of measurement error: the error *common* among the central bank *and* private firms, θ_t, and the error *specific* to the central bank, z_t. For example, the common error θ_t could be interpreted as the noise included in the productivity statistics *publicly* released. We assume that the actual realization of $a_t(CB)$ is unobservable by private firms. Likewise, equation (35) says that firm *i* observes technology with the common error θ_t and the idiosyncratic error (as before) $v_t(i)$. We also assume that the private signal of firm i $a_t(i)$ is not observed either by *other* firms or by the central bank.

$m_t = gt + a_t$, where *g* equals the rate of trend inflation. Note that the deterministic money growth is entirely neutral in our model.

The central bank controls the money supply based on its noisy signal $a_t(CB)$ in every period. For simplicity, and to focus on the essential aspects of the model, we consider the following policy rule:

$$m_t = \beta a_t(CB) = \beta(a_t + \theta_t + z_t). \tag{36}$$

This policy rule linearly maps the current signal of the central bank into the current level of the money supply. The key point to note is that β measures the extent to which the central bank accommodates the effects of technology shocks: As β becomes smaller, the central bank responds less actively to technology shocks by varying the money supply.

Under these assumptions, equation (23) implies the aggregate price level is expressed as

$$p_t = \sum_{k=0}^{\infty} \alpha(1-\alpha)^k \, \overline{\mathrm{E}}_t^{k+1}\left[\beta(a_t + \theta_t + z_t) - a_t\right].$$

Clearly, in the current setup, firms need to form the higher-order expectations of θ_t and z_t as well as a_t. The output gap in turn becomes[26]

$$\begin{aligned} y_t - y_t^f &= m_t - p_t - y_t^f \\ &= \beta(a_t + \theta_t + z_t) - \left[\sum_{k=0}^{\infty} \alpha(1-\alpha)^k \, \overline{\mathrm{E}}_t^{k+1}\left[\beta(a_t + \theta_t + z_t) - a_t\right]\right] - a_t \qquad (37) \\ &= (\beta - 1)(a_t - H_t^a) + \beta\left[(\theta_t - H_t^\theta) + (z_t - H_t^z)\right], \end{aligned}$$

where

$$H_t^x \equiv \sum_{k=0}^{\infty} \alpha(1-\alpha)^k \, \overline{\mathrm{E}}_t^{k+1}[x_t]$$

with $x \equiv a, \theta, z$. Here $x_t - H_t^x$ measures the discrepancy between the actual level of the shock x_t and a certain linear combination of the higher-order average expectations regarding that shock. It should be clear that given the value of β, the larger the discrepancy $a_t - H_t^a$, the stronger the contractionary effects of a technology shock. Note also that the discrepancies $\theta_t - H_t^\theta$ and $z_t - H_t^z$ have the destabilizing effects on the output gap. Thus, θ_t and z_t — which stem from the perception mistake by the monetary authority — act as "aggregate demand" or "monetary" shocks, which cause firms to have another uncertainty in setting their prices.

It is important to note that in the current setup, the central bank can no longer attain the equilibrium associated with the full employment level, unlike the previous full-information case. Indeed, the central bank now faces a serious tradeoff: As $\beta \to 1$, the central bank can counteract the contractionary effects of technology shocks more aggressively, but at the same time it suffers from more welfare losses that arise from the fluctuations driven by its own measurement errors: θ_t and z_t.

To consider the problem faced by the central bank more concretely, we suppose that technology follows a random walk:

$$a_t = a_{t-1} + u_t,$$

where $u_t \overset{i.i.d.}{\sim} N\left(0, \sigma_u^2\right)$.[27] In that case, the aggregate state of the economy, $\tilde{S}_t$, consists of the following six random variables:

$$\tilde{S}_t = \begin{bmatrix} a_t & \theta_t & z_t & H_t^a & H_t^\theta & H_t^z \end{bmatrix}'.$$

[26] Notice that in the current setup, the output gap equals employment, because the (averaged) production function implies $n_t = y_t - a_t = y_t - y_t^f$.

[27] Even if we allow for autocorrelation of technology change, most of our analysis holds except that we need to use numerical solutions in deriving the law of motion for the aggregate state of the economy.

That is, the state of the economy would be fully described by the three exogenous shocks and certain linear combinations of the higher-order average expectations of these shocks.

The key point to note is that under the assumptions we make, the monetary authority *cannot* affect the law of motion for the aggregate state itself. The reason for this is that private firms solve signal extraction problems based only upon the *history of the noisy signals on exogenous shocks*, regardless of the intervention by the monetary authority. If we allowed for the firms to infer the underlying shocks through observing *endogenous* variables, the monetary authority could affect the law of motion by changing the history of the actual realized values of these variables. Pursuing the implications of this sort of possibility would be of substantial interest, but is beyond the scope of this paper. Given that the law of motion for the state is independent of monetary policy, it turns out that the policymaker's problem consists of a sequence of *static* welfare-loss-minimization problems. Put another way, current policy choices do not constrain future policy choices and future choices do not affect current policy tradeoff.

We now derive the law of motion for the aggregate state. First, generalizing the method described in Subsection 3.2 and Appendix, we obtain the law of motion for $a_t - H_t^a$:[28]

$$\begin{aligned} a_t - H_t^a &= \left(1-\tilde{\kappa}_u\right)\left(a_{t-1} - H_{t-1}^a\right) + \left(1-\tilde{\kappa}_u\right)u_t \\ &= \sum_{j=0}^{\infty}\left(1-\tilde{\kappa}_u\right)^{j+1} u_{t-j}, \end{aligned}$$

where

$$\tilde{\kappa}_u = \frac{1}{2}\left(-\tilde{\zeta} + \sqrt{\tilde{\zeta}^2 + 4\tilde{\zeta}}\right) \quad \text{with} \quad \tilde{\zeta} = \frac{\alpha}{\left(\sigma_\theta^2 + \sigma_v^2\right)/\sigma_u^2}.$$

Similarly, the law of motion for $\theta_t - H_t^\theta$ is given by

$$\theta_t - H_t^{\theta} = \frac{1-\kappa_\theta}{1-(1-\alpha)\kappa_\theta}\theta_t,$$

where

$$\kappa_\theta = \frac{\sigma_\theta^2}{\sigma_u^2 + \sigma_\theta^2 + \sigma_v^2}.$$

Finally, note that the firm i 's estimate of the central-bank-specific noise z_t is always zero (i.e., *unconditional* expectation), and hence $\overline{\mathrm{E}}_t^k [z_t] = 0$ for all *t* and *k*. This occurs because private firms have *no* prior information regarding z_t due to a lack of correlation of z_t with their signal $a_t(i)$. As a result, the central-bank-specific error is always totally "surprising" for the firms and thus has the relatively large, destabilizing effects on the output gap.

Putting everything together, the output gap (37) can be expressed as

$$y_t - y_t^f = (\beta - 1)\sum_{j=0}^{\infty}(1-\tilde{\kappa}_u)^{j+1} u_{t-j} + \beta\left(\frac{1-\kappa_\theta}{1-(1-\alpha)\kappa_\theta}\theta_t + z_t\right).$$

Substituting this into equation (32), we obtain the following expression for the welfare loss function:

$$L_t = (\beta-1)^2 \frac{(1-\tilde{\kappa}_u)^2}{1-(1-\tilde{\kappa}_u)^2}\sigma_u^2 + \beta^2\left[\left(\frac{1-\kappa_\theta}{1-(1-\alpha)\kappa_\theta}\right)^2 \sigma_\theta^2 + \sigma_z^2\right]. \tag{38}$$

Hence, the optimal value of β is immediately given by

$$\beta^* = \frac{\dfrac{(1-\tilde{\kappa}_u)^2}{1-(1-\tilde{\kappa}_u)^2}\sigma_u^2}{\dfrac{(1-\tilde{\kappa}_u)^2}{1-(1-\tilde{\kappa}_u)^2}\sigma_u^2 + \left(\dfrac{1-\kappa_\theta}{1-(1-\alpha)\kappa_\theta}\right)^2 \sigma_\theta^2 + \sigma_z^2}. \tag{39}$$

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[28] Replacing σ_v^2 in equations (A-26)-(A-28) in the Appendix with $\sigma_\theta^2 + \sigma_v^2$ yields the expressions in the

It should be evident that it is only the *relative* size of the innovation variances that matters for the determination of β^*. A number of interesting results emerge from equation (39).

First and most obviously, the presence of the noise faced by the central bank in observing the state of the technology naturally leads to $\beta^* < 1$, which implies the "partial accommodation" of technology shocks. The intuition behind that result is fairly straightforward. By choosing positive values of β to counteract the contractionary effects of technology improvements, the central bank also inadvertently reacts to the noise processes. This introduces undesirable movements in the money supply, which feed back to the economy and generate unnecessary fluctuations in the output gap. As a result, the optimal policy that properly accounts for the noise seeks a balance and calls for *less* activism than would be appropriate to adopt with full information. Our result thus confirms the common finding in the literature that the limitations of the information available to policymakers in real time act as a counterweight to the highly responsive policy.

However, it should be also noted that there is certainly some room for short-run stabilizing policy, even considering the informational limitations. That is, as long as $\sigma_\theta^2 + \sigma_z^2$ is finite, β^* always takes on a positive value, and hence the central bank should vary the money supply to some extent in response to technology shocks. Note, moreover, that this result holds even if σ_z^2 is larger than σ_v^2, i.e., the central bank has *less* information on the state of the technology than the private firms. In this sense, the suggested strategy for monetary policy is neither at the extreme of total passivity like the "*k*-percent rule" nor at the alternative of highly aggressive activism.

Second, as the relative variance of the *central-bank-specific* error becomes larger, optimal policy requires the money supply to be less responsive to movements in technology. Put another

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text.

way, it would be dangerous for the central bank to conduct active "fine-tuning" when it does not have much knowledge of the underlying technology. This sort of policy prescription accords well with the monetarist spirits typically observed in Friedman (1968).

Finally, the effect of an increase in the relative variance of the *common* error on β^* is ambiguous, depending on the assumed parameter values.[29] In other words, when *both* the central bank and private firms become less informed about the state of the technology, the optimal degree of monetary policy responsiveness may rise or fall. This ambiguity arises because an increase in variability of the common error generates two opposing forces. On one hand, it has the same pressure on β^* as an increase in variability of the central-bank-specific error, naturally requiring optimal policy to be more cautious. On the other hand, an increase in the variance of the common error has the same effect as an increase in the variance of the firms' idiosyncratic error. The latter implies *less* information contained in the firms' perceptions of the state of the technology, thus leading to greater uncertainty about higher-order expectations. This in turn makes technology improvements more contractionary, and hence the incentive of the central bank to avoid such contraction becomes stronger. In general, either effect can dominate, and therefore an increase in variability of the common error can result in either a net rise or a net fall in the proper degree of accommodation of technology shocks.

6. Conclusion

In this paper, we develop a quantitative, general-equilibrium business cycle model with imperfect common knowledge regarding technology shocks, building on Phelps (1983) and Woodford (2003a). In the model, uncertainty about the higher-order average expectations of

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[29] This fact can be confirmed by partially differentiating equation (39) with respect to $\sigma_\theta^2 / \sigma_u^2$ with messy algebra.

technology shocks endogenously generates substantial nominal adjustment delays. We then show that the model has the ability to match several key estimates of the responses of macroeconomic variables to technological disturbances. In particular, the model can explain why improvements in production technology are found to reduce employment in the short run. Equally important, the model predicts that the rate of inflation responds to autocorrelated technology change in a delayed, hump-shaped way.

Of course, it is desirable for the central bank to accommodate technology improvements by keeping output near the full employment level. Recent empirical studies suggest, however, that the actual central bank does not adjust the nominal income sufficiently to changes in technology. Using the constructed model, we provide one interpretation for such a phenomena. In particular, we demonstrate that when the central bank controls the money supply based only upon the noisy signals on aggregate technology, the partial accommodation of the effects of technology shocks is indeed optimal policy in terms of model-based welfare. We find, moreover, that the informational noise faced by the central bank may have different effects on the optimal degree of accommodation, depending on whether the information available to private firms is subject to the same kind of noise or not.

This paper has several limitations. We emphasize two here. First, we do not allow the private firms and the central bank to infer the underlying technology shocks from realized *endogenous* variables: In our formulation, they are assumed to receive the noisy signals on exogenous shocks directly. Yet, for instance, it appears more realistic for the firms to make some inference about the underlying shocks through observing the variables involved in their own production activities and sales. Second, we abstract from endogenous variations in the economy's capital stock. Extension of the model to allow for investment spending would be particularly interesting, because Basu, Fernald, and Kimball (2002) find a significant *decline* in investment in response to a technology improvement.[30] Yet this task requires one to address an unresolved difficult question of whether *intertemporal* coordination of expectations remains feasible in the presence of differential information. We hope to address both limitations in future research.

[30] Since the seminal paper of Galí (1999), most studies about the empirical effects of technology shocks have focused on the negative response of employment to an improvement in technology as discussed in this paper. By contrast, Kimball (2003) emphasizes the importance of the decline in investment after the technology improvement that is found by Basu, Fernald, and Kimball (2002); He argues that the decline in investment is more decisive in rejecting a real business cycle model explanation of the transmission of technology shocks, because such a decline cannot be generated by the standard RBC model regardless of parameter values. See also the discussion in footnote 7.

Appendix. Solution of the Law of Motion for the Aggregate State

This appendix describes a detailed solution of the law of motion for the aggregate state discussed in Subsection 3.2, following similar steps to Woodford (2003a).

For convenience, we reproduce several key equations in the main text. The exogenous stochastic process of technology can be expressed as the following state space form:

$$X_t = DX_{t-1} + du_t, \tag{A-1}$$

where

$$X_t \equiv \begin{bmatrix} a_t \\ a_{t-1} \end{bmatrix}, \quad D \equiv \begin{bmatrix} 1+\rho & -\rho \\ 1 & 0 \end{bmatrix}, \quad d \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

where $0 \leq \rho < 1$ and $u_t \overset{i.i.d.}{\sim} N\left(0, \sigma_u^2\right)$.

We conjecture that the aggregate state of the economy evolves according to the law of motion given by

$$S_t = GS_{t-1} + gu_t, \tag{A-2}$$

for a certain matrix G and vector g that we have yet to specify, where

$$S_t \equiv \begin{bmatrix} X_t \\ H_t \end{bmatrix}$$

and

$$H_t \equiv \sum_{k=0}^{\infty} \alpha(1-\alpha)^k \, \overline{\mathrm{E}}_t^{k+1}[X_t]. \tag{A-3}$$

Note that given equation (A-1), the matrix G and vector g must be of the form:

$$\underset{4\times 4}{G} = \begin{bmatrix} \underset{2\times 2}{D} & \underset{2\times 2}{0} \\ \underset{2\times 2}{J} & \underset{2\times 2}{L} \end{bmatrix}, \quad g = \begin{bmatrix} \underset{2\times 1}{d} \\ \underset{2\times 1}{l} \end{bmatrix}. \tag{A-4}$$

Here D and d are already known. The law of motion for H_t is expressed as

$$H_t = JX_{t-1} + LH_{t-1} + lu_t. \tag{A-5}$$

We keep in mind that our ultimate goal is to solve for the parameters of *J*, *L*, and *l* in this equation.

Given the conjectured "state equation" (A-2), the firm's "observation equation" can be written as

$$a_t(i) = e_1' S_t + v_t(i), \tag{A-6}$$

where e_j denotes the *j*th unit vector. Now, the firm i's optimal estimate of S_t is given by a standard "Kalman filtering" equation:

$$\mathrm{E}_t^i[S_t] = \mathrm{E}_{t-1}^i[S_t] + \kappa\left(a_t(i) - \mathrm{E}_{t-1}^i\left[a_t(i)\right]\right), \tag{A-7}$$

where κ is a 4×1 vector of the Kalman gains.

Using the state equation (A-2) and the observation equation (A-6), the Kalman filtering equation (A-5) can be rewritten as

$$\mathrm{E}_t^i[S_t] = \kappa e_1' G S_{t-1} + G\mathrm{E}_{t-1}^i[S_{t-1}] - \kappa e_1' G \mathrm{E}_{t-1}^i[S_{t-1}] + \kappa e_1' g u_t + \kappa v_t(i).$$

Averaging both sides of this equation over *i* yields the law of motion for the *average expectation* of the state S_t:

$$\overline{\mathrm{E}}_t[S_t] = \kappa e_1' G S_{t-1} + G\overline{\mathrm{E}}_{t-1}[S_{t-1}] - \kappa e_1' G \overline{\mathrm{E}}_{t-1}[S_{t-1}] + \kappa e_1' g u_t. \tag{A-8}$$

Note that equation (A-3) can be rewritten as

$$\begin{aligned}
H_t &\equiv \sum_{k=0}^{\infty} \alpha(1-\alpha)^k\, \overline{\mathrm{E}}_t^{k+1}[X_t] \\
&= \alpha\overline{\mathrm{E}}_t[X_t] + (1-\alpha)\sum_{k=0}^{\infty}\alpha(1-\alpha)^k\,\overline{\mathrm{E}}_t^{k+2}[X_t] \\
&= \alpha\overline{\mathrm{E}}_t[X_t] + (1-\alpha)\overline{\mathrm{E}}_t[H_t] \\
&= Q\overline{\mathrm{E}}_t[S_t],
\end{aligned} \tag{A-9}$$

where $Q \equiv \left[\alpha I_2 \quad (1-\alpha) I_2\right]$. Combing equations (A-8) and (A-9) gives us

$$H_t = \tilde{\kappa} e_1' G S_{t-1} + Q G \overline{\mathrm{E}}_{t-1}\left[S_{t-1}\right] - \tilde{\kappa} e_1' G \overline{\mathrm{E}}_{t-1}\left[S_{t-1}\right] + \tilde{\kappa} e_1' g u_t,$$

where $\tilde{\kappa} \equiv Q\kappa$. Substituting (A-4) into the above equation, we obtain

$$H_t = \tilde{\kappa} D_1 X_{t-1} + \left[\alpha D + (1-\alpha) J - \tilde{\kappa} D_1\right] \overline{\mathrm{E}}_{t-1}\left[X_{t-1}\right] + (1-\alpha) L \overline{\mathrm{E}}_{t-1}\left[H_{t-1}\right] + \tilde{\kappa} u_t, \qquad \text{(A-10)}$$

where $D_1 \equiv \left[1+\rho \quad -\rho\right]$. Lagging (A-9) by one period yields

$$H_{t-1} = \alpha \overline{\mathrm{E}}_{t-1}\left[X_{t-1}\right] + (1-\alpha) \overline{\mathrm{E}}_{t-1}\left[H_{t-1}\right]$$

Using this expression, we substitute out for $\overline{\mathrm{E}}_{t-1}\left[H_{t-1}\right]$ in equation (A-10) to get

$$H_t = \tilde{\kappa} D_1 X_{t-1} + L H_{t-1} + \left[\alpha D + (1-\alpha) J - \tilde{\kappa} D_1 - \alpha L\right] \overline{\mathrm{E}}_{t-1}\left[X_{t-1}\right] + \tilde{\kappa} u_t. \qquad \text{(A-11)}$$

Now we can use the standard "method of undetermined coefficients" to solve for the unknown parameters of *J*, *L*, and *l*. That is, if our conjecture is correct, the two expressions for H_t, (A-5) and (A-11), must be equivalent:

$$J = \tilde{\kappa} D_1 \qquad \text{(A-12)}$$

$$0 = \alpha D + (1-\alpha) J - \tilde{\kappa} D_1 - \alpha L \qquad \text{(A-13)}$$

$$l = \tilde{\kappa}. \qquad \text{(A-14)}$$

Using these three matrix equations, we can uniquely identify *J*, *L*, and *l given the value of the Kalman gain vector κ* (recall that $\tilde{\kappa} \equiv Q\kappa$).[31]

The remaining task is thus to determine the vector *κ*: Once we know *κ*, we can in fact express all of the unknown parameters for the law of motion (A-2) in terms of model parameters. The Kalman gain vector is given in the usual way by

[31] (A-12) and (A-14) uniquely identify *J* and *l* once we know the value of *κ*. Substituting (A-12) into (A-13) yields $L = D - J$, so we have a unique solution for *L* as well.

$$
\begin{aligned}
\kappa &= \mathrm{Cov}\left[S_t - \mathrm{E}_{t-1}^i[S_t], a_t(i) - \mathrm{E}_{t-1}^i[a_t(i)]\right]\mathrm{Var}\left[a_t(i) - \mathrm{E}_{t-1}^i[a_t(i)]\right]^{-1} \\
&= \mathrm{Cov}\left[S_t - \mathrm{E}_{t-1}^i[S_t], e_1'\left(S_t - \mathrm{E}_{t-1}^i[S_t]\right) + v_t(i)\right]\mathrm{Var}\left[e_1'\left(S_t - \mathrm{E}_{t-1}^i[S_t]\right) + v_t(i)\right]^{-1} \\
&= \left(e_1'\Sigma e_1 + \sigma_v^2\right)^{-1}\Sigma e_1,
\end{aligned}
\tag{A-15}
$$

where Σ is the variance-covariance matrix of *prior* forecast errors:

$$\Sigma \equiv \mathrm{Var}\left[S_t - \mathrm{E}_{t-1}^i[S_t]\right].$$

Thus, equation (A-15) specifies κ as a function of Σ. Note that Σ and κ are the same for all firms, since the observation errors are assumed to follow the same stochastic process for each of them.

Using the state equation (A-2), the matrix Σ can be computed as

$$
\begin{aligned}
\Sigma &= \mathrm{Var}\left[G\left(S_{t-1} - \mathrm{E}_{t-1}^i[S_{t-1}]\right) + gu_t\right] \\
&= GVG' + \sigma_u^2 gg',
\end{aligned}
\tag{A-16}
$$

where V is the variance-covariance matrix of *posterior* forecast errors:

$$V \equiv \mathrm{Var}\left[S_t - \mathrm{E}_t^i[S_t]\right].$$

The matrix V in turn is given by

$$
\begin{aligned}
V &= \mathrm{Var}\left[S_t - \mathrm{E}_{t-1}^i[S_t] - \kappa e_1'\left(S_t - \mathrm{E}_{t-1}^i[S_t]\right) - \kappa v_t(i)\right] \\
&= \left(I_4 - \kappa e_1'\right)\Sigma\left(I_4 - \kappa e_1'\right)' + \sigma_v^2\kappa\kappa' \\
&= \Sigma - \left(e_1'\Sigma e_1 + \sigma_v^2\right)^{-1}\Sigma e_1 e_1'\Sigma,
\end{aligned}
\tag{A-17}
$$

where we use equation (A-15). Finally, combining equations (A-16) and (A-17), we obtain the following stationary "Riccati equation" for Σ:

$$\Sigma = G\Sigma G' - \left(e_1'\Sigma e_1 + \sigma_v^2\right)^{-1} G\Sigma e_1 e_1'\Sigma G' + \sigma_u^2 gg'. \tag{A-18}$$

Note that this equation depends on the elements of G and g, and hence on the elements of J, L, and l. The latter elements in turn depend on κ and hence on Σ. As a result, this Riccati equation turns out to be a function of only Σ. Thus we can identify the elements of Σ by solving for a fixed point of this nonlinear matrix equation. For the special case where $\rho = 0$ — i.e., technology follows a random walk —, we can indeed obtain an analytical solution for Σ as shown below. However, when $\rho > 0$ — i.e., technology change is autocorrelated —, we must resort to numerical solutions given the particular parameter values of ρ, α, and σ_v^2 / σ_u^2.

Now we derive the analytical solution for the case of $\rho = 0$. Note first that in this case, the exogenous state vector X_t may be reduced to the single element a_t. Thus, in equation (A-1) $D = 1$ and $d = 1$ become scalars. Furthermore, the law of motion for the aggregate state (A-2) can be simplified so that H_t is a scalar, and the blocks, J, L, and l of G and g are scalars as well. Equation (A-9) continues to apply, but now with the definition $Q \equiv [\alpha \quad 1-\alpha]$. Noting that $D = D_1 = 1$ and $\tilde{\kappa}$ are now scalars, Equations (A-12)-(A-14) can be re-expressed as

$$J = \tilde{\kappa}, \tag{A-19}$$

$$L = 1 - \tilde{\kappa}, \tag{A-20}$$

$$l = \tilde{\kappa}. \tag{A-21}$$

Substituting these expressions into the Riccati equation (A-18) yields

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \tilde{\kappa} & 1-\tilde{\kappa} \end{bmatrix} \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \begin{bmatrix} 1 & \tilde{\kappa} \\ 0 & 1-\tilde{\kappa} \end{bmatrix}$$
$$- \left(\Sigma_{11} + \sigma_v^2\right)^{-1} \begin{bmatrix} 1 & 0 \\ \tilde{\kappa} & 1-\tilde{\kappa} \end{bmatrix} \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \begin{bmatrix} 1 & \tilde{\kappa} \\ 0 & 1-\tilde{\kappa} \end{bmatrix} + \sigma_u^2 \begin{bmatrix} 1 & \tilde{\kappa} \\ \tilde{\kappa} & \tilde{\kappa}^2 \end{bmatrix}.$$

We can easily obtain the upper-left equation in this system:

$$\Sigma_{11} = \Sigma_{11} - \left(\Sigma_{11} + \sigma_v^2\right)^{-1} \Sigma_{11}^2 + \sigma_u^2,$$

or

$$\Sigma_{11}^2 = \sigma_u^2 \left(\Sigma_{11} + \sigma_v^2\right). \tag{A-22}$$

Clearly, this is a quadratic equation in Σ_{11}, which is independent of κ. Since the variance Σ_{11} must be nonnegative, the solution of this equation is uniquely given by

$$\Sigma_{11} = \frac{\sigma_u^2 + \sqrt{\left(\sigma_u^2\right)^2 + 4\sigma_u^2\sigma_v^2}}{2}. \tag{A-23}$$

With some algebraic manipulation, we obtain the lower-left equation as well:

$$\Sigma_{21} = \tilde{\kappa}\Sigma_{11} + (1-\tilde{\kappa})\Sigma_{21} - \left(\Sigma_{11} + \sigma_v^2\right)^{-1}\left[\tilde{\kappa}\Sigma_{11}^2 + (1-\tilde{\kappa})\Sigma_{11}\Sigma_{21}\right] + \sigma_u^2\tilde{\kappa},$$

which can be further simplified as

$$\left(\tilde{\kappa}\sigma_v^2 + \Sigma_{11}\right)\Sigma_{21} = \sigma_v^2\tilde{\kappa}\Sigma_{11} + \left(\Sigma_{11} + \sigma_v^2\right)\tilde{\kappa}\sigma_u^2.$$

Thus, given the values of Σ_{11} and $\tilde{\kappa}$, the solution for Σ_{21} is expressed as

$$\Sigma_{21} = \frac{\tilde{\kappa}\left[\left(\sigma_u^2 + \sigma_v^2\right)\Sigma_{11} + \sigma_u^2\sigma_v^2\right]}{\tilde{\kappa}\sigma_v^2 + \Sigma_{11}}. \tag{A-24}$$

Using equation (A-15), $\tilde{\kappa}$ can be written as

$$\begin{aligned}\tilde{\kappa} &= Q\left(e_1'\Sigma e_1 + \sigma_v^2\right)^{-1}\Sigma e_1 \\ &= \left(\Sigma_{11} + \sigma_v^2\right)^{-1}\left[\alpha\Sigma_{11} + (1-\alpha)\Sigma_{21}\right].\end{aligned}$$

Using equation (A-24), we first substitute out for Σ_{21} in this expression to obtain

$$\left(\Sigma_{11} + \sigma_v^2\right)\tilde{\kappa} = \alpha\Sigma_{11} + (1-\alpha)\frac{\tilde{\kappa}\left[\left(\sigma_u^2 + \sigma_v^2\right)\Sigma_{11} + \sigma_u^2\sigma_v^2\right]}{\tilde{\kappa}\sigma_v^2 + \Sigma_{11}},$$

which can be further rewritten as

$$\frac{\sigma_v^2}{\sigma_u^2}\left[\sigma_u^2\left(\Sigma_{11} + \sigma_v^2\right)\right]\tilde{\kappa}^2 + \left[\Sigma_{11}^2 - \sigma_u^2\left(\Sigma_{11} + \sigma_v^2\right) + \alpha\sigma_u^2\left(\Sigma_{11} + \sigma_v^2\right)\right]\tilde{\kappa} - \alpha\Sigma_{11}^2 = 0.$$

Substituting equation (A-22) into this, we obtain

$$\frac{\sigma_v^2}{\sigma_u^2}\tilde{\kappa}^2 + \alpha\tilde{\kappa} - \alpha = 0 \tag{A-25}$$

We can easily confirm that for any parameters $\alpha, \sigma_v^2, \sigma_u^2 > 0$, equation (A-25) has two real roots, one satisfying

$$0 < \tilde{\kappa} < 1,$$

another that is negative.

Substituting equations (A-19)-(A-21) into the law of motion for the aggregate state (A-2) yields

$$a_t - H_t = (1-\tilde{\kappa})(a_{t-1} - H_{t-1}) + (1-\tilde{\kappa})u_t. \tag{A-26}$$

This equation implies that $a_t - H_t$ — which measures the discrepancy between the actual level of technology and a certain linear combination of the higher-order average expectations regarding that technology — is a stationary random variable if and only if $|1-\tilde{\kappa}| < 1$. This requires that $\tilde{\kappa} > 0$, so we can exclude the negative root of equation (A-25). Hence,

$$\tilde{\kappa} = \frac{1}{2}\left(-\zeta + \sqrt{\zeta^2 + 4\zeta}\right), \tag{A-27}$$

where

$$\zeta = \frac{\alpha}{\sigma_v^2 / \sigma_u^2}. \tag{A-28}$$

Thus we obtain the analytical solutions for *J*, *L*, and *l* in the state equation (A-2).

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Table 1.1. Baseline Values for Calibration Parameters

	Parameter	Value
Frisch Labor Supply Elasticity	$1/\psi$	1
Steady-State Markup	$\mu(1) = \varepsilon^* / (\varepsilon^* - 1)$	1.1
Steady-State Elasticity of Demand	ε^*	11
Steady-State Returns to Scale	γ	1.1
Elasticity of the Desired Markup with respect to a Firm's Market Share	$\mu'(1)/\mu(1)$	2.5
Elasticity of the Full Employment Output with respect to Technology	χ	1.1
Elasticity of MC/MR with respect to Aggregate Output (Balanced Expansion)	Ω	1.9
Elasticity of MC/MR with respect to Firm-Level Output	ω	3.5
Slope of the Notional SRAS Curve	α	0.05
Autocorrelation Parameter of Technology Growth	ρ	0, 0.4, 0.8
Variance Ratio	σ_v^2 / σ_u^2	4

Figure 1.1. The Dynamic Effects of a Positive Technology Shock ($\rho = 0$)

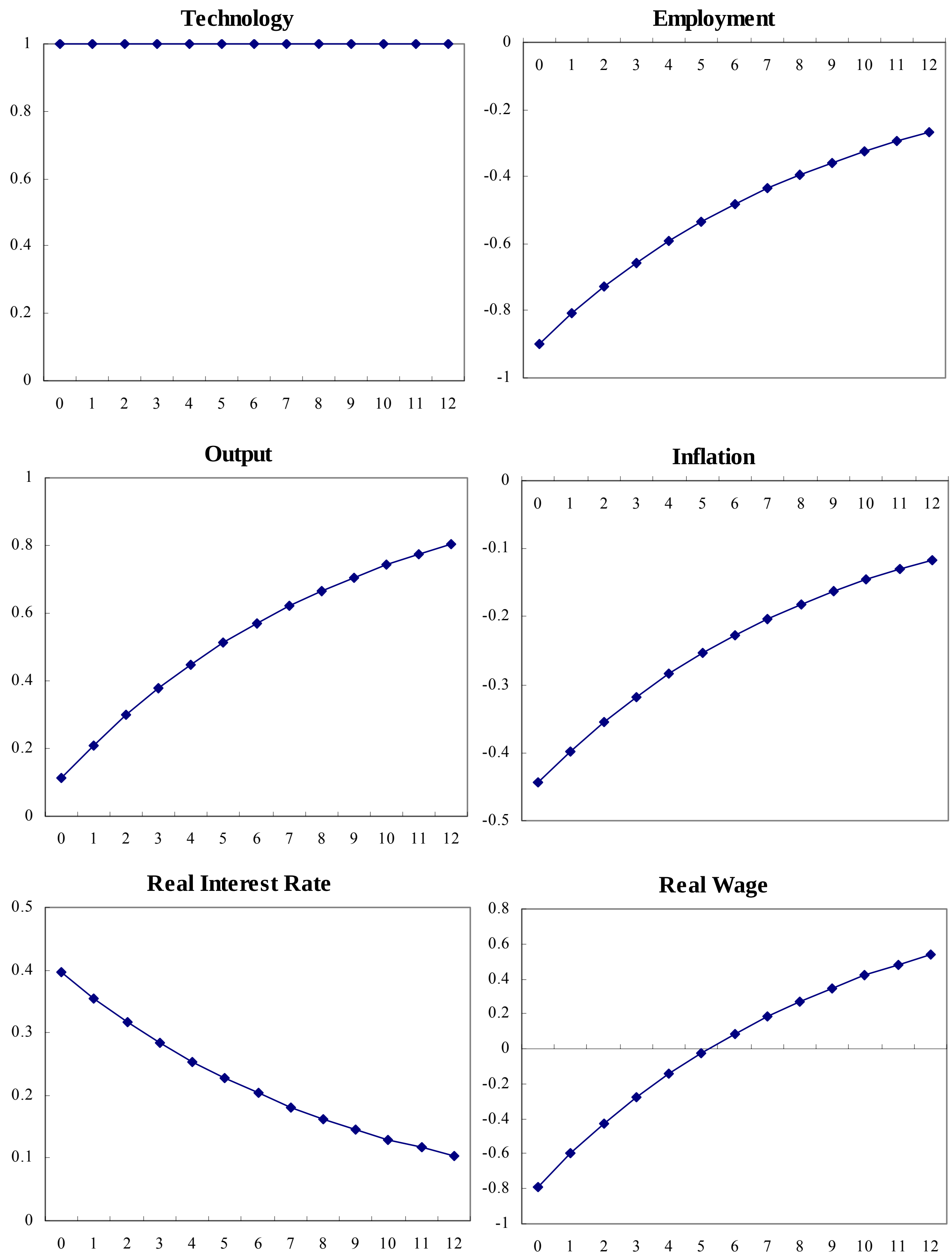

Figure 1.2. The Dynamic Effects of a Positive Technology Shock $(\rho = 0.4)$

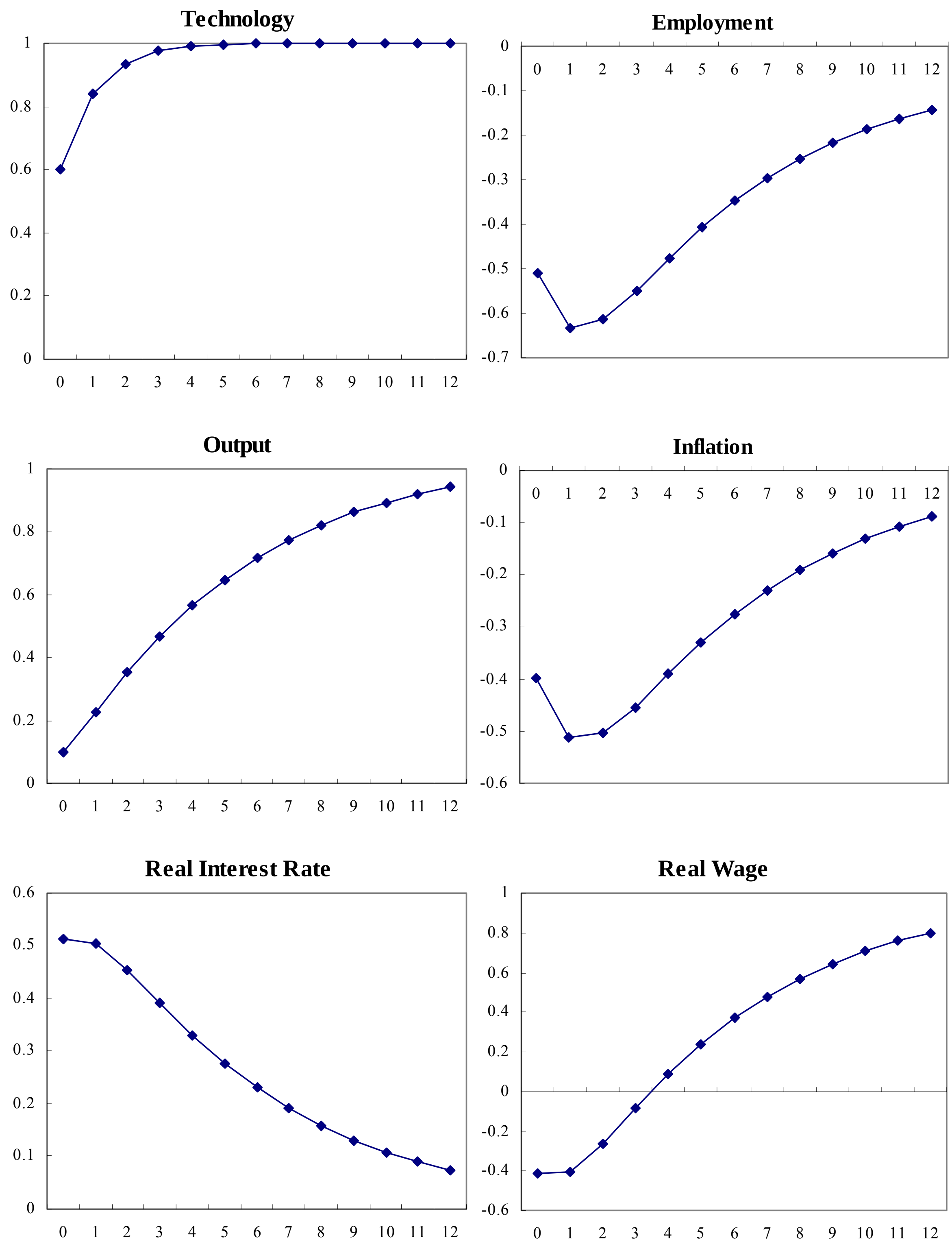

Figure 1.3. The Dynamic Effects of a Positive Technology Shock $(\rho = 0.8)$