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Discussion Paper No. 2003-E-8

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IMES Discussion Paper Series 2003-E-8

July 2003

Inflation Target as a Buffer against Liquidity Trap

Shin-Ichi NISHIYAMA*

Abstract

In times of low-inflation, conventional monetary policy is perpetually exposed to the risk of being caught by the liquidity trap. As a part of a pre-emptive monetary policy to avoid the liquidity trap, many economists have pointed out that this risk can be possibly circumvented by targeting a small but positive inflation rate - i.e., so-called the 'buffer' role of an inflation target. In this paper, based on the stylized framework of a central bank's linear-quadratic dynamic optimization problem taking into account the zero lower-bound constraint on the nominal interest rate, we analyze the role of an inflation target in reducing the long run stabilization cost stemming from the liquidity trap. We prove the existence of the 'buffer' role of small but positive inflation in the presence of a liquidity trap. Moreover, we analytically show that a central bank's loss function evaluated at the steady state is decreasing and convex function of an inflation target. Finally, these analytical properties of an inflation target are verified by a numerical method. Sensitivity analyses suggest that, in the presence of a liquidity trap, a central bank faced with volatile macroeconomic shocks should consider the 'buffer' role of an inflation target seriously.

Key words: Deflation; Inflation Target; Liquidity Trap; Zero-Bound

JEL classification: E52, E58, C63

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I would like to thank Steve Cecchetti, Kentaro Iwatsubo, Ryo Kato, Yukinobu Kitamura, Jun Nagayasu, Masao Ogaki, Akihisa Shibata, Raphael Solomon, Dan Thornton, and the staff members at the Institute for Monetary and Economic Studies and the Research and Statistics Department for their valuable comments. Any remaining errors are, of course, my own. The views expressed in this paper are those of the author and do not necessarily reflect those of the Bank of Japan or the Institute for Monetary and Economic Studies.

"··· there are several measures that the Fed (or any central bank) can take to reduce the risk of falling into deflation. First, the Fed should try to preserve a buffer zone for the inflation rate, that is, during normal times it should not try to push inflation down all the way to zero. Most central banks seem to understand the need for a buffer zone. For example, central banks with explicit inflation targets almost invariably set their target for inflation above zero, generally between 1 to 3 percent per year. Maintaining an inflation buffer zone reduces the risk that a large, unanticipated drop in aggregate demand will drive the economy far enough into deflationary territory to lower the nominal interest rate to zero." – Governor Ben S. Bernanke in remarks titled "Deflation: Making Sure "It" Doesn't Happen Here" (November 21, 2002).

"At the same time, the Governing Council agreed that in the pursuit of price stability it will aim to maintain inflation rates close to 2% over the medium term. This clarification underlines the ECB's commitment to provide a sufficient safety margin to guard against the risks of deflation." – ECB Press Release: The ECB's Monetary Policy Strategy (May 8, 2003).

1 Introduction

1.1 Motivation

Since the late 90's, many countries have been experiencing low inflation and in some cases, especially Japan, have experienced mild deflation. The trend of disinflation is not only common for the industrialized countries, but also for the emerging market economies (see Figure 1). For instance, China and Hong Kong have experienced mild, but persistent deflation during the late 90's. In an era when low inflation is the norm for many countries, the scenario of deflation is no longer a myth. As the possibility of deflation becomes a reality, central banks are now seriously exposed to the risk of falling into the liquidity trap (see Figure 2).[1] A liquidity trap is a situation in which a central bank can no longer rely on the nominal interest rate channel – the conventional monetary policy – to control the economy. Once caught by the liquidity trap, conventional monetary policy will be virtually impotent that, unless for some effective unconventional monetary policy measure, the economy will likely experience prolonged recession and deflation. In the hope of avoiding this unduly prolonged recession, a monetary policy measure to prevent falling into the liquidity trap – i.e., the pre-emptive monetary policy – has never been so important for central banks.

[1] The issue of the liquidity trap and its disasterous consequences has been resurrected by Krugman (1998). For a study that reviewed the predicaments caused by the liquidity trap taking the case of Japan's 'lost decade,' see Ahearne et al. (2002).

As a part of this pre-emptive monetary policy measure, it has been pointed out that the central bank can potentially circumvent the risk of falling into the liquidity trap by targeting small but positive inflation. This particular function of small but positive inflation can be interpreted as a social benefit aspect of inflation, in the sense that inflation can reduce the central bank's stabilization cost in the presence of the liquidity trap. This social benefit aspect of small but positive inflation is not just a theoretical possibility, but seems to be well understood by the central banks in practice. Indeed, many central banks that adopt an explicit inflation target, set the target in the range of 1% to 3% (see Table 1) and this behavior by the central banks could be seen as evidence that the social benefit role of inflation actually exists in the real world. Of course, one reason for targeting small but positive inflation comes from the measurement bias in inflation. As the Boskin Commission's (1996) seminal study reports, there exists an upward bias in inflation measurement ranging from 0.5 percent to 2 percent annually, with a mid-point estimate of 1.1 percent for the U.S. consumer price index. A similar study on the Japanese consumer price index was conducted by Shiratsuka (1999), who reports a mid-point estimate of upward bias of 0.9 percent annually. However, even taking into account the existence of a measurement bias in inflation (which is more or less 1 percent annually), still it seems to be that the 'true' inflation target is positive.

Table 1: Range of Inflation Targets Set by Central Banks

Country	**Initiated**	**Target Range***
New Zealand	1988	0~3%
Canada	1991	2±1%
United Kingdom	1992	2.5(±1)%
Sweden	1993	2±1%
Australia	1993	2~3%
South Korea	1998	2.5±1%
ECB	1998	~2%**

Source: Bernanke et al.. (1999), BOJ Policy Planning Office (2000a, b)

* As of year 2000. ** ECB sets the range in the context of 'price stability.' As of May 8, 2003, ECB announced that "it will aim to maintain inflation rates close to 2% over the medium term."

Turning to the theoretical development[2] of the social benefit aspect of small but

[2] Another theoretical justification for targeting small but positive inflation comes from the downward rigidity in nominal wages. As Akerlof et al. (1996) pointed out, when there exists downward rigidity in nominal wages, targeting too low an inflation rate will impair the adjustment of real wages, thereby distorting the efficiency of the labor market. To support this view, Kuroda and Yamamoto (2003a,b) found some empirical evidence on downward rigidity in nominal wages using Japanese micro data. However, in order to keep our model manageable, in this paper, we maintain our focus on the social

positive inflation, several researchers have pointed out its importance in the presence of the liquidity trap. Summers (1991) was perhaps the first to recognize the importance of such social benefit aspect of inflation. He argued that "the optimal inflation rate is surely positive, perhaps as high as 2 or 3 percent " (p.627). Summers' proposition of a positive inflation target has long been ignored or given little attention, probably because low inflation or deflation were hardly an issue during the early 90's. However, since the late 90's when many industrial countries have been experiencing low inflation, the side-effects stemming from the liquidity trap have once again started to gain attention. Fischer (1996) proposed an inflation target of 2% and emphasized that "the most important factor is the difficulty for monetary policy posed by the lower bound of zero on the nominal interest rate" (p.19). In the sequence of studies conducted by Bernanke and Mishkin (1997), Posen (1998), and Bernanke et al. (1999), they suggested to add a 1 percent "margin of safety"[3] on top of the measurement bias when setting the inflation target. In the same spirit, Blinder (2000) specifically warned that "zero inflation is something to be avoided" and suggested to set the inflation target "sufficiently high" to keep the risk of being caught by the liquidity trap low (pp.1093-1094). Svensson (1999), also being aware of the "potential drawbacks of too low an inflation target due to nonnegative nominal interest rates" (p.649), supported the view of setting a positive inflation target.

Several simulation studies were conducted to verify the social benefit aspect of inflation. Fuhrer and Madigan (1997) showed that the impulse response of the output gap and the inflation rate return to their steady states faster when the inflation target is set significantly positive. Orphanides and Wieland (1998), in their stochastic simulation study, showed that the probability of the economic state entering a liquidity trap will be lower when the inflation target is set higher and concluded that the social welfare loss can be reduced by setting a positive inflation target. Teranishi (2002), in his simulation study, considered the case of a hybrid New Keynsian Phillips curve and showed the social welfare loss to be minimized under positive inflation. Hunt and Laxton (2003), using MULTIMOD simulation model, showed that targeting too low an inflation rate will induce a central bank to be susceptible to a deflationary spiral and suggested to target the inflation rate higher than 2% in the long run.

Unfortunately, however, studies regarding the social benefit aspect of inflation stemming from the liquidity trap have been mainly conjectural or simulation studies. To the best knowledge of the author, there has been no analytical study regarding the social benefit aspect of inflation, let alone proof of its existence. The main contribution of this paper is that we provide an analytical foundation for the social benefit from targeting positive inflation in the presence of a liquidity trap – i.e, to borrow from Bernanke's

benefit aspect of inflation solely arising from the liquidity trap.

[3] See Bernanke et al. (1999, pp. 316-317).

(2002) terminology, the 'buffer' role of an inflation target - and to prove that, indeed, the stabilization cost in the long run is a decreasing-convex function of the inflation target. Further, employing a numerical rather than simulation method, we numerically approximate the steady state loss function in the presence of a liquidity trap and demonstrate that the loss function is, indeed, decreasing and convex in the inflation target. The strength of this numerical approach is that, unlike a simulation method, we can numerically approximate the true loss function of the central bank to the magnitude of arbitrary accuracy. In sum, we have analytically proved and numerically demonstrated the followings:

- The existence of the social benefit role (or the buffer role) of inflation in the presence of a liquidity trap.
- The steady state loss function of the central bank to be a decreasing-convex function of the inflation target.

1.2 Pre-emptive Monetary Policy and the 'Backward-looking' Economy

The analysis of pre-emptive monetary policy (including the buffer role of an inflation target) is usually discussed in the context of the 'backward-looking' economy – i.e., accelarationist Phillips Curve or Fuhrer and Moore's (1995) hybrid New Keynsian Phillips Curve.[4] Now, what about pre-emptive monetary policy in the context of a pure forward-looking economy? To the best knowledge of the author, pre-emptive monetary policy has never been associated with a forward-looking economy. And there is a reason for this.

In a pure forward-looking economy, where the expectation channel exists, the liquidity trap does not constitute a severe constraint on the conduct of monetary policy. This is because, theoretically, a central bank can escape from a deflationary spiral by exploiting the expectation channel through credible commitment. For instance, Krugman (1998) argues that the announcement of an inflation target may have an actual impact on the economy via the expectation channel. Eggertsson (2003), in the context of a dynamic general equilibrium (DGE) model with a sticky price setting, showed that a central bank can extricate the economy from a deflationary spiral by committing to higher money supply in the future.[5] In such a purely forward-looking economy, provided

[4] For instance, Reifschneider and Williams (2000) analyze pre-emptive monetary policy in the framework of an accelarationist Phillips Curve. Orphanides and Wieland (2000), Teranishi (2001), and Hunt and Laxton (2003) analyze pre-emptive monetary policy in the framework of a hybrid New Keynsian Phillips Curve.

[5] Theoretically speaking, however, as was pointed out by Benhabib et al. (2001), the issue of indeterminacy or sunspot equilibria can easily arise under both a flexible and sticky price DGE model. As

that an announcement or commitment is credible, expected inflation, and consequently current inflation, will instantly become positive allowing the economy to escape from a deflationary spiral. As such, the zero-bound constraint on the nominal interest rate will not particularly hinge on the conduct of monetary policy, giving only a weak *raison d'être* for pre-emptive monetary policy in a forward-looking economy – i.e., no need for an aggressive monetary policy conduct or no need for a buffer zone for an inflation target.

In contrast, in a backward-looking economy,[6] where the expectation channel is non-existent by construction, the conduct of pre-emptive monetary policy through the nominal interest rate channel[7] will be crucial. Since a central bank cannot rely on the expectation channel to extricate the economy from a deflationary spiral,[8] the existence of a liquidity trap will pose a serious threat to the conduct of monetary policy. As such, in a backward-looking economy, it is clearly in the interest of a central bank to precautiously avoid being caught in a liquidity trap both in the short-run and long-run, which gives a solid *raison d'être* for pre-emptive monetary policy. In this paper, in order to model pre-emptive monetary policy in the long-run – i.e., the buffer role of an inflation target, we frame our analysis in the context of a backward-looking economy with the nominal interest rate channel being the sole monetary transmission channel.

The remainder of this paper is organized as follows. In Section 2 we review the design of monetary policy in the presence of a liquidity trap. In Section 3, building upon the results stated in Section 2, we provide the analytical foundation for the social benefit role of inflation in the presence of a liquidity trap. In Section 4 we conduct a numerical analysis of the stabilization cost in the long run in the presence of a liquidity trap. Concluding remarks are presented in Section 5.

Carlstrom and Fuerst (2000) state, the avoidance of sunspot equilibria is considered to be a "necessary condition for any good monetary policy rule" (p.2). Thus, whenever considering monetary policy in the context of a DGE model, one should be careful to rule out the possibility of sunspot equilibria.

[6] In particular, what we have in mind here is sticky inflation. For the micro-foundation regarding sticky inflation, see Woodford (2001b) and Mankiw and Reis (2002).

[7] Of course, if a fiscal instrument (such as purchase of real assets) is readily available to a central bank or foreign exchange intervention is allowed, it may be possible for a central bank to rely on those instruments to extricate the economy from a deflationary spiral even when the nominal interest rate is binding at zero. In reality, however, since such policy options for a central bank are not readily available or are constrained by institutional regulations, the focus of this paper is on the nominal interest rate channel, which is readily available to a central bank.

[8] Here, one may question the effectiveness of an announcement or commitment in a backward-looking economy. As far as the model in this paper is concerned, the announcement of an inflation target or commitment to a higher money supply will have no effect on the economy. Under the backward-looking Phillips Curve setting, since the private sector's expected inflation is formed by the current inflation and output gap, the expected inflation rate will never be positive unless a central bank actually realizes a positive inflation or output gap. Nothing else matters.

2 Monetary Policy Rule and Liquidity Trap

2.1 Model Setup

The monetary authority is assumed to behave so as to minimize the squared deviation of the inflation from its target and squared output gap (the percent deviation of real GDP from its potential) over time by controlling for the policy instrument – i.e., the short-term nominal interest rate in our case. Mathematically speaking, the inflation rate and output gap can be considered as the state variables and the nominal interest rate as the control variable for the monetary authority. The period-by-period loss function of the monetary authority is assumed to take the standard form,

$$L_t = \frac{1}{2}\left[y_t^2 + \lambda(\pi_t - \pi^*)^2\right], \tag{1}$$

where π_t and y_t stand for the inflation rate and output gap at period t, respectively. π^* stands for the inflation target and λ is the preference parameter of the monetary authority. It should be noted that the inflation target, π^*, is set exogenously and, therefore, the monetary authority is assumed to minimize the loss function over time taking the level of the inflation target as given. Note further that the inflation target is assumed to be constant over time. Also, implicit in the loss function above, the monetary authority is assumed to target a zero output gap.

The economy is modeled as a conventional IS-LM, AD-AS type formulation following Svensson (1997), Ball (1999), and Reifschneider and Williams (2000). Namely, the state transition rule of the output gap is assumed to follow the simple IS type equation,

$$y_{t+1} = \rho y_t - \delta\left(i_t - E_t\pi_{t+1}\right) + \upsilon_{t+1}, \tag{2}$$

where i_t stands for the nominal interest rate[9] at period t. $E_t\pi_{t+1}$ represents the expected next period's inflation at period t, where E_t stands for the expectation operator conditioned upon the information set available at period t. Coefficient ρ represents the magnitude of output gap stickiness and is assumed to be $\rho > 0$. Further, the output gap at period t is assumed to be negatively related to the current real interest rate and, therefore, the coefficient δ, which controls the responsiveness of the output gap with respect to the real interest rate, is assumed to be $\delta > 0$. Similarly, the state transition process of the inflation is assumed to follow the simple AS type equation which can be interpreted as a backward-looking accelerationist Phillips curve,

$$\pi_{t+1} = \pi_t + \alpha y_t + \varepsilon_{t+1}. \tag{3}$$

[9] Note that we have implicitly assumed the equilibrium real interest rate, r^*, to be zero following Svensson (1997). However, this can be easily modified without altering the main propositions of this paper.

The stochastic disturbance vector representing IS and AS shocks, $(\upsilon_{t+1}, \varepsilon_{t+1})$, is independently and identically distributed with mean 0 and variance-covariance matrix Ω. Technically speaking, it should be noted that the monetary authority's control variable, i_t, enters only through the transition equation (2), but not directly inside the loss function.

Under this setup, the IS equation (2) contains a forward-looking variable, $E_t\pi_{t+1}$. However, from the AS equation (3), the expected inflation rate can be simply expressed as

$$E_t\pi_{t+1} = \pi_t + \alpha y_t. \tag{4}$$

In other words, the linear combination of the current inflation rate and current output gap constitutes sufficient statistics in forecasting the next period's inflation rate. Substituting eq. (4) for eq. (2), the IS equation can be transformed as

$$y_{t+1} = (\rho + \alpha\delta)y_t + \delta\pi_t - \delta i_t + \upsilon_{t+1}. \tag{5}$$

Given the state transition rules of the state variables output gap and inflation rate and loss function as above, we are now ready to describe the dynamic optimization problem of the monetary authority.

2.2 Monetary Policy in the absence of a Liquidity Trap: The Taylor Rule

We first consider the dynamic optimization problem of the monetary authority in the absence of a liquidity trap - i.e., no lower-bound constraint on the nominal interest rate.[10] It should be noted that, under this environment, a negative nominal interest rate is permitted. Formally, the dynamic optimization problem of the monetary authority can be described as follows.

$$\min_{\{i_{t+j}\}_{j=0}^{\infty}} \quad E_t\sum_{j=0}^{\infty}\beta^j L_{t+j} \ , \tag{6}$$

subject to

$$\begin{aligned} y_{t+j+1} &= (\rho + \alpha\delta)y_{t+j} + \delta\pi_{t+j} - \delta i_{t+j} + \upsilon_{t+j+1} \\ \pi_{t+j+1} &= \pi_{t+j} + \alpha y_{t+j} + \varepsilon_{t+j+1} \ , \end{aligned}$$

[10] Striclty speaking, the concept of a liquidity trap and zero-bound constraint on the nominal interest rate are different concepts. In principle, if the 'money demand' curve becomes perfectly elastic above the zero nominal interest rate, then it is possible that the nominal interest rate is bounded above zero, say $i_t \geq b$. However, since the main implications of this paper are unaltered even when the lower-bound is positive, for the sake of simplicity, we assume the liquidity trap to occur at a zero nominal interest rate.

for $\forall j \geq 0$, where the monetary authority's discount factor β is assumed to be sufficiently small for the Contraction Mapping Theorem to hold. Also, for this purpose, we have assumed the monetary authority's optimization horizon to be infinite. Under these conditions,[11] the above minimization problem can be restated as a recursive two-period problem using the Bellman equation as follows,

$$V(\pi_t, y_t) = \min_{i_t} \left\{ \frac{1}{2} \left[y_t^2 + \lambda(\pi_t - \pi^*)^2 \right] + \beta E_t V(g(y_t, \pi_t, \upsilon_{t+1}, \varepsilon_{t+1})) \right\}, \tag{7}$$

where

$$g(y_t, \pi_t, \upsilon_{t+1}, \varepsilon_{t+1}) = \begin{bmatrix} \rho + \alpha\delta & \delta \\ \alpha & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} - \begin{bmatrix} \delta \\ 0 \end{bmatrix} i_t + \begin{bmatrix} \upsilon_{t+1} \\ \varepsilon_{t+1} \end{bmatrix}.$$

Thus, the problem can be regarded as a stochastic linear-quadratic (LQ) control problem with linear state transition function. As is well known in the operation research literature, the optimal feedback rule will be a linear function of the state variables under the LQ stochastic control and the value function will be a quadratic function of the state variables. For the problem considered here, this implies that the optimal monetary policy reaction function is a linear function of y_t and π_t and the value function – representing the long run stabilization cost borne by the monetary authority –, $V(\pi_t, y_t)$, is a quadratic function. This was first pointed out by Svensson(1997) and Ball (1999). They further give an interpretation of its linearity with respect to theoutput gap and inflation in the context of the Taylor Rule.

By invoking the Riccatti equation as in Svensson (1997), the optimal monetary policy reaction function without zero-bound constraint (i.e., Taylor Rule) can be shown to be[12]

$$i^{Taylor}(\pi_t, y_t) = \pi_t + \left(\alpha + \frac{\rho\theta + \theta - 1}{\delta\theta} \right) y_t + \left(\frac{\theta - 1}{\alpha\delta\theta} \right) (\pi_t - \pi^*), \tag{8}$$

where

$$\theta = \frac{\alpha^2\beta\lambda + \beta + 1 + \sqrt{(\alpha^2\beta\lambda + \beta + 1)^2 - 4\beta}}{2} \text{ and } \theta > 1. \tag{9}$$

It should be noted that the Taylor Rule stated above allows the nominal interest rate to be negative, especially when the state of the economy is in severe recession or deflation.

2.3 Monetary Policy in the presence of a Liquidity Trap

In reality, however, the nominal interest rate cannot take a negative number.[13] Although the issue of a liquidity trap is hardly relevant during an inflationary period such as

[11] For the lists of regularity conditions regarding the Contraction Mapping Theorem, see Stokey and Lucas (1989). Throughout this paper, we simply assume that reguralit conditions are satisfied.

[12] The coefficients on the output gap and inflation gap in our formulation is slightly different from those of Svensson's (1997) due to a difference in model specification.

[13] As for the exceptional case, the overnight call rate on Japanese funds fell to negative 0.01 percent at the end of January, 2003 for the first time in Japanese history, according to Sanchanta and Swann (2003).

70's and 80's, in a period of disinflation (or, even worse, deflation), the existence of a liquidity trap poses a serious threat for the monetary authority.[14] In order to address the monetary authority's problem in a realistic manner, we now explicitly take into account the existence of a zero-bound constraint on the nominal interest rate. Formally, the additional constraint,

$$i_t \geq 0 \tag{10}$$

is imposed to the dynamic optimization problem (6). The Bellman equation in the case of this problem will be

$$\tilde{V}\left(\pi_t, y_t\right) = \min_{i_t \geq 0} \left\{ \frac{1}{2}\left[y_t^2 + \lambda(\pi_t - \pi^*)^2\right] + \beta E_t \tilde{V}(g(y_t, \pi_t, \upsilon_{t+1}, \varepsilon_{t+1})) \right\}, \tag{11}$$

which looks almost the same as eq. (7) except that the control variable is now bounded by zero. This zero-bound constraint on the nominal interest rate appears to be innocuous at first glance. However, this turns out to be misleading. As pointed out by Chmielewski and Manousiouthakis (1996), when the control variable is constrained, the optimal feedback rule is no longer a linear function of the state variables. In addition, the loss function of the monetary authority is no longer a quadratic function of the state variables, but will be a non-quadratic function. In the context of inflation targeting, this implies that monetary policy should react more aggressively to a change in inflation and output gap relative to the baseline Taylor Rule (8) as the threat of a liquidity trap becomes more likely.[15]

In such circumstances where the monetary authority needs to take into account this occasionally binding zero-bound constraint, Kato and Nishiyama (2001) derive the optimal monetary policy reaction function to be,

$$i^*(\pi_t, y_t) = \left\{ \begin{array}{l} \pi_t + \left(\alpha + \frac{\rho\theta + \theta - 1}{\delta\theta}\right) y_t + \left(\frac{\theta - 1}{\alpha\delta\theta}\right)(\pi_t - \pi^*) - h(\pi_t, y_t) \text{ for } (\pi_t, y_t) \ s.t. \ i^* > 0 \\ 0 \text{ for } (\pi_t, y_t) \ s.t. \ i^* \leq 0 \end{array} \right\}. \tag{12}$$

It should be noted that, except for the modification term, $h(\pi, y)$, the rest of the specification of monetary policy reaction is exactly the same as in the Taylor Rule laid out in

However, since this trade was settled between foreign banks that were constrained from depositing additional funds in the BOJ reserve account due to their bylaws, this case really should be considered as an aberration. Banks that are free from such bylaws are guaranteed to earn a zero interest rate by depositing their funds in the BOJ reserve account.

[14] For an influential paper that studied the predicaments of deflation taking the case of Japanese 'lost decade,' see Ahearne et al. (2002).

[15] This has been pointed out by Reifschneider and Williams (2000), Blinder (2000), Orphanides and Wieland (2000), Kato and Nishiyama (2001), and Hunt and Laxton (2003).

eq. (8). Thus, the function[16] $h(\pi, y)$ represents the deviation of the optimal monetary policy reaction in the presence of a liquidity trap from the Taylor Rule and, therefore, can be interpreted as the term representing the monetary authority's pre-emptiveness against the liquidity trap.

Although the closed form expression of $h(\pi, y)$ does not exist due to its highly nonlinear nature stemming from the occasionally binding constraints, Kato and Nishiyama (2001) and Kato (2002) neverthless prove that $h(\pi, y)$ is positive, decreasing and convex in both π and y. Since we build upon these qualitative properties of $h(\pi, y)$ in analyzing the buffer role of an inflation target in the next section, for the sake of convenience, we state three properties in the form of lemma.

Lemma 1 (Positive h) *The function $h : \Re^2 \rightarrow \Re$ is defined as $h(\pi, y) = i^{Taylor}(\pi, y) - i^*(\pi, y)$. Then for any state (π, y) where i^* is strictly greater than zero, h is at least greater than zero - i.e.,*

$$h(\pi, y) \geq 0 \text{ for } \forall(\pi, y) \text{ s.t. } i^*(\pi, y) > 0.$$

Proof. See 'Proposition 1' in Kato and Nishiyama (2001). ■

Lemma 2 (Decreasing h) *For any state (π, y) where i^* is strictly greater than zero, h is decreasing in both π and y - i.e.,*

$$h_\pi(\pi, y) \leq 0 \text{ and } h_y(\pi, y) \leq 0 \text{ for } \forall(\pi, y) \text{ s.t. } i^*(\pi, y) > 0.$$

Proof. See 'Proposition 2' in Kato and Nishiyama (2001). ■

Lemma 3 (Convex h) *For any state (π, y) where i^* is strictly greater than zero, h is convex in both π and y - i.e.,*

$$h_{\pi\pi}(\pi, y) \geq 0 \text{ and } h_{yy}(\pi, y) \geq 0 \text{ for } \forall(\pi, y) \text{ s.t. } i^*(\pi, y) > 0.$$

Proof. See Kato (2002). ■

3 The Buffer Role of an Inflation Target

So far we have discussed the design of monetary policy rule in the presence of a liquidity trap, taking the level of the inflation target as given. The natural next question is then, "What if the inflation target is variable? Can the central bank further reduce the stabilization cost by raising the inflation target?" As has been pointed out by several

[16] Also note that the function $h(\pi, y)$ is time-invariant. This indirectly follows from the assumption that the Contraction Mapping Theorem holds for the value function in eq. (11). Since the value function is time-invariant by assumption, as a collorary, the optimal reaction function also will be time-invariant.

researchers,[17] there is a potential for the central bank to circumvent the risk of falling into the liquidity trap by targeting some positive inflation rate which Bernanke (2002) termed the 'buffer' zone. This role can be thought to be a social benefit aspect of inflation, in the sense that the central bank can reduce the stabilization cost by targeting a positive inflation rate in the long run. The purpose of this section is to provide the analytical foundation for this social benefit aspect of inflation in the presence of a liquidity trap and to prove that, indeed, the central bank can reduce the stabilization cost in the long run by raising the inflation target – i.e., the buffer role of the inflation target.

3.1 Analytical Aspect of the Buffer Role of an Inflation Target

3.1.1 Steady State

As a preliminary step in analyzing the buffer role of an inflation target in the long-run, we first derive the steady state of the output gap and inflation rate as a function of the inflation target. Substituting the optimal monetary policy reaction function (12) into the IS equation (5), we obtain the (controlled) AD equation augmented for a liquidity trap as follows,

$$y_{t+1} = \left(\frac{1-\theta}{\theta}\right) y_t + \frac{1-\theta}{\alpha\theta}(\pi_t - \pi^*) + \delta h(\pi_t, y_t) + \upsilon_{t+1}, \tag{13}$$

which can interpreted as the optimized state transition equation of the next period's output gap as a function of the current output gap and inflation rate. Noticing that the coefficient on the inflation gap is negative and that the function h is decreasing with respect to inflation (from Lemma 2), we know that the next period's output gap is negatively related with current inflation (which is consistent with the downward sloping[18] AD curve found in any macroeconomics textbook). Further noticing that the coefficient on the current output gap is negative and the function h is decreasing with respect to the output gap, we know that the next period's and the current output gap have a negative relationship. This is due to the stabilizing force via the optimal monetary policy reaction function. In other words, if the economy is in expansion (recession) during the current period, then the monetary authority struggles to stabilize the economy toward its potential by raising (cutting) the nominal interest rate, which in turn places downward (upward) pressure on the next period's output gap.

[17] For instance, see Summers (1991), Fischer (1996), Bernanke and Mishkin (1997), Orphanides and Wieland (1998), Posen (1998), Bernanke et al. (1999), and Bernanke (2002) among others.

[18] This is only true for the state such that $i^*(\pi_t, y_t) > 0$. Once the monetary authority is caught in a liquidity trap, the AD equation will then be,

$$y_{t+1} = (\rho + \alpha\delta)y_t + \delta\pi_t + \upsilon_{t+1},$$

which yields an upward sloping AD curve in the (y_{t+1}, π_t) space. Indeed, this upward sloping AD curve will be the source of a 'deflationary spiral.'

Combining AD equation (13) with AS equation (3) and by recursive substitution, output gap at period $t+1$ can be expressed as a response function of past states and shocks as

$$\begin{aligned} y_{t+1} &= \left(\frac{1-\theta}{\theta}\right)\frac{1}{\theta^t}y_0 + \left(\frac{1-\theta}{\alpha\theta}\right)\frac{1}{\theta^t}(\pi_0-\pi^*) \\ &\quad +\delta\left(\frac{1-\theta}{\theta}\right)\sum_{j=0}^{t-1}\frac{1}{\theta^j}h(\pi_{t-1-j},y_{t-1-j}) + \delta h(\pi_t,y_t) \\ &\quad +\left(\frac{1-\theta}{\theta}\right)\sum_{j=0}^{t-1}\frac{1}{\theta^j}\left(\frac{1}{\alpha}\varepsilon_{t-j}+\upsilon_{t-j}\right)+\upsilon_{t+1}, \end{aligned} \tag{14}$$

and, by the same token, the response function of the inflation rate at period $t+1$ can be expressed as

$$\begin{aligned} \pi_{t+1}-\pi^* &= \frac{1}{\theta^t}(\pi_0-\pi^*) + \alpha\frac{1}{\theta^t}y_0 + \alpha\delta\sum_{j=0}^{t-1}\frac{1}{\theta^j}h(\pi_{t-1-j},y_{t-1-j}) \\ &\quad +\sum_{j=0}^{t-1}\frac{1}{\theta^j}(\varepsilon_{t-j}+\alpha\upsilon_{t-j})+\varepsilon_{t+1}, \end{aligned} \tag{15}$$

where (π_0, y_0) is an arbitrary chosen initial state such that $i^*(\pi_0, y_0) > 0$. Recalling that $\theta > 1$ from condition (9), the initial effect of both the output gap, y_0, and inflation gap, $\pi_0 - \pi^*$, die out asymptotically and will only have a transitory effect on the future inflation and output gap. By the same reasoning, the IS shock, υ_{t-i}, and AS shock, ε_{t-i}, will only have a transitory effect and have no effect on the steady state of the output gap and inflation rate.

Next, tending time t to infinity, the steady state (π^{SS}, y^{SS}) can be characterized by the following conditions,

$$\begin{aligned} y^{SS} &= \delta\left(\frac{1-\theta}{\theta}\right)\sum_{j=0}^{\infty}\frac{1}{\theta^j}h(\pi^{SS},y^{SS}) + \delta h(\pi^{SS},y^{SS}) \\ \pi^{SS} &= \pi^* + \alpha\delta\sum_{j=0}^{\infty}\frac{1}{\theta^j}h(\pi^{SS},y^{SS}), \end{aligned}$$

which, in turn, imply the steady state[19] to be

$$y^{SS} = 0 \tag{16}$$

$$\pi^{SS} = \pi^* + \frac{\alpha\delta\theta}{\theta-1}h(\pi^{SS},0). \tag{17}$$

As can be seen from condition (16), even in the presence of a liquidity trap, the steady state of the output gap is zero regardless of the level of the inflation target. In contrast, the steady state of the inflation rate is significantly affected by the presence of a liquidity trap. Some remarks should follow for the steady state inflation rate. First, it should be noted the steady state of the inflation rate is unique even in the presence of liquidity constraint. Since the function h is monotone decreasing with respect to the arguments (as stated in Lemma 2), this assures the uniqueness of the steady state inflation rate in the state space such that $i^*(\pi, y) > 0$. Second, since $\alpha > 0$, $\delta > 0$, $\theta > 1$ and h is positive (as stated in Lemma 1), this implies that steady state inflation is greater than the inflation target.[20] Of course, were it not for the fear of a liquidity trap, steady state inflation and the inflation target would be the same, since $h = 0$ in that case. Finally, as is obvious from eq. (17), steady state inflation is an implicit function of the inflation target – i.e., $\pi^{SS}(\pi^*)$. Thus, as the value of the inflation target varies, so does the steady state inflation rate.

3.1.2 Properties of Steady State Loss Function with respect to an Inflation Target

Having identified the steady state of the output gap and inflation rate, we are now in a position to analyze how a change in inflation target can affect the monetary authority's

[19] One can derive this steady state more directly. By presupposing the existence of unique steady state (π^{SS}, y^{SS}) and substituting them for eq. (3) and eq. (13), one obtains the following two conditions,

$$\pi^{SS} = \pi^{SS} + \alpha y^{SS}$$

$$y^{SS} = \frac{1-\theta}{\theta}y^{SS} + \frac{1-\theta}{\alpha\theta}(\pi^{SS} - \pi^*) + \delta h(\pi^{SS}, y^{SS}).$$

Obviously, the first steady state condition implies y^{SS} to be 0. Substituting $y^{SS} = 0$ into the second steady state condition, one obtains $\pi^{SS} = \pi^* + [\alpha\delta\theta/(\theta-1)]h(\pi^{SS}, 0)$ which is the exactly the same condition shown in eq. (17). Of course, a drawback of this approach is the presumption of the existence of a unique steady state. Moreover, this approach is silent about how the state converges from a transitory state to steady state. In order to gain some intuition about the convergence mechanism, it is essential to lay out the response function of the future state with respect to the initial state, (π_0, y_0).

[20] This result may appear to be surprising for some readers. A natural interpretation of this discrepancy between the steady state inflation rate and inflation target is that it represents the inflation bias of a central bank confronted with a liquidity trap. In other words, in order to avoid the risk of being caught in a liquidity trap, it is in the interest of a central bank to deliberlately 'miss' the target upward. The issue of inflation bias stemming from the liquidity trap is interesting and deserves a separate discussion. For the rest of this paper, however, we will maintain our focus on the relationship between an inflation target and the long run stabilization cost.

stabilization cost in the long run. Here, the term 'stabilization cost in the long run' may have different meanings for different researchers. To avoid ambiguity, we define 'stabilization cost in the long run' as the value of loss function evaluated at the steady state – or, steady state loss function for short. More precisely, the steady state loss function is defined as,

$$L^{SS} = \frac{1}{2}(y^{SS})^2 + \frac{1}{2}\lambda(\pi^{SS} - \pi^*)^2, \tag{18}$$

which can be interpreted as the steady state counterpart of eq. (1). Thus, in this paper, the relationship between the long run stabilization cost and inflation target is actually proxied by the relationship between the steady state loss function and inflation target. In particular, we say that there exist the buffer role of an inflation target when the steady state loss function is decreasing in the inflation target.

Keeping this in mind, we now state the main proposition of this paper – i.e., the existence of the buffer role of an inflation target in the presence of a liquidity trap.

Proposition 1 (Decreasing L^{SS} w.r.t. π^*) *The steady state loss function is decreasing in the inflation target – i.e.,*

$$\frac{\partial L^{SS}}{\partial \pi^*} \leq 0.$$

Proof. Differentiating steady state condition (17) with respect to π^* and invoking the implicit function theorem, it follows that

$$\frac{\partial \pi^{SS}(\pi^*)}{\partial \pi^*} = \frac{1}{1 - \frac{\alpha\delta\theta}{\theta-1}\frac{\partial h}{\partial \pi^{SS}}}. \tag{19}$$

Now, since $\partial h/\partial \pi^{SS} \leq 0$ from Lemma 2, the denominator of RHS of eq. (19) is greater than 1. This, in turn, implies that

$$0 \leq \frac{\partial \pi^{SS}}{\partial \pi^*} \leq 1. \tag{20}$$

Finally, differentiating the steady state loss function (18) with respect to π^* and recalling that y^{SS} is independent of π^* from steady state condition (16), it follows that

$$\frac{\partial L^{SS}}{\partial \pi^*} = \lambda\left(\pi^{SS} - \pi^*\right)\left(\frac{\partial \pi^{SS}}{\partial \pi^*} - 1\right).$$

Now, since $\lambda > 0$, $(\pi^{SS} - \pi^*) \geq 0$ from eq. (17), and $(\frac{\partial \pi^{SS}}{\partial \pi^*} - 1) \leq 0$, it follows that $\frac{\partial L^{SS}}{\partial \pi^*} \leq 0$. ■

Proposition 2 (Convexity of L^{SS} w.r.t. π^*) *The steady state loss function is a convex function of the inflation target – i.e.,*

$$\frac{\partial^2 L^{SS}}{\partial \pi^{*2}} \geq 0.$$

Proof. Differentiating steady state condition (17) with respect to π^* twice and invoking the implicit function theorem, it follows that

$$\frac{\partial^2 \pi^{SS}}{\partial \pi^{*2}} = \frac{\frac{\alpha\delta\theta}{\theta-1}\left[\frac{\partial^2 h}{\partial(\pi^{SS})^2}\left(\frac{\partial \pi^{SS}}{\partial \pi^*}\right)^2\right]}{1-\frac{\alpha\delta\theta}{\theta-1}\frac{\partial h}{\partial \pi^{SS}}} \geq 0, \tag{21}$$

where the final inequality follows from Lemma 2 and Lemma 3. Now, differentiating the steady state loss function (18) with respect to π^* twice and from the inequality condition (21), it follows that

$$\frac{\partial^2 L^{SS}}{\partial \pi^{*2}} = \lambda\left(\frac{\partial \pi^{SS}}{\partial \pi^*}-1\right)^2 + \lambda\left(\pi^{SS}-\pi^*\right)\frac{\partial^2 \pi^{SS}}{\partial \pi^{*2}} \geq 0.$$

■

Proposition 1 basically proves the existence of the buffer role of an inflation target. In other words, in the presence of a liquidity trap, as long as the central bank preserves the effectiveness of conventional monetary policy (i.e., the nominal interest rate is positive and not yet caught in a liquidity trap), there exists room for the central bank to reduce the stabilization cost in the long run by raising the inflation target (of course, in conjunction with the revision of the optimal monetary policy reaction function (12) accordingly). Proposition 2 proves the convexity of the steady state loss function with respect to the inflation target. This means that the significance of the buffer role of an inflation target dissipates as the inflation target is set higher.

The intuition of the buffer role of an inflation target is as follows. In the presence of a liquidity trap, the central bank is perpetually confronted with the risk of entering it. By taking a more expansionary position relative to the Taylor Rule (i.e., by adopting a pre-emptive monetary policy), it is possible for the central bank to circumvent the liquidity trap risk, which can be considered as a benefit for the central bank. However, the pre-emptive monetary policy proposed here is not a free-lunch, but comes with a cost. As a trade-off for taking such a pre-emptive position, a persistent inflation bias above target is created, which is a cost for the central bank. Taking the level of the inflation target as given, the central bank needs to strike a balance between the benefit of reduced liquidity trap risk and the cost of inflation bias by optimally choosing the magnitude of pre-emptiveness.

Now, suppose the inflation target is set at a relatively low level – say, π^*_{Low}. Then the risk of being caught in a liquidity trap will be relatively high that it induces the central bank to behave more pre-emptively in order to offset some of the liquidity trap risk. As a consequence of this significantly pre-emptive behavior, significant inflation bias is generated forcing the central bank to bear a high loss persistently – i.e., $L^{SS}_{Low} = (\pi^{SS}_{Low} - \pi^*_{Low})^2 > 0$. Next, consider the case where the inflation target is set at relatively high level – say, π^*_{High}. Then the risk of being caught in a liquidity trap will be relatively low that the central bank need not be too pre-emptive about the liquidity trap risk. Thanks to the less pre-emptive behavior by the central bank, the inflation bias, $(\pi^{SS}_{High} - \pi^{*}_{High})$, will remain relatively low compared to that of a low inflation target, $(\pi^{SS}_{Low} - \pi^{*}_{Low})$. As a result, when the target is set at a relatively high level, the steady state loss borne by the central bank will be relatively small – i.e., $L^{SS}_{Low} = (\pi^{SS}_{Low} - \pi^{*}_{Low})^2 > L^{SS}_{High} = (\pi^{SS}_{High} - \pi^{*}_{High})^2$. This is the basic mechanism whereby an increase in the inflation target can lower the stabilization cost in the long run.

Before we move on, some remarks are in order. First, if the central bank alone can freely choose the inflation target, there exists an incentive for a central bank to set an infinitely high inflation target in the presence of a liquidity trap (see Proposition 1). This is because the cost of inflation for the central bank is defined as the deviation from its target, rather than the level of inflation. In reality, however, there is obviously a social cost of inflation and, therefore, the social benefit aspect of the inflation (i.e., the buffer role) needs to be balanced. Second, in the absence of a liquidity trap, steady state loss will be invariant to the level of the inflation target since the marginal steady state loss with respect to the inflation target is zero. Therefore, in the absence of a liquidity trap, the central bank will be indifferent to where to set the target.

3.2 Phase Diagram Illustration: The Buffer Role of an Inflation Target

In the previous subsection, we have analytically shown the social benefit role of an inflation target. In other words, in the presence of a liquidity trap, the inflation target can function as a buffer to circumvent the risk of falling into a deflationary spiral, thereby reducing the stabilization cost in the long-run. In this subsection, we illustrate this buffer role of an inflation target using the phase diagram. This phase diagram approach to the dynamics of the inflation rate and output gap in the presence of a zero-bound was first taken by Reifschneider and Williams (2000). In their phase diagram, they show how the steady-state locus of the IS equation is altered in the presence of a liquidity trap and further show how a deflationary spiral zone arises as a consequence. In this paper, building upon Reifschneider and Williams (2000), we illustrate the risk of falling into a deflationary spiral zone and further show how the risk can be reduced by raising the inflation target.

Figure 3 shows the phase diagram of the controlled dynamics of the IS equation (5) and AS equation (3) where the nominal interest rate, i_t, in the IS equation is now substituted for the optimal monetary policy reaction function, $i^*_t = f(\pi_t, y_t)$, implied by the Bellman equation (11). The dashed curve in Figure 3 represents the set of states where the optimal monetary policy reaction function, $i^*_t = f(\pi_t, y_t)$, just binds at zero. The area above this dashed curve represents the set of states where i^*_t is positive. Thus, we can interpret this area as the place where 'conventional' monetary policy is taking effect. The area below the dashed curve represent the set of states where $i_t^* = 0$ (i.e., zero-bound constraint is binding). We adopt Krugman's (1998) terminology and refer to this area as the 'Liquidity Trap' Zone, in the sense that the monetary authority has exhausted the room for conventional monetary policy.

The bold vertical line and the kinked line in Figure 3, represent the loci such that $\Delta\pi = 0$ and $\Delta y = 0$, respectively. It should be noted that the locus such that $\Delta y = 0$ is negatively sloped in the region such that $i_t^* > 0$, yet is positively sloped within the Liquidity Trap Zone. This can be easily verified by substituting $i_t^* = 0$ in the IS equation (5). Point E_1 in Figure 3 represents the steady state of the economy, which is locally stable, and it occurs when the output gap is zero and the inflation rate is equal to π^{SS}. As pointed out by Reifschneider and Williams (2000), due to the zero-bound constraint, there exists a saddle path running through the saddle point E_2. The area above this saddle path represents the set of states where, in the absence of stochastic disturbances, the states eventually converge to the steady state E_1. For convenience, we call this area the Controllable Zone. In contrast, the area below the saddle path represents the set of states where both the inflation rate and output gap diverge to negative infinity, unless for a favorable shock that pushes the state back to the Controllable Zone. In referring to this area, we adopt the term 'Deflationary Spiral' Zone.[21] In other words, once the state is enmeshed in this region, there occurs a vicious cycle of run-away deflation and recession rendering the Cost-to-Go unaffordably high for the monetary authority to bear.[22]

The black area in Figure 3 represents the probability measure or the risk of falling into the Deflationary Spiral Zone from a steady state. The larger the area, the more the monetary authority exposed to the risk of a deflationary spiral and vice versa. Thus, in order to minimize the stabilization cost in the long-run, it is in the interest of the monetary authority to reduce this risk by some measure. If the inflation target is fixed, the best strategy for the monetary authority is to become more expansionary and aggressive as the state approaches the Deflationary Spiral Zone, so that it can contain the

[21] Note that Reifschneider and Williams (2000) refer it as 'Deflationary Trap.' However, since the term Deflationary Trap is not often used by economists and slightly fails to capture the nuance of run-away deflation and recession, we adopt the term Deflationary Spiral in this paper.

[22] Occasionally, the terms 'liquidity trap' and 'deflationary spiral' are used synonymously in the literature. However, as far as the model adopted in this paper is concerned, there is a subtle difference between the two concepts. 'Liquidity trap' simply refers to the state in which the nominal interest rate is binding at zero, yet this does not necessarily mean that the state of the economy is diverging (i.e. uncontrolable). Indeed, as can be seen from Figure 3, there exists a region between the dashed curve and the saddle path where the nominal interest rate is zero, but the state is still controlable in the sense that the economy can autonomously converge back to the steady-state. On the other hand, 'deflationary spiral,' in our definition, refers to a diverging state of the economy and this necessarily implies that the nominal interest rate is zero. In other words, the Deflationary Spiral Zone is a subset of the Liquidity Trap Zone, but the converse is not true.

state in the controllable region as much as possible. This was the reasoning behind the pre-emptive strategy that has been pointed out by Reifshcneider and Williams (2000), Orphanides and Wieland (2000), Kato and Nishiyama (2001), and Hunt and Laxton (2003).

Now, if the inflation target is variable – as considered in this paper –, the monetary authority can reduce the stabilization cost in the long-run by raising the inflation target. By setting a high inflation target, the steady state inflation rate, π^{SS}, will rise. Consequently, in the phase diagram, the steady state E_1 will move up along the vertical locus, thereby reducing the probability of the state falling into the Deflationary Spiral Zone – i.e., the black area will shrink as the inflation target moves up. This is the intuition behind the buffer role of an inflation target.

4 Numerical Illustration: A Quantitative Approach to the Buffer Role of an Inflation Target

4.1 Motivation and Methodology

The main purpose of this section is the quantitative illustration of the main propositions made in the previous section – i.e., the central bank's long-run stabilization cost is a decreasing-convex function of the inflation target. However, as was pointed out by Chmielewski and Manousiouthakis (1996), the LQ stochastic control problem no longer yields a linear feedback rule or a quadratic value function when a control variable is constrained, but rather yields a non-linear feedback rule and non-quadratic value function that does not have a closed form expression in general. In the context of inflation targeting with a zero-bound constraint, this implies that neither the optimal monetary policy reaction function (12) nor the monetary authority's value function (11) have a closed form expression.

Confronted with this difficulty, predecessors in this line of research have taken a rather different approach mainly relying on a simulation method. For instance, Fuhrer and Madigan (1997) showed that, in the presence of a liquidity trap, the impulse response of inflation and the output gap return to their steady states faster when the inflation target is set high. Orphanides and Wieland (1998), employing a stochastic simulation approach, showed the frequency of the nominal interest rate binding at zero to be high when the inflation target is set low and vice versa. Although these studies have eloquently demonstrated the predicament caused by the liquidity trap, their approach is rather indirect in illustrating the social benefit role of an inflation target. Moreover, since the monetary policy rules adopted in their studies are ad-hoc (i.e., not the optimal monetary policy reaction function), their assessments do not represent the 'true' social benefit role of an inflation target.

In this paper, we employ a rather direct approach in illustrating the social benefit role of an inflation target quantitatively – i.e., we approximate the optimal monetary reaction function and value function numerically. The strength of a numerical analysis is that it allows us to numerically approximate the 'true' optimal reaction function and value function to a degree of an arbitrary accuracy, where computational power is the limit. Although the quantitative implication of the numerical approach will be inherently parameter-dependent, we still can infer a significant amount of information from the numerical approximation and can learn further the properties of the loss function under the existence of a zero-bound constraint.

Some remarks regarding the numerical methodology adopted in this paper are in order. Since the Bellman equation, which we considered in eq. (11), is a member of the class of functional equations, the solution space of the dynamic programming problem is inherently a functional space whose dimension is infinite. The idea of a numerical approach in solving the dynamic programming is to reduce this infinite dimensional problem into a finite dimensional problem, a process also known as discretization of the space. In our paper we use a numerical technique, known as the Collocation method [Judd(1998) and Miranda and Fackler (2002)], in reducing the infinite-dimensional problem into finite dimensional problem. For further details regarding the Collocation method, see Appendix.

4.2 Preliminaries

4.2.1 Definition of Long-run Cost-to-Go

We first define the concept of 'long-run Cost-to-Go,'[23] which is used as a proxy concept for the central bank's long-run stabilization cost in this section. A few words of caution should follow. Strictly speaking, the concept of steady state Cost-to-Go (i.e., $V^{SS}(\pi^{SS}, 0; \pi^*) = \sum_{i=0}^{\infty} \beta^i L^{SS} = L^{SS}/(1-\beta)$, which is a proxy concept for the stabilization cost in the long-run in the previous section) and long-run Cost-to-Go (which is a proxy concept adopted in this section) are different concepts. The former cost concept, V^{SS}, emerges when the value function is evaluated at the *steady state* $(\pi^{SS}, 0)$, whereas the latter cost concept, V^{LR}, emerges when the value function is evaluated at the *target state* $(\pi^*, 0)$. The reason why we do not employ the steady state Cost-to-Go, V^{SS}, is purely a technical one. In the process of numerical analysis, we faced a difficulty in identifying precisely the value of the steady state $(\pi^{SS}, 0)$. In order to avoid this difficulty, we instead employ the long-run Cost-to-Go, V^{LR}, as a proxy concept for the stabilization cost in the long-run. Notice that, even when the initial state is at the target state $(\pi^*, 0)$, we know from eq. (14) and (15) that the state will eventually converge

[23] Cost-to-Go is the actual number of the central bank's value function evaluated at some certain state and represents the discounted sum of future losses given that certain state.

to the steady state $(\pi^{SS}, 0)$. Thus, when the target state and steady state are not too far apart, then the long-run Cost-to-Go and steady state Cost-to-Go should not be too different. We now state the definition of the long-run Cost-to-Go formally.

Definition 1 (Long-run Cost-to-Go) *Let $V(\pi_t, y_t; \pi^*)$ be the loss-function without a zero-bound constraint on i_t with an inflation target parameter π^*. Let $\tilde{V}(\pi_t, y_t; \pi^*)$ be the loss-function with a zero-bound constraint. Then the long-run Cost-to-Go without a zero-bound constraint is defined as the value of the loss-function evaluated at the $(\pi_t, y_t) = (\pi^*, 0)$; $V^{LR}(\pi^*, 0; \pi^*)$. Similarly, the long-run Cost-to-Go with a zero-bound constraint is defined as $\tilde{V}^{LR}(\pi^*, 0; \pi^*)$.*

In the remainder of this section, we will demonstrate that, indeed, the long-run Cost-to-Go in the presence of a liquidity trap, $\tilde{V}^{LR}$, is a decreasing-convex function of the inflation target by conducting a sensitivity analysis.

4.2.2 Parameter Values

As a preliminary step to compute the value of the long-run Cost-to-Go, we first numerically interpolate the central bank's value function with and without the zero-bound. We restrict the interpolation range of the value function to $-10 \leq \pi \leq 10$ and $-10 \leq y \leq 10$. Regarding the parameter values governing the IS and AS equations, as a benchmark, we take $\rho = 0.6$, $\delta = 0.4$, and $\alpha = 0.1$. For the parameter values governing the preference of the central bank, we set the preference parameter, λ, at 1 and the discount rate, β, at 0.7.[24] Further, we temporarily set the inflation target, π^*, at 0. For the sake of computational cost reduction, we specify the distribution of IS-AS shocks as $(\upsilon_t, \varepsilon_t)' \overset{iid}{\sim} N(0, \sigma I)$, where I stands for the 2×2 identity matrix. Thus, in this numerical illustration, IS and AS shocks have a standard deviation of σ and are uncorrelated with each other. Further, as a benchmark, we temporarily set the standard deviation, σ, at 1.

4.2.3 Interpolating Value Function

Figure 4 depicts the interpolated value function without a zero-bound constraint on the nominal interest rate. This can be regarded as a visualization of eq. (7). The

[24] The value of the discount rate may seem to be extremely low. However, this parameter choice is inevitable in a sense. Since the Cost-to-Go for the state inside the deflationary spiral zone tends to be extremely costly, the discounted sum of future losses tends to diverge under the conventional discount rate, such as 0.95. In order to keep the loss function visible under both a deflationary and a non-deflationary spiral regime, we have set the discount rate at 0.7. Also, considering that governor's term of appointment is no more than five years for most central banks, a discount rate of 0.7 in the infinite horizon model does not sound to be too unrealistic.

qualitative feature to be noted here is the symmetry of the value function.[25] This implies that, in the absence of a liquidity trap, the stabilization cost in a deflationary-recessionary state (i.e., third quadrant in π, y space) is symmetrically equal to that of inflationary-expansionary counterpart (i.e., first quadrant in π, y space).

Figure 5 depicts the interpolated value function in the presence of a zero-bound constraint. This can be regarded as a visualization of eq. (11). Unlike the symmetric feature we observed in Figure 4, the value function in the presence of the zero-bound reveals an asymmetric feature which entails the value function to be non-quadratic. The asymmetry is most obvious by observing the sharp rise of Cost-to-Go in a deflationary-recessionary state. This means that, with a liquidity trap present, the stabilization cost in a deflationary-recessionary state is much higher than that of the inflationary-expansionary counterpart. This is another exemplification why the central bank should act pre-emptively to avoid the Deflationary Spiral Zone.

4.2.4 Identifying the Long-run Cost-to-Go

Having interpolated the value function, we are now ready to evaluate the central bank's long-run Cost-to-Go. Again, the idea is to evaluate the value function at the target state, $(\pi, y) = (\pi^*, 0)$. For visual convenience in identifying the long-run Cost-to-Go, we rotate Figure 4 – the interpolated value function without a zero-bound – to offer the (magnified) horizontal viewpoint and is shown in Figure 6. As can be seen from the figure, the Cost-to-Go of the value function evaluated at the target state $(\pi^*, 0)$ is 2.61. This value represents the 'long-run Cost-to-Go' (which is a proxy of 'stabilization cost in the long run' in this section), which we defined earlier in this section. It should be noted that the long-run Cost-to-Go identified in Figure 6 was free of risk arising from a liquidity trap.

In the same way, we rotate Figure 5 – the interpolated value function with a zero-bound – to offer the (magnified) horizontal viewpoint and is shown in Figure 7. As can be seen from the figure, the Cost-to-Go of the value function evaluated at the target state $(\pi^*, 0)$ is 3.27, which represents the long-run Cost-to-Go in the presence of the zero-bound. Here we notice an important feature. Comparing the long-run Cost-to-Go without a zero-bound to that with the zero-bound, we notice that the cost in the presence of the zero-bound to be higher. This is exactly the feature we should expect from the argument made in Section 3. Namely, in the presence of the zero-bound, a central bank is exposed to the risk of being caught in a liquidity trap – an additional risk factor that is non-existent in the case without a zero-bound constraint. Reflecting the risk from a liquidity trap, the long-run Cost-to-Go with the zero-bound will necessarily

[25] Or to be more specific, the quadratic nature of the loss function. When the objective function is quadratic and the state transition function is linear, then the value function will be a quadratic function of the state variables. This is a well known result in the linear-stochastic control literature.

be costlier than that without it. Further, by subtracting the long-run Cost-to-Go in Figure 6 from Figure 7, we can completely separate out the cost emerging from the liquidity trap risk from the standard risk emerging from stochastic disturbances. In this particular case, where parameters have been set at $\pi^* = 0$ (i.e., inflation target) and $\sigma = 1$ (i.e., standard deviation of the stochastic disturbance terms), the portion of the long-run Cost-to-Go emerging solely from the liquidity trap risk can be computed as, $\tilde{V}(\pi^*, 0) - V(\pi^*, 0)$ =3.27 - 2.61 = 0.66.

4.3 Sensitivity Analysis I: Long-run Cost-to-Go and Inflation Target

Having identified the long-run Cost-to-Go emerging from the liquidity trap risk, we are now ready to illustrate the relationship between an inflation target and the long-run Cost-to-Go. In other words, in order to substantiate the proposition we have made in Section 2, we demonstrate that, indeed, the long-run Cost-to-Go is a decreasing-convex function of an inflation target.

Thus far, for simplicity, we have fixed the inflation target, π^*, at zero. We now conduct a sensitivity analysis of the long-run Cost-to-Go with respect to varying inflation targets. Before examining the numerical results, it is useful to review the intuition behind the decreasing-convex relationship between the long-run Cost-to-Go and the inflation target. Suppose the inflation target has been set at an extremely low value, say negative one percent. Since the risk of being caught in a liquidity trap will be high, we expect to see the long-run Cost-to-Go to be also high. On the other hand, setting a high inflation target will significantly mitigate the risk of being caught in a liquidity trap so that we expect the long-run Cost-to-Go to be low. Keeping this conjecture in mind, let us now turn to the numerical results.

Figure 8 depicts the relationship between the long-run Cost-to-Go with respect to varying levels of inflation targets from -1% to 5%. Note that we have interpolated the value functions for each inflation target and evaluated each long-run Cost-to-Go in depicting the graph shown in Figure 8. The solid line represents the relationship in the presence of the zero-bound and the dashed line represents the relationship without the zero-bound. Let us first turn to the solid line, the long-run Cost-to-Go in the presence of a liquidity trap. Conforming to our porpositions, the long-run Cost-to-Go tends to be high when the inflation target is set low and vice-versa. Interpreting the reduction of the long-run Cost-to-Go as a social benefit, **this solid line exactly captures the social benefit role (or the buffer role) of an inflation target in the presence of a liquidity trap – i.e., the stabilization cost in the long-run is a decreasing-convex function of the inflation target.**

Next, let us turn to the long-run Cost-to-Go without the risk of a liquidity trap, which is shown by the dashed line in Figure 8. Not surprisingly, the dashed line is horizontal implying that, without the fear of a liquidity trap, the long-run Cost-to-Go is constant regardless of the choice of inflation target. In other words, the long-run cost is invariant to the choice of inflation target that no matter how high the inflation target may be, it is not possible for a central bank to reduce the stabilization cost in the long-run. The reason is quite simple. In the absence of a zero-bound constraint, there will be no risk of being caught in a liquidity trap. Therefore the long-run Cost-to-Go is now composed of the standard stabilization cost arising from IS-AS shocks only. Since the variance of IS-AS shocks is invariant with respect to the choice of inflation target, the stabilization cost is constant regardless of the inflation target. This is the reason behind the horizontal line depicted in Figure 8. To summarize, **in the absence of a liquidity trap, there is no social benefit role (or the buffer role) of an inflation target – i.e., the stabilization cost in the long-run is invariant to the level of the inflation target.**

Some remarks are in order. As can be seen from Figure 8, the solid line always lies above the dashed line, which is trivially saying that the long-run Cost-to-Go under the risk of a liquidity trap is higher than that without it. This gap between the two lines thus represents the stabilization cost solely arising from the liquidity trap risk. It should be noted that this gap decreases as the inflation target increases implying that the risk of being caught in a liquidity trap is decreasing. One should also note the convergence of the solid line toward the dashed line as the inflation target gets higher. This implies that the probability of the state entering the liquidity trap zone tends to zero as the inflation target tends to infinity, which is a sensible result.

4.4 Sensitivity Analysis II: Long-run Cost-to-Go, Inflation Target and Volatility

In the previous sensitivity analysis, we have fixed the standard deviation of IS-AS shocks at one (i.e, $\sigma = 1$), in order to illustrate the relationship between the long-run Cost-to-Go and the inflation target. In this subsection, we relax this assumption and conduct a sensitivity analysis of the long-run Cost-to-Go with respect to the varying levels of volatility in IS-AS shocks.

Figure 9 depicts the long-run Cost-to-Go corresponding to various levels of inflation targets and volatilities without a zero-bound constraint. The interval of standard deviation has been set from 0.25 to 2.5. From a careful examination of Figure 9, we can make the following two observations. First, regardless of the magnitude of volatility, the long-run Cost-to-Go is invariant to the level of inflation target. To put it differently, the social marginal benefit with respect to the inflation target is zero regardless of the level of volatility. Second, as the magnitude of volatility increases, the long-run Cost-to-Go rises. Note that this increase in the long-run Cost-to-Go is simply capturing the standard stabilization cost and nothing more than that.

Let us now turn to Figure 10, which depicts the long-run Cost-to-Go in the presence of a liquidity trap corresponding to various levels of inflation targets and volatilities. Similar to the feature observed in Figure 9, we observe that the long-run Cost-to-Go increases as volatility rises. However, it should be noted that, for any magnitude of volatility, the long-run Cost-to-Go is no longer invariant to the level of the inflation target. Indeed, it seems that the gradient of the long-run Cost-to-Go with respect to the inflation target gets steeper as volatility rises, which is equivalent to saying that the buffer role of an inflation target becomes more substantial as volatility increases.

In order to clearly illustrate the effect of volatility on the liquidity trap risk, we calculate the difference between Figure 9 (which represents the long-run Cost-to-Go emerging from the standard stabilization cost only) and Figure 10 (which represents the cost emerging from both the standard stabilization cost and the liquidity trap risk). The calculated difference is shown in Figure 11. Again, note that Figure 11 depicts the component of long-run Cost-to-Go emerging solely from the liquidity trap risk. As can be vividly seen from Figure 11, the gradient of the long-run Cost-to-Go with respect to the inflation target gets steeper as volatility increases. In other words, **the social benefit role (or the buffer role) of an inflation target tends to be more significant as the volatility of IS-AS shocks rises.** This in turn implies that there is a stronger incentive for a central bank to set a higher inflation target as the economy becomes more uncertain.

So what is the intuition behind this result? Suppose the inflation target is set at some constant level. An increase in the volatility of IS-AS shocks will then increase the long-run Cost-to-Go of the central bank for two reasons. The first one is trivial. Due to an increase in the volatility of IS-AS shocks, naturally the standard stabilization cost will increase. This is true for both cases with or without the zero-bound. Another reason, which is more intriguing, is that as the volatility of IS-AS shocks rises, the probability of the state entering the Deflationary Spiral Zone will also rise, thereby increasing the long-run Cost-to-Go – i.e., the component of stabilization cost emerging from the risk of being caught in a liquidity trap. This is the reason why the buffer role of an inflation target becomes more significant as the economy becomes more volatile.

5 Concluding Remarks

In times of low inflation when the event of deflation is no longer a myth, central banks are seriously exposed to the risk of falling into a liquidity trap. Confronted with this reality, the design of monetary policy in avoiding a liquidity trap – so-called pre-emptive monetary policy – is now one of the most pressing issues for policymakers and researchers alike. As a part of this pre-emptive monetary policy, it has been pointed out that a central bank can potentially circumvent the risk of falling into a liquidity trap by targeting small but positive inflation in the long-run. This particular function of positive inflation can be regarded as a social benefit aspect of inflation, in the sense that it reduces the ex-ante long-run stabilization cost for a central bank. In reality, this social benefit aspect of inflation arising from avoiding liquidity trap risk seems to be well recognized by many central banks. Indeed, the central banks adopting an explicit form of inflation target are setting their targets in the range of 1 to 3 percent. However, the theoretical developments of a social benefit aspect of inflation arising from avoiding liquidity trap risk have been mainly conjectural or have relied on simulation evidence.

In this paper, based on the stylized framework of a central bank's dynamic optimization problem following Svensson (1997), we have attempted to provide the analytical foundation for a social benefit aspect of inflation stemming from the presence of a liquidity trap. In particular, we have shown analytically that the long-run stabilization cost of a central bank is a decreasing-convex function of an inflation target, implying that, in the presence of a liquidity trap, it is possible for a central bank to reduce the stabilization cost by setting a higher inflation target in the long-run. Putting it another way, we have shown that an inflation target can play a key role in circumventing the risk of falling into a liquidity trap, thereby reducing the stabilization cost in the long-run – i.e., the 'buffer' role of an inflation target.

Further, in order to demonstrate the analytical properties proven, employing the numerical method – also known as the Collocation method – , we have directly interpolated the (non-linear) optimal monetary policy reaction function and the (non-quadratic) value function. Conforming to the analytical properties, sensitivity analysis results have shown the stabilization cost in the long-run to be decreasing and convex in inflation target and also have shown the buffer role of an inflation target to be more significant as the volatility of the economy rises.

In sum, this paper has analytically and numerically clarified the mechanism by which an inflation can play a social benefit role in the presence of a liquidity trap, providing a theoretical justification for targeting small but positive inflation in the long-run. Of course, in reality the social benefit role of inflation is not confined to the 'buffer' role discussed in this paper, but also can arise from other factors. For instance, as was pointed out by Akerlof et al. (1996), when there exists a downward rigidity in nominal wages, too low an inflation rate will impair real wage adjustment, thereby disrupting the efficiency of the labor market. In this sense, as they point out, small but positive inflation can serve as a 'grease' for efficient clearing of a labor market. Thus, in this sense, the 'buffer' role of inflation should not be seen as *'the'* social benefit aspect of inflation, but rather should be regarded as *'a'* social benefit. With no doubt, the complete picture of the social benefit of inflation will include the 'grease' role as well as the 'buffer' role.

Before closing this paper, some remarks should be made regarding the optimal level of inflation target. As King (1999) states, "as inflation has fallen from earlier high levels toward something approaching price stability, the question of what is the optimal inflation rate has become more important" (p.15). Although we understand and share King's view, nevertheless, we have kept our focus on identifying the social benefit role of an inflation target in this paper, merely offering a qualitative justification for targeting positive inflation in the long-run, but have deliberately kept silent about where to target. This is not without reason. In discussing the optimal inflation rate, it is essential to identify the social benefit as well as cost of inflation. However, since the theory of the social cost of inflation is still in development and considering that the potential factors constituting the social cost of inflation cover a wide spectrum,[26] we are not yet aware of a model that incorporates all factors under one model. Conceding that it may be possible to incorporate some aspects of the social cost of inflation under one model and that it may be possible to derive the 'optimal' inflation rate, however, can we really be confident in calling it as the optimal inflation rate while being aware that some aspects of social cost are lacking? Nevertheless, we still share King's view and further research in pursuit of the optimal inflation rate is definitely important, especially in this era of low inflation.

[26] For instance, to name the few, Lucas (2000) discusses the 'shoe leather' cost of inflation, Feldstein (1997) discusses the 'tax distortion' cost caused by inflation, Woodford (2001a) discusses the 'relative price' distortion caused by inflation, and Cecchetti and Ehrmann (1999) point out that high inflation is historically associated with high volatility. For a survey on the costs of inflation (as well as on the benefits of inflation), see Shiratsuka (2001).

A Appendix: Collocation Method

In this appendix, we explain the numerical algorithm in approximating the value function in the presence of a zero-bound constraint. Specifically, we employ a numerical method known as the Collocation method[27] in solving the functional fixed-point problem posed by the Bellman equation.

For convenience, let us restate the Bellman equation (eqn (7)) suppressing the time subscripts as follows,

$$V(\pi, y) = \min_{x \geq 0} \{ f(\pi, y; \pi^*) + \beta EV(g(\pi, y, x, \upsilon, \varepsilon) \}, \tag{22}$$

where $f(\pi, y; \pi^*)$ stands for the period-by-period loss function with the inflation target parameter, π^*, and $g(\pi, y, x, \upsilon, \varepsilon)$ stands for the state transition function. Note that the nominal interest rate, denoted by x in this appendix, is constrained by the zero lower bound. The state transition function is linear in the state variables and the coefficient matrix is time-invariant, i.e.,

$$g(\pi, y, x, \upsilon, \varepsilon) = \begin{bmatrix} \rho + \alpha\delta & \delta \\ \alpha & 1 \end{bmatrix} \begin{bmatrix} y \\ \pi \end{bmatrix} - \begin{bmatrix} \delta \\ 0 \end{bmatrix} x + \begin{bmatrix} \upsilon \\ \varepsilon \end{bmatrix}.$$

Given the above specification of the Bellman equation and the state transition function, our goal is to interpolate the value function $V(\pi, y)$ in the interval of $-10 \leq \pi \leq 10$ and $-10 \leq y \leq 10$.

The Collocation method proceeds in the following steps. First, we discretize the state space by the set of interpolation nodes such that $Node = \{(\pi_{n_\pi}, y_{n_y}) | \; n_\pi = 1, 2, \cdots, N_\pi$ and $n_y = 1, 2, \cdots, N_y\}$.[28] Thus, we discretize the state space into a total of $N_\pi \times N_y$ interpolation nodes. Then we interpolate the value function $V(\cdot)$ using a cubic spline function[29] over these interpolation nodes as follows.

$$V(\pi_{n_\pi}, y_{n_y}) = \sum_{i=1}^{N_\pi} \sum_{j=1}^{N_y} c_{ij} \gamma_i^\pi(\pi_{n_\pi}) \gamma_j^y(y_{n_y}), \quad \text{for each } (\pi_{n_\pi}, y_{n_y}) \in Node. \tag{23}$$

[27] For complete elucidation regarding the Collocation method, see Judd (1998, Ch.11 and 12) and Miranda and Fackler (2002, Ch.8 and 9).

[28] There are several ways to discretize the state space. One example is Chebychev nodes. However, in order to preserve an exact solution of the value function and optimal policy reaction function at the equally distributed states, equally distributed interpolation nodes have been chosen in this paper.

[29] There are several other options for the basis function. One of the most frequently used basis functions is the Chebychev polynomial, which is known to possess superior properties when the curvature of the function to be interpolated is "nice and smooth." In contrast, the cubic spline function is known to possess superior properties when the function contains some "kinks." Since the value function and the optimal policy reaction function are kinked due to the presence of the zero lower bound in this paper, the cubic spline function will be our choice as a basis function. For more details regarding cubic spline interpolation, see Judd (1998, Ch.6), Cheney and Kincaid (1999), and Miranda and Fackler (2002, Ch.6).

The basis functions $\gamma_i^\pi(\pi_{n_\pi})$ and $\gamma_j^y(y_{n_y})$ take the form of cubic spline functions and are defined as

$$\gamma_i^\pi(\pi_{n_\pi}) = \left\{ \begin{array}{cc} \frac{2}{3}(1-6q_\pi^2(1-q_\pi)) & \text{if } q_\pi = \frac{|\pi_{n_\pi}-\pi_i|}{w} \leq 1 \\ \frac{4}{3}(1-q_\pi)^3 & \text{if } 1 \leq q_\pi \leq 2 \\ 0 & \text{otherwise} \end{array} \right\}$$

$$\gamma_j^y(y_{n_y}) = \left\{ \begin{array}{cc} \frac{2}{3}(1-6q_y^2(1-q_y)) & \text{if } q_y = \frac{|y_{n_y}-y_j|}{w} \leq 1 \\ \frac{4}{3}(1-q_y)^3 & \text{if } 1 \leq q_y \leq 2 \\ 0 & \text{otherwise} \end{array} \right\},$$

where $\pi_i = \underline{\pi} + wi$, where w is an equal step from the lower bound of state π (which is -10 in this paper) to the upper bound (which is 10 in this paper). The definition of y_j is similar. Interpolation equations (23) could be expressed compactly using the tensor product notation as follows,

$$\mathbf{v} = [\Gamma_\pi \otimes \Gamma_y] \cdot \mathbf{c}, \tag{24}$$

where $\mathbf{v}$ stands for $N_\pi N_y \times 1$ vector of the values of $V(\pi_{n_\pi}, y_{n_y})$ for each interpolation node, Γ_π stands for $N_\pi \times N_\pi$ matrix of the basis functions $\gamma_i^\pi(\pi_{n_\pi})$ (i.e., each matrix element is defined as $\Gamma_\pi[i, n_\pi] = \gamma_i^\pi(\pi_{n_\pi})$), Γ_y stands for $N_y \times N_y$ matrix of the basis functions $\gamma_j^y(y_{n_y})$, and $\mathbf{c}$ stands for $N_\pi N_y \times 1$ vector of the basis coefficients c_{ij}.

Next, we turn to the right-hand side of the Bellman equation (22). In approximating the expected value function, i.e., $E[V(g(\pi, y, x, \upsilon, \varepsilon)]$, we assume the distribution of the error terms (υ, ε) to be *i.i.d.* multivariate normal. Under the assumption of normal distribution, the expected value function can be approximated by the Gaussian-Hermite quadrature method[30] – a type of Gaussian quadrature method which is specifically used when the error terms are normally distributed. The Gaussian-Hermite quadrature method discretizes the random space with a set of quadrature nodes such that $QNode = \{(\upsilon_{h_\upsilon}, \varepsilon_{h_\varepsilon}) | h_\upsilon = 1, 2, \cdots, M_\upsilon \text{ and } h_\varepsilon = 1, 2, \cdots, M_\varepsilon\}$ with corresponding quadrature weights $\omega_{h_\upsilon h_\varepsilon}$. Thus, we discretize the random space into a total of $M_\upsilon \times M_\varepsilon$ quadrature nodes. Then by substituting the interpolation equation (23) for the value function $V(g(\pi, y, x, \upsilon, \varepsilon)$, the right-hand side of the Bellman equation can be approximated as

$$RHS_{n_\pi n_y}(\mathbf{c}) = \min_{x \geq 0} \left\{ f(\pi_{n_\pi}, y_{n_y}) + \beta \sum_{h_\upsilon=1}^{M_\upsilon} \sum_{h_\varepsilon=1}^{M_\varepsilon} \sum_{i=1}^{N_\pi} \sum_{j=1}^{N_y} \omega_{h_\upsilon h_\varepsilon} c_{ij} \gamma_{ij}(g(\pi_{n_\pi}, y_{n_y}, x, \upsilon_{h_\upsilon}, \varepsilon_{h_\varepsilon})) \right\}, \tag{25}$$

for each $(\pi_{n_\pi}, y_{n_y}) \in Node$ where γ_{ij} stands for the cross products of the basis function. The minimization of the above problem with respect to x can be attained using

[30] For more details regarding the Gaussian Quadrature method, see Judd (1998, Ch.7) and Miranda and Fackler (2002, Ch. 5).

a standard Quasi-Newton optimization method. It should be noted that when implementing this minimization problem, one should pay attention to the corner solution of the minimization problem due to the zero lower bound constraint on the control variable x.

Finally, by equating eqn (23) and eqn (25) for each interpolation node, we obtain the following approximation of the Bellman equation (22),

$$\sum_{i=1}^{N_\pi}\sum_{j=1}^{N_y} c_{ij}\gamma_i^\pi(\pi_{n_\pi})\gamma_j^y(y_{n_y}) = RHS_{n_\pi n_y}(\mathbf{c}) \text{ for each } (\pi_{n_\pi}, y_{n_y}) \in Node. \tag{26}$$

Using the tensor product notation, the above equation can be compactly expressed as

$$[\Gamma_\pi \otimes \Gamma_y]\mathbf{c} = \mathbf{RHS}(\mathbf{c}), \tag{27}$$

where $\mathbf{RHS}(\mathbf{c})$ stands for $N_\pi N_y \times 1$ vector of the values of $RHS_{n_\pi n_y}(\mathbf{c})$. Now the task is to find the unknown basis coefficient vector $\mathbf{c}$ from the above nonlinear equation system (27). The nonlinear equation system can be solved using an iterative nonlinear root-finding technique such as the Functional Iteration method, Newton's method or a Quasi-Newton method.[31] For computational ease, we have adopted the Functional Iteration method as the solution algorithm.[32]

Algorithm 3 *(Functional Iteration method)*
Step 1: Choose the degree of approximation N_π,N_y,M_υ, *and* M_ε. *Then set the appropriate interpolation nodes and quadrature nodes for the state space and random space, respectively. Guess the initial basis coefficients vector* $\mathbf{c}_0$.
Step 2: Update the basis coefficient vector by the following functional iteration;

$$\mathbf{c}_{k+1} \leftarrow [\Gamma_\pi^{-1} \otimes \Gamma_y^{-1}] \cdot \mathbf{RHS}(\mathbf{c}_k).$$

Step 3: Check for convergence. If $|c_{ij,k+1} - c_{ij,k}| < \tau$ *for any* i *and* j, *where* τ *is a convergence tolerance parameter, then stop. Otherwise, repeat step 2.*

Once convergence has been reached, interpolation of the value function $V(\pi, y)$ has been achieved. Of course, as a by-product of interpolating the value function, the approximation of the optimal policy function $x^*(\pi, y)$ will also be attained at the same time. It should be noted that one can attain the desired level of approximation by controlling the degree of interpolation nodes, quadrature nodes and convergence tolerance parameter τ with a trade-off of convergence speed.[33]

[31] For more details regarding the nonlinear root-finding technique, see Judd (1998, Ch.5) and Miranda and Fackler (2002, Ch.3).

[32] We also tried the Quasi-Newton method and obtained virtually the same result.

[33] In our paper, we have set the parameter values as follows; $N_\pi = 20$, $N_y = 20$, $M_\upsilon = 3$, $M_\varepsilon = 3$

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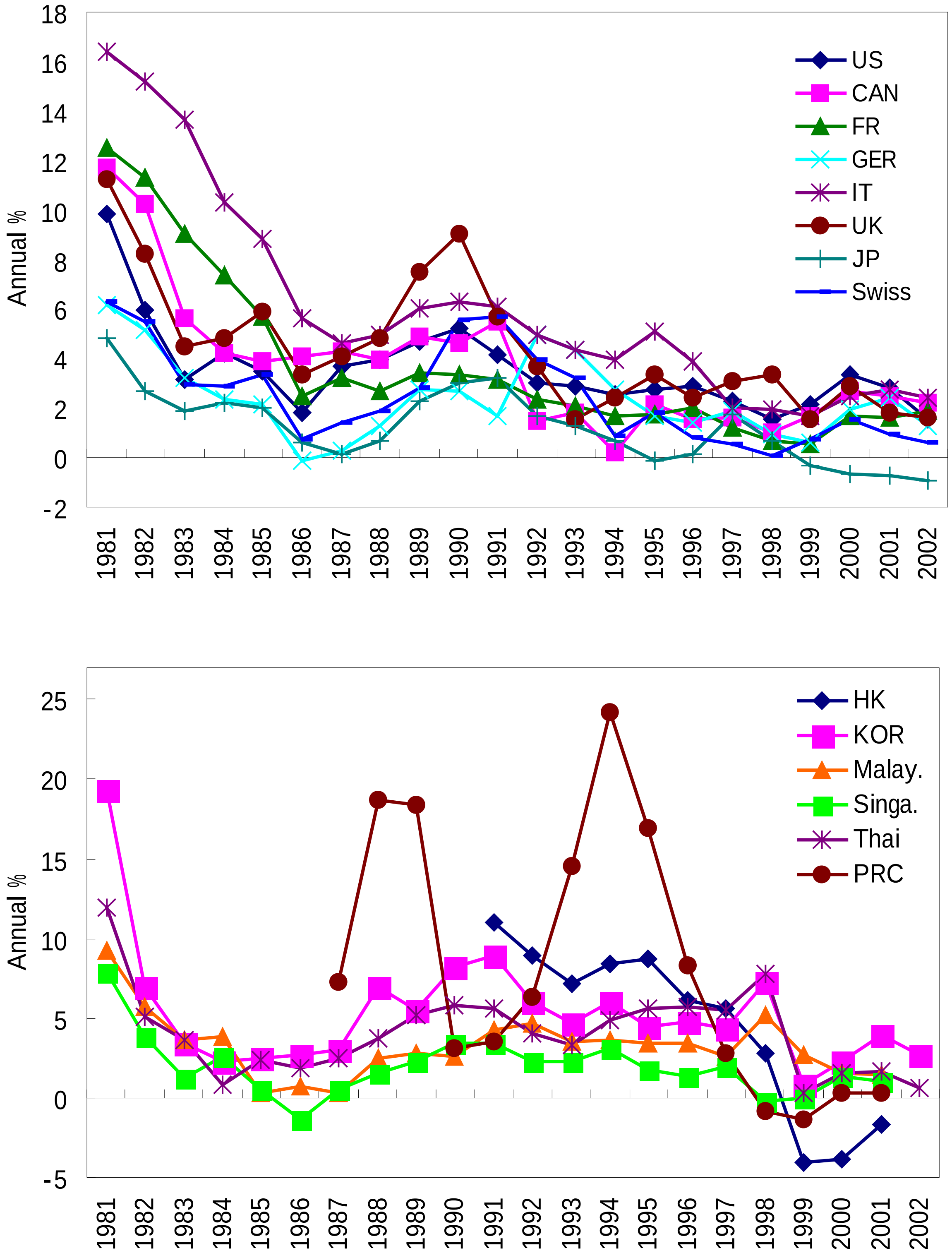
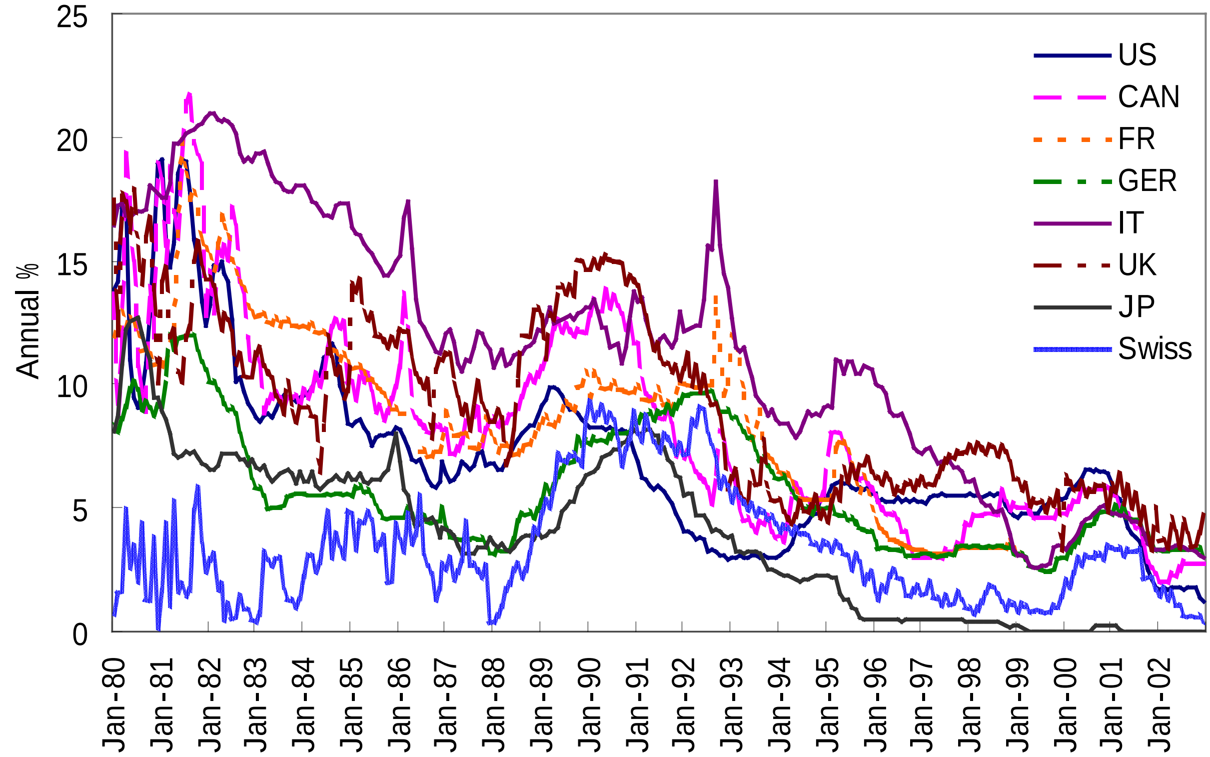
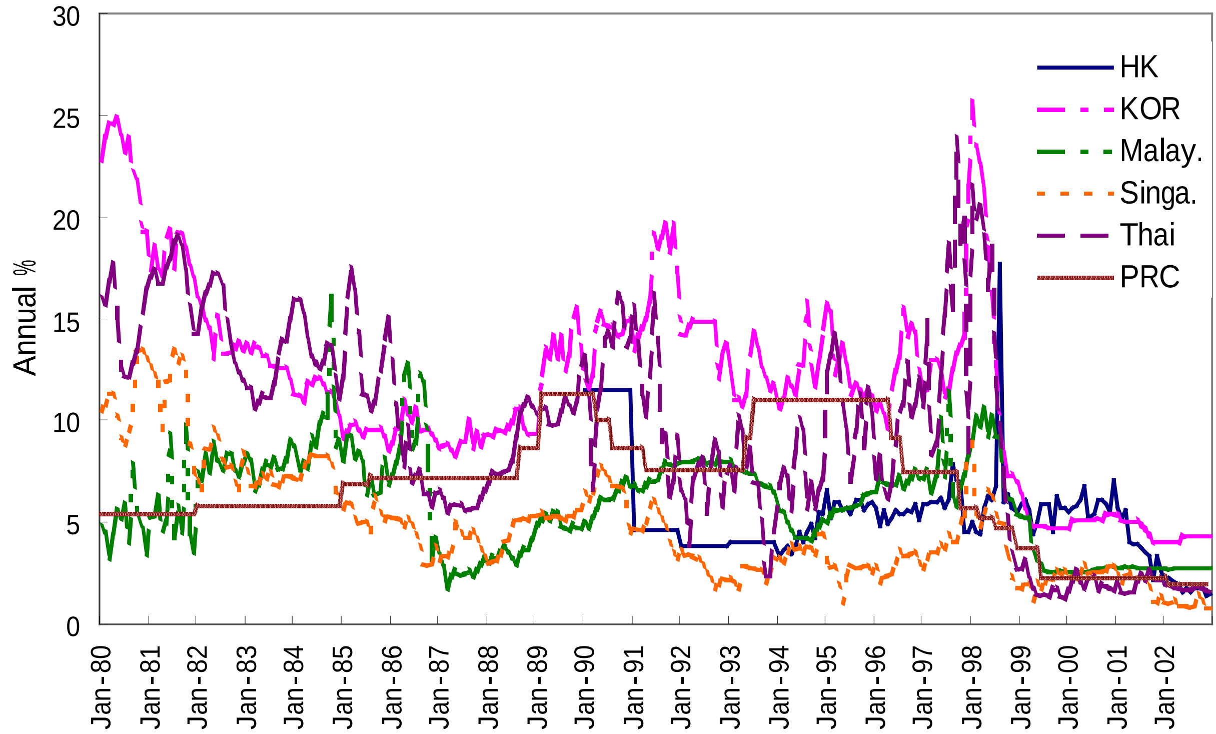
Figure 1: Inflation in Industrial and Emerging Market Economies



Source: IMF, International Financial Statistics

Figure 2: Money Market Rates in Industrial and Emerging Market Economies





Source: IMF, International Financial Statistics

Phase Diagram Illustration:
Intuition behind the Buffer Role of an Inflation Target

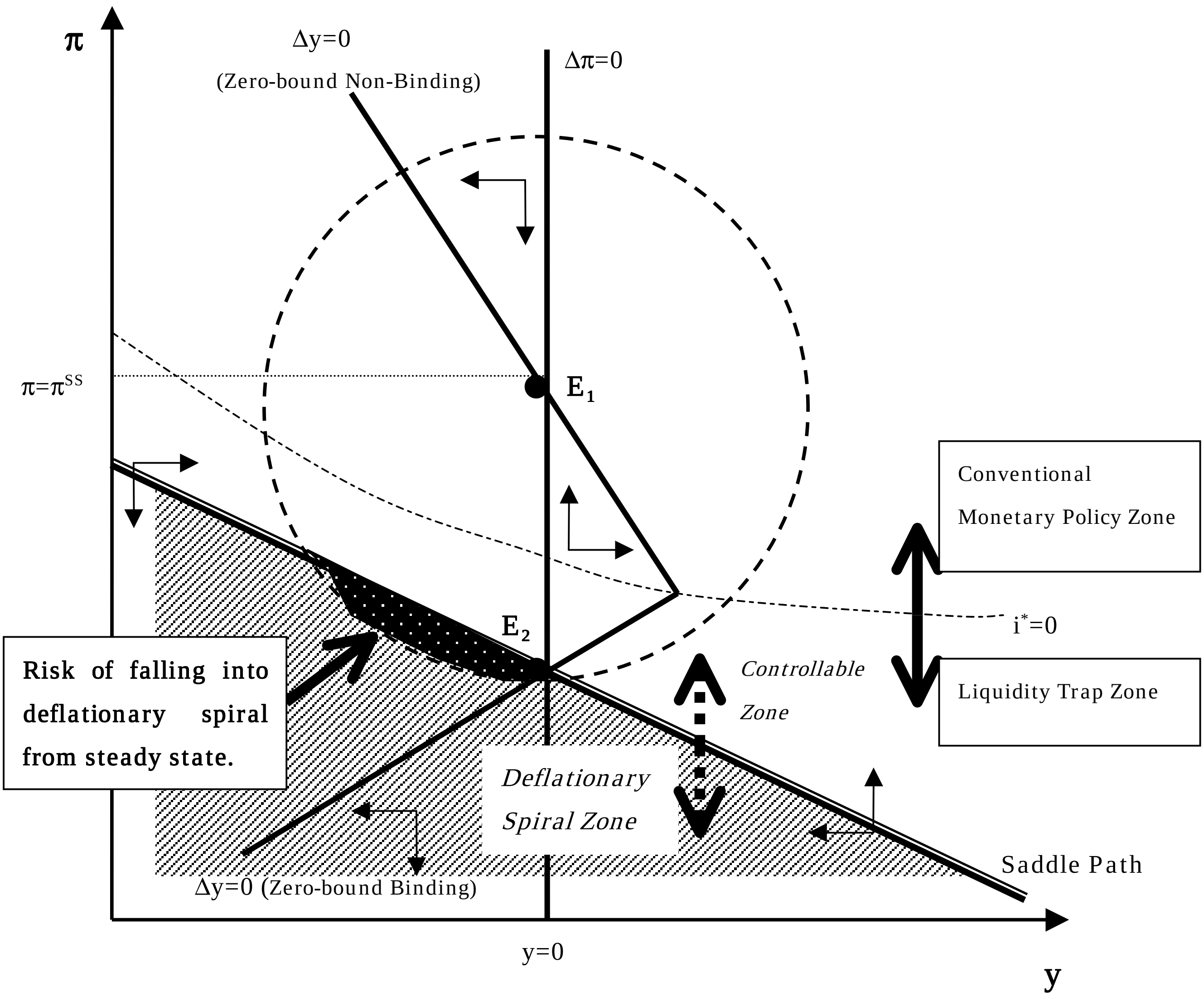


Figure 3

Note: The monetary authority can reduce the risk of falling into a deflationary spiral by raising the inflation target in the long-run. Note that steady-state inflation is an increasing function of the inflation target.

Figure 4: Interpolated Value Function without Zero-Bound (π^*=0, σ=1)

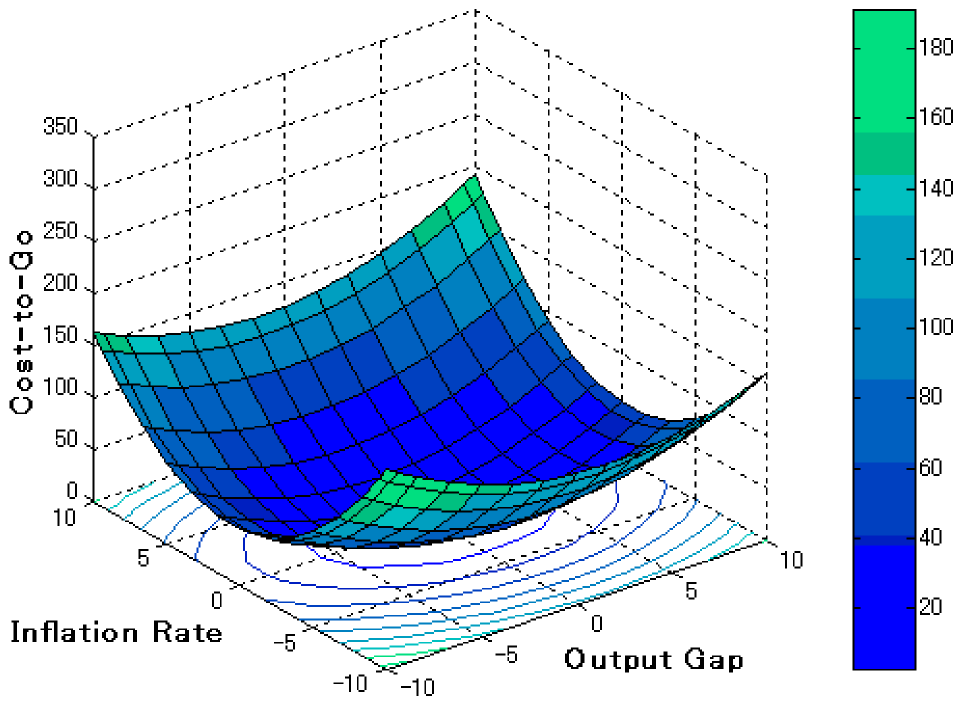


Figure 5: Interpolated Value Function with Zero-Bound (π^*=0, σ=1)

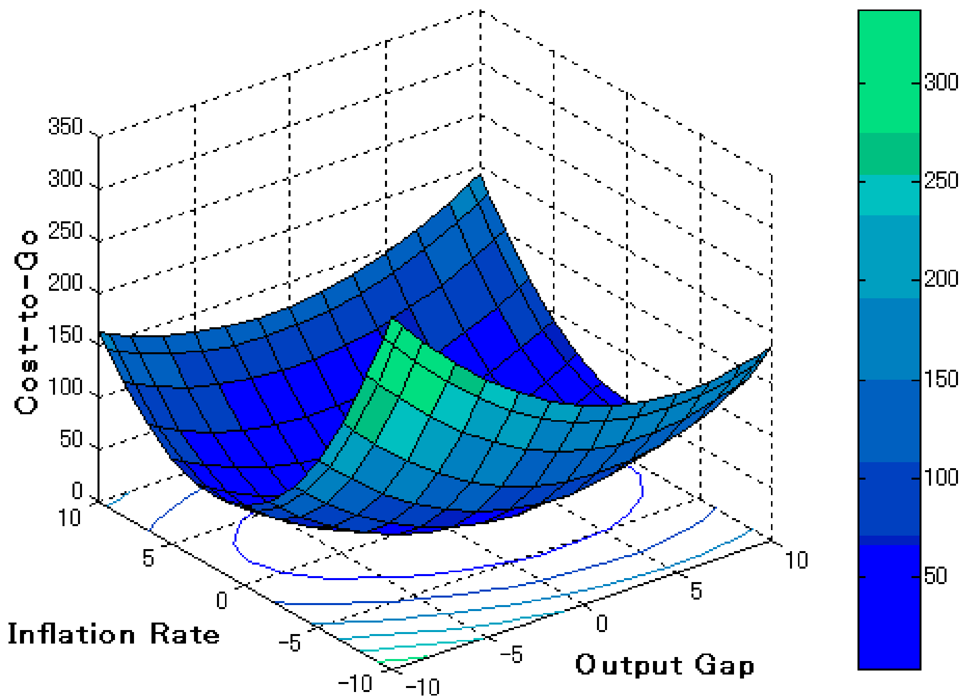


Figure 6: Horizontal View of Figure 4 (π^*=0, σ=1)

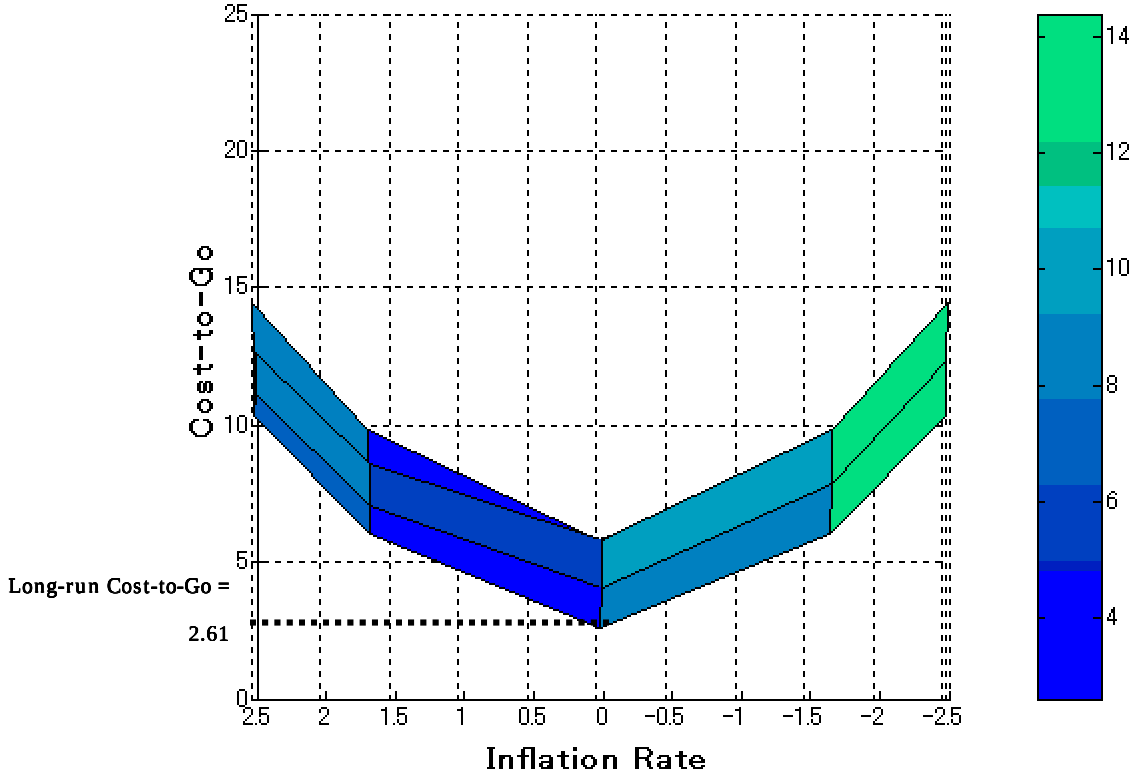


Figure 7: Horizontal View of Figure 5 (π^*=0, σ=1)

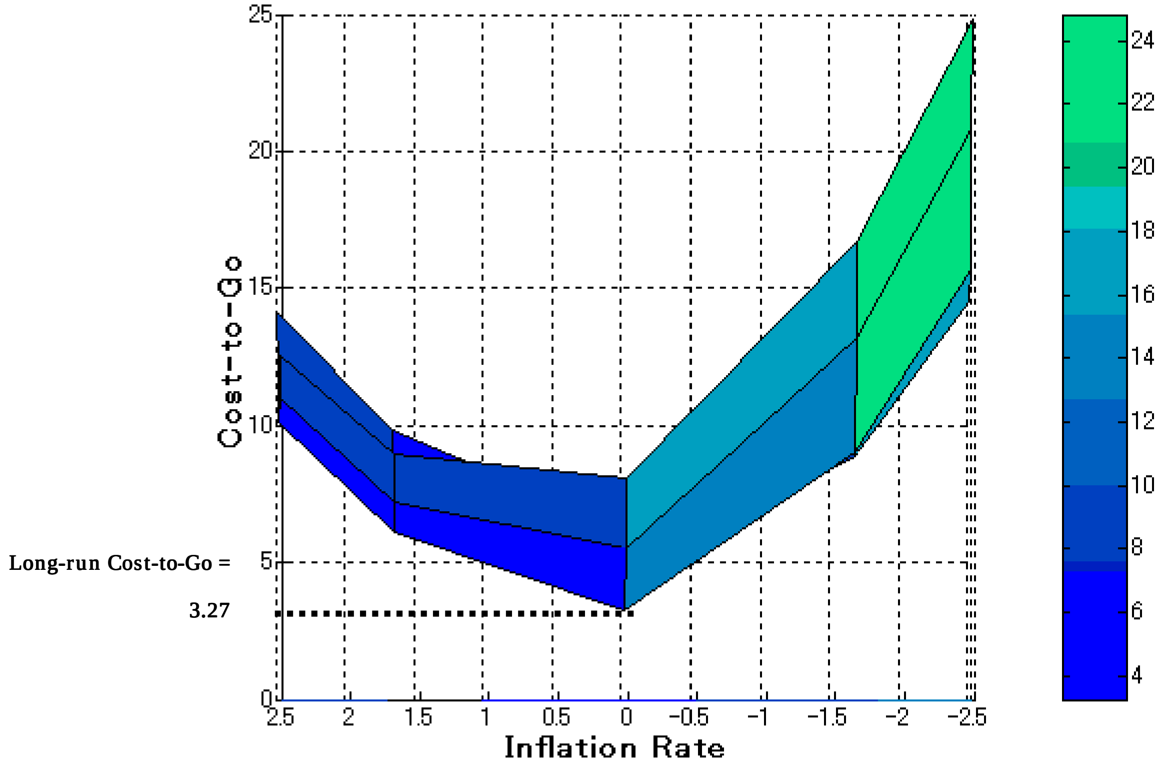
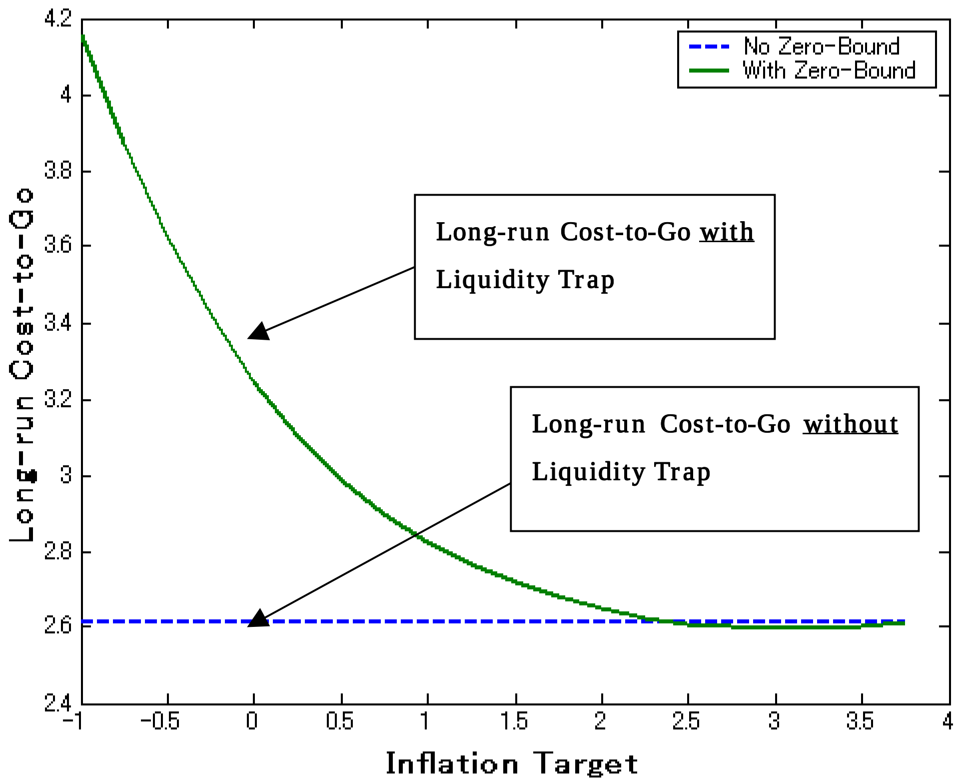


Figure 8: Relationship between Long-run Cost-to-Go and Inflation Target
(Given σ = 1)



Note: The solid curve represents the long-run Cost-to-Go in the presence of a liquidity trap as a function of the inflation target, while the dashed curve represents the long-run Cost-to-Go in the absence of a liquidity trap. As can be seen from the figure, the long-run Cost-to-Go is a decreasing-convex function of the inflation target, which implies that a central bank's stabilization cost could be reduced by setting a higher inflation target in the long-run – i.e., the buffer role of an inflation target. It should be noted that this buffer role of an inflation target is non-existent when there is no liquidity trap in the economy. In other words, the long-run Cost-to-Go is invariant to the level of the inflation target when there is no liquidity trap.

Figure 9: Relationship among LR Cost-to-Go, Inflation Target, and Volatility
(No Liquidity Trap)

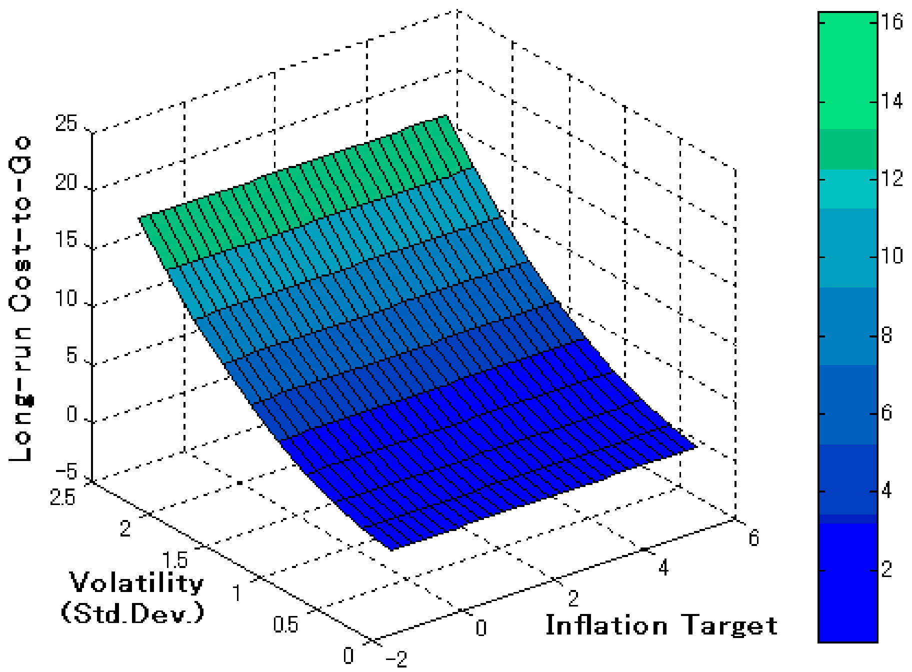


Figure 10: Relationship among LR Cost-to-Go, Inflation Target, and Volatility
(With Liquidity Trap)

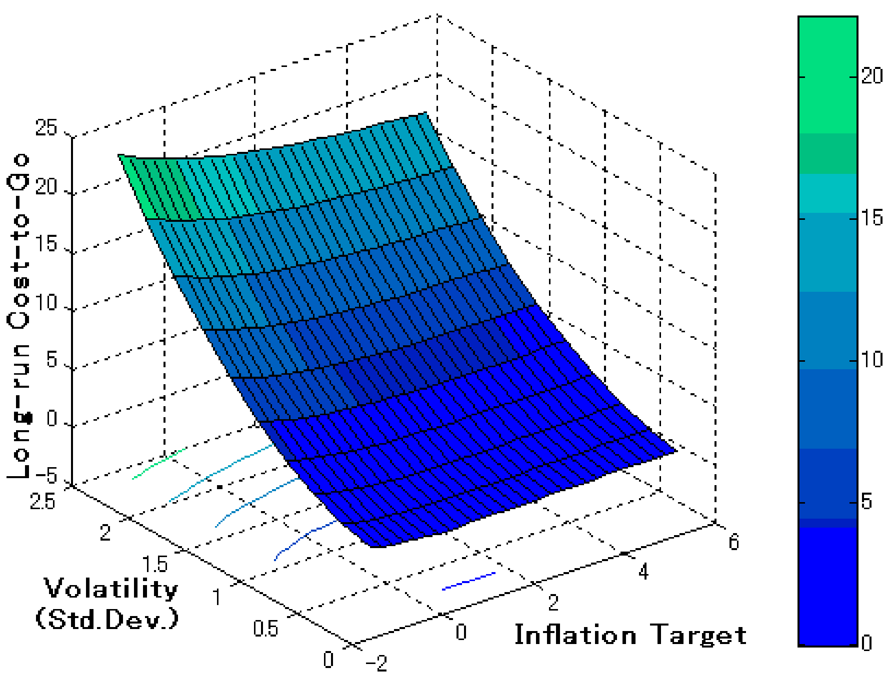
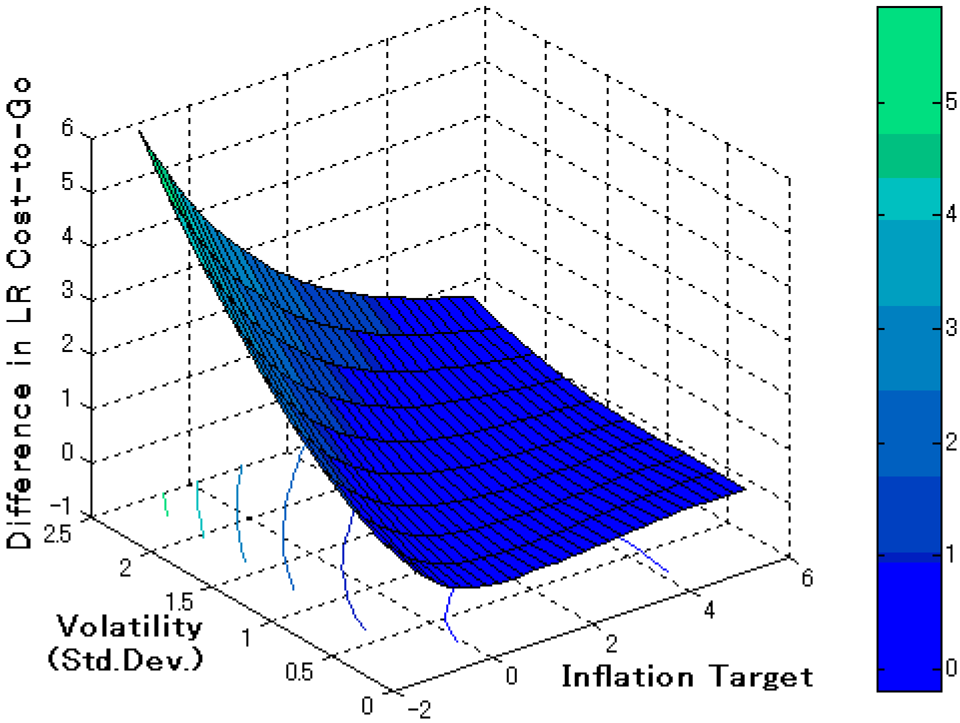


Figure 11: Difference in LR Cost-to-Go with and without Liquidity Trap



Note: The figure represents the portion of long-run Cost-to-Go solely arising from the risk of falling into a liquidity trap. It should be noted that this particular cost approaches zero, as the inflation target is set higher. As can be seen from the figure, the buffer role of an inflation target becomes more substantial as the economy becomes more volatile.