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Discussion Paper No. 2002-E-21

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**The Cross-Euler Equation Approach to Intertemporal
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Shinichi NISHIYAMA*

Abstract

We use the standard two-goods version of the Life Cycle/Permanent Income Model in analyzing the intertemporal aspect of import demand. The empirical dilemma in identifying and estimating the parameters governing the intertemporal elasticity of substitution (IES) for import demand is addressed. We propose a new concept, the Cross-Euler equation, for overcoming the empirical dilemma. IES parameters are estimated by exploiting the cointegrating restriction implied by the Cross-Euler equation and from the standard Euler equation using GMM. Further, by comparing the IES estimates from the Cross-Euler equation to those from the standard Euler equation, we formally test the hypothesis whether import demand is affected by nuisance factors, such as liquidity constraint or habit formation. Using U.S. non-durable goods expenditure data, we found expenditure on imported goods to be robust against nuisance factors, but expenditure on domestic goods were not. This empirical finding reveals the interesting (but puzzling) characteristic of import demand in contrast to demand for domestic goods.

Key words: Euler Equation, Intertemporal Substitution, Import Demand, Cointegration

JEL classification: C22, E21, F19

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This paper is a revised version of Chapter 2 of my OSU Ph.D. dissertation. I would like to especially thank Masao Ogaki (Ohio State University) for his advice and encouragement. I would like to thank C.Y. Choi (University of New Hampshire), Paul Evans (Ohio State University), Nelson Mark (Ohio State University), Kiyoshi Matsubara (Nagoya City University), Dongyu Sul (University of Auckland), and the staff members at Research Division 1, Institute for Monetary and Economic Studies, for their valuable comments. Any remaining errors are, of course, my own. The views expressed in this paper are those of the author and do not necessarily reflect those of the Bank of Japan or the Institute for Monetary and Economic Studies.

1 Introduction

Until recently, empirical research on import demand was confined to the static environment.¹ Under the static model, by construction, demand for imported goods will be determined by the relative price (or real exchange rate) of foreign to domestic goods and real income of the agents. Although it is undeniable that current relative price is one of the most important determining factors for import demand, this static approach is insufficient in the sense that it overlooks two other determinants of import demand - i.e. the real interest rate and expected future relative price between foreign and domestic goods. A change in real interest rate will alter the relative price of current consumption goods to future consumption goods and therefore alters the opportunity cost of current to future foreign goods. Similarly, a change in expected future relative price will also change future opportunity cost between domestic and foreign goods, thereby changing the opportunity cost of current to future foreign goods. The real interest rate and expected future relative price combined with the current relative price constitute the complete picture of the opportunity cost for current to future foreign goods. In response to a change in this opportunity cost, a forward-looking agent decides to intertemporally substitute future foreign goods consumption for current consumption. Without addressing the importance of the real interest rate and expected future relative price, our understanding of the intertemporal aspect of import demand will be nothing but incomplete. Thus, it is crucial for empirical researchers to frame their analysis in the context of dynamic optimization, taking into account the effects of the real interest rate and expected future relative prices.

Reflecting the need for dynamics, modern empirical research on import demand adopts the two-goods version of the life cycle/permanent income model (LCPIM) with rational expectation. Pioneering work has been done by Ceglowski (1991), who framed analysis in the context of LCPIM to estimate the intertemporal elasticity of substitution (IES) for imported non-durable goods. Clarida (1994) pointed out that, under the addilog utility function as in Houthakker (1960), the IES parameter for import demand can be estimated from the intratemporal optimality condition between foreign goods and domestic goods. Exploiting the cointegration restriction imposed on the intratemporal optimality condition, he estimated the IES of imported non-durable goods. Clarida (1996), Ogaki and Reinhart (1998) further estimated the IES of durable goods using the similar methodology. Amano and Wirjanto (1996) estimated the IES parameter of non-durable import demand from both the intratemporal and the intertemporal optimality

¹For instance, see the survey of Goldstein and Khan (1985).

conditions (i.e. Euler equations). They found the specification of the intratemporal condition between foreign goods and domestic goods to be robust. In contrast, for the intertemporal conditions, they found the specification to be fragile. Based on this observation, they concluded that some nuisance factors, such as liquidity constraints and/or time non-separability in the agent's preference, are affecting the specification of the Euler equations. In their subsequent study, Amano and Wirjanto (1998) weaken the assumption of the goods-separability in preference and allow for non-separability in domestic and foreign goods to estimate the IES for non-durable import demand. De la Croix and Urbain (1998) weaken the assumption of time-separability in preference and allow for the possibility of habit-formation. They estimate the curvature parameters on an instantaneous utility from the cointegration relationship implied by the intratemporal optimality condition and estimate the parameters governing habit-formation from the Euler equations. They found the habit-formation parameters to be statistically significant and concluded habit-formation to be an important factor of intertemporal substitution in import demand.

The focus of modern empirical literature on intertemporal substitution in import demand is to first estimate the IES parameter for import demand and then to compare the IES estimates from the intratemporal relationship to the Euler equation to check the robustness of the model. However, there is an empirical dilemma in pursuing this scheme. The objective is to estimate the IES parameters from both the intratemporal relationship and the Euler equations. When estimating the IES parameter from the intratemporal optimality condition, conventionally some kind of error term (such as preference shock or measurement error) is assumed inside the utility function as a preliminary step for the cointegration analysis. However, by assuming the error terms as such, the specification of the Euler equation will be non-standard that the GMM estimator of the IES parameters from the standard Euler equation will be inconsistent. On the other hand, if the model is built without error terms inside the utility function, the Euler equation will remain standard and therefore the GMM estimation will be applicable. But then, the intratemporal optimality condition will imply a deterministic relationship, rather than cointegration relationship, between domestic and foreign goods, which is obviously unrealistic. Thus, from a structural econometric viewpoint, the IES parameter estimated from the intratemporal optimality condition and Euler equations are estimated from the different model assumption that one cannot simply compare both estimates on the same ground, let alone hypothesis testing. This is the point where one experiences the dilemma.²

²Facing this dilemma, Clarida (1994) decided to focus solely on the intratemporal relationship in

In this paper, we propose the Cross-Euler equation approach as a prescription for this empirical dilemma. The Cross-Euler equation represents the optimal consumption pattern of a good in the current period to another good at a future period. It can be interpreted as the composite optimal condition that embeds both intertemporal and intratemporal optimal consumption relationships into one equation. Under the assumption that the agent's utility function is of the addi-log type, we show that the Cross-Euler equation between current imported goods to future domestic goods (by the same token, current domestic goods to future imported goods) implies a linear-cointegration relationship among current imported goods, future domestic goods, and the opportunity cost between them. We then further show that the cointegration restriction imposed by the Cross-Euler equation is robust to nuisance factors such as liquidity constraints and/or time non-separability in utility. By comparing the IES estimates from the Cross-Euler equation to estimates from the standard Euler equation, which will be misspecified under the existence of nuisance factors, we can indirectly infer how severely liquidity constraints or time non-separability are affecting intertemporal substitution in import demand. In other words, under the null hypothesis that nuisance factors are non-existent, the IES estimator from both Cross-Euler and standard Euler equations should asymptotically yield the same estimates. The virtue of this Cross-Euler equation approach is that it enables us to compare IES estimates without altering the assumption of the model from one estimation to another. Therefore, we can formally test the null hypothesis under the same model assumption.

Following previous empirical studies, we use U.S. non-durable goods expenditure data to investigate intertemporal substitution in import demand. We first estimate the IES parameters from a log-linearized Cross-Euler equation using Park's (1992) CCR. Our estimates turn out to be more or less similar to those of Ceglowski (1991), Clarida(1994), and Amano and Wirjanto (1996). Next, using Hansen's (1982) GMM, we estimate the same parameters from the standard Euler equations. The IES for domestic goods showed a wide variance in there estimates, while, in sharp contrast, the IES estimates for imported goods were considerably tighter. Finally, we conduct Cooley and Ogaki's (1996) likelihood ratio (LR) type test to formally test the null hypothesis that the IES estimate from the Cross-Euler equation is equal to that from the standard Euler equation. Interestingly, the null hypothesis was rejected frequently for domestic goods, but not for imported goods. Test results suggest that intertemporal substitution

order to estimate the IES parameter for import demand. Amano and Wirjanto (1996) estimated the IES parameters from both the intratemporal relationship and Euler equation by assuming a different model for each estimation scheme and informally compared the two parameter estimates.

in import demand is relatively robust against the nuisance factors compared to that of domestic goods demand.

The paper is organized as follows. In section 2, we describe the two-goods version of LCPIM modeling foreign and domestic goods. We also consider how the standard Euler equation is affected under the presence of nuisance factors, such as liquidity constraint. In section 3, we first address the issue of empirical dilemma. We then propose the Cross-Euler equation approach as a prescription. Section 4 is devoted to the estimation of IES parameters. We estimate IES parameters exploiting the cointegration restriction implied by the Cross-Euler equation. We also estimate the IES parameter from the standard Euler equation using GMM. In section 5, we formally compare the IES estimates from the Cross-Euler to standard Euler equation using Cooley and Ogaki's LR type test. Section 6 concludes with some future directions.

2 Model

This paper adopts the standard two-goods version of the Life Cycle/ Permanent Income Model (LCPIM) as in Ceglowski (1991), Clarida (1994), Amano and Wirjanto (1996), and Xu (2002). A representative agent is assumed to maximize his expected lifetime utility under his lifetime budget constraint. The dynamic optimization problem is formulated as follows:

$$\max E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}, M_{t+i}) \quad (1)$$

$$s.t. A_{t+1} = (1 + r_{t+1})A_t + Y_t - P_t^C C_t - P_t^M M_t \quad \text{for } \forall t \geq 0 \quad (2)$$

where C_t stands for domestic non-durable goods at period t , M_t stands for imported non-durable goods, A_t stands for the asset holding of the agent, Y_t stands for the labor income of the agent, r_t stands for the real interest rate from period $t - 1$ to t , P_t^C stands for the price of a domestic non-durable good, and P_t^M stands for the price of an imported non-durable good. Finally, we parameterize the agent's subjective discount rate as constant β .

We have assumed that period-by-period utility is time separable for this agent and have implicitly assumed additive-separability between durable goods and non-durable goods. Solving the above optimization problem yields the following first order conditions (FOC):

$$\frac{P_t^C}{P_t^M} = \frac{U_{C,t}}{U_{M,t}} \quad \text{for } \forall t \geq 0 \quad (3)$$

$$E_t \left[\beta \frac{U_{C,t+1}}{U_{C,t}} (1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^C} - 1 \right] = 0 \quad \text{for } \forall t \geq 0 \quad (4)$$

$$E_t \left[\beta \frac{U_{M,t+1}}{U_{M,t}} (1 + r_{t+1}) \frac{P_t^M}{P_{t+1}^M} - 1 \right] = 0 \quad \text{for } \forall t \geq 0. \quad (5)$$

Eq. (3) represents the intratemporal FOC for this representative agent. These FOC's follow if the agent is maximizing his utility given the current relative price of domestic non-durable consumption goods to imported non-durable consumption goods. In other words, a representative agent will equalize his intratemporal marginal rate of substitution (MRS) to current price ratio of two goods. Eq. (4) represents the intertemporal FOC (i.e. Euler equation) of domestic non-durable consumption goods. This FOC will follow if the agent is equalizing the intertemporal marginal rate of substitution (IMRS) to the discounted expected opportunity cost of domestic non-durable consumption goods from period t to period $t + 1$. In other words, by saving one unit of C_t at period t , then in the next period one expects to receive $(1 + r_t) \cdot P_t^C / P_{t+1}^C$ units of C_{t+1} in return. In order for the agent to be indifferent between the choice of saving and consumption, IMRS needs to be equal to $(1 + r_t) \cdot P_t^C / P_{t+1}^C$, otherwise there will be room for the agent to be better off either by saving more or consuming more. Eq. (5) holds by parallel logic.

Next, in order to make the model econometrically estimable, we parametrize the utility function. We specify the utility function as a standard addi-log function following Houthakker (1960). This specification was used in Ceglowski (1991), Clarida (1994), and Amano and Wirjanto (1996). De la Croix and Urbain (1998) basically used the same specification but allowed for habit formation making the period-by-period utility function time non-separable.

We specify the utility function as follow:

$$U(C_t, M_t) = \frac{C_t^{1-\alpha}}{1-\alpha} + K \cdot \frac{M_t^{1-\nu}}{1-\nu}. \quad (6)$$

It should be noted that under this addi-log specification, $1/\alpha$ and $1/\nu$ can be interpreted

as the intertemporal elasticity of substitution (IES) of C_t and M_t respectively.³ Under this specification, FOC's will then be as follows:

$$\frac{P_t^C}{P_t^M} = \frac{1}{K} \frac{C_t^{-\alpha}}{M_t^{-\nu}} \quad \text{for } \forall t \geq 0 \quad (7)$$

$$E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} (1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^C} - 1 \right] = 0 \quad \text{for } \forall t \geq 0 \quad (8)$$

$$E_t \left[\beta \left(\frac{M_{t+1}}{M_t} \right)^{-\nu} (1 + r_{t+1}) \frac{P_t^M}{P_{t+1}^M} - 1 \right] = 0 \quad \text{for } \forall t \geq 0. \quad (9)$$

Given these specifications, we are now ready to actually estimate and test the implication of the model.

Some remarks should follow for these FOC's. As was pointed out by Amano and Wirjanto (1996) and Ogaki and Park (1998), the specification of the intratemporal relationship eq. (7) turns out to be robust to several kinds of nuisance factors, such as liquidity constraint and/or habit formation in utility function. However, the specification of Euler equations is very sensitive to the presence of liquidity constraint⁴ or habit formation.⁵ In other words, specification of the intratemporal relationship is robust, but the specification of Euler equations is not. Conversely, if for any method we can

³This will not be the case if a utility function is time non-separable (e.g. allowing habit formation) or goods non-separable (e.g. CES type function). This was pointed out by Constantinides (1991). The general formula for deriving IES under a time non-separable utility function was shown by McLaughlin (1995).

⁴Zeldes (1989) shows that when the liquidity constraint (i.e. $A_t \geq 0$ for $\forall t$) is present and binding, then the Euler equation will take inequality as follows.

$$E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} (1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^C} - 1 \right] < 0$$

$$E_t \left[\beta \left(\frac{M_{t+1}}{M_t} \right)^{-\nu} (1 + r_{t+1}) \frac{P_t^M}{P_{t+1}^M} - 1 \right] < 0$$

⁵Amano and Wirjanto (1996) shows that when the agent has habit forming preference as follows

$$E_t \left[\sum_{i=0}^{\infty} \beta^i U(C_{t+i}^*, M_{t+i}^*) \right]$$

where $U(C_t^*, M_t^*)$ takes the addi-log type utility function where $C_t^* \equiv \sum_{j=0}^{\infty} \delta_j C_{t-j}$ and $M_t^* \equiv \sum_{j=0}^{\infty} \eta_j M_{t-j}$, then the intratemporal relationship will reveal a stochastic relationship (instead of deter-

find evidence that the Euler equation is correctly specified, that will be strong evidence against the presence of liquidity constraint or habit formation. This specification issue will be the central focus of the rest of this paper.

3 The Cross-Euler Equation Approach

If our model is correct, then the intratemporal optimality condition (7) and Euler equations (8) and (9) will be correctly specified. Therefore, the parameter estimates α and ν from eq. (7) and Euler equations (8) and (9) should be reasonably close. Thus, the main focus of modern empirical research on import demand is to test whether these estimates from different equations are statistically close enough or not. If the statistical test concludes that parameter estimates are significantly different from each other, then, by contrapositive logic, we should conclude that some of the assumptions we had made (i.e. addi-log type utility function, additive separability of durable and non-durable goods, non-existence of liquidity constraint, non-existence of habit formation, etc.) are implausible. Unfortunately, a statistical test will not be informative as to say exactly which assumption is wrong. Conversely, if a statistical test does not reject the null hypothesis that parameter estimates are equal, it will support or, at least, leave some possibility open for the joint assumption of addi-log utility specification without the presence of habit formation or liquidity constraint.

Thus, the empirical task is to first obtain the parameter estimates from the intratemporal relationship - i.e. eq. (7) - and from Euler equations - i.e. eq. (8) and eq. (9). Predecessors in this line of research have used cointegration analysis and/or GMM in estimating parameters. However, when the utility function is of the addi-log type, there will be an empirical complication in estimating parameters. In this section, we will first address this complication and then propose a prescription.

3.1 An Empirical Dilemma

Let us turn back to the three FOC's - i.e. eq. (7), (8), and (9) - implied by the model. Since the conditional moment condition is established for eq. (8) and (9), there is no

 ministic relationship) as follows

$$K \frac{P_t^C}{P_t^M} \frac{C_t^\alpha}{M_t^\nu} \sim I(0)$$

Thus, even when habit formation is present, the intratemporal relationship holds, albeit stochastic, but Euler equations (8) and (9) will be misspecified.

problem in applying GMM to these equations. If indeed these Euler equations are correctly specified, then GMM will yield $O_P(T^{-1/2})$ consistent estimate of α and ν . However, unfortunately, the complication will arise from intratemporal relationship eq. (7).

A natural way⁶ to estimate the parameters from the intratemporal relationship is to log-linearize and rearrange eq. (7) as follows.

$$\ln M_t + \text{const.} - \frac{1}{\nu} \ln \frac{P_t^C}{P_t^M} - \frac{a}{\nu} \ln C_t = 0$$

If the forcing variables $\ln M_t$, $\ln C_t$, and $\ln(P_t^C/P_t^M)$ follow the I(1) process, one is tempted to introduce some I(0) disturbance terms on RHS of the above equation to set up the cointegrating relationship among the forcing variables as follows:

$$\ln M_t + \text{const.} - \frac{1}{\nu} \ln \frac{P_t^C}{P_t^M} - \frac{a}{\nu} \ln C_t = \varepsilon_t \text{ where } \varepsilon_t \sim I(0) \text{ and } E(\varepsilon_t) = 0. \quad (10)$$

This error term can be an optimization error, measurement error, preference shock,⁷ etc. However, in order to maintain coherence within the model structure, one should also include this newly introduced error term into existing Euler equations (8) and (9). In other words, in addition to the forecast error embedded in Euler equations, one is now introducing another kind of error term which is an intrinsically different type of error. It can be shown that when a new error term is introduced to Euler equations, eq. (8) and eq. (9) are no longer correctly specified (see Appendix 1).

This is the point at which one experiences the dilemma. The objective is to estimate the parameters from both the intratemporal relationship and Euler equations. If one introduces some arbitrary error term to the intratemporal relationship in order to conduct the cointegration analysis, this newly introduced error term will affect the specification of Euler equation. Conducting GMM on standard Euler equations (8) and (9) will no longer yield a consistent estimates for α and ν . On the other hand, if one does not introduce any error term to the intratemporal relationship, Euler equations

⁶There will be a problem if one attempts to estimate the parameters without log-linearizing eq. (7), since the forcing variables in eq. (7) involve the I(1) processes. Suppose one attempts to estimate the parameter by GMM, which will be a non-linear estimation method in this case, then conditional moment will be $E_t \left[\frac{1}{K} \frac{P_t^M}{P_t^C} \frac{C_t^{-\alpha}}{M_t^{-\nu}} - 1 \right] = 0$ which includes I(1) processes in its forcing variables. This will violate the fundamental assumption of GMM (see Hall (1993) and Ogaki (1993)).

⁷Clarida (1994) and Amano and Wirjanto (1996) adopted preference shock in their model, making cointegration analysis possible under mild conditions. However, it should be noted that if one adopts a preference shock in their model, then, as a trade-off, the specification of the Euler equation will be non-standard so that GMM estimation on the standard Euler equation will be inconsistent.

(8) and (9) will be correctly specified and GMM on them will yield consistent estimates assuming that the model is correct. But then, since the error term is not present for the intratemporal relationship (i.e. intratemporal relationship will be deterministic), one faces an illegitimacy in exploiting the cointegration approach, which is a method based on stochastic relationship among forcing variables.⁸

So, is there any way to overcome this dilemma?

3.2 A Prescription: The Cross-Euler Equation Approach

This subsection proposes a prescription to the above empirical dilemma. The idea is to first define the concept called cross intertemporal marginal rate of substitution (CIMRS) and then to derive the corresponding first order condition which we will call the Cross-Euler equation. We then show how the Cross-Euler equation can be a prescription for the above empirical dilemma.

Defining CIMRS and deriving Cross-Euler equations

Definition 1 (CIMRS) *Let $V(x_1^1, \dots, x_1^K, \dots, x_T^1, \dots, x_T^K)$ be a utility function defined upon K goods with T periods. Then we call the following expression*

$$-\frac{\partial V(\bullet)/\partial x_{t+1}^i}{\partial V(\bullet)/\partial x_t^j}$$

the cross intertemporal marginal rate of substitution (CIMRS) between goods x_{t+1}^i and x_t^j where $i \neq j$ and $t = 1, \dots, T - 1$.

The concept of CIMRS is just a simple extension of IMRS. It can be easily conceptualized as the IMRS defined upon different goods instead of same goods.⁹ From the concept of CIMRS and from our model, we can derive the “alternative” FOC. For convenience we will call the following FOC the Cross-Euler equation.

$$E_t \left[\beta K \frac{M_{t+1}^{-\nu}}{C_t^{-\alpha}} (1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^M} - 1 \right] = 0 \quad (11)$$

The Cross-Euler equation represents the optimal consumption pattern of a good in the current period to another good at a future period. It can be interpreted as the

⁸For the basic assumptions of the cointegration approach in estimating the preference parameters, see Ogaki and Park (1998).

⁹Another way of saying this is that the IMRS of goods i is a special case of CIMRS between x_{t+1}^i and x_t^j where $i = j$.

composite optimal condition that embeds both intertemporal and intratemporal optimal consumption relationships into one equation.

We can see the intuition of the Cross-Euler equation by thinking of the situation where the agent is trading C_t to M_{t+1} . Now the marginal rate of substitution between M_{t+1} and C_t (or CIMRS in our terminology) is defined as $-\beta U_{M,t+1}/U_{C,t}$ and takes the form of $-\beta K M_{t+1}^{-\nu}/C_t^{-\alpha}$ under the addi-log utility function. Next, let us consider the opportunity cost of obtaining M_{t+1} in terms of C_t . By selling one unit of C_t at period t , the agent can obtain P_t^C of numeraire goods. By saving all of these numeraire goods at period t , the agent can obtain $(1+r_{t+1}) \cdot P_t^C$ of numeraire goods at period $t+1$. By using all of these to buy M_{t+1} , the agent can buy $(1+r_{t+1}) \cdot P_t^C/P_{t+1}^M$ units of M_{t+1} . Thus, the opportunity cost of M_{t+1} in terms of C_t is $(1+r_{t+1}) \cdot P_t^C/P_{t+1}^M$. If the agent is optimally trading C_t to M_{t+1} , then the agent is equalizing the opportunity cost to CIMRS between M_{t+1} and C_t , yielding the above Cross-Euler equation.¹⁰

In a similar fashion, we can derive the another version of the Cross-Euler equation as follow.

$$E_t \left[\frac{\beta}{K} \frac{C_{t+1}^{-\alpha}}{M_t^{-\nu}} (1+r_{t+1}) \frac{P_t^M}{P_{t+1}^C} - 1 \right] = 0 \quad (12)$$

In deriving the above Cross-Euler equation, we have equalized the CIMRS between C_{t+1} and M_t (i.e. $-(\beta/K)(C_{t+1}^{-\alpha}/M_t^{-\nu})$) to the opportunity cost of C_{t+1} in terms of M_t (i.e. $(1+r_{t+1})P_t^M/P_{t+1}^C$).

Cointegration relationship implied by the Cross-Euler equation

Returning to Cross-Euler equation (11), it follows that

$$\beta K \frac{M_{t+1}^{-\nu}}{C_t^{-\alpha}} (1+r_{t+1}) \frac{P_t^C}{P_{t+1}^M} = 1 + e_{t+1} \quad (13)$$

where we defined e_{t+1} as

$$e_{t+1} \equiv \beta K \frac{M_{t+1}^{-\nu}}{C_t^{-\alpha}} (1+r_{t+1}) \frac{P_t^C}{P_{t+1}^M} - E_t \left[\beta K \frac{M_{t+1}^{-\nu}}{C_t^{-\alpha}} (1+r_{t+1}) \frac{P_t^C}{P_{t+1}^M} \right]$$

Taking logarithm¹¹ on both sides of eq. (13) will yield

$$\text{const.} + \ln \left[(1+r_{t+1}) \frac{P_t^C}{P_{t+1}^M} \right] - \nu \ln M_{t+1} + \alpha \ln C_t = \ln(1 + e_{t+1})$$

¹⁰For a formal derivation, see Appendix 2.

¹¹As was pointed out by Carroll (1997), ideally speaking, it is preferable to estimate the parameters without log-linearization. However, since possibly I(1) processes are present inside the conditional moment equations (11) and (12), again, it is likely that the fundamental assumption of the GMM estimation method is violated. This forced us to log-linearize Cross-Euler equations.

Assuming that the growth rate of both domestic and imported non-durable goods consumption (i.e. C_{t+1}/C_t and M_{t+1}/M_t), the real interest rate (i.e. r_t), and the growth rate of the price level of both domestic and imported non-durable goods (i.e. P_{t+1}^C/P_t^C and P_{t+1}^M/P_t^M) are stationary,¹² it can be shown that $\ln(1 + e_{t+1})$ will also be stationary (see Appendix 3).

Exploiting the $I(0)$ process of $\ln(1 + e_{t+1})$, we can obtain the following cointegrating relationship¹³:

$$\ln M_{t+1} + \text{const.} - \frac{1}{\nu} \ln \left[(1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^M} \right] - \frac{\alpha}{\nu} \ln C_t \sim I(0). \quad (14)$$

In a similar fashion, we can derive the following cointegration relationship from eq. (12):

$$\ln M_t + \text{const.} - \frac{1}{\nu} \ln \left[\frac{1}{1 + r_{t+1}} \frac{P_{t+1}^C}{P_t^M} \right] - \frac{\alpha}{\nu} \ln C_{t+1} \sim I(0). \quad (15)$$

¹²Empirical evidence seems to support this assumption. See, for instance, Clarida (1994), Amano and Wirjanto (1996), and De la Croix and Urbain (1998).

¹³Since this cointegrating relationship was derived from the Cross-Euler equation, one might be concerned about the non-robustness of this relationship vis-à-vis liquidity constraint or time non-separability. However, this concern turns out to be groundless. Let us restate the contemporaneous relationship implied by the model.

$$\text{const.} - (\ln P_t^C - \ln P_t^M) - \alpha \ln C_t + \nu \ln M_t = 0$$

Since this equation was derived from the intratemporal relationship implied by the model, this equation is robust vis-à-vis liquidity constraints and time non-separability. Now, by adding $\nu \ln M_{t+1}, \ln P_{t+1}^M, -\ln(1 + r_{t+1})$ on both sides of this equation and subtracting $\nu \ln M_t, \ln P_t^M$ from both sides, we can obtain the following equation:

$$\begin{aligned} & \text{const.} - \ln(1 + r_{t+1}) - (\ln P_t^C - \ln P_{t+1}^M) - \alpha \ln C_t + \nu \ln M_{t+1} \\ = & \underbrace{\nu(\ln M_{t+1} - \ln M_t)}_{I(0)} + \underbrace{(\ln P_{t+1}^M - \ln P_t^M)}_{I(0)} - \underbrace{\ln(1 + r_{t+1})}_{I(0)}. \end{aligned}$$

Assuming the stationarity of the growth rate of M_t, P_t^M and the real interest rate, RHS of the above equation will be $I(0)$ and therefore LHS of the equation will also follow the $I(0)$ process. By further rearranging this equation, we can show the following cointegrating relationship, which is the cointegration relationship we derived in eq. (14):

$$\ln M_{t+1} + \text{const.} - \frac{1}{\nu} \ln \left[(1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^M} \right] - \frac{\alpha}{\nu} \ln C_t \sim I(0).$$

Thus, cointegrating equation (14) turns out to be robust to liquidity constraint and time non-separability. A similar argument holds for cointegrating equation (15).

Given these cointegrating relationships of log-linearized Cross-Euler equations, combined with GMM-estimable standard Euler equations (8) and (9), we now have firm ground for comparing the estimates of α and ν . To summarize, under the assumption which allows for the existence of liquidity constraint and/or a certain type of habit formation, log-linearized Cross-Euler equations (14) and (15) will yield super-consistent estimates for α and ν , while Euler equations (8) and (9) are not guaranteed to yield consistent estimates. On the other hand, under the assumption that liquidity constraints or habit formation are non-existent, both log-linearized Cross-Euler equations and standard Euler equations will yield super-consistent and consistent estimates of α and ν , respectively. This latter proposition, which basically states that the estimates of IES parameters from cointegration analysis and GMM to be close under the stronger assumption, is particularly important since we can formally test this proposition using statistical methods such as Cooley and Ogaki's (1996) LR type test. The following table summarizes the main idea of this section.

Table 1: Table Summarizing the Consistency of the Cross-Euler and Euler Approach

	Log-linearized Cross-Euler Equation (Method: Cointegration)	Standard Euler Equation (Method: GMM)
Habit Formation/Liq. Constraints	Super-consistent estimates for α and ν	Inconsistent
No Habit Formation/No Liq. Constraints	Super-consistent estimates for α and ν	Consistent estimates for α and ν

4 Estimation

This section explains the empirical method of this paper. First, we describe the source and the construction of the data set for our estimation. Next, we conduct Park's (1992) CCR on equations (14) and (15) to estimate parameters α and ν . Park's (1990) H(p,q) test will also be conducted to check for the cointegration which is implied by the theory. Finally, we conduct Hansen's (1982) GMM on Euler equations (8) and (9) to estimate parameters α and ν . Hansen's J statistics will also be reported to check for the specification of Euler equations.

4.1 Data Description and Pretesting of Difference Stationarity

The data we use in this paper are seasonally-adjusted quarterly U.S. data covering the period from 1967 Q1 to 1994 Q3 (111 observations). The data set spanning from 1967 Q1 to 1988 Q4 was constructed by Ceglowski (1991) and has been extended to period 1994 Q3 by De la Croix and Urbain (1998).¹⁴ The choice of non-durable goods differs among the researchers, but in this paper we follow the choice of Ceglowski (1991). The data for imported non-durable goods were constructed from a quarterly series of constant dollar imports of consumer non-durables and food.¹⁵ As for the data for domestic non-durable goods, since the consumption data in the National Income and Product Accounts (NIPA) does not distinguish between domestic and imported goods, the data are constructed by subtracting the value of imports from U.S. non-fuel personal consumption expenditures on non-durables. The source and construction of the price index is also due to Ceglowski (1991). The import price measure was used for the price index for imported non-durable goods. A price index for domestic non-durable goods was constructed as a geometric average of the implicit price deflators for domestic non-durable and food expenditures (see Ceglowski (1991) for further details). Finally, the real interest rate was constructed based on the 3-month U.S. Treasury Bill and U.S. Implicit Price Deflator for GDP.¹⁶

As for the preliminary step for cointegration analysis, we test the null of difference stationarity against the null of (trend) stationarity for the variables included in the cointegrating regressions. To be more specific, we tested the difference stationarity of the following four variables: log imported non-durable goods ($\ln M_t$), log domestic non-durable goods ($\ln C_t$), log relative price of current domestic non-durable goods in terms of future imported non-durable goods ($\ln[(1 + r_{t+1}) * P_t^C / P_{t+1}^M]$), and log relative price of future domestic non-durable goods in terms of current imported non-durable goods ($\ln[(1 + r)^{-1} * P_{t+1}^C / P_t^M]$). The results of the tests are reported in Table 2.

We used Said and Dickey's (1984) Augmented Dickey-Fuller (ADF) test, Phillips and Perron's (1988) PP test, and Park and Choi's (1988) J test for testing for the null of

¹⁴The data set is available on the Internet.

http://www.econ.ucl.ac.be/ires/csssp/home_papers/delacroix

¹⁵Ideally speaking, the data for imported non-durable goods should be constructed only from final non-durable goods, excluding intermediate goods. However, unfortunately, import data on NIPA does not distinguish between final and intermediate goods.

¹⁶Thus, real interest rate used in this paper is actually an ex post real interest rate. Not an ex ante real interest rate.

Table 2: Unit Root Test for Domestic and Imported Goods

Variable	ADF		PP		J-test	
	cst.	cst. & trd.	cst.	cst. & trd.	J(0,3)	J(1,5)
$\ln C_t$	-1.035	-2.625	-1.244	-2.323	115.05	0.933
$\ln M_t$	-0.456	-2.535	-0.600	-3.143	13.815	1.174
$\ln \left[(1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^M} \right]$	-1.877	-1.805	-1.732	-1.632	1.592	5.162
$\ln \left[\frac{1}{1+r_{t+1}} \frac{P_{t+1}^C}{P_t^M} \right]$	-1.656	1.329	-1.474	-1.224	2.247	7.229

Note: Lag order used for ADF test and PP test was four. The 10% critical values of ADF test and PP test with a constant is -2.581 and with constant and trend is -3.151. The critical values are due to MacKinnon (1991). The 10% critical values for J(0,3) test and J(1,5) test are 0.577 and 0.452, respectively. The critical values are due to Park and Choi (1988). It should be noted under the J-test, the null of difference stationarity is rejected when the statistics are smaller than the critical value.

difference stationarity. As can be seen from the table, the tests does not reject the null of difference stationarity at the 10% significance level for all variables. This evidence sets the ground for the following cointegration analysis.

4.2 Cointegration Analysis

4.2.1 Park's CCR and H(p,q) test

In this section we will apply Park's (1992) Canonical Cointegration Regression (CCR) on log-linearized Cross-Euler eq. (14) and eq. (15). For convenience both equations are restated below, respectively.

$$\ln M_{t+1} + \text{const.} - \frac{1}{\nu} \ln \left[(1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^M} \right] - \frac{\alpha}{\nu} \ln C_t \sim I(0)$$

$$\ln M_t + \text{const.} - \frac{1}{\nu} \ln \left[\frac{1}{1+r_{t+1}} \frac{P_{t+1}^C}{P_t^M} \right] - \frac{\alpha}{\nu} \ln C_{t+1} \sim I(0)$$

Given the difference stationary processes of log imported non-durable goods, log domestic non-durable goods, and relative prices, variables $\ln M_{t+1}$, $\ln \left[(1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^M} \right]$, and $\ln C_t$ in eq. (14) are cointegrated¹⁷ by vector $(1, -\frac{1}{\nu}, -\frac{\alpha}{\nu})'$, as well as variables $\ln M_t$, $\ln \left[\frac{1}{1+r_{t+1}} \frac{P_{t+1}^C}{P_t^M} \right]$ and $\ln C_{t+1}$ in eq. (15) are cointegrated by vector $(1, -\frac{1}{\nu}, -\frac{\alpha}{\nu})'$.

¹⁷To follow the terminology of Ogaki and Park (1997), here we mean deterministic cointegration.

Clearly, intertemporal substitution parameters α and ν are identified for both cointegrating equations.

If indeed both equations are cointegrated, Park's (1992) CCR will yield super-consistent and asymptotically efficient estimates for α and ν . By applying Park's (1990) $G(p,q)$ test on the residuals, we can obtain Park's $H(p,q)$ statistics. Under the null of cointegration, Park showed that $H(p,q)$ statistics are asymptotically χ^2 distributed with $q - p$ degrees of freedom. In particular, since we are interested in the deterministic cointegration relationship, we conducted the $H(0,q)$ test in this paper.

Some remarks are in order regarding the cointegration approach. Since we are constructing $I(0)$ error terms by leading the variables $\ln M, \ln C, \ln P_{t+1}^M, \ln P_t^C$, there will obviously be an endogeneity problem when estimating these cointegrating relationships. However, this problem could be handled by an estimation method such as Phillips and Hansen's (1991) FM-OLS or Park's (1992) CCR. Further, since the estimators in cointegrating regression will be super-consistent (i.e. $O_P(T^{-1})$ consistent), the endogeneity problem will not matter asymptotically. Thus, despite this endogeneity problem, we can still obtain consistent estimates for α and ν from eq. (14) or eq. (15).

4.2.2 Result

The results of Park's CCR estimates¹⁸ are reported in Table 3 for cointegrating equation (14) and in Table 4 for cointegrating equation (15). Since we are interested in the deterministic cointegration relationship, we conducted $H(0,q)$ test for both equation. Also, to check for the stochastic cointegration relationship, $H(1,q)$ test was also conducted.

Let us first turn to the estimation results of eq. (14). The implied parameter estimate for α was 1.8774 and ν was 1.4245. Thus, IES for domestic non-durable goods (i.e. $1/\alpha$) was 0.5327 and IES for imported non-durable goods (i.e. $1/\nu$) was 0.7020. Checking for the cointegration relationship, the test did not reject the null of cointegration, either deterministic or stochastic.

Next, turning to the estimation result for eq. (15), the estimate for α was 2.3372 and ν was 1.2173. Therefore, the implied IES for domestic non-durable goods was 0.4279 and for imported non-durable goods was 0.8215. We found that estimates of ν to be

¹⁸In estimating the long-run covariance matrix of error term, we used Andrews and Monahan's (1992) VAR prewhitened HAC estimator. The choice of kernel was QS kernel as suggested by Andrews (1991). Following the Monte Carlo study of Han (1996), third stage CCR estimates are reported and $H(p,q)$ statistics are based on fourth stage CCR.

Table 3: CCR Results: Cross-Euler Equation

$\ln M_{t+1} = \text{const.} + \frac{1}{\nu} \ln [(1 + r_{t+1})P_t^C / P_{t+1}^M] + \frac{\alpha}{\nu} \ln C_t + I(0)$					
Estimates			Implied Estimates		
const.	$1/\nu$	α/ν	α	γ	
-0.0508	0.07019	2.6747	1.8774	1.4245	
(0.0094)	(0.0675)	(0.0629)			
Test Statistics					
H(0,1)	H(0,2)	H(0,3)	H(1,2)	H(1,3)	H(1,4)
2.4698	2.4977	2.9773	0.0279	0.5075	0.8610
[0.116]	[0.287]	[0.395]	[0.867]	[0.776]	[0.835]

Note: Numbers in parenthesis stand for the estimated standard error. Numbers in square brackets stand for p-value. * denotes the rejection of null of cointegration at 5% level. ** denotes the rejection of null of cointegration at 1% level.

Table 4: CCR Results: Cross-Euler Equation

$\ln M_t = \text{const.} + \frac{1}{\nu} \ln [(1 + r_{t+1})^{-1}P_{t+1}^C / P_t^M] + \frac{\alpha}{\nu} \ln C_{t+1} + I(0)$					
Estimates			Implied Estimates		
const.	$1/\nu$	α/ν	α	γ	
-0.0654	0.8215	2.8452	2.3372	1.2173	
(0.0094)	(0.0675)	(0.0629)			
Test Statistics					
H(0,1)	H(0,2)	H(0,3)	H(1,2)	H(1,3)	H(1,4)
0.8433	1.8206	1.8596	0.9773	1.0163	2.6395
[0.358]	[0.402]	[0.602]	[0.323]	[0.602]	[0.451]

Note: Numbers in parenthesis stand for the estimated standard error. Numbers in square brackets stand for p-value. * denotes the rejection of null of cointegration at 5% level. ** denotes the rejection of null of cointegration at 1% level.

reasonably close between eq. (14) and (15). Testing for the cointegration relationship, again the test did not reject the null of cointegration, either deterministic and stochastic.

Thus, as one can see from this result, the cointegration relationship implied by economic theory seems to be supported by Park's H(p,q) test. Further, the estimates of intertemporal substitution parameters seem to be reasonable in the sense that they match with previous studies by Ceglowski (1991), Clarida (1994), and Amano and Wirjanto (1996).

4.3 GMM

In this subsection, we conduct Hansen's (1982) GMM on eq. (8) and (9). Parameters α and ν will be estimated under single equation and system equation contexts. We also discuss the choice of instrumental variables (IV) in this paper. Hansen's J test will also be reported.

4.3.1 Choice of Instruments and Lag Order

As was pointed out by Hall (1993) and Ogaki (1993), it is well known that the estimate of GMM is very sensitive to the choice of instrumental variables. To test for the robustness of the estimates vis-à-vis the choice of instruments, we estimated the parameters under several types of instruments with varying time lags. The first family of instrumental variables was chosen following the convention in applied GMM literature. As can be seen from the following table, six types of instrument sets were chosen.

Table 5: Types of Conventional IV's

IV Type	Euler Equation (8)	Euler Equation (9)
IV1	const., $\frac{C_{t+1}}{C_t}$	const., $\frac{M_{t+1}}{M_t}$
IV2	const., $\frac{P_{t+1}^C}{P_t^C}$	const., $\frac{P_{t+1}^M}{P_t^M}$
IV3	const., r_{t+1}	const., r_{t+1}
IV4	const., $\frac{C_{t+1}}{C_t}, r_{t+1}$	const., $\frac{M_{t+1}}{M_t}, r_{t+1}$
IV5	const., $\frac{C_{t+1}}{C_t}, \frac{P_{t+1}^C}{P_t^C}$	const., $\frac{M_{t+1}}{M_t}, \frac{P_{t+1}^M}{P_t^M}$
IV6	const., $\frac{C_{t+1}}{C_t}, \frac{P_{t+1}^C}{P_t^C}, r_{t+1}$	const., $\frac{M_{t+1}}{M_t}, \frac{P_{t+1}^M}{P_t^M}, r_{t+1}$

However, it is also known that conventional instruments suffer from a weak correlation with the forcing variables of the Euler equation. In order to remedy this weak

instruments problem, Cooley and Ogaki (1996) proposed using the financial instruments which have better properties compared to conventional one. Following their method, we chose a second family of instrument sets as below. We use common financial instruments for estimating Euler equations (8) and (9).

Table 6: Types of Financial IV's

IV Type	Euler equation (8) and (9)
FIV1	const., vwr_{t+1}
FIV2	const., div_{t+1}
FIV3	const., ysp_{t+1}
FIV4	const., vwr_{t+1} , div_{t+1}
FIV5	const., div_{t+1} , ysp_{t+1}
FIV6	const., vwr_{t+1} , div_{t+1} , ysp_{t+1}

Three financial series, in addition to the constant, have been chosen for the second instrument sets: value-weighted return on the stock market (denoted vwr), dividend yield (denoted div), and yield spread in the corporate bond market (denoted ysp). The value-weighted return has been calculated from the New York Stock Exchange (NYSE) and the American Stock Exchange (AMEX). The dividend yield is the value-weighted average of the dividend yields of stocks listed on NYSE and AMEX and has been seasonally adjusted using seasonal dummies. The yield spread is the yield-to-maturity difference between corporate bonds rated Baa and corporate bonds rated Aaa by Moody's Investor Services.¹⁹

Another issue in conducting GMM estimation is to choose the lag order of the error term when estimating the variance-covariance matrix of GMM disturbance terms. According to the rational expectation hypothesis, the forecast error will be serially uncorrelated. Since our model is based on the representative agent with rational expectation, economic theory suggests a lag order of zero.²⁰ Nevertheless, taking into account the time aggregation problem which was pointed out by Grossman et al. (1987) and Heaton (1995) among others, we choose a lag order of one in estimating the variance-covariance

¹⁹For further details in constructing these financial instruments, see Cooley and Ogaki (1996).

²⁰Of course, if the utility function of the agent is time non-separable as in De la Croix and Urbain (1998), forecast errors may be serially correlated depending on the magnitude of time non-separability. Also, if the model involves durable goods, forecast errors may be serially correlated, depending on the magnitude of the durability of the goods.

matrix of GMM disturbance terms²¹ following Hansen et al. (1996). Also, to be consistent with time aggregation issues, we have lagged instrumental variables for at least two periods when conducting GMM estimations.

4.3.2 Results

GMM estimation was conducted using a family of conventional instruments and a family of financial instruments. GMM estimation results for domestic goods Euler equation (8) are summarized in Table 7 and Table 8²². Similarly, GMM estimation results for imported goods Euler equation (9) are summarized in Table 9 and Table 10. Finally, system-equation GMM results for domestic and imported goods Euler equations are summarized in Table 11 and Table 12. Hansen’s J-statistics for each GMM estimation are also reported.

Let us first interpret the estimation results of the Euler equation for domestic non-durable consumption. We first observe a large dispersion in the estimates of α . The estimates for α range from -28.7769 to 17.739 under conventional instruments, and from -33.2406 to 34.5325 under financial instruments. This wide dispersion can also be confirmed from the estimated standard error for estimator $\hat{\alpha}$. Also, negative estimates of α were frequently encountered in the table. A negative estimate of α (which also implies a negative IES for domestic non-durable goods) is extremely counter-intuitive, since it means that the agent will buy more of C_{t+1} at the expense of C_t , even if the opportunity cost of C_{t+1} relative to C_t is rising.

We can think of two possibilities that have contributed to these odd estimation results. The first possibility is the weak instruments problem, i.e. if the instruments and the forcing variables in the regression are weakly correlated, the variance of the estimator will be large. It might be the case that in our GMM estimation, conventional instruments were weakly correlated to the forcing variables (in this case $(1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^C}$ and $\frac{C_{t+1}}{C_t}$), thus contributing to the wide variance of the estimator. Unfortunately financial instruments, which work better in some other literature, does not seem to reduce the variance of the estimator in this case. Financial instruments may also be weakly correlated with the forcing variables.

²¹Since the lag order was explicitly chosen, we will use a HAC estimator with truncated kernel when estimating the variance-covariance matrix of GMM disturbance terms.

²²Letter “a” indicates that conventional instruments were used. Letter “b” indicates that financial instruments were used.

Table 7: GMM Results: Euler Equation for Domestic Goods using Regular IV

$E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} (1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^C} - 1 \right] = 0$								
IV Type	Lag	β	s.e.	α	s.e.	J-statistics	p-value	D.F.
IV1	(-2)	0.996	(0.01)	0.983	(3.55)	--	--	Just Identified
	(-3)	0.994	(0.02)	0.524	(4.34)	--	--	Just Identified
	(-4)	Did not converge				--	--	Just Identified
IV2	(-2)	1.067	(0.08)	17.739	(21.50)	--	--	Just Identified
	(-3)	1.014	(0.01)	5.009	(3.15)	--	--	Just Identified
	(-4)	1.046	(0.07)	12.772	(16.55)	--	--	Just Identified
IV3	(-2)	0.863	(0.07)	-25.579	(14.83)	--	--	Just Identified
	(-3)	0.871	(0.06)	-23.820	(12.65)	--	--	Just Identified
	(-4)	0.848	(0.13)	-28.776	(25.68)	--	--	Just Identified
IV4	(-2)	0.880	(0.06)	-22.164	(12.44)	0.293	[0.58]	1
	(-3)	0.886	(0.01)	-20.672	(2.16)	0.424	[0.51]	1
	(-4)	0.861	(0.12)	-26.400	(23.89)	0.081	[0.77]	1
IV5	(-2)	0.969	(0.03)	-3.914	(7.42)	2.362	[0.12]	1
	(-3)	1.015	(0.01)	5.083	(3.15)	0.485	[0.48]	1
	(-4)	1.048	(0.07)	13.228	(16.53)	0.018	[0.89]	1
IV6	(-2)	0.902	(0.03)	-17.601	(5.68)	0.709	[0.70]	2
	(-3)	0.954	(0.01)	-6.896	(2.16)	6.145*	[0.04]	2
	(-4)	0.952	(0.04)	-7.553	(7.53)	4.187	[0.12]	2

Note: * denotes the rejection of null at 5% level. ** denotes the rejection of null at 1% level.

Table 8: GMM Results: Euler Equation for Domestic Goods using Financial IV

$E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} (1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^C} - 1 \right] = 0$								
IV Type	Lag	β	s.e.	α	s.e.	J-statistics	p-value	D.F.
FIV1	(-2)	1.078	(0.09)	18.942	(20.51)	--	--	Just Identified
	(-3)	0.954	(0.05)	-7.754	(11.41)	--	--	Just Identified
	(-4)	1.013	(0.01)	4.853	(4.31)	--	--	Just Identified
FIV2	(-2)	0.828	(0.12)	-31.213	(23.65)	--	--	Just Identified
	(-3)	0.817	(0.11)	-33.240	(22.61)	--	--	Just Identified
	(-4)	0.875	(0.08)	-24.074	(16.56)	--	--	Just Identified
FIV3	(-2)	1.119	(0.10)	29.670	(28.41)	--	--	Just Identified
	(-3)	1.101	(0.09)	25.251	(23.65)	--	--	Just Identified
	(-4)	1.063	(0.05)	17.288	(13.41)	--	--	Just Identified
FIV4	(-2)	1.119	(0.06)	28.649	(14.53)	0.199	[0.65]	1
	(-3)	1.120	(0.07)	32.076	(19.17)	0.503	[0.47]	1
	(-4)	0.966	(0.01)	-4.752	(2.60)	5.761*	[0.01]	1
FIV5	(-2)	1.138	(0.08)	34.532	(23.50)	0.062	[0.80]	1
	(-3)	1.122	(0.07)	30.795	(18.42)	0.085	[0.77]	1
	(-4)	0.979	(0.01)	-2.426	(3.63)	9.615**	[0.00]	1
FIV6	(-2)	1.119	(0.06)	28.618	(14.52)	0.210	[0.90]	2
	(-3)	1.093	(0.06)	24.831	(17.02)	1.668	[0.43]	2
	(-4)	0.974	(0.01)	-3.524	(2.45)	7.822*	[0.02]	2

Note: * denotes the rejection of null at 5% level. ** denotes the rejection of null at 1% level.

Table 9: GMM Results: Euler Equation for Imported Goods using Regular IV

		$E_t \left \beta \left(\frac{M_{t+1}}{M_t} \right)^{-\nu} (1 + r_{t+1}) \frac{P_t^M}{P_{t+1}^M} - 1 \right = 0$						
IV Type	Lag	β	s.e.	ν	s.e.	J-statistics	p-value	D.F.
IV1	(-2)	0.999	(0.29)	4.452	(26.15)	--	--	Just Identified
	(-3)	0.990	(0.01)	-0.231	(0.59)	--	--	Just Identified
	(-4)	0.993	(0.02)	0.045	(1.79)	--	--	Just Identified
IV2	(-2)	1.008	(0.008)	1.736	(1.04)	--	--	Just Identified
	(-3)	1.008	(0.008)	2.479	(2.86)	--	--	Just Identified
	(-4)	1.004	(0.01)	1.254	(1.11)	--	--	Just Identified
IV3	(-2)	1.009	(0.009)	2.548	(1.32)	--	--	Just Identified
	(-3)	1.007	(0.01)	2.940	(1.34)	--	--	Just Identified
	(-4)	0.999	(0.03)	4.088	(2.75)	--	--	Just Identified
IV4	(-2)	1.009	(0.009)	2.537	(1.32)	0.013	[0.90]	1
	(-3)	0.948	(0.03)	-2.243	(1.61)	0.912	[0.33]	1
	(-4)	0.917	(0.04)	-3.544	(1.79)	0.000	[0.99]	1
IV5	(-2)	1.008	(0.008)	1.800	(1.03)	0.043	[0.83]	1
	(-3)	0.979	(0.009)	-0.424	(0.54)	3.901*	[0.04]	1
	(-4)	1.004	(0.01)	1.333	(1.13)	0.136	[0.71]	1
IV6	(-2)	1.009	(0.009)	2.185	(1.07)	0.367	[0.83]	2
	(-3)	0.955	(0.02)	-1.786	(1.06)	1.492	[0.47]	2
	(-4)	1.012	(0.007)	2.334	(1.55)	2.218	[0.32]	2

Note: * denotes the rejection of null at 5% level. ** denotes the rejection of null at 1% level.

A second possibility is misspecification in the Euler equation. A casual way to check for this is to examine Hansen's J statistics. However, to our surprise, Hansen's J test does not reject the null hypothesis that Euler equation (8) is correctly specified in most cases. We only found four rejections out of a total 18 tests. Does this mean that Euler equation (8) is correctly specified? Statistically speaking, we cannot deny this possibility. But then the odd estimates of α as in Table 7 and 8 do not conform with the results of Hansen's J test. Or, it might be the case that the low power of Hansen's J test resulted in the under-rejection of the null. As such, we propose to use the likelihood ratio type test proposed by Cooley and Ogaki (1996), which will be the topic of the next section.

Table 10: GMM Results: Euler Equation for Imported Goods using Financial IV

$E_t \left[\beta \left(\frac{M_{t+1}}{M_t} \right)^{-\nu} (1 + r_{t+1}) \frac{P_t^M}{P_{t+1}^M} - 1 \right] = 0$								
IV Type	Lag	β	s.e.	ν	s.e.	J-statistics	p-value	D.F.
FIV1	(-2)	0.999	(0.006)	0.582	(0.25)	--	--	Just Identified
	(-3)	1.000	(0.008)	0.746	(0.77)	--	--	Just Identified
	(-4)	0.995	(0.15)	0.274	(14.28)	--	--	Just Identified
FIV2	(-2)	1.005	(0.01)	2.796	(0.60)	--	--	Just Identified
	(-3)	1.005	(0.01)	2.271	(0.34)	--	--	Just Identified
	(-4)	1.005	(0.006)	1.810	(0.36)	--	--	Just Identified
FIV3	(-2)	1.005	(0.005)	1.463	(0.51)	--	--	Just Identified
	(-3)	1.004	(0.005)	1.405	(0.46)	--	--	Just Identified
	(-4)	1.005	(0.007)	1.758	(0.79)	--	--	Just Identified
FIV4	(-2)	1.018	(0.005)	1.886	(0.34)	7.043**	[0.00]	1
	(-3)	1.005	(0.01)	2.352	(0.34)	0.397	[0.39]	1
	(-4)	1.005	(0.01)	1.806	(0.55)	0.000	[0.98]	1
FIV5	(-2)	1.016	(0.01)	2.510	(0.59)	3.143	[0.07]	1
	(-3)	0.996	(0.008)	1.758	(0.40)	1.685	[0.19]	1
	(-4)	1.005	(0.009)	1.797	(0.46)	0.004	[0.94]	1
FIV6	(-2)	1.013	(0.008)	1.933	(0.44)	5.610	[0.06]	2
	(-3)	0.998	(0.009)	1.940	(0.28)	1.697	[0.42]	2
	(-4)	1.005	(0.009)	1.789	(0.50)	0.003	[0.99]	2

Note: * denotes the rejection of null at 5% level. ** denotes the rejection of null at 1% level.

In sharp contrast to the estimation results of the Euler equation for domestic non-durable goods, the estimation results of the Euler equation for imported non-durable goods have a sensible result. Let us first refer to Table 9 which reports GMM results under conventional instruments. First of all, we can observe relative tightness in the estimates of ν . The estimates of ν range from -3.5445 to 4.4522. However, looking at the estimated standard error of estimator $\hat{\nu}$ we still observe relatively high variance, though not as severe as in the previous table. This, again, may result from the weak instruments problem. We also used financial instruments in estimating ν , the result for which is shown in Table 10. As one can observe from this table, the dispersion of the estimates of ν are even tighter compared to the estimates under conventional instruments. Estimates of ν range from 0.2742 to 2.796. No negative estimates were encountered. Also, looking at the estimated standard error of the estimator $\hat{\nu}$, we found conspicuously low variance compared to the estimator $\hat{\nu}$ under conventional instruments. This may be due to the better correlation of financial instruments to the forcing variables in Euler equation (9).

Now, turning to Hansen's J test, we generally did not reject the null hypothesis except for two cases. However, considering the low power of Hansen's J test, it may well be the case that the null hypothesis was under-rejected. As such, although the result looks encouraging for the Euler equation (9), we wait for Cooley and Ogaki's LR type test in judging whether the Euler equation (9) is correctly specified or not.

Finally, we have also estimated α and ν in the context of system equation by stacking eq. (8) and (9) as conditional moments. The virtue of the system equation approach is the gain of efficiency in estimating the parameters. However, the trade-off is the loss of robustness. In other words, if there are some conditional moments that are misspecified, then these will 'contaminate' the consistency of other conditional moments. Keeping this characteristic of system equation approach, let us now look at Table 11 and 12. As we can see from Table 11, the estimates of α ranged from -21.5729 to 5.7089 revealing large dispersion, while estimates for ν ranged from -4.3608 to 7.713, also revealing considerable dispersion. By simply comparing the results with the single equation approach, we observe that dispersion of estimates for α became tighter in system equation approach, but dispersion of estimates for ν became wider. It should be noted that there were quite a few negative estimates for both α and ν . Turning to Table 12, the estimates of α ranged from -0.6202 to 7.6425 revealing considerable tightness in comparison to Table 8 or 11, whereas the estimates of ν ranged from -0.0347 to 2.802

Table 11: GMM Results: System of Euler Equations for Domestic and Imported Goods for Regular IV

$$E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} (1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^C} - 1 \right] = 0$$

$$E_t \left[\beta \left(\frac{M_{t+1}}{M_t} \right)^{-\nu} (1 + r_{t+1}) \frac{P_t^M}{P_{t+1}^M} - 1 \right] = 0$$

IV Type	Lag	β	s.e.	α	s.e.	ν	s.e.	J-stat.	p-value	D.F.
IV1	(-2)	0.996	(0.01)	0.983	(3.56)	4.658	(3.20)	0.000	[0.99]	1
	(-3)	0.989	(0.01)	-0.454	(2.06)	-0.260	(0.61)	0.074	[0.78]	1
	(-4)	0.994	(0.03)	0.450	(7.70)	0.106	(2.89)	0.012	[0.91]	1
IV2	(-2)	1.016	(0.008)	5.708	(1.31)	2.725	(1.40)	1.426	[0.23]	1
	(-3)	1.010	(0.008)	4.050	(1.32)	2.680	(3.03)	0.126	[0.72]	1
	(-4)	1.002	(0.009)	2.747	(1.91)	1.059	(1.00)	1.246	[0.26]	1
IV3	(-2)	0.887	(0.06)	-21.372	(12.21)	-4.360	(2.05)	0.370	[0.54]	1
	(-3)	0.886	(0.05)	-21.572	(10.83)	-4.470	(2.05)	0.326	[0.56]	1
	(-4)	0.905	(0.10)	-17.970	(20.78)	7.713	(3.02)	0.727	[0.39]	1
IV4	(-2)	0.931	(0.02)	-12.235	(5.68)	-2.790	(1.41)	2.783	[0.42]	3
	(-3)	0.922	(0.02)	-13.864	(5.48)	-3.267	(1.38)	2.042	[0.56]	3
	(-4)	0.917	(0.04)	-15.450	(9.16)	-3.453	(1.72)	0.724	[0.86]	3
IV5	(-2)	1.010	(0.008)	4.486	(1.78)	1.682	(0.75)	5.746	[0.12]	3
	(-3)	0.979	(0.005)	-1.286	(0.86)	-0.238	(0.13)	5.935	[0.11]	3
	(-4)	1.004	(0.009)	3.238	(1.88)	1.311	(1.09)	1.791	[0.61]	3
IV6	(-2)	0.986	(0.004)	-1.521	(0.82)	-0.457	(0.27)	15.076*	[0.01]	5
	(-3)	0.970	(0.01)	-3.907	(1.45)	-1.018	(0.46)	12.287*	[0.03]	5
	(-4)	0.937	(0.02)	-10.073	(4.95)	-2.561	(1.05)	3.869	[0.56]	5

Note: * denotes the rejection of null at 5% level. ** denotes the rejection of null at 1% level.

Table 12: GMM Results: System of Euler Equations for Domestic and Imported Goods for Financial IV

$$E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} (1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^C} - 1 \right] = 0$$

$$E_t \left[\beta \left(\frac{M_{t+1}}{M_t} \right)^{-\nu} (1 + r_{t+1}) \frac{P_t^M}{P_{t+1}^M} - 1 \right] = 0$$

IV Type	Lag	β	s.e.	α	s.e.	ν	s.e.	J-stat.	p-value	D.F.
IV1	(-2)	0.994	(0.006)	0.746	(0.65)	0.319	(0.14)	1.348	[0.24]	1
	(-3)	0.990	(0.007)	0.075	(0.79)	0.049	(0.19)	1.543	[0.21]	1
	(-4)	Did not converge						--	--	1
IV2	(-2)	1.004	(0.006)	0.615	(1.28)	1.703	(0.37)	10.773**	[0.001]	1
	(-3)	1.027	(0.01)	7.642	(3.12)	2.802	(0.69)	8.338**	[0.003]	1
	(-4)	1.007	(0.005)	1.282	(0.55)	0.498	(0.22)	13.600**	[0.000]	1
IV3	(-2)	1.002	(0.006)	1.356	(0.65)	0.408	(0.17)	5.909*	[0.01]	1
	(-3)	1.003	(0.006)	1.580	(0.76)	0.391	(0.20)	6.233*	[0.01]	1
	(-4)	1.013	(0.006)	5.826	(1.73)	1.385	(0.58)	4.991*	[0.02]	1
IV4	(-2)	1.003	(0.006)	0.942	(0.55)	0.391	(0.11)	12.972**	[0.004]	3
	(-3)	0.992	(0.006)	-0.620	(0.69)	-0.034	(0.17)	13.174**	[0.004]	3
	(-4)	1.027	(0.008)	5.794	(2.30)	1.415	(0.54)	21.379**	[0.000]	3
IV5	(-2)	1.003	(0.005)	0.819	(0.65)	0.311	(0.16)	12.171**	[0.006]	3
	(-3)	1.001	(0.006)	0.328	(0.73)	0.141	(0.16)	12.416**	[0.006]	3
	(-4)	1.022	(0.007)	4.711	(2.34)	1.055	(0.47)	25.296**	[0.000]	3
IV6	(-2)	1.001	(0.005)	0.606	(0.59)	0.344	(0.12)	12.753**	[0.02]	5
	(-3)	1.003	(0.005)	0.513	(0.78)	0.120	(0.15)	13.234**	[0.02]	5
	(-4)	1.015	(0.005)	2.896	(1.16)	0.683	(0.26)	17.885**	[0.003]	5

Note: * denotes the rejection of null at 5% level. ** denotes the rejection of null at 1% level.

showing relative tightness relative to Table 11 but are slightly more dispersed compared to Table 10. Here again, we can draw the same observation. Compared to the single equation approach, the estimates of α have a tighter dispersion in a system equation approach, while the estimates of ν show slightly wider dispersion in a system equation approach. From these results, we can casually see that the conditional moment eq. (8) is ‘contaminating’ conditional moment eq. (9). Based on this observation, we cast doubt on the specification for eq. (8) while the specification for eq. (9) needs further investigation. In order to formally test the specification of eq. (8) and eq. (9) we will adopt Cooley and Ogaki’s (1996) LR type test. This will be the topic of the next section.

5 Test

In this section, we will discuss why Cooley and Ogaki’s test best suits for our purpose and also report the results of the test. Before we discuss Cooley and Ogaki’s LR type test, it may be useful to review the standard LR type test in GMM literature. For simplicity, we impose some linear restriction on the GMM estimator. In the most general linear form, the null hypothesis can be expressed as follows:

$$H_o : \mathbf{R}\hat{\boldsymbol{\theta}}_{GMM} = \mathbf{q}$$

where \mathbf{q} is $q \times 1$ vector of constant and \mathbf{R} is some $q \times k$ matrix. Then LR type statistics, denoted as QLR , are defined as follows and can be shown to be asymptotically χ^2 distributed with q degrees of freedom.

$$QLR = T \cdot J_{restricted} - T \cdot J_{unrestricted} \xrightarrow{d} \chi^2(q)$$

where T stands for the number of observations and J stands for the minimized objective function under GMM. Now, it should be noted that under the standard LR type test, \mathbf{q} was simply a vector of *constants*.

The punch line of Cooley and Ogaki’s LR type test is that they replaced \mathbf{q} with the *estimator* of cointegrating vector $\hat{\mathbf{q}}_{coint}$. By exploiting the super-consistency of $\hat{\mathbf{q}}_{coint}$, they show that QLR will again be asymptotically χ^2 distributed with q degrees

of freedom.²³ Restating mathematically,

$$H_o : \mathbf{R}\hat{\boldsymbol{\theta}}_{GMM} = \hat{\mathbf{q}}_{coint} \text{ and } QLR \xrightarrow{d} \chi^2(q).$$

Since our model involves cointegration analysis and GMM in estimating the parameters α and ν , Cooley and Ogaki's LR type test seems to be the best candidate for our specification test.

5.1 Economic Rationale Behind the Test

If the model is correct under the assumption that there is no liquidity constraint or habit formation, log-linearized Cross-Euler equations (14) and (15) will be correctly specified with cointegrating restriction. At the same time, standard Euler equations (8) and (9) will also be correctly specified. Consequently, parameter estimates of α and ν from cointegration analysis and GMM estimation should be statistically close. In testing whether those estimates are close to each other or not, we use Cooley and Ogaki's LR type test. Under the test, the null hypothesis will be

$$H_0 : \hat{\alpha}_{GMM} = \hat{\alpha}_{coint} \text{ and } \hat{\nu}_{GMM} = \hat{\nu}_{coint}.$$

The rejection of the null suggests that there seems to exist at least one assumption that is violated. Unfortunately, the rejection of the null does not provide us much information of which the assumption was violated. On the other hand, the non-rejection of the null suggests that joint assumption of there being no liquidity constraint and no habit formation is not an implausible assumption. Simply put, the addi-log utility function without the presence of the liquidity constraint is an 'OK' assumption if the test does not reject the null.

5.2 Results

In this section we report the results of Cooley and Ogaki's LR type test. In conducting the test, the same instruments from section 4 were used for both restricted and unrestricted GMM. We basically tested three types of null hypothesis. The first null hypothesis is $H_0^1 : \hat{\alpha}_{GMM} = \hat{\alpha}_{coint}$ and results are reported in Table 13. The second null hypothesis is $H_0^2 : \hat{\nu}_{GMM} = \hat{\nu}_{coint}$ and results are reported in Table 14. Finally, the third null hypothesis is $H_0^3 : \hat{\alpha}_{GMM} = \hat{\alpha}_{coint} \text{ and } \hat{\nu}_{GMM} = \hat{\nu}_{coint}$ and results are

²³If, instead, the estimator $\hat{\mathbf{q}}$ were only consistent (i.e. $O(T^{-1/2})$ consistent), then one would have to calculate the covariance of $\hat{\boldsymbol{\theta}}_{GMM}$ and $\hat{\mathbf{q}}$ in order to conduct the statistical inference. For details, see Ogaki (1993).

Table 13: LR test results for Domestic Goods: Regular IV (Left) and Financial IV (Right)

$H_o : \alpha_{GMM} = \alpha_{coint}$							
IV Type	Lag	QLR	p-value	IV Type	Lag	QLR	p-value
IV1	(-2)	0.054	[0.816]	FIV1	(-2)	1.476	[0.224]
	(-3)	0.073	[0.786]		(-3)	1.772	[0.183]
	(-4)	0.015	[0.901]		(-4)	0.766	[0.381]
IV2	(-2)	2.711	[0.099]	FIV2	(-2)	9.766**	[0.001]
	(-3)	1.183	[0.276]		(-3)	12.711**	[0.000]
	(-4)	1.539	[0.214]		(-4)	16.021**	[0.000]
IV3	(-2)	15.253**	[0.000]	FIV3	(-2)	5.959*	[0.014]
	(-3)	16.198**	[0.000]		(-3)	6.412*	[0.011]
	(-4)	15.072**	[0.000]		(-4)	5.889*	[0.015]
IV4	(-2)	15.260**	[0.000]	FIV4	(-2)	12.606**	[0.000]
	(-3)	16.023**	[0.000]		(-3)	14.929**	[0.000]
	(-4)	17.548**	[0.000]		(-4)	10.507**	[0.001]
IV5	(-2)	4.677*	[0.030]	FIV5	(-2)	10.011**	[0.001]
	(-3)	3.091	[0.078]		(-3)	13.223**	[0.000]
	(-4)	3.063	[0.080]		(-4)	6.881**	[0.008]
IV6	(-2)	15.188**	[0.000]	FIV6	(-2)	12.725**	[0.000]
	(-3)	10.309**	[0.001]		(-3)	13.910**	[0.000]
	(-4)	14.193**	[0.000]		(-4)	8.719**	[0.003]

Note: * denotes the rejection of null at 5% level. ** denotes the rejection of null at 1% level.

reported in Table 15. Note again, if indeed eq. (7), eq. (8) and eq. (9) are all well specified, then the test is likely to accept all three null hypotheses. We will interpret the results under three different nulls one by one.

First, let us turn to the results under the null of $H_0^1 : \hat{\alpha}_{GMM} = \hat{\alpha}_{coint}$. Under conventional instruments (Table 13 left-side panel), although the results are not clear-cut, we found some evidence against the null. The test rejected 10 out of 18 cases. Especially for cases where over-identifying restrictions were imposed, the result showed clear evidence against the null. Turning to financial instruments (Table 13 right-side panel), we found stronger evidence against the null. Indeed, the test rejected 15 out of 18 cases. For both just and over-identified restrictions, the majority of QLR statistics

Table 14: LR test results for Imported Goods: Regular IV (Left) and Financial IV (Right)

$H_o : \nu_{GMM} = \nu_{coint}$							
IV Type	Lag	QLR	p-value	IV Type	Lag	QLR	p-value
IV1	(-2)	0.080	[0.776]	FIV1	(-2)	2.353	[0.125]
	(-3)	1.051	[0.305]		(-3)	0.237	[0.626]
	(-4)	0.123	[0.725]		(-4)	0.0003	[0.984]
IV2	(-2)	0.132	[0.715]	FIV2	(-2)	7.286**	[0.006]
	(-3)	0.407	[0.523]		(-3)	5.318*	[0.021]
	(-4)	0.019	[0.888]		(-4)	1.392	[0.238]
IV3	(-2)	1.796	[0.180]	FIV3	(-2)	0.006	[0.938]
	(-3)	2.990	[0.083]		(-3)	0.001	[0.966]
	(-4)	4.448*	[0.034]		(-4)	0.242	[0.622]
IV4	(-2)	1.798	[0.179]	FIV4	(-2)	0.735	[0.391]
	(-3)	4.159*	[0.041]		(-3)	6.132*	[0.013]
	(-4)	5.390*	[0.020]		(-4)	0.649	[0.420]
IV5	(-2)	0.256	[0.612]	FIV5	(-2)	5.533*	[0.018]
	(-3)	-2.630	- -		(-3)	1.126	[0.288]
	(-4)	-0.008	- -		(-4)	0.837	[0.359]
IV6	(-2)	1.460	[0.226]	FIV6	(-2)	2.093	[0.147]
	(-3)	4.596*	[0.032]		(-3)	3.155	[0.075]
	(-4)	3.320	[0.068]		(-4)	0.701	[0.402]

Note: * denotes the rejection of null at 5% level. ** denotes the rejection of null at 1% level.

were bigger than the nominal critical value²⁴ of 1%. Overall, we found strong evidence against the null of $\hat{\alpha}_{GMM} = \hat{\alpha}_{coint}$, which suggests that eq. (7) and/or eq. (8) are misspecified.

Next, we turn to results under the null of $H_0^2 : \hat{\nu}_{GMM} = \hat{\nu}_{coint}$. Let us first interpret the results under conventional instruments (Table 14 left-side panel). Despite for some rejections, we did not find strong evidence against the null. Out of 16 valid cases,²⁵

²⁴In this paper, every test conducted was an asymptotic test. It is not clear whether the sample size of 100 or so qualifies as a ‘large’ sample. Ideally speaking, the finite sample critical value should be used. But this requires the simulation technique such as moving-block bootstrap, which is beyond the scope of this paper.

²⁵We encountered two negative QLR statistics in the test under conventional instruments. Since we

Table 15: Joint LR test results for Domestic and Imported Goods: Regular IV (Left) and Financial IV (Right)

$H_o : \alpha_{GMM} = \alpha_{CCR}$							
IV Type	Lag	QLR	p-value	IV Type	Lag	QLR	p-value
IV1	(-2)	1.187	[0.552]	FIV1	(-2)	10.611**	[0.004]
	(-3)	4.295	[0.116]		(-3)	7.703*	[0.021]
	(-4)	2.572	[0.276]		(-4)	--	--
IV2	(-2)	9.387**	[0.009]	FIV2	(-2)	-0.007	--
	(-3)	5.158	[0.075]		(-3)	4.739	[0.093]
	(-4)	7.025*	[0.029]		(-4)	3.301	[0.191]
IV3	(-2)	15.609**	[0.000]	FIV3	(-2)	3.843	[0.146]
	(-3)	15.746**	[0.000]		(-3)	3.234	[0.198]
	(-4)	14.453**	[0.000]		(-4)	3.186	[0.203]
IV4	(-2)	15.899**	[0.000]	FIV4	(-2)	3.035	[0.219]
	(-3)	14.393**	[0.000]		(-3)	3.865	[0.144]
	(-4)	33.541**	[0.000]		(-4)	-5.856	--
IV5	(-2)	5.287	[0.071]	FIV5	(-2)	1.206	[0.547]
	(-3)	6.875*	[0.032]		(-3)	5.466	[0.065]
	(-4)	7.720*	[0.021]		(-4)	-9.047	--
IV6	(-2)	6.657*	[0.035]	FIV6	(-2)	2.996	[0.223]
	(-3)	10.370**	[0.005]		(-3)	24.760**	[0.000]
	(-4)	18.049**	[0.000]		(-4)	-0.63	--

Note: * denotes the rejection of null at 5% level. ** denotes the rejection of null at 1% level.

the test rejected the null in only for four cases. Turning to the test under financial instruments (Table 14 right-side panel), we found similar results as in conventional instruments. Again, we were not able to find strong evidence against the null. Out of 18 cases,²⁶ the test rejected the null in only for four cases. In sharp contrast to the null of $\hat{\alpha}_{GMM} = \hat{\alpha}_{coint}$, overall we found less evidence against the null of $\hat{\nu}_{GMM} = \hat{\nu}_{coint}$. This suggest that both eq. (7) and eq. (9) may be well specified.

Finally, in order to supplement the above results, let us turn to the results under

do not know how to interpret the negative QLR, which may happen in the finite sample unfortunately we will regard these two results as invalid. However, it should be noted that negative QLR will not happen in a large sample.

²⁶Under financial instruments, we did not encounter any negative QLR.

the null of $H_0^3 : (\hat{\alpha}_{GMM}, \hat{\nu}_{GMM})' = (\hat{\alpha}_{coint}, \hat{\nu}_{coint})'$. It should be noted that null is now the equality of a vector of the estimators. This, in turn, means that if there exists at least one estimator which is not equal, then the null will be rejected, even if other estimators are equal. Since we have rejected the null that $\hat{\alpha}_{GMM} = \hat{\alpha}_{coint}$ frequently, we naturally expect to see a frequent rejection of H_0^3 . This expectation is confirmed in the left-side panel of Table 15. Indeed 13 out of 18 tests rejected the null. Even among five accepted cases, the call for three cases was marginal. On the contrary to our intuition, however, the result in the right-side panel of Table 15 does not conform with our expectation. Simply looking at *QLR* statistics, the test only rejects three out of 13 valid cases. However, more careful inspection of the result reveals that among 10 accepted, cases *QLR* statistics were larger than 3 for eight cases. Further, we can observe that the majority of low *QLR* statistics resulted from unusually high unrestricted *J* statistics. Since high *J* statistics implies misspecification of the conditional moment according to Hansen's *J* test and further Cooley and Ogaki's LR type test assumes the specification of unrestricted conditional moment under the null, *QLR* statistics in the right-side panel of Table 15 probably have no meaning. It may well be the case that *QLR* statistics under the alternative hypothesis (which is highly likely considering the high unrestricted *J* statistics) happened to have a distribution similar to χ^2 distribution.²⁷

5.3 Interpretation

The interesting observation from the test is the contrasting results between the two null hypothesis: $H_0^1 : \hat{\alpha}_{GMM} = \hat{\alpha}_{coint}$ and $H_0^2 : \hat{\nu}_{GMM} = \hat{\nu}_{coint}$. The test frequently rejected the null hypotheses of $\hat{\alpha}_{GMM} = \hat{\alpha}_{coint}$, while the rejection of the null hypothesis $\hat{\nu}_{GMM} = \hat{\nu}_{coint}$ was infrequent. The question here is why did the test reject the specification of the Euler equation for domestic non-durable goods so frequently, while accepting it for imported non-durable goods so frequently.

One explanation, as De la Croix and Urbain (1998) argue, is that habit formation is an important factor in non-durable goods consumption. This can at least explain why the test frequently rejected the specification of the Euler equation for domestic non-durable goods. But what it cannot explain is the non-rejection of the Euler equation for imported non-durable goods. If indeed habit formation is so important regarding people's preference for non-durable goods, then the test should also reject the specification of the Euler equation for imported non-durable goods. Unless we firmly believe

²⁷An unusually large negative number such as -5.8564 or -9.0472 supports our claim here.

that people form habits only for domestic non-durable goods and not for imported non-durable goods, the habit formation argument cannot successfully explain our test results.

Another possible explanation is that the liquidity constraint is present for non-durable goods consumption. This argument can explain the frequent rejection of the Euler equation for non-durable goods. But, again, what it cannot explain is the non-rejection of the Euler equation for imported non-durable goods. If the liquidity constraint argument is true, then the test should also reject the specification of the Euler equation for imported non-durable goods as frequently as it does for domestic non-durable goods. However, from our result, we did not observe this implication. Unless we firmly believe that people are facing liquidity constraints only for domestic non-durable goods and not for imported non-durable goods, then the liquidity constraint argument does not seem to be successful in explaining the result.

Finally, one can cast doubt on the auxiliary assumptions of the model such as the addi-log specification of the utility function or the existence of the representative agent. However, if these auxiliary assumptions are indeed violated in reality, then the test should uniformly reject the specification of the Euler equation for both domestic and imported non-durable goods. On the contrary, what we observe is the non-uniform rejection of the Euler equations; frequent rejections of the domestic Euler equation and infrequent rejection of the imported Euler equation.

Thus, our test results pose a serious question for economists. What can explain the rejection of the Euler equation for domestic the non-durable goods and non-rejection of the Euler equation for imported non-durable goods at the same time?

6 Conclusion

In this paper, we adopted the standard two goods version of the Life Cycle/ Permanent Income Model to investigate intertemporal aspects of U.S. imported non-durable goods expenditure and domestic non-durable goods expenditure. In the process of investigation, previous researchers often faced an empirical dilemma in estimating the intertemporal elasticity of substitution (IES) parameters. By modeling the preference shock or measurement error to the intratemporal relationship between domestic and imported non-durable goods, it will be possible to estimate the IES parameters from the intratemporal relationship. However, by doing so will induce the Euler equation to be non-standard so that the usual GMM estimation on a standard Euler equation will be inconsistent.

The main contribution of this paper is the introduction of a new concept called the Cross-Euler equation, using which we successfully overcame the empirical dilemma that previous researchers faced. By exploiting the cointegrating restriction implied by the Cross-Euler equations defined upon current domestic non-durable goods expenditure to future imported non-durable goods (and vice-versa), we made it possible to compare the estimates of intertemporal substitution parameters for domestic (i.e. α) and imported (i.e. ν) non-durable consumption goods with estimates from standard Euler equations. We first estimated parameters α and ν from log-linearized Cross-Euler equations implied by the model using Park's CCR. Our estimates were more or less similar to previous studies by Ceglowski (1991), Clarida (1994), Amano and Wirjanto (1996), and De la Croix and Urbain (1998). Next, using Hansen's GMM, we estimated the same parameters from the standard Euler equations implied by the model. Under the assumption that there is no liquidity constraint or habit formation, the model predicts that estimates from Cross-Euler equations and from standard Euler equations to be reasonably close. In order to test this implication formally, we conducted Cooley and Ogaki's (1996) LR type test. The null hypothesis of $\alpha_{CCR} = \alpha_{GMM}$ was rejected frequently, but the null of $\nu_{CCR} = \nu_{GMM}$ was rejected infrequently. These results suggest that the Euler equation for domestic non-durable goods is misspecified, but somehow support the specification of the Euler equation for imported non-durable goods.

The results of the test posed a serious question for economists. What can explain the rejection of the Euler equation for domestic non-durable goods and non-rejection of the Euler equation for imported non-durable goods at the same time? The liquidity constraint, which is often thought of as a culprit for the rejection of the Euler equation in consumption literature, does not seem to satisfactorily explain this result. If it was indeed the reason, then we would expect to see the rejection of both the domestic and imported Euler equations. Unless we believe that the representative agent is only constrained for domestic non-durable goods and not for imported non-durable goods, the result of the test remains puzzling. The existence of habit formation, which can also lead to the rejection of the Euler equation, does not seem to explain this puzzle either, unless we believe that the representative agent is forming the habit only for domestic non-durable goods, not for imported non-durable goods. Finally, violation of auxiliary assumptions of the model such as the addi-log type utility function, separability of non-durable goods and durable goods, or the existence of a representative agent can also lead to the rejection of the Euler equation. But what cannot be explained is the contrast between the rejection of the domestic Euler equation and the non-rejection of the imported Euler equation.

One possible explanation for resolving this puzzle is an approach from a heterogenous agent model *à la* Campbell and Mankiw (1989). Suppose there are two agents in the economy, rich and poor. Further suppose that domestic non-durable goods are mainly composed of necessities and imported non-durable goods are mainly composed of luxuries. Because the poor agent is likely to buy more domestic non-durable goods and also likely to face liquidity constraints, whereas the rich agent is likely to buy more imported non-durable goods without facing liquidity constraints, then it makes sense why the Euler equation for imported goods is correctly specified while the Euler equation for domestic goods is not. Investigation through this channel is left for future research.

A Appendix 1

Here we prove the proposition that when some kind of an error is introduced to the intratemporal relationship, eq. (7), then at least one of the Euler eq. (8) or (9) will be misspecified. For notational simplicity, let us define the terms in eq.(7), (8), and (9) as follows.

$$\begin{aligned} A_t &\equiv \frac{1}{K} \frac{C_t^{-\alpha} P_t^C}{M_t^{-\nu} P_t^M} \\ B_{t+1} &\equiv \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} (1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^C} \\ \Gamma_{t+1} &\equiv \beta \left(\frac{M_{t+1}}{M_t} \right)^{-\nu} (1 + r_{t+1}) \frac{P_t^M}{P_{t+1}^M} \end{aligned}$$

Observe that under the optimization behavior of the representative agent,

$$\begin{aligned} A_t &= 1 \\ B_{t+1} &= 1 + \varepsilon_{t+1}^B \\ \Gamma_{t+1} &= 1 + \varepsilon_{t+1}^C \end{aligned}$$

where

$$\begin{aligned} \varepsilon_{t+1}^B &\equiv \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} (1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^C} - E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} (1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^C} \right] \\ \varepsilon_{t+1}^C &\equiv \beta \left(\frac{M_{t+1}}{M_t} \right)^{-\nu} (1 + r_{t+1}) \frac{P_t^M}{P_{t+1}^M} - E_t \left[\beta \left(\frac{M_{t+1}}{M_t} \right)^{-\nu} (1 + r_{t+1}) \frac{P_t^M}{P_{t+1}^M} \right] \end{aligned}$$

Before we prove the proposition, it is useful to prove the following lemma.

Lemma 1 *Let X and Y be any two random variables. Then, in general¹,*

$$E \left(\frac{X}{Y} \right) \neq \frac{E(X)}{E(Y)}.$$

Proof. Let us first observe that

$$E \left(\frac{X}{Y} \right) = E(X) E \left(\frac{1}{Y} \right) + Cov \left(X, \frac{1}{Y} \right).$$

¹ Except for the special case when $Cov \left(X, \frac{1}{Y} \right) = E(X) \left[\frac{1}{E(Y)} - E \left(\frac{1}{Y} \right) \right]$.

We prove the lemma by way of contradiction. Suppose the statement $E(X/Y) = E(X)/E(Y)$ is true. Then the following relationship must be true.

$$E(X) E\left(\frac{1}{Y}\right) + Cov\left(X, \frac{1}{Y}\right) = \frac{E(X)}{E(Y)}$$

Dividing both sides by $E(X)$, we get the following expression.

$$E\left(\frac{1}{Y}\right) + \frac{Cov(X, 1/Y)}{E(X)} = \frac{1}{E(Y)}$$

Now, this equation have to be true for any two random variables X and Y . In particular, let us choose the case when random variables X and Y are independent of each other. Then $Cov(X, 1/Y) = 0$. This implies the following equation.

$$E\left(\frac{1}{Y}\right) = \frac{1}{E(Y)}$$

But by Jensen's Inequality, the above equation cannot be true. A contradiction. ■

A.1 Case 1: Introducing Additive Error to A_t

Proposition 2 *When an additive stationary error term is introduced to A_t , i.e. $A_t = 1 + e_t$ where $e_t \sim I(0)$, at least one moment condition of $E(B_{t+1}) = 1$ or $E(\Gamma_{t+1}) = 1$ will be violated.*

Proof. Suffice to show one inequality. Let us first note the following algebraic relationship

$$\begin{aligned} \Gamma_{t+1} &= \frac{A_t}{A_{t+1}} B_{t+1} \\ &= \frac{(1 + e_t)}{(1 + e_{t+1})} (1 + \varepsilon_{t+1}^B) \end{aligned}$$

Applying conditional expectation operator $E_t(\cdot)$ on both sides,

$$E_t(\Gamma_{t+1}) = (1 + e_t) E_t \left[\frac{1 + \varepsilon_{t+1}^B}{1 + e_{t+1}} \right]$$

By lemma 1, the following inequality holds.

$$\begin{aligned} E_t(\Gamma_{t+1}) &\neq (1 + e_t) \frac{E_t(1 + \varepsilon_{t+1}^B)}{E_t(1 + e_{t+1})} \\ &\neq \frac{1 + e_t}{1 + E_t(e_{t+1})} \end{aligned}$$

Applying unconditional expectation operator $E(\cdot)$ on both sides,

$$\begin{aligned} E(\Gamma_{t+1}) &\neq E\left[\frac{1+e_t}{1+E_t(e_{t+1})}\right] \\ &\neq \frac{E(1+e_t)}{E[1+E_t(e_{t+1})]} = \frac{1+E(e_t)}{1+E(e_{t+1})} \end{aligned}$$

Now, by the stationarity of e_t , $[1+E(e_t)]/[1+E(e_{t+1})] = 1$. Therefore, in general, $E(\Gamma_{t+1}) \neq 1$. ■

A.2 Case 2: Introducing Multiplicative Error² to A_t

Proposition 3 *When a multiplicative error term is introduced to A_t , i.e. $A_t = e_t$ where $e_t \sim I(0)$, at least one moment condition of $E(B_{t+1}) = 1$ or $E(\Gamma_{t+1}) = 1$ will be violated.*

Proof. Suffice to show one inequality. Again, by the algebraic relationship,

$$\begin{aligned} \Gamma_{t+1} &= \frac{A_t}{A_{t+1}} B_{t+1} \\ &= \frac{e_t}{e_{t+1}} (1 + \varepsilon_{t+1}^B) \end{aligned}$$

Applying conditional expectation operator $E_t(\cdot)$ on both sides,

$$\begin{aligned} E_t(\Gamma_{t+1}) &= e_t E_t\left[\frac{1+\varepsilon_{t+1}^B}{e_{t+1}}\right] \\ &\neq e_t \frac{E_t(1+\varepsilon_{t+1}^B)}{E_t(e_{t+1})} = \frac{e_t}{E_t(e_{t+1})} \end{aligned}$$

Applying unconditional expectation operator $E(\cdot)$ on both sides,

$$\begin{aligned} E(\Gamma_{t+1}) &\neq E\left[\frac{e_t}{E_t(e_{t+1})}\right] \\ &\neq \frac{E(e_t)}{E[E_t(e_{t+1})]} = \frac{E(e_t)}{E(e_{t+1})} = 1 \end{aligned}$$

Thus, in general, $E(\Gamma_{t+1}) \neq 1$. ■

²The import demand model with preference shocks adopted by Clarida (1994) and Amano et al. (1996) is a special case of this multiplicative error.

B Appendix 2

Here we formally show how to derive the Cross-Euler equations. For convenience, let us restate the representative agent's problem.

$$\begin{aligned} & \max E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}, M_{t+i}) \\ \text{s.t. } & A_{t+1+i} = (1+r_t)A_{t+i} + Y_{t+i} - P_{t+i}^H C_{t+i} - P_{t+i}^F M_{t+i} \text{ for } \forall i \geq 0 \end{aligned}$$

By constructing a lifetime budget constraint from period-by-period budget constraints, we can reformulate the above optimization problem as follows.

$$\begin{aligned} & \max E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}, M_{t+i}) \\ \text{s.t. } & A_t + \sum_{i=0}^{\infty} \left(\prod_{j=0}^i \frac{1}{1+r_{t+j}} \right) Y_{t+i} = \sum_{i=0}^{\infty} \left(\prod_{j=0}^i \frac{1}{1+r_{t+j}} \right) (P_{t+i}^H C_{t+i} + P_{t+i}^F M_{t+i}) \end{aligned}$$

The left-hand side of the constraint can be considered as the present value of a life-time wealth of the agent, and the right-hand side of the constraint represents the present value of life-time consumption. Formulating the Lagrangian,

$$\begin{aligned} \mathfrak{L} = & E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}, M_{t+i}) \\ & + \lambda \left[A_t + \sum_{i=0}^{\infty} \left(\prod_{j=0}^i \frac{1}{1+r_{t+j}} \right) Y_{t+i} - \sum_{i=0}^{\infty} \left(\prod_{j=0}^i \frac{1}{1+r_{t+j}} \right) (P_{t+i}^H C_{t+i} + P_{t+i}^F M_{t+i}) \right] \end{aligned}$$

Then, the FOC's for C_t , M_t , C_{t+1} , and M_{t+1} will be as follows.

$$U_{C_t} = \lambda P_t^H \tag{B.1}$$

$$U_{M_t} = \lambda P_t^F \tag{B.2}$$

$$E_t (\beta U_{C_{t+1}}) = \lambda \frac{1}{1+r_{t+1}} P_{t+1}^C \tag{B.3}$$

$$E_t (\beta U_{M_{t+1}}) = \lambda \frac{1}{1+r_{t+1}} P_{t+1}^M \tag{B.4}$$

From eq.(B.1.) and (B.4.), we obtain

$$E_t \left[\beta \frac{U_{M_{t+1}}}{U_{C_t}} (1 + r_{t+1}) \frac{P_t^C}{P_{t+1}^M} \right] = 1$$

which is the Cross-Euler equation (11) in the paper. From eq.(B.2.) and (B.4.), we get,

$$E_t \left[\beta \frac{U_{C_{t+1}}}{U_{M_t}} (1 + r_{t+1}) \frac{P_t^M}{P_{t+1}^C} \right] = 1$$

which is the Cross-Euler equation (12) in the paper.

C Appendix 3

Here we prove the (strict) stationarity of the forecast error embedded in the Cross-Euler equation (11). The strict stationarity of the forecast error from the Cross-Euler equation (12) is similar and will therefore be omitted.

Proposition 4 *Let C_{t+1}/C_t , r_t , and P_{t+1}^C/P_t^C be strictly stationary processes. The forecast error e_t is defined as*

$$e_t = \xi_t - E_{t-1}(\xi_t)$$

where

$$\xi_t \equiv \beta K \frac{M_t^{-\nu}}{C_{t-1}^{1-\alpha}} (1 + r_t) \frac{P_{t-1}^H}{P_t^M}$$

Then, $\ln(1 + e_t)$ is a strictly stationary process with $E(e_t) = 0$ and $E(e_t e_{t-j}) = 0$ for $\forall j \neq 0$.

Proof. First, let us prove the proposition $E(e_t)$. Applying unconditional expectation operator $E(\cdot)$ on both sides of $e_t = \xi_t - E_{t-1}(\xi_t)$,

$$\begin{aligned} E(e_t) &= E(\xi_t) - E[E_{t-1}(\xi_t)] \\ &= E(\xi_t) - E(\xi_t) = 0 \end{aligned}$$

Thus, $E(e_t) = 0$.

Next, let us prove the proposition $E(e_t e_{t-j}) = 0$ for $\forall j \neq 0$. Without loss of generality, consider the case where $j \geq 1$. Since $E_{t-1}(e_t) = 0$ and e_{t-j} is inside the information set available at period $t - 1$ for any $j \geq 1$, this will imply $E_{t-1}(e_t e_{t-j}) = 0$

for any $j \geq 1$. Applying unconditional expectation operator $E(\cdot)$ on both sides of $E_{t-1}(e_t e_{t-j}) = 0$,

$$E[E_{t-1}(e_t e_{t-j})] = 0$$

$$\Rightarrow E(e_t e_{t-j}) = 0 \text{ for } \forall j \geq 1$$

Finally, let us prove the strict stationarity of $\ln(1 + e_t)$. Since the logarithmic function is a continuous and monotone function, it suffices to show the strict stationarity of e_t . Recalling the definition of A_t and B_{t+1} from Appendix 1, we can observe the following algebraic relationship.

$$\xi_t = \frac{B_t}{A_{t-1}}$$

By the strict stationarity assumption of C_{t+1}/C_t , $(1 + r_t)$ and P_{t+1}^C/P_t^C , B_t is strictly stationary. Also, since $A_{t-1} = 1$ from Appendix 1, this implies the strict stationarity of ξ_t . Now, since $e_t = \xi_t - E_{t-1}(\xi_t)$ and by the strict stationarity of ξ_t , e_t will be strictly stationary. ■

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