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Abstract

In the dynamic factor model, a single unobserved factor common to some macroeconomic variables is defined as a composite index to measure business cycles. This model has recently been developed by combining with the regime-switching model so that the mean growth of the index may shift depending on whether the economy is in the boom regime or in the recession regime. An advantage of this dynamic Markov switching factor model is that estimating the model by a Bayesian method produces the posterior probabilities that the economy is in the recession regime, which can be used to date the business cycle turning points. This article estimates the dynamic Markov switching factor model using some macroeconomic variables in Japan. The model comparison using Bayes factor does not provide strong evidence that the mean growth of the index shifts, but the dynamic Markov switching factor model is found to produce the estimates of turning points close to the reference dates by the Economic and Social Research Institute in Cabinet Office unless only weakly correlated variables are used.

Key words: Business cycles, Factor model, Gibbs sampling, Marginal Likelihood, Markov switching, Particle filter.

JEL classification: C11, C32, E32

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1 Introduction

How should we measure business cycles? This problem has long attracted the attention of many macroeconomists and econometricians, and several methods have been proposed. A well-known method is the one based on dynamic factor models proposed by Stock and Watson (1989,1991). They define the composite index of coincident economic indicators to measure the state of overall economic activity as a single unobserved factor common to several macroeconomic variables using a dynamic factor model. Because their model can be estimated by the maximum likelihood method via the Kalman filter, their composite index can be estimated by running the Kalman filter or smoother given the maximum likelihood estimates of the parameters.

Kim and Nelson (1998) extend the dynamic factor model of Stock and Watson (1989,1991) so that the mean growth rate of the composite index may vary depending on whether the economy is in the recession regime or in the boom regime. They specify the mean growth rate of the index using the regime-switching model of Hamilton (1989). One advantage of their model is that it produces not only the composite index but also the probabilities that the economy is in the recession regime, which can be utilized to date the business cycle turning points. It is, however, difficult to evaluate the likelihood in their model, so that they apply a Bayesian method via the Gibbs sampler. Specifically, the model parameters, the latent factor, and the regime are sampled from their posterior distribution using the Gibbs sampler, and simulated draws are used for Bayesian posterior analysis.

This article applies the Kim and Nelson (1998) model to macroeconomic data in Japan. While several researchers such as Ohkusa (1992), Mori, Satake, and Ohkusa (1993), Kanoh and Saito (1994), and Fukuda and Onodera (2001) have already applied the Stock and Watson (1989,1991) model to the analysis of business cycles in Japan, there are few who have applied the Kim and Nelson (1998) model. The only exception is Kaufman (2000), who applies the Kim and Nelson (1998) model to eight countries including Japan.[1] While she uses the

[1] Kasuya and Shinki (2001) have applied the Kim and Nelson (1998) model to forecasting the turning points of consumer price index in Japan.

quarterly data for real GDP, consumption, and investment, we use the monthly data selected from ten macroeconomic variables (see Table 1(A)) used by the Economic Planning Agency (EPA), which was reorganized as Economic and Social Research Institute (ESRI) in Cabinet Office after January 2001, to construct its composite index.

Following Kim and Nelson (1998), we estimate the composite index and the probabilities that the economy is in the recession as well as the model parameters using a Bayesian method via the Gibbs sampler. We also analyze whether the regime-shift occurs in the mean growth rate of the composite index by comparing the Kim and Nelson (1998) model with the Stock and Watson (1989,1991) model. Classical test statistics such as the likelihood ratio statistics are not directly applicable to this analysis (see Hansen (1992) and Garcia (1998)). In a Bayesian framework, model comparisons are conducted based on the posterior odds, that is the ratio of the marginal likelihood, which does not cause any problem in analyzing whether the regime-shit occurs or not. We adopt this method and calculate the marginal likelihood following the method proposed by Chib (1995). A diagnostic checking is also conducted.

The model comparison using Bayes factor does not provide strong evidence that the Kim and Nelson (1998) model is favored over the Stock and Watson (1989,1991) model. In addition, no major differences between the composite indices produced by the two models are found. On the other hand, the Kim and Nelson (1998) model produces the estimates of turning points close to the reference dates by the Economic and Social Research Institute in Cabinet Office unless only weakly correlated variables are used.

The rest of this article is organized as follows. Section 2 explains the Kim and Nelson (1998) model and a Bayesian method for analyzing this model. Section 3 fits the model to macroeconomic data in Japan and summarizes the results. Conclusions are given in Section 4.

2 Econometric Methodology

2.1 Dynamic Factor Models

Since our analysis is based on the dynamic factor models proposed by Stock and Watson (1988,1991) and developed by Kim and Nelson (1998), we start with a brief review of these models.

Suppose that we have data on n macroeconomic variables from period 0 to T. Let ΔY_{it} ($i = 1, \ldots, n$; $t = 1, \ldots, T$) denote the growth rate of the ith macroeconomic variable defined as the first difference of the log of the ith variable at time t. In the simplest version of the dynamic factor models, ΔY_{it} is specified as follows.

$$\Delta Y_{it} = D_i + \lambda_i \Delta C_t + e_{it},$$

where D_i and λ_i are constants, ΔC_t is the component common to all variables, which is interpreted as the first difference of the composite index of coincident economic indicators C_t, and e_{it} is the idiosyncratic component of the ith variable.
ΔY_{it} may depend on not only the current value of ΔC_t but also the past values. To allow for this possibility, we use the following specification.

$$\Delta Y_{it} = D_i + \lambda_{i0} \Delta C_t + \lambda_{i1} \Delta C_{t-1} + \cdots \lambda_{ir_i} \Delta C_{t-r_i} + e_{it}. \tag{1}$$

The idiosyncratic component e_{it} is assumed to follow an autoregressive (AR) processes with mean zero, i.e.,

$$e_{it} = \psi_1 e_{i,t-1} + \cdots + \psi_{q_i} e_{i,t-q_i} + \epsilon_{it}, \quad \epsilon_{it} \sim i.i.d.N(0, \sigma_i^2), \tag{2}$$

where error term ϵ_{it} is assumed to follow a serially independent normal distribution.

The difference between the Stock and Watson (1989, 1991) and the Kim and Nelson (1998) models is the specification of the common factor ΔC_t. While Stock and Watson (1989, 1991) specify ΔC_t as a simple AR process, Kim and Nelson (1989) extend it so that the mean growth of the composite index may shift depending on whether the economy is in a recession or in a boom as follows.

$$\Delta C_t = \delta + \mu_{s_t} + \phi_1(\Delta C_{t-1} - \delta - \mu_{s_{t-1}}) + \cdots + \phi_p(\Delta C_{t-p} - \delta - \mu_{s_{t-p}}) + \nu_t, \quad \nu_t \sim i.i.d.N(0,1), \quad (3)$$

where δ is the long-run growth of the composite index, which is constant, and μ_{s_t} is the deviation from that long-run growth, which may shift depending on whether the economy is in a recession or in a boom. Error term ν_t is assumed to follow a serially independent normal distribution. The variance of ν_t is normalized to unity for identification of the model. Error terms ν_t and ϵ_{is} are assumed to be mutually independent for all i, t, s.

Using the variable S_t that takes zero when the economy is in the recession regime and one when the economy is in the boom regime, Kim and Nelson (1998) specify μ_{s_t} as follows.

$$\mu_{s_t} = \mu_0 + \mu_1 S_t, \quad \mu_0 < 0, \ \mu_1 > 0. \quad (4)$$

The reason to assume that $\mu_1 > 0$ is that the mean growth of the composite index will be greater in a boom regime ($S_t = 1$) than that in a recession regime ($S_t = 0$). Although Kim and Nelson (1998) do not assume that $\mu_0 < 0$, we assume it because, otherwise, the average of μ_{s_t}, which is the deviation from the long-run growth, would be positive. They assume that S_t follows a Markov process with transition probabilities

$$\begin{aligned} &P(S_t = 1|S_{t-1} = 1) = \pi_{11}, \qquad P(S_t = 0|S_{t-1} = 1) = 1 - \pi_{11}, \\ &P(S_t = 0|S_{t-1} = 0) = \pi_{00}, \qquad P(S_t = 1|S_{t-1} = 0) = 1 - \pi_{00}. \end{aligned} \quad (5)$$

Equations (1)–(5) constitute the Kim and Nelson (1998) model, which collapses to the Stock and Watson (1989, 1991) model if $\mu_{s_t} = 0$ in equation (3). If ΔC_t is an observed macroeconomic variable instead of the growth of the composite index, equations (3)–(5) constitutes the regime switching model proposed by Hamilton (1989). Therefore, the Kim and Nelson (1998) model is a synthesis of the Stock and Watson (1989, 1991) and the Hamilton (1989) models. The Stock and Watson (1989, 1991) model produces the estimates of the composite index C_t but does not produce the probabilities of a recession ($S_t = 0$). The regime switching model of Hamilton (1989) produces the probability of a recession, which can be used to date the business cycle turning points, but does not produce the estimates of the composite index. An advantage of the Kim and Nelson (1998) model is that it produces the both estimates of the composite index and probabilities of a recession.

D_i in equation (1) and δ in equation (3) are usually removed because they are not identified. To do so, define

$$\begin{aligned}\Delta c_t &= \Delta C_t - E(\Delta C_t) = \Delta C_t - \delta,\\ \Delta y_{it} &= \Delta Y_{it} - E(\Delta Y_{it}) = \Delta Y_{it} - D_i - (\lambda_{i0} + \cdots \lambda_{ir_i})\delta.\end{aligned}$$

Then, equations (1) and (3) can be written as

$$\Delta y_{it} = \lambda_{i0}\Delta c_t + \lambda_{i1}\Delta c_{t-1} + \cdots \lambda_{ir_i}\Delta c_{t-r_i} + e_{it}, \tag{1'}$$

$$\Delta c_t = \mu_{s_t} + \phi_1(\Delta c_{t-1} - \mu_{s_{t-1}}) + \cdots + \phi_p(\Delta c_{t-p} - \mu_{s_{t-p}}) + \nu_t, \quad \nu_t \sim i.i.d.N(0,1). \tag{3'}$$

The demeaned growth rate $\Delta Y_{it} - \overline{\Delta Y_i}$, where $\overline{\Delta Y_i}$ is the sample average of $\Delta Y_{i1},\ldots,\Delta Y_{iT}$, is used for Δy_{it}. In what follows, we consider the model that consists of equations (1'), (2), (3'), (4), and (5) as the Kim and Nelson (1998) model.

2.2 Bayesian Estimation via the Gibbs Sampler

The Stock and Watson (1989, 1991) model can be represented by a linear Gaussian state space model. The likelihood of the linear Gaussian state space model can be evaluated by executing the Kalman filter. The likelihood of the Hamilton (1989) model can also be evaluated by executing the filter proposed by Hamilton (1989). Hence, the parameters in these models can be estimated using the conventional maximum likelihood method. The likelihood of the Kim and Nelson (1998) model cannot, however, be evaluated analytically, so that the parameters cannot be estimated using the maximum likelihood method. The estimation of the Kim and Nelson (1998) model requires other estimation methods. Kim and Nelson (1998) apply a Bayesian method via the Gibbs sampler.

Let $\boldsymbol{\theta}$ denote the set of the unknown parameters. The conventional Bayesian method proceeds as follows.

1. Set the prior distribution $f(\boldsymbol{\theta})$, which is the distribution the researcher have in mind before observing the data.

2. Convert the prior distribution to the posterior distribution $f(\boldsymbol{\theta}|\text{data})$, which is the distribution conditional on the data, using the Bayes theorem

$$f(\boldsymbol{\theta}|\text{data}) = \frac{f(\text{data}|\boldsymbol{\theta})f(\boldsymbol{\theta})}{\displaystyle\int f(\text{data}|\boldsymbol{\theta})f(\boldsymbol{\theta})d\boldsymbol{\theta}}. \tag{6}$$

3. Estimate the parameters $\boldsymbol{\theta}$ using the posterior distribution.

Notice that $f(\text{data}|\boldsymbol{\theta})$ in the right-hand-side of the Bayes theorem (6) is the likelihood. Therefore, the conventional Bayesian method cannot be applied to the models such as the Kim and Nelson (1998) model whose likelihood cannot be obtained analytically. In such models, the above 2 and 3 are replaced by

2' Sample $\boldsymbol{\theta}$ from the posterior distribution $f(\boldsymbol{\theta}|\text{data})$.

3' Estimate the parameters $\boldsymbol{\theta}$ using the draws sampled in 2'.

Some readers may think it impossible to sample from the posterior distribution that cannot be obtained analytically. It is the Gibbs sampler that makes it possible.

The Gibbs sampler is a Monte Carlo method for sampling from a joint distribution using conditional distributions. Suppose that it is impossible to obtain the joint posterior distribution $f(\boldsymbol{\theta}|\text{data})$ analytically using the Bayes theorem, but $\boldsymbol{\theta}$ can be divided into k partitions $(\theta_1, \ldots, \theta_k)$, where θ_i may be a scalar or a vector, such that, for all $i = 1, 2, \cdots, k$, it is possible to obtain conditional distribution $f(\theta_i|\{\theta_j\}_{j\neq i}, \text{data})$ analytically and sample θ_i from it by some methods. The Gibbs sampler is used in such cases and works as follows. Starting from an arbitrary set of initial value $(\theta_2^{(0)}, \ldots, \theta_k^{(0)})$, we draw $\theta_1^{(1)}$ from $f(\theta_1|\theta_2^{(0)}, \theta_3^{(0)}, \ldots, \theta_k^{(0)}, \text{data})$, $\theta_2^{(1)}$ from $f(\theta_2|\theta_1^{(1)}, \theta_3^{(0)}, \ldots, \theta_k^{(0)}, \text{data})$, and so on up to $\theta_k^{(1)}$ from $f(\theta_k|\theta_1^{(1)}, \theta_2^{(1)}, \ldots, \theta_{k-1}^{(1)}, \text{data})$. Let us call this procedure one iteration. After l such iterations, we obtain $(\theta_1^{(l)}, \theta_2^{(l)}, \ldots, \theta_k^{(l)})$. Under mild conditions, it converges in distribution to be a set of random variables from $f(\theta_1, \theta_2, \ldots, \theta_k|\text{data})$ as $l \to \infty$. Therefore, for a sufficiently large M, $(\theta_1^{(l)}, \theta_2^{(l)}, \ldots, \theta_k^{(l)})$ $(l = M+1, M+2, \cdots, M+N)$ can approximately be regarded as a sample from the joint posterior distribution $f(\theta_1, \theta_2, \ldots, \theta_k|\text{data})$. Hence, the first M draws, which is called "burn-in," are discarded and the last N draws are used for parameter estimation. For instance, the expectation of a function of the parameters, $g(\theta_1, \theta_2, \ldots, \theta_k)$, is estimated by the sample average

$$E[g(\theta_1, \theta_2, \ldots, \theta_k)] \approx \frac{1}{N} \sum_{l=M+1}^{M+N} g(\theta_1^{(l)}, \theta_2^{(l)}, \ldots, \theta_k^{(l)}). \tag{7}$$

The unknown parameters in the Kim and Nelson (1998) model that consists of equations (1′),(2),(3′),(4), and (5) are: $\boldsymbol{\lambda}_i = [\lambda_{i0}, \ldots, \lambda_{ir_i}]$ $(i = 1, \ldots, n)$, $\boldsymbol{\psi}_i = [\psi_{i1}, \ldots, \psi_{iq_i}]$ $(i = 1, \ldots, n)$, σ_i^2 $(i = 1, \ldots, n)$, $\boldsymbol{\phi} = [\phi_1, \ldots, \phi_p]$, $\boldsymbol{\mu} = (\mu_0, \mu_1)'$, $\boldsymbol{\pi} = (\pi_{00}, \pi_{11})'$. As well as these parameters, latent variables $\Delta\boldsymbol{c}_T = [\Delta c_1, \ldots, \Delta c_T]$ and $\boldsymbol{S}_T = [S_1, \ldots, S_T]$ are also treated as if they were unknown parameters. Then, all we have to do to sample from the joint posterior distribution using the Gibbs sampler is to sample from the following conditional distributions.

$$f(\boldsymbol{\lambda}_i|\boldsymbol{\theta}_{/\boldsymbol{\lambda}_i}, \Delta\boldsymbol{c}_T, \boldsymbol{S}_T, \Delta\boldsymbol{y}_T), \quad i = 1, \ldots, n \tag{8}$$

$$f(\boldsymbol{\psi}_i|\boldsymbol{\theta}_{/\boldsymbol{\psi}_i}, \Delta\boldsymbol{c}_T, \boldsymbol{S}_T, \Delta\boldsymbol{y}_T), \quad i = 1, \ldots, n \tag{9}$$

$$f(\sigma_i^2|\boldsymbol{\theta}_{/\sigma_i^2}, \Delta\boldsymbol{c}_T, \boldsymbol{S}_T, \Delta\boldsymbol{y}_T), \quad i = 1, \ldots, n \tag{10}$$

$$f(\boldsymbol{\phi}|\boldsymbol{\theta}_{/\boldsymbol{\phi}}, \Delta\boldsymbol{c}_T, \boldsymbol{S}_T, \Delta\boldsymbol{y}_T) \tag{11}$$

$$f(\boldsymbol{\mu}|\boldsymbol{\theta}_{/\boldsymbol{\mu}}, \Delta\boldsymbol{c}_T, \boldsymbol{S}_T, \Delta\boldsymbol{y}_T) \tag{12}$$

$$f(\boldsymbol{\pi}|\boldsymbol{\theta}_{/\boldsymbol{\pi}}, \Delta\boldsymbol{c}_T, \boldsymbol{S}_T, \Delta\boldsymbol{y}_T) \tag{13}$$

$$f(\boldsymbol{S}_T|\boldsymbol{\theta}, \Delta\boldsymbol{c}_T, \Delta\boldsymbol{y}_T) \tag{14}$$

$$f(\Delta\boldsymbol{c}_T|\boldsymbol{\theta}, \boldsymbol{S}_T, \Delta\boldsymbol{y}_T) \tag{15}$$

where $\boldsymbol{\theta}_{/\omega}$ represents the set of all parameters except ω, which does not include latent variables, $\Delta\boldsymbol{y}_{iT} = [\Delta y_{i1}, \ldots, \Delta y_{iT}]$, and $\Delta\boldsymbol{y}_T = [\Delta\boldsymbol{y}_{1T}, \ldots, \Delta\boldsymbol{y}_{nT}]$.

As for the prior distributions of the unknown parameters, we may use any distributions but it is convenient to assume the following distributions.

$$\boldsymbol{\lambda}_1' \sim N\left(M_{\lambda_1}^{(0)}, \Sigma_{\lambda_1}^{(0)}\right) I[\lambda_{10} > 0] \tag{16}$$

$$\boldsymbol{\lambda}_i' \sim N\left(M_{\lambda_i}^{(0)}, \Sigma_{\lambda_i}^{(0)}\right), \quad i = 2, \ldots, n \tag{17}$$

$$\boldsymbol{\psi}_i' \sim N\left(M_{\psi_i}^{(0)}, \Sigma_{\psi_i}^{(0)}\right) I_{S(\boldsymbol{\psi}_i)}, \quad i = 1, \ldots, n \tag{18}$$

$$\boldsymbol{\phi}' \sim N\left(M_{\phi}^{(0)}, \Sigma_{\phi}^{(0)}\right) I_{S(\boldsymbol{\phi})} \tag{19}$$

$$\boldsymbol{\mu}' \sim N\left(M_{\mu}^{(0)}, \Sigma_{\mu}^{(0)}\right) I[\mu_0 < 0, \mu_1 > 0] \tag{20}$$

$$\sigma_i^2 \sim IG\left(\nu^{(0)}/2, \delta^{(0)}/2\right), \quad i = 1, \ldots, n \tag{21}$$

$$\pi_{11} \sim beta(u_{11}^{(0)}, u_{10}^{(0)}), \quad \pi_{00} \sim beta(u_{00}^{(0)}, u_{01}^{(0)}) \tag{22}$$

where $I[\cdot]$ is the indicator function that takes one if the condition in the bracket is satisfied and zero otherwise, and $I_{S(\boldsymbol{\psi}_i)}$ (or $I_{S(\boldsymbol{\phi})}$) is the indicator function that takes one if the roots of the polynomial $1 - \psi_{i1}L - \cdots - \psi_{iq_i}L^{q_i} = 0$ (or $1 - \phi_1 L - \cdots \phi_p L^p = 0$) lie outside the unit circle and zero otherwise. At least, one parameter among λ_{ij} ($i = 1, \ldots, n$; $j = 1, \ldots, n$) is assumed to be positive for identification of the model. Here, we assume that $\lambda_{10} > 0$. Hence, the prior of $\boldsymbol{\lambda}_1'$ is set to be the truncated normal whose density is zero unless $\lambda_{10} > 0$, and that of $\boldsymbol{\lambda}_i'$ ($i = 2, \ldots, n$) is set to be the normal. Under the assumption that equations (2) and (3) are stationary, the priors of $\boldsymbol{\psi}_i'$, $\boldsymbol{\phi}'$ are set to be the truncated normal whose density is zero outside the stationary region. The prior of $\boldsymbol{\mu}'$ is the truncated normal whose density is zero unless $\mu_0 < 0$ and $\mu_1 > 0$. The prior of σ_i^2 is set to be the inverted gamma, which means that $1/\sigma_i^2$ follows the gamma distribution. The priors of π_{11} and π_{00} are set to be beta distributions.

Under these priors, it is straightforward to obtain the conditional distributions (8)–(13), which have the same forms as the priors (16)–(22), and sample from those distributions (see Appendix A).

The condition of (14) includes $\Delta \boldsymbol{c}_T$. Given $\Delta \boldsymbol{c}_T$, equations (3'), (4), and (5) constitute the regime switching model proposed by Hamilton (1989). Thus, sampling $\boldsymbol{S}_T$ from (14) can be conducted using the Hamilton (1989) filter. Applying the Hamilton (1989) filter to the model that consists of equations (3'), (4), and (5) produces $p(S_t|\Delta\boldsymbol{c}_t)$ and $p(S_t|\Delta\boldsymbol{c}_{t-1})$ for $t = 1, \ldots, T$. Then, starting with S_T sampled from $p(S_T|\Delta\boldsymbol{c}_T)$, we can proceed backwards in time. Specifically, given S_{t+1}, S_t is generated using the probability

$$p(S_t|\Delta\boldsymbol{c}_t, S_{t+1}) = \frac{p(S_{t+1}|S_t)p(S_t|\Delta\boldsymbol{c}_t)}{p(S_{t+1}|\Delta\boldsymbol{c}_t)}$$

where $p(S_{t+1}|S_t)$ is the transition probability given by (5), and $p(S_t|\Delta\boldsymbol{c}_t)$ and $p(S_{t+1}|\Delta\boldsymbol{c}_t)$ are obtained from the Hamilton (1989) filter.

The condition of (15) includes $\boldsymbol{S}_T$. Once $\boldsymbol{S}_T$ are given, the Kim and Nelson (1998) model can be represented by a linear Gaussian state space model. Suppose that $n = 5$, $p = 3$, and $r_i = q_i = 1$ $(i = 1, \ldots, n)$. Let $\Delta y^*_{it} = \Delta y_{it} - \psi_{i1}\Delta y_{i,t-1}$ $(i = 1, \ldots, n)$ and $\Delta y^*_t = [\Delta y^*_{1t}, \cdots, \Delta y^*_{nt}]'$. Then, the Kim and Nelson (1998) model may be represented as

$$\Delta y_t = \Lambda z_t + \epsilon_t, \quad \epsilon_t \sim i.i.d.N(0, \Sigma_\epsilon), \tag{23}$$

$$z_t = M_{s_t} + \Phi z_{t-1} + u_t, \quad v_t \sim i.i.d.N(0, \Sigma_v). \tag{24}$$

Then, z_t, Λ, M_{s_t}, Φ, Σ_ϵ, and Σ_v are given by

$$z_t = [\Delta c_t, \Delta c_{t-1}, \Delta c_{t-2}]'$$

$$\Lambda = \begin{bmatrix} \lambda_{10} & -\lambda_{10}\psi_{11} + \lambda_{11} & -\lambda_{11}\psi_{11} \\ \lambda_{20} & -\lambda_{20}\psi_{21} + \lambda_{21} & -\lambda_{21}\psi_{21} \\ \lambda_{30} & -\lambda_{30}\psi_{31} + \lambda_{31} & -\lambda_{31}\psi_{31} \\ \lambda_{40} & -\lambda_{40}\psi_{41} + \lambda_{41} & -\lambda_{41}\psi_{41} \\ \lambda_{50} & -\lambda_{50}\psi_{51} + \lambda_{51} & -\lambda_{51}\psi_{51} \end{bmatrix}$$

$$\Phi = \begin{bmatrix} \phi_1 & \phi_2 & \phi_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$M_{s_t} = [\phi(L)\mu_{S_t}, 0, 0]'$$

$$v_t = [\nu_t, 0, 0]'$$

$$\Sigma_\epsilon = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5^2 \end{bmatrix}$$

$$\Sigma_v = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where $\phi(L)\mu_{s_t} = \mu_{S_t} - \phi_1\mu_{S_{t-1}} - \phi_2\mu_{S_{t-2}} - \phi_3\mu_{S_{t-3}}$.

Since equations (23) and (24) constitute the linear Gaussian state space model, it is straightforward to sample $\Delta\boldsymbol{c}_T$ from (15) using the Kalman filter and smoother. Once $\Delta\boldsymbol{c}_T$ are obtained, they can be transformed into the composite index $\boldsymbol{C}_T = [C_1, \ldots, C_T]$ as

$$C_t = \Delta c_t + C_{t-1} + \delta$$

where δ is the long-run growth of the composite index, which can be estimated using the steady-state Kalman gain (see Kim and Nelson (1988,1999)).

2.3 Model Comparison

2.3.1 Marginal Likelihood

It is important to examine whether the mean growth of the composite index shifts depending on whether the economy is in the boom regime or the recession regime by comparing the Kim and Nelson (1998) model with the Stock and Watson (1989,1991) model. Kaufman (2000) proposes a method for comparing these two models. As is a usual practice in Bayesian model comparison, his method is based on the posterior odds ratio. Posterior odds ratio between model i, M_i, and model j, M_j, is given by

$$\begin{aligned} \text{POR} &= \frac{f(M_i|\Delta\boldsymbol{y}_T)}{f(M_j|\Delta\boldsymbol{y}_T)} \\ &= \frac{f(\Delta\boldsymbol{y}_T|M_i)}{f(\Delta\boldsymbol{y}_T|M_j)}\frac{f(M_i)}{f(M_j)} \end{aligned}$$

where $\frac{f(\Delta\boldsymbol{y}_T|M_i)}{f(\Delta\boldsymbol{y}_T|M_j)}$ and $\frac{f(M_i)}{f(M_j)}$ are called Bayes factor and prior odds ratio respectively. If POR is greater than one, M_i is favored over M_j.

The prior odds ratio is usually set to be one, so that the posterior odds ratio is equal to the Bayes factor. To evaluate the Bayes factor, we must calculate $f(\Delta \boldsymbol{y}_T|M_i)$ and $f(\Delta \boldsymbol{y}_T|M_j)$ called marginal likelihoods. The log of the marginal likelihood of model M_i can be written as

$$\ln f(\Delta \boldsymbol{y}_T|M_i) = \ln f(\Delta \boldsymbol{y}_T|M_i, \boldsymbol{\theta}_i) + \ln f(\boldsymbol{\theta}_i|M_i) - \ln f(\boldsymbol{\theta}_i|M_i, \Delta \boldsymbol{y}_T), \tag{25}$$

where $\boldsymbol{\theta}_i$ is the set of unknown parameters for model M_i, $f(\Delta \boldsymbol{y}_T|M_i, \boldsymbol{\theta}_i)$ is the likelihood, $f(\boldsymbol{\theta}_i|M_i)$ is the prior density, and $f(\boldsymbol{\theta}_i|M_i, \Delta \boldsymbol{y}_T)$ is the posterior density. The above identity holds for any value of $\boldsymbol{\theta}_i$, but Chib (1995) proposes to set $\boldsymbol{\theta}_i$ equal to its posterior mean $\hat{\boldsymbol{\theta}}_i$ calculated using the Gibbs draws. In what follows, subscript i and M_i are omitted.

The Kim and Nelson (1998) model is more general than the Stock and Watson (1989, 1991) model in the sense that setting $mu_{s_t} = 0$ in the Kim and Nelson (1998) model leads to the Stock and Watson (1989, 1991) model. Hence, the likelihood of the Kim and Nelson (1998) model cannot be smaller than that of the Stock and Watson (1989, 1991) model. Notice, however, that the marginal likelihood of the Kim and Nelson (1998) model may be smaller than that of the Stock and Watson (1989, 1991) model.

2.3.2 Prior Density

If prior distributions are given by (16)–(22) where $\boldsymbol{\lambda}_i$ $(i = 1, \ldots, n)$, $\boldsymbol{\psi}_i$ $(i = 1, \ldots, n)$, $\boldsymbol{\sigma}_i^2$ $(i = 1, \ldots, n)$, $\boldsymbol{\phi}$, $\boldsymbol{\mu}$, $\boldsymbol{\pi}$ are mutually independent, we have

$$f(\hat{\boldsymbol{\theta}}) = f(\hat{\boldsymbol{\lambda}}_1)\times\cdots\times f(\hat{\boldsymbol{\lambda}}_n)\times f(\hat{\boldsymbol{\psi}}_1)\times\cdots\times f(\hat{\boldsymbol{\psi}}_n)\times f(\hat{\boldsymbol{\sigma}}_1^2)\times\cdots\times f(\hat{\boldsymbol{\sigma}}_n^2)\times f(\hat{\boldsymbol{\phi}})\times f(\hat{\boldsymbol{\mu}})\times f(\hat{\boldsymbol{\pi}}). \tag{26}$$

It is straightforward to evaluate $f(\hat{\boldsymbol{\lambda}}_i)$ $(i = 2, \ldots, n)$, $f(\hat{\boldsymbol{\mu}})$, and $f(\hat{\boldsymbol{\pi}})$. It may possibly be difficult to evaluate the other terms in (26) analytically because truncation may make it difficult to calculate the normalizing constant. Even in such cases, there are some numerical methods available to evaluate the normalizing constant (see Chen et. al.(2000)).

2.3.3 Posterior Density

Kaufman (2000) uses the method proposed by Chib (1995) to evaluate the posterior density. The posterior density is written as

$$f(\hat{\boldsymbol{\theta}}|\Delta \boldsymbol{y}_T) = f(\hat{\boldsymbol{\pi}}|\Delta \boldsymbol{y}_T) \times f(\hat{\boldsymbol{\lambda}}_1, \ldots, \hat{\boldsymbol{\lambda}}_n|\hat{\boldsymbol{\pi}}, \Delta \boldsymbol{y}_T) \times \cdots \times f(\hat{\boldsymbol{\mu}}|\hat{\boldsymbol{\theta}}_{/\boldsymbol{\mu}}, \Delta \boldsymbol{y}_T), \tag{27}$$

and evaluates each term separately using the Gibbs sampler.

The first term can be written as

$$f(\hat{\boldsymbol{\pi}}|\Delta \boldsymbol{y}_T) = \int f(\hat{\boldsymbol{\pi}}|\boldsymbol{S}_T, \Delta \boldsymbol{y}_T) f(\boldsymbol{S}_T|\Delta \boldsymbol{y}_T) d\boldsymbol{S}_T. \tag{28}$$

The Gibbs sampler explained above produces draws from $f(\boldsymbol{S}_T|\Delta \boldsymbol{y}_T)$. All we have to do is to sample from the conditional distributions (8)–(15) sequentially. Given M draws $(\boldsymbol{S}_T^{(1)}, \ldots, \boldsymbol{S}_T^{(M)})$ from $f(\boldsymbol{S}_T|\Delta \boldsymbol{y}_T)$, equation (28) can be estimated by

$$f(\hat{\boldsymbol{\pi}}|\Delta \boldsymbol{y}_T) \approx \frac{1}{M} \sum_{m=1}^{M} f(\hat{\boldsymbol{\pi}}|\boldsymbol{S}_T^{(m)}, \Delta \boldsymbol{y}_T). \tag{29}$$

This is not true for the other terms because some parameters included in the conditions are fixed at their posterior means. For example, the second term can be written as

$$\begin{aligned} & f(\hat{\boldsymbol{\lambda}}_1, \ldots, \hat{\boldsymbol{\lambda}}_n|\hat{\boldsymbol{\pi}}, \Delta \boldsymbol{y}_T) \\ = & \int f(\hat{\boldsymbol{\lambda}}_1, \ldots, \hat{\boldsymbol{\lambda}}_n|\hat{\boldsymbol{\pi}}, \sigma_1^2, \ldots, \sigma_n^2, \Delta \boldsymbol{c}_T, \Delta \boldsymbol{y}_T) \\ & \times f(\sigma_1^2, \ldots, \sigma_n^2, \Delta \boldsymbol{c}_T|\hat{\boldsymbol{\pi}}, \Delta \boldsymbol{y}_T) d\sigma_1^2 \cdots d\sigma_n^2 d\Delta \boldsymbol{c}_T. \end{aligned} \tag{30}$$

To sample from $f(\sigma_1^2, \ldots, \sigma_n^2, \Delta \boldsymbol{c}_T|\hat{\boldsymbol{\pi}}, \Delta \boldsymbol{y}_T)$ using the Gibbs sampler, we must sample from the following conditional distributions.

$$f(\boldsymbol{\lambda}_i|\hat{\boldsymbol{\pi}}, \boldsymbol{\theta}_{/(\boldsymbol{\lambda}_i, \boldsymbol{\pi})}, \Delta \boldsymbol{c}_T, \boldsymbol{S}_T, \Delta \boldsymbol{y}_T), \quad i = 1, \ldots, n$$

$$f(\boldsymbol{\psi}_i|\hat{\boldsymbol{\pi}}, \boldsymbol{\theta}_{/(\boldsymbol{\psi}_i, \boldsymbol{\pi})}, \Delta \boldsymbol{c}_T, \boldsymbol{S}_T, \Delta \boldsymbol{y}_T), \quad i = 1, \ldots, n$$

$$f(\sigma_i^2|\hat{\boldsymbol{\pi}}, \boldsymbol{\theta}_{/\sigma_i^2, \boldsymbol{\pi})}, \Delta \boldsymbol{c}_T, \boldsymbol{S}_T, \Delta \boldsymbol{y}_T), \quad i = 1, \ldots, n$$

$$f(\boldsymbol{\phi}|\hat{\boldsymbol{\pi}}, \boldsymbol{\theta}_{/(\boldsymbol{\phi}, \boldsymbol{\pi})}, \Delta \boldsymbol{c}_T, \boldsymbol{S}_T, \Delta \boldsymbol{y}_T)$$

$$f(\boldsymbol{\mu}|\hat{\boldsymbol{\pi}}, \boldsymbol{\theta}_{/(\boldsymbol{\mu}, \boldsymbol{\pi})}, \Delta \boldsymbol{c}_T, \boldsymbol{S}_T, \Delta \boldsymbol{y}_T)$$

$$f(\boldsymbol{S}_T|\hat{\boldsymbol{\pi}}, \boldsymbol{\theta}_{/\boldsymbol{\pi}}, \Delta\tilde{c}_T, \Delta\tilde{y}_T)$$

$$f(\Delta\boldsymbol{c}_T|\hat{\boldsymbol{\pi}}, \boldsymbol{\theta}_{/\boldsymbol{\pi}}, \boldsymbol{S}_T, \Delta\boldsymbol{y}_T).$$

Given M draws from $f(\sigma_1^2, \ldots, \sigma_n^2, \Delta\boldsymbol{c}_T|\hat{\boldsymbol{\pi}}, \Delta\boldsymbol{y}_T)$, equation (30) can be estimated by

$$\begin{aligned} & f(\hat{\boldsymbol{\lambda}}_1, \ldots, \hat{\boldsymbol{\lambda}}_n|\hat{\boldsymbol{\pi}}, \Delta\boldsymbol{y}_T) \\ \approx & \ \frac{1}{M}\sum_{m=1}^{M} f(\hat{\boldsymbol{\lambda}}_1, \ldots, \hat{\boldsymbol{\lambda}}_n|\hat{\boldsymbol{\pi}}, \sigma_1^{2(m)}, \ldots, \sigma_n^{2(m)}, \Delta\boldsymbol{c}_T^{(m)}, \Delta\boldsymbol{y}_T). \end{aligned}$$

The other terms in (27) can be evaluated similarly.

2.3.4 Likelihood

Kaufman (2000) uses a particle filter to evaluate the likelihood. A particle filter is the algorithm to sample from the filtering density $f(z_t, S_t|\Delta\boldsymbol{y}_T, \boldsymbol{\theta})$ sequentially starting from $t = 0$ (see Pitt and Shephard (1999)) where z_t is the state variable that appears in equations (23) and (24).

Suppose that we have M draws $\left\{z_{t-1}^{(m)}, S_{t-1}^{(m)}\right\}$ $(m = 1, \ldots, M)$ sampled from the density $f(z_{t-1}, S_{t-1}|\Delta\boldsymbol{y}_{t-1}, \boldsymbol{\theta})$. Then, we can sample $\left\{z_t^{(m)}, S_t^{(m)}\right\}$ from the density $f(z_t, S_t|\Delta\boldsymbol{y}_t, \boldsymbol{\theta})$ as follows (see Appendix B for details).

Step 1. Select a (S_t, m) from $2 \times M$ combinations $(S_t = 0, 1;\ m = 1, \ldots, M)$ with probability proportional to

$$\eta_{S_t,m} = |\Sigma_\epsilon|^{-1/2} \exp\left(-\frac{1}{2}e_t'\Sigma_\epsilon^{-1}e_t\right) p(S_t|S_{t-1}^{(m)}).$$

Step 2. Using the (S_t, m) selected in Step 1, sample from $N(\mu_{t|t}^{(m)}, \Sigma_{t|t}^{(m)})$, where

$$\begin{aligned} e_t &= \Delta y_t - \Lambda(M_{s_t} + \Phi z_{t-1}^{(m)}), \\ \mu_{t|t}^{(m)} &= M_{s_t} + \Phi z_{t-1}^{(m)} + \Sigma_v\Lambda'\Sigma_\epsilon^{-1}e_t, \\ \Sigma_{t|t}^{(m)} &= \Sigma_v - \Sigma_v\Lambda'\Sigma_\epsilon^{-1}\Lambda\Sigma_v. \end{aligned}$$

The likelihood can be expressed as

$$L = \prod_{t=1}^{T} f(\Delta y_{t+1}|\Delta\boldsymbol{y}_t, \boldsymbol{\theta}),$$

where

$$
\begin{aligned}
& f(\Delta y_{t+1}|\Delta \boldsymbol{y}_t, \boldsymbol{\theta}) \\
& \quad = \int f(\Delta y_{t+1}|z_{t+1}, \boldsymbol{\theta}) f(z_{t+1}|z_t, S_{t+1}, \boldsymbol{\theta}) \\
& \qquad \times p(S_{t+1}|S_t, \boldsymbol{\theta}) f(z_t, S_t|\Delta \boldsymbol{y}_t, \boldsymbol{\theta}) dS_t dz_t dS_{t+1} dz_{t+1}. \qquad (31)
\end{aligned}
$$

Given M draws $\left\{z_t^{(m)}, S_t^{(m)}\right\}$ $(m = 1, \ldots, M)$ from the density $f(z_t, S_t|\Delta \boldsymbol{y}_t, \boldsymbol{\theta})$ using the above particle filter, we can evaluate $f(\Delta y_{t+1}|\Delta \boldsymbol{y}_t, \boldsymbol{\theta})$ as follows.

Step 3. Sample $S_{t+1}^{(m)}$ using the transition probability $p(S_{t+1}^{(m)}|S_t)$.

Step 4. Using $S_{t+1}^{(m)}$ sampled in Step 3, sample $z_{t+1}^{(m)}$ from

$$
z_{t+1}^{(m)}|z_t^{(m)}, S_{t+1}^{(m)} \sim N(M_{s_t} + \Phi z_t^{(m)}, \Sigma_v)
$$

Based on M draws on S_{t+1} and z_{t+1} sampled in Step 3 and 4, $f(\Delta y_{t+1}|\Delta \boldsymbol{y}_t, \boldsymbol{\theta})$ can be estimated by

$$
f(\Delta y_{t+1}|\Delta \boldsymbol{y}_t, \boldsymbol{\theta}) \approx \frac{1}{M} \sum_{m=1}^{M} f(\Delta y_{t+1}|z_{t+1}^{(m)}, S_{t+1}^{(m)}, \boldsymbol{\theta})
$$

2.4 Diagnostics

Draws on S_{t+1} and z_{t+1} sampled in Step 3 and 4 can be used also for a diagnostic test. The probability that $\Delta y_{i,t+1}$ will be less than the observed value $y_{i,t+1}^o$ conditional on $\Delta \boldsymbol{y}_t$ and $\boldsymbol{\theta}$ can be written as

$$
\begin{aligned}
& P(\Delta y_{i,t+1} \leq \Delta y_{i,t+1}^o|\Delta \boldsymbol{y}_t, \boldsymbol{\theta}) \\
& \quad = \int P(\Delta y_{i,t+1} \leq \Delta y_{i,t+1}^o|z_{t+1}, S_{t+1}, \Delta \boldsymbol{y}_t, \boldsymbol{\theta}) f(z_{t+1}, S_{t+1}|\Delta \boldsymbol{y}_t, \boldsymbol{\theta}) dz_{t+1} S_{t+1} \\
& \quad \approx \frac{1}{M} \sum_{m=1}^{M} P(\Delta y_{i,t+1} \leq \Delta y_{i,t+1}^o|z_{t+1}^{(m)}, S_{t+1}^{(m)}, \boldsymbol{\theta})
\end{aligned}
$$

Let $u_{i,t+1}^M = \frac{1}{M} \sum_{m=1}^{M} P(\Delta y_{i,t+1} \leq \Delta y_{i,t+1}^o|z_{t+1}^{(m)}, S_{t+1}^{(m)}, \boldsymbol{\theta})$. Under the null of a correctly specified model, $u_{i,t}^M$ converges in distribution to independently and identically distributed uniform random variables as $M \to \infty$ (Rosenblatt (1952)). This provides a valid basis for diagnostic checking. These variables can be mapped into the normal distribution, by using the inverse of the normal distribution function $n_{i,t}^M = F^{-1}(u_{i,t}^M)$ to give a standard sequence of independent and identically distributed normal variables.

3 Application to Macroeconomic Data in Japan

3.1 Data Description

Economic and Social Research Institute (ESRI) uses eleven macroeconomic variables to construct its Coincident Index (see Table 1(A) for definitions of these eleven variables). Among them, "Business Profit" (ZBOAS) is quarterly data and the other ten variables are monthly data. We obtained the raw data for these ten variables from Jan. 1975 to Dec. 2000 and transformed them into seasonally adjusted ones by the Census-X11 method. The use of all ten variables to estimate the Stock and Watson (1989,1991) and the Kim and Nelson (1998) model is, however, computationally costly. Hence, our analysis is based on the following two datasets, both of which consist of five variables selected by Fukuda and Onodera (2001).

Dataset 1: (1) IIP95P (2) SCI95 (3) ESRAO (4) HWINMF (5) CELL9

Dataset 2: (1) IIP95P (2) SMSALE (3) HWINMF (4) IIP95O (5) IIP95M

The both datasets were selected based on the principle to use variables related not only to production but also to trade sales and labor market. On one hand, dataset 1 includes "Index of Wholesale Sales" (SCI95) as a trade sales variable and "Ratio of Job Offers to Applicants" (ESRAO) and "Index of Non-Scheduled Hours Worked" (HWINMF) as labor market variables. On the other hand, dataset 2 includes "Sales of Small and Medium Size Companies" (SMSALE) as a trade sales variable and HWINMF as a labor market variable. These two datasets, however, differ in the sense that dataset 1 includes variables that are less correlated with each other while all variables except HWINMF in dataset 2 are highly correlated with each other. Table 1 (B) reports the contemporaneous correlation of the growth rate of the ten variables, showing that "Index of Industrial Production" (IIP95P), "Index of Raw Materials Consumption" (IIP95M), "Index of Operating Rate" (IIP95O), and "Sales of Small and Medium Size Companies" (SMSALE) have large positive correlations with each other. Dataset 2 includes all these variables. The correlations between variables in dataset 1 are less than 0.5 except the ones between IIP95P and "Electric Power Consumption of Large Users" (CELL9) and between IIP95P and SCI95. In addition, Table 2 (C) shows the serial correlation of the growth rate of the ten variables, indicating that two labor market variables HWINMF and ESRAO have positive serial correlations and the other variables have negative serial correlations. Dataset 1 includes the both of these two variables while dataset 2 includes only HWINMF.

The sifts of the mean growth of the composite index create a correlation between macroeconomic variables. Hence, we can expect that such shifts may be observed only in dataset 2 in which variables are highly correlated with each other.

3.2 Estimation Details

Following Fukuda and Onodera (2001), we set $p = 3$ and $q_i = 1$ $(i = 1, \ldots, 5)$ for the both datasets. While Fukuda and Onodera (2001) assume $r_i = 0$, we set it equal to one.

For parameter estimation, we conduct the Gibbs sampler with 12,000 iterations for each model. The first 2,000 draws are discarded and then the next 10,000 are recorded. Using these 10,000 draws for each of the parameters, we calculate the posterior means, the standard errors of the posterior means, the 95 percent intervals, and the convergence diagnostic (CD) statistics proposed by Geweke (1992). The posterior means are computed by averaging the simulated draws. The standard errors of the posterior means are computed using a Parzen window with a bandwidth of 1,000. The 95 percent intervals are calculated using the 2.5th and 97.5th percentiles of the simulated draws. The convergence of the Gibbs sampler can be assessed using the method proposed by Geweke (1992). He suggests to compare values early in the sequence with those late in the sequence. Let $\theta^{(i)}$ be the ith draw of a parameter in the recorded 10,000 draws, and let $\bar{\theta}_A = \frac{1}{n_A}\sum_{i=1}^{n_A}\theta^{(i)}$ and $\bar{\theta}_B = \frac{1}{n_B}\sum_{i=10{,}001-n_B}^{10{,}000}\theta^{(i)}$. Using these values, Geweke (1992) proposes the following statistic called *convergence diagnostics* (CD).

$$\mathrm{CD} = \frac{\bar{\theta}_A - \bar{\theta}_B}{\sqrt{\hat{\sigma}_A^2/n_A + \hat{\sigma}_B^2/n_B}}, \tag{32}$$

where $\sqrt{\hat{\sigma}_A^2/n_A}$ and $\sqrt{\hat{\sigma}_B^2/n_B}$ are standard errors of $\bar{\theta}_A$ and $\bar{\theta}_B$. If the sequence of $\theta^{(i)}$ is stationary, it converges in distribution to the standard normal. We set $n_A = 1,000$ and $n_B = 5,000$ and compute $\hat{\sigma}_A^2$ and $\hat{\sigma}_B^2$ using Parzen windows with bandwidths of 100 and 500 respectively.

In calculating the marginal likelihood, we set the number of iterations to evaluate the both posterior densities and the likelihood equal to $2,000$.

3.3 Estimation Results

Table 2 shows the estimation results for dataset 1. Table 2 (A) and (B) are the results for the Kim and Nelson (1998) model and the Stock and Watson (1989,1991) model respectively. According to the CD values, the null hypothesis that the sequence of 10,000 draws is stationary is accepted at the 5 percent significance level for all parameters in the both models. The log marginal likelihood of the Kim and Nelson (1998) model of -2714.16 is smaller than that of the Stock and Watson (1989,1991) model of -2713.31, indicating that the latter model is favorable over the former model although the difference of log marginal likelihoods is small.

Table 2 (C) shows the results of diagnostic checking based on variables $n_{i,t}^M$ explained in Section 2. The Table shows the mean, the standard deviation, the skewness, the kurtosis, and the Ljung-Box statistics to test the null hypothesis of no serial correlation up to the sixth lag, where the numbers in brackets show the standard errors. If the model is correctly specified, the asymptotic distribution of $n_{i,t}^M$ is the standard normal. For SCI95, ESRAO, and HWINM, the null hypothesis of no serial correlation is rejected at the 1 percent level. For all variables, the kurtosis is significantly larger than three.

Figures 1 (A) depicts the composite indices (CIs) estimated by the Kim and Nelson (1998) model and the Stock and Watson (1989,1991) model jointly with that by the ESRI.

The shaded areas represent the periods of Economic and Social Research Institute (ESRI) recessions (from peak to through). There is no major difference between the CIs estimated by the both models, which is consistent with the result that the difference of log marginal likelihoods is small. Figure 1 (B) depicts the posterior probabilities that the economy is in the recession state in each month as inferred from the Kim and Nelson (1998) model. These probabilities can be calculated simply by averaging 10,000 draws of the state S_t sampled from its posterior distribution.

Table 3 shows the results for dataset 2. According to the CD values, the null hypothesis that the sequence of 10,000 draws is stationary is accepted at any standard level for all parameters in the both models. The log marginal likelihood of the Kim and Nelson (1998) model of -2202.10 is slightly larger than that of the Stock and Watson (1989,1991) model of -2203.91, providing evidence, although weak, that the mean growth rate shifts depending on whether the economy is in a recession or in a boom.

Table 3 (C) shows the results of diagnostic checking. Except for HWINMF, the null hypothesis of no serial correlation is rejected at the 1 percent level. The kurtosis is still significantly larger than three for all variables.

Figure 2 (A) depicts the CIs estimated by the Kim and Nelson (1998) model and the Stock and Watson (1989,1991) model jointly with that by the ESRI. No major difference between the CIs estimated by the both models is found again. Figure 2 (B) depicts the posterior probabilities that the economy is in the recession state in each month as inferred from the Kim and Nelson (1998) model. In contrast to the probabilities based on dataset 1, they move in a wider range between 0 percent and 100 percent, compared to Figure 1 (B).

We further estimate the Kim and Nelson (1998) model by using the following dataset.

Dataset 3: (1) IIP95P (2) SCI95 (3) ESRAO (4) HWINMF

This dataset is the one in which CELL9 is excluded from dataset 1. These four variables are used to construct the Nikkei Business Index. This index is the CI constructed by the Nihon

Keizai Shimbun, Inc. using the Stock and Watson (1989, 1991) model. These four variables correspond to the four variables used by the Department of Commerce (DOC) to construct its CI: "industrial production," "total personal income less transfer payments in 1987 dollars," "employees on nonagricultural payrolls," and "total manufacturing and trade sales in 1987 dollars". We only report the posterior probabilities of a recession, which is depicted in Figure 3. Unlike datasets 1 and 2, the posterior probabilities move in a very narrow range around 50 percent, so that they cannot be used to date the business cycle turning points. This may be attributed to the fact that the four variables in dataset 3 are weakly correlated with each other.

In dataset 2, the null hypothesis of no serial correlation in the diagnostic statistic is rejected for HWINMF. This may be attributed to the facts that HWINMF has positive serial correlation while all other variables in dataset 2 have negative serial correlation and that HWINMF is weakly correlated with other variables. Hence, we also analyze dataset 2 without HWINMF, that is,

Dataset 4: (1) IIP95P (2) SMSALE (3) IIP95O (4) IIP95M

The posterior probabilities of a recession calculated by fitting the Kim and Nelson (1998) model to dataset 4 are depicted in Figure 4. Figure 2(C) and Figure 4 look alike, demonstrating that HWINMF does not play an important role in dataset 2.

Following Kaufman (2000), we date the turning points by defining period t as a peak if the posterior probability $P(S_t = 1|\Delta\tilde{y}_T) > 0.5$ and $P(S_{t+1} = 1|\Delta\tilde{y}_T) < 0.5$ and a trough if $P(S_t = 1|\Delta\tilde{y}_T) < 0.5$ and $P(S_{t+1} = 1|\Delta\tilde{y}_T) > 0.5$. As mentioned, the posterior probabilities estimated using dataset 3 move in a very narrow range around 50 percent, so that they cannot be used to date the business cycle turning points. Therefore, we estimate the turning points using datasets 1, 2, and 4. The estimated turning points are shown in Table 4 jointly with the reference date by the ESRI. The difference of the turning points among the three datasets is at most one month except the peak and through in 1981, which are detected only by dataset 1 and 4, and the peak in 2000, which are detected only by dataset 2 and 4. The difference of the turning points estimated by the Kim and Nelson (1998) model from the reference date by the ESRI is larger, but at most three months except the trough in 1997, the peak and through in 1981, and the peak in 2000.

Thus far, $\beta(18, 2)$ is used as a prior distribution for the transition probabilities π_{00} and π_{11}. It is tight because the mean and standard deviation of $\beta(18, 2)$ are 0.9 and 0.065 respectively. To examine how the results are sensitive to the prior distribution of π_{00} and π_{11}, we estimate the Kim and Nelson (1989) model using dataset 1, 2, and 4 under the diffuse prior $\beta(1, 1)$, which corresponds to a uniform distribution in $[-1, 1]$. The estimated posterior probabilities of a recession are depicted in Figures 5 (dataset 1), 6 (dataset 2), and 7 (dataset 4). Figure 5 shows that the posterior probabilities estimated using dataset 1 move in a very narrow range around 50 percent, indicating that dataset 1, in which five variables are weakly correlated with each other, requires a tight prior for the transition probabilities π_{00} and π_{11} to date the business cycle turning points. Figures 6 and 7 show that this is not true for datasets 2 and 4, in which the correlations of variables are not so weak. The turning points estimated using dataset 2 and 4 with the diffuse prior for π_{00} and π_{11} are shown in Table 5. The effects of a prior for π_{00} and π_{11} on the estimated turning points are small especially for dataset 4. When dataset 4 is used, the turning points estimated with the diffuse prior are the same as those with the tight prior except the through in 2000, which is not detected when the tight prior is used, and the through in 1985, whose difference is only one month. The conclusion must be that the estimation results of the Kim and Nelson (1989) model are insensitive to the prior distribution when highly correlated variables are used, but it is not true when weakly correlated variables are used.

The ESRI announces the date of a turning point one year and a few months after the date of the turning point and may revise the date few months after the first announcement. For example, the ESRI announced the peak in 1997 as Mar. 1997 in Jun. 1998, and revised it as May 1997 in Dec. 2001. The trough in 1999 was first announced as Apr. 1994 in Jun. 2000 and revised as Apr. 1999 in Dec. 2001. The peak in 2000 was announced as Oct. 2001 in Dec. 2001 and may possibly be revised in the future. We examine how quickly the Kim and Nelson (1989) model can detect the date of turning point. Specifically, we examine when the peaks in 1997 and 2000 and the trough in 1999 are first detected by estimating the Kim and Nelson (1989) model using the dataset 4 up to one to three months after those turning points. Surprisingly enough, all three turning points are detected only two months after the dates of turning points, and the detected dates are the same as those estimated using the data up to the three months after the turning points and those using the full sample.

We find that the Kim and Nelson (1989) model performs well when dataset 4 (or 2) are used. All variables except SMSALE in dataset 4 are production-related variables. Hence, the Kim and Nelson (1989) using only IIP95P, which is representative of production-related variables, may also perform well. To examine whether this is true or not, we estimate the Kim and Nelson (1989) using only IIP95P. The estimated posterior probabilities of a recession move in a narrow range around 50 percent. This result indicates that the Kim and Nelson (1989) does not perform well when only IIP95P and requires other production-related variables.

4 Conclusions

This article fits the Markov switching dynamic factor model proposed by Kim and Nelson (1998) to some macroeconomic variables in Japan. We do not find strong evidence that the Kim and Nelson (1998) model is favored over the Stock and Watson (1989,1991) model nor major differences between the composite indices estimated by the two models. The Kim and Nelson (1998) model, however, produces the estimates of turning points close to the reference dates by the Economic and Social Research Institute in Cabinet Office unless only weakly correlated variables are used.

In this article, we focus on the in-sample fit of the models. Needless to say, it is worthwhile examining the out-of-sample forecasting ability.

APPENDIX A: Sampling from Conditional Distributions (8)–(13)

Conditional distributions (8)–(12) can be derived based on the following theorem, which is well known in Bayesian econometrics.

Theorem. *Consider the linear regression model*

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{u}, \quad \boldsymbol{u} \sim N\left(0, \sigma^2 \boldsymbol{I}_T\right), \tag{A.1}$$

where $\boldsymbol{Y}$ is the $T \times 1$ vector of dependent variable, $\boldsymbol{X}$ is the $T \times k$ matrix of independent variables, $\boldsymbol{\beta}$ is the $k \times 1$ vector of regression coefficients, $\boldsymbol{u}$ is the $T \times 1$ vector of error term which follows the independent normal distribution with variance σ^2, and $\boldsymbol{I}_T$ is the $T \times T$ identity matrix.

Under the prior distributions

$$\boldsymbol{\beta} \sim N(M^{(0)}, \Sigma^{(0)}), \quad \sigma^2 \sim IG(\nu^{(0)}/2, \delta^{(0)}/2),$$

the conditional distributions $f(\boldsymbol{\beta}|\sigma^2, \boldsymbol{X}, \boldsymbol{Y})$ and $f(\sigma^2|\boldsymbol{\beta}, \boldsymbol{X}, \boldsymbol{Y})$ are given by

$$\boldsymbol{\beta}|\sigma^2, \boldsymbol{Y} \sim N(M^{(1)}, \Sigma^{(1)}), \tag{A.2}$$

where

$$\begin{aligned} M^{(1)} &= \left(\Sigma^{(0)-1} + \sigma^{-2}\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\left(\Sigma^{(0)-1}M^{(0)} + \sigma^{-2}\boldsymbol{X}'\boldsymbol{Y}\right), \\ \Sigma^{(1)} &= \left(\Sigma^{(0)-1} + \sigma^{-2}\boldsymbol{X}'\boldsymbol{X}\right)^{-1}, \end{aligned}$$

and

$$\sigma^2|\boldsymbol{\beta}, \boldsymbol{Y} \sim IG(\nu^{(1)}/2, \delta^{(1)}/2), \tag{A.3}$$

where

$$\begin{aligned} \nu_1 &= \nu_0 + T, \\ \delta_1 &= \delta_0 + (\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta})'(\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta}). \end{aligned}$$

Proof.

$$
\begin{aligned}
& f(\boldsymbol{\beta}|\sigma^2, \boldsymbol{Y}) \\
& \quad \propto \; f(\boldsymbol{Y}|\boldsymbol{\beta}, \sigma^2) f(\boldsymbol{\beta}) \\
& \quad \propto \; \exp\left[-\frac{1}{2\sigma^2}(\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta})'(\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta})\right] \exp\left[-\frac{1}{2}(\boldsymbol{\beta} - M^{(0)})'\Sigma_0^{-1}(\boldsymbol{\beta} - M^{(0)})\right] \\
& \quad \propto \; \exp\left[-\frac{1}{2}(\boldsymbol{\beta} - M^{(1)})'\Sigma^{(1)-1}(\boldsymbol{\beta} - M^{(1)})\right].
\end{aligned}
$$

$$
\begin{aligned}
& f\left(\frac{1}{\sigma^2}|\boldsymbol{\beta}, \boldsymbol{Y}\right) \\
& \quad \propto \; f\left(\boldsymbol{Y}|\boldsymbol{\beta}, \sigma^2\right) f\left(\frac{1}{\sigma^2}\right) \\
& \quad \propto \; \left(\frac{1}{\sigma^2}\right)^{(T/2)} \exp\left[-\frac{1}{2\sigma^2}(\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta})'(\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta})\right] \left(\frac{1}{\sigma^2}\right)^{\nu_0/2-1} \exp\left[-\frac{\delta_0}{2\sigma^2}\right] \\
& \quad \propto \; \left(\frac{1}{\sigma^2}\right)^{\nu_1/2-1} \exp\left[-\frac{\delta_1}{2\sigma^2}\right].
\end{aligned}
$$

Conditional Distribution (8)

The condition of (8) includes $\boldsymbol{\psi}_i$, $\Delta \boldsymbol{y}_T$, and $\Delta \boldsymbol{c}_T$. Given them, we can calculate

$$
\begin{aligned}
\Delta y_{it}^* &= \Delta y_{it} - \psi_{i1}\Delta y_{i,t-1} - \cdots - \psi_{iq_i}\Delta y_{i,t-q_i} \\
\Delta c_t^* &= \Delta c_t - \psi_{i1}\Delta c_{t-1} - \cdots - \psi_{iq_i}\Delta c_{t-q_i}.
\end{aligned}
$$

Using them, define

$$
\begin{aligned}
\boldsymbol{Y} &= \left[\Delta y_{i,q_i+r_i}^*, \ldots, \Delta y_{iT}^*\right]' \\
\boldsymbol{X} &= \begin{bmatrix} \Delta c_{q_i+r_i}^* & \cdots & \Delta c_{q_i+1}^* \\ \vdots & \ddots & \vdots \\ \Delta c_T^* & \cdots & \Delta c_{T-r_i+1}^* \end{bmatrix} \\
\boldsymbol{\beta} &= \boldsymbol{\lambda}_i' \\
\boldsymbol{u} &= [\epsilon_{i,q_i+r_i}, \ldots, \epsilon_{i,T}]' \\
\sigma^2 &= \sigma_i^2.
\end{aligned}
$$

Then, equation (1') will be the linear regression model (A.1) in the above theorem, so that the conditional distribution (8) is given by equation (A.2). Specifically,

$$\lambda_1|\cdot \quad \sim \quad N(M_{\lambda_1}^{(1)}, \Sigma_{\lambda_1}^{(1)}) I[\lambda_{10} > 0], \tag{A.4}$$

$$\lambda_i|\cdot \quad \sim \quad N(M_{\lambda_i}^{(1)}, \Sigma_{\lambda_i}^{(1)}), \quad i = 2, \ldots, n, \tag{A.5}$$

where

$$\begin{aligned} M_{\lambda_i}^{(1)} &= (\Sigma_{\lambda_i}^{(0)-1} + \sigma_i^{-2}\boldsymbol{X}'\boldsymbol{X})^{-1}(\Sigma_{\lambda_i}^{(0)-1} M_{\lambda_i}^{(0)} + \sigma_i^{-2}\boldsymbol{X}'\boldsymbol{Y}) \\ \Sigma_{\lambda_i}^{(1)} &= (\Sigma_{\lambda_i}^{(0)-1} + \sigma_i^{-2}\boldsymbol{X}'\boldsymbol{X})^{-1}. \end{aligned}$$

It is straightforward to sample from the normal distribution (A.5). We can sample from the truncated normal distribution (A.4) by sampling from the normal distribution $N(M_{\lambda_1}^{(1)}, \Sigma_{\lambda_1}^{(1)})$ and accepting it only if it is positive.

Conditional Distributions (9) and (10)

The conditions of (9) and (10) include $\boldsymbol{\lambda}_i$, $\Delta \boldsymbol{y}_T$, and $\Delta \boldsymbol{c}_T$. Given them, we can calculate

$$e_{it} = \Delta y_{it} - \lambda_{i0}\Delta c_t - \lambda_{i1}\Delta c_{t-1} - \cdots \lambda_{ir_i}\Delta c_{t-r_i}, \quad t = r_i + 1, \ldots, T.$$

Define

$$\begin{aligned} \boldsymbol{Y} &= [e_{i,r_i+q_i+2}, \ldots, e_{i,T}]' \\ X &= \begin{bmatrix} e_{i,r_i+q_i+1} & \cdots & \epsilon_{i,r_i+1} \\ \vdots & \ddots & \vdots \\ e_{i,T-1} & \cdots & e_{i,T-q_i-1} \end{bmatrix} \\ \boldsymbol{\beta} &= \boldsymbol{\psi}_i' \\ \boldsymbol{u} &= [\epsilon_{i,r_i+q_i+2}, \ldots, \epsilon_{i,T}]' \\ \sigma^2 &= \sigma_i^2. \end{aligned}$$

Then, equation (2) will be the linear regression model (A.1), so that the conditional distributions (9) and (10) are given by equations (A.2) and (A.3). Specifically,

$$\psi_i|\cdot \quad \sim \quad N(M_{\psi_i}^{(1)}, \Sigma_{\psi_i}^{(1)}) I_{S(\boldsymbol{\psi}_i)}, \quad i = 2, \ldots, n, \tag{A.6}$$

$$\sigma_i^2|\cdot \quad \sim \quad IG(\nu^{(1)}/2, \delta^{(1)}/2), \quad i = 2, \dots, n, \tag{A.7}$$

where

$$\begin{aligned} M_{\psi_i}^{(1)} &= (\Sigma_{\psi_i}^{(0)-1} + \sigma_i^{-2}\boldsymbol{X}'\boldsymbol{X})^{-1}(\Sigma_{\psi_i}^{(0)-1}M_{\psi_i}^{(0)} + \sigma_i^{-2}\boldsymbol{X}'\boldsymbol{Y}) \\ \Sigma_{\psi_i}^{(1)} &= (\Sigma_{\psi_i}^{(0)-1} + \sigma_i^{-2}\boldsymbol{X}'\boldsymbol{X})^{-1}. \end{aligned}$$

We can sample from the truncated normal distribution (A.6) by sampling from the normal distribution $N(M_{\psi_i}^{(1)}, \Sigma_{\psi_i}^{(1)})$ and accepting it only if it is in the stationary region. It is straightforward to sample from the gamma distribution (see Ripley (1987)). Thus, we can sample σ_i^2 from the inverted gamma distribution (A.7) by sampling $1/\sigma_i^2$ from the gamma distribution and taking its reciprocal.

Conditional Distribution (11)

The condition of (11) includes $\Delta \boldsymbol{c}_T$ and $\boldsymbol{S}_T$. Given them, we can calculate

$$\begin{aligned} \boldsymbol{Y} &= \left[(\Delta c_{p+1} - \mu_{s_{p+1}}), \dots, (\Delta c_T - \mu_{s_T})\right]', \\ \boldsymbol{X} &= \begin{bmatrix} (\Delta c_p - \mu_{s_p}) & \cdots & (\Delta c_1 - \mu_{s_1}) \\ \vdots & \ddots & \vdots \\ (\Delta c_{T-1} - \mu_{s_{T-1}}) & \cdots & (\Delta c_{T-p} - \mu_{s_{T-p}}) \end{bmatrix}, \end{aligned}$$

If we further define

$$\boldsymbol{\beta} = \boldsymbol{\phi}', \quad \boldsymbol{u} = [\nu_{p+1}, \dots, \nu_T]', \quad \sigma^2 = 1,$$

equation (3') will be the linear regression model (A.1). Hence, conditional distribution (11) is given by

$$\boldsymbol{\phi}|\cdot \sim N(M_\phi^{(1)}, \Sigma_\phi^{(1)}) I_{S(\boldsymbol{\phi})},$$

where

$$\begin{aligned} M_\phi^{(1)} &= (\Sigma_\phi^{(0)-1} + \boldsymbol{X}'\boldsymbol{X})^{-1}(\Sigma_\phi^{(0)-1}M_\phi^{(0)} + \boldsymbol{X}'\boldsymbol{Y}), \\ \Sigma_\phi^{(1)} &= (\Sigma_\phi^{(0)-1} + \boldsymbol{X}'\boldsymbol{X})^{-1}. \end{aligned}$$

Conditional Distribution (12)

The condition of (12) includes $\boldsymbol{\phi}$, $\Delta \boldsymbol{c}_T$, and $\boldsymbol{S}_T$. Given them, we can calculate

$$\begin{aligned} \boldsymbol{Y} &= \begin{bmatrix} \Delta c_{p+1} - \phi_1 \Delta c_p - \cdots - \phi_p \Delta c_1 \\ \vdots \\ \Delta c_T - \phi_1 \Delta c_{T-1} - \cdots - \phi_p \Delta c_{T-1} \end{bmatrix}, \\ \boldsymbol{X} &= \begin{bmatrix} 1 - \phi_1 - \cdots - \phi_p & S_{p+1} - \phi_1 S_p - \cdots - \phi_p S_1 \\ \vdots & \vdots \\ 1 - \phi_1 - \cdots - \phi_p & S_T - \phi_1 S_{T-1} - \cdots - \phi_p S_{T-1} \end{bmatrix}. \end{aligned}$$

If we further define

$$\boldsymbol{\beta} = \boldsymbol{\mu}', \quad \boldsymbol{u} = [\nu_{p+1}, \ldots, \nu_T]', \quad \sigma^2 = 1,$$

equation (3') will be the linear regression model (A.1). Hence, the conditional distribution (12) is given by

$$\boldsymbol{\mu}'|\cdot \sim N(M_\phi^{(1)}, \Sigma_\phi^{(1)}) I_{S(\phi)} I[\mu_0 < 0, \mu_1 > 0],$$

where

$$\begin{aligned} M_\mu^{(1)} &= (\Sigma_\mu^{(0)-1} + \boldsymbol{X}'\boldsymbol{X})^{-1} (\Sigma_\mu^{(0)-1} M_\mu^{(0)} + \boldsymbol{X}'\boldsymbol{Y}), \\ \Sigma_\mu^{(1)} &= (\Sigma_\mu^{(0)-1} + \boldsymbol{X}'\boldsymbol{X})^{-1}. \end{aligned}$$

Conditional Distribution (13)

Conditional distribution (13) can be written as

$$f(\pi_{00}, \pi_{11}|\cdot) \propto f(\boldsymbol{S}_T|\pi_{00}, \pi_{11}) f(\pi_{00}, \pi_{11}),$$

where

$$f(\pi_{00}, \pi_{11}) \propto \pi_{00}^{u_{00}} (1 - \pi_{00})^{u_{01}} \pi_{11}^{u_{11}} (1 - \pi_{11})^{u_{10}}.$$

Once $\boldsymbol{S}_T$ are given, we can obtain the number of transitions from $S_{t-1} = i$ to $S_t = j$, which is denoted by n_{ij}. Then,

$$f(\boldsymbol{S}_T|\pi_{00}, \pi_{11}) \propto \pi_{00}^{n_{00}} (1 - \pi_{00})^{n_{01}} \pi_{11}^{n_{11}} (1 - \pi_{11})^{n_{10}}.$$

Hence,

$$\pi_{00}|\cdot \sim beta(u_{00}+n_{00}, u_{01}+n_{01}),$$

$$\pi_{11}|\cdot \sim beta(u_{11}+n_{11}, u_{10}+n_{10}).$$

We can sample from $beta(\alpha_1, \alpha_2)$ as the ratio $x_1/(x_1+x_2)$ where x_1 and x_2 are draws sampled from $gamma(\alpha_1, 1)$ and $gamma(\alpha_2, 1)$ respectively.

APPENDIX B: Particle Filter

The filter density $f(z_t, S_t|\Delta \boldsymbol{y}_{t-1})$, where $\boldsymbol{\theta}$ in the condition is omitted, can be written as

$$\begin{aligned}
&f(z_t, S_t|\Delta \boldsymbol{y}_{t-1}) \\
&\quad\propto\quad f(\Delta y_t|z_t) f(z_t, S_t|\Delta \boldsymbol{y}_{t-1}) \\
&\quad=\quad f(\Delta y_t|z_t)\int f(z_t, S_t|z_{t-1}, S_{t-1}) f(z_{t-1}, S_{t-1}|\Delta \boldsymbol{y}_{t-1}) dz_{t-1} dS_{t-1}.
\end{aligned} \tag{B.1}$$

Suppose that we have M draws $\left\{z_{t-1}^{(m)}, S_{t-1}^{(m)}\right\}$ $(m = 1, \ldots, M)$ sampled from the density $f(z_{t-1}$,$S_{t-1}|\Delta \boldsymbol{y}_{t-1})$. Using these draws, the integral in (B.1) can be estimated as follows.

$$\int f(z_t, S_t|z_{t-1}, S_{t-1}) f(z_{t-1}, S_{t-1}|\Delta \boldsymbol{y}_{t-1}) dz_{t-1} dS_{t-1} \approx \frac{1}{M}\sum_{l=1}^{M} f(z_t, S_t|z_{t-1}^{(m)}, S_{t-1}^{(m)}). \tag{B.2}$$

Substituting (B.2) into (B.1) yields

$$f(z_t, S_t|\Delta \boldsymbol{y}_t) \approx \frac{1}{M}\sum_{l=1}^{M} f(\Delta y_t|z_t) f(z_t, S_t|z_{t-1}^{(m)}, S_{t-1}^{(m)}), \tag{B.3}$$

where $f(\Delta y_t|z_t) f(z_t, S_t|z_{t-1}^{(m)}, S_{t-1}^{(m)})$ in the right-hand-side can be written as

$$\begin{aligned}
f(z_t, S_t|\Delta \tilde{y}_t) &\propto f(\Delta y_t|z_t) f(z_t, S_t|z_{t-1}^{(m)}, S_{t-1}^{(m)}) \\
&\propto f(\Delta y_t|z_t) f(z_t|S_t, z_{t-1}^{(m)}) p(S_t|S_{t-1}^{(m)}) \\
&= f(\Delta y_t, z_t|S_t, z_{t-1}^{(m)}) p(S_t|S_{t-1}^{(m)}) \\
&= f(z_t|S_t, z_{t-1}^{(m)}, \Delta y_t) f(\Delta y_t|z_{t-1}^{(m)}) p(S_t|S_{t-1}^{(m)}) \\
&= \eta_{s_t,m} f(z_t|S_t, z_{t-1}^{(m)}, \Delta y_t),
\end{aligned} \tag{B.4}$$

where

$$
\begin{aligned}
\eta_{s_t,m} &= f(\Delta y_t | z_{t-1}^{(m)}) p(S_t | S_{t-1}^{(m)}) \\
&\propto |\Sigma_\epsilon|^{-1/2} \exp\left(-\frac{1}{2} e_t' \Sigma_\epsilon^{-1} e_t\right) p(S_t | S_{t-1}^{(m)}).
\end{aligned}
$$

Substituting (B.4) into (B.3) yields

$$
f(z_t, S_t | \Delta \boldsymbol{y}_t) \approx \frac{1}{M} \sum_{m=1}^{L} \eta_{s_t,l} f(z_t | S_t, z_{t-1}^{(l)}, \Delta y_t). \tag{B.5}
$$

We can sample from this mixture distribution by first selecting the indices (S_t, m) with probability proportional to $\eta_{S_t,m}$ and then sampling from $f(z_t | S_t, z_{t-1}^{(m)}, \Delta y_t)$, which is the normal whose mean and variance are given by

$$
\begin{aligned}
\mu_{t|t}^{(m)} &= M_{s_t} + \Phi z_{t-1}^{(m)} + \Sigma_v \Lambda' \Sigma_\epsilon^{-1} e_t \\
\Sigma_{t|t}^{(m)} &= \Sigma_v - \Sigma_v \Lambda' \Sigma_\epsilon^{-1} \Lambda \Sigma_v
\end{aligned}
$$

where $e_t = \Delta y_t - \Lambda(M_{s_t} + \Phi z_{t-1}^{(m)})$.

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Table 1. Eleven Variables used to construct by the Economics and Social Research Institute to Construct its Composite Index

(A) Definition of Variables

1	IIP95M	Index of Raw Materials Consumption, Mfg.
2	IIP95O	Index of Operating Rate, Mfg.
3	HWINMF	Index of Non-scheduled Hours Worked, Mfg
4	ESRAO	Ratio of Job Offers to Applicants
5	SDS	Sales of Department Stores
6	CELL9	Electric Power Consumption of Large Users
7	IIP95S	Index of Producers' Shipments, Investment Goods
8	SCI95	Index of Wholesale Sales
9	SMSALE	Sales of Small and Medium Size Companies
10	IIP95P	Index of Industrial Production, Mining and Mfg.
11	ZBOAS	Business Profit, All Industries

Note: ZBOAS is quarterly data and the others are monthly data.

(B) Contemporaneous Correlations of the Growth Rate of the Ten Variables

	IIP95M	IIP95O	HWINMF	ESRAO	SDS	CELL9	IIP95S	SCI95	SMSALE	IIP95P
IIP95M	1.0000									
IIP95O	0.8820	1.0000								
HWINMF	0.3321	0.3152	1.0000							
ESRAO	0.2401	0.2429	0.4157	1.0000						
SDS	-0.0671	-0.0943	-0.0656	-0.0054	1.0000					
CELL9	0.6507	0.6196	0.2549	0.2059	-0.0467	1.0000				
IIP95S	0.5250	0.5810	0.2088	0.1738	0.0298	0.4408	1.0000			
SCI95	0.5038	0.5059	0.1158	0.0974	0.3845	0.4562	0.4444	1.0000		
SMSALE	0.6843	0.6632	0.2728	0.2288	0.0348	0.5221	0.6334	0.6070	1.0000	
IIP95P	0.8673	0.8872	0.2524	0.2364	-0.0727	0.6822	0.6624	0.6096	0.7756	1.0000

(C) Serial Correlations of the Growth Rate of Ten Variables

Variabels	IIP95M	IIP95O	HWINMF	ESRAO	SDS	CELL9	IIP95S	SCI95	SMSALE	IIP95P
correlation	-0.3492	-0.4002	0.4176	0.5574	-0.5628	-0.2286	-0.4604	-0.3561	-0.3550	-0.4227

TABLE 2. Estimation Results for Dataset 1

(A) Kim and Nelson Model

Marginal Likelihood = -2714.16

Parameter	Mean	Standard Error	95% Interval	CD
ΔC_t				
π_{00}	0.9045	0.0023	[0.7578,0.9807]	0.64
π_{11}	0.9108	0.0038	[0.7648,0.9815]	-0.89
ϕ_1	-0.0489	0.0164	[-0.3711,0.2305]	1.05
ϕ_2	0.1034	0.0080	[-0.1170,0.2604]	0.87
ϕ_3	0.3572	0.0056	[0.1795,0.4989]	1.60
μ_0	-0.3460	0.0246	[-0.9584,-0.0144]	1.55
μ_1	0.5424	0.0405	[0.0156,1.4138]	-1.59
y_{1t}				
λ_{10}	1.0739	0.0026	[0.9301,1.2141]	1.50
λ_{11}	-0.4708	0.0059	[-0.6210,-0.3100]	-0.77
ψ_1	-0.3803	0.0030	[-0.5192,-0.2328]	-0.89
σ_1^2	0.5208	0.0066	[0.3256,0.7779]	-0.36
y_{2t}				
λ_{20}	0.9507	0.0028	[0.7782,1.1313]	1.11
λ_{21}	-0.4521	0.0055	[-0.6406,-0.2628]	-1.24
ψ_2	-0.3075	0.0010	[-0.4232,-0.1922]	-0.98
σ_2^2	2.0372	0.0031	[1.6911,2.4346]	-0.98
y_{3t}				
λ_{30}	0.4653	0.0031	[0.2540,0.6779]	1.11
λ_{31}	0.3589	0.0013	[0.1601,0.5556]	0.69
ψ_3	0.4102	0.0013	[0.2863,0.5319]	-0.82
σ_3^2	3.1206	0.0054	[2.6375,3.6731]	-1.20
y_{4t}				
λ_{40}	0.5369	0.0022	[0.3850,0.6914]	1.22
λ_{41}	0.4272	0.0025	[0.2679,0.5886]	0.92
ψ_4	0.1370	0.0025	[-0.0049,0.2846]	-1.02
σ_4^2	1.6903	0.0051	[1.4014,2.0190]	-0.66
y_{5t}				
λ_{50}	0.7666	0.0014	[0.6472,0.8872]	0.86
λ_{51}	-0.2366	0.0060	[-0.3932,-0.0800]	-1.06
ψ_5	-0.2060	0.0013	[-0.3359,-0.0773]	-0.15
σ_5^2	0.7474	0.0042	[0.6030,0.9132]	0.85

Note: y_{1t}, y_{2t}, y_{3t}, y_{4t}, y_{5t}represent IIP95P, SCI95, ESRAO, HWINMF, and CELL9 respectively. The first 2,000 draws are discarded and then the next 10,000 are used for calculating the posterior means, the standard errors of the posterior means, 95 percent interval, and the convergence diagnostic (CD) statistics proposed by Geweke (1992). The posterior means are computed by averaging the simulated draws. The standard errors of the posterior means are computed using a Parzen window with a bandwidth of 1,000. The 95 percent intervals are calculated using the 2.5th and 97.5th percentiles of the simulated draws. The CD is computed using equation (32), where we set $n_A = 1,000$ and $n_B = 5,000$ and compute $\hat{\sigma}_A^2$ and $\hat{\sigma}_B^2$ using a Parzen window with bandwidths of 100 and 500 respectively.

(B) Stock and Watson Model

Marginal Likelihood = -2713.31

Parameter	Mean	Standard Error	95% Interval	CD
ΔC_t				
ϕ_1	0.0164	0.0044	[-0.1725,0.2087]	-0.93
ϕ_2	0.1526	0.0008	[0.0362,0.2683]	-0.28
ϕ_3	0.3904	0.0006	[0.2716,0.5026]	0.46
y_{1t}				
λ_{10}	1.304	0.0035	[0.9832,1.2702]	1.65
λ_{11}	-0.5063	0.0026	[-0.6642,-0.3023]	0.93
ψ_1	-0.3893	0.0019	[-0.5293,-0.2335]	0.80
σ_1^2	0.4775	0.0057	[0.2632,0.7090]	-1.28
y_{2t}				
λ_{20}	0.9936	0.0013	[0.8220,1.1708]	0.90
λ_{21}	-0.4840	0.0028	[0.4776,0.9502]	0.75
ψ_2	-0.3025	0.0007	[-0.4179,-0.1868]	1.05
σ_2^2	2.0207	0.0025	[1.6806,2.4113]	-0.73
y_{3t}				
λ_{30}	0.4735	0.0015	[0.2629,0.6904]	-0.23
λ_{31}	0.3372	0.0010	[0.1386,0.5358]	0.85
ψ_3	0.4481	0.0013	[0.3224,0.5679]	0.81
σ_3^2	3.0486	0.0030	[2.5843,3.5953]	1.43
y_{4t}				
λ_{40}	0.5615	0.0014	[0.4061,0.7167]	1.21
λ_{41}	0.4258	0.0023	[0.2692,0.5835]	1.10
ψ_4	0.1391	0.0009	[0.0027,0.2802]	-0.12
σ_4^2	1.6958	0.0030	[1.4137,2.0224]	1.43
y_{5t}				
λ_{50}	0.8001	0.0015	[0.6848,0.9222]	0.47
λ_{51}	-0.2616	0.0030	[-0.4082,-0.1138]	1.03
ψ_5	-0.1975	0.0010	[-0.3266,-0.0666]	1.19
σ_5^2	0.7397	0.0028	[0.5969,0.9018]	1.55

Note: y_{1t}, y_{2t}, y_{3t}, y_{4t}, y_{5t}represent IIP95P, SCI95, ESRAO, HWINMF, and CELL9 respectively. The first 2,000 draws are discarded and then the next 10,000 are used for calculating the posterior means, the standard errors of the posterior means, 95 percent interval, and the convergence diagnostic (CD) statistics proposed by Geweke (1992). The posterior means are computed by averaging the simulated draws. The standard errors of the posterior means are computed using a Parzen window with a bandwidth of 1,000. The 95 percent intervals are calculated using the 2.5th and 97.5th percentiles of the simulated draws. The CD is computed using equation (32), where we set $n_A = 1,000$ and $n_B = 5,000$ and compute $\hat{\sigma}_A^2$ and $\hat{\sigma}_B^2$ using a Parzen window with bandwidths of 100 and 500 respectively.

(C) Diagnostic Check for the Kim and Nelson Model

	IIP95P	SCI95	ESRAO	HWINMF	CELL9
Mean	0.0304	0.0178	0.0230	0.0356	0.0287
	(0.0581)	(0.0565)	(0.0570)	(0.0558)	(0.0584)
St. dev.	1.0223	0.9950	1.0027	0.9829	1.0274
Skewness	-0.2690	-0.1454	-0.1355	0.3565	0.1320
	(0.1391)	(0.1391)	(0.1391)	(0.1391)	(0.1391)
Kurtosis	4.0703	7.9968	5.7695	5.0004	4.8018
	(0.2782)	(0.2782)	(0.2782)	(0.2782)	(0.2782)
LB(6)	8.31	18.17	44.17	61.14	13.43

Note: Numbers in bracket are standard errors. LB(6) is the Ljung-Box statistic including six lags. The critical values for LB(6) are: 10.64 (10%), 12.59 (5%), 16.81 (1%).

TABLE 3. Estimation Results for Dataset 2.

(A) Kim and Nelson Model

Marginal Likelihood = -2202.10

Parameter	Mean	Standard Error	95% Interval	CD
ΔC_t				
π_{00}	0.9178	0.0010	[0.8334,0.9698]	-0.42
π_{11}	0.9368	0.0018	[0.8401,0.9779]	0.96
ϕ_1	-0.2761	0.0057	[-0.4595,-0.0597]	-0.00
ϕ_2	0.0008	0.0049	[-0.1641,0.2059]	-0.31
ϕ_3	0.2657	0.0033	[0.1236,0.4273]	-0.29
μ_0	-0.6341	0.0149	[-0.9535,-0.0767]	-0.80
μ_1	1.0687	0.0242	[0.1435,1.5009]	0.97
y_{1t}				
λ_{10}	1.0930	0.0041	[0.9875,1.2135]	-0.79
λ_{11}	-0.4251	0.0042	[-0.5467,-0.2959]	-1.33
ψ_1	-0.4006	0.0010	[-0.5269,-0.2681]	0.34
σ_1^2	0.2649	0.0010	[0.1997,0.3434]	-1.32
y_{2t}				
λ_{20}	0.8629	0.0035	[0.7463,0.9936]	-0.76
λ_{21}	-0.2101	0.0031	[-0.3285,-0.0920]	-1.39
ψ_2	-0.3417	0.0006	[-0.4509,-0.2308]	-0.44
σ_2^2	0.9226	0.0009	[0.7738,1.0903]	-0.78
y_{3t}				
λ_{30}	0.5736	0.0021	[0.4377,0.7180]	-1.05
λ_{31}	0.4670	0.0022	[0.3356,0.6070]	-0.89
ψ_3	0.0848	0.0017	[-0.0478,0.2173]	-0.38
σ_3^2	1.6668	0.0032	[1.4080,1.9668]	-1.01
y_{4t}				
λ_{40}	1.1547	0.0045	[1.0146,1.2890]	-0.85
λ_{41}	-0.4274	0.0045	[-0.5554,-0.2921]	-1.28
ψ_4	-0.4088	0.0014	[-0.5588,-0.2520]	-0.86
σ_4^2	0.3516	0.0008	[0.2717,0.4494]	-0.41
y_{5t}				
λ_{50}	1.1268	0.0043	[1.0180,1.2518]	-1.21
λ_{51}	-0.3209	0.0044	[-0.4478,-0.1810]	-1.39
ψ_5	-0.2604	0.0013	[-0.3971,-0.1215]	-1.09
σ_5^2	0.3233	0.0008	[0.2504,0.4097]	1.26

Note: y_{1t}, y_{2t}, y_{3t}, y_{4t}, y_{5t} represent IIP95P, SMSALE, HWINMF, IIP95O, and IIP95P. The first 2,000 draws are discarded and then the next 10,000 are used for calculating the posterior means, the standard errors of the posterior means, 95 percent interval, and the convergence diagnostic (CD) statistics proposed by Geweke (1992). The posterior means are computed by averaging the simulated draws. The standard errors of the posterior means are computed using a Parzen window with a bandwidth of 1,000. The 95 percent intervals are calculated using the 2.5th and 97.5th percentiles of the simulated draws. The CD is computed using equation (32), where we set $n_A = 1,000$ and $n_B = 5,000$ and compute $\hat{\sigma}_A^2$ and $\hat{\sigma}_B^2$ using a Parzen window with bandwidths of 100 and 500 respectively.

(B) Stock and Watson Model

Marginal Likelihood = -2203.91

Parameter	Mean	Standard Error	95% Interval	CD
ΔC_t				
ϕ_1	-0.0869	0.0028	[-0.2229 , 0.0499]	1.23
ϕ_2	0.1715	0.0012	[0.0627 , 0.2783]	0.71
ϕ_3	0.3785	0.0007	[0.2728 , 0.4830]	-0.91
y_{1t}				
λ_{10}	1.2304	0.0028	[1.1229 , 1.3471]	0.01
λ_{11}	-0.4982	0.0031	[-0.6302 , -0.3704]	-1.24
ψ_1	-0.3874	0.0009	[-0.5164 , -0.2544]	-0.16
σ_1^2	0.2329	0.0007	[0.1743 , 0.3009]	0.00
y_{2t}				
λ_{20}	0.9695	0.0021	[0.8475 , 1.0997]	-0.08
λ_{21}	-0.2596	0.0025	[-0.3910 , -0.1335]	-1.21
ψ_2	-0.3395	0.0007	[-0.4512 , -0.2278]	-1.05
σ_2^2	0.9042	0.0009	[0.7572 , 1.0738]	-1.40
y_{3t}				
λ_{30}	0.6409	0.0014	[0.4902 , 0.7936]	-0.37
λ_{31}	0.5017	0.0018	[0.3556 , 0.6511]	-0.49
ψ_3	0.0814	0.0013	[-0.0476 , 0.2171]	-1.01
σ_3^2	1.6608	0.0026	[1.4044 , 1.9578]	-1.00
y_{4t}				
λ_{40}	1.3007	0.0030	[1.1849 , 1.4245]	-0.06
λ_{41}	-0.5049	0.0033	[-0.6453 , -0.3658]	-1.16
ψ_4	-0.3640	0.0008	[-0.4894 , -0.2363]	-0.78
σ_4^2	0.3063	0.0007	[0.2389 , 0.3831]	0.07
y_{5t}				
λ_{50}	1.2589	0.0030	[1.1434 , 1.3777]	-0.01
λ_{51}	-0.3726	0.0010	[-0.5166 , -0.2282]	-1.24
ψ_5	-0.2816	0.0013	[-0.4095 , -0.1515]	-1.42
σ_5^2	0.3159	0.0009	[0.2479 , 0.3945]	0.58

Note: y_{1t}, y_{2t}, y_{3t}, y_{4t}, y_{5t} represent IIP95P, SMSALE, HWINMF, IIP95O, and IIP95P. The first 2,000 draws are discarded and then the next 10,000 are used for calculating the posterior means, the standard errors of the posterior means, 95 percent interval, and the convergence diagnostic (CD) statistics proposed by Geweke (1992). The posterior means are computed by averaging the simulated draws. The standard errors of the posterior means are computed using a Parzen window with a bandwidth of 1,000. The 95 percent intervals are calculated using the 2.5th and 97.5th percentiles of the simulated draws. The CD is computed using equation (32), where we set $n_A = 1,000$ and $n_B = 5,000$ and compute $\hat{\sigma}_A^2$ and $\hat{\sigma}_B^2$ using a Parzen window with bandwidths of 100 and 500 respectively.

(C) Diagnostic Check for the Kim and Nelson (1997) Model

	IIP95P	SMSALE	HWINMF	IIP95O	IIP95P
Mean	0.0129	0.0107	0.0263	0.0210	0.0160
	(0.0590)	(0.0571)	(0.0559)	(0.0591)	(0.0586)
St. dev.	1.0384	1.0051	0.9843	1.0406	1.0318
Skewness	-0.3275	0.0174	0.3323	-0.2062	-0.2058
	(0.1391)	(0.1391)	(0.1391)	(0.1391)	(0.1391)
Kurtosis	4.2933	4.4246	4.7422	3.5241	3.6938
	(0.2782)	(0.2782)	(0.2782)	(0.2787)	(0.2787)
LB(6)	4.93	11.92	69.36	10.82	4.94

Note: Numbers in bracket are standard errors. LB(6) is the Ljung-Box statistic including six lags. The critical values for LB(6) are: 10.64 (10%), 12.59 (5%), 16.81 (1%).

Table 4. Estimates of Business Cycle Turning Points Based on the Kim and Nelson Model

	ESRI	KN		
		Dataset1	Dataset2	Dataset4
P	Jan. 1977	Mar. 1977	Jan. 1977	Jan. 1977
T	Oct. 1977	Apr. 1977	Mar. 1977	Mar. 1977
P	Feb. 1980	Feb. 1980	Feb. 1980	Feb. 1980
T		Mar. 1981		Jun. 1981
P		Oct. 1981		Oct. 1981
T	Feb. 1983	Dec. 1982	Dec. 1982	Dec. 1982
P	Jun. 1985	May 1985	May 1985	May 1985
T	Nov. 1986	Nov. 1986	Nov. 1986	Nov. 1986
P	Feb. 1991	Dec, 1990	Dec. 1990	Jan. 1991
T	Oct. 1993	Jan. 1994	Jan. 1994	Jan. 1994
P		Mar. 1995	Apr. 1995	Apr. 1995
T		Sep. 1995	Sep. 1995	Sep. 1995
P	Mar. 1997	Mar. 1997	May 1997	May 1997
T	Apr. 1999	Feb. 1999	Jan. 1999	Jan. 1999
P			Aug. 2000	Aug. 2000

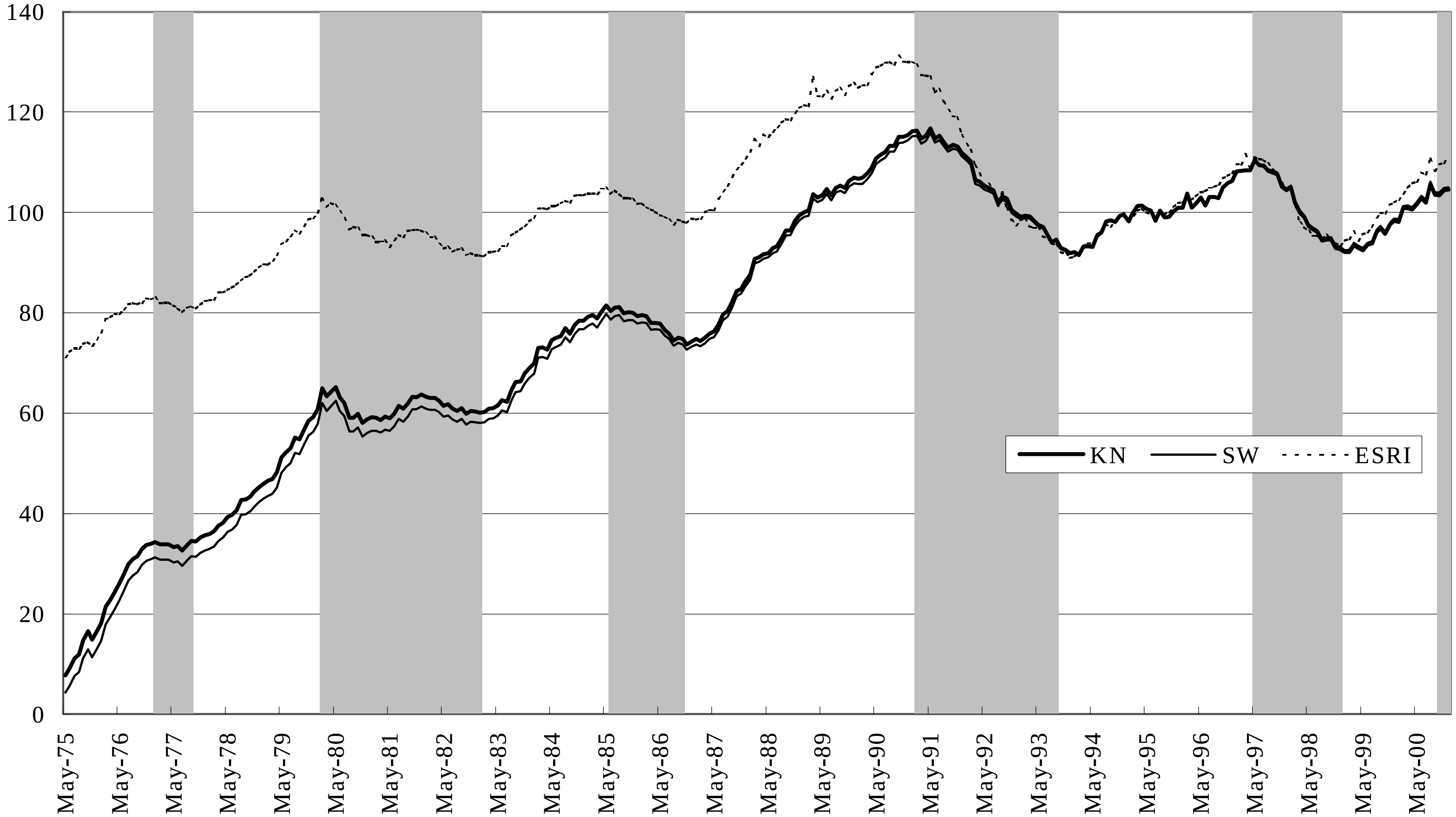
Note: "P"(peak) indicates the date when the posterior probability $P(S_t = 1|y^T) > 0.5$ and $P(S_{t+1} = 1|y^T) < 0.5$. "T"(trough) indicates the date when the posterior probability $P(S_t = 1|y^T) < 0.5$ and $P(S_{t+1} = 1|y^T) > 0.5$. The column "KN" is the estimates of turning point based on the Kim and Neslson (1998) model. "ESRI" is the reference date by the Economic and Social Research Institute in Cabinet office.

Table 5. Estimates of Business Cycle Turning Points Based on the Kim and Nelson Model (Diffuse Prior for Transition Probabilities π_{00} and π_{11})

	KN	
	Dataset2	Dataset4
P		Jan. 1977
T		Mar. 1977
P	Feb. 1980	Feb. 1980
T	Mar. 1981	May 1981
P	Nov. 1981	Oct. 1981
T	Nov. 1982	Dec. 1982
P	May. 1985	May 1985
T	Nov. 1986	Nov. 1986
P	Jan. 1991	Jan. 1991
T	Dec. 1993	Jan. 1994
P	Apr. 1995	Apr. 1995
T	Jul. 1995	Sep. 1995
P	May 1997	May 1997
T	Dec. 1998	Jan. 1999
P	Aug. 2000	Aug. 2000
T	Oct. 2000	Nov. 2000

Note: "P"(peak) indicates the date when the posterior probability $P(S_t = 1|y^T) > 0.5$ and $P(S_{t+1} = 1|y^T) < 0.5$. "T"(trough) indicates the date when the posterior probability $P(S_t = 1|y^T) < 0.5$ and $P(S_{t+1} = 1|y^T) > 0.5$.

Figure 1(A). Composite Indexes: Dataset 1



* The shaded areas represent the the periods of Economic and Social Research Institute(ESRI) recessions(from peak to through).

Figure 1(B). Posterior probability of a recession: Dataset 1

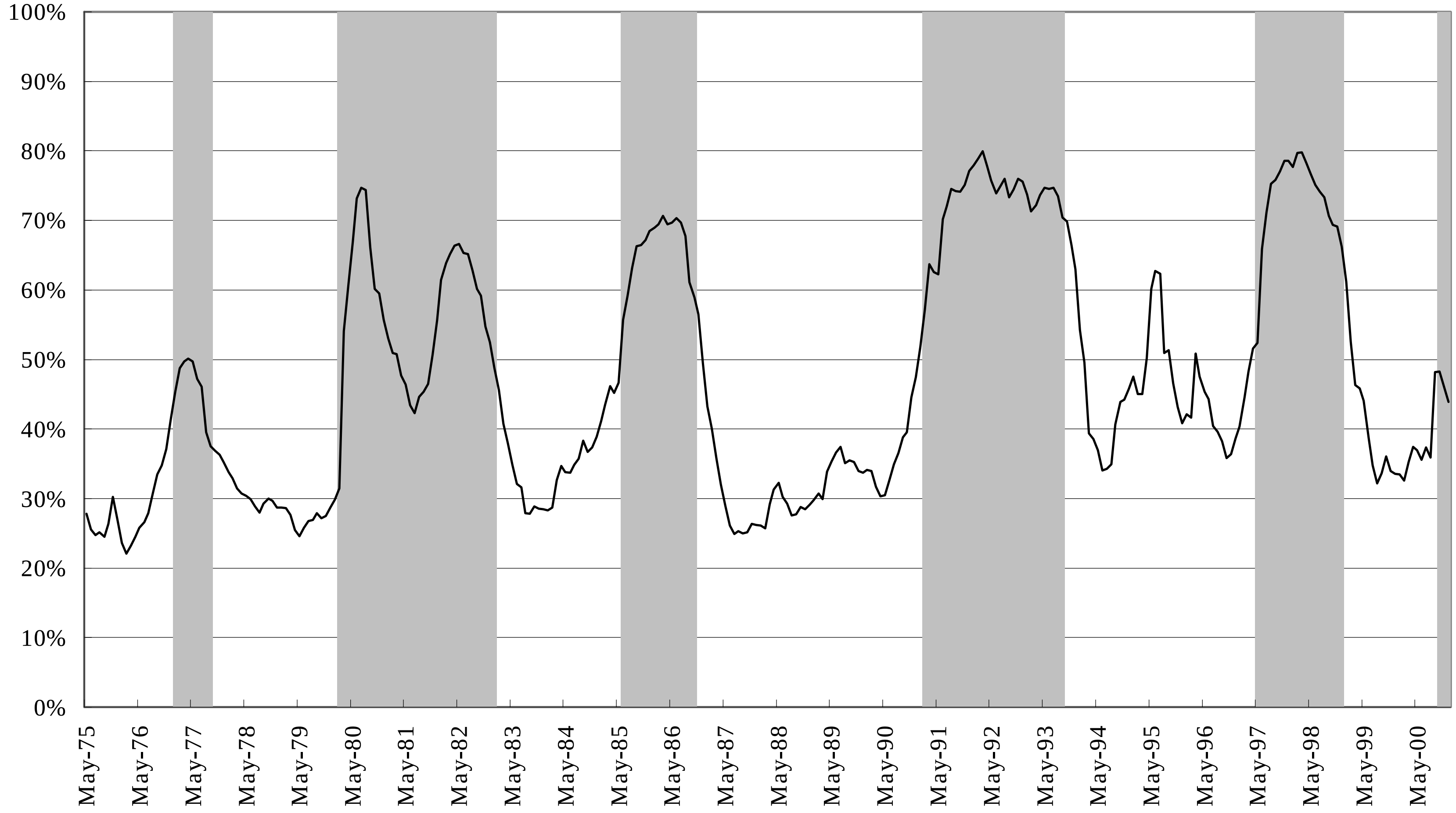


Figure 2(A). Composite Indexes: Dataset 2

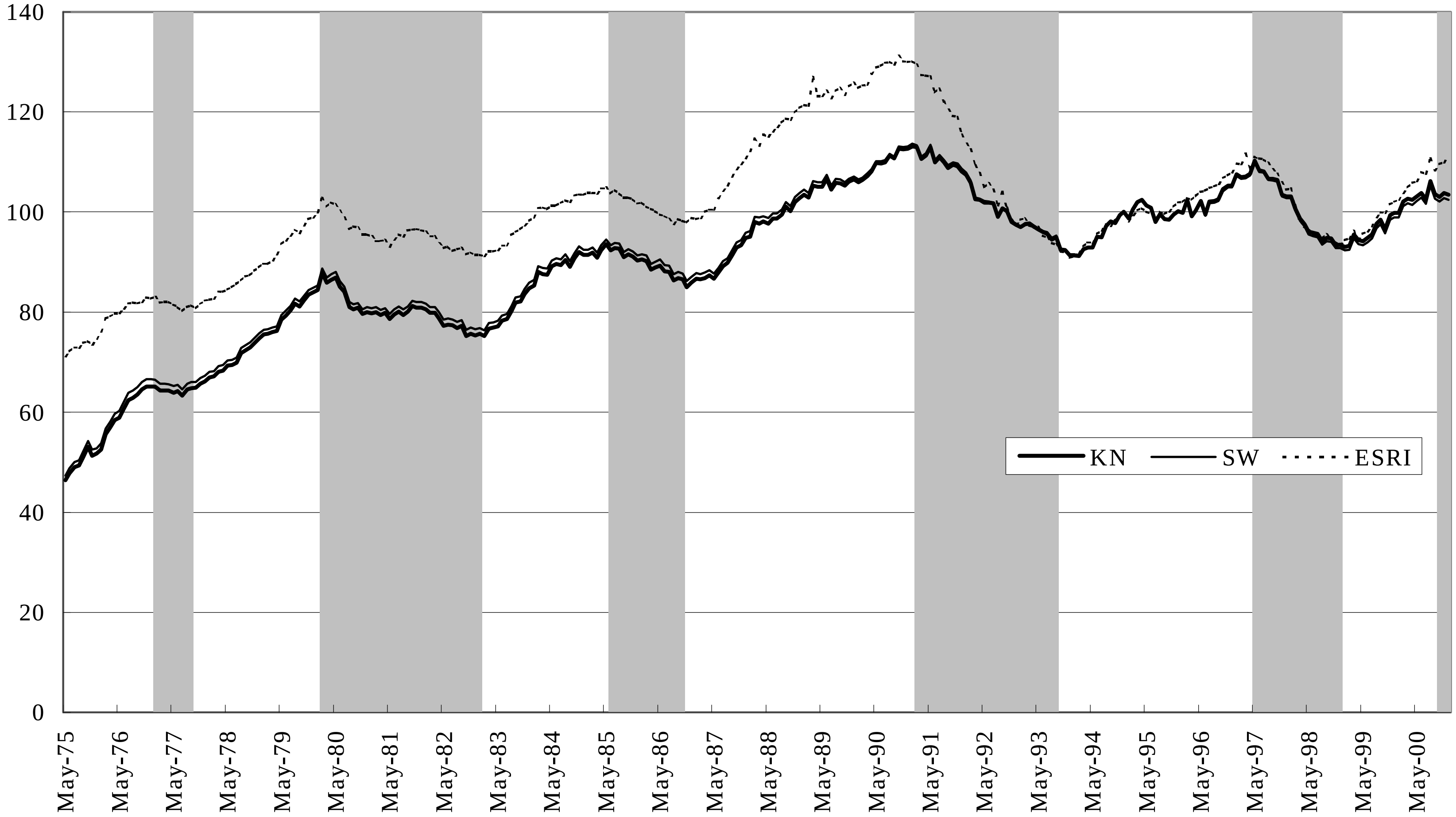


Figure 2(B). Posterior probability of a recession: Dataset 2

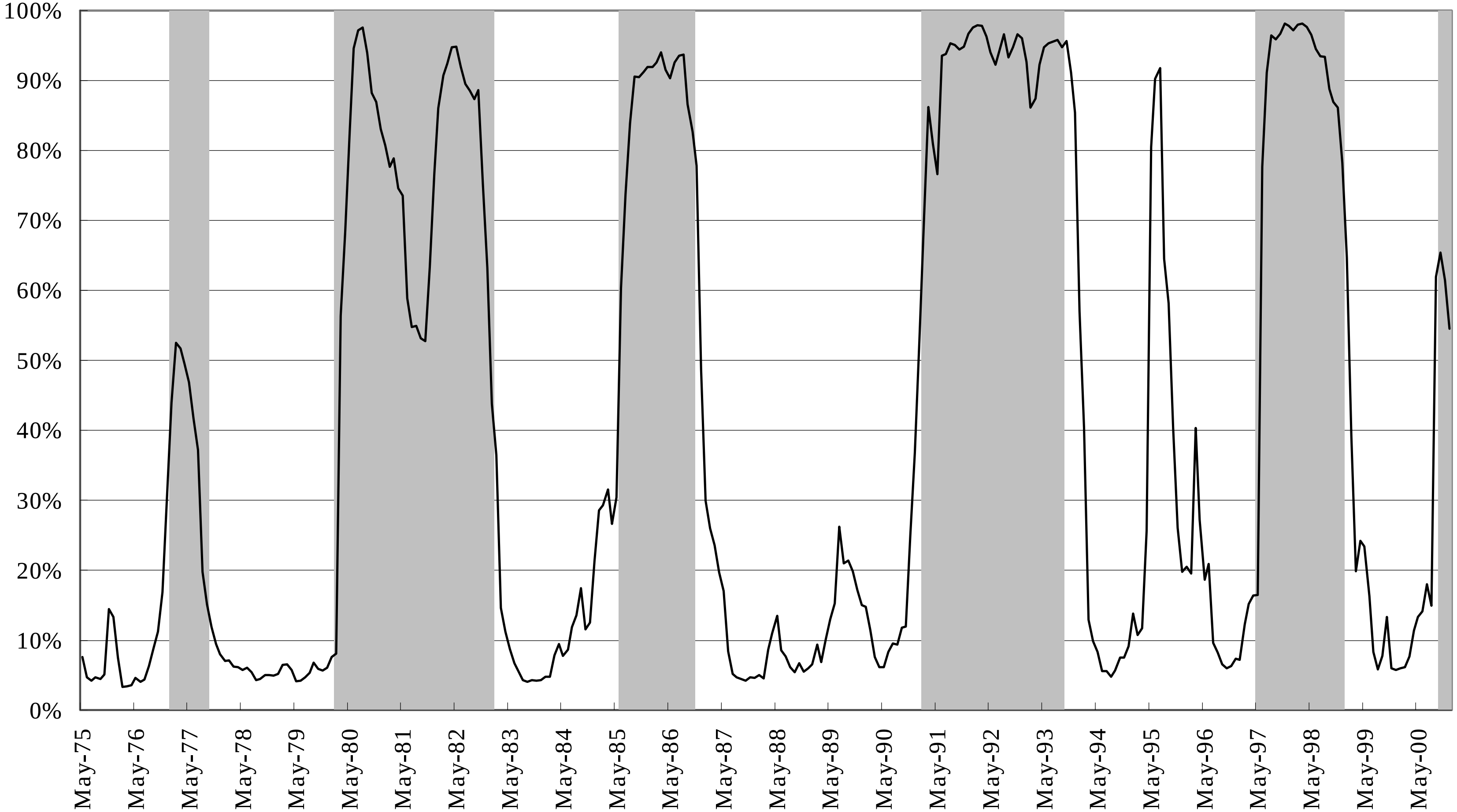


Figure 3. Posterior probability of a recession: Dataset 3

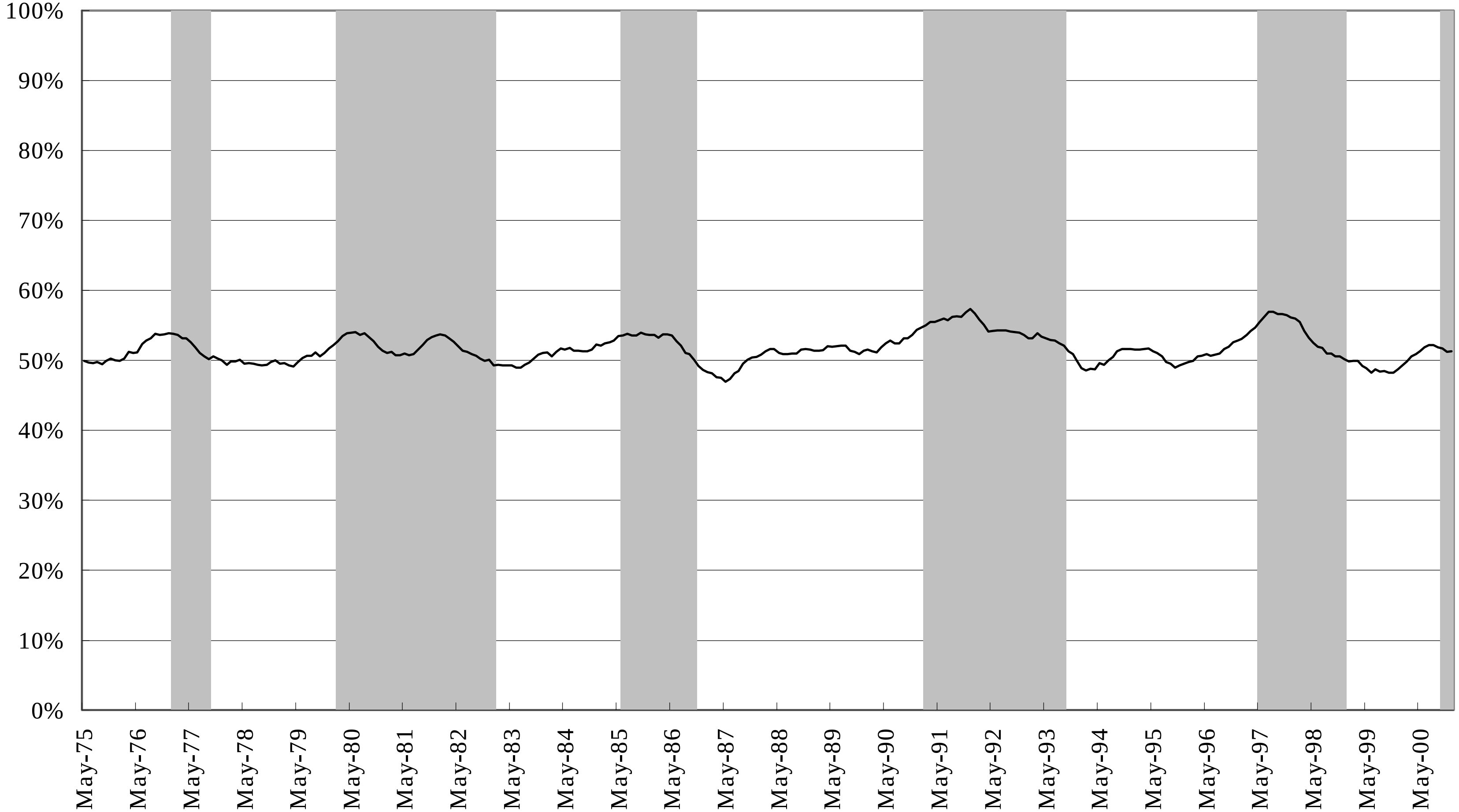


Figure 4. Posterior probability of a recession: Dataset 4

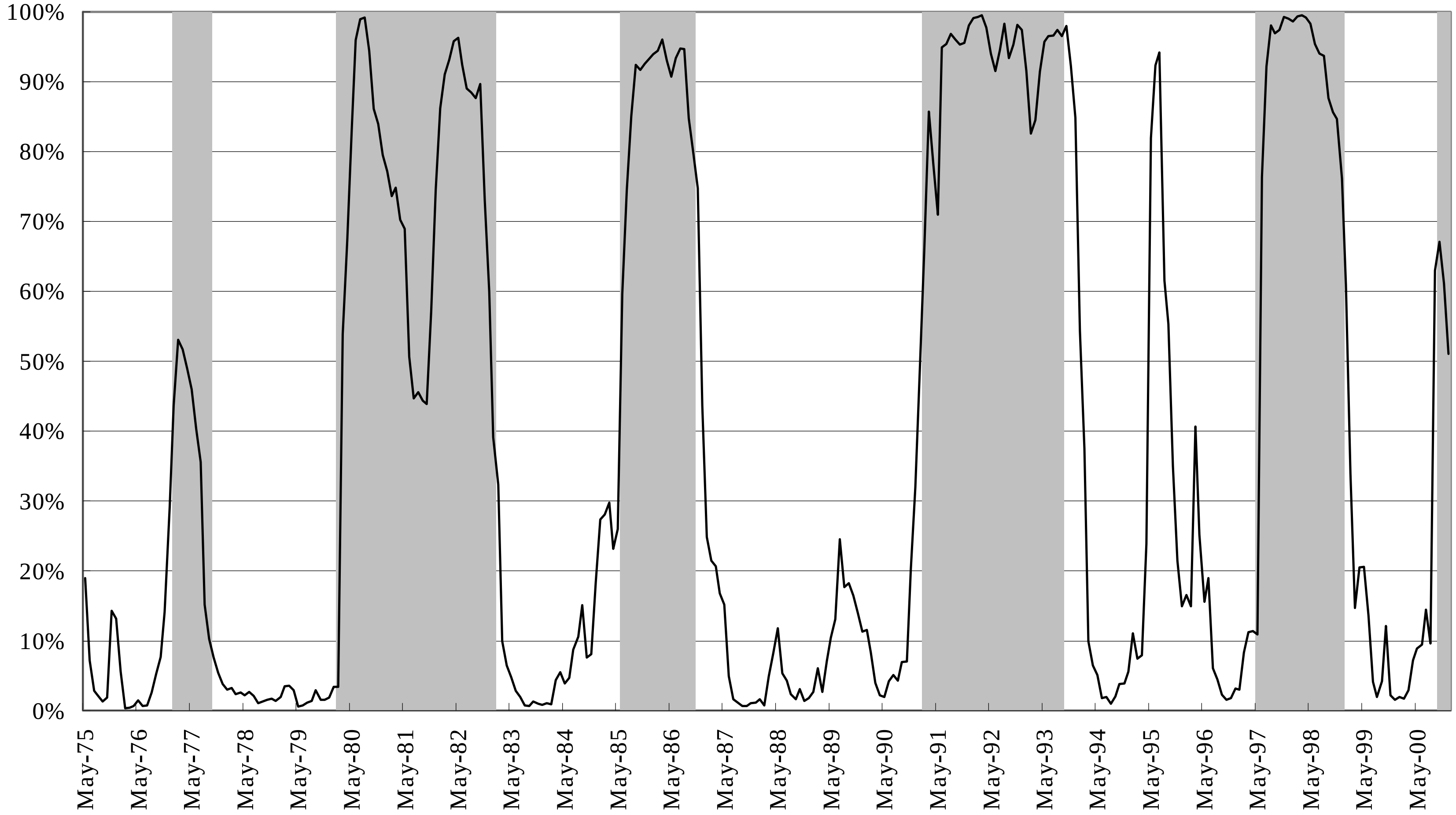


Figure 5. Posterior probability of a recession (diffuse prior for the transition probabilities): Dataset 1

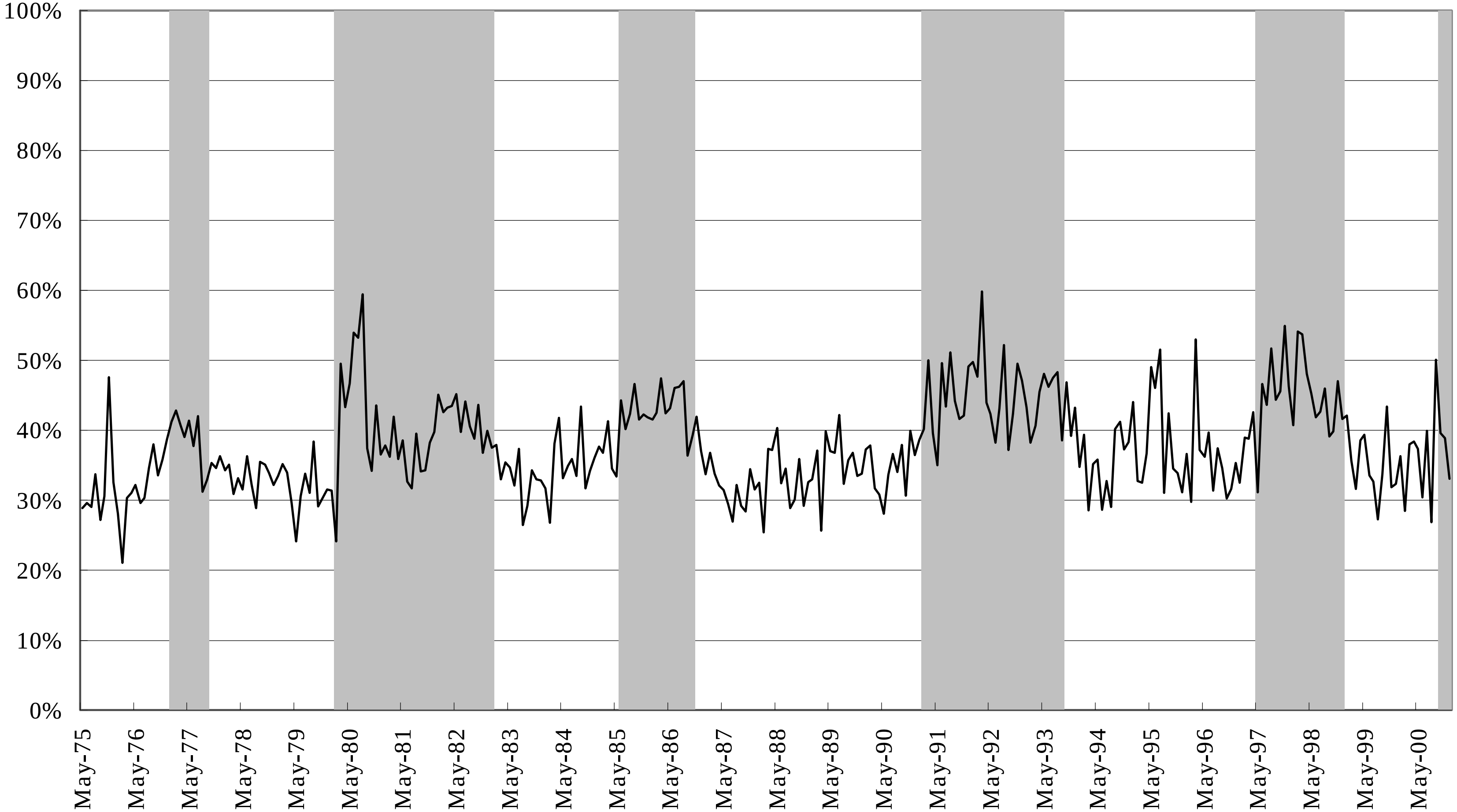


Figure 6. Posterior probability of a recession (diffuse prior for the transition probabilities): Dataset 2

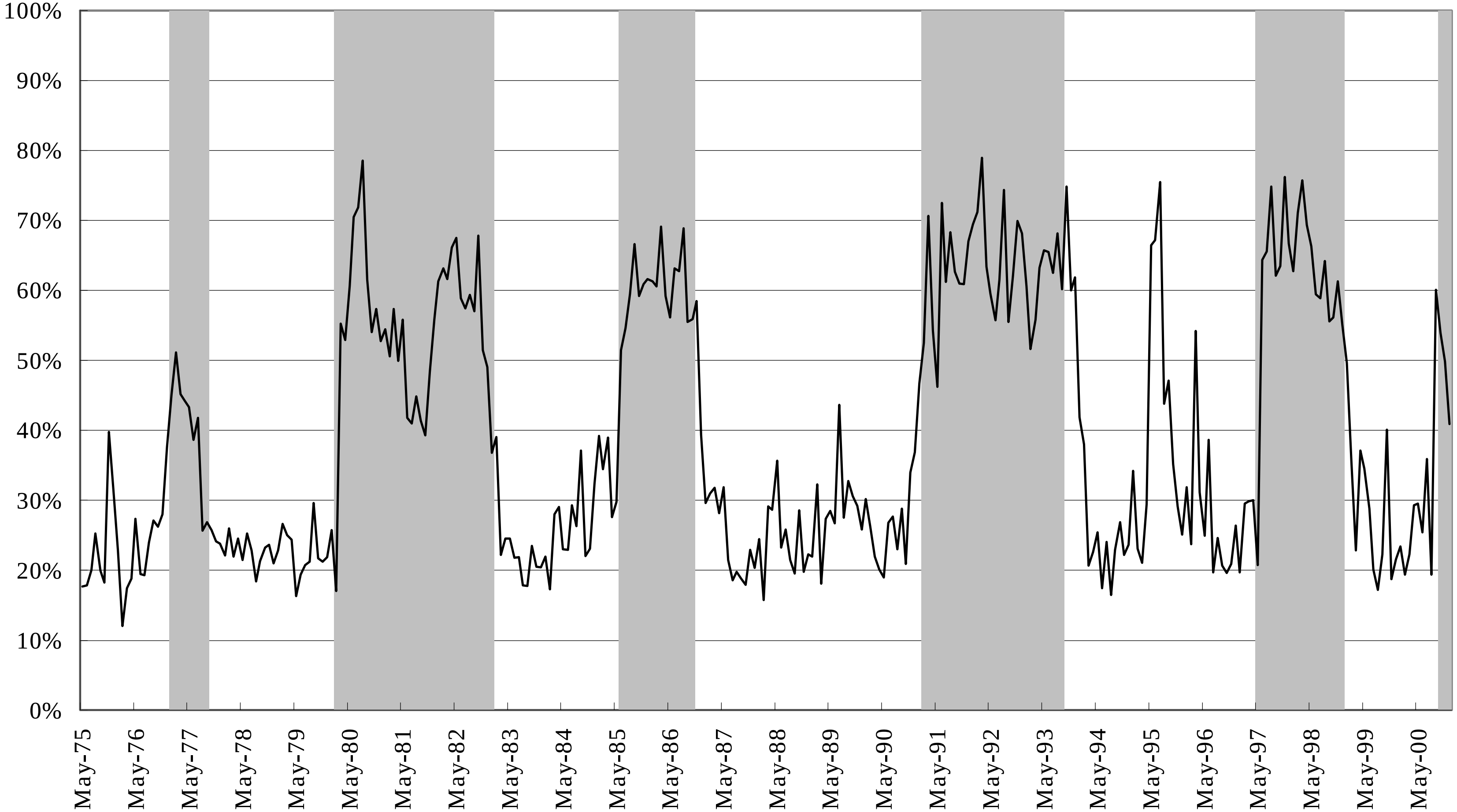


Figure 7. Posterior probability of a recession (diffuse prior for the transition probabilities): Dataset 4