Pricing-to-Market (PTM) and the International Transmission Effect of Monetary Policy: The New Open-Economy Macroeconomics Approach

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ABSTRACT

Empirical analyses on firms’ price setting behavior show that while the exchange rate pass-through of Japanese firms is low (many Japanese firms adopt pricing-to-market [PTM]), the export prices charged by U.S. firms nearly perfectly reflect foreign exchange rate fluctuations. This paper analyzes how the difference in domestic and foreign firms’ price setting behavior affects the domestic and international transmission effect of monetary policy by using a model that explicitly incorporates differences in the price setting behavior of domestic and foreign firms. This model is constructed by adopting the framework of the “new open-economy macroeconomics” which has been the subject of numerous research papers in recent years. The findings demonstrate that the transmission effects of domestic and foreign monetary policies are greatly different when domestic and foreign firms adopt different price setting behaviors. This indicates that central banks have to give sufficient attention to firms’ price setting behavior for the implementation of monetary policies.

Additionally, model simulations based on Japan and U.S. data show that the international transmission effect of Japanese monetary policy is negligible compared with that of U.S. monetary policy due to the PTM price-setting behavior of Japanese firms.

Key Words: New open economy macroeconomics, PPP (purchasing power parity), PTM (pricing-to-market), Monetary policy, Beggar-thy-neighbor effect.

JEL classification: D4, E12, E52, F3, F4, L13

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I. Introduction

By the early 1990s, the Mundell-Fleming model has served as the main analytical framework for international macroeconomics. However, Obstfeld and Rogoff [1995] proposed their dynamic general equilibrium open economy model (hereinafter referred to as “the O-R model”) with monopolistic competition and sticky price. Since then, the “new open-economy macroeconomics” has attracted the attention of a great many researchers, and recently, research has been conducted on extending some aspects of the O-R model. This line of research includes numerous papers on the effects of firms’ price setting behavior.1 Previously, the O-R model assumed that the Law of One Price always holds because firms set their export prices at the foreign-currency equivalents of their domestic sales prices, based on producer’s currency pricing (PCP). However, many researchers have amended this assumption by explicitly incorporating firms’ pricing-to-market (PTM) price setting behavior into their models. These research efforts are generating useful findings regarding exchange rate volatility and the international transmission effect of monetary policy.

Nevertheless, the research to date attempting to incorporate PTM into “new open-economy macroeconomics” has still room for improvement. For example, many of the researchers such as Devereux and Engel [1998] assume that all domestic and foreign firms adopt PTM for their price setting.2 Under this extreme assumption, the exchange rate pass-through on to export prices is zero for an entire nation, and the depreciation of a nation’s currency from monetary easing improves the nation’s terms of trade and worsens foreign countries’ terms of trade. However, Obstfeld and Rogoff [2000] question the practical validity of the PTM model by noting that, in the real world, the exchange rate pass-through is not zero and depreciation of a nation’s currency actually worsens its terms of trade.

As one approach to overcoming this problem, Otani [2001] proposes the introduction of international asymmetric price setting behavior. Specifically, the empirical analyses on PTM conducted by Knetter [1993] and others show that Japanese firms absorb approximately half of all exchange rate fluctuations by changing their mark-up ratios.3

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1 These research efforts have not been confined to PTM. See Lane [2001], Sarno [2001], and Otani [2001] for surveys of other issues.

2 Exceptions include Betts and Devereux [2000], as discussed below.

3 From the macroeconomic perspective, the pass-through ratio is 50% for an entire nation either when the pass-through ratios of all the firms in a country are 50% or when half of the firms adopt PTM and the other half adopt PCP. Therefore, the simulation analysis in Section 6 assumes that half of the Japanese firms set their export prices based on PTM.
while the exchange rate pass-through of U.S. firms is 100% (that is, virtually all U.S. firms adopt PCP). When this asymmetry in price setting behavior between U.S. and Japanese firms suggested by the empirical studies is introduced into the model, depreciation of a given nation’s currency deteriorates its terms of trade. Thus, this approach can overcome the empirical criticism made by Obstfeld and Rogoff [2000].

In this paper we incorporate the asymmetric price setting behavior of domestic and foreign firms in the new open-economy macroeconomics model, based on the approach presented in Otani [2001]. Then we analyze the effect of asymmetric price setting behavior on exchange rate volatility and on the international transmission effect of monetary policy. This paper expands the model developed by Betts and Devereux [2000], which assumes that a fraction \(s\) of both domestic and foreign firms adopt PTM for their price setting. More specifically, the model in this paper assumes that a fraction \(s\) of domestic firms and a fraction \(s^*\) of foreign firms adopt PTM price setting (hereinafter, these are referred to as “PTM firms”). Under these assumptions, the ratio of domestic and foreign firms that adopt PCP price setting (hereinafter referred to as “PCP firms”) becomes \(1-s\) and \(1-s^*\), respectively. Thus, it becomes possible to incorporate the asymmetry in price-setting behavior between domestic and foreign firms into the model.

The introduction of asymmetric price setting behavior has several advantages. First, it enables research consistent with the firms’ price setting behavior based on available empirical evidences. Second, it demonstrates that the exchange rate pass-through is not 0 and that depreciation of the home currency deteriorates the home country’s terms of trade, overcoming the criticism by Obstfeld and Rogoff [2000]. Finally, the model includes both the O-R model \((s=s^*=0)\) and Betts and Devereux [2000] \((s=s^*>0)\) as special cases. Thus, the model presented in this paper well illustrates the extent to which the results of prior research are changed when the domestic and foreign firms’ price setting behaviors are different.

While the model examined in this paper is a slightly generalized model of Betts and Devereux [2000], to the best of my knowledge virtually no research to date has explicitly incorporated firms’ asymmetric price setting behavior into the new open-economy macroeconomics framework.

One might wonder if the assumption of exogenously predetermined percentages of PTM firms in the domestic and foreign countries might not be plausible under changes in macroeconomic economic policies (see for example, Taylor [2000]). However, the

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4 Taylor [2000] states that exchange rate pass-through is on a downward trend in recent years due to low world-wide inflation rates and stable monetary policies. Devereux and Engel [2001] analyze
empirical evidences on exchange rate pass-through presented by Campa and Goldberg [2001] show that it is not a stable macroeconomic environment but rather structural changes has brought about the decline in exchange rate pass-through. Since this paper analyzes a model without any structural changes, it may be reasonable to assume that firms’ price setting behavior is exogenous and constant, based on the conclusions reached in Campa and Goldberg [2001].

Following this Introduction, Section II explains the framework for the model adopted herein. Section III incorporates price rigidity into the model and derives the equilibrium under price rigidity. Based on this, Section IV explicates how PTM influence the effect of monetary policy and Section V conducts welfare-based analysis on the international transmission effect of monetary policy. Section VI presents simulations based on Japanese and U.S. data, leading to various implications for the implementation of Japanese monetary policy. Finally, Section VII summarizes the findings of this paper. Appendices 1 and 2 define the long run and short run equilibria discussed in Section III.

II. The Model Set-up

The model is structured around those presented in Obstfeld and Rogoff [1995, 1996] and Betts and Devereux [2000]. There are two countries: home and foreign. Foreign country variables are denoted by an asterisk. Each country consists of the household, firm and government sectors. The households in both countries are continuously distributed within the range of 0 and 1, and consume differentiated goods produced by firms of both countries. The home country household is continuously distributed within the range of 0 to $n$, and the foreign country household within the range of $n$ to 1 (where $0<n<1$). Then, $n$ and 1-$n$ also represent the home and foreign country populations, respectively. Let index $h$ denote household.

Every household owns one distinctive firm that produces one differentiated good using only its labor. Let index $j$ designate good. This paper introduces the asymmetric price setting behaviors, then, it is assumed that the percentage of PTM firms

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Taylor’s conjecture using the framework of new open economy macroeconomics. They show that exporters generally set prices in the currency of the economy with the most credible monetary policy. Therefore, firms in the country with credible monetary policy and low inflation rate adopt PCP, but firms in the country with incredible monetary policy and high inflation rate adopt PTM.

$^5$ Structural changes include the rise in the share of manufactured goods and the corresponding decline in that of agricultural products, raw materials, and mineral fuels in world trade.
in the home country is \(s\) and that in the foreign country is \(s'\). The results using the model in this paper are consistent with Betts and Devereux [2000] in the case when \(s = s'\) and with the O-R model in the case when \(s = s' = 0\).

A. Households

1. Utility function

The representative agent in the home country has the following utility function.\(^6\)

\[
U_t^h = \sum_{t=0}^{\infty} \beta^t \left[ \log C_t^h + \chi \log \frac{M_t^h}{P_t} - \frac{\kappa}{2} (l_t^h)^2 \right]
\]

(1)

Here, \(\beta\) expresses the discount ratio, subscript \(t\) denotes time, superscript \(h\) is the index designated household, and \(C^h\) is the real consumption index, where \(c_t^h(j)\) shows consumption of good \(j\) by household \(h\), as detailed below.

\[
C_t^h = \left[ \int_0^1 c_t^h(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{\theta}}, \ \theta > 1.
\]

(2)

The price index \(P\), which corresponds to the real consumption index in Equation (2) in the sense that \(P\) minimizes the nominal expenditure at any given real consumption level, is expressed as follows.

\[
P_t = \left[ \int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}
\]

(3)

Where \(p_t(j)\) is the nominal price of good \(j\), as explained in detail later.

The term \(M_t^h/P\) in this utility function expresses the real money holding at the beginning of period \(t+1\). Since an increase in money holding reduces transaction costs, it increases utility. The final term in Equation (1) represents the disutility from labor input \(l_t^h\) and \(\kappa\) of the coefficient of labor input is the positive parameter.

\(^6\) Betts and Devereux [2000] use the following utility function

\[
U_t^h = \sum_{t=0}^{\infty} \beta^t \left[ \log C_t^h + \gamma \left( \frac{M_t^h}{P_t} \right)^{1-\epsilon} + \eta \log (1 - l_t^h) \right]
\]

while this paper uses the utility function of Obstfeld and Rogoff [1996] just as it is, as shown in Equation (1). This is because the utility function of Equation (1) is more easily solved and provides less complex solutions, and because one of the purposes of this paper is to synthesize the open-economy macroeconomic model developed herein based on PCP price setting behavior (the O-R model) and the new research on incorporating PTM into open economy macroeconomics.

The difference in the findings when Equation (1) is used rather than the utility function in Betts and Devereux [2000] is essentially limited to the potential that the foreign exchange rate may overshoot depending upon the value of \(\epsilon\) (see Footnote 12), and has almost no influence on the international transmission effects of monetary policy.
2. Firms’ Price Setting and Price Indices

PCP firms set their export prices at the foreign currency equivalent of their domestic sales prices. PTM firms set their domestic sales prices in domestic currency and their export prices in foreign currency, so they price-discriminate across countries.

Let \( p_t(h) \) and \( q_t(h) \) represent the home-currency and foreign-currency prices of a good made by domestic PTM firm \( h \). The home-currency price of a domestic PCP firm’s good is also \( p_t(h) \). This reflects the fact that, as shown in Section II C., the home-currency prices of domestic PTM firms are always the same as those of domestic PCP firms. Moreover, the foreign-currency price (export price) of a domestic PCP firm’s good is \( p_t(h)/e_t \), assuming that \( e_t \) is the nominal exchange rate (the home-currency price of foreign currency).

Likewise, let \( p_t^*(h^*) \) and \( q_t^*(h^*) \) denote the home-currency and foreign-currency prices of a good made by a foreign PTM firm \( h^* \). Then, the foreign-currency price of the foreign PCP firm’s good is \( q_t^*(h^*) \) and its export price is \( e_t q_t^*(h^*) \).

Equation (3) can now be amended using these notations and the ratios of domestic and foreign PTM firms, \( s \) and \( s^* \). The domestic and foreign general price indices, \( P_t \) and \( P_t^* \), become as follows.\(^7\)

\[
P_t = \left[ \int_0^n p_t(h)^{1-\theta} dh + \int_n^{n+(1-n)s} p_t^*(h^*)^{1-\theta} dh^* + \int_{n+(1-n)s}^l (e_t q_t^*(h^*))^{1-\theta} dh^* \right]^{1/(1-\theta)}
\]

\[
P_t^* = \left[ \int_0^{ns} q_t(h)^{1-\theta} dh + \int_{ns}^{ns+(1-n)s} \left( \frac{p_t(h)}{e_t} \right)^{1-\theta} dh + \int_{ns+(1-n)s}^l q_t^*(h^*)^{1-\theta} dh^* \right]^{1/(1-\theta)}
\]

Similarly, the home export price and import price indices \( \Gamma_t \) and \( \Gamma_t^* \) (both denominated in the home currency) can be expressed as follows.

\[
\Gamma_t = \left[ \int_0^n (e_t q_t(h))^{1-\theta} dh + \int_n^{n+(1-n)s} p_t(h)^{1-\theta} dh \right]^{1/(1-\theta)}
\]

\[
\Gamma_t^* = \left[ \int_n^{ns+(1-n)s} p_t^*(h^*)^{1-\theta} dh^* + \int_{ns+(1-n)s}^l (e_t q_t^*(h^*))^{1-\theta} dh^* \right]^{1/(1-\theta)}
\]

3. International Bond Markets

Households can trade one-period nominal discounted bonds denominated in the home currency.

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\(^7\) Obstfeld and Rogoff [1995, 1996] assume that all firms are PCP firms \( (s = s^* = 0) \). In this case, it is easy to show that the domestic general price index, Equation (4), and the foreign general price index, Equation (5) lead to \( P_t = e_t P_t^* \), that is, PPP (purchasing power parity) always holds even if prices are rigid. In the model presented in this paper, since PTM firms exist, PPP holds only when prices are flexible (see Section II C. and Section III B.), and PPP does not hold when prices are rigid.
currency in a completely integrated international asset market. Foreign-currency
denominated bonds do not exist. Let $B^h_t$ be the total value of nominal bonds held by
domestic households, which were issued at the end of period $t-1$ to finance the current
account deficit in period $t-1$ and mature in period $t$. And let $B^h_{-t}$ express the total value
of bonds held by foreign households. If $B^h_t$ or $B^h_{-t}$ are negative, the figure becomes the
outstanding balance of the bond issuance. Since the households in the country with a
current account deficit issue bonds and the households in the other country hold all of
them, $nB^h_t + (1-n)B^h_{-t} = 0$ always holds true.

In addition, let $d_t$ be the home-currency price of the bond which matures at period $t$.
Then, $d_t$ is the inverse of one plus the nominal interest rate (the gross nominal interest
rate) as follows:

$$d_t = \frac{1}{1 + i_t},$$

where $i_t$ denotes the nominal interest rate between period $t-1$ and $t$. Letting $r_t$ be the
real interest rate, the Fisher equation, $1 + i_t = (1 + r_t)(e_t / e_{t-1})$, holds between $i_t$ and $r_t$.
Since there is free trade between the countries in nominal bonds, uncovered interest rate
parity, $1 + i_t = (1 + i^*_t)(e_t / e_{t-1})$, also holds between $i_t$ and $i^*_t$.

4. Household’s Optimization Behavior

The household $h$ in the home country receives income from wages, $w_t l_t^h$, profits on its
ownership of domestic firm, $\pi_t^h$, and transfers $\tau_t^h$ from the government (negative figure
indicates a lump-sum tax) at period $t$. It is assumed that the household holds the total
value of nominal bonds, $B^h_t$, and the nominal money balance $M_{t-1}^h$ from period $t-1$.
Then, the period $t$ budget constraint for household $h$ can be written as follows.

$$d_{t+1} B_{t+1}^h + M_t^h = B_t^h + M_{t-1}^h + w_t l_t^h + \pi_t^h - P_t C_t^h + P_t \tau_t^h$$

The household maximize its lifetime utility defined by Equation (1) subject to the
above budget constraint to determines its consumption, labor, and money and bond
holdings as follows.

$$C_{t+1}^h = \beta(1 + r_{t+1})C_t^h$$

$$\frac{M_t^h}{P_t} = \chi C_t^h \left( 1 + i_{t+1}^h \right)$$

$$\kappa l_t^h = \frac{w_t}{P_t C_t^h}$$

Equation (8) is the Euler equation, Equation (9) is the real money demand function,
and Equation (10) shows that the marginal disutility from additional one unit of labor is
equal to the marginal utility from consuming the added wage income.

The situation of foreign household is the mirror image of home household.

B. Government

Government in each country conducts both monetary policy and fiscal policy. All the seigniorage revenues are assumed to be rebated to its own citizens as transfers (and possibly in the form of lump-sum taxes). Thus, the government’s per-capita budget constraint is expressed as follows.

\[ P_t x_t^h = M_t^h - M_{t-1}^h \]

The foreign government’s per capita budget constraint is defined analogously.

C. Enterprises

We assume that a home firm \( h \)'s production function is expressed as \( y_t^h = l_t(h) \).

The real consumption index defined as Equation (2) leads to home household \( h \)'s demand for good \( j \) as:

\[ c_t^h(j) = \left[ \frac{v(j)}{P_t} \right]^\theta C_t^h \]

where \( v(j) \) denotes the home currency price of good \( j \). \( v(j) \) is equal to either \( p_t(j) \), \( p_t^*(j) \), or \( e_t q_t^*(j) \), depending upon which category (domestic good, foreign PTM good, or foreign PCP good) good \( j \) falls within.

When the home firm is a PCP firm, it determines its employment and \( p_t(h) \) to maximize its profit defined as follows.

\[ \pi_t^h = p_t(h) y_t^h - w_t y_t^h \]

where \( y_t^h \) denotes the domestic PCP firm’s production level.

Assuming that prices are flexible, given the demand function expressed in Equation (11), the optimal price is determined as follows.

\[ p_t(i) = \frac{\theta}{\theta - 1} w_t \]

Equation (12) states that monopolistic firms set their goods’ prices at the marginal cost \( (w_t) \) multiplied by the mark-up ratio \( (\theta / (\theta - 1)) \). Note that the export price of PCP firm \( h \) is \( p_t(h)/e_t \).

On the other hand, when the home firm is a PTM firm, its profit can be expressed as follows.

\[ \pi_t^h = p_t(h) x_t^h + e_t q_t(h) z_t^h - w_t (x_t^h + z_t^h) \]

Here, \( x_t^h \) and \( z_t^h \) express the home PTM firm’s production level for the domestic and
Based on these same considerations, the sales price of this PTM firm can be expressed as shown in Equation (13), when prices are flexible.

\[ p_t(h) = e_t q_t(h) = \frac{\theta}{\theta - 1} w_t \]  

(13)

From Equations (12) and (13), the sales price of a PTM firm is equal to that of a PCP firm. Therefore, even if the domestic and foreign markets are segmented and firms set their export prices in different currencies, the Law of One Price holds for every good as long as prices are flexible.

### III. Equilibrium under Price Rigidity

This section introduces price rigidity to the model presented in the previous section following Obstfeld and Rogoff [1995, 1996], to examine how equilibrium is determined under price rigidity.

We assume that the goods’ prices \((p_t(h), q_t(h), p^*_t(h^*), \text{and } q^*_t(h^*))\) in the current period (period \(t\)) are set one period in advance and do not change even if an unexpected shock occurs in the current period. Prices are adjusted in the following period (period \(t+1\)) to incorporate the effects of such shocks. The PCP firms change their export prices in response to exchange rate changes even during period \(t\), while PTM firms’ export prices are constant in period \(t\). Therefore, the effect of exchange rate changes on the general price level in period \(t\) can vary, depending on the ratio of PTM firms.

Households change their consumption schedule following changes in the general price level and bond holdings even in period \(t\). Since monopolistic firms can earn more profit through increased sales even though their sales prices are constant, they will raise their production level to meet any additional increase in demand. In period \(t+1\), the long run effect of goods price changes also emerges.

We can define period \(t\), when households and firms change their consumption and production schedule given constant prices, as the short run, and period \(t+1\), when prices are changed, as the long run. Since agents optimize their intertemporal resource allocations, future changes in consumption and production influence current behavior. Therefore, to examine the short run influence of shocks on the economy, it is necessary to simultaneously consider their long run influence as well.

In this paper, we consider the effects of permanent expansionary monetary policy as

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8 For foreign firm \(h^*\), \(x_t^{h^*}\) and \(z_t^{h^*}\) denote its production level for the domestic and foreign market, respectively.
one example of such shocks. However, we can obtain a closed form solution in this model only at the steady state where the nominal bond holdings are zero. So, we evaluate the effects of monetary policy by obtaining the rate of change from the initial steady state before the shock occurs.

In this section, we first define the initial steady state where the nominal bond holdings are zero and then clarify the relationship between monetary policy and equilibrium conditions in the short run and the long run.

**A. Initial Steady State**

In the initial steady state, consumption and production are both constant without any shock. It is assumed that the activities of individual agents can be represented by the representative agent and are symmetric. Under this assumption, since the first-order conditions for every household and firm are all the same, we drop superscript $h$ and $h^*$ in Equations (8)-(10). Similarly, we do not need to distinguish each firm’s output, since the export prices of PCP and PTM firms are identical. Hereafter in this paper, overbars indicate a symmetric steady state.

From Equation (8), the real interest rate in the steady state is determined by the subjective discount rate as follows.

$$\bar{\delta} = \frac{1 - \beta}{\beta} \equiv \delta$$

Moreover, the budget constraint of representative household as well as goods and bond market equilibrium conditions lead to Equations (14) and (15) held in the steady state.

$$n\bar{C} = n(1 - \bar{d})\bar{B} + n\left(1 - s\right)\bar{p}\bar{y} + s\bar{p}(\bar{x} + \bar{q}\bar{z})$$

(14)

$$(1 - n)\bar{C}^* = -n(1 - \bar{d})\bar{B} + (1 - n)\frac{(1 - s^*)\bar{q}^*\bar{y}^* + s^*\bar{p}^*\bar{x}^* + \bar{q}^*\bar{z}^*)}{\bar{p}^*}$$

(15)

The first terms of the right-hand sides of Equations (14) and (15) express the interest revenues from bond holdings (or interest payments), and the second terms indicate the revenues from the production and sales of goods. Both terms are expressed in real terms. In addition, the nominal bond price $\bar{d}$ can be expressed as $\bar{d} = \beta = 1/(1 + \delta)$ since the inflation rate is zero and the real interest rate equals the nominal interest rate in the steady state.

When prices are flexible, it can easily be shown that $P_t = e_tP_t^*$ holds between
domestic and foreign general price levels, by inserting the Law of One Price for individual goods and the identity of every good’s price into Equations (4) and (5). Therefore, PPP holds in the steady state.

Next, we consider the initial preshock steady state, where the current account balances and bond holdings are zero, in order to obtain a closed form solution. Let 0 subscripts on the barred variables denote this steady state.

In this steady state, \( \overline{B}_0 = 0 \) holds by definition, and the equilibrium is completely symmetric across countries. It is straightforward to show the following closed form solution for labor input, consumption, exchange rate hold.

\[
\bar{C}_0 = \bar{C}_0^* = \bar{l}_0 = \bar{l}_0^* = \left( \frac{\theta - 1}{\theta \kappa} \right)^{\frac{1}{3}}
\]

\[
\bar{e}_0 = \frac{\bar{M}_0}{M^*_0}
\]

Hereafter, for any variable \( U \), the rate of change from the initial steady state, \( \Delta U_0 \), can be expressed as \( \dot{U} = (U - \bar{U}_0)/\bar{U}_0 \). By this method, we can examine the short run and long run effect of monetary policy.

B. Long Run Equilibrium

Suppose permanent monetary easing, \( \hat{\dot{M}}_t - \hat{\dot{M}}^*_t = \hat{\dot{M}}_{t+1} - \hat{\dot{M}}^*_{t+1} \), is implemented in the steady state examined above. The equilibrium in period \( t+1 \) can then be expressed by the following six conditions: (i) the Euler equation, (ii) the money demand function, (iii) the labor supply equation, (iv) the balance of payment equation\(^9\), (v) the market clearing conditions for domestic and foreign goods\(^10\), and (vi) PPP (\( P_{t+1} = e_{t+1}P^*_{t+1} \)).

Based on these conditions, the relationship between money supplies changes (which are an exogenous variable) and changes in consumption, the current account and the foreign exchange rate in period \( t+1 \) can be expressed as shown in Equations (16) and (17). Appendix 1 shows the details regarding the derivation of these equations.

10 The balance of payment equation indicates that the current account balance is equal to the difference between the revenues and the expenditures.

10 In the long run equilibrium, the market clearing conditions for PCP goods are identical to those for PTM goods, since the export prices of PCP firms are equal to those of PTM firm.
The rate of change in each variable is evaluated based on its initial steady state level. Note that the change in bond holdings from the initial steady state \((\hat{B}_{t+1})\) is calculated based on the nominal consumption expenditure in the initial steady state \((\vec{P}_0\vec{C}_0)\), since the current account balances and \(\bar{B}_0 = 0\) hold in the initial steady state.

Equation (16) is the long run money market equilibrium condition and shows that when the domestic money supply increases faster than domestic consumption, the general domestic price level has to rise and, thus, the domestic currency is depreciated. Equation (17) is the long run goods market equilibrium condition. It indicates that a home country current account surplus in period \(t\) will raise home consumption relative to foreign consumption in the long run, since the current account surplus increases bonds holdings, and therefore, interest revenues in period \(t+1\).

C. Short Run Equilibrium

1. Short Run Equilibrium Conditions

Next we examine the equilibrium conditions in period \(t\). It is assumed that firms do not change their goods’ prices \(p_t(h), q_t(h), p_t^*(h^*), \text{and } q_t^*(h^*)\) in response to an expansionary monetary policy shock, and that they passively change their production schedules in period \(t\) in response to the foreign exchange rate changes resulting from the monetary shock. Additionally, households revise their consumption schedule in period \(t\) responding to changes in PCP goods’ prices following the exchange rate fluctuations and increases in money holdings. Based on these assumptions, the short run equilibrium conditions are changed from the long run equilibrium conditions as follows. To begin with, Equation (10), which is the labor supply equations, is no longer valid because of price rigidity. Because PCP and PTM firms set different export prices, market clearing conditions for both PCP goods and PTM goods become necessary. Additionally, PPP does not hold due to the existence of PTM firms, since when the exchange rate changes after \(p_t(h)\) and \(q_t(h)\) were set one period ahead, \(p_t(h)\) is different from \(e_tq_t(h)\) and the Law of One Price does not hold. Nevertheless, uncovered interest rate parity still holds.

To summarize, the equilibrium conditions in period \(t\) are expressed by (i) the Euler equation, (ii) the real money demand function, (iii) the balance of payment equation, (iv) the market clearing conditions for PCP goods, (v) the market clearing condition for PTM goods, and (vi) uncovered interest rate parity. See Appendix 2 for the details of these equations.
2. Short Run Money Market Equilibrium and Goods Market Equilibrium

From the definition of general price indices, it is easy to show that
\[ \hat{P}_t = (1 - n)(1 - s^*) \hat{e}_t \] (18)
\[ \hat{P}_t^* = -n(1 - s) \hat{e}_t \] (19)

With sticky prices, the country’s general price level depends on the firms’ price setting behavior in the other country. That is, the response of the country’s general price level to exchange rate change is lower, the higher the percentage of PTM firms in the other country. In the limit, if all foreign firms are PTM firms \((s^* = 1)\), domestic general price remains constant in the short run. Additionally the foreign general price level does not change at all when all the domestic firms are PTM firms.

From the uncovered interest rate parity, the real interest rate differential between the home and foreign countries becomes Equation (20).
\[ \frac{\delta}{1 + \delta} (\hat{r}_{t+1} - \hat{r}_{t+1}^*) = -[ns + (1 - n)s^*] \hat{e}_t \] (20)

Equation (20) demonstrates that the home and foreign real interest rate differential increases in proportion to the ratio of PTM firms (regardless of whether they are domestic or foreign).11

We can now derive the short run relationship between money supply (which is a policy variable) and the model’s endogenous variables (changes in the foreign exchange rate and consumption).

From Equations (8) and (18)-(20), the short run and long run differentials between the rates of change in domestic and foreign consumption can be expressed by the rate of change in the exchange rate as in Equation (21).
\[ \hat{C}_{t+1} - \hat{C}_{t+1}^* = (\hat{C}_t - \hat{C}_t^*) - [ns + (1 - n)s^*] \hat{e}_t \] (21)

This implies that the short run and long run differentials between the rates of

11 The rates of changes in the domestic and foreign real interest rates can be derived as follows. From Equations (9), (18)-(20), and (27), \( \hat{r}_{t+1} \) and \( \hat{r}_{t+1}^* \), which denote the rate of changes in the domestic and foreign real interest rates between \( t \) and \( t+1 \), can be expressed as
\[ \hat{r}_{t+1} = -\frac{1 + \delta}{\delta} \left\{ C^w_t + (1 - n)[ns + (1 - n)s^*] \hat{e}_t \right\} \]
\[ \hat{r}_{t+1}^* = \frac{1 + \delta}{\delta} \left\{ C^w_t - n[ns + (1 - n)s^*] \hat{e}_t \right\} \]
where \( C^w_t = n \hat{C}_t + (1 - n) \hat{C}_t^* \). In case of \( \hat{e}_t > 0 \), it is easily verified that \( \hat{r}_{t+1} \) and \( \hat{r}_{t+1}^* \) are negative, but the foreign real interest rate does not decline so much compared with the domestic real interest rate.
changes in domestic and foreign consumption are not identical as long as the percentage of PTM firms is not zero, because home and foreign real interest rates diverge.

Substituting Equations (18)-(20) into the short run equilibrium conditions, the short run money and goods markets equilibrium conditions can be expressed as shown in Equations (22) and (23).

\[
(1 - ns - (1 - n)s^*) \frac{1}{\delta} e_i - \frac{1}{\delta} e_i = (\hat{M}_r - \hat{M}_r^*) - (\hat{C}_r - \hat{C}_r^*)
\]

(22)

\[
\frac{1}{(1 - n)(1 + \delta)} \hat{B}_{t+1} = \left( \theta[1 - (1 - n)s - ns^*] + (s + s^* - 1) \hat{e}_i - (\hat{C}_r - \hat{C}_r^*) \right)
\]

(23)

**D. Changes from the Initial Steady State**

In this sub-section, we examine how the endogenous variables are determined after the permanent money supply changes \( \hat{M}_r - \hat{M}_r^* = \hat{M}_{t+1} - \hat{M}_{t+1}^* \). Let the change in variable \( X \) all over the world be \( \hat{X}^W = n\hat{X} + (1 - n)\hat{X}^* \).

First, the money market equilibrium condition in both the short run and the long run, Equation (24), is derived from Equations (16), (21), and (22).

\[
[1 - ns - (1 - n)s^*] e_i = (\hat{M}_r - \hat{M}_r^*) - (\hat{C}_r - \hat{C}_r^*)
\]

(24)

Additionally, from Equations (17), (21), and (23), the goods market equilibrium condition both in the short run and the long run equilibrium can be expressed as follows.

\[
\hat{e}_i = \frac{[2\theta + \delta(\theta + 1)](\hat{C}_r - \hat{C}_r^*)}{\delta(\theta^2 - 1) + 2\theta[ns + (1 - n)s^*] - \delta(\theta + 1)(\theta[(1 - n)s + ns^*] - (s + s^*))}
\]

(25)

Eliminating \( \hat{C}_r - \hat{C}_r^* \) from Equations (24) and (25), the exchange rate change can then be expressed by the rates of changes in the money supplies as follows.

\[
\hat{e}_i = \frac{[2\theta + \delta(\theta + 1)](\hat{M}_r - \hat{M}_r^*)}{\delta\theta(\theta + 1) + 2\theta - \delta(\theta^2 - 1)[(1 - n)s + ns^*]}
\]

(26)

From Equations (22) and (24), we find \( \hat{e}_i = \hat{e}_{t+1} \). Note that Obstfeld and Rogoff [1995, 1996] show that the exchange rate does not over-shoot but rather immediately jumps to the long-term equilibrium level in response to the money supply changes.

\[ \text{12 Betts and Devereux [2000] use the utility function presented in Footnote 6, and show that the exchange rate over-shoots when } \varepsilon > 1. \text{ When } \varepsilon = 1, \text{ they find that the exchange rate does not over-shoot.} \]
This paper also demonstrates that there is no over-shooting of the exchange rate.

Second, the changes in world consumption, $\hat{C}^W$, can be derived as follows from Equations (8), (9), (18), and (19).

$$\hat{C}^W_t = \hat{M}_t^W - n(1-n)(s-s^*)\hat{e}_t$$  \hspace{1cm} (27)

The change in domestic consumption in period $t$ can be shown as follows from Equations (22) and (27), and $\hat{e}_t = \hat{e}_{t+1}$.

$$\hat{C}_t = \hat{M}_t - (1-n)(1-s^*)\hat{e}_t$$  \hspace{1cm} (28)

To derive this equation, we use the relationships $\hat{C}_t = \hat{C}^W_t + (1-n)(\hat{C}_t - \hat{C}_t^*)$ and $\hat{C}_t^* = \hat{C}^W_t - n(\hat{C}_t - \hat{C}_t^*)$.

Then, by substituting Equation (26) into Equation (28), the change in domestic consumption can be expressed by the changes in domestic and foreign money supplies as follows.\(^{13}\)

$$\hat{C}_t = \left[1 - \frac{(1-n)(1-s^*)[2\theta + \delta(\theta + 1)]}{\delta\theta(\theta + 1) + 2\theta - \delta(\theta^2 - 1)(1-n)s + ns^*}ight]{\hat{M}_t} + \left[1 - \frac{(1-n)(1-s^*)[2\theta + \delta(\theta + 1)]}{\delta\theta(\theta + 1) + 2\theta - \delta(\theta^2 - 1)(1-n)s + ns^*}ight]{\hat{M}_t^*}$$  \hspace{1cm} (29)

Noting that $\theta > 1$, the coefficients of $\hat{M}_t$ and $\hat{M}_t^*$ in the above equation are both positive (when $s^* = 1$ then the coefficient of $\hat{M}_t^*$ is 0). Therefore, Equation (29) shows that either domestic or foreign monetary easing increases domestic consumption.\(^{14}\)

Third, let us see how domestic and foreign monetary easing influences domestic production. Because there are domestic and foreign PTM and PCP firms in this model, the aggregate domestic production ($Y$) is defined as $s(x + z) + (1-s)y$, and the

\(^{13}\) Similarly, the change in foreign consumption can be expressed by the domestic and foreign money supply changes by the following equation.

$$\hat{C}_t^* = \frac{n(1-s)[2\theta + \delta(\theta + 1)]}{\delta\theta(\theta + 1) + 2\theta - \delta(\theta^2 - 1)(1-n)s + ns^*} {\hat{M}_t} + \left[1 - \frac{n(1-s)[2\theta + \delta(\theta + 1)]}{\delta\theta(\theta + 1) + 2\theta - \delta(\theta^2 - 1)(1-n)s + ns^*}ight]{\hat{M}_t^*}$$

\(^{14}\) When $s + s^* < 1$, the reason for this result is as follows. First, when expansionary monetary policy is implemented in the home country, the terms of trade of the home country deteriorates by home currency depreciation. On the other hand, since the production increases in response to the improvement in domestic firms’ competitiveness and money holding by domestic households increases, the real income of domestic households increases. In addition, the domestic real interest rate falls. Therefore, domestic consumption increases by domestic monetary easing. Next, when the foreign country implements monetary easing, the terms of trade of the home country improve by the appreciation of the home currency and the domestic real interest rate falls. Therefore, domestic consumption also increases under this case.
aggregate foreign production \((Y^*)\) is defined as \(s'(x^* + z^*) + (1 - s')y^*\). As with the rate of change in domestic consumption, the rate of change in domestic production \((\hat{Y}_t)\) is equal to \(\hat{Y}_t^w + (1 - n)(\hat{Y}_t - \hat{Y}_t^*)\).

The rate of change in world production is equal to that in world consumption in Equation (23). The differential between the rate of changes in domestic and foreign production can be derived as follows from the market clearing conditions for PTM goods and PCP goods.\(^{15}\)

\[
\hat{Y}_t - \hat{Y}_t^* = \theta [1 - (1 - n)s - ns^*] \hat{e}_t
\]

Domestic production can then be expressed by the rates of changes in domestic and foreign money supplies as Equation (30) utilizing the above equation.\(^{16}\)

\[
\hat{Y}_t = \left[ 1 + \frac{2\theta(\theta - 1)[1 - (1 - n)s - ns^*] - [2\theta + \delta(\theta + 1)]s}{\delta\theta(\theta + 1) + 2\theta - \delta(\theta^2 - 1)(1 - n)s + ns^*} \right] \hat{M}_t + (1 - n) - \frac{2\theta(\theta - 1)[1 - (1 - n)s - ns^*] + [2\theta + \delta(\theta + 1)]s}{\delta\theta(\theta + 1) + 2\theta - \delta(\theta^2 - 1)(1 - n)s + ns^*} \hat{M}_t^*
\]

Simple calculations show that the coefficient of \(\hat{M}_t\) is positive, and that the coefficient of \(\hat{M}_t^*\) is either positive or negative depending on the values of \(s\) and \(s^*\). Consequently, while domestic monetary easing always increases domestic production, foreign monetary easing may either increase or decrease domestic production.\(^{17}\)

Finally, the domestic current account balance in period \(t\) is equal to the total amount of fund raising by bond issuance in period \(t\), \(d_{rt}B_{rt+1}\). The change in the current account balance can be expressed as \(\beta \hat{B}_{rt+1}\) by log-linearizing \(d_{rt}B_{rt+1}\), and \(\beta \hat{B}_{t+1}\) is shown as

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\(^{15}\) The differential between the rate of changes in home and foreign production expresses the substitution effect between home and foreign goods of exchange rate changes. This can be easily shown from Equations (6) and (7).

\(^{16}\) Similarly, the change in foreign production can be expressed as follows by the rates of changes in the domestic and foreign money supplies.

\[
\hat{Y}_t^* = n - \frac{2\theta(\theta - 1)[1 - (1 - n)s - ns^*] + [2\theta + \delta(\theta + 1)]s^*}{\delta\theta(\theta + 1) + 2\theta - \delta(\theta^2 - 1)(1 - n)s + ns^*} \hat{M}_t + \left[ 1 + n - \frac{2\theta(\theta - 1)[1 - (1 - n)s - ns^*] - [2\theta + \delta(\theta + 1)]s^*}{\delta\theta(\theta + 1) + 2\theta - \delta(\theta^2 - 1)(1 - n)s + ns^*} \right] \hat{M}_t^*
\]

\(^{17}\) The reason of this conclusion is as follows. First, when expansionary monetary policy is implemented in the home country, the domestic production increases since the depreciation of home currency improves the competitiveness of domestic firms and the domestic monetary easing boosts consumption in the home and foreign countries. Next, when monetary easing is implemented in the foreign country, the home currency appreciation hurts the competitiveness of domestic firms. On the other hand, world demand expands. Therefore, whether the domestic production increases or decreases depends on the degree of deterioration in competitiveness and the degree of the increase in world consumption.
Equation (31) by substituting Equations (24) and (26) into Equation (23).

\[ \beta \hat{B}_{t+1} = \frac{(1 - n)^2 \theta (\theta - 1) [(1 - (1 - n)s - ns^*)]}{\theta (\theta + 1) + 2\theta - \delta (\theta^2 - 1) [(1 - n)s + ns^*]} \left( \hat{M}_t - \hat{M}^*_t \right) \]

(31)

The coefficient of \( \hat{M}_t - \hat{M}^*_t \) in Equation (31) is positive, so domestic monetary easing causes a current account surplus in the home country, while foreign monetary easing causes a current account deficit in the home country.\(^{18}\)

IV. PTM and the International Transmission Effect of Monetary Policy

We now consider in details how PTM influences the effect of monetary policy on endogenous variables defined in the previous section.

A. Effects on Exchange Rate Changes

In the previous section, we show that there is no over-shooting of exchange rate in this model. Then, in this sub-section, we move on to other characteristics of exchange rate changes, that is, the relationship between PTM and the exchange rate changes shown in Equation (26). Betts and Devereux [2000] incorporate PTM price setting behavior into O-R model, but they assume domestic and foreign firms’ price setting behaviors are symmetric (\( s = s^* > 0 \)). They show that the rate of depreciation increases from domestic monetary easing when the percentages of domestic and foreign PTM firms rise at the same tempo. However, in the model presented in this paper, there is no need to assume that the PTM firm ratios rise with \( s = s^* \). From the denominator of the right-hand side of Equation (26), even if \( s \) and \( s^* \) rise at a different tempo, the rate of depreciation increases from domestic monetary easing. That is, Equation (26) generalizes the characteristics of the exchange rate changes shown by Betts and Devereux [2000]. The main contribution of this paper which incorporates asymmetric price setting behavior is to show that the rate of depreciation is not only dependent on the firms’ price setting behavior but also on the economic scales of the home and foreign countries. To consider this point, differentiating the exchange rate changes from the money supply changes with respect to the economic scale of the home country,

\(^{18}\) When expansionary monetary policy is implemented in the home country, home income increases in the short run. This causes the current account surplus in the home country, since the home increases its consumption by less than the increase in income for intertemporal consumption smoothing. Conversely, foreign monetary easing bring about the current account deficit in the home country.
\[ n, \text{ we derive the following equation.} \]
\[
\frac{d(\hat{\sigma_i}/\partial(\hat{M}_i - \hat{M}_i^*))}{dn} = \frac{(s^* - s)(2\theta + \delta(\theta + 1))\delta(\theta^2 - 1)}{(\delta\theta(\theta + 1) + 2\theta - \delta(\theta^2 - 1))(1-\theta)s + n s^*)}\]

(32)

The denominator of the right-hand side and the coefficient of \( s - s^* \) on the numerator of Equation (32) are respectively positive. Therefore, it is clear that the relationship between the exchange rate changes from permanent money supply changes and the economic scale of the home country, \( n \), depend on the relative size of \( s \) and \( s^* \).

Imposing \( s = s^* \) on the right-hand side of Equation (32) following Betts and Devereux [2000], the change in \( n \) does not influence the exchange rate changes from the permanent money supply changes. However, when \( s > s^* \), the exchange rate changes from the permanent money supply changes increase \( n \) approaches 0.\(^{19}\) One intuitive explanation regarding this point is as follows.

Domestic currency depreciation usually causes a shift in demand from foreign goods to home goods. However, since the percentage of domestic PTM firms is lower than that of foreign PTM firms, this effect is weak. Under these circumstances, when \( n \) declines, the number of domestic goods to which demands shift from foreign goods, becomes lower, making it difficult for the demand to shift from foreign goods to domestic goods in accordance with the reduction in \( n \). Therefore, for exchange rate changes to have a given effect on the real economy, the rate of depreciation must be far greater as \( n \) declines.

This result implies that when firms in a small country set their export prices based on the large country’s currency under floating exchange rate regime, the influence of a certain level of monetary easing or tightening on the exchange rate changes increases as the size of the small country gets smaller. The framework of this model is limited in that the country size and the PTM firm ratios are exogenous. Nevertheless, this result implies that the effect of monetary policy on exchange rates is likely to be emphasized in the small country when it uses the large country’s currency as its settlement currency, as in the case of trade between the U.S. and Asian economies.

B. Effects on Consumption

Equation (29) shows that the effect of monetary policy on consumption depends on the value of \( s \) and \( s^* \). This relationship can be summarized as follows.

First, the positive effect of domestic monetary easing on consumption weakens as \( s \)

\(^{19}\) Similarly, when \( s < s^* \), the exchange rate changes from the permanent money supply changes decrease as \( n \) approaches 1.
rises, while it conversely strengthens as $s^*$ rises.

Let us explain the reason why $s$ and $s^*$ exert opposite effects on consumption. When $s$ is high, the exchange rate change does not affect the foreign-currency denominated export prices of domestic goods so much and its effect in shifting world demand from foreign goods towards domestic goods weakens. Moreover, the higher $s$ is, the less the rate of increase of foreign consumption is (see Equation (27)). All of these effects lower the rates of increase in exports, real income, and ultimately in consumption in the home country. On the other hand, when $s^*$ is high, regardless of the depreciation of the home currency, the import prices cease to rise and the terms of trade do not deteriorate by much in the home country, making monetary easing directly increase domestic real income. As a result, the rate of increase in domestic consumption is raised by an increase in $s^*$. In particular, when all the foreign firms are PTM firms ($s^* = 1$), domestic monetary easing exerts a great influence on domestic consumption but foreign monetary easing does not effect domestic consumption at all, as clarified by the fact that Equation (29) becomes $\dot{C}_t = \dot{M}_t$.

Similarly, when the monetary easing is implemented in the foreign country, the rate of increase in domestic consumption rises with higher $s$ and it falls with higher $s^*$. See Figure 1 for these effects of PTM on consumption.

C. Effects on Production

From Equation (30), the positive effect from domestic monetary easing on domestic production diminishes and that from foreign monetary easing on domestic production strengthens (or the negative impact weakens) with a rise in both $s$ and $s^*$. The reason is that an increase in the PTM firm ratios (regardless of whether they are domestic or foreign) reduces the response of local currency denominated export prices to exchange rate changes, leading to a weaker demand shift between domestic goods and foreign goods (it lowers the substitution effect). In the special case where all the domestic and foreign firms are PTM firms (where $s = s^* = 1$), both nations’ export prices remained unchanged regardless of exchange rate changes, so production in both nations expands by the same amount in line with the expansion in world consumption. \(^{20}\) See Figure 2 shows this relationship.

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\(^{20}\) This result is clarified by following equations.

$$\dot{Y}_t = n\dot{M}_t + (1-n)\dot{M}_t^w$$

$$\dot{Y}_t^* = n\dot{M}_t^* + (1-n)\dot{M}_t^w$$

These equations are derived by substituting $s = s^* = 1$ into the change in domestic production in Equation (30) and the change in foreign production shown in Footnote 16.
D. Effects on the Changes in Current Account Balance

As in the cases of consumption and production, Equation (31) reveals that the effect of monetary easing on current account balances can also change depending on $s$ and $s^*$. As the PTM firm ratio rises (regardless of whether firms are domestic or foreign), the domestic current account surplus by domestic monetary easing and the domestic current account deficit by foreign monetary easing diminish. Figure 3 summarizes this relationship.

The intuitive reasons behind these results can be explained using the case where the monetary easing is implemented in the home country. First, as $s$ rises, the rates of increase in production and real income decline in the home country. Therefore, the income, which will be spent for future consumption from the perspective of consumption smoothing, decreases and the current account surplus diminishes. Next, as $s^*$ rises, the import prices cease to rise regardless of the depreciation of the home currency, leading to an increase in the short run real income, although the rate of increase in domestic production declines. So, the domestic households try to save for the future consumption. However, since the foreign real interest rate does not fall by as much as the home real interest rate, the foreign households are not eager to borrow funds from the home country, and increase their short run consumption. As a result, even with a rise in $s^*$, the home current account surplus lessens.

The current account surplus leads to an increase in consumption and a decrease in production in the long run.\(^ {21} \)

\(^{21}\) Specifically, the rate of changes in domestic and foreign consumption in the long run can be expressed by $B$ as follows

\[
\dot{C}_{i+1} = \frac{\theta + 1}{2\theta} \frac{\delta}{1+\delta} \dot{B}_{i+1}
\]

\[
\dot{C}^*_i = -\frac{n \theta + 1}{1-n} \frac{\delta}{2\theta} \frac{1+\delta}{1+\delta} \dot{B}_{i+1}
\]

These equations are derived from the domestic and foreign labor supply equation in the long run, Equations (1-5) and (1-6) in Appendix 1, and the differential between rates of changes in domestic and foreign consumption in the long run, Equation (17). These equations imply that compared with the steady state levels, the domestic consumption increases and foreign consumption decreases in the long run in response to the domestic current account surplus (the foreign current account deficit) from domestic monetary easing.

As for the change in production in the long run, we can derive following equations from Equations (1-5) through (1-10) in Appendix 1.

\[
\dot{Y}_{i+1} = -\frac{\delta}{2(1+\delta)} \dot{B}_{i+1}
\]

\[
\dot{Y}^*_{i+1} = -\frac{n \delta}{1-n} \frac{1+\delta}{2(1+\delta)} \dot{B}_{i+1}
\]

That is, compared with the initial steady state levels, the domestic production decreases and the foreign production increases in the long run in response to the domestic current account surplus (the...
increase in the both countries’ PTM firm ratios, because the current account surplus decreases. When \( s = s^* = 1 \), the monetary easing has no long run effect whatsoever, since the current account always balances.

V. Welfare Based Analysis on the International Transmission Effect of Monetary Policy with Special Emphasis on PTM

Because the model in this paper has a micro foundation including consumers’ utility maximization, the effects of economic policies can be evaluated on a welfare basis. The objective of this section is to clarify how the existence of domestic and foreign PTM firms influences the international transmission effect of monetary policy from the perspective of its effect on economic welfare.

A. Influence of Monetary Policy on Economic Welfare

Monetary policy has both a short run and a long run influence on macroeconomic variables. For this reason, to understand the influence of monetary policy on economic welfare, it is necessary to consider the changes in utility levels in both the short run and the long run. Accordingly, in the following analyses the sum of (i) the change in the utility from the initial steady state in the short run and (ii) the present discounted value of the change in the utility from the initial steady state in the long run is used to determine the overall changes in economic welfare.

The derivation method used for calculating economic welfare follows Obstfeld and Rogoff [1995, 1996]. We assume that the utility derived from money holding is so small that it can be ignored (\( \chi \to 0 \)). Additionally utilizing the fact that labor inputs equal production, and that a new steady state is attained one period after shocks, the changes in welfare levels can be expressed as Equation (33) by the changes in consumption and production both in the short run and the long run.

\[
dU = \hat{C}_t - \frac{\theta - 1}{\theta} \hat{Y}_t + \frac{1}{\delta} \left( \hat{C}_{t+1} - \frac{\theta - 1}{\theta} \hat{Y}_{t+1} \right)
\]

Substituting Equations (29) and (30) into Equation (33) and making use of the fact that the changes in consumption and production in the long run can be expressed by the changes in bond holdings, the changes in home welfare can now be shown by the

foreign current account deficit) from domestic monetary easing.
domestic and foreign money supply changes as follows.

\[
dU = \left[ \frac{n}{\theta} + \frac{(1-n)}{\theta} \left\{ \frac{(\theta-1)[ns + (1-n)s^*] + s^*}{\theta} \right\}^{2\theta + \delta(\theta + 1)} \right] \hat{M}_t + \frac{1-n}{\theta} \left\{ \frac{(\theta-1)[ns + (1-n)s^*] + s^*}{\theta} \right\}^{2\theta + \delta(\theta + 1)} \hat{M}^*_t
\]

(34)

From Equation (34), we know that the coefficient of \( \hat{M}_t \) is always positive and that the coefficient of \( \hat{M}^*_t \) may be positive or negative depending on the values of \( s \) and \( s^* \). In other words, while domestic monetary easing always improves domestic utility, foreign monetary easing can deteriorate domestic utility, depending on the PTM firm ratios.

Moreover, it follows from Equation (34) that as the PTM firm ratio rises (regardless of whether firms are domestic or foreign), the domestic welfare improves more from a given extent of monetary easing and diminishes (or the deterioration of domestic welfare accelerates) from a given extent of foreign monetary easing. Figure 4 summarizes this relationship.

Let us now consider the reasons for these conclusions.

In the case of domestic monetary easing, a rise in \( s \) lowers the rate of increase in consumption in the short run and the current account surplus (leading to a decline in the long run rate of increase in domestic consumption and a reduction in the long run rate of decrease in domestic production). However, the rate of increase in short run production lessens (the rate of decrease in leisure eases). The reductions in the rate of increase in consumption and the current account surplus do have negative impacts on welfare, and the decline in the rate of increase in production exerts positive impacts on welfare. However, this negative effect is outweighed by the positive effects, and thus, on the whole, as \( s \) rises, the improvement in domestic welfare accelerates. Meanwhile, an increase in \( s^* \) also has the negative effect of reducing the home country’s current account surplus and the positive effect of raising the rate of increase in consumption and reducing the rate of increase in production. However, in this case as well, the positive effects outweigh the negative effects. Consequently, when \( s^* \) rises, domestic monetary easing enhances domestic welfare.

Next we turn to the case of foreign monetary easing. Here, an increase in \( s \) expands the positive effect of raising the short run rate of increase in consumption and reducing the current account deficit, which leads to reductions in the rate of decrease in consumption and the rate of increase in production in the long run. But these positive effects are outweighed by the negative effect of expansion of the rate of increase in
production (or reduction in the rate of decrease of production). Thus, an increase in \( s \) slows the rate of growth (or accelerates the rate of decline) in home welfare. Meanwhile, an increase in \( s^* \) does reduce the home country’s current account deficit, but this is outweighed by the negative effects of lowering the short run consumption growth rate and increasing the production growth rate (or reducing the rate of production decline). Thus it decreases the rate of growth in domestic welfare (or accelerates the rate of decline in domestic welfare).

B. PTM and the International Transmission Effects of Domestic and Foreign Monetary Policy

We now examine the international transmission effects of domestic and foreign monetary policies from the perspective of welfare, based on the above considerations.

First we define beggar-thy-neighbor effect policies as those that reduce the welfare of the foreign country.

Under this definition, it follows from Equation (34) that foreign monetary policy has a beggar-thy-neighbor effect as long as \( s \) and \( s^* \) meet the following condition.

\[
s > -\frac{\delta\theta(\theta + 1) + 2\theta[1 + (\theta - 1)(1 - n)]}{\delta(\theta^2 - 1) + 2\theta(\theta - 1)n} s^* + \frac{\delta\theta(\theta + 1) + 2\theta}{\delta(\theta^2 - 1) + 2\theta(\theta - 1)n} \tag{35}
\]

Similarly, from the relationship between changes in domestic and foreign money supplies and the changes in foreign utility,\(^{22}\) domestic monetary policy has a beggar-thy-neighbor effect when as \( s \) and \( s^* \) satisfy the following condition.

\[
s > -\frac{\delta\theta(\theta^2 - 1) + 2\theta(\theta - 1)(1 - n)}{\delta\theta(\theta + 1) + 2\theta[(\theta - 1)n + 1]} s^* + \frac{\delta\theta(\theta + 1) + 2\theta}{\delta\theta(\theta + 1) + 2\theta[(\theta - 1)n + 1]} \tag{36}
\]

Figure 5 shows the relationship between the domestic and foreign PTM firm ratios and the international transmission effects of domestic and foreign monetary policies from the perspective of the economic welfare on the \((s, s^*)\) dimension.

In this figure, when the values of \( s \) and \( s^* \) fall within Quadrant (I), the domestic and foreign monetary policies have positive international transmission effects. When the values of \( s \) and \( s^* \) fall within Quadrant (IV), the domestic and foreign monetary policies

\(^{22}\) Specifically, the changes in foreign utility can be expressed as follows.

\[
dU^* = \left[\frac{n}{\theta} \cdot \left\{\theta - 1\right\}[ns + (1 - n)s^*] + s^* + \frac{2\theta + \delta\theta(\theta + 1)}{\theta + 2\theta + \delta\theta(\theta + 1) - \delta(\theta^2 - 1)(1 - n)s + ns^*}] M^*_t + \right.
\]

\[
\left[\frac{1 - n}{\theta} + \frac{n}{\theta} \cdot \left\{\theta - 1\right\}[ns + (1 - n)s^*] + s^* + \frac{2\theta + \delta\theta(\theta + 1)}{\theta + 2\theta + \delta\theta(\theta + 1) - \delta(\theta^2 - 1)(1 - n)s + ns^*}] M^*_t, \right.
\]
have beggar-thy-neighbor effects.

However, when the values of $s$ and $s^*$ fall within Quadrant (II), that is, when the home PTM firm ratio is high and the foreign PTM firm ratio is low, the domestic monetary policy has a beggar-thy-neighbor effect while the foreign monetary policy has a positive transmission effect on the home country. Finally, when the values of $s$ and $s^*$ fall within Quadrant (III), in other words, when the domestic PTM firm ratio is low but the foreign PTM firm ratio is high, domestic monetary policy has a positive transmission effect on the foreign country and foreign monetary policy has a beggar-thy-neighbor effect. This shows that the international transmission effects of home and foreign monetary policies can be asymmetric if $s$ and $s^*$ fall within Quadrant (II) or (III).23

The origin included in Quadrant (I) corresponds to the case of $s = s^* = 0$ which Obstfeld and Rogoff [1995, 1996] assume. The positive international transmission effect of monetary policy in Quadrant (I) is consistent with the conclusions reached in Obstfeld and Rogoff [1995, 1996]. Moreover, the case of domestic and foreign firms’ symmetric price setting behaviors, which Betts and Devereux [2000] consider, corresponds to the 45-degree line in Figure 5. The negative international transmission effect in Quadrant (IV) is consistent with the conclusions reached in Betts and Devereux [2000] that monetary policy can have a beggar-thy-neighbor-effect when the PTM firm ratios are sufficiently high. The new findings of this paper are that the international transmission effects of monetary policies can be asymmetric when $s$ and $s^*$ are in Quadrant (II) or (III).

The intuitive reasons why the international transmission effects of monetary policies can be asymmetric when the price setting behaviors of domestic and foreign firms are asymmetric may be summarized as follows.

First, in Quadrant (II), because the domestic PTM firm ratio is high while the foreign PTM firm ratio is low, domestic monetary easing increases production in the short run and causes a current account deficit in the foreign country. Similarly, foreign

23 It can easily be verified that Quadrants (II) and (III), where the international transmission effects of domestic and foreign monetary policies are asymmetric, always exist. First, the $s$ intercept of Equation (35) and the $s^*$ intercept of Equation (36) can be expressed respectively as follows.

\[
\begin{align*}
    s^* &= \frac{\theta(\theta+1)+2\theta}{\theta(\theta+1)+2[\theta(\theta-1)(1-n)+1]} \\
    s &= \frac{\theta(\theta+1)+2\theta}{\theta(\theta+1)+2[\theta(\theta-1)n+1]}
\end{align*}
\]

Since $\theta > 1$, these intercepts are less than 1. Additionally, Equation (35) and (36) are both downward sloping to the right, and the slope of Equation (35) is more acute than that of Equation (36). Consequently, it follows that Quadrants (II) and (III) always exist.

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monetary easing enhances production in the short run and leads to current account
deficit in the home country. However, as is clear from the considerations thus far,
when the domestic PTM firm ratio is high, the domestic monetary easing results in a
deceleration in foreign consumption growth, and foreign monetary easing results in an
acceleration in domestic consumption growth. This shows the reason why the positive
effect of domestic monetary easing on foreign welfare is smaller than that of foreign
monetary easing on domestic welfare.

To sum up these effects, it can be concluded that in Quadrant (II) domestic monetary
policy has a beggar-thy-neighbor-effect, while foreign monetary policy improves
domestic welfare because of their asymmetric effects on consumption.

Note that monetary policies have asymmetric international transmission effects so
long as the domestic PTM firm ratio stays within a certain range. The case where the
foreign PTM firm ratio is low and the domestic PTM firms ratio is extremely high
corresponds to Quadrant (IV). In Quadrant (IV), both domestic and foreign monetary
policies have beggar-thy-neighbor-effects.

Behind the transmission effect of monetary policy in Quadrant (IV) lies the
following mechanism. As the domestic PTM firm ratio rises, foreign monetary easing
accelerates the growth of home production. That is, in Quadrant (IV) the positive
effect of foreign monetary easing (an increase in domestic consumption) is outweighed
by its negative effect (an increase in domestic production), so foreign monetary policy
reduces the domestic utility level. Thus in this quadrant both domestic and foreign
monetary policies have beggar-thy-neighbor-effects.

VI. Implications for Japanese Monetary Policy

According to the empirical findings of Knetter [1993] and others, Japanese firms absorb
approximately half of all foreign exchange rate changes by changing their mark-up
ratios, while U.S. firms almost all set their export prices based on PCP.

This section begins by setting the model parameters to numerically analyze the
influence of Japanese and U.S. monetary policies on consumption, production and the
current account balances based on former empirical results. The international
transmission effects of Japanese and U.S. monetary policies are then examined from an
economic welfare perspective, and some considerations are given to the optimal
implementation of monetary policy from an international perspective.
A. Setting of Parameter Values

To begin with, we posit Japan as the home country, the U.S. as the foreign country, and adopt the values $s = 0.5$ and $s^* = 0$ based on the prior research. From the Japanese and U.S. nominal GDP in 2000, we set the economic scale ($n$) of the home country (Japan) at 0.3, and the real interest rate at 4% (this corresponds to $\beta = 0.96$ on an annual basis).

The elasticity of substitution ($\theta$) is also an important variable, and its value significantly changes the simulation results. In the prior research, the elasticity of substitution between domestic goods and foreign goods is often set at 1~2, and the elasticity of substitution among domestic goods is frequently set at a high level of around 11 (for example, by Betts and Devereux [1999]).

This paper, however, assumes the same elasticity of substitution among all goods, regardless of the distinction between domestic goods and foreign goods, and thus we set the range of the value of $\theta$ based on the estimated actual mark-up ratios. For example, Betts and Devereux [1996] set the elasticity of substitution at 6 based on the assertion made by Rotemberg and Woodford [1992] that the U.S. mark-up ratio is 1.2. Among research on the mark-up ratio in Japan, Baba [1995] shows that the elasticity of substitution for the Japanese manufacturing industry is around 1.3. This implies that the elasticity of substitution is around 4-5. Therefore, this paper adopts a figure of 4-6 for the elasticity of substitution.

B. International Transmission Effects of Japanese and U.S. Monetary Policies and Implications for the Implementation of Monetary Policy


Based on the above parameter values, we now move on to numerical analyses on the international transmission effects of Japanese and U.S. monetary policies, and in particular, on the effects on macroeconomic variables. These analyses adopt the

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24 Under given PTM firm ratio and country scale figures, changes in the value of elasticity of substitution change the intercepts and slopes of Equations (35) and (36) in Figure 8. For example, as the elasticity of substitution approaches 1, both equations move away from the origin. Therefore the area of Quadrant (I) expands, and Japanese monetary policy improves the welfare of the U.S. with high probability. As the elasticity of substitution becomes larger, both equations approach the origin, and the area where monetary policy has a beggar-thy-neighbor effect expands. Since this paper examine the case of $s^* = 0$, the sizes of the $s$-intercepts of Equations (35) and (36) are related to whether or not monetary policy has a beggar-thy-neighbor-effect.

25 Chari, Kehoe, and MacGrattan [1998] state that almost all of the reliable research papers adopt a figure of 1-2 for the U.S. elasticity of substitution between domestic and foreign goods.

26 Mark-up ratios can be expressed as $\frac{\theta}{(\theta - 1)}$ in terms of the elasticity of substitution.
assumption taken by Obstfeld and Rogoff [1995, 1996] that all firms are PCP firms as the benchmark case.

Figure 6 examines the influence of Japanese and U.S. monetary policies on consumption. The transmission effect of Japanese monetary easing on the U.S. consumption is approximately 1/2 of that under the benchmark case where all the Japanese and U.S. firms are PCP firms. Thus, the world demand creation effect falls to about 2/3 of that under the benchmark case. Conversely, U.S. monetary easing works to increase U.S. and Japanese consumption growth rates because of the existence of Japanese PTM firms, and thus its world demand creation effect is amplified.

Next, turning to the influence on production in Figure 7, both Japanese and U.S. monetary easing increase domestic production but decrease production in the other country. The positive and negative effects are significantly diminished compared with the benchmark case, since the Japanese PTM firm ratio is high.

Finally, as for the influence on the current account balances, which is examined in Figure 8, monetary easing in either country causes a (per capita) current account surplus in the country and a (per capita) current account deficit in the other. However, the extent of this influence is greatly diminished in comparison with the benchmark case.


We now turn to the effect of Japanese and U.S. monetary policy in terms of economic welfare (Figure 9). Compared with the benchmark case, monetary easing in either country works to increase that country’s utility, but simultaneously lessens the improvement in utility, or even diminishes the utility, in the other country. It follows from the simulation result in Figure 9 that U.S. monetary easing increases Japanese utility, while the effect of Japanese monetary easing on U.S. utility is so small that it can be neglected. Additionally, under Japanese monetary easing when the elasticity of substitution ($\theta$) is assumed to be 5 or 6, U.S. economic welfare actually worsens slightly. The reason for this result can be explained as follows, using Figure 5.

When the Japanese PTM firm ratio is 0.5 and the U.S. PTM firm ratio is 0, the relationship between $s = 0.5$ on the vertical axis and the $s$-intercepts of Equations (35) and (36) represents Quadrants (I), (II), or (IV), to which the simulation results of the cases $\theta = 4, 5,$ and 6 correspond. Therefore, we can judge whether monetary policies in

27 Although there is a large current account imbalance between U.S. and Japan in the real world, this simulation estimates the changes in economic welfare from unexpected monetary easing in the steady state where the current account balances are zero.
both countries have beggar-thy-neighbor effects or not by comparing the $s$-intercepts of both equations and $s = 0.5$.

First, when the real interest rate is 4%, the value of the $s$-intercept of Equation (36), which is the boundary between Quadrants (I) and (II), is less than 0.5 if the elasticity of substitution is more than 4.72. In this case, Japanese monetary policy has a beggar-thy-neighbor effect. Therefore, we can confirm $\theta = 4$ results in a different conclusion from the case where $\theta$ is more than 5 in terms of whether or not Japanese monetary policy has a beggar-thy-neighbor effect.

Next, when the real interest rate is 4%, the $s$-intercept of Equation (35), which is the border between Quadrants (II) and (IV), is less than 0.5 if the elasticity of substitution is more than 8.37. When $s = 0.5$, both Japanese and U.S. monetary policies have beggar-thy-neighbor effects. Therefore, if we assume that the elasticity of substitution is 11 as in Betts and Devereux [1999], both Japanese and U.S. monetary policies have beggar-thy-neighbor effects even if we incorporate the asymmetry of $s$ and $s^*$.

3. Implications for the Implementation of Monetary Policy

Based on these simulation results, how should monetary policy then be conducted from an international perspective?

First of all, if we assume that the elasticity of substitution ($\theta$) is 4-6, Japanese and U.S. monetary policies do not simultaneously have beggar-thy-neighbor effects. Suppose that the value of $\theta$ is within the range of 5-6 and that Japanese monetary easing exerts a small negative influence on the U.S. through the depreciation of yen as shown in Figure 9. Then, even if the U.S. implements monetary easing to offset this negative impact, this policy change does not have a negative effect on Japan according to Figure 9. Therefore, competitive devaluation through a chain of easy monetary policies, in the sense that Japan conducts more monetary easing responding to U.S. policy changes, cannot occur within the range of monetary easing which is valid under the log-linearization approach among the steady states adopted in this paper.

Second, based on the relatively high Japanese PTM firm ratio, Japanese monetary policy has a miniscule effect on U.S. demand creation, and its effect on U.S. utility is nearly zero, so its international transmission effect is extremely small. For example, according to Figure 6, the effect of Japanese monetary policy on world demand is 3/7 of the effect of U.S. monetary policy (this is equal to the ratio of the two nations’ economic scales) when all the Japanese and U.S. firms are PCP firms. However, Japanese monetary policy has only 1/4 of the effect of U.S. monetary policy when half of the Japanese firms are PTM firms. This means that Japan should consider that the
international transmission effect of Japanese monetary policy is small in its conduct of monetary policy.

Third, the international transmission effect of U.S. monetary policy is bigger than that of Japanese monetary policy since the U.S. PTM firm ratio is relatively low. Because of this, other conditions being constant, U.S. monetary policy should hopefully give a greater emphasis to the international perspective compared with Japanese monetary policy.

VII. Conclusions

This paper incorporates asymmetrical price-setting behavior between the home country and the foreign country into the “new open economy macroeconomics” framework proposed by Obstfeld and Rogoff, and then examines how this affects the international transmission effect of monetary policy.

The findings of this paper can be summarized as follows:

(1) This paper shows that the exchange rate does not over-shoot but rather jumps to its long run equilibrium level immediately after the implementation of monetary policy. However, this result depends on the structure of the utility function.

(2) This paper confirms that a rise in the percentage of PTM firms magnifies the response of the exchange rate to monetary policy, consistent with Betts and Devererux [2000]. Moreover, the exchange rate volatility is dependent on the percentages of domestic and foreign PTM firms together with the sizes of the domestic and foreign economies. For example, the exchange rates between large and small countries become more volatile in proportion with the percentage of PTM firms in the small country. This finding indicates that the exchange rate volatility between large and small countries is high when firms in the small country use the currency of the large country as their invoice currency, as long as a floating exchange rate regime is in place. It may apply to the exchange rate fluctuation characteristics between the U.S. and Asian economies or between the EU and East European countries.

(3) The international transmission effects of domestic and foreign monetary policies depend on firms’ price setting behavior. Consequently, the international transmission effect of domestic monetary policy can be asymmetric against that of foreign monetary policy when domestic and foreign firms’ price setting behaviors are different. For example, if the ratio of PTM firms in the home country is high
and that in the foreign country is low, domestic monetary policy may have a beggar-thy-neighbor-effect in the sense that it deteriorates the welfare of the foreign country. On the other hand, foreign monetary policy can enhance the welfare of the home country.

(4) The simulation analysis based on Japanese and U.S. data shows that the difference between Japan and U.S. firms’ price setting behavior results in a weak transmission effect of Japanese monetary policy on the U.S. economy and a strong transmission effect of U.S. monetary policy on the Japanese economy. The following mechanism provides the background to this result. Even if the yen/dollar exchange rate fluctuates due to changes in Japanese monetary policy, Japanese firms pass only a portion of the exchange rate fluctuation on to their export prices to the U.S. Thus the change in U.S. import prices is limited and U.S. demand is not greatly influenced by the Japanese monetary policy. Conversely, U.S. firms’ export prices change by as much as the exchange rate fluctuations, hence the Japanese economy is more strongly affected by the U.S. monetary policy.

These simulation results illustrate that we should not overvalue the international transmission effect of Japanese monetary policy if the current percentage of Japanese PTM firms remains reasonably stable.
Appendix 1: The Long Run Relationship between Foreign Exchange Rate Changes and Macroeconomic Variables

Appendix 1 explains the long run equilibrium conditions and the derivations of Equations (16) and (17). The equilibrium conditions at period $t+1$ can be expressed by the following eleven equations.

(Euler Equation)
\[
C_{t+2} = \beta (1 + r_{t+2}) C_{t+1}
\]  
(1- 1)
\[
C^*_{t+2} = \beta (1 + r^*_{t+2}) C^*_{t+1}
\]  
(1- 2)

(Real Money Demand Function)
\[
\frac{M_{t+1}}{P_{t+1}} = \chi C_{t+1} \left( \frac{1 + i_{t+2}}{i_{t+2}} \right)
\]  
(1- 3)
\[
\frac{M^*_{t+1}}{P^*_{t+1}} = \chi C^*_{t+1} \left( \frac{1 + i^*_{t+2}}{i^*_{t+2}} \right)
\]  
(1- 4)

(Labor Supply Equation)
\[
-\kappa l_{t+1} + \frac{w_{t+1}}{P_{t+1}} C_{t+1} = 0
\]  
(1- 5)
\[
-\kappa l^*_{t+1} + \frac{w^*_{t+1}}{P^*_{t+1}} C^*_{t+1} = 0
\]  
(1- 6)

(Balance of Payments Equation)
\[
d_{t+2} B_{t+2} = B_{t+1} + P_{t+1} y_{t+1} - P_{t+1} C_{t+1}
\]  
(1- 7)
\[
\frac{d_{t+2}}{e_{t+1}} B^*_{t+2} = B^*_{t+1} + q_{t+1} y^*_{t+1} - P^*_{t+1} C^*_{t+1}
\]  
(1- 8)

(Market Clearing Condition for Goods)
\[
y_{t+1} = x_{t+1} + z_{t+1} = \left( \frac{P_{t+1}}{P_{t+1}} \right)^\theta \left[ nC_{t+1} + (1-n)C^*_{t+1} \right]
\]  
(1- 9)
\[
y^*_{t+1} = x^*_{t+1} + z^*_{t+1} = \left( \frac{e_{t+1} q_{t+1}}{P_{t+1}} \right)^\theta \left[ nC_{t+1} + (1-n)C^*_{t+1} \right]
\]  
(1- 10)

(PPP)
\[
P_{t+1} = e_{t+1} P^*_{t+1}
\]  
(1- 11)
Among these, Equations (1-1)-(1-6) are directly derived from the first-order conditions for households, Equations (8)-(10).

Equations (1-7) and (1-8) show that the nominal bond holdings are determined by the difference between the sum of nominal bond holdings in the previous period and the income derived from production and consumption expenditures.

Equations (1-9) and (1-10) are derived from the aggregation of the demand for the good $j$ across all agents. Note that since prices are flexible at period $t+1$, the prices set by PTM firms and PCP firms are identical. Thus, the market clearing conditions for PTM and PCP firms are the same, PPP always holds, and the domestic and foreign real interest rates are equalized.

Finally, as for the derivations of Equations (16) and (17), Equation (16) is obtained by converting Equations (1-1) and (1-2) to log-linear expressions, and utilizing the fact that PPP is maintained from period $t+1$. Equation (17) is derived by log-linearizing Equations (1-3)-(1-10), and then deleting the terms $y_{t+1}$, $y^*_{t+1}$, and $e_{t+1}$ from them, using the fact that $B_{t+1}$ equals $B_{t+2}$. 
Appendix 2: Short Run Equilibrium Conditions

The short run equilibrium conditions can be expressed by the following thirteen equations.

(Euler Equation)
\[ \frac{C_{t+1}}{C_t} = \beta (1 + r_{t+1}) \]  
\[ \frac{C_{t+1}^*}{C_t^*} = \beta (1 + r_{t+1}^*) \]  

(Real Money Demand Function)
\[ \frac{M_t}{P_t} = \chi C_t \left( \frac{1 + i_{t+1}}{i_{t+1}} \right) \]  
\[ \frac{M_t^*}{P_t^*} = \chi C_t^* \left( \frac{1 + i_{t+1}^*}{i_{t+1}^*} \right) \]  

(Balance of Payments Equation)
\[ d_{t+1}B_{t+1} = B_t + (1 - s)p_t y_t + s(p_t x_t + e_t q_t) - P_t C_t \]  
\[ \frac{d_{t+1}B_{t+1}^*}{e_t} = \frac{B_t^*}{e_t} + (1 - s)q_t^* y_t^* + s(\frac{p_t^*}{e_t} x_t^* + q_t^* z_t^*) - P_t^* C_t^* \]  

(Market Clearing Condition for PCP Goods)
\[ y_t = \left( \frac{p_t}{P_t} \right)^{-\theta} nC_t + \left( \frac{p_t}{e_t P_t} \right)^{-\theta} (1 - n)C_t^* \]  
\[ y_t^* = \left( \frac{e_t q_t^*}{P_t} \right)^{-\theta} nC_t + \left( \frac{q_t^*}{P_t^*} \right)^{-\theta} (1 - n)C_t^* \]  

(Market Clearing Condition for PTM Goods)
\[ x_t = \left( \frac{p_t}{P_t} \right)^{-\theta} nC_t \]  
\[ x_t^* = \left( \frac{p_t^*}{P_t^*} \right)^{-\theta} nC_t \]  
\[ z_t = \left( \frac{q_t}{P_t^*} \right)^{-\theta} (1 - n)C_t^* \]
\[ z_i^* = \left( \frac{q_i^*}{P_i^*} \right)^{-\theta} (1 - n)C_t^* \]  

(2-12)

(Uncovered Interest Rate Parity)

\[(1 + i_{t+1}) = \frac{e_{t+1}}{e_t} (1 + i_{t+1}^*) \]  

(2-13)

These thirteen equations contain twelve unknown variables \((C_t, C_t^*, i_{t+1}, i_{t+1}^*, e_t, x_t, x_t^*, z_t, z_t^*, y_t, y_t^*, \text{ and } B_{t+1})\). However, from Walras’ law one of these equations is not independent, so the number of equations and the number of unknown variables becomes equal, showing that these equations do express the equilibrium at period \(t\).

In addition, the difference between these equations and the long run equilibrium conditions in the previous appendix is as follows. Since prices are assumed to be rigid in the short run, the prices set by PCP and PTM firms are not identical. Thus, Equations (1-9) and (1-10) have to be substituted by Equations (2-7) through (2-12). Additionally, the labor supply equation does not hold in the short run.

Although Equation (2-13) holds even in the short run because of bond transactions in the international bond market, PPP does not hold due to the existence of PTM firms, so Equation (1-11) is excluded from the equilibrium conditions. Moreover, because the general price levels in both countries do not change symmetrically, the domestic and foreign real interest rates are not equalized.

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28 Assuming that the terms \(C_{t+1}, C_{t+1}^*, e_{t+1}, B_t, \text{ and } r_t\) are given.


Otani, Akira, “Atarashii Kaihou Makuro Keizaigaku ni tsuite – PTM (Pricing-to


Figure 1: PTM Effect for the Influence of Monetary Policy on Consumption

<table>
<thead>
<tr>
<th></th>
<th>Domestic Monetary Easing</th>
<th>Foreign Monetary Easing</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Domestic Consumption ( (C_t) )</strong></td>
<td>Positive ((\partial \hat{C}_t / \partial \hat{M}_t &gt; 0))</td>
<td>Positive (including (0)) ((\partial \hat{C}_t / \partial \hat{M}_t^* \geq 0))</td>
</tr>
<tr>
<td>Effect of an increase in (s)</td>
<td>Decline in rate of increase</td>
<td>Acceleration in rate of increase</td>
</tr>
<tr>
<td>Effect of an increase in (s^*)</td>
<td>Acceleration in rate of increase (in case of (s^* = 1, \hat{C}_t = \hat{M}_t))</td>
<td>Decline in rate of increase (in case of (s^* = 1, \hat{C}_t = 0))</td>
</tr>
<tr>
<td><strong>Foreign consumption ( (C_t^*) )</strong></td>
<td>Positive (including (0)) ((\partial \hat{C}_t^* / \partial \hat{M}_t \geq 0))</td>
<td>Positive ((\partial \hat{C}_t^* / \partial \hat{M}_t^* &gt; 0))</td>
</tr>
<tr>
<td>Effect of an increase in (s)</td>
<td>Decline in rate of increase (in case of (s = 1, \hat{C}_t^* = 0))</td>
<td>Acceleration in rate of increase (in case of (s = 1, \hat{C}_t^* = \hat{M}_t^*))</td>
</tr>
<tr>
<td>Effect of an increase in (s^*)</td>
<td>Acceleration in rate of increase</td>
<td>Decline in rate of increase</td>
</tr>
</tbody>
</table>
Figure 2: PTM Effect for the Influence of Monetary policy on Production

<table>
<thead>
<tr>
<th></th>
<th>Domestic Monetary Easing</th>
<th>Foreign Monetary Easing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic Production $(Y_t)$</td>
<td>Positive $(\partial \hat{Y}_t / \partial \check{M}_t &gt; 0)$</td>
<td>Positive or negative $(\partial \hat{Y}_t / \partial \check{M}_t^* &gt; 0$ or $&lt;0)$</td>
</tr>
<tr>
<td>Effect of an increase in $s$</td>
<td>Decline in rate of increase</td>
<td>Acceleration in rate of increase (or decline in rate of decrease)</td>
</tr>
<tr>
<td>Effect of an increase in $s^*$</td>
<td>Decline in rate of increase</td>
<td>Acceleration in rate of increase (or decline in rate of decrease)</td>
</tr>
<tr>
<td>Foreign Production $(Y_t^*)$</td>
<td>Positive or negative $(\partial \hat{Y}_t^* / \partial \check{M}_t &gt; 0$ or $&lt;0)$</td>
<td>Positive $(\partial \hat{Y}_t^* / \partial \check{M}_t^* &gt; 0)$</td>
</tr>
<tr>
<td>Effect of an increase in $s$</td>
<td>Acceleration in rate of increase (or decline in rate of decrease)</td>
<td>Decline in rate of increase</td>
</tr>
<tr>
<td>Effect of an increase in $s^*$</td>
<td>Acceleration in rate of increase (or decline in rate of decrease)</td>
<td>Decline in rate of increase</td>
</tr>
</tbody>
</table>
Figure 3: PTM Effect for the Influence of Monetary Policy on the Current Account Balance

<table>
<thead>
<tr>
<th>Home Current Account Balance ( (d_{n+1}, B_{n+1}) )</th>
<th>Domestic Monetary Easing</th>
<th>Foreign Monetary Easing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surplus (in case of ( s = s^* = 1 ), it can balance)</td>
<td>Decline in surplus</td>
<td>Decline in deficit</td>
</tr>
<tr>
<td>( \frac{\partial B_{n+1}}{\partial M} \geq 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deficit (in case of ( s = s^* = 1 ), it can balance)</td>
<td>Decline in surplus</td>
<td>Decline in deficit</td>
</tr>
<tr>
<td>( \frac{\partial B_{n+1}}{\partial M^*} \leq 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effect of an increase in ( s )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effect of an increase in ( s^* )</td>
<td></td>
<td></td>
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</tbody>
</table>
**Figure 4: PTM Effect for the Influence of Monetary Policy on the Economic Welfare**

<table>
<thead>
<tr>
<th>Domestic Utility</th>
<th>Domestic Monetary Easing</th>
<th>Foreign Monetary Easing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effect of an increase in $s$</td>
<td>Positive ($\frac{\partial dU}{\partial \hat{M}_t} &gt; 0$)</td>
<td>Positive or negative ($\frac{\partial dU}{\partial \hat{M}_t} &gt; 0$ or $&lt; 0$)</td>
</tr>
<tr>
<td>Effect of an increase in $s^*$</td>
<td>Acceleration of improvement</td>
<td>Decline in improvement (or decline in deterioration)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Foreign Utility</th>
<th>Domestic Monetary Easing</th>
<th>Foreign Monetary Easing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effect of an increase in $s$</td>
<td>Positive or negative ($\frac{\partial dU^*}{\partial \hat{M}_t} &gt; 0$ or $&lt; 0$)</td>
<td>Positive ($\frac{\partial dU^<em>}{\partial \hat{M}_t}^</em> &gt; 0$)</td>
</tr>
<tr>
<td>Effect of an increase in $s^*$</td>
<td>Decline in improvement (or decline in deterioration)</td>
<td>Acceleration of improvement</td>
</tr>
<tr>
<td>Effect of an increase in $s^*$</td>
<td>Decline in improvement (or decline in deterioration)</td>
<td>Acceleration of improvement</td>
</tr>
</tbody>
</table>
Quadrant I: Both domestic and foreign monetary policy increase the other country’s utility.

Quadrant II: Domestic monetary policy diminishes the foreign country’s utility, but foreign monetary policy increases domestic utility (asymmetric monetary policy effect).

Quadrant III: Domestic monetary policy increases the foreign country’s utility, but foreign monetary policy diminishes domestic utility (asymmetric monetary policy effect).

Quadrant IV: Both domestic and foreign monetary policy decrease the other country’s utility.
Figure 6: Influence of Japanese and U.S. Monetary Easing on Consumption

<Effect of 1% increase in Money Supply>

(percentage change from the initial steady state)

<table>
<thead>
<tr>
<th>Elasticity of Substitution</th>
<th>$\theta = 4$</th>
<th>$\theta = 5$</th>
<th>$\theta = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japanese Monetary Easing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effect on Japan</td>
<td>0.332</td>
<td>0.340</td>
<td>0.348</td>
</tr>
<tr>
<td></td>
<td>(0.348)</td>
<td>(0.360)</td>
<td>(0.372)</td>
</tr>
<tr>
<td>Effect on the U.S.</td>
<td>0.143</td>
<td>0.141</td>
<td>0.140</td>
</tr>
<tr>
<td></td>
<td>(0.280)</td>
<td>(0.274)</td>
<td>(0.269)</td>
</tr>
<tr>
<td>Global total effect</td>
<td>0.200</td>
<td>0.201</td>
<td>0.202</td>
</tr>
<tr>
<td></td>
<td>(0.300)</td>
<td>(0.300)</td>
<td>(0.300)</td>
</tr>
<tr>
<td>U.S. Monetary Easing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effect on Japan</td>
<td>0.668</td>
<td>0.660</td>
<td>0.652</td>
</tr>
<tr>
<td></td>
<td>(0.652)</td>
<td>(0.640)</td>
<td>(0.628)</td>
</tr>
<tr>
<td>Effect on the U.S.</td>
<td>0.857</td>
<td>0.859</td>
<td>0.860</td>
</tr>
<tr>
<td></td>
<td>(0.720)</td>
<td>(0.726)</td>
<td>(0.731)</td>
</tr>
<tr>
<td>Global total effect</td>
<td>0.800</td>
<td>0.799</td>
<td>0.798</td>
</tr>
<tr>
<td></td>
<td>(0.700)</td>
<td>(0.700)</td>
<td>(0.700)</td>
</tr>
</tbody>
</table>

Note: Figures in parentheses show the transmission effect under the benchmark case where all Japanese and U.S. firms are PCP firms.
Figure 7: Influence of Japanese and U.S. Monetary Easing on Production

<Effect of 1% increase in Money Supply>

(percentage change from the initial steady state)

<table>
<thead>
<tr>
<th>Elasticity of Substitution</th>
<th>$\theta = 4$</th>
<th>$\theta = 5$</th>
<th>$\theta = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japanese Monetary Easing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effect on Japan</td>
<td>1.937</td>
<td>2.345</td>
<td>2.744</td>
</tr>
<tr>
<td></td>
<td>(2.909)</td>
<td>(3.500)</td>
<td>(4.070)</td>
</tr>
<tr>
<td>Effect on the U.S.</td>
<td>-0.545</td>
<td>-0.718</td>
<td>-0.887</td>
</tr>
<tr>
<td></td>
<td>(-0.818)</td>
<td>(-1.071)</td>
<td>(-1.316)</td>
</tr>
<tr>
<td>Global total effect</td>
<td>0.200</td>
<td>0.201</td>
<td>0.202</td>
</tr>
<tr>
<td></td>
<td>(0.300)</td>
<td>(0.300)</td>
<td>(0.300)</td>
</tr>
<tr>
<td>U.S. Monetary Easing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effect on Japan</td>
<td>-0.937</td>
<td>-1.345</td>
<td>-1.744</td>
</tr>
<tr>
<td></td>
<td>(-1.909)</td>
<td>(-2.500)</td>
<td>(-3.070)</td>
</tr>
<tr>
<td>Effect on the U.S.</td>
<td>1.545</td>
<td>1.718</td>
<td>1.887</td>
</tr>
<tr>
<td></td>
<td>(1.818)</td>
<td>(2.071)</td>
<td>(2.316)</td>
</tr>
<tr>
<td>Global total effect</td>
<td>0.800</td>
<td>0.799</td>
<td>0.798</td>
</tr>
<tr>
<td></td>
<td>(0.700)</td>
<td>(0.700)</td>
<td>(0.700)</td>
</tr>
</tbody>
</table>

Note: Figures in parentheses show the transmission effect under the benchmark case where all Japanese and U.S. firms are PCP firms.
Figure 8: Influence of Japanese and U.S. Monetary Easing on the Current Account Balance

<Effect of 1% increase in Money Supply>

(percentage change from the initial steady state)

<table>
<thead>
<tr>
<th>Elasticity of Substitution</th>
<th>$\theta = 4$</th>
<th>$\theta = 5$</th>
<th>$\theta = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Japanese Monetary Easing</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effect on Japan</td>
<td>1.271</td>
<td>1.675</td>
<td>2.070</td>
</tr>
<tr>
<td></td>
<td>(1.909)</td>
<td>(2.500)</td>
<td>(3.070)</td>
</tr>
<tr>
<td>Effect on the U.S.</td>
<td>-0.545</td>
<td>-0.718</td>
<td>-0.887</td>
</tr>
<tr>
<td></td>
<td>(-0.818)</td>
<td>(-1.071)</td>
<td>(-1.316)</td>
</tr>
<tr>
<td><strong>U.S. Monetary Easing</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effect on Japan</td>
<td>-1.271</td>
<td>-1.675</td>
<td>-2.070</td>
</tr>
<tr>
<td></td>
<td>(-1.909)</td>
<td>(-2.500)</td>
<td>(-3.070)</td>
</tr>
<tr>
<td>Effect on the U.S.</td>
<td>0.545</td>
<td>0.718</td>
<td>0.887</td>
</tr>
<tr>
<td></td>
<td>(0.818)</td>
<td>(1.071)</td>
<td>(1.316)</td>
</tr>
</tbody>
</table>

Note: Figures in parentheses show the transmission effect under the benchmark case where all Japanese and U.S. firms are PCP firms.
Figure 9: Influence of Japanese and U.S. Monetary Easing on the Economic Welfare

<Effect of 1% increase in Money Supply>

(percentage change from the initial steady state)

<table>
<thead>
<tr>
<th>Elasticity of Substitution</th>
<th>$\theta = 4$</th>
<th>$\theta = 5$</th>
<th>$\theta = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Japanese Monetary Easing</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effect on Japan</td>
<td>0.150</td>
<td>0.139</td>
<td>0.131</td>
</tr>
<tr>
<td></td>
<td>(0.075)</td>
<td>(0.060)</td>
<td>(0.050)</td>
</tr>
<tr>
<td>Effect on the U.S.</td>
<td>0.007</td>
<td>-0.002</td>
<td>-0.008</td>
</tr>
<tr>
<td></td>
<td>(0.075)</td>
<td>(0.060)</td>
<td>(0.050)</td>
</tr>
<tr>
<td><strong>U.S. Monetary Easing</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effect on Japan</td>
<td>0.100</td>
<td>0.061</td>
<td>0.035</td>
</tr>
<tr>
<td></td>
<td>(0.175)</td>
<td>(0.140)</td>
<td>(0.117)</td>
</tr>
<tr>
<td>Effect on the U.S.</td>
<td>0.243</td>
<td>0.202</td>
<td>0.175</td>
</tr>
<tr>
<td></td>
<td>(0.175)</td>
<td>(0.140)</td>
<td>(0.117)</td>
</tr>
</tbody>
</table>

Note: Figures in parentheses show the transmission effect under the benchmark case where all Japanese and U.S. firms are PCP firms.