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Optimal Timing in Banks' Write-off Decisions under the Possible Implementation of a Subsidy Scheme: A Real Options Approach

Naohiko Baba*

Abstract

This paper provides a formal model that investigates optimal timing in banks' writing off their non-performing loans. The motivation comes from the recent episodes of Japanese banks, which have been slow to clean up their non-performing loans after the collapse of the "bubble economy" in the early 1990s. A real options approach is employed to measure the value of the rationality of the "forbearance policy". It is assumed that uncertainty will arise from the following sources: (i) the reinvestment return from freeing up funds though write-offs, (ii) liquidation losses, (iii) the possible implementation of a subsidy scheme, and (iv) the reputational repercussions from *not* immediately writing off their non-performing loans. This paper attaches particular importance to the uncertainty from the possible implementation of the subsidy scheme to explore its desirable features. Also, this paper examines the possible role of monetary policy in boosting the banks' incentive to write off.

Key Words: Write-off, Non-Performing Loans, Dynamic Programming, Real Options, Reputation, Forbearance Policy

JEL Classification: G21; G28

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I. Introduction

The Japanese economy has been in a prolonged recession, with many banks weighed down by large-scale non-performing loans. The primary cause has been a sharp fall in land prices that began in late 1991. The Ministry of Finance (MOF), once Japan's primary regulatory agency[1], was slow in reacting to this problem[2]. Recently, harsh criticism has been directed to the MOF's so-called "forbearance" or "buy-time" policy, which allowed banks to keep non-performing loans on their balance sheets in the hope that the economy and real estate market would recover in the not-too-distant future[3].

As argued by many authors including Cargill (2000), the failure to promptly solve the non-performing loan problem generated a credit crunch. It has contributed to stagnant or declining real GDP growth for almost a decade and has interfered with the Bank of Japan's (BOJ) efforts to stimulate the economy.

Note, however, that purely from the banks' perspective, the forbearance policy itself can be a rational choice. This is because under the stochastic circumstances with potentially large losses associated with write-offs, the option to wait (delay write-offs) should have some value. Hence, in deciding whether to write off their non-performing loans immediately the banks should weigh between the value of the option to wait and the (net) value of carrying out write-off immediately.

Hoshi (2000) pointed out that under the condition of banks not required by the authorities to disclose the true magnitude of their non-performing loans, there is no incentive to dispose of their non-performing loans. Rather, they tend to increase their lending to riskier projects. The true problem caused by non-performing loans, Hoshi (2000) argues, is that banks lose their incentive to lend to (possibly manufacturing) corporations with prospective projects, which might damage the intermediation function of banks[4]. If the social costs caused by the damage of banks' financial intermediary function outweigh the subsidy costs, then it might

[1] In June 1998, the Financial Supervisory Agency (FSA) was established, directly under the prime minister and independent of the MOF. The functions of monitoring financial markets and supervising financial institutions were transferred from the MOF to the FSA. In July 2000, the FSA was upgraded to the Financial Agency (FA) responsible for wide-ranging matters related to the financial system. The MOF became mainly in charge of budgetary and taxation matters.

[2] Ito (2000) pointed out that bank analysts at brokerage firms began to discuss a potential non-performing loan problem in 1992-93, but the MOF was reluctant to force banks to disclose the specific amounts of their non-performing loans.

[3] As explained by Ueda (2000), with the exception of a brief period around 1975, postwar Japan had never experienced a decline in land prices. Thus, no time series analysis of Japanese land price through the late 1980s would have given a high probability of a sharp fall in land prices in 1990.

[4] Hoshi (2000) also argues that even if the economy is in the state of a liquidity trap, the credit channel

justified to use public funds to push self-help efforts on banks to clean up their non-performing loans[5]. At last, in March 1998 and 1999, the Japanese government injected public funds into some banks as capital support[6]. The use of public funds was justified on the judgment that a prompt resolution of the non-performing loan problem benefits the economy as a whole in the long run.

As pointed out by Corbett and Mitchell (2000), however, one of the most puzzling and interesting facts regarding Japan's recent bank rescue package is that the government's offer was not welcomed by the rescued banks. The key to understanding this seemingly puzzling fact lies in the existence of asymmetric information and a possible reputation problem.

Typically, asymmetric information exists between banks and the public regarding the true magnitude of non-performing loans on their balance sheets[7]. More specifically, "the public" means shareholders, depositors, and other participants in financial asset markets. This asymmetric information creates an incentive for banks to roll over their non-performing loans to disguise their true financial standing. The government, which is generally in a position to grasp the true standing of the banks' balance sheets through bank examinations and monitoring on a regular basis, is able to rescue the banks by providing capital supports.

Corbett and Mitchell (2000) further argue that banks may decline rescue offers due to reputation concerns. This is because accepting such offers may force banks to write off non-performing loans, revealing the hidden information to the public.

More generally, however, a bank's reputation depends on what market participants infer from its write-off decision. In fact, on many occasions, stock prices rose for Japanese banks[8] that announced they would increase write-offs. Such episodes suggest there could be reputational repercussions from *not* writing off non-performing loans. This paper models reputational concerns as a fear of rising fund-raising (outside finance) costs[9]. And based on

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works to ensure the effectiveness of monetary policy.

[5] It should be noted, however, that bank regulators may pursue self-interests rather than social welfare. Boot and Thakor (1993) examine this possibility by introducing uncertainty in a regulators' ability to monitor banks' asset choices.

[6] In March 1998, an initial of wave of less than 2 trillion yen capital injection was provided to 21 banks. The size of the injection, in the form of subordinated loans and preferred stocks, was almost uniform across the banks, yet insufficient. In April 1998, "the prompt corrective action (PCA) rule" took effect, requirings the banks with capital ratios below certain levels to restructure or even cease operations. Also in October 1998, an agreement to appropriate 60 trillion yen of public funds to strengthen the financial system and recapitalize the banks was enacted. Recapitalization tool place in March 1999. See Ito (2000) for further details.

[7] In fact, attempts to identify the scale of the problem have been hampered by lack of disclosure and frequent changes in the definition of 'non-performing' loans.

[8] For example, recall a surge in the stock prices of Sanwa, Tokai, and Asahi after they released news of large-scale write-offs for FY 2000.

[9] In this regard, see Boot and Greenbaum (1993) as an example.

these episodes, I introduce the fear only in specifying the value of the option to wait (delay write-offs), not in specifying the value of immediate write-offs.

Motivated by the discussion above, this paper attempts to evaluate (i) optimal timing in banks' write-off decisions[10] and (ii) how much compensation or subsidization is needed for banks to carry out write-offs as self-help efforts. This paper uses the so-called real options approach to measure the value of the option to wait. More specifically, the paper treats banks' optimal write-off decisions as continuous-time problems within an infinite horizon[11].

This paper recognizes that the capital injections from the Japanese government in 1998 and 1999 were not literally a subsidy for write-offs. Nevertheless, it is of some benefit to regulatory authorities to conduct this kind of theoretical experiments as far as they believe that cleaning up the non-performing loan problem is a prerequisite to stabilizing the financial system as a whole.

The sources of uncertainty in this paper are as follows (see Figure 1):

(i) the reinvestment return from freeing up funds[12] (collected by liquidating collateral) : the banks can lend (invest) the funds to prospective projects (in financial assets);

(ii) the loss caused by carrying out write-offs: this is closely linked to land prices since real estate collateral has been extensively used when bank loans were contracted;

(iii) the future implementation of a government subsidy scheme;

(iv) the reputational repercussions from not writing off immediately, which takes the form of an upward jump in fund-raising costs.

Another (secondary) aim of this paper is to provide a solid microeconomic foundation to the question of why loans to manufacturing corporations have stagnated in Japan. As explained by Hoshi (2001), during the bubble period, banks eagerly shifted into collateralized lending. For them, lending to real estate and construction corporations was particularly promising, because they owned real estate collateral. Thus, when land prices collapsed in the early 1990s, a non-negligible portion of the loans to such industries became non-performing.

Figure 2 shows that bank loans to real estate and construction industries relative to the size of real economic activity has remained almost unchanged even after the bursting of the

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[10] In reality, the write-off procedure consists of two steps. The first step is known as an indirect write-off. In this step, the bank only reports an estimated loss, but does not actually liquidate collateral. The second step is a direct write-off, which makes the bank liquidate collateral and the actual amount of the associated loss is fixed. This paper's approach skips the first step of indirect write-offs for simplicity.

[11] A simpler class of models with only two or three discrete decision points might suffice for our qualitative analysis needs. It should be noted, however, that such a class of models is based on the unrealistic assumption that there is no uncertainty after two or three units of time. In most markets, future returns are always uncertain, and the degree of uncertainty increases with the time horizon.

[12] In other words, this is an opportunity cost arising from keeping non-performing loans on the balance

bubble economy, while loans to the manufacturing industry have declined significantly. In the meantime, Figure 3 shows that until recently, profitability measured by the ratio of current profits to sales had been markedly lower in the real estate industry than in other industries. These facts, viewed in conjunction, imply that Japanese banks have rolled over non-performing loans to the real estate industry and thus have not had any incentive to explore prospective projects in the manufacturing industry.

The rest of the paper is organized as follows. Section II describes the theoretical foundation using dynamic programming technique. Section III numerically analyzes the model. Section IV discusses some policy implications. Lastly, Section V concludes the study.

II. Theoretical Foundation

A. Assumptions

The problem that a typical bank faces is whether or not the bank exercises the option to write off its non-performing loans. Figure 4 illustrates a simplified bank's balance sheet on the premise that the bank lends all collected money to good projects[13]. It shows that once the bank writes off its non-performing loans, it can lend collected money, denoted $L_B + S - L$, to prospective projects[14]. Here, L denotes the amount of losses associated with the write-off defined as the difference between the value of non-performing loans (L_B) and the current value of the collateral. S is the government subsidy. At the same time, net worth changes from N to $N + S - L$.

There are informational asymmetries regarding the true magnitude of non-performing loans between bank managers and the public including its shareholders[15]. The bank's incentive to write off lies in the fact that the bank can lend money collected from liquidating collateral to other borrowers with prospective projects (or re-invest it in the financial markets). The return from the lending (reinvestment) after netting out a possible rise in fund-raising cost from not

sheet.

[13] Alternatively, the bank can re-invest collected money in financial assets markets.

[14] If one assumes that the bank lends collected money to other projects, then the loss L should include monitoring costs.

[15] Some recent studies in corporate governance suggest that bank managers in Japan probably do not act to maximize bank share price. Instead, they engage in activities to entrench themselves. Particularly, Claessens, Djankov, and Lang(2000), and Morck and Nakamura(1999) documented that Japanese banks engage in substantial cross-shareholdings with other companies as an explicit defense against competitive changes in corporate control. Through crossholdings, companies can acquire control rights in banks without getting cash flow rights. Such controlling shareholders could have incentive to encourage bank management to do things that maximize their own wealth, like continue to finance their company's unprofitable operations, which does not maximize the bank's stock price.

writing off immediately is denoted R.

This paper assumes that the reinvestment return itself follows the standard geometric Brownian motion. However, due to possible reputation problems when the bank delays write-off, there is a possibility that the net reinvestment return will fall as a result of rising fund-raising costs in the future. Hence, this paper assumes that there is a probability, denoted λ, that the reinvestment return net of fund-raising costs exhibits a downward jump.

Also, the bank suffers losses denoted L in carrying out write-offs, particularly due to a fall in the value of collateral. It should be noted, however, that L itself moves stochastically because it mainly reflects land prices[16]. L also follows the standard geometric Brownian motion.

The stochastic processes of the reinvestment return and the write-off losses can be summarized as

$$dR = \alpha_R R dt + \sigma_R R dz_R - R dq \,, \tag{1}$$

and
$$dL = \alpha_L L dt + \sigma_L L dz_L \,, \tag{2}$$

where α_R (α_L) denotes the expected growth rate of R (L), σ_R (σ_L) the standard deviation parameter of R (L), dz_R (dz_L) the increment of a Wiener process of R (L), dq the increment of a Poisson (jump) process with the probability λdt. The paper assumes $E[(dz_R)(dq)] = 0$, that is, dz_R and dq are assumed to be independent of each other. Also, it is assumed $E[(dz_R)^2] = E[(dz_L)^2] = dt$ and $E[(dz_R)(dz_L)] = \rho dt$, implying the correlation ρ between R and L can be considered in the following analysis. Lastly, equation (1) states that when a jump occurs, R falls by sum fixed ratio ϕ ($0 \le \phi \le 1$). For computational facility, this paper assumes $\phi = 1$ throughout the paper.

Further, the bank faces another source of uncertainty from the future implementation of the government's rescue scheme under which the government subsidizes the bank's write-off. The paper's strategy is to express the possible policy implementation by a Poisson jump[17]. Also, for simplicity, the subsidy amount is proportional to the loss from the write-off.

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[16] The majority of non-performing loans are in the real estate and construction industries.

[17] This strategy basically follows Hasset and Metcalf (1999). They examine the case where there is one underlying stochastic variable and a possible Poisson jump-type policy intervention, and analyze the impact of uncertain tax policy on investment decisions. The model in this paper augments their model by using two underlying stochastic variables, one entailing a possible downward jump risk, as well as a Poisson jump-type policy intervention.

When the subsidy scheme is not in effect, the probability that the government will implement it in the next short period dt is denoted $\lambda_1 dt$. In such cases, the amount of the subsidy is θL. On the other hand, when the scheme is in effect, the probability that it will be removed in the next short period dt is $\lambda_0 dt$. In sum, the subsidy process is given by the following equation of motion:

$$d\theta = \begin{cases} \begin{cases} \Delta\theta & \textit{with probability} \quad \lambda_1 dt \\ 0 & \textit{with probability} \quad 1-\lambda_1 dt \end{cases} \\ \begin{cases} -\Delta\theta & \textit{with probability} \quad \lambda_0 dt \\ 0 & \textit{with probability} \quad 1-\lambda_0 dt \end{cases} \end{cases}. \tag{3}$$

In what follows, first, I will examine the case without the possible implementation of the subsidy scheme, and then, consider the government subsidy.

B. The Case without the Subsidy Scheme

(i) Basic Setup

First, the value of (immediate) write-offs is given by considering only the part of the standard geometric Brownian motion of equation (1) such that[18]

$$V(R) = E\left[\int_0^\infty R(t)\mathrm{e}^{-\mu_R t}\,dt\right] = \int_0^\infty R\,\mathrm{e}^{-(\mu_R-\alpha_R)t} = \frac{R}{\mu_R-\alpha_R} = \frac{R}{\delta_R}, \tag{4}$$

where the relationship $\mu_R \equiv \alpha_R + \delta_R = r + \upsilon\rho(R,M)\sigma_R$ is assumed to hold[19] as in Dixit and Pindyck (1994)[20]. Here, μ_R denotes the risk-adjusted discount rate, δ_R the rate of return shortfall in R (hereafter, shortfall rate), r the risk-free interest rate, υ the market price of risk, and $\rho(R,M)$ the coefficient of correlation between R and the market return M. For the

[18] Notice that once write-offs are carried out, further uncertainty associated with losses are irrelevant, hence the value of write-offs depends only on the return from reinvesting the collected money.
[19] The relationship $\mu_R = r + \upsilon\rho(R,M)\sigma_R$ is derived from the CAPM(Capital Asset Pricing Model). To get this relationship, one needs the stochastic fluctuations in R to be spanned by financial markets. Also, implicitly, I assume that the jump risk is non-systematic, that is, uncorrelated with the market portfolio.
[20] See Chapter 4 for details.

value of write-off $V(R)$ to be bounded, the condition $\delta_R > 0$ must hold[21]. Otherwise, the bank would never carry out write-offs irrespective of uncertainty and sunk costs.

Second, let $F(R, L)$ denote the value of keeping the option to write off alive in the future (hereafter, the value of waiting). The Bellman equation can be written as[22]

$$\mu F(R, L)dt = E[dF(R, L)]. \tag{5}$$

Expanding $dF(R, L)$ in equation (5) by Ito's Lemma for the combined geometric Brownian motion and Poisson jump[23] yields

$$\mu F(R, L) = \frac{1}{2}\left(\sigma_R^2 R^2 F_{RR} + 2\rho\sigma_R\sigma_L RLF_{RL} + \sigma_L^2 L^2 F_{LL}\right) + \alpha_R RF_R + \alpha_L LF_L + \lambda\{F(0, L) - F(R, L)\} \tag{6}$$

where $F_{RR} \equiv \partial^2 F/\partial R^2$, $F_{LL} \equiv \partial^2 F/\partial L^2$, and $F_{RL} \equiv \partial^2 F/\partial R \partial L$.

Now, boundary conditions can be written as

$$F(\hat{R}, \hat{L}) = V(\hat{R}) - \hat{L} = \frac{\hat{R}}{\delta_R} - \hat{L}, \tag{7}$$

$$F_R(\hat{R}, \hat{L}) = V'(\hat{R}) = \frac{1}{\delta_R}, \tag{8}$$

and $$F_L(\hat{R}, \hat{L}) = -1, \tag{9}$$

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[21] In understanding the role of δ_R, it is helpful to draw upon the analogy with a financial call option in which R corresponds to the price of a share of common stock, and δ_R the dividend rate. Thus, the total expected rate on the stock is written as $\mu_R = \alpha_R + \delta_R$. In such a case, if the dividend rate δ_R were zero, the call option would always be held to maturity, and never exercised since the opportunity cost to keep the option alive is zero.

[22] In what follows, for simplicity, I drop the subscript R for μ_R.

[23] In general, if the stochastic process is

$$dx = a(x, t)dt + b(x, t)dz + g(x, t)dq ,$$

then the expected value of the change in any function $H(x, t)$ can be given by

$$E[dH] = \left[\frac{\partial H}{\partial t} + a(x, t)\frac{\partial H}{\partial x} + \frac{1}{2}b^2(x, t)\frac{\partial^2 H}{\partial x^2}\right]dt + E_\phi\{\lambda[H(x + g(x, t)\phi, t) - H(x, t)]\}dt .$$

For more details, see Dixit and Pindyck (1994), Chpter 3, pp. 86.

where condition (7) is the value-matching condition and conditions (8) and (9) are both the smooth-pasting conditions. Also, $\hat{R}$ and $\hat{L}$ indicate the threshold values of R and L at which the bank becomes indifferent between writing off and waiting.

(ii) The Free boundary Problem

The problem in the last section is called a “free boundary” problem[24]. In such a case, it is very difficult to obtain clear-cut analytical solutions. Nevertheless, the property of homogeneity of the net value function $V(R)-L$ [25] allows one to reduce the problem to one dimension. Thus, the optimal decision only depends on the ratio $r_R \equiv R/L$, which implies that the value of waiting $F(R, L)$ should also be homogeneous of degree one with respect to R and L. That is, the following set of relationships holds:

$$F(R, L) = Lf\left(\frac{R}{L}\right) \equiv Lf(r_R). \tag{10}$$

$$F_R(R, L) = f'(r_R), \tag{11}$$

$$F_L(R, L) = f(r_R) - r_R f'(r_R), \tag{12}$$

$$F_{RR}(R, L) = f''(r_R)/L, \tag{13}$$

$$F_{RL}(R, L) = -r_R f''(r_R)/L, \tag{14}$$

and

$$F_{LL}(R, L) = (r_R)^2 f''(r_R)/L. \tag{15}$$

Using equations (10)-(15), equation (6) can be rewritten as

$$\frac{1}{2}\left(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2\right)(r_R)^2 f''(r_R) + (\alpha_R - \alpha_L) r_R f'(r_R) + (\alpha_L - \mu - \lambda) f(r_R) = 0. \tag{16}$$

The solution to the second-order differential equation (16) takes the form:

$$f(r_R) = A(r_R)^{\beta}, \tag{17}$$

[24] For more details, see Dixit and Pindyck (1994), Chapter 6, pp. 209.
[25] Note that if the current values of both R and L are doubled, it will double the value V and the cost

where A and β are coefficients to be determined.

Direct substitution of solution (17) into equation (16) yields

$$\frac{1}{2}\left(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2\right)\beta(\beta-1) + (\alpha_R - \alpha_L)\beta + (\alpha_L - \mu - \lambda) = 0\,. \tag{18}$$

Thus, β can be solved analytically as

$$\beta = \frac{1}{2} - \frac{(\alpha_R - \alpha_L)}{\left(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2\right)} + \sqrt{\left[\frac{(\alpha_R - \alpha_L)}{\left(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2\right)} - \frac{1}{2}\right]^2 + \frac{2(\mu + \lambda - \alpha_L)}{\left(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2\right)}}\,. \tag{19}$$

Now boundary conditions (7)-(9) can be rewritten as

$$f(\hat{r}_R) = A(\hat{r}_R)^{\beta} = \frac{\hat{r}_R}{\delta_R} - 1\,, \tag{20}$$

$$f'(\hat{r}_R) = A\beta(\hat{r}_R)^{\beta-1} = \frac{1}{\delta_R}\,, \tag{21}$$

and $$f(\hat{r}_R) - \hat{r}_R f'(\hat{r}_R) = A(\hat{r}_R)^{\beta}(1-\beta) = -1\,, \tag{22}$$

where $\hat{r}_R$ denotes the threshold ratio. Note that of these three boundary conditions, no single one is independent of the other two.

Equations (20) and (21) jointly imply

$$\hat{r}_R = \frac{\beta}{\beta - 1}\delta_R\,. \tag{23}$$

Figure 5 illustrates a free boundary $\hat{r}_R$ of the ratio of reinvestment return to write-off losses. In regime I, the current value of r_R is below the threshold value $\hat{r}_R$ so that the bank prefer waiting to writing off now. Also, Figure 6 depicts boundary conditions for $f(r_R)$ and the determination of $\hat{r}_R$. At the threshold ratio $\hat{r}_R$, the value from write-offs meets the value of waiting tangentially.

L simultaneously.

(iii) The Relationship between $\hat{r}_R$ and the Required Reinvestment Rate of Return $\overline{rr}$

Note that the threshold ratio $\hat{r}_R$ is in terms of write-off losses, not in terms of the amount of reinvested funds. Thus, in evaluating $\hat{r}_R$ in line with a realistic economic situation, it is helpful to translate $\hat{r}_R$ into a usual rate of return form.

Figure 7 shows the relationship between $\hat{r}_R$ and the required reinvestment rate of return $\overline{rr}$. The following relationship is evident:

$$\begin{cases} if\ L \leq \frac{1}{2}\overline{L_B}, & then\ \overline{rr} = \frac{L}{L_B - L}\hat{r}_R \leq \hat{r}_R \\ otherwise, & \overline{rr} > \hat{r}_R \end{cases}. \tag{24}$$

For example, when a loss amounts to a quarter of the non-performing loan, $\overline{rr}$ is equal to $1/3\hat{r}_R$. And when the loss is three quarters, $\overline{rr}$ is $3\hat{r}_R$.

C. The Case with the Possible Implementation of a Subsidy Scheme

Now let me consider the case in which the bank expects the implementation of a subsidy by the government with some probability. To begin, let $F_0(R, L) = Lf_0(r_R)$ denote the value of waiting in the absence of a subsidy scheme and let $F_1(R, L) = Lf_1(r_R)$ denote the value in the presence of the scheme, respectively. In this setting, one can divide the decision rule of the bank into the following three regimes, implying the existence of two threshold ratios. See Figures 8 and 9 for the illustration of the three regimes.

First, over the interval of low values of r_R, denoted $(0, \underline{r_R})$, the bank will not write off irrespective of whether or not the subsidy scheme is in effect.

Second, over the interval denoted $(\underline{r_R}, \overline{r_R})$, the bank will write off if the subsidy scheme is in effect. Otherwise, the bank will prefer to wait in the hope that the subsidy scheme will be implemented and/or land prices will recover so that liquidation losses will decrease in the future.

Third, over the interval denoted $(\overline{r_R}, \infty)$, the bank is willing to write off irrespective of a subsidy scheme. Referring to Hassett and Metcalf (1999), let me find the two thresholds, $\overline{r_R}$ and $\underline{r_R}$ below.

(i) Regime 1 $\left(0, \underline{r_R}\right)$: No Write-off Irrespective of the Subsidy Scheme

Over the interval $\left(0, \underline{r_R}\right)$, the bank prefers to wait irrespective of a subsidy scheme, and each regime can switch to the other. Thus, the following pair of equations holds[26]:

$$\frac{1}{2}\left(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2\right)\left(r_R\right)^2 f_0''\left(r_R\right) + \left(\alpha_R - \alpha_L\right)r_R f_0'\left(r_R\right) + \left(\alpha_L - \mu - \lambda\right)f_0\left(r_R\right) + \lambda_1\left[f_1\left(r_R\right) - f_0\left(r_R\right)\right] = 0 \tag{25}$$

$$\frac{1}{2}\left(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2\right)\left(r_R\right)^2 f_1''\left(r_R\right) + \left(\alpha_R - \alpha_L\right)r_R f_1'\left(r_R\right) + \left(\alpha_L - \mu - \lambda\right)f_1\left(r_R\right) + \lambda_0\left[f_0\left(r_R\right) - f_1\left(r_R\right)\right] = 0 \tag{26}$$

Since both value functions $f_0\left(r_R\right)$ and $f_1\left(r_R\right)$ appear in each equation, one needs to consider the two linear combinations that can be solved easily. For example, consider new value functions $f_a\left(r_R\right)$ and $f_b\left(r_R\right)$ such that

$$f_a\left(r_R\right) = \frac{f_0\left(r_R\right)}{\lambda_1} + \frac{f_1\left(r_R\right)}{\lambda_0}, \tag{27}$$

and $$f_b\left(r_R\right) = f_1\left(r_R\right) - f_0\left(r_R\right). \tag{28}$$

Then, equations (25) and (26) can be rewritten in terms of $f_a\left(r_R\right)$ and $f_b\left(r_R\right)$ as

$$\frac{1}{2}\left(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2\right)\left(r_R\right)^2 f_a''\left(r_R\right) + \left(\alpha_R - \alpha_L\right)r_R f_a'\left(r_R\right) + \left(\alpha_L - \mu - \lambda\right)f_a\left(r_R\right) = 0, \tag{29}$$

$$\frac{1}{2}\left(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2\right)\left(r_R\right)^2 f_b''\left(r_R\right) + \left(\alpha_R - \alpha_L\right)r_R f_b'\left(r_R\right) + \left(\alpha_L - \mu - \lambda - \lambda_0 - \lambda_1\right)f_b\left(r_R\right) = 0.$$

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[26] The derivation of equation (25) is made in the following way. When the subsidy is not in effect, over the next short interval of time dt, the probability that the subsidy will be implemented is $\lambda_1 dt$. In this case, the value of the option to write off is $F_1\left(R + dR, L + dL\right)$. Otherwise, it is $F_0\left(R + dR, L + dL\right)$. Hence,

$F_0\left(R + dR, L + dL\right) = e^{-\mu dt}\left\{\left(\lambda_1 dt\right)E\left[F_1\left(R + dR, L + dL\right)\right] + \left(1 - \lambda_1 dt\right)E\left[F_0\left(R + dR, L + dL\right)\right]\right\}$

follows. Expanding the preceding equation by Ito's Lemma and using the assumption of homogeneity

(30)

The solutions to the second-order differential equations take the forms[27]:

$$f_a(r_R) = B(r_R)^{\beta_1}, \tag{31}$$

and $$f_b(r_R) = C(r_R)^{\beta_2}, \tag{32}$$

where B, C, β_1, and β_2 are coefficients to be determined. Here note that β_1 is the positive root of

$$\frac{1}{2}\left(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2\right)\beta(\beta-1) + (\alpha_R - \alpha_L)\beta + (\alpha_L - \mu - \lambda) = 0, \tag{33}$$

and β_2 is the positive root of

$$\frac{1}{2}\left(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2\right)\beta(\beta-1) + (\alpha_R - \alpha_L)\beta + (\alpha_L - \mu - \lambda - \lambda_0 - \lambda_1) = 0. \tag{34}$$

With this background information, the solutions for $f_0(r_R)$ and $f_1(r_R)$ over the interval $(0, \underline{r_R})$ are given by

$$f_0(r_R) = \frac{\lambda_0\lambda_1 B(r_R)^{\beta_1} - \lambda_1 C(r_R)^{\beta_2}}{\lambda_0 + \lambda_1}, \tag{35}$$

and $$f_1(r_R) = \frac{\lambda_0\lambda_1 B(r_R)^{\beta_1} + \lambda_0 C(r_R)^{\beta_2}}{\lambda_0 + \lambda_1}. \tag{36}$$

yields equation (25). Equation (26) can be derived in the same way.

[27] Note that since the interval of r_R extends to zero, only the positive root of a quadratic equation matters.

(ii) Regime 2 $\left(\underline{r_R}, \overline{r_R}\right)$: Write-off Now if the Subsidy Scheme is in Effect

Over the interval $\left(\underline{r_R}, \overline{r_R}\right)$, the bank will write off non-performing loans immediately if the subsidy scheme is in effect. Otherwise, the bank will not. Thus $f_1(r_R)$ is given by[28]

$$f_1(r_R) = \frac{r_R}{\delta_R} - (1-\theta), \tag{37}$$

where θ denotes the portion of the subsidy[29] in the loss.

On the other hand, $f_0(r_R)$ is found in the same way as equations (25) and (26).

$$\frac{1}{2}\left(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2\right)(r_R)^2 f_0''(r_R) + (\alpha_R - \alpha_L) r_R f_0'(r_R) + (\alpha_L - \mu - \lambda) f_0(r_R) + \lambda_1 [f_1(r_R) - f_0(r_R)] = 0 \tag{38}$$

Using equation (37), equation (38) can be rewritten as

$$\begin{aligned}&\frac{1}{2}\left(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2\right)(r_R)^2 f_0''(r_R) + (\alpha_R - \alpha_L) r_R f_0'(r_R)\\ &\qquad + (\alpha_L - \mu - \lambda - \lambda_1) f_0(r_R) + \frac{\lambda_1}{\delta_R} r_R - \lambda_1(1-\theta) = 0\end{aligned} \tag{39}$$

The general solution takes the form:

$$f_0(r_R) = D(r_R)^{\beta_3} + E(r_R)^{\beta_4} + \frac{\lambda_1 r_R}{\delta_R(\mu + \lambda + \lambda_1 - \alpha_R)} - \frac{\lambda_1(1-\theta)}{\mu + \lambda + \lambda_1 - \alpha_L}, \tag{40}$$

where D and E are constants to be determined and β_3 and β_4 are the positive and negative roots of the quadratic function of the form:

$$\frac{1}{2}\left(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2\right)\beta(\beta-1) + (\alpha_R - \alpha_L)\beta + (\alpha_L - \mu - \lambda - \lambda_1) = 0. \tag{41}$$

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[28] Notice that equation (37) is equivalent to $F_1(R, L) = R/\delta_R - (1-\theta)L$.

(iii) Regime 3 $\left(\overline{r_R}, \infty\right)$: Write-off Now Irrespective of the Subsidy Scheme

Over the interval $\left(\overline{r_R}, \infty\right)$, the bank always writes off non-performing loans, irrespective of the subsidy scheme. Thus the following relationships hold:

$$f_0(r_R) = \frac{r_R}{\delta_R} - 1, \tag{42}$$

and

$$f_1(r_R) = \frac{r_R}{\delta_R} - (1 - \theta). \tag{43}$$

(iv) Boundary Conditions Linking each Regime[30]

Now that each solution form has been found, the next step is to find boundary conditions, which relate to each of the above-derived value functions.

First, at the threshold $\underline{r_R}$, the bank will write off if the subsidy scheme is in effect. Thus for the expressions for $f_1(r_R)$, equations (36) and (37) yield,

$$\frac{\lambda_0 \lambda_1 B\left(\underline{r_R}\right)^{\beta_1} + \lambda_0 C\left(\underline{r_R}\right)^{\beta_2}}{\lambda_0 + \lambda_1} = \frac{\underline{r_R}}{\delta_R} - (1 - \theta), \tag{44}$$

and

$$\frac{\lambda_0 \lambda_1 B \beta_1 \left(\underline{r_R}\right)^{\beta_1 - 1} + \lambda_0 C \beta_2 \left(\underline{r_R}\right)^{\beta_2 - 1}}{\lambda_0 + \lambda_1} = \frac{1}{\delta_R}, \tag{45}$$

where equations (44) and (45) denote value-matching and smooth-pasting conditions.

Second, for $f_0(r_R)$, although this is not actually associated with a decision threshold, the function has to be continuously differentiable across it. Thus equations (35) and (40) yield

$$\frac{\lambda_0 \lambda_1 B\left(\underline{r_R}\right)^{\beta_1} - \lambda_1 C\left(\underline{r_R}\right)^{\beta_2}}{\lambda_0 + \lambda_1} = D\left(\underline{r_R}\right)^{\beta_3} + E\left(\underline{r_R}\right)^{\beta_4} + \frac{\lambda_1 \underline{r_R}}{\delta_R(\mu + \lambda + \lambda_1 - \alpha_R)} - \frac{\lambda_1(1 - \theta)}{\mu + \lambda + \lambda_1 - \alpha_L},$$

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[29] See equation (3) for its definition.

[30] Notice that the case without the government subsidy corresponds to a special case where $\lambda_0 = \lambda_1 = \theta = 0$ in this model.

(46)

$$\frac{\lambda_0 \lambda_1 B\beta_1 \left(\underline{r_R}\right)^{\beta_1 - 1} - \lambda_1 C\beta_2 \left(\underline{r_R}\right)^{\beta_2 - 1}}{\lambda_0 + \lambda_1} = D\beta_3 \left(\underline{r_R}\right)^{\beta_3 - 1} + E\beta_4 \left(\underline{r_R}\right)^{\beta_4 - 1} + \frac{\lambda_1}{\delta_R \left(\mu + \lambda + \lambda_1 - \alpha_R\right)} . \quad (47)$$

Third, at the threshold $\overline{r_R}$, the expressions for $f_0(r_R)$ should satisfy the value-matching and smooth-pasting conditions. Hence, equations (40) and (42) yield

$$D\left(\overline{r_R}\right)^{\beta_3} + E\left(\overline{r_R}\right)^{\beta_4} + \frac{\lambda_1 \overline{r_R}}{\delta_R \left(\mu + \lambda + \lambda_1 - \alpha_R\right)} - \frac{\lambda_1 (1 - \theta)}{\mu + \lambda + \lambda_1 - \alpha_L} = \frac{\overline{r_R}}{\delta_R} - 1, \quad (48)$$

and $$D\beta_3 \left(\overline{r_R}\right)^{\beta_3 - 1} + E\beta_4 \left(\overline{r_R}\right)^{\beta_4 - 1} + \frac{\lambda_1}{\delta_R \left(\mu + \lambda + \lambda_1 - \alpha_R\right)} = \frac{1}{\delta_R}, \quad (49)$$

where one can analytically find β s from equations (33), (34), and (41) as follows:

$$\begin{cases} \beta_1 = \frac{1}{2} - \frac{F}{G} + \sqrt{\left(\frac{F}{G} - \frac{1}{2}\right)^2 + \frac{2\left(\mu + \lambda - \alpha_L\right)}{G}} \\ \beta_2 = \frac{1}{2} - \frac{F}{G} + \sqrt{\left(\frac{F}{G} - \frac{1}{2}\right)^2 + \frac{2\left(\mu + \lambda + \lambda_0 + \lambda_1 - \alpha_L\right)}{G}} \\ \beta_3 = \frac{1}{2} - \frac{F}{G} + \sqrt{\left(\frac{F}{G} - \frac{1}{2}\right)^2 + \frac{2\left(\mu + \lambda + \lambda_1 - \alpha_L\right)}{G}} \\ \beta_4 = \frac{1}{2} - \frac{F}{G} - \sqrt{\left(\frac{F}{G} - \frac{1}{2}\right)^2 + \frac{2\left(\mu + \lambda + \lambda_1 - \alpha_L\right)}{G}} \end{cases} \quad (50)$$

$F = \alpha_R - \alpha_L$, and $G = \sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2$. (51)

It is evident that the relation $\beta_4 < 0 < 1 < \beta_1 \le \beta_3 \le \beta_2$ holds. In sum, there are six equations to determine two thresholds $\underline{r_R}$ and $\overline{r_R}$ and four constants B, C, D, and E [31].

Figures 10 and 11 illustrate these boundary conditions and the determination of the

[31] This paper uses a Levenberg-Marquardt method included in Mathcad 2000 Professional as a solving algorithm. It is a variant of the usual quasi-Newton method. To make the Levenberg-Marquardt method more efficient, Mathcad modifies the following points:
(i)The first time the solver stops at a point that is not a solution, Mathcad adds a small random amount to all the variables and attempts again. This helps avoid dead ends in local minima and other points from which there is no preferred direction.
(ii) If inequality constraints are included as in the case of mine, Mathcad first solves the subsystem consisting of only the inequalities before adding the equality constraints and attempting a full solution.

threshold ratios $\underline{r_R}$ and $\overline{r_R}$. They show that at the threshold ratio $\underline{r_R}$, two expressions of $f_0(r_R)$ representing regimes 1 and 2 (equations (35) and (40)) meet tangentially and at the same time, two expressions of $f_1(r_R)$ representing regimes 1 and 2 (equations (36) and (37)) meet in the same manner. And at the threshold ratio $\overline{r_R}$, only the two expressions of $f_0(r_R)$ representing regimes 2 and 3 meet tangentially to ensure continuity.

III. Numerical Analysis

This section reports the results of numerical analysis based on the theoretical model described in the last section. The baseline parameters are set in annual terms as follows:

$$\alpha_R = 0.02,\ \ \alpha_L = -0.02,\ \ \sigma_R = 0.2,\ \ \sigma_L = 0.3,\ \ \delta_R = 0.02,\ \ \rho = 0.0, \text{ and } \ \lambda = 0.0 .$$

Here, the negative value of the expected growth rate of the write-off loss α_L reflects the banks' optimistic expectations about future conditions in the real estate market[32] and the relative magnitude between σ_R and σ_L reflects larger volatility in the real estate market. As for the correlation term ρ and the probability of a jump in fund-raising costs λ, it is difficult to find "plausible" values. Thus, this paper tentatively sets both values at zero in the baseline case and changed them over a wide range. Table 1 summarizes the qualitative results of numerical analysis and I will see the resulting details.

A. The Case without a Subsidy Scheme

Figures 12 (i)-(v) show the dependence of the threshold ratio $\hat{r}_R = \hat{R}/\hat{L}$ on various parameter values. Before examining the detailed numerical results, note that a very large rate of return is required for the banks to immediately write off their non-performing loans under various settings. For example, a value of $\hat{r}_R$=0.05 means more than a 5 percent annual reinvestment rate of return if the loss is more than half of the non-performing loans. Judging from the low current investment rate of return in the Japanese financial markets[33], obtaining such a high

[32] With the benefit of hindsight, we know that this expectation was not realized, but as Cargill, Hutchison, and Ito(1997) argue, the MOF also had such an optimistic view of the real estate market, not to mention the Japanese banks .

[33] For example, in December 1999 , the average contracted interest rate on new loans and discounts was

return seems almost impossible in reality.

Now, let me check how the basic model works. First, Figure 12 (i) shows that the more and more uncertain the bank's economic environment becomes (larger values of σ_R and σ_L), the larger the threshold ratio $\hat{r}_R$ becomes[34]. This result holds for uncertainty in both underlying stochastic variables, reinvestment return R and the loss from the write-off L.

Second, consider the effect of a rise in the coefficient of correlation ρ between R and L. As shown in Figure 12 (ii), a greater ρ results in a lower $\hat{r}_R$. This result directly follows the fact that the variance of r_R under the assumption of homogeneity of degree one can be expressed as $\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2$ so that a larger value of ρ implies a smaller volatility, which increases the incentive to immediately write off.

Third, look at the effects of a change in expected growth parameters, α_R and α_L. It turns out that the effect of α_L is more straightforward than that of α_R. As shown in Figures 12 (ii) and (iii), the larger (smaller) the expected losses from future write-offs, the greater the (weaker) banks' incentive to immediately write off. This result corresponds to the familiar story about the forbearance policy taken by the government regarding write-offs of non-performing loans in the Japanese banking industry.

For the effect of α_R, one should be careful about the results obtained under alternative assumptions regarding which parameter is adjustable, μ or δ_R. Under the assumption that μ changes exactly as much as α_R[35](Figure 12 (iii) (a)), the threshold ratio $\hat{r}_R$ rises as a result of a rise in α_R. Intuitively, the underlying reason is that the present value of write-off losses carried out in the future is discounted by the risk-adjusted discount rate μ,[36] while the present value of the reinvestment of collected money is discounted by δ_R[37], which is assumed to be constant. Hence, an increase in α_R (thus μ) reduces the present value of future write-off costs, but does not reduce its payoff.

On the other hand, under the alternative assumption that μ is constant while

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1.82 percent and the yield of 10-year government bonds, which occupy a non-negligible part of Japanese banks' working asset was 1.84 percent (both in annual terms).

[34] As pointed out by Dixit and Pindyck (1994)(see Chapter 5, pp.153), an interesting point here is that write-off (investment) decisions are highly sensitive to volatility in write-off (project) values, irrespective of investors' or managers' risk preferences. Thus, if one assumes that a bank is risk-neutral, the same result will follow.

[35] Recall the relationship $\mu \equiv \alpha_R + \delta_R$.

[36] See equation (5).

[37] See equation (4).

adjusting δ_R in response to changes in α_R, the direction of the effect of rises in α_R does the opposite of the preceding case. The reason can be found using previous logic; while the present value of the future write-off losses is invariant, reinvestment payoff will rise.

Fourth, how does an increase in the downward jump risk of the reinvestment return influence the optimal decision-making of rational banks? Generally, the effects of a positive value of the probability of the downward jump risk λ can be stated as follows. First, it reduces the expected rate of capital gain on R, which decreases the value of waiting. On the other hand, it increases the variance of changes in R and thus raises the value of waiting. It turns out that under normal circumstances, the former effect is more dominant. Figure 12 (iv) lays out the result that the former effect is much larger than the latter, thereby reducing the threshold ratio $\hat{r}_R$. Further, notice that a small increase in λ lead to a substantial decline in $\hat{r}_R$, prompting the bank to immediately write off.

Fifth, look at the effect of an increase in the shortfall rate δ_R on the threshold ratio of $\hat{r}_R$. Figure 12 (v) shows the result under the assumption that the risk-adjusted discount rate μ moves in response to a change in δ_R[38]. As explained earlier, the profit flow is discounted by δ_R, while the future write-off losses are discounted by μ. In the present model, although the effect via μ have an effect through β (equation (19)), the effect via δ_R has greater direct influence as shown by equation (23). Thus, the net effect increases the threshold ratio $\hat{r}_R$.

B. The Case with the Possible Implementation of a Subsidy Scheme

Now let me examine the case with the possible implementation of a subsidy scheme by the government[39]. As mentioned earlier, this case generalizes the last case in that λ_0, λ_1, and θ take on positive values. In fact, the threshold ratios $\overline{r_R}$ and $\underline{r_R}$ should converge to $\hat{r}_R$ as the values of λ_0, λ_1, and θ approach zero. Thus, in this subsection, I focus on the numerical results when one changes the values of λ_0, λ_1, and θ.

First, consider the situation where the scheme is not currently in effect. Figure 13 (i) shows that as the probability of the implementation λ_1 increases, the threshold ratio $\overline{r_R}$

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[38] For the results under the alternative assumption that μ is held constant while letting α_R freely adjust , see the results where α_R changes while μ is held constant.

[39] The results of the effects of changes in parameters other than those of policy uncertain on $\underline{r_R}$ and $\overline{r_R}$

increases. This result is very intuitive. The prospect of reduced write-off losses inevitably increases the value of waiting. One of the most impressive aspects to note here is the magnitude of the effect of an increase in λ_1. When the bank is 100% sure[40] of the implementation of the subsidy scheme[41] in the next period, the threshold ratio $\overline{r_R}$ more than doubles from when $\lambda_1 = 0.1$. Also, notice that even when the implementation of the scheme is being discussed and is still uncertain, the effect is to strongly discourage the incentive to immediately write off. Another important point is that even in the absence of the scheme, the threshold ratio $\overline{r_R}$ is influenced by the probability λ_0 of the scheme's removal.

Next consider the situation in the presence of the scheme. Figure 13 (ii) shows that the threshold ratio $\underline{r_R}$ decreases as λ_0 increases. This result is also intuitive because it is natural that the prospect of losing the scheme should induce bank to immediately write off. Further, this figure shows that an increase in λ_1 also increases $\underline{r_R}$.

Now let me examine the effect of an increase in the ratio of the subsidy to the loss θ. Figure 13 (iii) shows the dependence of $\overline{r_R}$ and $\underline{r_R}$ on the value of θ. This figure shows that both threshold ratios $\overline{r_R}$ and $\underline{r_R}$ are inversely related to θ for low values of θ. The effect is also much stronger on $\underline{r_R}$ than on $\overline{r_R}$.

An interesting point is that the threshold ratio $\overline{r_R}$ in the absence of a scheme is also influenced by θ. Numerical analysis suggests that there are two competing channels through which θ can influence $\overline{r_R}$. One channel increases the incentive to wait by lowering the last term of $f_0(r_R)$ (equation (40)). The other channel works in the opposite direction through a fall in D in the same equation. Which force is stronger depends on the range of the parameter θ. Generally, as shown in Figure 13 (iii), when θ is small, the latter effect is greater than the former effect, but at some value of θ, the net effect reverses direction, and a rise in θ raises the threshold ratio $\overline{r_R}$.

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are the same as in the case without the possible subsidy.

[40] Note that the 100% probability of a subsidy enactment is just in the bank's expectation and does not imply that the policy will really be enacted within the next year.

[41] One can rephrase this condition as saying "if the regulatory authorities can make a fully credible commitment to implement the subsidy scheme in the near future".

IV. Some Policy Discussions

A. Implications for the Implementation of a Subsidy Scheme

Uncertainty about the implementation of a subsidy scheme will give banks incentive to delay write-offs. In other words, if the government will aim to accelerate banks' self-help efforts towards reducing their non-performing loans, sound policy should have properties of low λ_1, high λ_0, and large θ. The government should immediately implement the subsidy scheme immediately, giving banks a credible threat to immediately abolish it and pledging never to restore it, although we cannot imagine such a threat in reality.

B. Possible Implications for Monetary Policy

Until quite recently, the BOJ has controlled short-term interest rates such as the overnight call rate for purposes of influencing the real economy[42]. The over-night call rate is considered to be riskless. Under this paper's assumption of $\mu \equiv \alpha_R + \delta_R = r + \upsilon\rho(R, M)\sigma_R$, a rise in the call rate by the BOJ leads to a rise in the risk-adjusted discount rate μ of banks either via a rise in α_R or δ_R, other things being equal.

The analysis shows that a rise in the risk-adjusted discount rate μ via either α_R or δ_R makes banks more hesitant to immediately dispose of their non-performing loans. Hence, if the central bank would like to accelerate banks' efforts to clean up their balance sheets, it should lower the interest rate up to a point where banks regards it as long-lasting so they revise downward their perceived risk-adjusted discount rate. In this regard, recent monetary policy conducted by the BOJ is worthy of attention. To stimulate a depressed economy, the BOJ lowered short-term interest rates, including the call rate, to almost zero from 1995[43], which might have some effects to raise the incentive for the banks to write off[44].

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[42] In March 2001, the BOJ changed the main operating target for money market operations from the overnight call rate to the outstanding balance of the current accounts at the BOJ.

[43] Monetary easing of this nature can be characterized as a policy to "buy time", that is, to buy time until the structural policy bears the fruit. In fact, when it lowered the official discount rate to 0.5 percent in September 1995, the BOJ Policy Board issued a statement stressing that such monetary easing would only be effective if it were accompanied by structural policies.

[44] However, it is also true that the enlargement of profit margin resulting from the so-called zero-interest – rate policy gave banks room to retain their non-performing loans. This paper pays no attention to the monetary policy effect on the interaction between the cost structure and the incentive to dispose of non-performing bank loans. Thus, we should be careful when evaluating implications for monetary policy.

V. Concluding Remarks

This paper has investigated how rational banks' optimal timing of write-offs is influenced by uncertainty stemming from various sources. A real options approach was employed to evaluate the value of the option to delay write-offs, that is the value of forbearance policy.

Numerical analysis shows that under normal circumstances, a very large rate of reinvestment return is required for the banks to immediately write off their non-performing loans. Another important result is that uncertainty about the implementation of a subsidy scheme gives banks incentive to wait. This is contrary to the government's intention. If the government aims to encourage banks' self-help efforts towards reducing non-performing loans, it should immediately enact a subsidy scheme, giving them a legitimate threat to immediately abolish it and pledging never to restore it in the future. This kind of policy is theoretically possible, but almost impossible in reality.

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Table 1: Summary of Numerical Analysis

Exogenous Variables		Endogenous Variables $\hat{r}_R$	$\left(\underline{r_R}, \overline{r_R}\right)$
Underlying Parameters			
Volatility			
σ_R		+	+ +
σ_L		+	+ +
Correlation			
ρ		–	– –
Expected Growth			
α_R	Case (i)	+	+ +
	Case (ii)	–	– –
α_L		–	– –
Shortfall Rate			
δ_R		+	+ +
Reputation Problem			
Risk Premium			
λ		–	– –
Policy Uncertainty			
Probability about the Scheme			
λ_1		----------	+ +
λ_0		---------	– –
Portion of the Subsidy			
θ		----------	– ±

Notes: 1. + indicates that the endogenous variable rises when the exogenous variable rises.
– indicates vice versa. ± denotes that the direction depends on other parameter values.
2. The case (i) denotes the case in which δ_R is held constant, while letting μ adjust freely.
The case (ii) denotes the case in which μ is held constant while letting δ_R adjust freely.

Figure 1: Sources of Uncertainty Influencing Bank Decisions

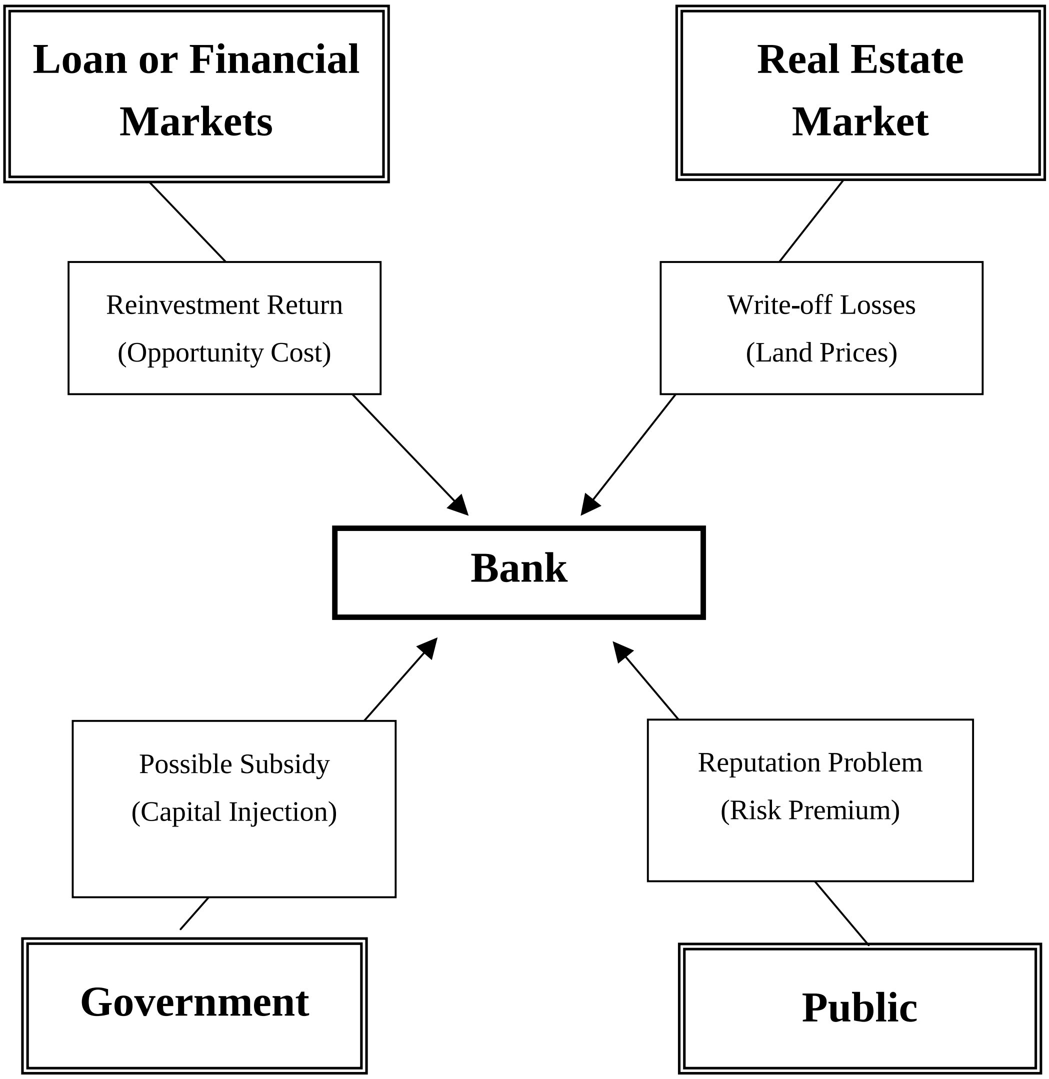
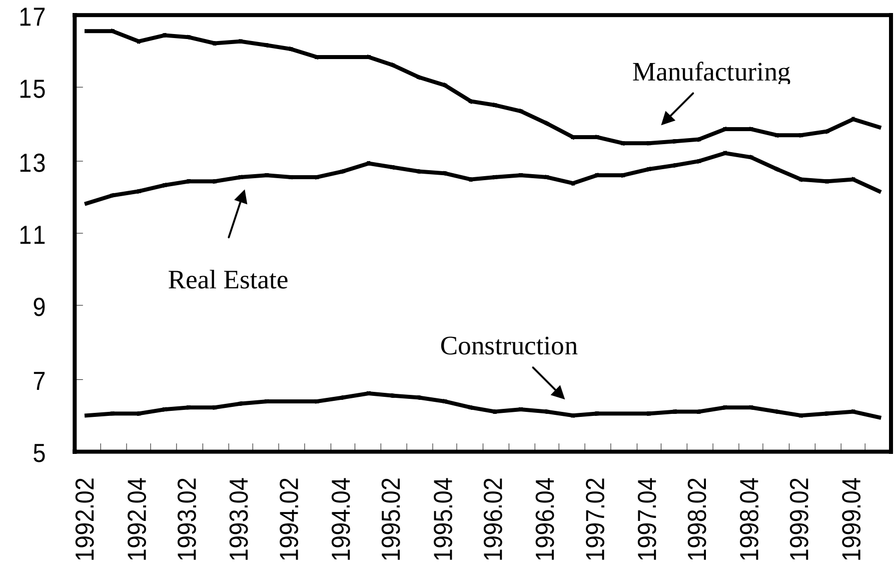


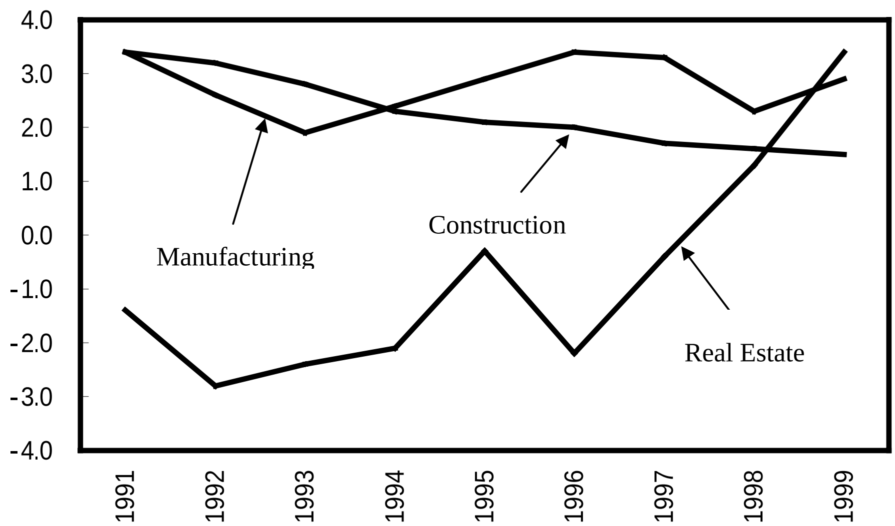
Figure 2: Loans and Discounts Outstanding by Industry

(Percent of Nominal GDP)



Note: The definition of the domestic banks changed in 92/1Q. Thus, I used data after 92/2Q. The data was taken from the Bank of Japan's Financial and Economic data CD-ROM.

Figure 3: Ratio of Current Profits to sales



Note: The data was taken from Corporate Business Statistics quarterly issued by the MOF.

Figure 4: A Simplified Balance Sheet

A. Before Write-Off

Assets	Liabilities
Loans ($L_G + L_B$ **)**	**Funds (** F_U **)**
Good Loans (L_G **)**	
Bad Loans (L_B **)**	
Other Assets (OA **)**	**Net Worth (** N **)**

B. After Write-Off

Assets	Liabilities
Loans **(** $L_G + L_B + S - L$ **)**	**Funds (** F_U **)**
Good Loans **(** $L_G + L_B + S - L$ **)**	
Other Assets (OA **)**	**Net Worth (** $N + S - L$ **)**

Notes: 1. S : Government subsidy, L : Liquidation losses

2. The figures are based on the assumption that collected money is lent to profitable projects.

Figure 5: Free Boundary of $\hat{r}_R$ without a Subsidy Scheme

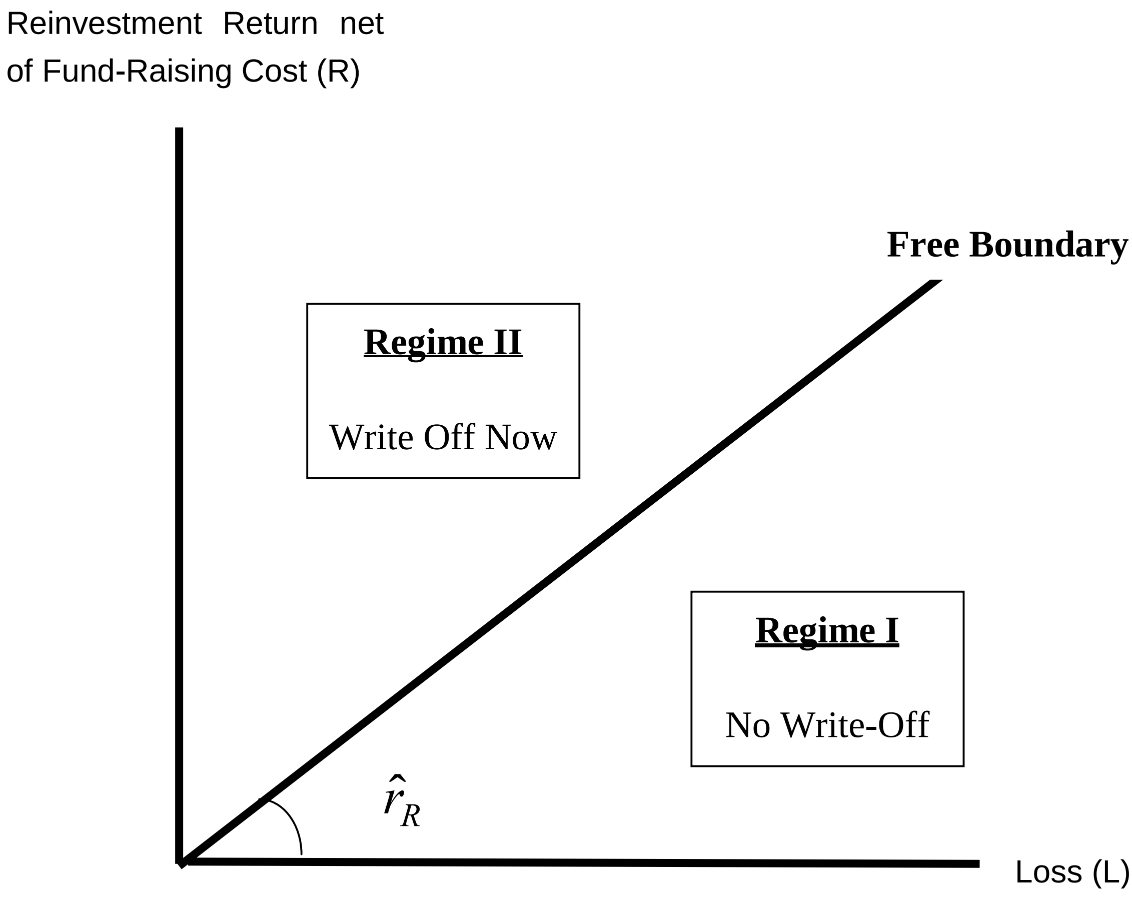
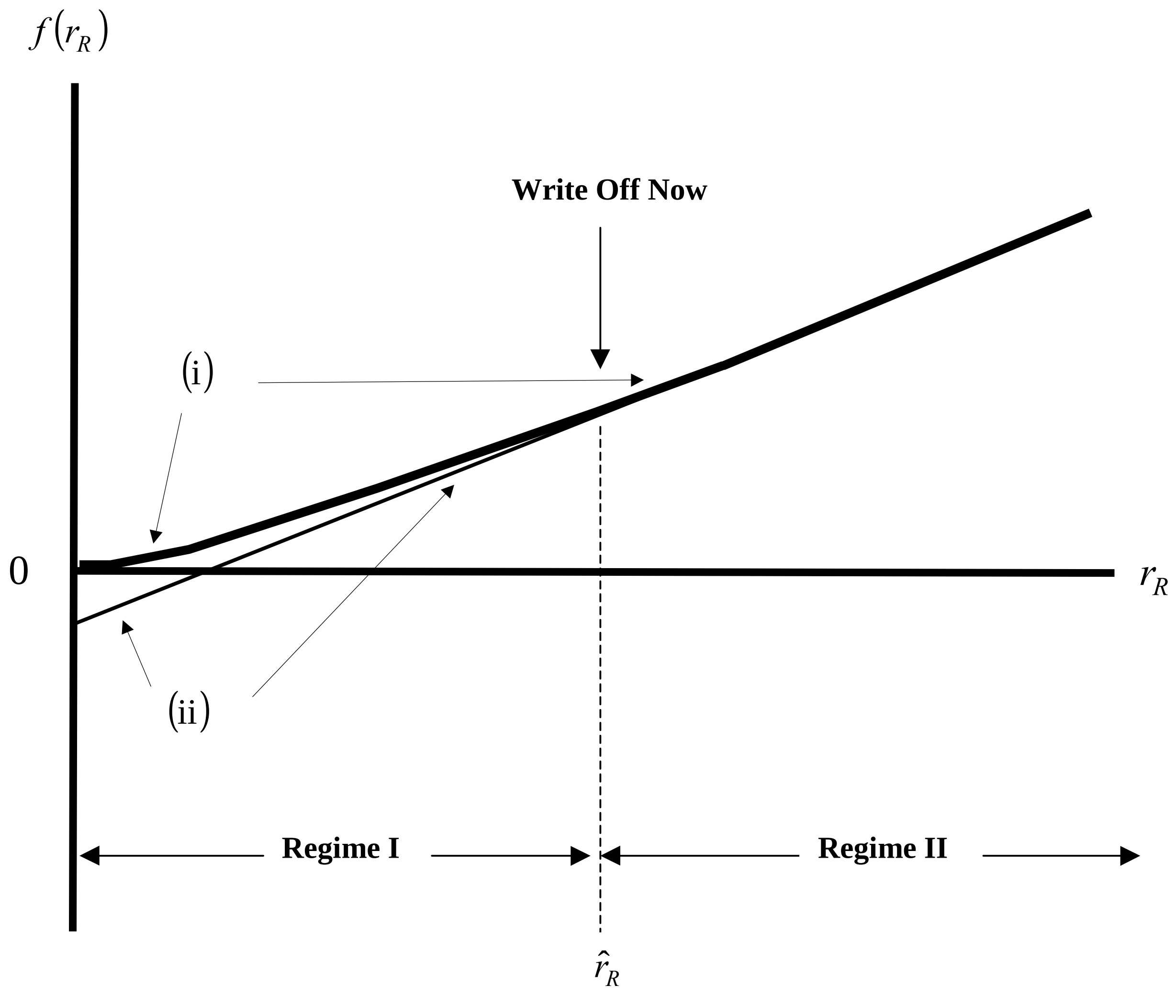


Figure 6: Boundary Conditions for $f(r_R)$ and the Determination of $\hat{r}_R$



$(\text{i}): f(r_R) = A(r_R)^{\beta}$ (*equation* (17))

$(\text{ii}): f(r_R) = \frac{r_R}{\delta_R} - 1$ (*right – hand side of equation* (20))

Figure 7: The Relationship between $\hat{r}_R$ and the Required Rate of Return $\overline{rr}$

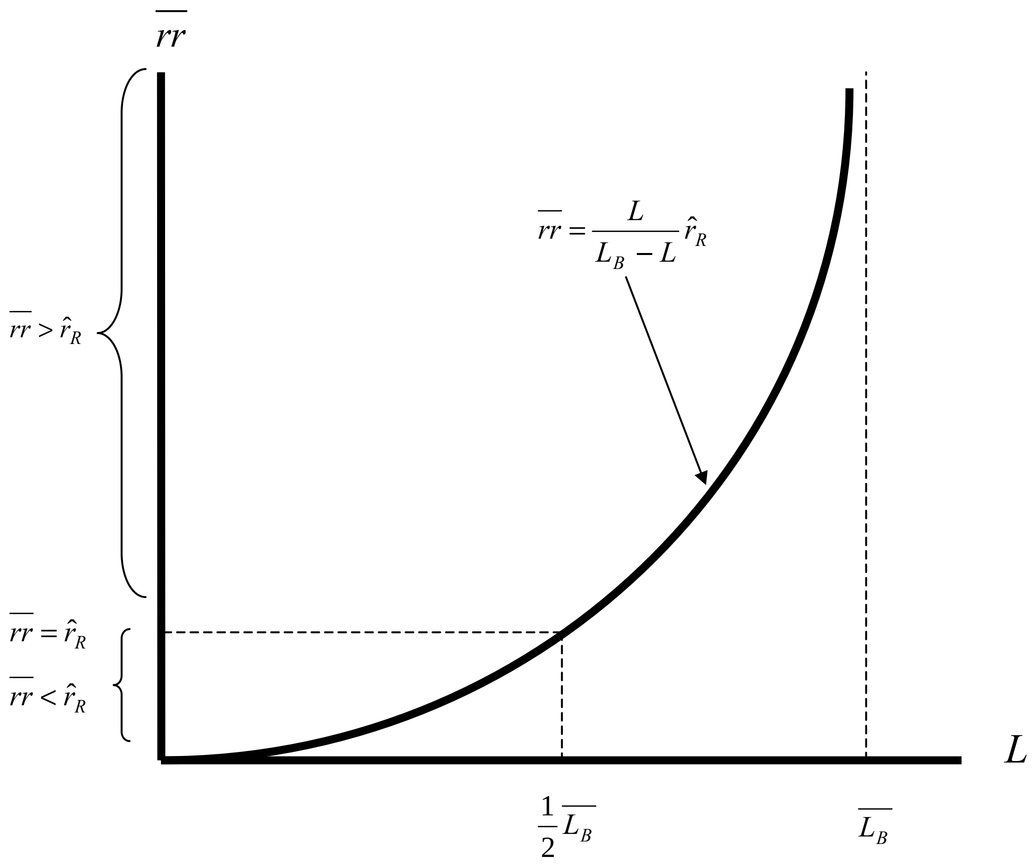


Figure 8: Three Regimes of Bank's Optimal Decisions

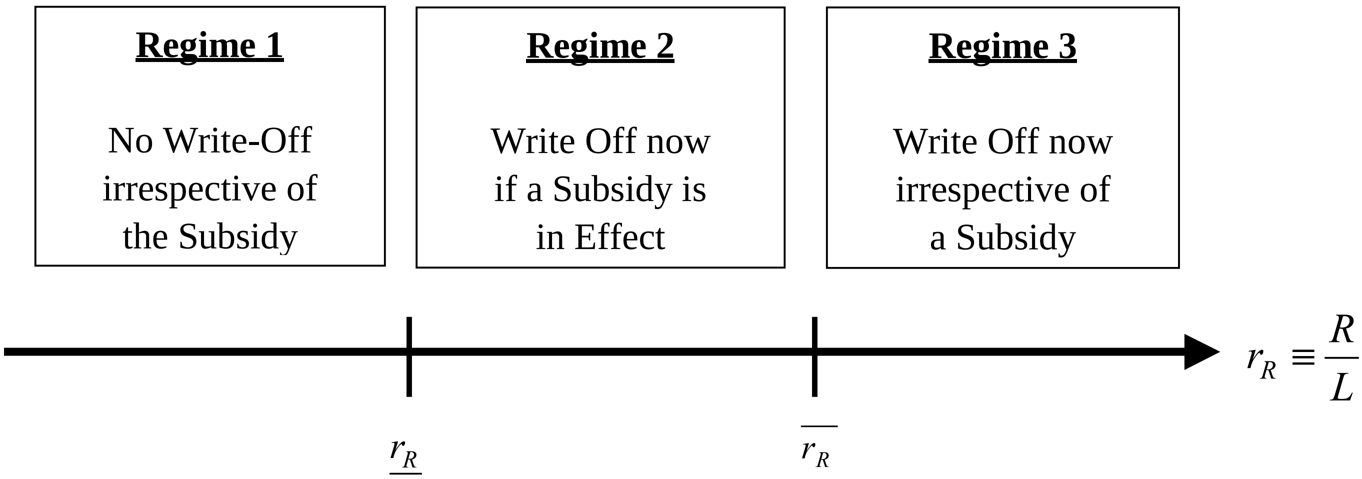


Figure 9: Free Boundaries of the Ratio of the Reinvestment Return to Write-off Losses

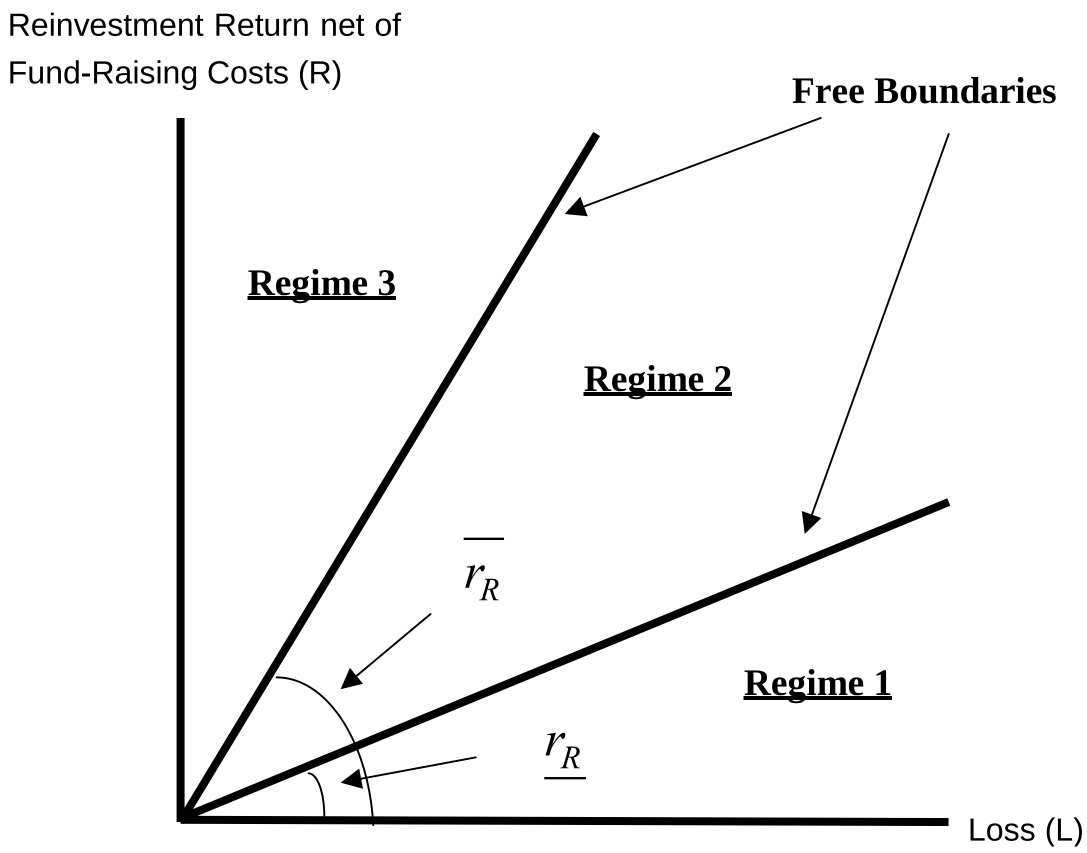
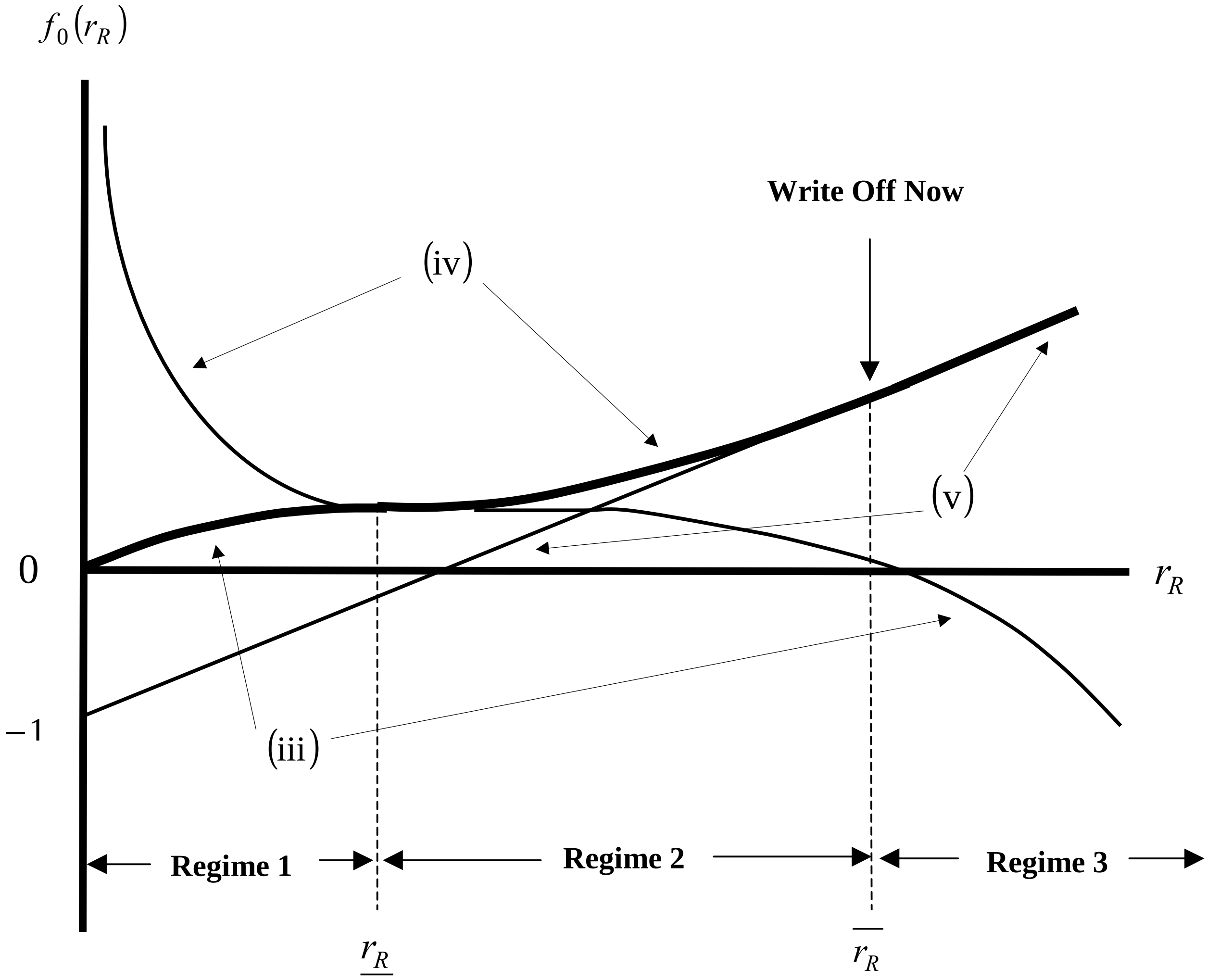


Figure 10: Boundary Conditions for $f_0(r_R)$ and the Determination of $\underline{r_R}$ and $\overline{r_R}$

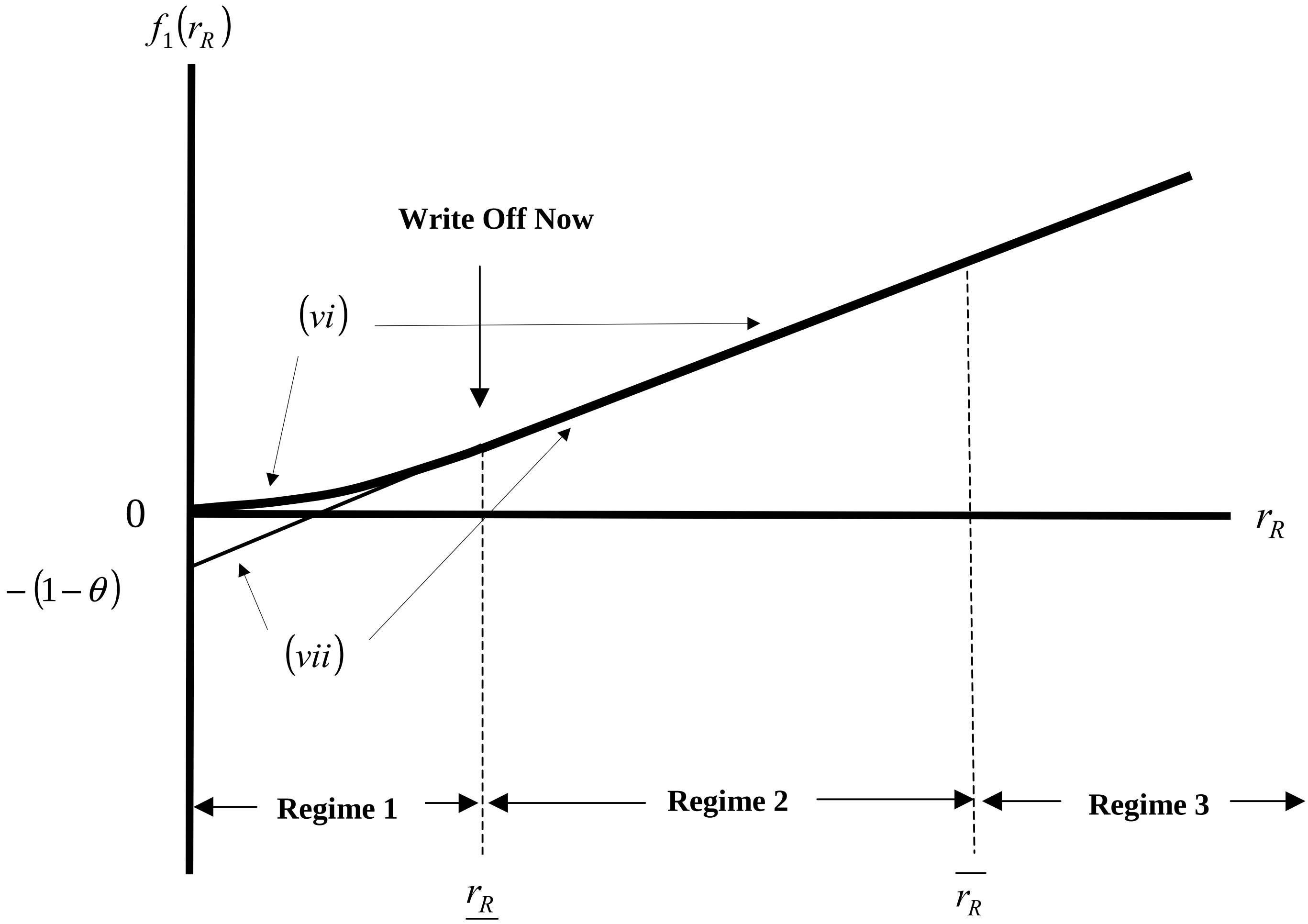


$$(\text{iii}): f_0(r_R) = \frac{\lambda_0 \lambda_1 B(r_R)^{\beta_1} - \lambda_1 C(r_R)^{\beta_2}}{\lambda_0 + \lambda_1} \quad (\textit{equation } (35))$$

$$(\text{iv}): f_0(r_R) = D(r_R)^{\beta_3} + E(r_R)^{\beta_4} + \frac{\lambda_1 r_R}{\delta_R(\mu + \lambda + \lambda_1 - \alpha_R)} - \frac{\lambda_1(1-\theta)}{\mu + \lambda + \lambda_1 - \alpha_L} \quad (\textit{equation } (40))$$

$$(\text{v}): f_0(r_R) = \frac{r_R}{\delta_R} - 1 \quad (\textit{equation } (42))$$

Figure 11: Boundary Conditions for $f_1(r_R)$ and the Determination of $\underline{r_R}$

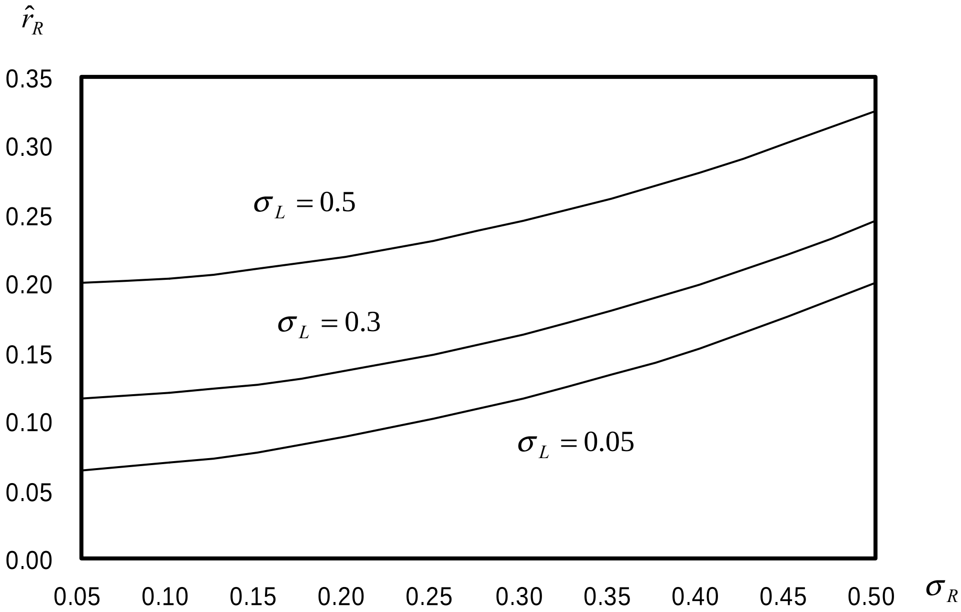


$$(vi): f_1(r_R) = \frac{\lambda_0 \lambda_1 B(r_R)^{\beta_1} + \lambda_0 C(r_R)^{\beta_2}}{\lambda_0 + \lambda_1} \quad (equation\ (36)\ )$$

$$(vii): f_1(r_R) = \frac{r_R}{\delta_R} - (1-\theta) \quad (equations (37)\ and\ (43)\ )$$

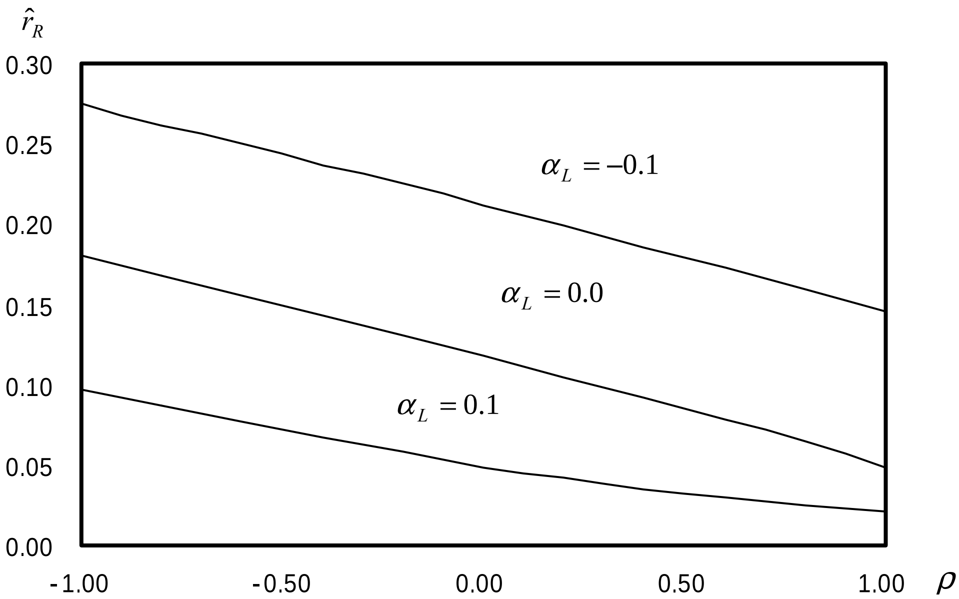
Figure 12: Threshold Value $\hat{r}_R$ as a Function of Parameters

(i) Dependence of $\hat{r}_R$ on σ_R and σ_L



Note: $\alpha_R = 0.02$, $\alpha_L = -0.02$, $\delta_R = 0.02$, $\rho = 0.0$, and $\lambda = 0.0$.

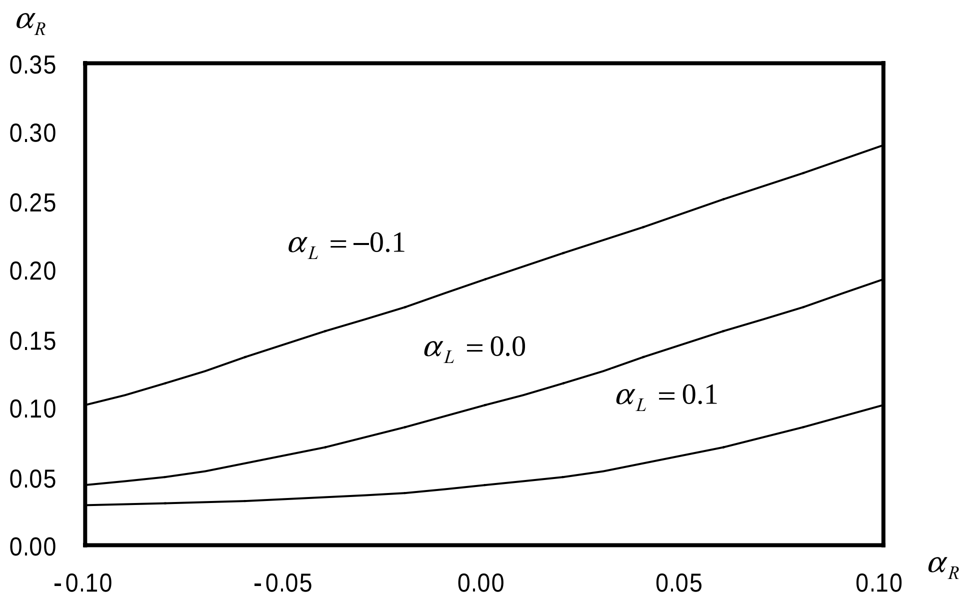
(ii) Dependence of $\hat{r}_R$ on ρ and α_L



Note: $\alpha_R = 0.02$, $\delta_R = 0.02$, $\sigma_R = 0.2$, $\sigma_L = 0.3$, and $\lambda = 0.0$.

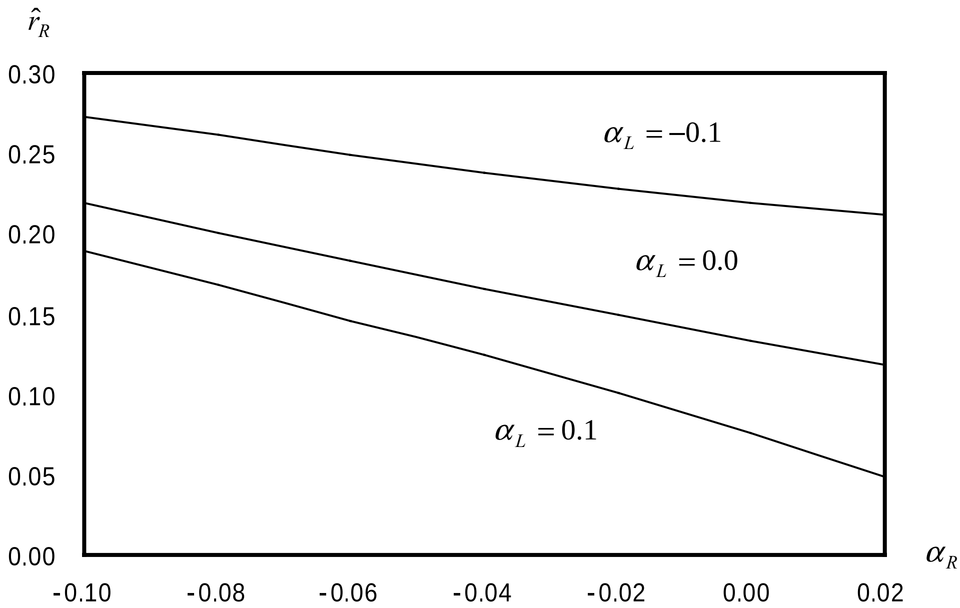
(iii) Dependence of $\hat{r}_R$ on α_R and α_L

(a) In the Case of Adjustable μ and Fixed δ_R



Note: $\delta_R = 0.02$, $\sigma_R = 0.2$, $\sigma_L = 0.3$, $\rho = 0.0$, and $\lambda = 0.0$.

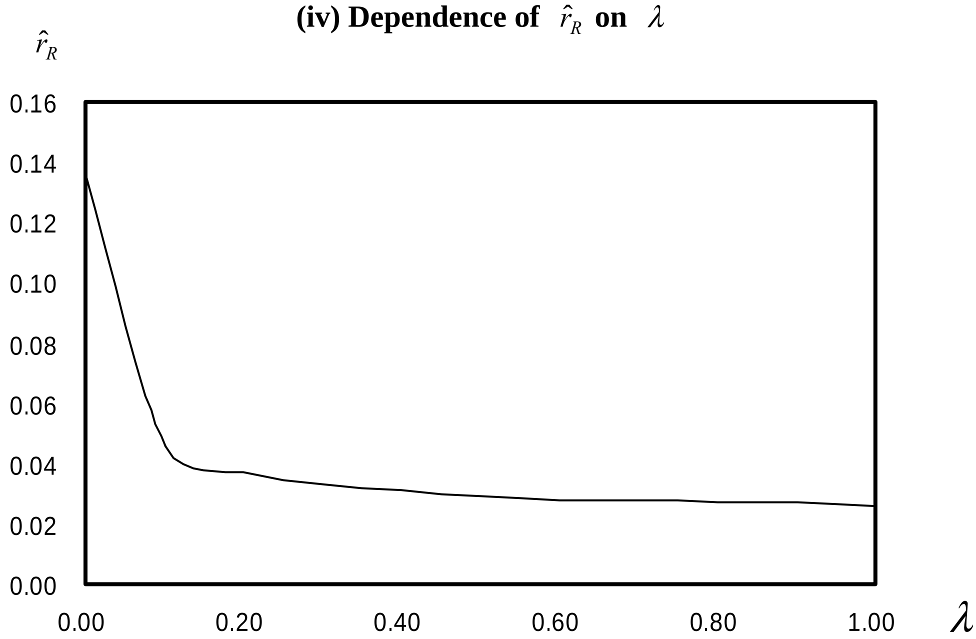
(b) In the Case of Adjustable δ_R and Fixed μ



Notes: 1. $\mu = 0.04$, $\sigma_R = 0.2$, $\sigma_L = 0.3$, $\rho = 0.0$, and $\lambda = 0.0$.

2. The range of α_R must satisfy the constraint $\delta_R \geq 0$.

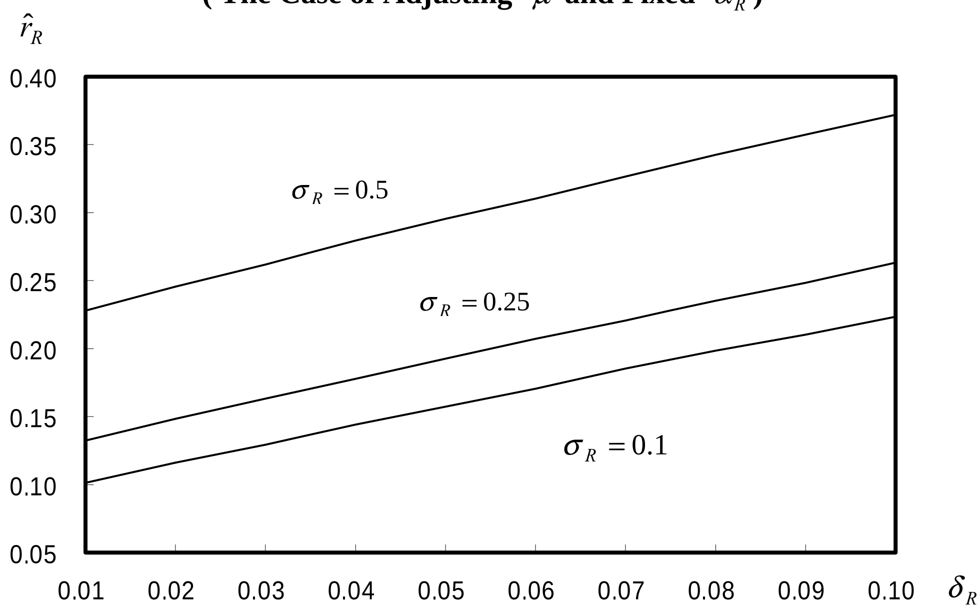
(iv) Dependence of $\hat{r}_R$ on λ



Note: Calculations are under the assumption that α_R and μ are fixed irrespective of the level of λ. Parameters are set as follows: $\mu = 0.04$, $\alpha_R = 0.02$, $\alpha_L = -0.02$, $\delta_R = 0.02$, $\sigma_R = 0.2$, $\sigma_L = 0.3$, and $\rho = 0.0$.

(v) Dependence of $\hat{r}_R$ on δ_R and σ_R

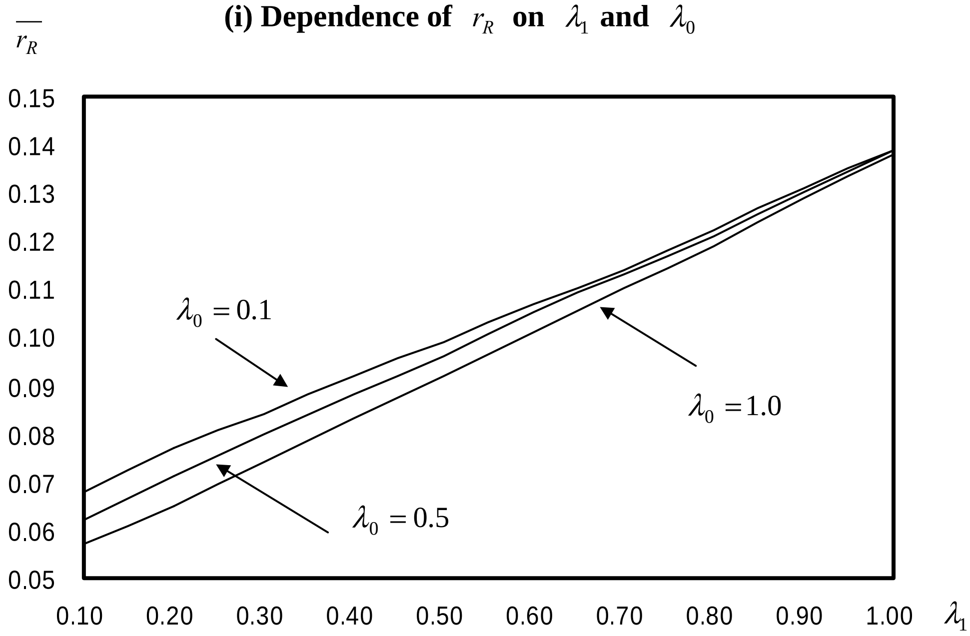
(The Case of Adjusting μ and Fixed α_R)



Note: $\alpha_R = 0.02$, $\alpha_L = -0.02$, $\sigma_L = 0.3$, $\rho = 0.0$, and $\lambda = 0.0$.

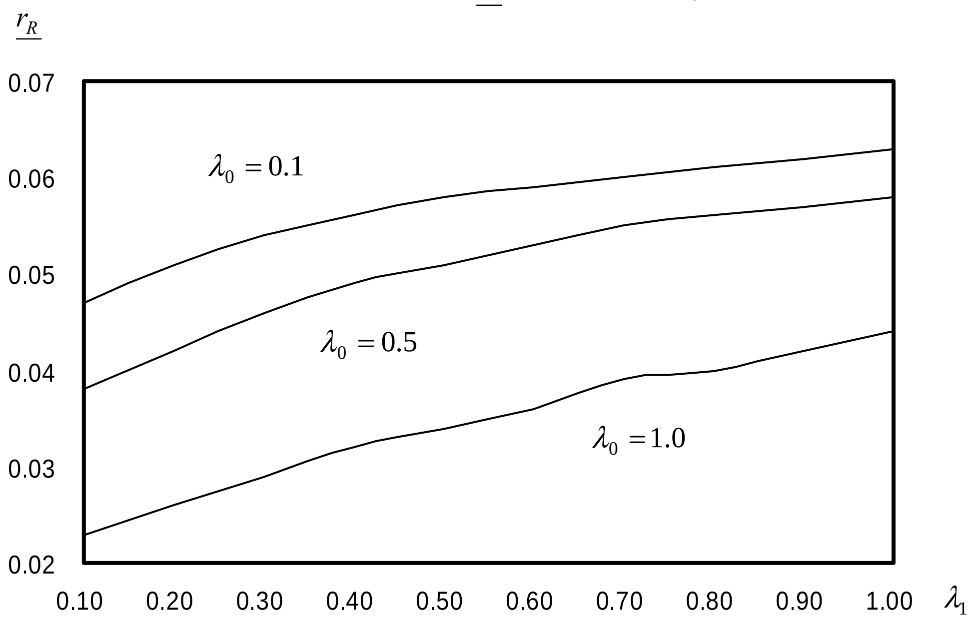
Figure 13: Threshold Values $\overline{r_R}$ and $\underline{r_R}$ as a Function of Parameters

(i) Dependence of $\overline{r_R}$ on λ_1 and λ_0



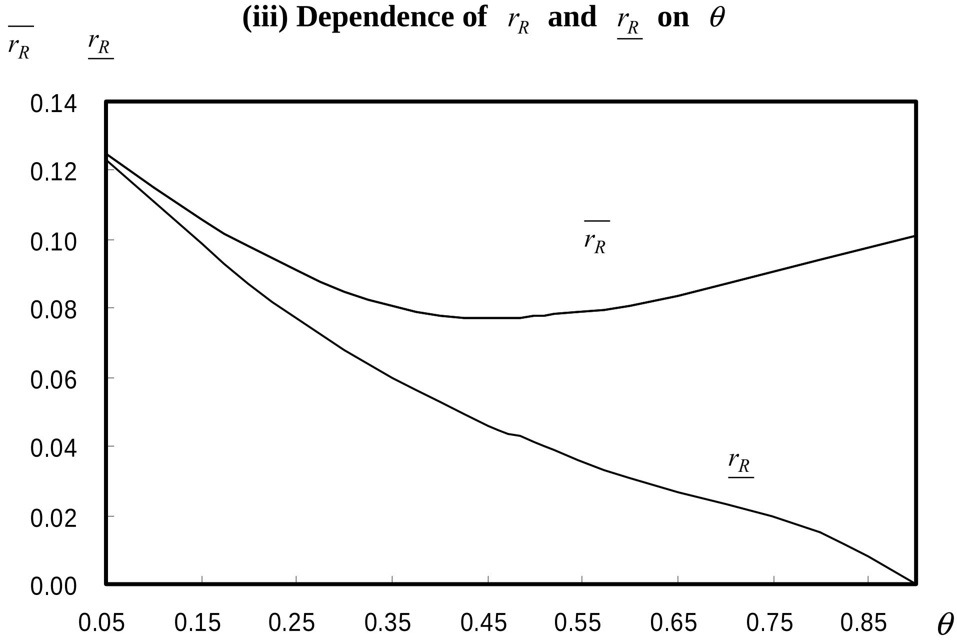
Note: $\alpha_R = 0.02$, $\alpha_L = -0.02$, $\sigma_R = 0.2$, $\sigma_L = 0.3$, $\rho = 0.0$, $\delta_R = 0.02$, $\lambda = 0.1$, and $\theta = 0.5$.

(ii) Dependence of $\underline{r_R}$ on λ_1 and λ_0



Note: $\alpha_R = 0.02$, $\alpha_L = -0.02$, $\sigma_R = 0.2$, $\sigma_L = 0.3$, $\rho = 0.0$, $\delta_R = 0.02$, $\lambda = 0.1$, and $\theta = 0.5$.

(iii) Dependence of $\overline{r_R}$ and $\underline{r_R}$ on θ



Note: $\alpha_R = 0.02$, $\alpha_L = -0.02$, $\sigma_R = 0.2$, $\sigma_L = 0.3$, $\rho = 0.0$, $\delta_R = 0.02$, $\lambda = 0.1$, $\lambda_0 = 0.3$, and $\lambda_1 = 0.3$.