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A Game-theoretic Real Options Approach

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Uncertainty, Monitoring Costs, and Private Banks' Lending Decisions in a Duopolistic Loan Market:

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Abstract

This paper explores banks' entry decisions into a duopolistic loan market in the hope of shedding some light on the recent slump in the Japanese loan market. The game-theoretic real options approach is used to analyze the effect of uncertainty on lending decisions. Numerical analysis shows that a rise in the demand volatility raises the threshold values of demand for both banks. In contrast, the direction of the effect stemming from a rise in the expected growth of demand depends on the assumption regarding which parameter is adjustable, the discount rate or the dividend rate. The similar tendency is found in the effect via the probability of bankruptcy of a borrowing firm. Also, a rise in the *ex ante* loan share of the leader raises the threshold demand for the leader. Further, a relative increase in irreversible monitoring costs to be paid by the follower raises the threshold demand for the follower, but lowers it for the leader.

Key words: Bank Lending, Uncertainty, Entry Decision, Monitoring Costs, Duopolistic Market, Real Options, Stochastic Game

JEL classification: C73, G21, L13

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I. Introduction

Since 1985, the Japanese economy has experienced unprecedented fluctuations. In particular, a significant surge in private bank[1] loans during the so-called bubble period, a subsequent sharp fall starting in 1991, and a declining trend that continues up to today are widely recognized as noteworthy characteristics[2].

There are of course two sides to the recent slump in bank lending. One is acceleration of loan repayments by corporations reflecting their desire to reduce interest-bearing liability as part of restructuring plans. The other is a decrease in new bank loans which reflects both a decline in loan demand by corporations and a prudent attitude on the part of banks toward extending new loans. Figure 1 shows the change in loans and discounts by domestically licensed banks as a percentage of nominal GDP, which indicates that it has been fluctuating below zero percent since around 1993[3]. And, based on recent data, Figure 2 shows a steady downward trend in new loans for equipment funds after a temporary pickup in 1995[4]. This paper focuses on the latter aspect of lending, that is, the decline in new loans.

In this regard, in addition to the direct impact of the prolonged recession, many other plausible causes have already been pointed out for the slump in loan demand, including a downward shift in investment planning on the part of non-financial corporations. From a structural point of view, a series of liberalization measures in the capital market have undoubtedly played a role. It is widely believed that such measures have prompted a switch from indirect financing to direct financing including equity financing and the issue of corporate bonds, particularly among leading corporations[5].

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[1] I use the word 'private' for the purpose of distinguishing between domestically licensed (private) banks and government-related financial organizations such as Development Bank of Japan and Housing Loan Corporation. The financial organizations in the latter category are expected to play the role of funding the fields that are not profitable enough in the perspective of private banks, but might provide benefits from a social point of view.

[2] For more details, see Ogawa and Kitasaka (1999), for example.

[3] Note that data shown in Figure 1 cannot specify which factor has contributed more to the decline in bank lending. One interesting point here, however, is that although previously a decrease in bank loans relative to the size of nominal GDP was only seen during periods of tight monetary policy, suggesting the important role of demand, the recent slump occurred despite an unprecedented easy monetary policy.

[4] Unfortunately, this data is not available before 1993.

[5] Moreover, the recent entry of many companies that are not originally categorized under the banking industry is likely to intensify competition in the already reduced loan market.

Also, turning to the supply side of bank loans, some argue that one of the potential reasons for the slump in bank lending lies in the fact that real estate has been extensively used as collateral, which is especially the case for new borrowers[6]. Here, it is often pointed out that, particularly during the bubble period, banks could neglect rigorous monitoring efforts due to an almost religious faith that potential losses in the future could be sufficiently covered by the real estate that borrowers put up as collateral.

As will be explained, in some sense, monitoring ability can be viewed as a kind of fixed capital that takes a long time to accumulate. Hence, if accumulation of monitoring ability is neglected for a prolonged period, then its recovery (re-accumulation) cannot be done in a short period and thus banks will suffer from larger costs than before in order to properly judge the creditworthiness of potential borrowers[7]. Thus, a delay in accumulating monitoring ability is likely to have something to do with the recent overly prudent attitude of banks toward extending loans to new borrowers.

As suggested by Aoki (1994), transactions involving funds between firms that plan to undertake a project and intermediaries like banks entail a high degree of information asymmetry[8]. To overcome this problem to some extent[9], there need to be some mechanisms for assessing the creditworthiness of projects. Monitoring is one such mechanism, and, from the perspective of banks, induces sunk costs in the sense that they cannot be retrieved once they are actually paid in advance of lending decisions[10].

To be more specific, monitoring can be categorized into three kinds. The first is *ex ante* monitoring that aims to assess a corporation's creditworthiness regarding projects and to screen them. The second is interim monitoring, the purpose of which is to closely observe management in order to alleviate the problem of moral hazard. The last is *ex post* monitoring, which tries to verify a firm's financial condition and apply appropriate punitive and corrective action. Among these three, this paper focuses on the first, *ex ante*

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[6] See Higano (1987) and Ogawa and Kitasaka (1999) in this regard.

[7] In this regard, the credit guarantee system is considered to have facilitated lending to small and medium-sized firms by Japanese banks by reducing monitoring costs. But, the possibility should not be overlooked that the system itself weakened the incentive of banks to accumulate monitoring ability.

[8] See Akerlof (1970) for the original discussion about information asymmetry.

[9] It should be noted here that even if the problem of information asymmetry is completely eliminated, uncertainty inherent in projects themselves remains.

[10] In other words, monitoring efforts are irreversible. Dixit and Pindyck (1994) define the term 'irreversible' as follows: *investment expenditures are sunk costs or irreversible when they are firm or industry specific*. For typical examples, they argue that most investments in marketing and advertising are sunk costs because they cannot be recovered.

monitoring, which means it concentrates on the decisions made by banks as to whether they extend a new loan or not.

As emphasized by Sheard (1994), regarding monitoring activity, one of the most salient features in Japan is that most large firms maintain a close relationship with a bank. Such close bank-corporation ties are often termed the 'main bank system'. The main bank is, in most cases, a principal shareholder in the firm and plays a decisive role in monitoring it.

Monitoring is sometimes said to be delegated to the leader bank[11]. In other words, as explained by Higano (1987), the leader bank plays the role of 'bell cow' or 'bellwether' in the sense that other banks follow its decisions and behavior, because information regarding the screening process effected by the leader bank is revealed (or sent as a signal) via its actual lending decisions[12].

Another important aspect of actual lending is that the leader bank has the largest loan share, but often it is not the sole lender so that the loan market for a specific potential borrower can be reasonably approximated to be an oligopolistic market[13]. Also, it is a well-known fact that the loan syndicate led by the leader bank is hierarchical in terms of proportionate loan shares. The leader bank decides loan shares in advance, and then the follower banks judge whether participation in the loan syndicate is really beneficial to them. Hence, if a researcher takes a perfectly competitive or monopolist market structure as given in analyzing bank lending decisions, he or she might miss some important aspects.

Further, under the assumption of uncertainty and irreversibility, it is natural to think that banks[14] should seriously consider the option to wait before extending a new loan. This is a typical setting of a so-called real options approach first applied by McDonald and Siegel (1984) and later extensively reviewed by Dixit and Pindyck (1994).

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[11] The delegated monitoring theory was first developed by Diamond (1984). The theory says that monitoring typically involves increasing returns to scale, implying that specialized banks are more efficient in handling it. Therefore, individual lenders tend to delegate monitoring activity instead of performing it themselves.

[12] It should be noted, however, that this information activity entails the problem of free-riding by the follower bank. One possible solution to this is to impose fees on the part of the follower bank so as to internalize the externality. Although in the Japanese case, in particular, this kind of information fee might not have been explicit, it is often said that the monopolization of some profitable businesses such as domestic and foreign exchange operations by the leader bank has fulfilled the role.

[13] In the words of Sheard (1994), there is *exclusivity in monitoring with non-exclusivity in lending.*

[14] Throughout the paper, uncertainty means that the best one can do is to assess the subjective probabilities of the alternative outcomes that entail greater or smaller profit (or loss) for a project.

As emphasized by Trigeorgis (1993) and Dixit and Pindyck (1994), among others, the options approach helps explain why actual investment decisions made by the business sector cannot be explained by conventional wisdom such as the net present value (NPV) approach. In reality, firms make investment decisions that are expected to yield a return well in excess of the required rate of return[15].

Indeed, the adoption of the real options framework is likely to provide an important insight into the role of uncertainty and sunk costs in the recent slump in bank lending in Japan. Specifically, within the real options framework, in general, one can see the change in the value of the option to wait and see as one changes the values of such variables as sunk costs (monitoring costs), the discount rate, uncertainty (volatility), the expected growth rate of demand, and the subjective probability of future bankruptcy of the borrowing firm, the last three being associated with the assumed stochastic process[16]. Thus, one can numerically assess lending decisions directly in terms of uncertainty and monitoring ability.

Motivated by the discussion above, this paper attempts to analyze lending (entry) decisions made by banks within the real options framework in the hope of shedding some light on the recent slump in the Japanese bank loan market[17]. Specifically, it focuses on the duopolistic loan market in which the leader bank makes entry decisions taking the reaction of followers into consideration and then, given the leader bank's action, follower banks determine whether to enter the loan market or not.

The rest of the paper is organized as follows: Section II describes the basic theoretical framework of the game-theoretic real options approach. Section III numerically analyzes lending decisions in a duopolistic loan market, and Section IV concludes by linking the insight of the real options approach to episodes of the recent bank lending situation in Japan.

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[15] On the downside, firms continue to stay in business for a long time although operating profit is well below operating costs so that they lose.

[16] Actually, I regard the shift parameter of loan demand as a stochastic variable instead of return itself because of the assumption of a duopolistic loan market. I adopt a combined geometric Brownian motion and (Poisson) jump process as an underlying stochastic process.

[17] Note that although this paper is motivated by the literature on the main bank system, the aim of this paper is not necessarily to directly analyze the main bank system itself, but to examine the role of uncertainty in extending a new loan in a duopolistic loan market setting.

II. Theoretical Framework

(i) Basic Setup

Introduction of an oligopolistic market structure into a stochastic dynamic setting usually gives rise to many practical difficulties. In fact, applications of the game-theoretic option theory are quite recent. Under such circumstances, Smets (1993) developed a very simplified version of this kind of model in which there are a predetermined leader and a follower[18] in order to analyze the decision-making between exporting and foreign direct investment.

In contrast to the modeling difficulties, the essence of the model is actually not too difficult to state. The existence of both uncertainty and irreversibility implies that there is some value to an option to wait, and the higher the degree of uncertainty, the greater the hesitancy on the part of both players. The fear of preemption by a rival, however, indicates that the leader needs to make decisions without delay. Which of these considerations is more relevant depends on the underlying parameters and the current state of the stochastic variable.

In this paper, the strategic interaction is assumed as follows. First, before actually paying monitoring costs, the leader bank declares the *ex ante* loan shares (amounts)[19] of each bank. Then, both banks judge strategically whether they will enter the market depending on current demand (return) conditions.

The theoretical framework itself basically follows Smets (1993), but the following modifications are made for the purpose of enriching implications for bank lending behavior in a duopolistic market:

(i) the demand curve is specified such that it is downward-sloping and its demand elasticity is constant in any region instead of generic form.

[18] For other works on the trade-off between the strategic incentive to invest early in an oligopoly and the value of flexibility under uncertainty, see Appelbaum and Lim (1985), Spencer and Brander (1992), and Kulatilaka and Perotti (1992), for example.

[19] Here, for simplicity, I assume that the leader bank can decide the exact loan amounts to be extended by both banks. Of course, one can determine optimal new loan amounts under the framework of the Stackerberg model instead of giving them *ad hoc* values. Since, by assumption, lending amounts are constant throughout future periods, the maximization problem for the leader bank turns out to be maximizing current profits (revenues less sunk costs) taking account of the optimal reaction taken by the follower bank. In my setting, lending amounts of banks are determined by the value of sunk costs and demand structure. But due to the non-linearity of derived first-order conditions, it is not easy to find the solution. As long as one assumes that lending amounts are constant over time, however, implications derived from the real options approach are the same so that I adopt the exogenously given amounts of new loans for the sake of computational facility in comparing the result when loan shares change.

(ii) *ex ante* loan shares can be arbitrarily changed to investigate the relationship between *ex ante* loan shares and the threshold values of current demand for entry.

(iii) sunk costs of both banks can be separately specified to explicitly take the leader bank's informational cost advantage into consideration.

(iv) a combined geometric Brownian motion and Poisson downward jump process is adopted to the demand shift parameter instead of the standard geometric Brownian motion in order to take the possibility of bankruptcy of the borrowing company[20] into consideration.

Now, let me proceed to the structure of the model. First, consider the value of the follower bank contemplating entry to the loan market[21]. Let $v_f(\Pi_f)$ denote the value of the follower bank's future cash flow net of operational cost from actual lending, where $\Pi_f \equiv r_L L_f$ and L_f is the amount of a new loan extended by the follower bank.

I assume that profit margin r_L that is common to both banks can be specified as

$$r_L = Y(L_l + L_f)^{-\varepsilon}, \tag{1}$$

where ε denotes the inverse of the elasticity of loan demand with respect to the interest rate (net of operating costs)[22]. Here, by loan demand I mean demand by a specific potential borrower[23]. Demand uncertainty is assumed to follow the following combined geometric Brownian motion and Poisson downward jump process such as

$$dY = \alpha Y dt + \sigma Y dz - Y dq, \tag{2}$$

where α denotes the expected growth rate parameter that is relevant only in the Brownian motion part, σ the volatility parameter, dz the increment of the standard

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[20] In fact, after the bursting of the bubble economy, the liabilities of bankrupt corporations as a proportion of total financial liabilities held by private non-financial corporations rose from about 0.25% in 1990 to 2.5% in 1998 according to a survey by Tokyo Teikoku Bank, although in 1999 the figure dropped to about 0.6% due to the adoption of stabilization measures under the credit guarantee system.

[21] This 'backward solution' is a familiar method in analyzing the dynamic duopolistic strategy.

[22] In the original model by Smets (1993), demand is assumed to be sufficiently elastic to ensure capacity production, implying that total output is either zero, one, or two depending on the number of active firms.

[23] In this paper, for simplicity, I assume that a borrowing company is passive in the sense that it does not have any bargaining power in making loan contracts. Introducing the game-theoretic interaction between borrowers and lenders is one of my future tasks.

Wiener process, and dq the increment of a Poisson process with mean arrival time rate λ. By assumption, $E[(dz)(dq)]=0$ holds. Also, equation (2) states that if an event occurs, Y falls by some fixed percentage ϕ ($0 \le \phi \le 1$) with probability one[24].

Clearly, equation (2) implies that

$$d\Pi_f = \alpha\Pi_f dt + \sigma\Pi_f dz - \Pi_f dq \tag{3}$$

holds since $(L_l + L_f)^{-\varepsilon}$ is assumed to be fixed.

It is important to note that the expected rate of change in Π_f is not α as in the case of the standard geometric Brownian motion, but

$$\frac{E[d\Pi_f/\Pi_f]}{dt} = \alpha - \lambda\phi \,. \tag{4}$$

Hence, given the value of ϕ, an increase in λ decreases the expected rate of capital gains on Π_f by increasing the chance of a sudden downward jump in Π_f. Also, note that since a Poisson event occurs infrequently, most of the time the variance of $d\Pi_f/\Pi_f$ over a short interval of time dt is just that of the part governed by the Brownian motion $\sigma^2 dt$. If the jump happens, however, it gives rise to a large deviation, so its contribution to the variance cannot be neglected.

(ii) Solving the Maximization Problem by Dynamic Programming

First, suppose that the leader has already entered the market. Next, I consider the entry decision made by the leader taking account of the follower's response. Here, note that it does not make a sense unless $v_f(0)=0$ holds because if profits are zero in the geometric Brownian motion, they will remain zero forever.

Now let $F_f(\Pi_f)$ denote the follower's value of the option to lend. For simplicity,

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[24] Formally, one can write $dY = \begin{cases} \alpha Y dt + \sigma Y\sqrt{dt} & \textit{with probability} \;\; (1-\lambda dt)/2 \\ \alpha Y dt - \sigma Y\sqrt{dt} & \textit{with probability} \;\; (1-\lambda dt)/2 \\ -\phi Y & \textit{with probability} \;\; \lambda dt \end{cases}$.

As will be discussed later, the case of ϕ =1 can be thought of as the case of bankruptcy of the borrower.

I assume that there is no fixed finite time horizon. I can now proceed to solve for the optimal lending rule by dynamic programming. The Bellman equation can be written as[25]

$$\rho F_f(\Pi_f) = \underset{\theta}{Max}\, E\left[\frac{1}{dt} dF_f(\Pi_f)\right], \qquad (5)$$

where ρ denotes the discount rate and θ the control (decision) variable of the bank. Applying Ito's Lemma for the combined geometric Brownian motion and jump process[26][27] yields

$$\rho F_f(\Pi_f)dt = \alpha \Pi_f F_f'(\Pi_f)dt + \frac{1}{2}\sigma^2 \Pi_f^2 F_f''(\Pi_f)dt - \lambda\{F_f(\Pi_f) - F_f[(1-\phi)\Pi_f]\}dt \text{ ,} \qquad (6)$$

where $F_f'(\Pi) \equiv \partial F_f / \partial \Pi_f$ and $F_f''(\Pi) \equiv \partial^2 F_f / \partial \Pi_f^2$. Equation (6) can be rewritten as

$$\frac{1}{2}\sigma^2 \Pi_f^2 F_f''(\Pi_f) + (\rho - \delta)\Pi_f F_f'(\Pi_f) - (\rho + \lambda)F_f(\Pi_f) + \lambda F_f[(1-\phi)\Pi_f] = 0\,. \qquad (7)$$

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[25] Formally, derivation of equation (5) goes as follows. First, the original form of the Bellman equation can be expressed as

$$F_f(\Pi_f, t) = \underset{\theta}{Max}\left\{\frac{1}{1+\rho\Delta t} E[F_f(\Pi_f, t+\Delta t) \,|\, \theta]\right\}.$$

Multiplying this equation by $(1+\rho\Delta t)$ and rearranging yields

$$\rho F_f(\Pi_f, t) = \underset{\theta}{Max}\, E\left[\frac{1}{dt} dF_f(\Pi_f)\right],$$

where note that I let Δt approach zero and $(1/dt)E[dF_f]$ denotes the limit of $E[\Delta F/\Delta t]$.

[26] In general, if the stochastic process is

$$dx = a(x,t)dt + b(x,t)dz + g(x,t)dq \text{ ,}$$

then the expected value of the change in any function $H(x,t)$ can be given by

$$E[dH] = \left[\frac{\partial H}{\partial t} + a(x,t)\frac{\partial H}{\partial x} + \frac{1}{2}b^2(x,t)\frac{\partial^2 H}{\partial x^2}\right]dt + E_\phi\{\lambda[H(x+g(x,t)\phi, t) - H(x,t)]\}dt \text{ ,}$$

where ϕ is the size of the jump when the event happens. For more details, see Dixit and Pindyck (1994).

[27] Inclusion of a jump process is advantageous because it enables one to describe a more realistic situation, but from a practical viewpoint, there are some problems. Among them, the most important problem is that the adoption of a jump process makes building a perfect hedge impossible. This implies that, in general, it is not possible to build a riskless portfolio as in Black-Scholes-Merton type contingent claims analysis. This is why I use dynamic programming with an exogenous discount rate ρ instead of contingent claims analysis. To avoid such a disadvantage, one sometimes assumes that the jump-risk is non-systematic, that is, uncorrelated with the market portfolio, which enables one to construct a risk-free portfolio. In such a case, for example, equation (6) can also be derived by contingent claims analysis.

Note that in deriving equation (7), I use the relationship of $\rho = \alpha + \delta$, where δ denotes the dividend rate. As suggested by Dixit and Pindyck (1994), in such a case, the solution[28] is known to have a form such that

$$F_f(\Pi_f) = A(\Pi_f)^{\beta}, \tag{8}$$

where A and β are constants to be determined. The expression for $\beta > 1$ can be found by solving the following fundamental quadratic equation:

$$\frac{1}{2}\sigma^2 \beta(\beta - 1) + (\rho - \delta)\beta - (\rho + \lambda) + \lambda(1 - \phi)^{\beta} = 0\,. \tag{9}$$

Unfortunately, however, equation (9) is so non-linear that one cannot find any closed form solutions. Hence, in what follows, I consider the special case of $\phi = 1$, which means that once jump happens, it removes full value of Π and remains at zero forever. That is, one can think of the event as abrupt bankruptcy. In such a special case, the positive root β of equation (9) can be found by

$$\beta = \left[\frac{1}{2} - \frac{\rho - \delta}{\sigma^2}\right] + \sqrt{\left[\frac{\rho - \delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2(\rho + \lambda)}{\sigma^2}} > 1\,. \tag{10}$$

Now, consider the boundary conditions[29] that must be satisfied at the threshold value $\overline{\Pi}_f$ to close the model. First, the value-matching condition can be written as

$$F_f(\overline{\Pi}_f) = v_f(\overline{\Pi}_f) - L_f I_f\,, \tag{11}$$

where $v_f(\overline{\Pi}_f) = \overline{\Pi}_f / \delta$ and I_f denotes monitoring cost per loan. Equation (11) states

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[28] Generally speaking, one must write the solution as

$$F_f(\Pi_f) = A_1(\Pi_f)^{\beta_1} + A_2(\Pi_f)^{\beta_2} \quad (\beta_1 > 1 \; and \; \beta_2 < 0)$$

instead of equation (8). The condition of $F_f(0) = 0$, however, enables one to omit the second term on the right-hand side of the equation to assure continuity of the function.

[29] Generally, the value-matching and smooth-pasting conditions, together with the condition

that the value of the option should equal the net value from exercising it.

Second, the smooth-pasting condition is

$$F_f'\left(\overline{\Pi}_f\right) = v_f'\left(\overline{\Pi}_f\right), \tag{12}$$

which implies that the graphs of $F_f\left(\Pi_f\right)$ and $v_f\left(\Pi_f\right) - L_f I_f$ should meet tangentially at the threshold value $\overline{\Pi}_f$.

Specifically, value-matching condition (11) and smooth-pasting condition (12) can be rewritten as

$$A\left(\overline{\Pi}_f\right)^{\beta} = \frac{\overline{\Pi}_f}{\delta} - L_f I_f \,, \tag{13}$$

and $$\beta A\left(\overline{\Pi}_f\right)^{\beta - 1} = \frac{1}{\delta} \,. \tag{14}$$

Solving $\overline{\Pi}_f$ from conditions (13) and (14) yields

$$\overline{\Pi}_f = \frac{\beta}{\beta - 1} \delta L_f I_f \,. \tag{15}$$

Now let $\overline{Y}_f$ be the threshold value of Y at which the follower bank decides to enter the market. It is easy to find $\overline{Y}_f$ such that

$$\overline{Y}_f = \frac{\beta}{\beta - 1} \frac{\delta I_f}{\left(L_l + L_f\right)^{-\varepsilon}} \,. \tag{16}$$

Hence, ignoring the time subscription, the follower's value of the option to lend can be summarized as

$$\begin{cases} if\ Y \geq \overline{Y}_f, \quad then \quad F_f\left(Y\right) = \left(\frac{1}{\delta}\right) L_f Y \left(L_l + L_f\right)^{-\varepsilon} - L_f I_f \,. \\ otherwise, \qquad F_f\left(Y\right) = \left(\frac{Y}{\overline{Y}_f}\right)^{\beta} \left[\left(\frac{1}{\delta}\right) L_f \overline{Y}_f \left(L_l + L_f\right)^{-\varepsilon} - L_f I_f\right]. \end{cases} \tag{17}$$

$F_f\left(0\right) = 0$ consist of so-called boundary conditions.

Here, it should be noted that if one sets the quantity of a loan to be extended by the leader bank such that $L_l = 0$, the follower's value depending on Y can be regarded as the monopolist's value denoted $F_m(Y)$. Figure 3 graphically depicts the entry decision to be made by the monopolist bank. At the point where $Y = \overline{Y}(=\overline{Y}_f)$, $F_m(Y)$, and $v_m(Y) - L_m I_m$ meet tangentially, which is suggested by boundary conditions (11) and (12).

Next, consider the lending decision to be made by the leader bank. If $Y \geq \overline{Y}_f$, then the follower bank will lend immediately and the leader bank's cash flow will be $L_l Y (L_l + L_f)^{-\varepsilon}$. On the other hand, if $Y < \overline{Y}_f$, then the follower bank will prefer waiting until period T when $\overline{Y}_f$ is first hit to extending a new loan now. Hence, the leader bank will have cash flow that is equivalent to $L_l(YL_l^{-\varepsilon})$, implying that its expected (gross) value before netting out the monitoring cost can be expressed as

$$E\left[\int_{s=0}^{T} e^{-\rho s} L_l (YL_l^{-\varepsilon}) ds\right] + E[e^{-\rho T}] \frac{L_l \overline{Y}_f (L_l + L_f)^{-\varepsilon}}{\delta}. \tag{18}$$

Hence, the leader bank's value of the option to lend can be summarized as

$$\begin{cases} if\ Y \geq \overline{Y}_f, \quad then \quad F_l(Y) = \left(\frac{1}{\delta}\right) L_l Y (L_l + L_f)^{-\varepsilon} - L_l I_l. \\ otherwise, \qquad F_l(Y) = \left(\frac{1}{\delta}\right) L_l (YL_l^{-\varepsilon}) \left[1 - \left(\frac{Y}{\overline{Y}_f}\right)^{\beta - 1}\right] + \left(\frac{Y}{\overline{Y}_f}\right)^{\beta} \left[\left(\frac{1}{\delta}\right) L_l \overline{Y}_f (L_l + L_f)^{-\varepsilon}\right] - L_l I_l. \end{cases} \tag{19}$$

And the threshold value of Y denoted $\overline{Y}_l$ at which the leader bank makes an entry decision must satisfy the condition:

$$F_l(\overline{Y}_l) = F_f(\overline{Y}_l) > 0, \tag{20}$$

which implies that at $Y = \overline{Y}_l$, both banks are indifferent about which role they assume, leader or follower[30].

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[30] Note that in the region of Y, which satisfies $F_l(Y) < F_f(Y)$, the leader prefers waiting because in such a region the leader does not have an incentive to become a leader. In an analogy to financial

The decisions made by both banks can be described in Figures 4 (i)-(iii). First, Figure 4 (i) graphically demonstrates the interaction between the leader and follower banks when they are identical in every aspect except for the predetermined roles they assume. Two curves representing the leader's and follower's values cross each other at $\bar{Y}_l$ and meet tangentially at $\bar{Y}_f$.

Second, Figure 4 (ii) illustrates the case in which the sunk cost of the leader bank is lower than that of the follower bank, that is, $I_l < I_f$. In this case, neither curve actually meets at $\bar{Y}_f$, although their slopes are the same from this point on. This fact can be confirmed by comparing equations in (17) and (19) under the conditions that $L_l = L_f$ and $I_l < I_f$.

Lastly, Figure 4 (iii) demonstrates the case in which the *ex ante* loan share of the leader is larger than that of the follower, although sunk costs are the same, that is, $I_l = I_f$ and $L_l > L_f$. In this case, at $\bar{Y}_f$, both curves do not meet as in the second case, and they diverge in the region of $Y \geq \bar{Y}_f$, due to the fact that the slope of the leader's value is steeper than that of the follower's.

(iii) Stochastic Version of Tobin's q

Here, note that equation (15) can be modified as

$$v_f\left(\overline{\Pi}_f\right) = \frac{\beta}{\beta - 1} L_f I_f \,. \qquad (21)$$

One can interpret equation (21) as saying there is an edge between investment (sunk cost), denoted $L_f I_f$, and the expected value of the follower bank's cash flow from actual lending net of operating cost which is denoted $v_f\left(\overline{\Pi}_f\right)$.

Thus, the edge can be defined as

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options, the investment (monitoring) opportunity described in this paper corresponds to a call option on a common stock. It gives one an option that is *in the money*, which means that if it were exercised today it would yield a positive net payoff. In contrast, an option is said to be *out of the money* if exercising it today yields a negative net payoff.

$$\bar{q} \equiv \frac{\beta}{\beta - 1} > 1 . \qquad (22)$$

The index $\bar{q}$ captures a very similar and comparable notion introduced by Tobin (1969). It should be noted, however, that $\bar{q}$ [31] as defined in equation (22) depends on uncertainty[32] about future demand (and hence profit) conditions such as the drift (expected growth rate) term α, the volatility term σ and the subjective probability of bankruptcy of the borrowing company λ. Hence, under uncertainty it is always the case that as Π_f fluctuates stochastically, there will be periods when the conventionally measured $\bar{q}$ exceeds 1 without attracting investment.

From solution (10) of β, the theoretical relationship between each parameter of the stochastic process (α, σ, and λ) and $\bar{q}$ can be explained as follows[33]. First, a rise in σ raises the value of $\bar{q}$ since a high degree of volatility about future demand prompts the investor to wait for uncertainty to disappear as suggested by the real options theory. Thus, it also raises the threshold value of $\bar{Y}_f$.

Second, a rise in α raises the value of $\bar{q}$ because it makes it relatively more beneficial to wait and enter the market later due to the fact that it raises the expected future appreciation in the value of the project.

Lastly, a rise in λ decreases $\bar{q}$ by raising β. The reason is that it is more beneficial for the follower bank to enter now before the potential borrower actually goes bankrupt.

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[31] Conceptually, this version of $\bar{q}$ is called the value of assets in place notion in contrast to the value of the firm notion in which $\bar{q}$ is defined as $\left[v(\Pi_f) - F(\Pi_f)\right] / L_f I_f$, which is net of the option value. For details, see Dixit and Pindyck (1994).

[32] This is why I call $\bar{q}$ the stochastic version of Tobin's q.

[33] As will be discussed later, parameter ρ is directly linked to α, so suffice it to say the relationship to α.

III. Numerical Analysis

As shown in the preceding section, bank values are highly non-linear so that it is generally a difficult task to draw clear-cut qualitative implications from ordinary comparative static analysis. Thus, in this section, I conduct numerical analysis by changing each parameter in succession, holding others fixed at plausible values.

In what follows, I divide numerical analysis into two stages. The first stage assumes away the possibility of a Poisson jump, that is, it imposes a condition of λ =0 and the second stage deals with the case of positive λ.

(i) The Case without a Poisson Jump

As the baseline case, let me set some parameter values such that $\varepsilon = 2$, $I_l = 0.2$, and $Y = 5$. Also, I deal with cases in which (L_l, L_f)=(15, 5) and (10, 10) to analyze the effects of a change in the *ex ante* loan shares on entry decisions. Hence, when both banks make entry decisions, the current return from lending is 1.25%. On the other hand, when only the leader bank makes an entry decision, the current return turns out to be 2.22% if L_l =15 and 5.00% if L_l =10[34].

Actual data states that the average contracted interest rate on new long-term loans and discounts extended by domestically licensed banks fell from about 5.09% per annum in January 1993[35] to about 2.38% in December 1999. Since, for example, the uncollaterized call rate was about 3.88% in January 1993, and 0.05% in December 1999, the baseline parameter values of ε and (L_l, L_f) I set in this paper might not be so unrealistic.

Table 1 summarizes the main results of numerical analysis. It shows that a rise in the volatility of the future demand condition σ raises the threshold values of current demand for both the leader and follower bank, $\bar{Y}_l$ and $\bar{Y}_f$, holding other parameters constant. This result is quite sensible since the existence of uncertainty and irreversibility of investment yields the value of the option to wait.

In contrast, the direction of the effect of a rise in the expected growth parameter

[34] Recall that the inverse loan demand function is specified as $r_L = Y(L_l + L_f)^{-\varepsilon}$.

[35] The figure can be obtained from only January 1993. For details, see various issues of *Financial and Economic Statistics Monthly* (Bank of Japan).

α on lending decisions depends on the presumption regarding which parameter is adjustable, the discount rate ρ or the dividend rate δ when α changes[36]. When one assumes that ρ adjusts to accommodate the rise in α, it raises both $\overline{Y}_l$ and $\overline{Y}_f$. On the other hand, if one assumes that δ adjusts to offset the rise in α, holding ρ constant, the rise in α lowers both $\overline{Y}_l$ and $\overline{Y}_f$ [37].

Next, an increase in the leader bank's share $L_l/(L_l + L_f)$ raises $\overline{Y}_l$, although it has no impact on $\overline{Y}_f$ and $\overline{q}$.

Lastly, as is easily expected, a uniform increase in sunk costs paid by both banks raises both $\overline{Y}_l$ and $\overline{Y}_f$. On the other hand, a relative increase in I_f raises $\overline{Y}_f$, but lowers $\overline{Y}_l$. In this case, however, $\overline{q}$ remains constant unlike the other cases. In what follows, we look at each result in more detail.

A. Dependence of $\overline{Y}_l$ (F_l), $\overline{Y}_f$ (F_f), and $\overline{q}$ on α and σ (Table A-1)

Table A-1 in the Appendix 1 shows the interaction between (i) threshold values $(\overline{Y}_l, \overline{Y}_f)$ and $\overline{q}$ and (ii) the parameters of the underlying stochastic process, α and σ. Here, the first thing to note is that $\overline{Y}_f$ and $\overline{q}$ are irrespective of *ex ante* loan shares.

Although $\overline{Y}_l$ and $\overline{Y}_f$ rise resulting from a rise in volatility σ, holding other parameters fixed, the speed at which the threshold value rises is much greater for $\overline{Y}_f$ than for $\overline{Y}_l$. The same tendency is observed if one looks at the effect of the expected growth parameter α on $(\overline{Y}_l, \overline{Y}_f)$ holding other parameters constant. Also note that the values of the banks, F_l and F_f, monotonically increase with a rise in volatility σ when the follower bank does not enter the market, although they take the same values despite the rise in volatility when the follower bank makes an entry decision.

Next, look at the difference in reaction of $\overline{Y}_l$ and $\overline{Y}_f$ when the *ex ante* loan shares change while the total amount of lending extended by both banks is the same as before. An interesting point to note here is that although $\overline{Y}_f$ is the same irrespective of *ex*

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[36] Recall the relationship $\rho = \alpha + \delta$.

[37] It is because the smaller the cash flow relative to total return, the less one forgoes by holding an option to invest rather than investing in monitoring (information) activity to make a lending decision now.

ante loan shares, $\bar{Y}_l$ falls when the *ex ante* loan share changes from (L_l, L_f)=(15, 5) to (10, 10). The reason for this result is that in the region $\bar{Y}_l \le Y < \bar{Y}_f$, the follower bank does not lend, so the total amount of lending is nothing but L_l. A small value for L_l implies high profits from lending[38]. Hence, the threshold value of $\bar{Y}_l$ becomes smaller than in the case of a larger lending amount[39].

Lastly, one can confirm that the effect of a rise in α on the threshold values of $\bar{Y}_l$ and $\bar{Y}_f$ goes in the opposite direction between the two cases (i) and (ii) in Table A-1, as explained earlier. Under the assumption of fixed δ and adjustable ρ (case (i)), a rise in α raises the threshold values of $\bar{Y}_l$ and $\bar{Y}_f$. On the other hand, under the assumption of fixed ρ and adjustable δ, a rise in α lowers threshold values.

B. Dependence of $\bar{Y}_l$ (F_l), $\bar{Y}_f$ (F_f), and $\bar{q}$ on α and (I_l, I_f) (Table A-2)

Table A-2 details the interaction between (i) $(\bar{Y}_l, \bar{Y}_f)$ and $\bar{q}$ and (ii) α and (I_l, I_f). Here it should be noted that while $\bar{Y}_f$ rises in accordance with an increase in I_f, $\bar{q}$ remains constant irrespective of the value of I_f. In contrast, an increase in I_f lowers $\bar{Y}_l$ because it shifts down the value function of the follower bank. In the case of a uniform increase in (I_l, I_f), however, both $\bar{Y}_l$ and $\bar{Y}_f$ rise as is easily expected[40].

Also, one can see the same qualitative results regarding the effects of a rise in α in Table A-2 as in Table A-1.

C. Dependence of $\bar{Y}_l$, $\bar{Y}_f$, and $\bar{q}$ on ε (Table A-3)

Table A-3 shows the effect of a rise in the inverse of the elasticity of loan demand with respect to the interest rate ε on $\bar{Y}_l$, $\bar{Y}_f$, and $\bar{q}$. A striking point here is that the level of

[38] It should be noted that the elasticity of demand with respect to the profit margin is assumed to be $1/\varepsilon$ =0.5. Thus, a decrease in the amount of lending implies an increase in the profits of the leader bank.

[39] Confirm that when both banks enter the market, F_l is equal to F_f due to the assumption of the same share and the same sunk costs.

[40] Likewise, one should notice the direct effect of an increase in monitoring costs on the values of both banks.

$\overline{q}$ has nothing to do with the level of ε, while $\overline{Y}_l$ and $\overline{Y}_f$ change if one changes ε, holding other things constant.

Both $\overline{Y}_l$ and $\overline{Y}_f$ rise together with a rise in ε, since given the value of L_l and L_f, a larger value of ε implies a smaller value for profit margin r_L, other things equal. It should be noted, however, that as the value of ε falls so that the loan market becomes more competitive, the values of $\overline{Y}_f$ and $\overline{Y}_l$ (in the case of the same lending share) converge.

Also, note that around the point where $\varepsilon \cong 1$, the values of $\overline{Y}_l$ in the case of both (L_l, L_f)=(15,5) and (10,10) become almost the same, while above the point where ε >1, the value of $\overline{Y}_l$ when (L_l, L_f)=(15,5) is larger than when (L_l, L_f)=(10,10), and below the point where ε <1, and vice versa. This result is consistent with a conventional relationship between the elasticity of loan demand with respect to the interest rate and the level of revenues when the quantity of supply changes.

(ii) The Case with a Poisson Jump (Table A-4)

Generally speaking, effects of a positive value for the bankruptcy probability of potential borrowing company λ can be stated in the following ways[41]. First, it reduces the expected rate of capital gain on Π, which in turn decreases the value of the option to wait. Second, it increases the variance of changes in Π and thus raises the value of the option to wait.

Table A-4 (i) in Appendix 2 shows that the net effect is to reduce the threshold value of Y for both banks. It should be noted that this qualitative result is obtained by assuming that an increase in λ has nothing to do with the value of the expected growth parameter α and the discount rate ρ.

Table A-4 (ii) reports the result under an alternative assumption that an increase in λ raises the value of α by the same amount, which means that an increase in λ is approximately equivalent to an increase in the discount rate ρ. In such a case, an increase in λ leads to an increase in the value of the option to wait, and thus the threshold value of Y for both banks.

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[41] Note that in this paper, costs stemming from the real bankruptcy procedure such as the loss from liquidation are not taken into consideration. These considerations are likely to raise the value of the option to wait.

IV. Concluding Remarks

This paper has explored lending (entry) decisions in a duopolistic loan market, where both leader and follower banks are under uncertainty regarding future loan demand (return) conditions and face the irreversibility of monitoring costs that are absolutely necessary in advance of extending a new loan to any potential borrower. The game-theoretic real options insight suggests that unlike the case of a single monopolist lender, the leader bank cannot take a wait-and-see option long enough until uncertainty disappears because of the possibility of preemption by the follower bank.

Then, what kinds of implications can be derived from the analysis thus far regarding the recent slump in the Japanese loan market? In what follows, I will explore the link between the actual bank lending situation in Japan and each insight derived from my approach.

First, it is often said that compared with the bubble period, the public's expected growth forecasts about future general demand conditions have bent downward. It is not strange to think that the same tendency in the public's expectation occurred in the bank loan market judging from the significant role of lending in the fund raising of Japan's non-financial business sector. This hypothesis can be roughly verified by looking at the data reported in various issues of the *Short-Term Economic Survey of Enterprises in Japan* (Bank of Japan), according to which Japanese companies have actually decreased[42] their borrowing from financial institutions because they have revised their fixed investment plans downward[43].

In this regard, the real options theory suggests that if one assumes that the discount rates for private banks are constant over time[44], a fall in the expected growth parameter α raises the threshold values of current loan demand $\overline{Y}_l$ and $\overline{Y}_f$, which implies that the incentives for supplying loans on the side of banks weaken given the current demand situation.

Second, various surveys state that Japanese as a whole face a much higher

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[42] The same source also reports that Japanese firms plan to restrain borrowing in the future.

[43] Also, the continuing structural trend to shift from indirect finance like bank borrowings to direct finance such as direct debt is thought to contribute to a fall in the expected growth rate of future loan demand, especially among large firms.

[44] This assumption implicitly presumes that the risk-free interest rate, variance, risk price, and beta of the return are constant.

degree of uncertainty about future business conditions than before, which in turn might lead to a rise in uncertainty on the side of banks about corporate borrowing demand. In this regard, the options theory says that a rise in the uncertainty (volatility) parameter σ definitely deters the incentive to lend to new potential borrowers. Related to this point, a rise in the perceived risk of the bankruptcy of potential borrowers raises the threshold values of current demand for both banks under the assumption of the flexible expected growth parameter.

Third, it might be reasonable to think that as perceived future uncertainty expands, the follower bank will have an incentive to lower the share of loans to borrowers who are the main customers of other banks. This is because an informationally inferior bank naturally thinks that it should concentrate its lending on firms for which it is the main bank. If this is the case[45], the options theory suggests that a relative rise in the leader bank's *ex ante* loan share, holding the total amount the same as before, leads to a rise in the threshold value of current demand for the leader bank, leaving that for the follower bank to remain the same as before[46].

Fourth, the options theory suggests that a uniform rise in the monitoring costs of both the leader and follower yields a rise in the threshold value of current demand for both banks, although a unilateral rise in the follower's monitoring costs will lower the threshold value of current demand for the leader, raising that for the follower bank.

If the story that the monitoring ability of banks weakened due to excessive dependence on real estate collateral during the bubble period is right, then this theoretical implication seems very relevant. In particular, if the syndicated loan system does not work any more in face of reduced expected future demand, banks are obliged to monitor potential borrowers independently because the leader bank might have lost the incentive to be delegated monitoring efforts. If such a situation occurs, the informationally superior bank (leader bank) might tend to monopolize the market *ex ante*, eliminating the follower's share. Also, regarding this point, reinforcement of the credit guarantee system should play a role in boosting bank lending as long as it does not weaken the incentive of banks to accumulate monitoring ability.

Lastly, it is often said that the elasticity of loan demand with respect to the

[45] *Economic Survey of Japan FY 1998* published by the Economic Planning Agency reports that borrowing firms tend to return to their main banks due to the recent passive attitude of financial institutions toward lending to borrowers whose main banks are not themselves.

[46] Recall that in this paper the total amount of loans is assumed to be fixed over time so that the demand situation influences the profit margin obtained from lending.

interest rate[47] has lowered recently. This kind of remark generally reflects the prolonged depressed condition of the loan market despite the significantly lowered lending interest rate. Although it is difficult to quantitatively distinguish between two closely-related phenomena, that is, a fall in the interest rate elasticity of loan demand and/or an inward shift of the demand schedule, it might be reasonable to suppose that both factors are at least working to some extent.

If this is the case, the real options theory suggests that a fall in the interest rate elasticity of loan demand should cause the threshold values of current demand for both banks to rise, which implies a higher probability of not making an entry decision.

At least to my knowledge, attempts to apply the options theory to bank lending decisions particularly in the oligopolistic market structure are quite rare thus far. I sincerely hope that this paper provides a starting point for future discussions in this field.

[47] Note that in this paper ε denotes the inverse of interest rate elasticity.

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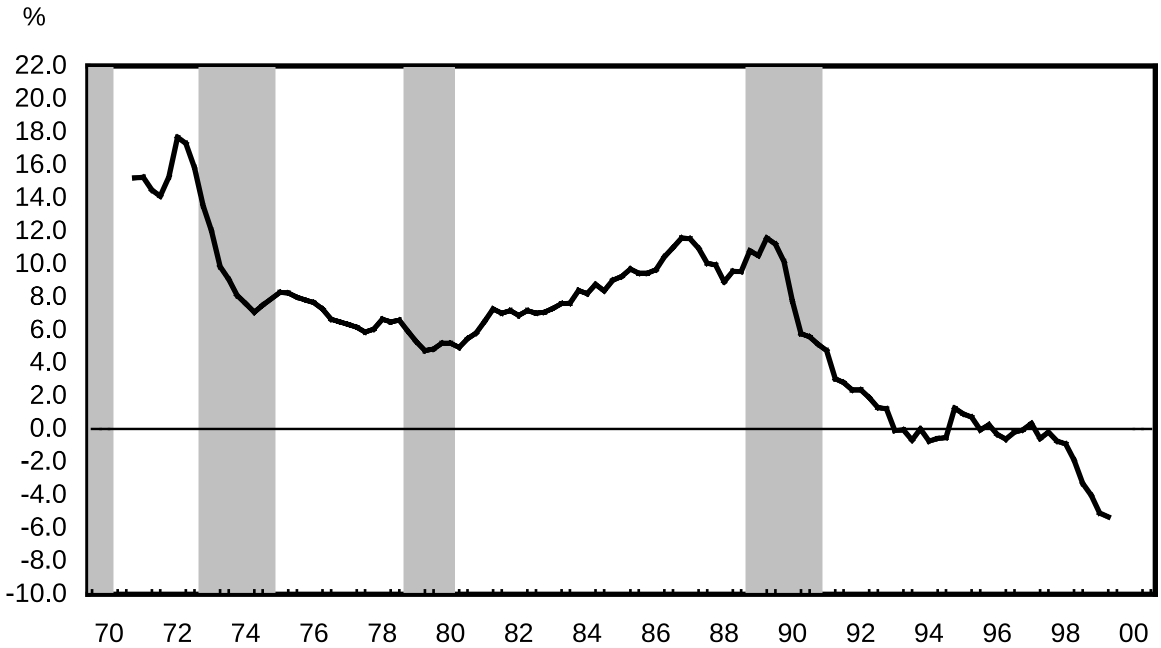
Table 1: Summary of Numerical Analysis

Exogenous Variables		Endogenous Variables $\overline{Y}_l$	$\overline{Y}_f$	$\overline{q}$
Stochastic Process				
Volatility σ		+	+	+
Expected Growth α	Case(i)	+	+	+
	Case(ii)	–	–	+
Probability of Bankruptcy λ	Case(a)	–	–	–
	Case(b)	+	+	+
***Ex Ante* Loan Share** $L_l/(L_l+L_f)$		+	O	O
Monitoring Costs				
I_l & I_f		+	+	O
I_f		–	+	O
Inverse of Demand Elasticity ε		+	+	O

Notes: 1. $\overline{Y}_l$, $\overline{Y}_f$, and $\overline{q}$ are defined as equations (20), (16), and (22), respectively.

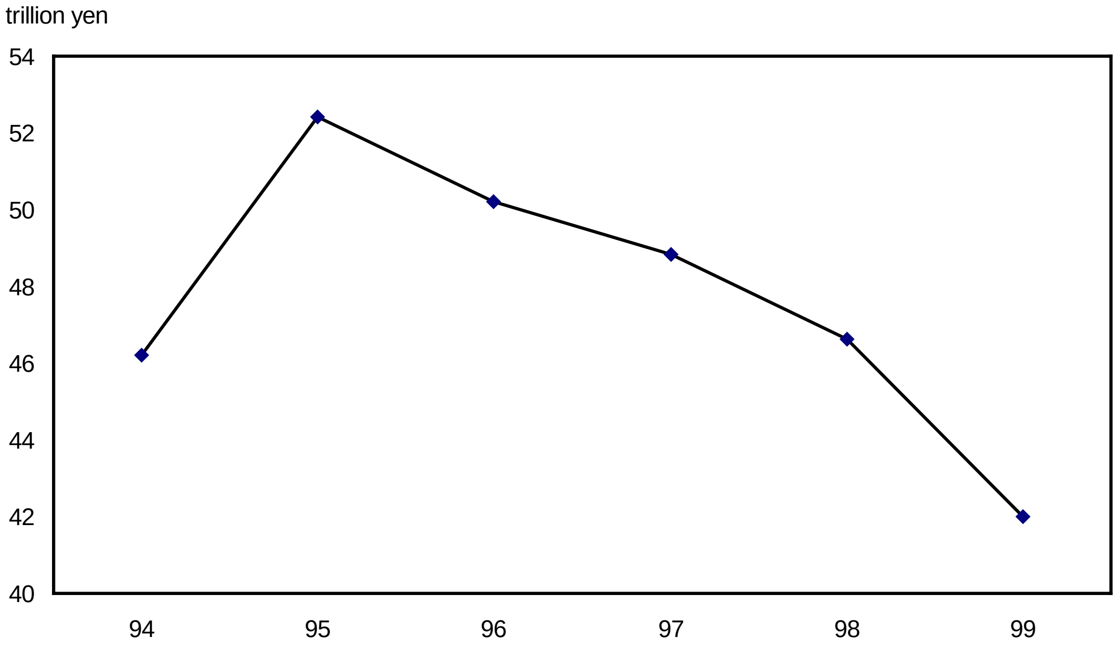
2. + indicates that the endogenous variable goes up when the exogenous variable rises. – indicates vice versa. O denotes no effect.
3. Case (i) denotes the case in which δ is held constant, while letting ρ adjust freely. Case (ii) denotes the case in which ρ is held constant while letting δ adjust freely.
4. Case (a) denotes the case in which α and ρ are fixed whatever the value of λ. Case (b) denotes the case in which α and ρ increase by the same amount as λ.
5. For comparative ease, total lending amount $L_l + L_f$ is always fixed at 20.

Figure 1: Increase in Loans and Discounts by Domestically Licensed Banks



Notes: 1. The data is in terms of percent ratio of an increase in loans and discounts by domestically licensed banks to nominal GDP. It is seasonally adjusted by taking three-quarter moving average. The data source is *Financial and Economic Data CD-ROM* issued by the Bank of Japan.
2. Shaded intervals indicate the period during which official discount rate was raised.
3. The data includes both banking accounts and trust accounts of domestically licensed banks.

Figure 2: New Loans for Equipment Funds by domestically Licensed Banks



Notes: 1. The data is new loans for equipment funds extended by domestically licensed banks. The data source is *Financial and Economic data CD-ROM* issued by the Bank of Japan.

2. The data includes both banking accounts and trust accounts of domestically licensed banks.

Figure 3: Entry Decision by a Monopolist Bank

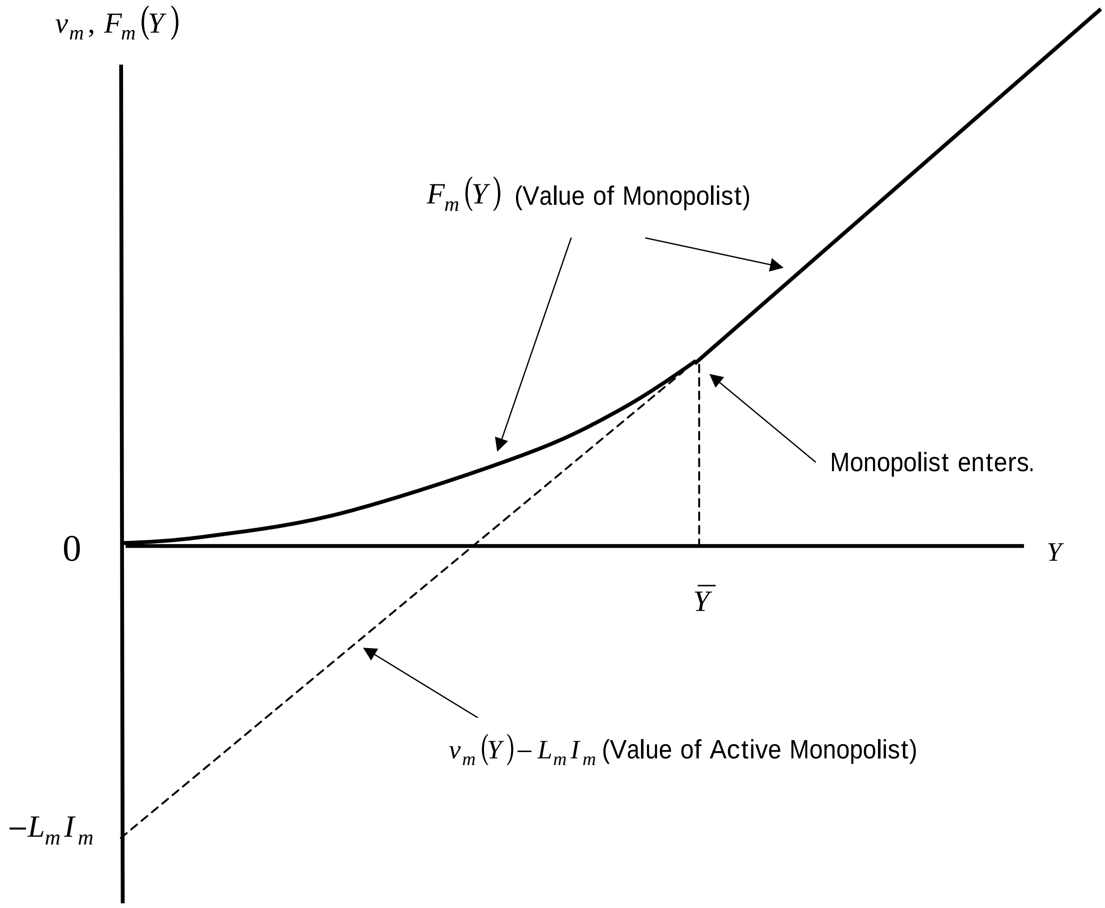
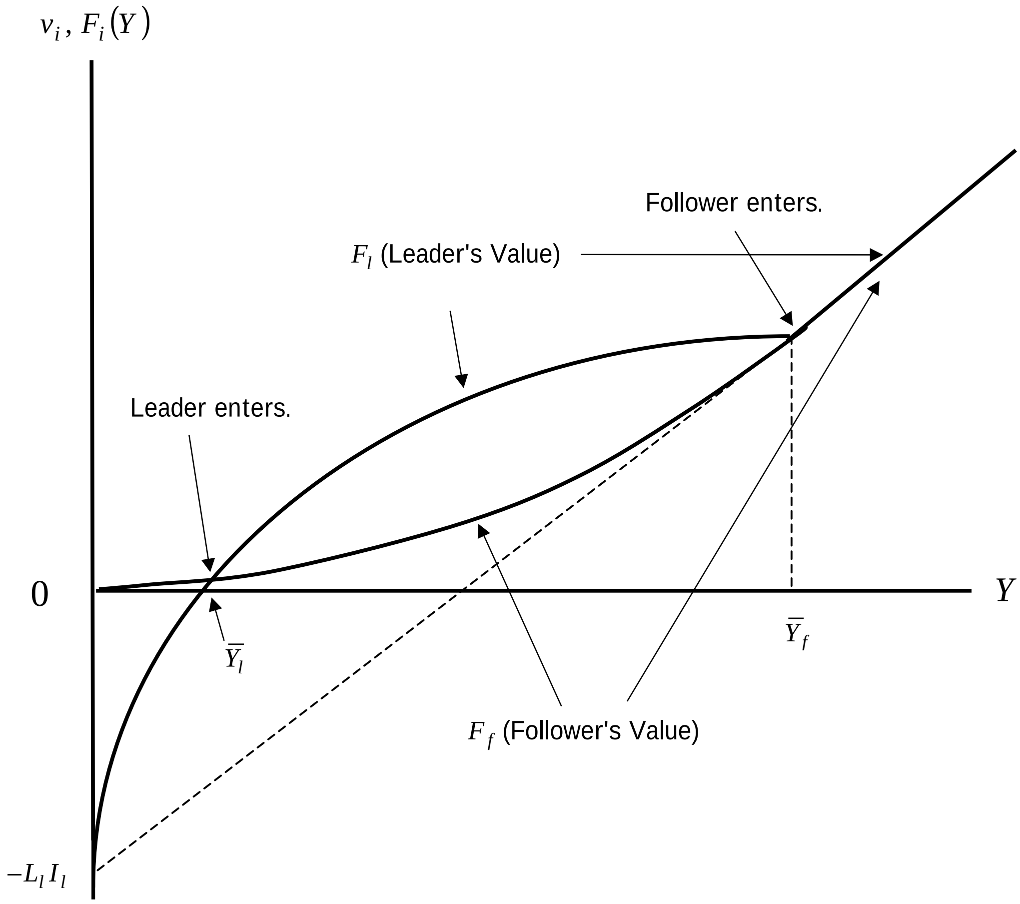


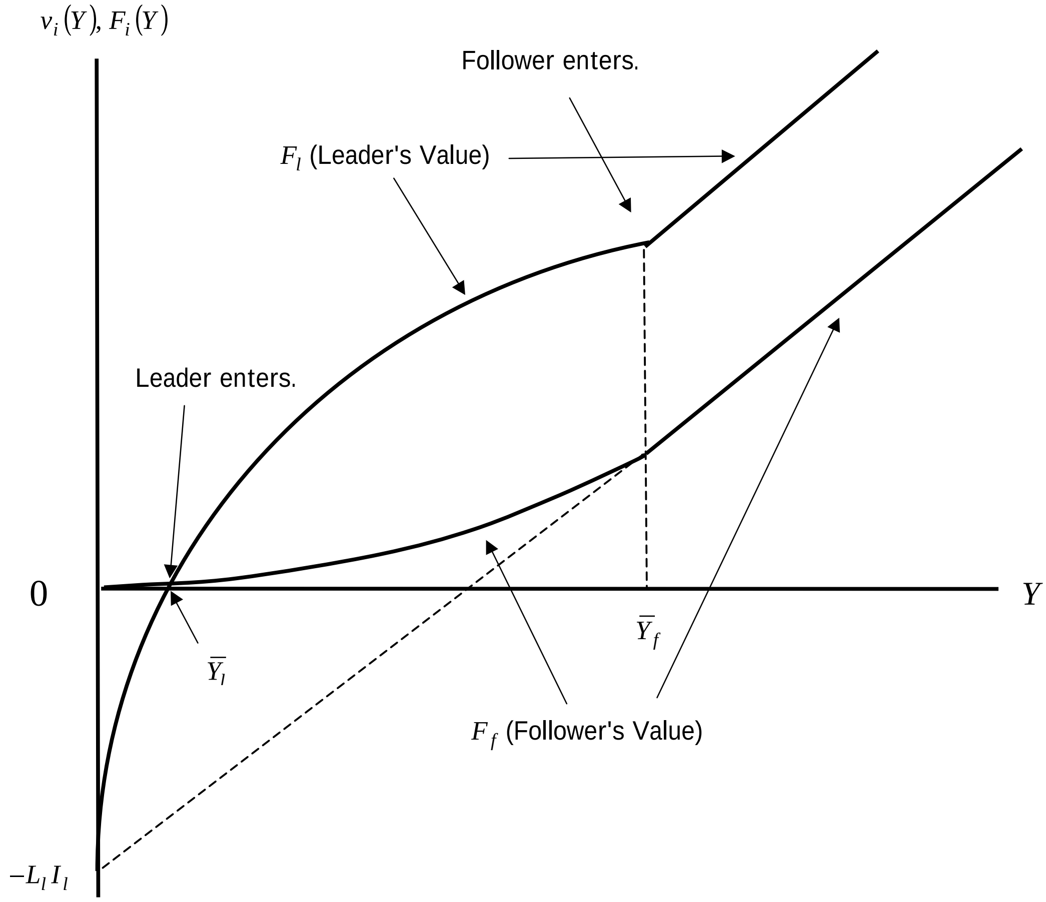
Figure 4: Values of the Leader and Follower

(i) The Case of Identical Banks



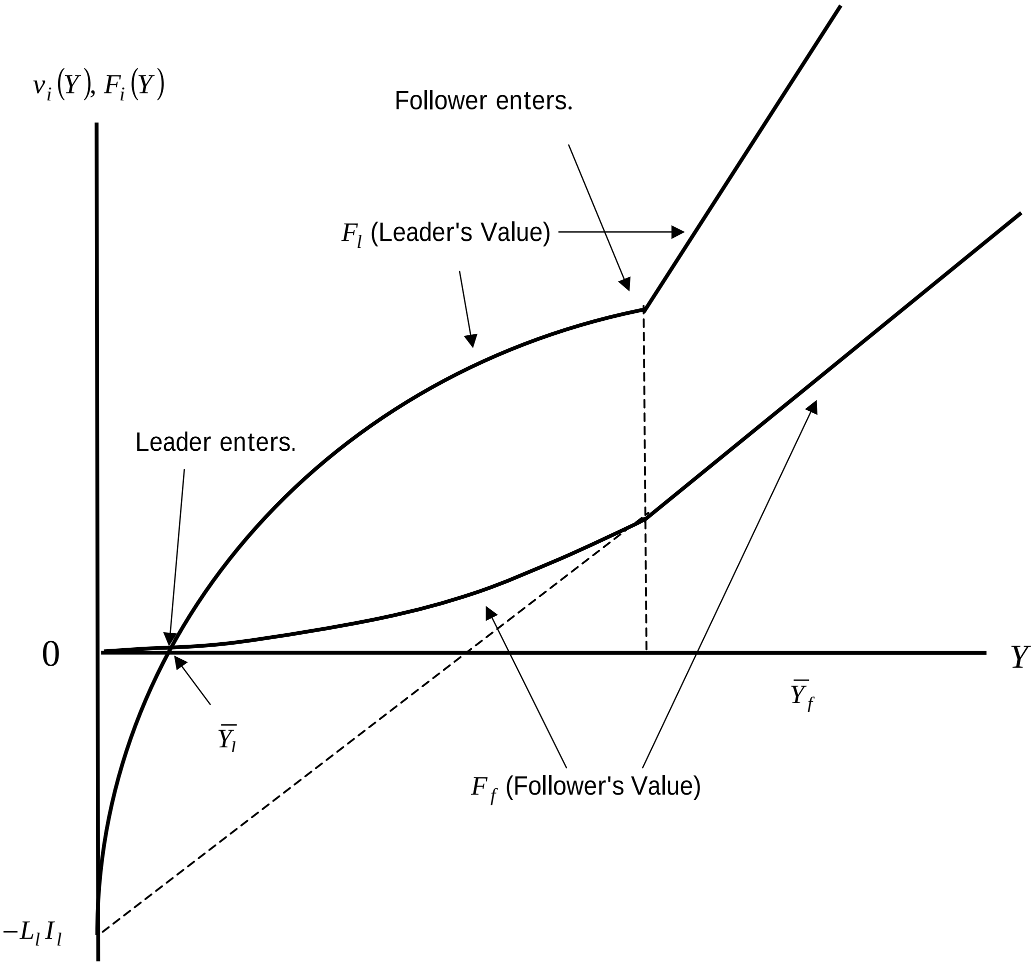
Note: The figure is drawn under the assumption that both banks are identical except for the predetermined roles they assume.

(ii) The Case of Different Sunk Costs



Note: The figure is drawn under the assumption that the sunk cost of the leader is lower than that of the follower.

(iii) The Case of Different *Ex Ante* Loan Shares



Note: The figure is drawn under the assumption that the *ex ante* loan share of the leader is larger than that of the follower.

Appendix 1: Detailed Results for the Case without a Poisson Jump

Table A-1: Dependence of $\overline{Y}_l$ (F_l), $\overline{Y}_f$ (F_f), and $\overline{q}$ on α and σ

(i) The Case of Fixed δ: δ=3% (Adjustable ρ)

A. (L_l, L_f)=(15, 5), Y=5, ε=2, (I_l, I_f)=(0.2, 0.2)

Parameters		Leader Bank		Follower Bank		
α	σ	$\overline{Y}_l$	F_l	$\overline{Y}_f$	$\overline{q}$	F_f
-2.0	5.0	1.352	3.250	2.546	1.061	1.083
	10.0	1.374	3.250	2.949	1.229	1.083
	20.0	1.591	3.250	4.360	1.817	1.083
0.0	5.0	1.371	3.250	2.942	1.226	1.083
	10.0	1.472	3.250	3.600	1.500	1.083
	20.0	1.717	3.489	5.317	2.215	1.087
2.0	5.0	1.574	3.250	4.231	1.763	1.083
	10.0	1.651	3.250	4.800	2.000	1.083
	20.0	1.841	3.949	6.530	2.721	1.128
4.0	5.0	1.766	3.722	5.771	2.405	1.099
	10.0	1.812	3.881	6.249	2.604	1.117
	20.0	1.951	4.129	7.898	3.898	1.188

B. (L_l, L_f)=(10, 10), Y=5, ε=2, (I_l, I_f)=(0.2, 0.2)

Parameters		Leader Bank		Follower Bank		
α	σ	$\overline{Y}_l$	F_l	$\overline{Y}_f$	$\overline{q}$	F_f
-2.0	5.0	0.600	2.167	2.546	1.061	2.167
	10.0	0.602	2.167	2.949	1.229	2.167
	20.0	0.654	2.167	4.360	1.817	2.167
0.0	5.0	0.600	2.167	2.942	1.226	2.167
	10.0	0.616	2.167	3.600	1.500	2.167
	20.0	0.723	2.783	5.317	2.215	2.173
2.0	5.0	0.647	2.167	4.231	1.763	2.167
	10.0	0.687	2.167	4.800	2.000	2.167
	20.0	0.823	3.963	6.530	2.721	2.257
4.0	5.0	0.762	3.380	5.771	2.405	2.198
	10.0	0.800	3.790	6.249	2.604	2.233
	20.0	0.948	4.428	7.898	3.898	2.376

Note: $\overline{Y}_l$, F_l, $\overline{Y}_f$, $\overline{q}$, and F_f are given by equations (20), (19), (16), (22), and (17), respectively.

(ii) The Case of Fixed ρ : ρ =7% (Adjustable δ)

A. (L_l, L_f)=(15, 5), Y =5, ε =2, (I_l, I_f)=(0.2, 0.2)

Parameters		Leader Bank		Follower Bank		
α	σ	$\bar{Y}_l$	F_l	$\bar{Y}_f$	$\bar{q}$	F_f
-2.0	5.0	4.049	0.703	7.582	1.053	0.000
	10.0	4.074	0.632	8.400	1.167	0.004
	20.0	4.386	0.374	10.594	1.471	0.045
0.0	5.0	3.162	1.392	6.400	1.143	0.020
	10.0	3.260	1.161	7.310	1.305	0.060
	20.0	3.609	0.933	9.498	1.696	0.146
2.0	5.0	2.441	1.612	5.909	1.477	0.285
	10.0	2.552	1.766	6.612	1.653	0.322
	20.0	2.827	1.847	8.593	2.148	0.417
4.0	5.0	1.766	3.722	5.771	2.405	1.099
	10.0	1.812	3.881	6.249	2.604	1.117
	20.0	1.951	4.129	7.898	3.291	1.188

B. (L_l, L_f)=(10, 10), Y =5, ε =2, (I_l, I_f)=(0.2, 0.2)

Parameters		Leader Bank		Follower Bank		
α	σ	$\bar{Y}_l$	F_l	$\bar{Y}_f$	$\bar{q}$	F_f
-2.0	5.0	1.797	3.554	7.582	1.053	0.000
	10.0	1.798	3.370	8.400	1.167	0.009
	20.0	1.833	2.708	10.594	1.471	0.090
0.0	5.0	1.402	4.191	6.400	1.143	0.040
	10.0	1.404	3.598	7.310	1.305	0.120
	20.0	1.491	3.012	9.498	1.696	0.292
2.0	5.0	1.022	2.715	5.909	1.477	0.569
	10.0	1.051	3.111	6.612	1.653	0.644
	20.0	1.186	3.320	8.593	2.148	0.834
4.0	5.0	0.762	3.380	5.771	2.405	2.198
	10.0	0.800	3.790	6.249	2.604	2.233
	20.0	0.948	4.428	7.898	3.291	2.376

Note: $\bar{Y}_l$, F_l, $\bar{Y}_f$, $\bar{q}$, and F_f are given by equations (20), (19), (16), (22), and (17), respectively.

Table A-2: Dependence of $\overline{Y}_l$ (F_l), $\overline{Y}_f$ (F_f), and $\overline{q}$ on α (I_l, I_f)

(i) The Case of Fixed δ: δ=3% (Adjustable ρ)

A. (L_l, L_f)=(15, 5), Y=5, ε=2, σ=10%

Parameters			Leader Bank		Follower Bank		
α	I_l	I_f	$\overline{Y}_l$	F_l	$\overline{Y}_f$	$\overline{q}$	F_f
-2.0	0.2	0.2	1.374	3.250	2.949	1.229	1.083
		0.3	1.353	3.250	4.423	1.229	0.583
		0.4	1.351	5.750	5.898	1.229	0.188
	0.3	0.3	2.061	1.750	4.423	1.229	0.583
		0.4	2.035	4.250	5.898	1.229	0.188
0.0	0.2	0.2	1.472	3.250	3.600	1.500	1.083
		0.3	1.399	3.943	5.400	1.500	0.595
		0.4	1.375	5.767	7.200	1.500	0.335
	0.3	0.3	2.209	2.443	5.400	1.500	0.595
		0.4	2.117	4.267	7.200	1.500	0.335
2.0	0.2	0.2	1.651	2.167	4.800	2.000	1.083
		0.3	1.519	3.256	7.200	2.000	0.723
		0.4	1.469	5.579	9.600	2.000	0.543
	0.3	0.3	2.209	3.235	7.200	2.000	0.723
		0.4	2.117	4.079	9.600	2.000	0.543
4.0	0.2	0.2	1.812	3.881	6.249	2.604	1.117
		0.3	1.659	4.826	9.374	2.604	0.867
		0.4	1.600	5.365	12.499	2.604	0.725
	0.3	0.3	2.718	3.326	6.249	2.604	0.867
		0.4	2.556	3.866	9.374	2.604	0.725

B. (L_l, L_f)=(10, 10), Y=5, ε=2, σ=10%

Parameters			Leader Bank		Follower Bank		
α	I_l	I_f	$\overline{Y}_l$	F_l	$\overline{Y}_f$	$\overline{q}$	F_f
-2.0	0.2	0.2	0.602	2.167	2.949	1.229	2.167
		0.3	0.597	2.167	4.423	1.229	1.167
		0.4	0.596	8.595	5.898	1.229	0.377
	0.3	0.3	0.902	1.167	4.423	1.229	1.167
		0.4	0.896	7.595	5.898	1.229	0.377
0.0	0.2	0.2	0.616	2.167	3.600	1.500	2.167
		0.3	0.605	3.950	5.400	1.500	1.191
		0.4	0.598	8.639	7.200	1.500	0.670
	0.3	0.3	0.923	2.950	5.400	1.500	1.191
		0.4	0.914	7.639	7.200	1.500	0.670
2.0	0.2	0.2	0.687	2.167	4.800	2.000	2.167
		0.3	0.655	5.986	7.200	2.000	1.447
		0.4	0.634	8.156	9.600	2.000	1.085
	0.3	0.3	1.030	4.986	7.200	2.000	1.447
		0.4	0.989	7.156	9.600	2.000	1.085
4.0	0.2	0.2	0.800	3.790	6.249	2.604	2.233
		0.3	0.741	6.219	9.374	2.604	1.734
		0.4	0.712	7.606	12.499	2.604	1.450
	0.3	0.3	1.200	5.219	9.374	2.604	1.734
		0.4	1.125	6.606	12.499	2.604	1.450

Note: $\overline{Y}_l$, F_l, $\overline{Y}_f$, $\overline{q}$, and F_f are given by equations (20), (19), (16), (22), and (17), respectively.

(ii) The Case of Fixed ρ: ρ=7% (Adjustable δ)

A. (L_l, L_f)=(15, 5), Y=5, ε=2, σ=10%

Parameters			Leader Bank		Follower Bank		
α	I_l	I_f	$\overline{Y}_l$	F_l	$\overline{Y}_f$	$\overline{q}$	F_f
-2.0	0.2	0.2	4.074	0.632	8.400	1.167	0.004
		0.3	4.057	0.697	12.600	1.167	0.000
		0.4	4.019	0.703	16.800	1.167	0.000
	0.3	0.3	6.110	-0.803	12.600	1.167	0.000
		0.4	6.082	-0.797	16.800	1.167	0.000
0.0	0.2	0.2	3.260	1.161	7.310	1.305	0.060
		0.3	3.180	1.603	10.965	1.305	0.016
		0.4	3.158	1.700	14.620	1.305	0.006
	0.3	0.3	4.890	0.103	10.965	1.305	0.016
		0.4	4.781	0.200	14.620	1.305	0.006
2.0	0.2	0.2	2.552	1.766	6.612	1.653	0.322
		0.3	2.390	2.645	9.919	1.653	0.173
		0.4	2.328	3.009	13.225	1.653	0.111
	0.3	0.3	3.829	1.145	9.919	1.653	0.173
		0.4	3.637	1.509	13.225	1.653	0.111
4.0	0.2	0.2	1.812	3.881	6.249	2.604	1.117
		0.3	1.659	4.826	9.374	2.604	0.867
		0.4	1.600	5.365	12.499	2.604	0.725
	0.3	0.3	2.718	3.326	9.374	2.604	0.867
		0.4	2.550	3.865	12.499	2.604	0.725

B. (L_l, L_f)=(10, 10), Y=5, ε=2, σ=10%

Parameters			Leader Bank		Follower Bank		
α	I_l	I_f	$\overline{Y}_l$	F_l	$\overline{Y}_f$	$\overline{q}$	F_f
-2.0	0.2	0.2	1.798	3.370	8.400	1.229	0.009
		0.3	1.789	3.539	12.600	1.229	0.001
		0.4	1.787	3.553	16.800	1.229	0.000
	0.3	0.3	2.709	2.539	12.600	1.229	0.001
		0.4	2.705	2.553	16.800	1.229	0.000
0.0	0.2	0.2	1.404	3.598	7.310	1.500	0.120
		0.3	1.393	4.734	10.965	1.500	0.032
		0.4	1.389	4.983	14.620	1.500	0.012
	0.3	0.3	2.105	3.734	10.965	1.500	0.016
		0.4	2.091	3.983	14.620	1.500	0.012
2.0	0.2	0.2	1.051	3.111	6.612	2.000	0.644
		0.3	1.032	5.372	9.919	2.000	0.346
		0.4	1.018	6.308	13.225	2.000	0.223
	0.3	0.3	1.577	4.372	9.919	2.000	0.346
		0.4	1.547	5.308	13.225	2.000	0.223
4.0	0.2	0.2	0.800	3.790	6.249	2.604	2.233
		0.3	0.741	6.219	9.374	2.604	1.734
		0.4	0.712	7.606	12.499	2.604	1.450
	0.3	0.3	1.200	5.219	9.374	2.604	1.734
		0.4	1.125	6.606	12.499	2.604	1.450

Note: $\overline{Y}_l$, F_l, $\overline{Y}_f$, $\overline{q}$, and F_f are given by equations (20), (19), (16), (22), and (17), respectively.

Table A-3: Dependence of $\overline{Y}_l$ (F_l), $\overline{Y}_f$ (F_f), and $\overline{q}$ on ε

δ =3%, ρ =7% , Y =5, (I_l, I_f)=(0.2, 0.2), σ =10%

Parameter	Leader Bank				Follower Bank			
ε	$\overline{Y}_l$		F_l		$\overline{Y}_f$	$\overline{q}$	F_f	
	(L_l, L_f)		(L_l, L_f)				(L_l, L_f)	
	(15, 5)	(10, 10)	(15, 5)	(10, 10)			(15, 5)	(10, 10)
0.1	0.009	0.018	1849.836	1233.224	0.021	2.604	616.612	1233.224
0.2	0.012	0.022	1370.201	913.467	0.028	2.604	456.734	913.467
0.4	0.021	0.031	751.272	500.848	0.052	2.604	250.424	500.848
0.6	0.037	0.045	411.307	274.205	0.094	2.604	137.102	274.205
0.8	0.066	0.067	224.571	149.714	0.172	2.604	74.857	149.714
1.0	0.115	0.099	122.000	81.333	0.312	2.604	40.667	81.333
1.2	0.200	0.149	65.660	43.773	0.569	2.604	21.887	43.773
1.4	0.349	0.225	34.714	23.142	1.036	2.604	11.571	23.142
1.6	0.605	0.341	17.715	11.810	1.886	2.604	5.905	11.810
1.8	1.050	0.522	8.379	5.586	3.433	2.604	2.793	5.586
2.0	1.812	0.800	3.881	3.790	3.790	2.604	1.117	2.233
3.0	27.247	7.124	-2.317	-0.529	6.249	2.604	0.009	0.017
4.0	399.961	67.493	-2.951	-1.837	2499.756	2.604	0.000	0.000
5.0	5799.434	649.937	-2.997	-1.983	49995.121	2.604	0.000	0.000

Note: $\overline{Y}_l$, F_l, $\overline{Y}_f$, $\overline{q}$, and F_f are given by equations (20), (19), (16), (22), and (17), respectively.

Appendix 2: Detailed Results for the Case with a Poisson Jump

Table A-4: Dependence of $\bar{Y}_l$, $\bar{Y}_f$, and $\bar{q}$ on λ

(i) The Case of Fixed α

Parameter	Leader Bank	Follower Bank	
λ (%)	$\bar{Y}_l$	$\bar{Y}_f$	$\bar{q}$
0	1.472	3.600	1.500
10	1.369	2.919	1.216
20	1.357	2.781	1.159
30	1.354	2.714	1.131
50	1.351	2.645	1.102
100	1.351	2.573	1.072

Note: Calculation is made under the assumption that the expected growth parameter α is fixed at zero. Other parameters are set as follows: δ =3% (thus ρ =3%), σ =10%, Y =5, ε =2, (L_l, L_f)=(15, 5), and $(I_{l,} I_f)$=(0.2, 0.2).

(ii) The Case of Flexible α

Parameter	Leader Bank	Follower Bank	
λ (%)	$\bar{Y}_l$	$\bar{Y}_f$	$\bar{q}$
0	1.472	3.600	1.500
10	1.605	4.467	1.853
20	1.627	4.596	1.915
30	1.635	4.657	1.940
50	1.639	4.716	1.963
100	1.645	4.754	1.981

Note: Calculation is made under the assumption that the expected growth parameter α and ρ adjust by the same amount in the same direction as λ. Other parameters are set as follows: δ =3% σ =10%, Y =5, ε =2, (L_l, L_f)=(15, 5), and $(I_{l,} I_f)$=(0.2, 0.2).