A Study of the Mechanism of Stress/shock Movements in Financial Markets*

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Abstract

Clarifying the mechanism of financial market stress/shock movements is an age-old topic for economists and traders and various approaches have been made. This report introduces our attempt to model its mechanisms and outlines the recent interim results. We start by defining the financial market as a non-linear paradigm and apply the idea of complexity science to it. First, we classify markets by the relative ease of stress movement occurrence (magnitude and frequency). Second, we construct a market rate-path-generation model based on the interaction among market participants' strategy; *i.e. a trader's present strategy would be affected by others' past strategies and past price movements (feedback system)*, and examine whether it is experimentally possible to generate the market fluctuation characteristics that were obtained from previous studies.

There are still numerous outstanding issues regarding each analysis, and we have not reached a definitive conclusion. The results up to the present are 1) the law of power might group historical stress movements into three major types, 2) our model successfully generates extreme shocks as are observed in the real world but that are not generated by normal stochastic random simulation, and some of those can be classified into the given major three types.

1 Preface

What is the mechanism by which market stress movements such as Black Monday and the Mexican currency crisis occur? The goal of this paper is to find clues

* The first draft of this research was written while Shigeru Yoshifuji was visiting the Institute for Monetary and Economic Studies, Bank of Japan. And this second draft was written with Hiroaki Demizu. The authors wish to thank Kunihiko Kaneko of Tokyo University for his helpful comments. The views expressed in this paper are those of the author and do not necessarily represent those of the Bank of Japan or the Bank of Tokyo-Mitsubishi.

toward answering this age-old question. First of all, do the foreign exchange, money, stock, and various other markets all show the same level of sensitivity to stress? This paper begins by identifying the characteristics of fluctuations in each market to determine whether or not markets can be classified by the relative ease of stress movement occurrence. We then construct a market rate-path-generation model based on the interaction among market participants (the adjustment of one's own market outlook based on the outlook of other participants, etc.), and examine whether the above classification (markets where stress movements easily occur versus those where they do not) can be obtained by changing the trading strategy and the level of influence from other participants.

This paper is organized in the following format. Chapter 2 identifies the characteristics of market fluctuation, and attempts to classify markets by the relative ease of stress movement occurrence. In Chapter 3, we construct a market rate-path-generation model based on the interaction among market participants, and examine whether it is possible to generate the market fluctuation characteristics obtained from the empirical analyses in Chapter 2 by changing the trading strategy and the level of influence from other participants. Chapter 4 presents a brief summary of our work.

As this paper is only an interim report, there are still outstanding issues in the analyses, and we do not draw any definitive conclusions. Our results to date may be summarized as follows.

- We analyzed data from 14 markets from January 1985 through November 1996. The level of stress movement varies substantially by market, and we confirmed that it is possible to classify markets by the relative ease of stress movement occurrence using frequency distribution tables and the shape of the connected scatter diagrams.
- In order to explain this difference among markets, we constructed a market rate-path-generation model based on the interaction among market participants. This model generated markets that are not formed through a random process (markets with extreme stress movements, etc.), and we demonstrated the model's effectiveness for clarifying the stress occurrence mechanism.

2 Market classification

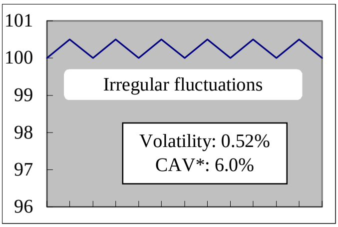
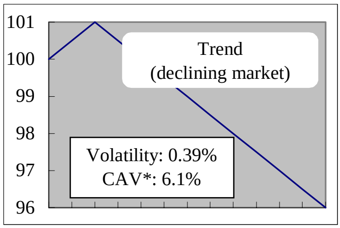
2.1 Indices expressing market fluctuations

For the analyses in this chapter, we first examined what indices best express the characteristics of market fluctuations. Considering the two points explained below, our conclusion was to adopt "the cumulative annual value of the absolute rate of return”[1] as the object for our analyses.

- **Different risk for dealers (proprietary trading) and investors (portfolio investment)**

The extent of market fluctuations is often measured using volatility, and volatility is calculated after removing the influence of trends. For dealers who can freely increase or decrease their position based on their market outlook, markets with irregular fluctuations are risky (while they also provide an opportunity to realize profits), and in this respect volatility can serve as an indicator of risk. Investors, however, fundamentally take a long position. Thus for investors, the greatest risk is a downward trend,[2] volatility after removing the influence of trends is not particularly meaningful, and a different parameter may be required. Figure 1 presents two market patterns in which the market fluctuation per period is 0.50 yen. Although the volatility value is relatively low in the trend pattern, the cumulative annual value of the absolute rate of return is almost the same for both patterns. Considering the risk to investors, the cumulative annual value of the absolute rate of return may provide a better expression of market characteristics.[3]

Figure 1: Traders vs. investors

	Traders	Investors
Position	Can freely increase or decrease their position	Fundamentally a long position
Time Horizon	Short-term	Long-term
Risk		

* **C**umulative **a**nnual **v**alue of the absolute rate of return

[1] We adopted the absolute values of the monthly rates of return, and added them to determine the cumulative annual values.

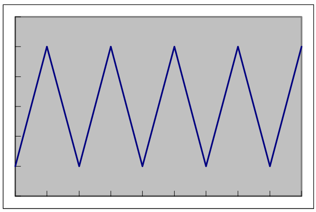
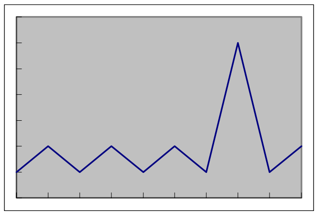
[2] Also in markets with an upward trend, in a certain sense investors who do not expand their positions suffer a loss. The point is that for investors failure to ride market trends as they appear is a key risk.

[3] Focusing on the fact that market participants gain a return in exchange for taking risks, the cumulative annual value of the absolute rate of return may provide a better expression of market characteristics as an indicator reflecting the riskiness. Considering investors only, markets with irregular fluctuations in which the range does not change significantly are less risky than trend markets, and the “simple sum of rates of return” may be more appropriate than the “cumulative annual value of the absolute rate of return” as a risk indicator. However, as this analysis covers both investors and dealers, the “cumulative annual value of the absolute rate of return” is adopted here.

- **Markets where stress movements easily occur**

Using the general concept of "complexity science" introduced below, market fluctuations may be viewed as an energy release phenomenon (not unlike an earthquake). Under this concept, there may be two main market patterns (refer to Figure 2): markets that regularly release stress through minor fluctuations and thus do not build up stores of stress energy, and markets that normally do not fluctuate significantly but which suddenly experience drastic movements in a release of accumulated stress (that is, markets where significant stress movements easily occur). From this perspective, indices that denote the sense of accumulated energy may prove effective.

Figure 2: Market fluctuation patterns

	Markets where stress movements do not easily occur	Markets where stress movements easily occur
Market Fluctuation Pattern		

2.2 Market classification: extent of stress

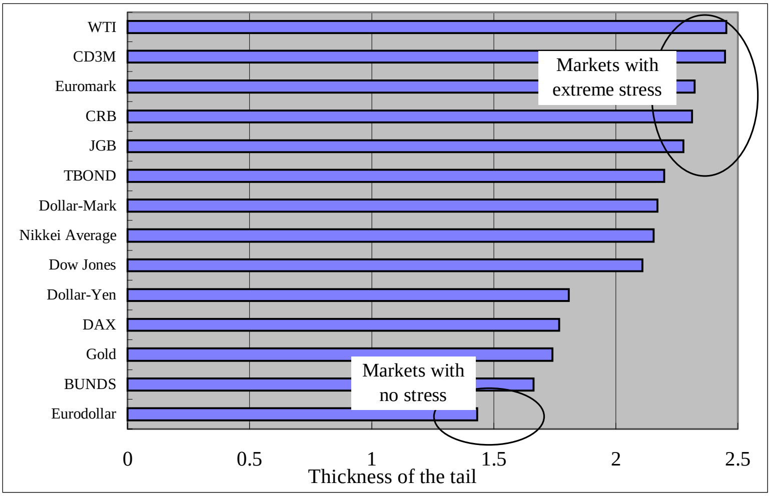
The above-mentioned market fluctuation indicator (the cumulative annual value of the absolute rate of return) was analyzed for 14 markets to investigate the fluctuation characteristics. Specifically, monthly data was used for the following 14 markets: short-term interest rates: 3 markets (Japan [CD3M], the U.S. [Euro-dollar], and Germany [Euromark]); long-term interest rates: 3 markets (Japan [JGB], the U.S. [TBOND], and Germany [BUNDS]); stocks: 3 markets (Japan [Nikkei Average], U.S. [Dow Jones], and Germany [DAX]); foreign exchange: 2 markets (dollar-yen and dollar-mark); and commodities: 3 markets (Gold, WTI, and CRB).[4] The analysis period was from January 1985 through November 1996

[4] There are some minor problems with the data accuracy (some figures are monthly averages, while others are monthly closings), and this issue will require further consideration. The specific attributes of the data are as follows—CD3M: price quotations for new contracts, monthly closing; Eurodollar: London monthly closing; Euromark: London monthly closing; JGB: futures interest rates, monthly closing; TBOND: monthly closing; BUNDS: monthly average (no data for 1985, data from 1986 forward); Nikkei Average: monthly average; Dow Jones: monthly average; DAX: monthly closing; dollar-yen: New York monthly closing; dollar-mark: New York monthly closing; Gold: London monthly settlements average; WTI: futures, monthly closing; and CRB: index, monthly closing. Also for the interest rate data, alternatively used rate of return on a price basis which is synthesized with a modified duration and interest rate levels then.

(refer to Appendix 1 for the original time series data and the market fluctuation indices).

Here, the extent of stress is defined by the thickness of the tail on the market fluctuation index distributions. Specifically, the greater the ratio of the 5% tile point to the standard deviation,[5] the more intense the stress. Figure 3 shows each of the markets arranged by the size of this ratio, with a wide distribution ranging from 1.43 for the Eurodollar market to 2.45 for WTI, indicating a broad range of stress levels on different markets. To further clarify this differential among markets, we then combined this analysis with investigations of market characteristics from other perspectives, specifically (1) the frequency distribution tables and (2) the shape of the connected scatter diagrams.[6]

Figure 3: Thickness of the tail by market



(1) Shape of the frequency distribution

Frequency distribution tables (refer to Appendix 2) for the market fluctuation indicator (the cumulative annual value of the absolute rate of return) reveal three

[5] The market fluctuation indicators (the cumulative annual values of the absolute rates of return) were arranged in order, the average value was subtracted from the value at the fifth percentile, and the remainder was divided by the standard deviation. In a normal distribution, this ratio would be 1.645.

[6] This is a simple plotting method with prior term values (or values from several terms before) on the horizontal axis and current term values on the vertical axis, but this makes it easy to identify periodicity and other characteristics. Using this method, trend data appears as a straight line, while cyclical data appears in an elliptical pattern.

main patterns. The Eurodollar market is a representative example of the first pattern (the "nervous foot-tapping type"), whereby the frequency does not decline significantly even when the scale of market fluctuations becomes large. The gold market is a representative example of the second pattern (the "critical state type"), whereby the frequency gradually declines as the scale of market fluctuations increases. The WTI market is a representative example of the third pattern (the "sudden wake-up call" type)—which has a thick distribution curve tail—whereby the frequency suddenly declines as the scale of market fluctuations increases. Typical frequency distribution curves for the three patterns are presented in the middle column of Figure 4.[7]

We also calculated the logarithms for the scale and frequency of market fluctuations to make it easier to grasp the relationship between these two factors, as presented in the right-hand column of Figure 4, and the resulting logarithms for the "critical state type" markets approximate a straight line declining to the right. This means that when the scale of the market fluctuations is plotted as X, the frequency Y is inversely proportional to X^a (where "a" is an arbitrary constant).[8] For the "nervous foot-tapping type" and "sudden wake-up call type" markets, the approximation toward a straight line is poor (that is, the coefficient of determination is low). Figure 5 shows the size of the coefficient of determination[9] and the thickness of the tail (ratio of the 5% point/standard deviation) for the 14 markets plotted in two dimensions, and the three market categories fall in order along a parabolic curve. At the present time, we have not yet interpreted the significance of this relationship (we will examine this in our subsequent research), but it may be said that we have identified one of the characteristics of market fluctuations. In brief, our research shows that there are markets for which the relationship "the frequency Y is inversely proportional to the a^{th} power of the scale X" holds true and others for which it does not, and that the latter group can be divided into markets with extreme stress and markets with no stress.

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[7] In the actual classification of markets, we first categorized all markets for which the coefficient of determination exceeds 0.8 in approximating the relational formula (explained below) "the frequency Y is inversely proportional to the a^{th} power of the scale X" as "critical state type" markets. The remaining markets were categorized as "nervous foot-tapping type" or "sudden wake-up call type" based on the shape of the frequency distribution curve.

[8] In the "complexity science" field, which is attracting a great deal of attention recently, this relationship is referred to as the "law of power" (the B. Gutenberg C. F. Richter law), and the moniker "critical state" also derives from this. Self-organized criticality designates a system that arrives at a certain equilibrium from internal factors only, without applying any external shocks, and in nature when a system arrives at a status of self-organized criticality the occurrence frequency of a given phenomenon may be exponentially inversely proportional to the scale of the phenomenon (the law of power). A simple example of this is the relation between small and large earthquakes whereby while small earthquakes occur frequently large earthquakes seldom occur. The possibility of expanding our research into the "complexity science" field will be considered later on.

[9] Because the coefficient of determination is not an interval scale (or a ratio scale), the Y axis is not statistically significant. Nevertheless, we have adopted the coefficient of determination as the Y axis so that the relationship between the two factors can be grasped visually.

Figure 4: The three market categories

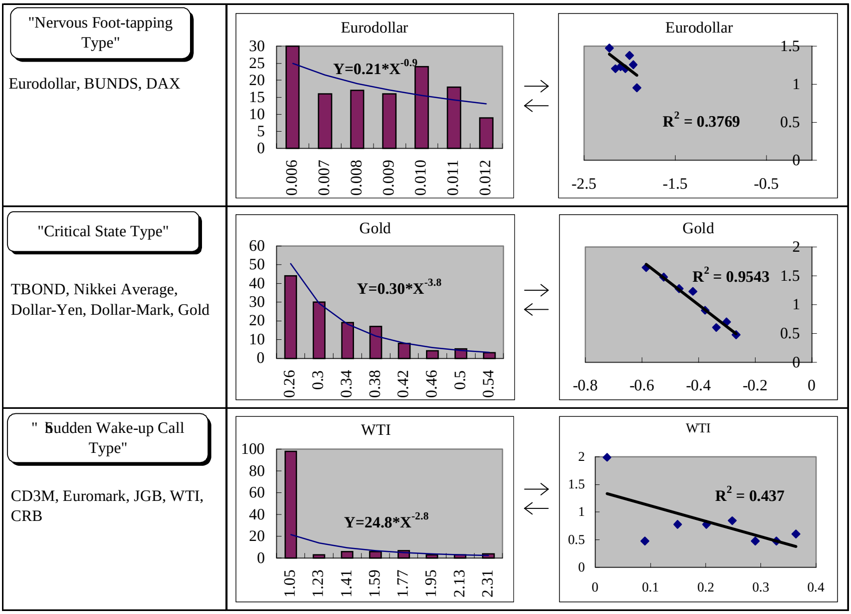
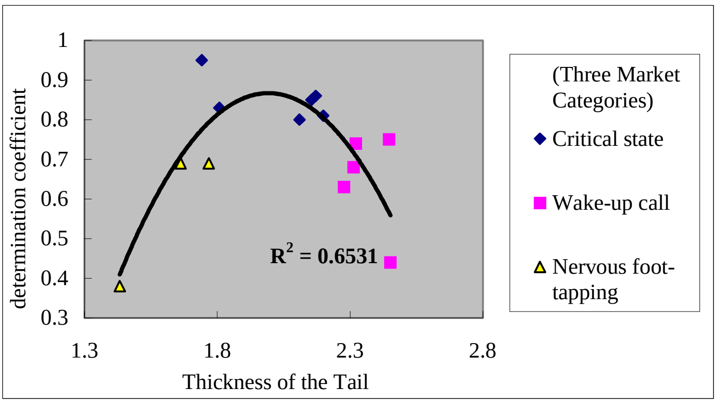


Figure 5: Relationship between the determination coefficient and the thickness of the tail



(2) Shape of the connected scatter diagrams

Connected scatter diagrams[10] (refer to Appendix 3) for the market fluctuation indicator (the cumulative annual value of the absolute rate of return) reveal two main patterns. The dollar-yen market is a representative example of the first pattern (the "strange attractor type"), which exhibits complex behavior.[11] The WTI market is a representative example of the second pattern (the "limit cycle type"), which exhibits cyclical behavior. Typical connected scatter diagrams for the two patterns are presented in Figure 6. The relationship between these two patterns and the thicknesses of the tails is shown in Figure 7, and there appears to be a trend whereby markets exhibiting a limit cycle pattern have thick tails (in other words, these are markets with extreme stress). For reference, Figure 6 divides the time period in two to make the time series changes of the attractors' behavior visible: the data from February 1986 through May 1991 appear as a straight line, and the data from June 1991 through November 1996 as a dotted line.[12] All of the limit cycle pattern markets show a large movement of the attractors from the earlier data (the straight-line portion) toward the latter data (the dotted-line portion), and they appear to be degenerating into strange attractors.

[10] For this paper we have adopted the value from four months prior as the horizontal axis and the value for the current month as the vertical axis, but the decision on which month's value to adopt for the horizontal axis depends on the periodicity of the system. The periodicity could be calculated separately for each market using spectral analysis or other techniques, but here we have adopted a uniform time frame (four months prior) for convenience. The analysis of diverse combinations is an issue for future study.

[11] This name also derives from "complexity science." In chaos, which is one type of complexity science, attractors appear when movements are expressed in topological space diagrams such as connected scatter diagrams. The attractors exhibit strange behavior as they move about without converging on a single point (a fixed point) or settling into a periodic movement (a limit cycle). These are referred to as "strange attractors," and were given this name in the thesis entitled "On the nature of turbulence" written by Ruelle and Takens (1971), in which they are defined as having three properties: stability, low-dimensionality, and non-periodicity. For information on chaos, refer to Appendix 4.

[12] Methods to specify the point in time where structural changes occur such as the Chow test could also be considered, but here we have simply divided the time period in two.

Figure 6: Market classification by connected scatter diagrams

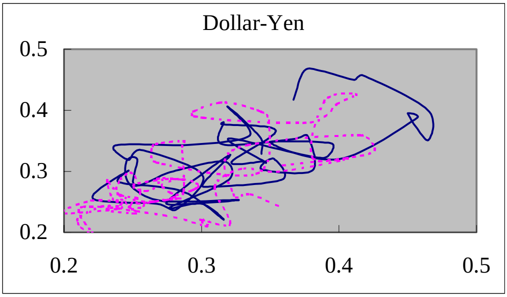
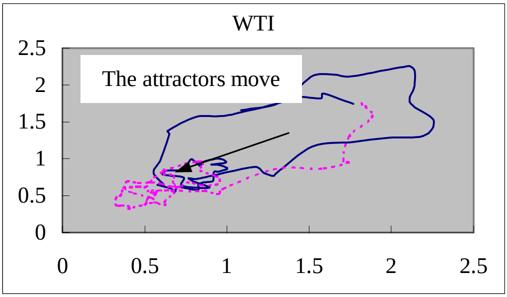
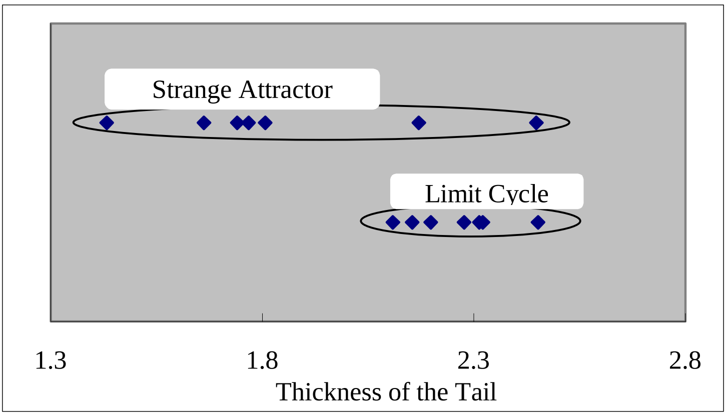
"Strange Attractors"	
CD3M, Eurodollar, BUNDS, DAX, Dollar-Yen, Dollar-Mark, Gold	
"Limit Cycle"	
Euromark, JGB, TBOND, Nikkei Average, Dow Jones, WTI, CRB	

Figure 7: Relationship between attractor behavior and the thickness of the tail



2.3 Summary of chapter 2 and certain considerations

(Summary)
In this chapter we identified market fluctuation characteristics using actual market data to determine whether or not markets can be classified by the relative ease of stress movement occurrence. Our results to date may be summarized as follows.

We analyzed data from 14 markets from January 1985 through November 1996, and clarified that the level of stress varies widely by market. We demonstrated that market classification is possible to a certain extent by combining the shape and other characteristics of the frequency distribution tables, connected scatter diagrams and other analytical tools. Specifically, we clarified two tendencies: (1) there are markets for which the relationship "the frequency Y is inversely proportional to the ath power of the scale X" holds true and others for which it does not, and the latter group can be divided into markets where extreme stress occurs and markets with no stress; and (2) according to the shape of the connected scatter diagrams, extreme stress can easily occur in limit cycle type markets.

(Certain considerations)
As our analysis demonstrated that the level of stress varies greatly by market, we now briefly consider the causes of this difference among markets. Figure 8 examines the thickness of the tail (ratio of the 5% point/standard deviation) for three markets where a direct comparison can be made among Japan, the U.S. and Germany: short-term interest rates, long-term interest rates, and stock prices. This figure shows that for all three markets the ratio (level of stress) tends to be higher in Japan. Although internationally dispersed investment is advancing, a "home bias"[13] still remains. Considering this, the characteristics of market participants in each nation could conceivably be the cause of the difference among markets.[14] For example, a high percentage of positive feedback traders[15] or distinctive characteristics such as a strong herd instinct whereby market participants are strongly influenced by other participants' market outlooks might be relevant factors. To verify this, in Chapter 3 we construct a market rate-path-generation model based on the interaction among agents (market participants), and conduct simulations.

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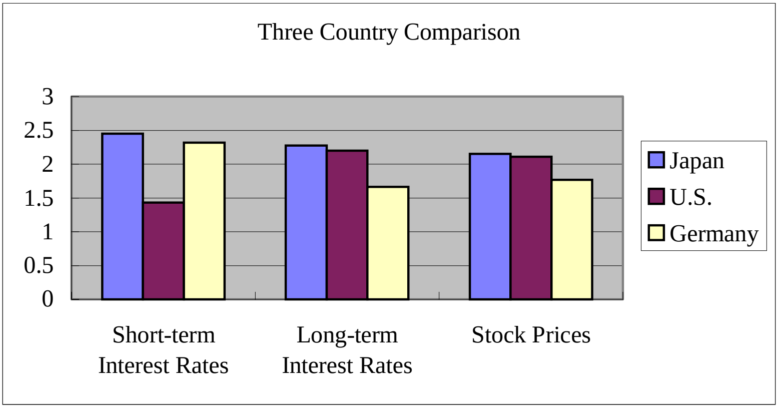
[13] That is, the inclination of investors to invest in domestic assets. For example, according to French and Poterba [1991], the percentage of domestic investment in stock portfolios is 94% in the U.S., 98% in Japan, and 82% in the United Kingdom. For further details, refer to Shiratsuka and Nakamura [1997].

[14] Of course, within any given country the characteristics of market participants may also vary greatly by market, such as the division between participants in short-term and long-term interest markets. Here, however, we take the stance that the characteristics of market participants greatly depends on national characteristics, such as the herd instinct. Additionally, stop limits and conventional trading practices could also be relevant factors, and these could be incorporated into the model constructed in Chapter 3.

[15] These are traders who adopt a strategy of purchasing if the market goes up. If the percentage of such traders is high, a cycle can easily be established whereby traders purchase because the market goes up and the market then moves up because traders are purchasing, and this is considered as one of the factors creating market trends.

Figure 8: Comparison of the thickness of the tail (ratio of the 5% point/standard deviation) in three nations



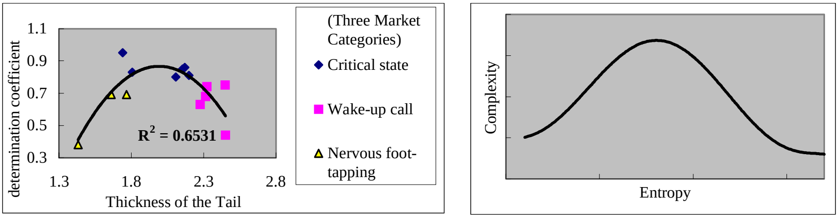
(Outstanding issues: concerning the potential for applying "complexity science" theory)

In our analysis, we utilized frequency distribution tables and the shape of the connected scatter diagrams to identify market characteristics, and this idea originates from "complexity science" theory. Market fluctuations are complex, and in the pricing model and risk management fields the irregularity of markets has often been described as a probability phenomenon. Recently, however, the science of "complexity science"—which grasps irregular market fluctuations not as a probability phenomenon but rather as a deterministic irregularity[16]—has attracted a great deal of attention. There are diverse ideas in the science of "complexity science" (refer to Appendix 4 for an explanation of complexity science). Here, we have presented our results regarding (1) the validity of the "law of power" (the B. Gutenberg C. F. Richter law) for certain frequency distribution diagrams and (2) the behavior of attractors in the connected scatter diagrams. We have also at-

[16] In the past, determinism and stochastic process (non-deterministic laws) were viewed as divergent concepts. Determinism, which is expressed by linear equations, has the following characteristics: (1) once the initial value is set, only a single future value is sought, (2) if minute changes are made to the initial value, the future value only changes minutely, and (3) the changes in the time series (behavior) are simple. Therefore, for systems that exhibit complex behavior and for which it is difficult to predict the future (such as markets), probability has been applied exclusively, without using determinism. (For example, in the pricing model for options and in risk management techniques, market fluctuations are viewed as probability phenomena). However, if non-linear characteristics are introduced into determinism, characteristics (2) and (3) no longer hold true and complex behavior appears. Namely, while still using the equations from determinism minute differences in the initial value result in behavior that cannot be predicted. This is referred to as "chaos."

tempted (3) to calculate the fractal dimension[17] and (4) to verify the symmetry of the time reversal,[18] but we have yet to reach any definitive conclusions regarding these items, and our efforts in these areas are not included in this paper (refer to Appendix 5 and Appendix 6 for a summary of our work and conclusions to date). Nevertheless, we have received the impression that the concept of "complexity science" may well prove helpful in various future analytical work toward elucidating the stress occurrence mechanism.

Figure 9: Relationship between complexity and entropy



For example, the parabolic shape of the relationship between the determination coefficient and the thickness of the tail (the stress level) as seen in the frequency distribution table for (1) (the left-hand graph of Figure 9) resembles the relationship between complexity and entropy (the right-hand graph of Figure 9) advocated by Crutchfield [1989, 1994]. Entropy is an indicator for measuring randomness, and the relationship between complexity and entropy shows that if an orderly state is adopted as the starting point up until a certain level complexity increases as randomness increases, but when this level is passed the complexity is lost and the condition becomes disorderly. Concerning this relationship, Crutchfield argues that a high-entropy condition is random and not that complex, while a low-entropy condition is orderly, and therefore also not that complex.[19] If we consider the size of the determination coefficient as a representative variable

[17] According to one definition of chaos, chaotic variables have a non-integral fractal dimension. Although a linear representation is in one dimension, curves with extremely complex forms are similar to a plane, so complex market fluctuations are expected to have a non-integral dimension that is greater than one dimension but less than two (simple fluctuations are close to one dimension, and complex fluctuations are close to two).

[18] For probability phenomena (noise), the accuracy of future predictions and past predictions is the same, but chaos predictions are dependent upon the past path so future predictions are expected to be more accurate than past predictions (according to Naito et al. [1997]). Strictly speaking, however, probability phenomena are not all time-symmetrical, so certain fixed conditions such as linearity and stationarity are required.

[19] For details, refer to Kaneko and Tsuda [1996]. However, Kaneko and Tsuda suggest the opposite relationship between complexity and entropy to that proposed by Crutchfield. That is, they propose that entropy itself is expressed in the complexity function (in a low-entropy condition, there are both complex and simple factors).

for complexity and the thickness of the tail as representing the extent of entropy, we might paraphrase Crutchfield's assertion by saying that "sudden wake-up call" type markets have high entropy and are not complex, while "nervous foot-tapping" type markets have low entropy and are also not complex. Of course, at the present time this is only an analogy. In the market analysis field, research on complexity science will advance in the future, and the relationship between complexity and entropy may be clarified. In examining chaos, high frequency data is required,[20] so the use of daily data or minute-by-minute data will be a high-priority issue for our chaos research.

3 Construction of a market rate-path-generation model

3.1 Outline of the model

In this section, we construct a market rate-path-generation model based on the interaction among agents (market participants) to examine the hypothesis considered in Chapter 2 of explaining market stress by the characteristics of the market participants. In the subsequent section, we conduct simulations to see what types of markets are formed when we change the characteristics of the market participants in various ways.

Our market rate-path-generation model is based on "genetic algorithms." The concept of "genetic algorithms" was derived from the process whereby living creatures evolve through the recombination of genes. The model is an optimization method whereby the ideal solution is sought by repeatedly recombining the most outstanding items (genes). This idea is being applied in diverse fields,[21] including research of the schooling or flocking behavior of fish and birds. Greg Reynals prepared a "boid" (abbreviation for "birdroid") simulation model which assigns three simple behavioral rules to the agents (boids): (1) a certain minimum distance is maintained from other objects (including other boids); (2) the boids attempt to synchronize their speed with nearby boids; and (3) the boids perceive the center of mass of the nearby boids, and move toward this center. This boid simulation model successfully generated the emergence of "flocking movement" (the appearance of flocking behavior without a direct rule to form a flock).[22] This concept of "emergence"[23] may be useful for research on market fluctuations as

[20] According to Kuratsu [1996], at least 10,000 units of data are required to verify a chaotic state.

[21] Prior examples of research applying genetic algorithms to stock market analysis include Takabayashi [1995] and Iwasawa [1996].

[22] For a detailed review of developments in this field, refer to M.M. Waldrop [1992]. Models based on fish schooling behavior include the FISH model developed by Kamihara et al.

[23] "Emergence" is one of the characteristics of complexity science whereby a property that was not artificially buried in the system beforehand appears.

well. The bandwagon effect[24] as a bubble phenomenon may be a type of flocking behavior, and market aphorisms[25] also treat other participants' market outlooks as important information (that is, changing one's own investment behavior due to the influence of other participants). In this chapter, we applied this boid model to construct a model whereby market outlooks change based on the interaction among agents, resulting in market fluctuations.

(1) Base model

First, let us explain the specific content of the model that will be used as a starting point (hereinafter referred to as the "base model"). The model is a closed system whereby the market level is determined exclusively by the market outlook of the agents. The model assumes that there are three types of agents: positive-feedback, negative-feedback, and noise.[26] In considering the interaction among the agents, the following three rules are set to govern investment behavior: (1) agents attempt to distance themselves when their market outlook is too close to that held by other agents (centrifugal behavior), (2) agents attempt to move closer when their market outlook is too far from that held by other agents (centripetal behavior), and (3) there are rules for position limits and cutting losses (when accumulated losses surpass a certain level, the agents close their positions).[27]

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[24] When a wagon which is carrying a band passes by, everyone is drawn by the excitement and files along without thinking. When others see this they wonder what is going on and also join the parade. This results in the inflation of the bubble (excerpted from Tanaka [1992]).

[25] "Follow those who are successful," "Move in the opposite direction from those who are failing," "If everyone is bullish, become a fool and sell rice," "If everyone from the mountains to the fields is bearish, become a fool and buy rice," etc. (excerpted from The Secrets of Market Professionals).

[26] Positive-feedback agents buy when the market goes up. Negative-feedback agents sell when the market goes up. Noise agents buy and sell without any fixed rule. For the influence on market stress from the relative proportion of these three types of agents, see Shimizu and Yamashita [1996].

[27] The expressions "distance themselves" and "move closer" in (1) and (2) are used assuming that the distances between the market outlooks held by the agents are expressed numerically (+5 or -3, etc.). A similar approach is applied in the model expressing fish schooling behavior (Kamihara [1997]) and the model for the emergence of edge cities (Krugman); for a detailed review of developments in this field, refer to Kitamura [1997].

Figure 10: Conceptual diagram of the base model

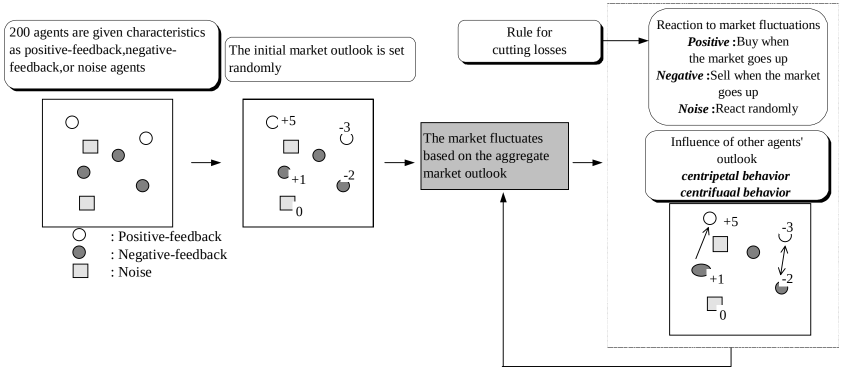

Figure 10 presents a conceptual diagram of the base model. Specifically, the market is formed via the following procedure.

Step 0: Initial settings (rules for position limits[28] and cutting losses, triggers for centrifugal and centripetal behavior).[29]

Step 1: Assign characteristics to the 200 agents as positive-feedback, negative-feedback, or noise agents (determine the initial relative proportion of each type of agent, and use random numbers later on; these characteristics are fixed for the entire simulation period).

Step 2: Randomly assign the initial market outlook to each agent (ranging from -4.5 to +4.5 in increments of 0.5).

Step 3: Calculate the aggregate market outlook of the agents, and use this value as the basis for the market formation.[30]

Step 4: Calculate the reaction of each agent to the market fluctuation.

(1) Reaction based on agent type (positive-feedback, negative-feedback, or noise).

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[28] The numerical expression of the market outlook is taken as the position (if the market outlook is +3, the position is also +3), and limits are set on this position.

[29] Reflecting market aphorisms such as “If everyone is bullish, become a fool and sell rice,” and “If everyone from the mountains to the fields is bearish, become a fool and buy rice,” it is possible to change the triggers in accordance with the extent of concentration of the agents’ market outlooks (for example, when the number of agents with a similar market outlook increases, the trigger for centripetal behavior can be lowered).

[30] $\Delta P = aXe^{\frac{|X|}{B}}$ (Δ P = range of market fluctuation; X = aggregate value of the market outlook; a, B = constants). Here, this equation was adopted assuming that the relationship between the market outlook bias and the range of market fluctuation is not linear (the stronger the market outlook bias, the higher the additional costs = a larger market fluctuation), but many other variations are possible. For example, order tables could be prepared for each agent by market price, and the prices could be set so that the total market supply and demand are always equal (refer to Shimizu and Yamada [1996]).

(2) Random observance of other agents' market outlooks, and adoption of centrifugal and centripetal behavior based on the distance from the other agents' outlooks.
(3) Follow the rules for position limits and cutting losses when they come into play.

Step 5: Repeat steps 3 and 4.[31]

(2) Advanced model

Next, we attempted to improve the base model by assigning professional market participant characteristics to the three types of agents (hereinafter, this is referred to as the "advanced model"). Specifically, we introduced two cases—(a) attributing a learning effect to the agents and (b) participation by agents with a different time horizon—and then formed markets following the same steps used for the base model.

(a) Case where the agents are given a learning effect.
We changed the strategy for when the accumulated losses surpass a certain level. The positive-feedback agents were given a negative-feedback strategy and the negative-feedback agents were given a positive-feedback strategy, while the strategy of the noise agents was left unchanged.

(b) Case with participation by agents with a different time horizon
In the case, the agents are assumed to be investors (with a long-term strategy). The investors adjust the volume of their positions based on changes in the moving average of the market. Specifically, when the moving average rises "N" times consecutively they increase their positions, and when it decreases "N" times consecutively they decrease their positions. Additionally, the investors have no loss limits, and their position limits as well as the size of the positions they can take at one time are set larger than those for other, regular agents.

3.2 Simulations

In this section, we conducted simulations to clarify the following two points: a) by changing the characteristics of the agents (market participants), are changes generated in the stress levels, frequency distribution tables, and shape of the connected scatter diagrams that would be visible on actual markets, and b) if markets with different stress levels, frequency distribution tables and connected scatter diagram shapes are generated, what are the conditions for markets with extreme stress (such as the markets categorized as "sudden wake-up call" type markets in Chapter 2).

For the base model, we conducted simulations to determine what type of markets occur from changing three parameters: (1) the relative proportion of

[31] In this simulation, these steps were repeated 150 times.

positive-feedback, negative-feedback, and noise agents, (2) the extent of influence from other agents (the trigger levels for centrifugal and centripetal behavior),[32] and (3) the position-limit levels and the rules for cutting losses. For the advanced model, we conducted simulations changing two parameters: (1) the trigger levels for changing strategies, and (2) the time horizon[33] of the investors.

(1) Base model simulations

(a) Relative proportions of positive-feedback, negative-feedback, and noise agents

The relative proportions of positive-feedback, negative-feedback, and noise agents were set for the nine cases shown in Figure 11 and simulations were conducted.[34] Figure 12 shows the relationship between *the size of the coefficient of determination* where the frequency distribution table for the market fluctuation indicator (the cumulative annual value of the absolute rate of return) satisfies the function "the frequency Y is inversely proportional to the a^{th} power of the scale X" (the law of power), and *the thickness of the tail* (ratio of the 5% point/standard deviation). Mimicking actual market conditions, the thickness of the tail was set over a wide range from 1.44 to 2.22 (refer to Appendix 9 for the price movements and other details of the generated markets). The relationship between the size of the coefficient of determination and the thickness of the tail showed a gentle parabolic curve, and it appears possible to classify the nine generated markets as "critical state," "sudden wake-up call," or "nervous foot-tapping" type markets. However, no simple rules were discovered for factors that determine the resulting classification (such as "markets with a high proportion of positive-feedback agents tend to be 'sudden wake-up call' type markets"). We also examined the behavior of attractors for representative examples of the three categories (critical state [E], sudden wake-up call [B], and nervous foot-tapping [I]): refer to Appendix 9 for the results.

Figure 11: Simulation settings

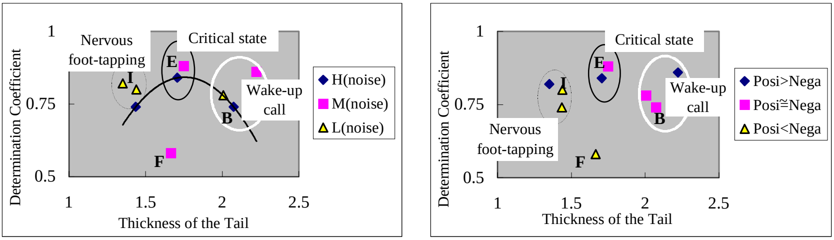
		Ratio of **Positive** vs. **Negative**		
		Positive>Negative	Positive□Negative	Positive<Negative
Ratio of **noise**	High (40%)	**A**	**B**	**C**
	Middle (30%)	**D**	**E**	**F**
	Low (20%)	**G**	**H**	**I**

[32] In other words, the herd instinct.

[33] Specifically, how many months are used for calculating the moving averages when the investors set their strategies.

[34] Refer to Appendix 8 for the details of the simulation settings.

Figure12: Relationship between the determination coefficient and the thickness of the tail (left-hand graph: ratio of noise, right-hand graph: ratio of positive vs. negative)



[Symbol] H: high, M: middle, L: low, Posi: positive-feedback, Nega: negative-feedback

(b) Extent of influence from other agents

Simulations were conducted as summarized in Figure 13 to view the influence from the trigger levels for centrifugal and centripetal behavior on the market characteristics.[35] The relationship between the size of the coefficient of determination and the thickness of the tail (just as in (a)) is shown in Figure 14 (refer to Appendix 10 for the detailed results). Diverse markets were generated by changing the trigger levels and the proportion of positive-feedback agents, and it appears possible to classify these as “critical state,” “sudden wake-up call,” or “nervous foot-tapping” type markets (however, some of the generated markets such as I and G are difficult to classify).

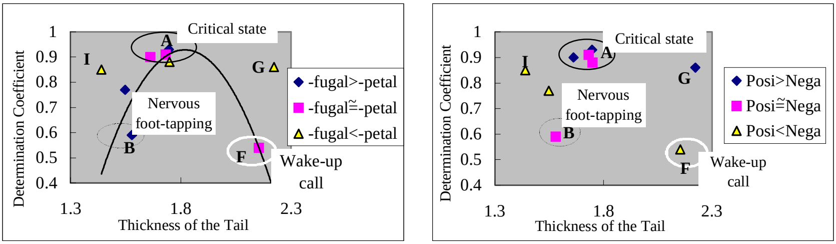
Figure 13: Simulation settings[36]

		Ratio of **Positive** vs. **Negative**		
		Positive>Negative	Positive≅Negative	Positive<Negative
Trigger level	centrifugal>centripetal	**A**	**B**	**C**
	centrifugal≅centripetal	**D**	**E**	**F**
	centrifugal<centripetal	**G**	**H**	**I**

35 Refer to Appendix 8 for the details of the simulation settings.

36 The size of the trigger level expresses the extent of the range in which the behaviors occur.

Figure 14: Relationship between the determination coefficient and the thickness of the tail (left-hand graph: trigger level, right-hand graph: ratio of positive vs. negative)



[Symbol] –fugal: centrifugal, -petal: centripetal,
Posi: positive-feedback, Nega: negative-feedback

(c) Rules for position limits and cutting losses

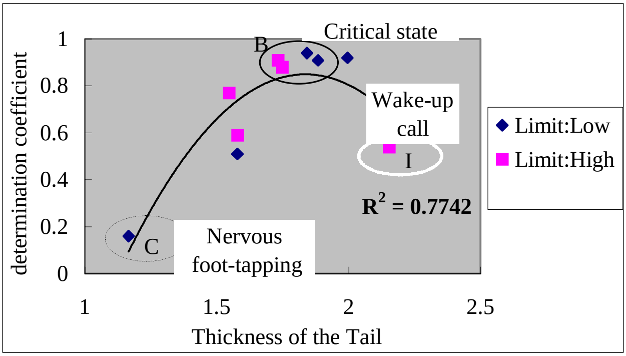
The rules for position limits and cutting losses were set for the 10 cases shown in Figure 15 to examine the influence of these parameters on the market characteristics, and simulations were conducted. The results are shown in Figure 16 (refer to Appendix 11 for the detailed results). Indeed, it is not possible to explain the market characteristics only by the size of the limit levels. However, various types of markets were generated by changing the trigger levels in combination with the proportion of positive-feedback agents.

Figure 15: Simulation settings

	(Trigger level)	-fugal > -petal	-fugal > -petal	-fugal □ -petal	-fugal □ -petal	-fugal < -petal
	(Ratio of Posi vs. Nega)	posi □ nega	posi < nega	posi □ nega	posi < nega	posi □ nega
	(Ratio of noise)	0.3	0.2	0.3	0.2	0.3
Limit level	Low	**A**	**B**	**C**	**D**	**E**
	High	**F**	**G**	**H**	**I**	**J**

[Symbol] –fugal: centrifugal, -petal: centripetal,
Posi: positive-feedback, Nega: negative-feedback

Figure 16: Relationship between the determination coefficient and the thickness of the tail



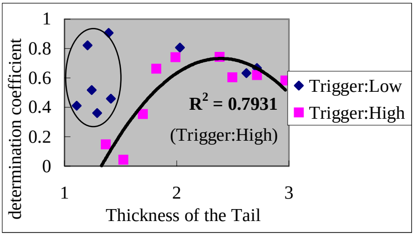
(2) Advanced model simulations

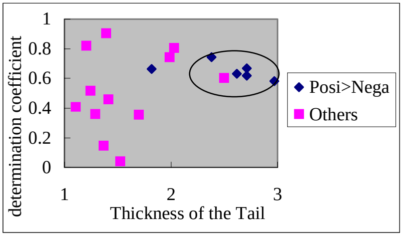
(a) Trigger levels for changing strategies

To view the influence of introducing a "learning effect," simulations were conducted for two cases of trigger levels for changing strategies.[37] The results are shown in Figure 17. Although a clear relationship between the parameters and the market classification could not be discovered, while low trigger levels for changing strategies (conditions whereby it is easy to change strategies frequently) tended not to fulfill the "law of power," the thickness of their tails tended to be small (left-hand side of Figure 17). Also, with this setting of parameters, in many cases the final conditions showed strong inclinations towards either positive-feedback or negative-feedback strategies, and in the cases with final conditions showing inclinations toward positive-feedback strategies the thickness of the tail (right-hand side of Figure 17) tended to be large (≅ "sudden wake-up call" type).

[37] Refer to Appendix 9 for the details of the simulation settings.

Figure 17: Relationship between the determination coefficient and the thickness of the tail (left-hand graph: trigger level, right-hand graph: ratio of positive vs. negative)



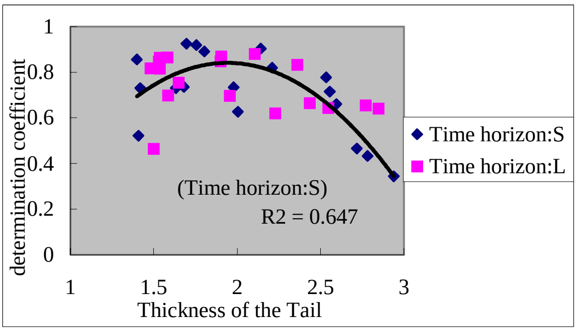


[Symbol] Posi: positive-feedback, Nega: negative-feedback

(b) Time horizon of the investors

To view the influence of introducing "investors," simulations were conducted for two cases of time horizons (the periods used for calculating the moving averages when investors set their strategies).[38] The results are shown in Figure 18. While short time horizons tended to better fulfill the "law of power," this simulation also could not demonstrate any clear relationship between the parameters and the market classification.

Figure 18: Relationship between the determination coefficient and the thickness of the tail



[Symbol] S: Short, L: Long

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[38] Refer to Appendix 9 for the details of the simulation settings.

3.3 Summary of chapter 3

In this chapter, we constructed a market rate-path-generation model (the base model) derived from the "boid model" in an effort to answer the following two questions: a) by changing the characteristics of the agents (market participants), are changes generated in the stress levels, frequency distribution tables, and connected scatter diagram shapes that would be visible on actual markets, and b) if markets with different stress levels, frequency distribution tables and connected scatter diagram shapes are generated, what are the conditions for markets with extreme stress (such as the markets categorized as "sudden wake-up call" type markets in Chapter 2). We then conducted simulations to determine the influence on market characteristics from changing three parameters: (1) the relative proportion of positive-feedback, negative-feedback, and noise agents, (2) the extent of influence from other agents (the trigger levels for centrifugal and centripetal behavior), and (3) the position-limit levels and the rules for cutting losses. We also experimented with reforming the model (into an advanced model) for two cases—(1) introducing a learning effect and (2) introducing participation by agents with a different time horizon—and conducted simulations following the same steps used for the base model.

The left-hand side of Figure 19 once again shows the coefficient of determination plotted against the thickness of the tail for the 22 samples[39] generated using the base model. To evaluate the model results, the right-hand side of the figure shows the same relationship for 10 markets generated randomly.[40] The markets generated by the base model show a wide range of stress levels, the relationship between the size of the coefficient of determination and the thickness of the tail shows a parabolic curve when expressed by the relationship (the law of power) "the frequency Y is inversely proportional to the a^{th} power of the scale X" (question "a"), and the generated markets may be considered closer to real markets (refer to Figure 9) than those generated randomly. In particular, the base model generated "sudden wake-up call" type markets (markets with extreme stress) and "nervous foot-tapping" type markets that did not appear in the markets generated randomly, so the base model may be considered effective in efforts toward clarifying the stress mechanism.[41]

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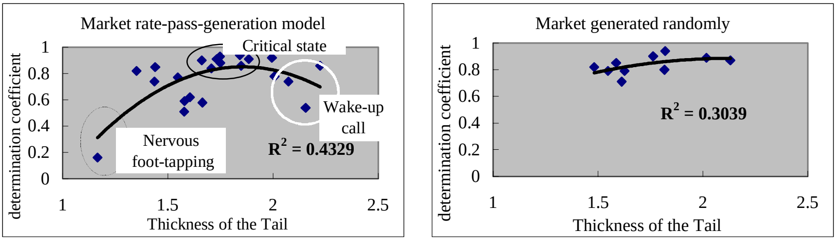
[39] These include some repetition of the samples shown in Figures 12, 14, and 16.

[40] First the series for the rate of return was generated using standard normal random numbers, and then the price series was prepared.

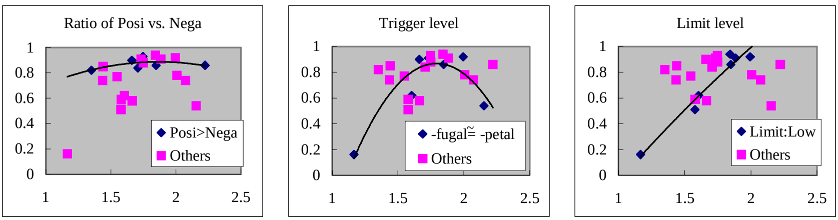
[41] However, no difference was seen versus the random generation for the "critical state" type which is observed in the herd model and on actual markets. In other words, it has not been determined whether "critical state" type markets are "complexity science" or simple probability phenomena.

Figure 19: Relationship between the determination coefficient and the thickness of the tail



We were unable to find answers for question “b.” That is, we did not discover any laws regarding the conditions whereby stress easily occurs, such as “stress easily occurs in markets with a high proportion of positive-feedback agents” (the same conclusion was reached for the advanced model). Figure 20 groups the left-hand side of Figure 19 by (1) whether or not the proportion of positive-feedback agents is high, (2) whether or not the trigger levels are even for the centrifugal and centripetal behavior areas, and (3) whether or not the limit levels are small. The parabolic curve (the law of power) for the relationship between the size of the coefficient of determination and the thickness of the tail (the stress level) for each group is shown in this figure. From Figure 20, we gained the impression that among the three parameters, (2) the extent of influence from other agents (the trigger levels for centrifugal and centripetal behavior) is supposed to be an effective factor in elucidating market stress mechanisms, since it appears to show a parabolic curve shape.

Figure 20: Relationship between the determination coefficient and the thickness of the tail (3 patterns)



4 Conclusion

In our research, we implemented diverse analyses to discover clues toward elucidating the market stress occurrence mechanism. There are still numerous outstanding issues regarding each of the analyses, and we did not draw any definitive conclusions. This chapter summarizes our results to date and reviews the future issues.

(Results of our research)
We analyzed data from 14 markets from January 1985 through November 1996, and clarified that the level of stress varies widely by market. We demonstrated that market classification is possible to a certain extent by combining the shape and other characteristics of the frequency distribution tables, connected scatter diagrams and other analytical tools. Specifically, we clarified two tendencies: (1) there are markets for which the relationship “the frequency Y is inversely proportional to the a^{th} power of the scale X” holds true and others for which it does not, and the latter group can be divided into markets where extreme stress occurs and markets with no stress; and (2) according to the shape of the connected scatter diagrams, extreme stress can easily occur in limit cycle type markets.

In order to explain this difference among markets, we constructed a market rate-path-generation model based on the interaction among market participants. This model generated markets that are not formed through a random process (markets with extreme stress movements, etc.), and we demonstrated the model’s effectiveness for clarifying the stress occurrence mechanism.

(Future Issues)
Improved market models should be constructed and additional simulations conducted to answer the question “What are the conditions whereby extreme stress occurs?” which was not resolved in this paper. Specific ideas include the following.

- Simulate the effect whereby the trigger levels change based on the market outlook bias (the greater the market bias, the more the agents flock together [that is, adopt centripetal behavior], or alternatively, the more the agents act in accordance with market aphorisms such as “If everyone is bullish, become a fool and sell rice” [that is, adopt centrifugal behavior]).
- The learning effect introduced in our research was based on the logic of changing strategy based on past earnings performance, but a pattern whereby the strategy is changed based on the market atmosphere could also be considered. For example, when the scale of changes in the market outlook surpasses a certain level and the market outlook changes radically, the agents could abandon the strategies they had pursued thus far and adopt the same strategy temporarily. Then, after the turning point has been reached,

the agents would revert to their previous, regular strategies. (This represents a model that would place greater emphasis on the bandwagon effect).

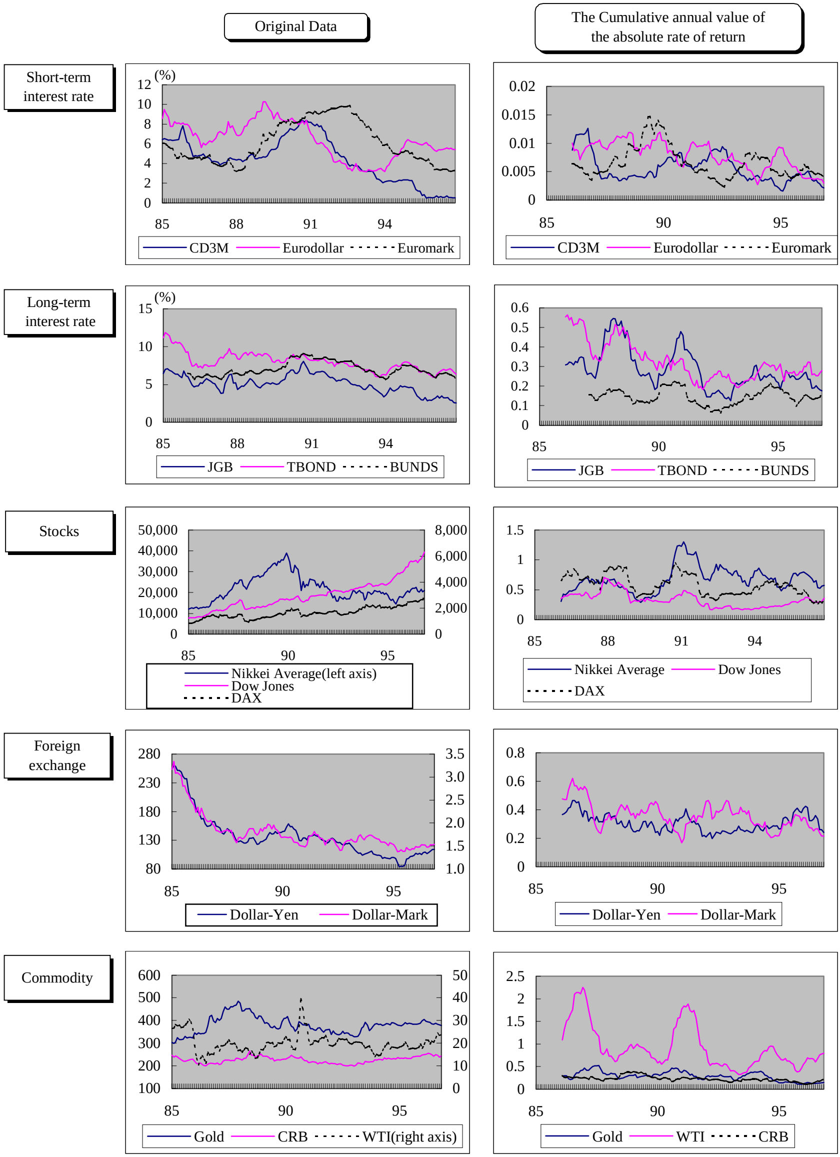
- When the agents go to check other agents' market outlooks, they could check with multiple agents at the same time, or place different weights on the market outlooks of different agents (because one may assume that in actual markets, the influence of other agents' market outlooks varies by agent).
- Change the logic whereby price changes are derived from the market outlook. In the model adopted for this paper, the price changes were calculated based on the imbalance between supply and demand, but order tables could be prepared for each agent by market price, and the prices could be set so that the total market supply and demand are always equal. (In this case, one option would be to introduce a buffer function, such as that provided by market makers).
- Other financial markets could be generated and the interaction among markets considered (two types of agents—domestic and global—might be included).

Through our various analyses during this research we gained the impression that the theory of "complexity science" may prove useful for elucidating the stress occurrence mechanism. At the current point in time, however, this is still in the realm of analogy and we have not identified any clear relationship between complexity science theory and market stress. In the future, we would like to clarify this relationship through further research. At that time, the use of daily data or minute-by-minute data[43] will be the first issue to raise the accuracy to the level required for statistical verification.

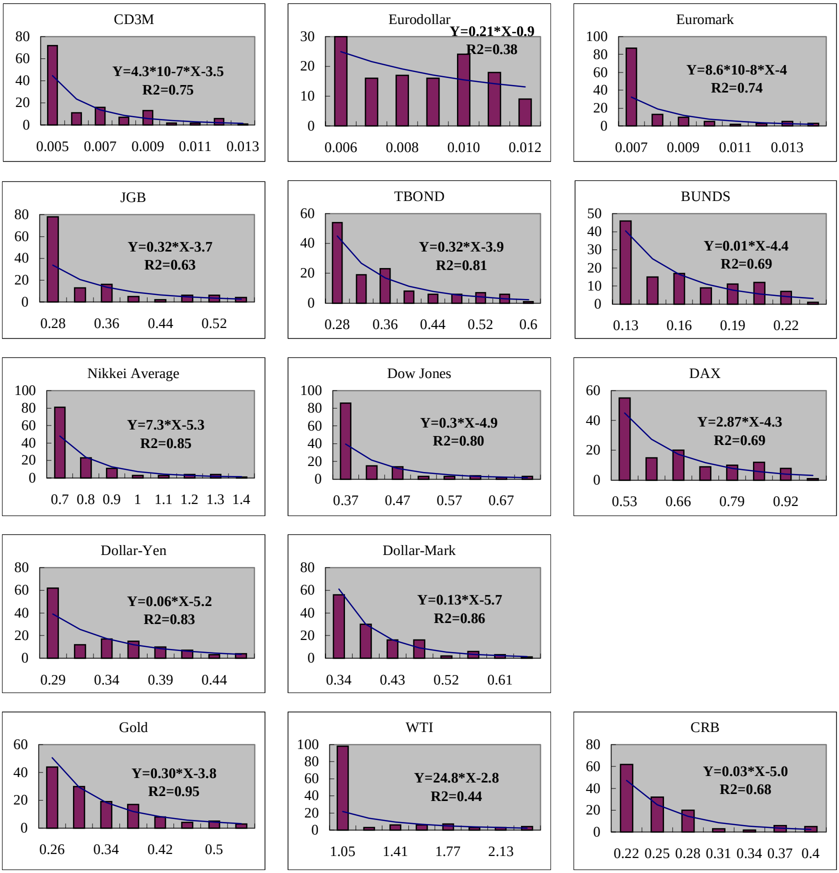
[43] The analysis of raw data should also be considered (market fluctuation indices are a type of moving average, and from the perspective of complexity the real characteristics of the markets may be weakened through the use of such indices).

Appendix

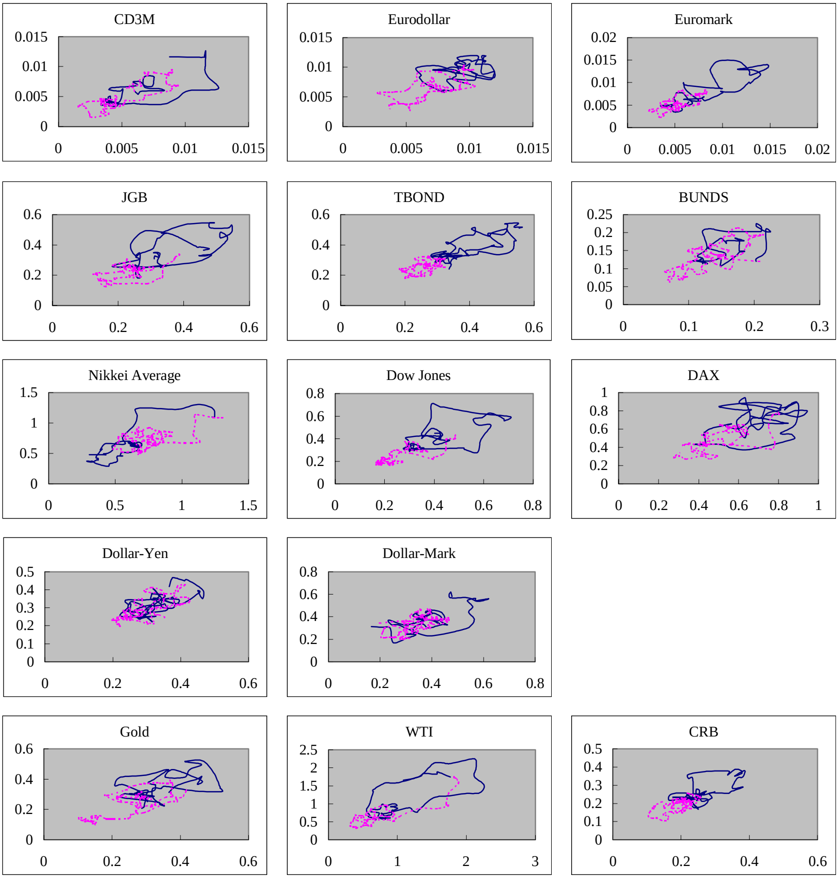
Appendix 1: Original time series data and market fluctuation indices (the cumulative annual value of the absolute rate of return) (Figure 21)



Appendix 2: Frequency distribution tables (Figure 22)



Appendix 3: Connected scatter diagrams (Figure 23)



* the straight line : Feb.86 - May.91
the dotted line : Jun.91 - Nov.96

Appendix 4: The theory of "complexity science"

In this appendix, we provide a simple introduction to the theory of "complexity science," including chaos.

- **Chaos**

No discussion of complexity science is possible without mentioning chaos, yet there is no clear definition of exactly what chaos is. According to R. M. Goodwin

[1989], the greatest characteristic of chaos is that it brings forth unpredictable behavior in a strictly deterministic system. If we look at the article of chaos, (for example, Aihara [1994]) the first item we encounter is the “logistic map.” This is a typical example whereby a very simple law (determinism) brings forth complex behavior, as in the following equation.

$$X(n+1) = r \cdot x(n) \cdot (1 - x(n))$$

[where x(n) is a variable (0~1), r is a constant (0~4), and n is time]

Let us examine the characteristics of chaos using this “logistic map” as an example. The left-hand graph of Figure 24 shows two time series where r = 3.7, with a minute difference in the initial values. The case where x(0) = 0.8 is shown as a solid line, and the case where x(0) = 0.8001 as a dotted line. The right-hand side of the figure is a graph known as a connected scatter diagram[44] with the previous period values on the horizontal axis and the current values on the vertical axis. This diagram makes it easy to see the periodicity and other characteristics.

Figure 24: Logistic map

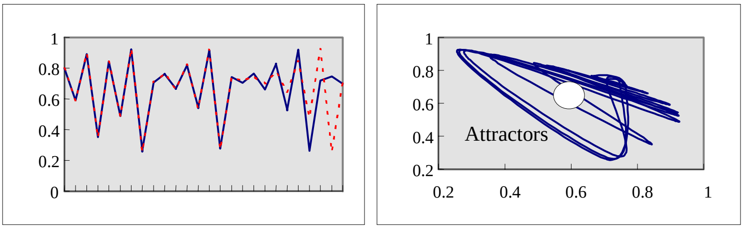


Figure 24 reveals the following two characteristics. (1) Hypersensitivity to the initial value, which is also called the butterfly effect.[45] As shown in the graph, a minute difference between two initial values results in a gradual slippage to the point that the resulting patterns are completely dissimilar. Because of this hypersensitivity to the initial value, in chaotic systems it is very difficult to predict the near future, and absolutely impossible to predict the distant future. (2) The exis-

[44] Using this plotting method, trend data appear as straight lines while cyclical data appear as ellipses.

[45] This was discovered by E. Lorenz while he was conducting computer simulations for a weather forecast model [1961]. The concept is that if a butterfly flaps his wings and stirs the air in Beijing today, this may change the way in which storms develop in New York next month. Refer to J. Gleick for a detailed review of developments in this field.

tence of strange attractors.[46] Attractors appear when movements are expressed in topological space diagrams such as connected scatter diagrams. The attractors exhibit strange behavior as they move about without converging on a single point (a fixed point) or settling into a periodic movement (a limit cycle).

Other properties of chaos such as non-linearity and self-similarity have been pointed out, but there is no clear definition of chaos. In this paper, we distinguish the presence of chaos based on two definitions. (1) The computation of a fractal dimension.[47] It is known that chaotic variables have a non-integral fractal dimension. A straight line is in one dimension, but curves with extremely complex forms are similar to a plane, so chaotic variables are believed to have a non-integral dimension that is greater than one dimension but less than two (simple fluctuations are close to one dimension, and complex fluctuations are close to two). (2) Verification of the symmetry of the time reversal. For noise (a probability phenomenon), the accuracy of future predictions and past predictions is the same, but chaos predictions are dependent upon the past path so future predictions are expected to be more accurate than past predictions.[48]

- **Self-organized criticality**

This concept was proposed by Bak. P,[49] and it is easy to grasp if one thinks about making a small sand hill by dropping grains of sand down onto the ground. As one drops the grains of sand the hill gradually grows higher, but eventually the hill reaches a "critical state," and will not grow any higher (this is a type of "self-organization" in that the hill itself determines its shape without any external control). This is a state of equilibrium in which the volume of sand that falls off the slopes is equal to the volume of sand that is being dropped from above. If additional sand is released from above, various scales of avalanches occur, and it is known that the "law of power" (frequency is inversely proportional to a certain power [exponent] of scale) describes the relationship between the scale and the frequency of these avalanches. Simply stated, the relationship is that small avalanches occur frequently while large avalanches seldom occur. It has been reported that the "law of power" holds true for earthquakes and other sporadic outbreak phenomena.[50] For market fluctuations as well, stress may be considered as a

[46] "Strange attractors" were name in the thesis entitled "On the nature of turbulence" written by Ruelle and Takens (1971), in which they are defined as having three properties: stability, low-dimensionality, and non-periodicity.

[47] Refer to Appendix 5 for information on fractal dimensions. There are other numerical definitions, such as "the maximum Lyapunov exponent is positive." However, because many have pointed out that the maximum Lyapunov exponent is highly arbitrary, the fractal dimension definition is adopted here.

[48] A proposal by Naito et al. [1997]. Strictly speaking, however, probability phenomena are not all time-symmetrical, so certain fixed conditions such as linearity and stationarity are required.

[49] Bak. P, Tang. C, and Wiesenfeld. K discovered self-organized criticality when they were researching electric charge density waves [1987]. For a detailed review of developments in this field, refer to M. M. Waldrop [1992]. Models of self-organized criticality include the "Antlion" (ant lion nesting hole) model prepared by Kashimori et al. [1997].

[50] The geologists Ben Gutenberg and Charles Richter observed that in all localities the number of earthquakes per year that release at least a certain magnitude of energy is inversely proportional

"sporadic outbreak" phenomenon and the concept of "self-organized criticality" may prove useful for elucidating the stress occurrence mechanism.

Appendix 5: Fractal dimensions

- **What are "fractal dimensions?"**

There are various ways of explaining fractal dimensions. One method that is easy to understand is to consider the relation between the rate of minification and the divisor for a geometrical shape with self-similarity.[51, 52] When a straight line is divided into N segments, the rate of minification is $\frac{1}{N}$. To divide a rectangle into N segments to make it into a geometrical shape with self-similarity, the rate of minification is $\frac{1}{N^{\frac{1}{2}}}$. To divide a parallelopipedon into N segments to make it into a geometrical shape with self-similarity, the rate of minification is $\frac{1}{N^{\frac{1}{3}}}$. If the divisor is N and the rate of minification r(N), then the relationship between these two factors may be expressed as

$$r(N) = \frac{1}{N^{\frac{1}{D}}} \left(\rightarrow D = \frac{\log N}{\log\left(\frac{1}{r}\right)} \right)$$

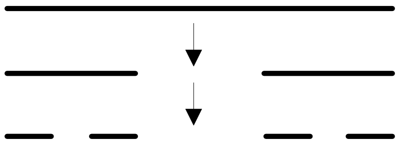
where D is the fractal dimension. For straight lines, rectangles, and parallelopipedons, the integers 1, 2, and 3 correspond to integral dimensions. Let us now consider one method of creating a random progression using deterministic laws: the Cantor ternary set. As in the following diagram, we remove the central 1/3 from a segment with a length of 1. In the same manner, we then remove the central 1/3 from the two resulting segments, and repeat this process infinitely. This is called a Cantor ternary set, and the result is a random progression. The set is chaotic in as much as a deterministic law is used to create a random result. Using the above equation, the fractal dimension is log2 / log3 = 0.63, a non-integral dimension (the divisor is 2 and the rate of minification is 1/3).

---

to a certain power (exponent) of the energy released (the empirical data suggest the power is approximately 3/2) [1956].

[51] These are referred to as fractal figures, and they have a structure whereby one portion fits inside another. Certain portions of these figures are miniaturized versions of the entire figure, and there are many complex items that have this characteristic (Yamaguchi [1986]).

[52] For details, refer to Yamaguchi [1986].



- **Fractal dimension calculation methods**

It is difficult to derive a deterministic law for time series data that appear to be random (in the above example, this would be the divisor and the rate of minification). Diverse methods have been considered for estimating the fractal dimension for this purpose. In this paper we adopt the Grassberger-Procaccia law (the G-P law). Specifically, following the method adopted by Dainichi [1995], our calculations are made according to the following steps.

<u>Step 1</u>: Calculate $C(\varepsilon)$ for each embedding dimension k (k = 1, 2, ... 20)[53]

$$C(\varepsilon) = \frac{2}{N^2} \lim_{N \to \infty} \sum_{k=1}^{N} \sum_{t=1}^{N-k} H(\varepsilon_j - \left|\log P_{t+k} - \log P_t\right|)$$

$$\varepsilon_j = a^j$$

$$H(z) = 1(z \geq 0)$$

$$H(z) = 0(z < 0)$$

<u>Step 2</u>: Repeat Step 1 for each j (j = 1, 2, ... 30) and estimate α_{0k} and α_{1k} in the following equation using the ordinary least squares.

$$\log C\left(\varepsilon_j\right)_k = \alpha_{0,k} + \alpha_{1,k} \log \varepsilon_j$$

<u>Step 3</u>: Estimate β_0 in the following equation using the ordinary least squares.[54] This β_0 is adopted as the fractal dimension.

$$\alpha_{1,k} = \beta_0 + \frac{\beta_1}{k}$$

The calculation results when a = 0.75 (in the second line of the Step 1 equation)[55] are presented in the following table.

---

[53] Strictly speaking, the calculations should be made for all k from 1~N, but we have restricted our calculations to k = 1~20 because of the computation limits.

[54] Because the calculations are restricted to k = 1~20 because of the computation limits, this work is conducted to estimate the limit when $k \to \infty$.

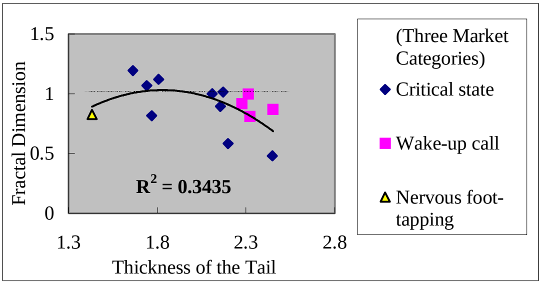
[55] If a is changed, the calculation results differ. For example, when a = 0.9 the dollar-yen fractal dimension decreases to 0.988. The level at which α should be set is an issue for further consideration.

CD3M	Euro-dollar	Euro-mark	JGB	TBOND	BUNDS	Nikkei Average
0.48	0.826	0.809	0.916	0.583	1.191	0.895
NY Dow	DAX	Dollar-Yen	Dollar-Mark	Gold	WTI	CRB
0.999	0.817	1.119	1.014	1.067	0.87	0.998

- **Certain considerations regarding the results**

It is known that chaotic variables have non-integral fractal dimensions,[56] and complex market fluctuations are expected to have a non-integral dimension that is greater than one dimension but less than two (simple fluctuations are close to one dimension, and complex fluctuations are close to two).[57] However, our results were unfavorable in that only four of the markets (BUNDS, dollar-yen, dollar-mark, and gold)[58] show a fractal dimension higher than 1.0. This may be due to the limitations of adopting monthly data, so the use of daily data or minute-by-minute data should be considered in the future. Nevertheless, as shown in Figure 25, the relationship between the fractal dimension and the thickness of the tail is a gentle parabolic curve. As shown in the graph which categorizes the markets by type, the "sudden wake-up call" and "nervous foot-tapping" type markets have fractal dimensions less than 1.0.

Figure 25: Relationship between fractal dimension and the thickness of the tail



[56] There are other numerical definitions, such as "the maximum Lyapunov exponent is positive." However, because many have pointed out that the maximum Lyapunov exponent is highly arbitrary, the fractal dimension definition is adopted here.

[57] Dainichi used daily data (for every working day on which transactions were conducted) from January 5, 1980 through April 30, 1992, and calculated the fractal dimensions for 79 different stocks. All of the stocks had fractal dimensions of over 1.0 (the lowest was 1.41, but the fractal dimensions for two of the stocks were above 2.0). Also, Kuratsu reported that "the fractal dimension for U.S. shares is 2.33," and "the fractal dimension for the dollar-yen market is 2.20."

[58] However, it cannot be stated with statistical significance that these four markets have fractal dimensions greater than 1.0. For example, the 95% confidence interval for the dollar-mark market, which showed a fractal dimension slightly over 1.0, is [1.000, 1.028].

Appendix 6: Verifying the symmetry of the time reversal

For probability phenomena (noise), the accuracy of future predictions and past predictions is the same, but for chaos phenomena predictions are dependent upon the past path so future predictions are expected to be more accurate than past predictions.[59]

In this regard, we devised a simple composite time series model for the market fluctuation indicator (the cumulative annual value of the absolute rate of return) with trend sections expressed using the exponential functions and the surrounding oscillations described using the Sin functions. This model was then used to make past and future predictions (the past predictions were made by reversing the time series data sequence, and the future predictions were made versus this), and the precision of these estimates was compared.

- **Model construction**

Step 1: We conducted spectral analysis[60] for the 64 months of data from October 1988 through January 1994, and identified the cycle periods.

Step 2: We constructed a composite time series model by combining the Sin function of the identified cycle periods with the exponential function (the coefficients were determined using the ordinary least squares).

Step 3: We classified the period from February 1986 through September 1988 (32 months) as "the past" and the period from February 1994 through September 1996 (32 months) as "the future," used the Step 2 model for each of these periods, and prepared the future estimates.[61]

This can be explained by using the Nikkei average as an example. The models derived for "future predictions" and "past predictions" are as follows (refer to Figure 24 for the time series data).

(For Future Predictions)

$$y = 0.97 + (-0.16 \quad -0.20 \quad 0.07 \quad -0.06 \quad -0.05)\begin{pmatrix} S_{64} \\ S_{32} \\ S_{21.3} \\ S_{16} \\ S_{9.1} \end{pmatrix} - 0.16e^{0.0116k}$$

[59] A proposal by Naito et al. [1997]. Strictly speaking, however, probability phenomena are not all time-symmetrical, so certain fixed conditions such as linearity and stationarity are required.

[60] Refer to Appendix 7 for details.

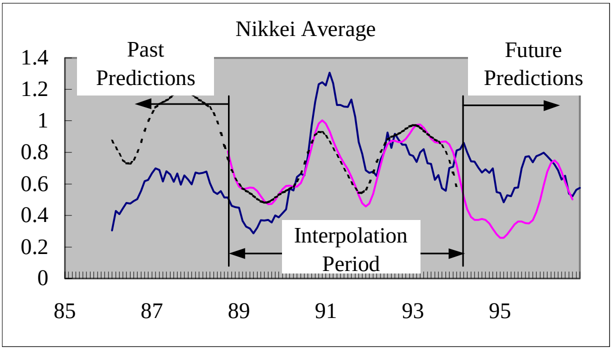
[61] The past predictions were made by reversing the time series sequence. In the construction of the Step 2 model, the coefficients were also determined by reversing the time series sequence.

(For Past Predictions)

$$y = 1.07 + (0.21 \quad 0.19 \quad -0.05 \quad 0.05 \quad 0.02)\begin{pmatrix} S_{64} \\ S_{32} \\ S_{21.3} \\ S_{16} \\ S_{9.1} \end{pmatrix} - 0.49e^{-0.0116k}$$

* y is the cumulative annual value of the absolute rate of return, S_t is the Sin function for a frequency of t months, and k is the time (number of months).

Figure 26: Prediction model



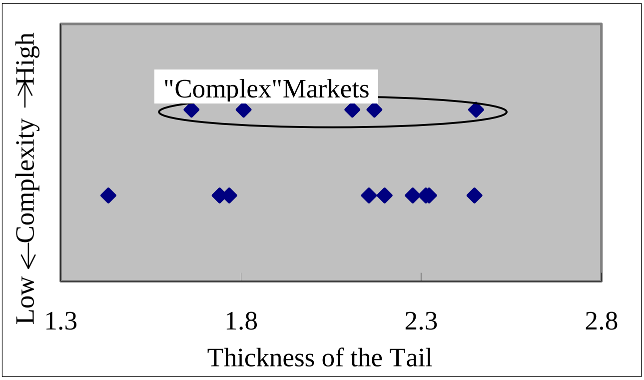
- **Certain considerations regarding the results**

We defined "complex" markets based on two standards: (1) the future predictions are superior,[62] and (2) the predictions become more difficult as the prediction period becomes longer.[63] We then examined the relationship versus the thickness of the tail (Figure 27). Unfortunately, no clear relationship was apparent.

[62] We compared the sum of the squares of the differentials between the predicted values and the actual values for the future predictions and the past predictions, and judged ratios higher than 1.0 as satisfactory. Statistically, it is dangerous to draw conclusions based on a single prediction period, so it will be necessary to follow procedures such as successively sliding the prediction periods and taking the average. In our research we made judgments based on a single fixed period due to limitations on the number of FFT-method data and other factors. Extension to multiple periods is left as an outstanding issue.

[63] In chaotic systems, because of the hypersensitivity to the initial value, even if the immediate predictions are satisfactory, the longer the time frame the more difficult the future predications are likely to be. Accordingly, we examined the correlation between the prediction period (the passage of time) and the prediction error. Of course, because a linear relationship cannot be expected between the passage of time and the prediction error, the statistical significance of this is weak. Nevertheless, we adopted this standard as one type of yardstick.

Figure 27: Relationship between the symmetry of the time reversal and the thickness of the tail[64]



Appendix 7: Spectral analysis

The basis of spectral analysis[65] is the Fourier transform. The Fourier transform is easily understood through an analogy to a light spectrum. When a ray of sunlight strikes a triangular prism, a seven-colored spectrum ranging from red to violet is emitted. The sunlight bundles together different types of light with different wavelengths, but by passing the light through a prism they line up in order due to the differences in the refraction ratio caused by the differences in wavelength. If one substitutes irregular fluctuations for the ray of sunlight and the Fourier transform for the prism, this becomes spectral analysis. In other words, spectral analysis is an analytical method to determine the wave energy distribution (spectral) for each frequency. While there are many types of spectral estimation methods, in this paper we adopt the fast Fourier transform (the FFT method). Specifically, we calculate the spectral value according to the following procedure. We take the waves with a high frequency in the spectral value as shown by the frequency diagram as the constituent elements (for reference, the dollar-mark spectral is presented in the following diagram).

<u>Step 1</u>: Calculate the Fourier components from the random variables x(t) using the FFT method (the Excel fast Fourier transform).[66]

[64] The graph shows markets that meet the two criteria as having a complexity of 1, and those that do not as having a complexity of 0. There is no significance to these numbers themselves, they are simply used to facilitate a visual grasp of the conditions.

[65] For details, refer to Hino [1977].

[66] The number of random variables must be an even-numbered power of 2.

Random Variables

(N variables)

x(0)

x(1)

⋮

x(n-1)

$(\, j = 0, 1, \cdots, n-1)$

FFT →

Fourier Components

(N/2 components)

$A_0 + i\,B_0$

$A_1 + i\,B_1$

⋮

$A_{\frac{n}{2}-1} + i\,B_{\frac{n}{2}-1}$

$\left(k = 0, 1, \cdots, \frac{n}{2} - 1 \right)$

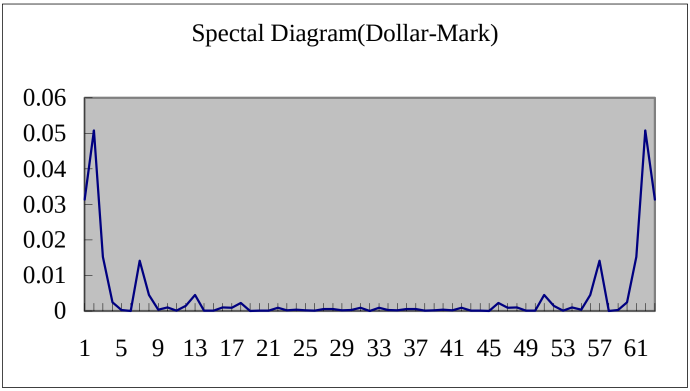
<u>Step 2</u>: Calculate the Fourier transforms X(k) for the random variables x(t).

$$X_k = \sum_{j=0}^{N-1} x(j) \exp\left[-i2\pi \frac{jk}{N} \right] \frac{T}{N} = A_k + iB_k$$

<u>Step 3</u>: Calculate the spectral P(f).

$$P(f) = \frac{\Delta t}{N} \left[A_k^2 + B_k^2 \right]$$

$$f = k\Delta f, \Delta f = \frac{1}{T}, \Delta t = \frac{T}{N}$$



<u>Appendix 8: Detailed settings for the simulations using the base model</u>

Reflecting the actual conditions of market transactions, the following effects were considered for the simulations.

(1) The trading volume varies depending on earnings conditions. (For example, a

case whereby market transactions increase substantially when earnings are favorable.)

(2) The trading volume varies depending on the market outlook. (For example, a contraction of the trading volume in volatile markets.)

(3) The level of influence from other agents changes depending on the bias of the market outlook. (For example, a case where the market aphorism "If everyone is bullish, become a fool and sell rice" applies.)

The specific settings for each of the simulations were as follows.

(1) Relative proportions of positive-feedback,negative-feedback,and noise agents

Reaction to market fluctuations

	P/L:Usual	P/L:Favorable		
Fluctuation:quiet	1	1.25	Fluctuation Criterion	4
Fluctuation:volatile	0.5	0.75	P/L Criterion	100

Influence of other agents' outlook

	Bias:Small	Bias:Big		
Region of Centrifugal	less than 0.51	less than 2.51	Bias Criterion	200
Region of Centripetal	more than 2.01	more than 5.01		
Trade Size	1	0.5		

Cutting Losses, Position Limit

	Fluct:quiet	Fluct:volatile		
Cutting Losses	-100		Fluctuation Criterion	4
Position Limit	7	5		

* Position limit is unchanged by the market fluctuations.

(2) Extent of influence from other agents

Reaction to market fluctuations

	P/L:Usual	P/L:Favorable		
Fluctuation:quiet	1	1.25	Fluctuation Criterion	4
Fluctuation:volatile	0.5	0.75	P/L Criterion	100

Influence of other agents' outlook

Centrifugal > Centripetal

	Bias:Small	Bias:Big
Region of Centrifugal	less than 0.51	less than 2.51
Region of Centripetal	more than 2.01	more than 5.01
Trade Size	1	0.5
Bias Criterion	200	

Centrifugal ≒ Centripetal

	Bias:Small	Bias:Big
Region of Centrifugal	less than 3.51	less than 3.51
Region of Centripetal	more than 3.51	more than 3.51

Centrifugal < Centripetal

	Bias:Small	Bias:Big
Region of Centrifugal	less than 0.51	less than 2.51
Region of Centripetal	more than 2.01	more than 5.01

Cutting Losses, Position Limit

	Fluct:quiet	Fluct:volatile		
Cutting Losses	-100		Fluctuation Criterion	4
Position Limit	7	5		

* Position limit is unchanged by the market fluctuations.

(3) Rules for Position Limits and Cutting losses

Reaction to market fluctuations

	P/L:Usual	P/L:Favorable		
Fluctuation:quiet	1	1.25	Fluctuation Criterion	4
Fluctuation:volatile	0.5	0.75	P/L Criterion	100

Influence of other agents' outlook

Centrifugal > Centripetal

	Pattern 1	Pattern2
Region of Centrifugal	less than 2.51	less than 3.51
Region of Centripetal	more than 3.51	more than 3.51
Trade Size	1	0.5
Bias Criterion	200	

Centrifugal ≒ Centripetal

	Pattern 1	Pattern2
Region of Centrifugal	less than 1.01	less than 2.51
Region of Centripetal	more than 4.01	more than 2.51

Centrifugal < Centripetal

	Bias:Small	Bias:Big
Region of Centrifugal	less than 0.51	less than 2.51
Region of Centripetal	more than 2.01	more than 5.01

* In the case of Limit level is low,the specific setting of simulations are as abaove.
In the case of Limit level is high,these are as setting of (2).

Cutting Losses, Position Limit

Limit level:Low

	Fluct:quiet	Fluct:volatile
Cutting Losses	-30	
Position Limit	5	5
Fluctuation Criterion	4	

Limit level:High

	Fluct:quiet	Fluct:volatile
Cutting Losses	-100	
Position Limit	7	5

* Position limit is unchanged by the market fluctuations.

Appendix 9: Detailed settings for the simulations using the advanced model

(1) Learning effect:trigger levels for changing strategies

Trigger levels for changing strategies

Trigger:Low	-30
Trigger:High	-100

Reaction to market fluctuations

	P/L:Usual	P/L:Favorable		
Fluctuation:quiet	1	1.25	Fluctuation Criterion	4
Fluctuation:volatile	0.5	0.75	P/L Criterion	100

Influence of other agents' outlook

Centrifugal > Centripetal

	Bias:Small	Bias:Big
Region of Centrifugal	less than 2.51	less than 2.51
Region of Centripetal	more than 5.01	more than 5.01
Trade Size	1	0.5
Bias Criterion	200	

Centrifugal ≒ Centripetal

	Bias:Small	Bias:Big
Region of Centrifugal	less than 3.51	less than 3.51
Region of Centripetal	more than 3.51	more than 3.51

Centrifugal < Centripetal

	Bias:Small	Bias:Big
Region of Centrifugal	less than 0.51	less than 2.51
Region of Centripetal	more than 2.01	more than 5.01

Cutting Losses, Position Limit

	Fluct:quiet	Fluct:volatile		
Cutting Losses	-100		Fluctuation Criterion	4
Position Limit	7	5		

* Position limit is unchanged by the market fluctuations.

Proportions of positive-feedback, negative-feedback, and noise agents

	A	B	C	D	E	F	G	H	I
Noise	80	80	80	60	60	60	40	40	40
Positive	75	60	45	85	70	55	95	80	65
Negative	45	60	75	55	70	85	65	80	95

(2) Time horizon of the investors

Time horizon of the investors

* How many months are used for calculating the moving averages when the investors set their strategies.

Time horizon:Short	3	months
Time horizon:Long	6	months

Reaction to market fluctuations

[Traders]

	P/L:Usual	P/L:Favorable		
Fluctuation:quiet	1	1.25	Fluctuation Criterion	4
Fluctuation:volatile	0.5	0.75	P/L Criterion	100

Influence of other agents' outlook

Centrifugal > Centripetal

[Traders]

	Bias:Small	Bias:Big
Region of Centrifugal	less than 2.51	less than 2.51
Region of Centripetal	more than 5.01	more than 5.01
Trade Size	1	0.5
Bias Criterion	200	

[Investors]

	Bias:Small	Bias:Big
Region of Centrifugal	less than 3.51	less than 3.51
Region of Centripetal	more than 7.01	more than 7.01
Trade Size	2	

Centrifugal ≒ Centripetal

[Traders]

	Bias:Small	Bias:Big
Region of Centrifugal	less than 3.51	less than 3.51
Region of Centripetal	more than 3.51	more than 3.51

[Investors]

	Bias:Small	Bias:Big
Region of Centrifugal	less than 5.01	less than 5.01
Region of Centripetal	more than 5.01	more than 5.01

Centrifugal < Centripetal

[Traders]

	Bias:Small	Bias:Big
Region of Centrifugal	less than 0.51	less than 2.51
Region of Centripetal	more than 2.01	more than 5.01

[Investors]

	Bias:Small	Bias:Big
Region of Centrifugal	less than 2.01	less than 2.01
Region of Centripetal	more than 4.01	more than 4.01

Cutting Losses, Position Limit

[Traders]

	Fluct:quiet	Fluct:volatile		
Cutting Losses	-100		Fluctuation Criterion	4
Position Limit	7	5		
Changing strategy level	-30			

[Investors]

Position Limit	10

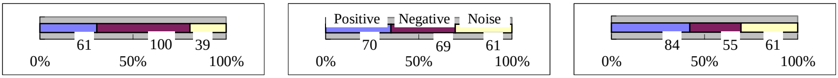
Proportions of positive-feedback, negative-feedback, noise agents, and investors

	A		B		C		D		E		F		G		H		I	
Noise		80		80		80		60		60		60		40		40		40
Positive	65	45	50	30	35	15	75	55	60	40	45	25	85	65	70	50	55	35
Investor	10	30	10	30	10	30	10	30	10	30	10	30	10	30	10	30	10	30
Negative		45		60		75		55		70		85		65		80		95

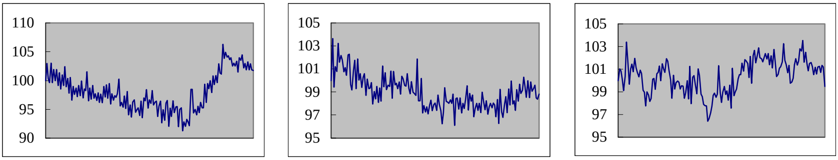
Appendix 10: Details of simulation outcomes < (1) Relative proportions of positive-feedback, negative-feedback, and noise agents>

"Nervous Foot-tapping Type"	"Critical State Type"	"Sudden Wake-up Call Type"

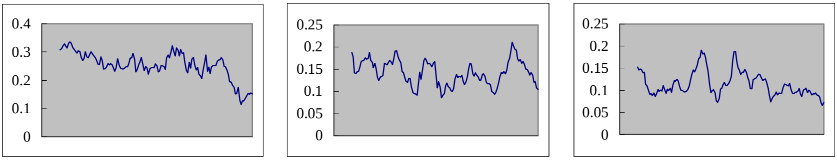
Ratio of Agent Type



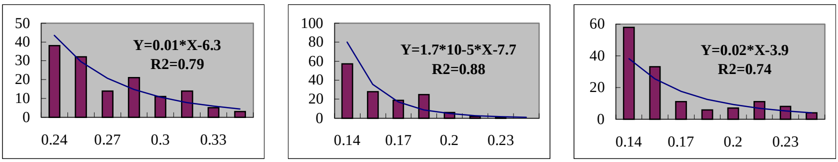
Price Change



CAV Change

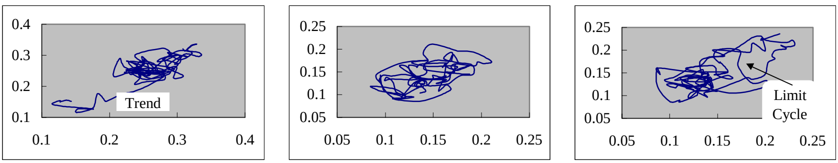


Frequency Distribution Table



Thickness of the Tail (Ratio of the 5% point /standard deviation)	Thickness of the Tail (Ratio of the 5% point /standard deviation)	Thickness of the Tail (Ratio of the 5% point /standard deviation)
1.44	**1.75**	**2.22**

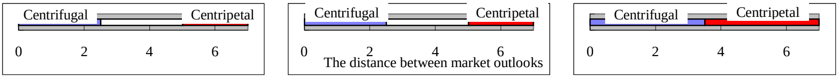
Attractor Behavior



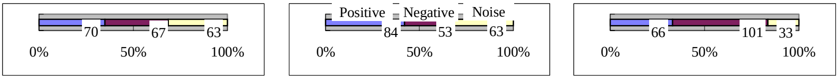
Appendix 11: Details of simulation outcomes <(2) Extent of influence from other agents>

"Nervous Foot-tapping Type" | "Critical State Type" | "Sudden Wake-up Call Type"

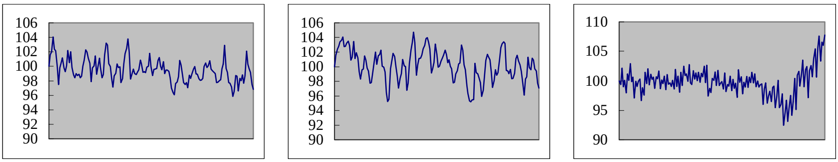
Trigger Level of Other Agents' Influence



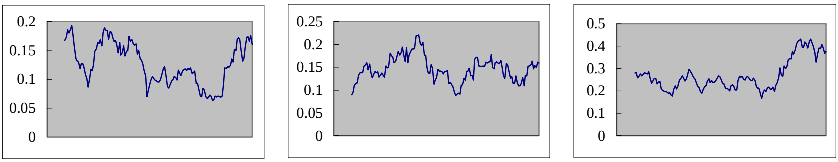
Ratio of Agent Type



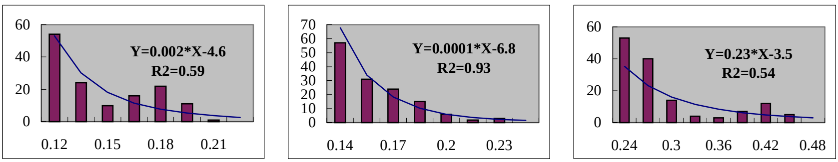
Price Change



CAV Change

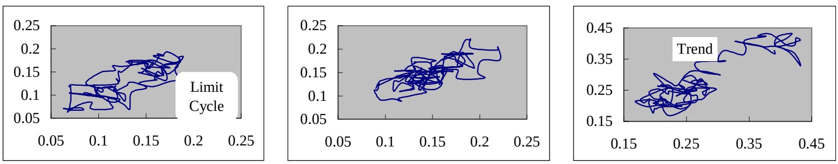


Frequency Distribution Table



Thickness of the Tail (Ratio of the 5% point /standard deviation)	Thickness of the Tail (Ratio of the 5% point /standard deviation)	Thickness of the Tail (Ratio of the 5% point /standard deviation)
1.58	**1.75**	**2.15**

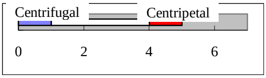
Attractor Behavior

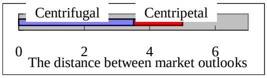


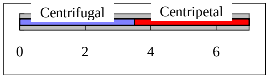
Appendix 12: Details of simulation outcomes <(3) Rules for position limits and cutting losses>

"Nervous Foot-tapping Type"	"Critical State Type"	"Sudden Wake-up Call Type ħ

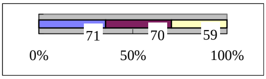
Trigger Level of Other Agents' Influence

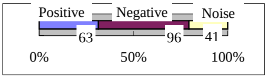


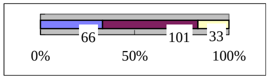




Ratio of Agent Type



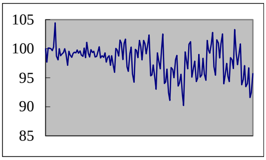


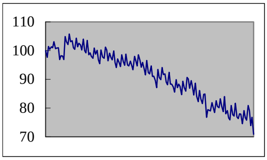


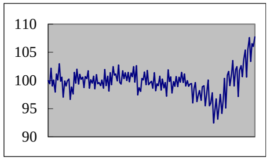
Position Limits

Position limit: ±5 Cutting losses: -30	Position limit: ±5 Cutting losses: -30	Position limit: ±7 Cutting losses: -100

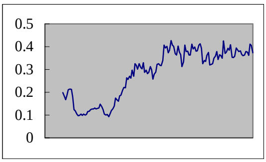
CAV Change

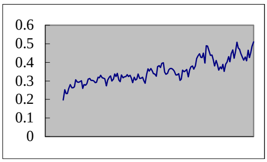


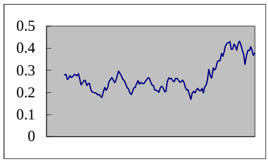




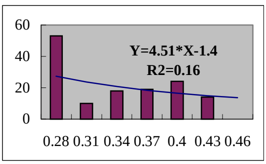
Price Change

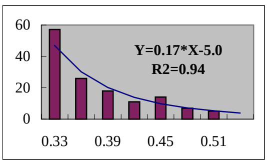


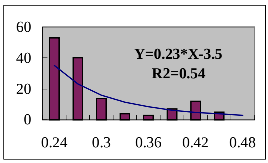




Frequency Distribution Table

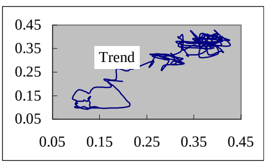


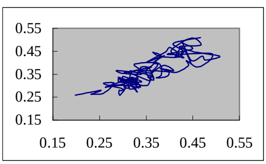


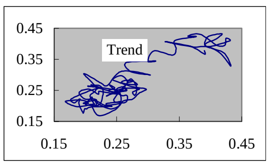


Thickness of the Tail (Ratio of the 5% point /standard deviation) **1.17**	Thickness of the Tail (Ratio of the 5% point /standard deviation) **1.84**	Thickness of the Tail (Ratio of the 5% point /standard deviation) **2.15**

Attractor Behavior







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