Can cross-border financial markets create good collateral in a crisis?*

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Abstract

In this paper, we explore whether markets can create endogenously good collateral in a crisis by analyzing a simple model where a country-specific catastrophic shock is shared between two countries in the presence of solvency constraints. In this model, due to severe solvency constraints, realized catastrophic shocks cannot be covered fully by *ex ante* arrangements. However, most uninsured shocks can be financed *ex post* by the collateral asset created endogenously through the simultaneous expansion of financial balance sheets in both countries. A nondamaged country finances risky loans by issuing relatively safe bonds to a damaged country. Such safe bonds in turn serve as high quality collateral on which a damaged country issues risky bonds (loans) to finance uncovered losses without violating solvency constraints. The above bilateral lending works to achieve almost perfect insurance outcomes.

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1 Introduction

In this paper, we explore whether cross-border financial markets can create endogenously good collateral when it is urgently needed in a crisis. If it is possible, then we examine how high quality collateral is created from voluntary transactions among private agents. We finally explore some policy implications from the theoretical exercise.

We often observe that collateral creation mechanisms are triggered by large-scale interventions of governments or central banks when a national economy suffers from catastrophic events such as natural disasters, political turmoils, economic depressions, and financial crises. For example, the government bonds that are issued to finance reconstruction loans can serve as high quality collateral for agents in serious need of liquidity. During a financial crisis, a central bank extensively lends their owned government bonds to private agents who are eager to hold high quality collateral. In addition, a central bank actively swaps safe bonds at hand with the risky bonds in possession of private agents in order to help their borrowing. To facilitate such operations on a large scale, central banks are required to aggressively expand their own financial balance sheets.

In this paper, we step back for the moment from an economic environment where public institutions play an active role in collateral provision. Instead, we construct a simple model in which collateral creation emerges endogenously in equilibrium. If we there find some equilibrium phenomena quite similar to those associated with public collateral provision, such as the lending of safe bonds, the exchange between safe bonds and risky bonds, and the expansion of financial balance sheets, then we finally inquire into which kinds of economic functions are carried out by public institutions in terms of collateral provision.

Employing a simple two-country setup, we investigate theoretically how country-specific catastrophic shocks are shared between countries in the presence of solvency constraints. In this model, given severe solvency constraints, the solvency of an insurer country is extremely limited. Accordingly, a realized catastrophic shock cannot be covered fully by *ex ante* arrangements such as catastrophe insurance. By simulation, we demonstrate that good collateral assets (relatively safe bonds) are created endogenously through the simultaneous expansion of financial balance sheets of both damaged and nondamaged countries *only* in the aftermath, and that a damaged country can finance uncovered shocks *ex post* on such collateral assets.

More concretely, a nondamaged country finances reconstruction loans (risky loans to a damaged country) by issuing relatively safe bonds to a damaged country. With relatively stable payoffs, the bonds issued by a nondamaged country in turn serve as high quality collateral on which a damaged country issues risky bonds (loans) to a nondamaged country for financing uncovered losses. That is, the relatively safe bonds issued by a nondamaged country is exchanged for the risky bonds issued by a damaged country. In equilibrium, a damaged country as a net debtor can finance uninsured losses, while a nondamaged country as a net creditor can smooth consumption. Without violating solvency constraints, consequently, the above *ex post* bilateral lending between countries in the aftermath works to achieve almost perfect insurance outcomes and reduce substantially equity premiums.[1]

The most essential element that underlies the above mechanism is that because of binding solvency constraints, the arbitrage opportunity associated with Lucas trees may not be exploited completely. Consequently, the constrained competitive equilibrium may sustain the price of risky bonds (Lucas trees) that is slightly high relative to the price dictated by arbitrage conditions. Given comparably expensive Lucas trees, a damaged country can issue risky bonds on more favorable terms. Thanks to good collateral assets issued by a nondamaged country, a damaged country can exploit the above arbitrage opportunity to some extent by making large-scale short positions in Lucas trees. At the same time, a nondamaged country can utilize a considerable investment in risky bonds as an effective instrument to smooth consumption.

The model employed in this paper is a simple two-country exchange economy with country-specific catastrophic shocks, in which markets are complete, but each country is subject to solvency constraints, following Lustig and Nieuwerburgh (2005), Lustig (2007), and Chien and Lustig (2010). In this setup, contingent claims and Lucas trees are traded between two countries in terms of either long or short positions. Here, the solvency constraint is binding once the net position of financial assets is non-positive for a certain future state. A country-specific catastrophic shock may be persistent on labor

[1] It does not necessarily imply that solvency constraints never matter in either resource allocations or asset pricing. In Lustig (2007), solvency constraints have large impacts on equity premiums under the assumption that the volatility of idiosyncratic shocks is extremely large and counter-cyclical.

endowment in level terms, but not permanent.

The above type of solvency constraints have two potential impacts on a two-country model embedded with complete markets. First, the solvency of an insurer country may be limited by this constraint. With extremely severe solvency constraints, accordingly, realized catastrophic shocks may not be insured fully by *ex ante* arrangements such as catastrophe insurance. Accordingly, limited insurance is treated not exogenously, but endogenously within the current framework.

Second, with solvency constraints satisfied, the financial portfolio of each country involves both long positions in one financial instrument and short positions in another. In this regard, the solvency constraint employed in our paper differs substantially from the standard borrowing constraint that is often introduced into an incomplete market setup. In the latter constraint, an agent (country) issues non-contingent bonds, and borrow resources on a part of human capital as collateral.[2] Accordingly, there is not any room for financial instruments to be utilized as collateral at all. In the former constraint, on the other hand, collateral assets may be created endogenously through active financial transactions among countries.

One advantage of the two-country setup is that it is possible to make each country's portfolio problem explicit. As in Lustig (2007) and others, we solve numerically a constrained competitive equilibrium using a representative agent framework with time-varying Negishi weights. A portfolio problem is always implicit in representative agent models. Unlike in the continuum agent setup of Lustig (2007), however, the two-country setup allows us to easily recover optimal portfolio problems from the computed equilibrium asset prices and resource allocations.[3] In so doing, we can investigate which financial transactions between countries would make the sharing of catastrophic shocks quite effective even in the presence of severe solvency constraints.

[2]Using the theoretical framework constructed by Bewley (1986), many papers including Telmer (1993), Lucas (1994), and Aiyagari (1994) investigate how transitory idiosyncratic shocks are insured against among agents in a setup where insurance markets do not exist, and borrowing constraints are exogenous. With mild borrowing constraints, transitory shocks are insured against almost perfectly through self-insurance, while a severe borrowing constraint reduces the self-insurance ability among agents. On the other hand, as shown by Constantinides and Duffie (1996) and others, in a case where idiosyncratic shocks are permanent, self-insurance does not work at all regardless of whether borrowing constraints are present.

[3]Using an extremely advanced technique, Kubler and Schmedders (2003) successfully solve portfolio problems in the presence of collateral constraints in incomplete markets.

The literature on international finance have intensively investigated the possible effects of solvency constraints because enforcement problems are quite serious in international financial arrangements. They initially addressed whether sovereign debts are sustainable in the presence of solvency constraints,[4] and later investigated how enforcement constraints influence borrowing ability as well as risk-sharing capacity.[5]

One closely related paper is Kehoe and Perri (2002). They apply the framework proposed by Alvarez and Jermann (2000) to a two-country model. The way solvency constraints are endogenized differs between Alvarez and Jermann (2000) and Lustig (2007). In the former model, once a country is in default, it is excluded forever from international financial trading, while in the latter, even if a country defaults, it can remain in international financial markets after its collateral is confiscated completely. However, both models are similar to each other in that those financial contracts are made such that effective default (insolvency) never occurs in equilibrium; in the latter model, for example, a borrowing country is indifferent between default and repayment even when solvency constraints are binding.

This paper is organized as follows. Section 2 constructs a two-country model with solvency constraints, while Section 3 presents the calibration results. Section 4 offers concluding comments.

2 Model

In this section, we construct a two-country exchange economies with solvency constraints following the framework proposed by Kehoe and Perri (2002), Lustig (2007), and Chien and Lustig (2010). Our goal in this paper is to analyze how effectively catastrophic shocks are shared between countries in the presence of solvency constraints. In particular, our simulation result based on the two-country model demonstrates that the expanded

[4] Eaton and Gersovitz (1981) analyze how sovereign debts are sustained by a reputation mechanism, while Bulow and Rogoff (1989) present a case in which a reputation mechanism does not necessarily make sovereign bonds sustainable when a borrowing country is allowed to make savings. More recently, Hellwig and Lorenzoni (2009) demonstrate that sovereign bonds are still sustainable given a general equilibrium effect of limited commitment on interest rates. Eaton and Fernandez (1995) survey the issue regarding sovereign bonds.

[5] Using a small country setup, Atkeson and Ríos-Rull (1996) construct a model with exogenous constraints on borrowing from foreign countries, while Caballero and Krishnamurthy (2001) explore the possible effects of collateral constraints on international borrowing.

financial balance sheet of a nondamaged country helps a damaged country to *ex post* finance catastrophic losses with solvency constraints satisfied. That is, a nondamaged country finances loans to a damaged country by issuing financial assets, while those assets issued by a nondamaged country in turn serve as collateral for a damaged country.

In this setup, the labor endowment of each country is subject to country-specific catastrophic shocks; country-specific catastrophic shocks on the level of the labor endowment may be persistent, but not permanent. On the other hand, dividends on Lucas trees are proportional to the world endowment. In terms of market structures, markets are complete with respect to country-specific catastrophic shocks. However, each country is subject to solvency constraints in the sense that net financial positions cannot be negative in every possible future state. That is, either Lucas trees or contingent claims can be used as collateral in cross-border financial transactions; however, nonfinancial assets (human capital) cannot be.

2.1 Setup

Labor endowment and Lucas trees A world economy consists of infinite-horizon exchange economies of country i ($i = 1$ or 2) in a discrete time setup. It is assumed that the labor endowment is homogeneous within each country, but heterogeneous between the two countries. Each country receives labor endowments subject to country-specific catastrophic shocks. There is a fixed supply of Lucas trees whose dividends are subject to world common shocks. The quantity of Lucas trees is standardized to one.

A set of states of country-specific labor endowment is defined as $y \in Y = \{y_1, \ldots, y_m\}$, while a set of states of dividends on Lucas trees is denoted as $z \in Z = \{z_1, \ldots, z_n\}$. A combination of country-specific and world common states is expressed by $s_t = (y_t, z_t)$, where s_t is in $S = Y \times Z$. In addition, $s^t = (y^t, z^t)$ denotes a history from time 0 up to time t, while $s^j \succeq s^t$ represents a continuation history from s^t.

Furthermore, we assume that the dividend on Lucas trees $d(z)$ is proportional to the total labor endowment $(e^1(y) + e^2(y))$. Therefore, a combination of country-specific shocks constitutes aggregate states. Hereafter, $e^i(y_t)$ denotes the labor endowment of country i, and $d(z_t)$ denotes the dividend on Lucas trees. The transition probability of the above state variables $\pi(y', z'|y, z)$ evolves according to the following Markov process:

$\pi(z'|z) = \sum_{y' \in Y} \pi(y', z'|y, z)$, $\forall z \in Z$, $\forall y \in Y$. Given the above processes of labor endowment and dividends, optimal policy functions of consumption and portfolios depend on state z_t.

Preferences and resource constraints Country i maximizes expected lifetime utility with respect to consumption at state s^t ($c^i(s^t)$) as follows:

$$U(\{c^i(s^t)\}_{t=0}^{\infty})(s_0) = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t|s_0) u \left[c^i(s^t)\right], \ i \in \{1, 2\},$$

where a preference is characterized as utility with constant relative risk aversion, or $u\left[c^i(s^t)\right] = \frac{c^i(s^t)^{1-\gamma}}{1-\gamma}$, γ denotes the degree of relative risk aversion, and β represents the rate of time preference.

The resource constraint of the world economy is given by:

$$e(z_t) = e^1(y_t) + e^2(y_t) + d(z_t).$$

Hereafter, α denotes the ratio of dividends to total labor endowment:

$$\alpha = \frac{d(z_t)}{e(z_t)}.$$

Complete markets In this economy with complete markets, both the shares of Lucas trees and one period contingent claims are traded between the two countries. $\theta^i(s^t)$ denotes the share of Lucas trees held by country i in time t, while $a^i(s^t, s')$ represents the holding of claims on one unit of goods at state s' in the next period given the current state s^t. $p(z^t)$ is the price of Lucas trees, and $q(s^t, s')$ is the price of a contingent claim.

The market clearing conditions hold as follows:

$$\theta^1(s^t) + \theta^2(s^t) = 1, \tag{1}$$

$$a^1(s^t, s') + a^2(s^t, s') = 0, \ \text{for all } s' \in S. \tag{2}$$

The budget constraint of country i in time t is expressed by:

$$c^i(s^t) + p(z^t)\theta^i(s^t) + \sum_{s' \in S} q(s^t, s')a^i(s^t, s') \leq w^i(s^t), \tag{3}$$

where $w^i(s^t)$ denotes the initial wealth in time t, which is described below.

Collateral constraints In this economy, the outstanding liability in short positions is enforceable up to the value of financial assets as collateral. That is, net positions of financial assets cannot be negative in every possible one-period ahead state:

$$\left[p(z^{t+1}) + d(z_{t+1})\right] \theta^i(s^t) \geq -a^i(s^t, s'), \ \forall s' \in S. \tag{4}$$

We call the above enforcement constraint a *collateral constraint*. This formulation of enforcement constraints implies that when equation (4) is binding, a debtor country is indifferent between default with confiscation and full repayment at maturity.

If a collateral constraint or equation (4) is binding for country i, then the possessed financial assets are exhausted for repayment (or they are confiscated), and the next period's wealth ($w^i(s^{t+1})$) consists of only the labor endowment:

$$w^i(s^{t+1}) = e^i(y_{t+1}),$$

otherwise, it is equal to:

$$w^i(s^{t+1}) = e^i(y_{t+1}) + \left[p(z^{t+1}) + d(z_{t+1})\right] \theta^i(s^t) + a^i(s^t, s_{t+1}).$$

We make some remarks on the above type of collateral constraints. First, an insurer country has to back catastrophe insurance payments ($-a^i(s^t, s')$) by his holdings of Lucas trees ($\left[p(z^{t+1}) + d(z_{t+1})\right] \theta^i(s^t)$). In other words, a country can offer catastrophe insurance capacity only up to the value of Lucas trees at hand. Second, equation (4) does not impose any upper limit on short positions in contingent claims. A country can issue contingent bonds as long as he can repay bond obligations by Lucas trees as collateral. Third, equation (4) does not exclude any short position in Lucas trees. A country can make short positions as long as he carries long positions in contingent bonds. Here, we assume that any short position is settled by cash or netting; that is, any delivery of Lucas trees is not involved in trading short positions. In this regard, making short positions in Lucas trees may be interpreted as issuing risky bonds whose repayment is proportional to the price of Lucas trees.

In a two-country setup, the last two aspects of collateral constraints may work together to simultaneously expand the financial balance sheets of the two countries in equilibrium. That is, one country may make large long positions in Lucas trees and short positions in contingent bonds, while the other country may make large-scale reverse positions in Lucas trees and contingent bonds.

Asset pricing in collateral-constrained economy Once a collateral constraint is binding at a certain state in time $t+1$ (s^{t+1}), then

$$q(s^t, s_{t+1})u'\left[c^i(s^t)\right] - \beta\pi(s_{t+1}|s_t)u'\left[c^i(s^{t+1})\right] > 0 \tag{5}$$

holds as intertemporal efficiency conditions. These inequalities are often called *Euler inequalities.*

The Euler inequality implies that the stochastic discount factor ($\beta\pi(s_{t+1}|s_t)\frac{u'\left[c^i(s^{t+1})\right]}{u'\left[c^i(s^t)\right]}$, hereafter, SDF) of a country subject to a collateral constraint at a certain state in time $t+1$ becomes irrelevant to the asset pricing behavior in time t. From equation (5), we have

$$\beta\pi(s_{t+1}|s_t)\frac{u'\left[c^i(s^{t+1})\right]}{u'\left[c^i(s^t)\right]} < \beta\pi(s_{t+1}|s_t)\frac{u'\left[c^{i'}(s^{t+1})\right]}{u'\left[c^{i'}(s^t)\right]} = q(s^t, s_{t+1})$$

for a constrained country (country i) and a unconstrained country (country i'). Thus, the SDF of a unconstrained country is larger than that of a constrained country in equilibrium.

One thing to be noticed is that the equilibrium price of Lucas trees may not be evaluated precisely according to arbitrage conditions in the presence of collateral constraints. That is, we may have the following inequality in equilibrium:

$$p(z^t) \neq \sum_{s^{t+1}\succeq s^t} q(s^t, s^{t+1})\left[p(z^{t+1}) + d(z_{t+1})\right].$$

In particular, when it is impossible to make flexibly short positions in Lucas trees due to binding collateral constraints, the arbitrage opportunity such as

$$p(z^t) > \sum_{s^{t+1}\succeq s^t} q(s^t, s^{t+1})\left[p(z^{t+1}) + d(z_{t+1})\right]$$

may not be exploited to a full extent upon the realization of fundamental shocks.

Competitive equilibrium with collateral constraints In sequential trading, each country maximizes expected lifetime utility subject to budget constraint (3) and collateral constraint (4):

$$\max_{\{c^i\},\{\theta^i\},\{a^i\}} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t|s_0) u\left[c^i(s^t)\right], \tag{6}$$

$$\text{s.t. } c^i(s^t) + p(z^t)\theta^i(s^t) + \sum_{s' \in S} q(s^t, s')a^i(s^t, s') \leq w^i(s^t),$$

$$\left[p(z^{t+1}) + d(z_{t+1})\right]\theta^i(s^t) \geq -a^i(s^t, s'),\ \forall s' \in S.$$

A collateral constrained competitive equilibrium is defined as follows. Given the initial wealth $\{w_0^1, w_0^2\}$, the trading strategy $\{a^i(s^t, s_t')\}$, $\{c^i(s^t)\}$, $\{\theta^i(s^t)\}$, the pricing function $\{q(s^t, s')\}$ and $\{p(z^t)\}$, each country maximizes (6) subject to equations (3) and (4), and market clearing conditions (1) and (2) are satisfied.

2.2 Construction of a representative agent model with time-varying Negishi weights

2.2.1 Time 0 cost minimization problem

It is not possible to directly solve the above sequential trading problem because of the presence of collateral constraints. Following Lustig (2007), we thus construct a representative agent model with time-varying Negishi weights (Negishi, 1960), and compute the constrained competitive equilibrium using stochastic discount factors derivable from the representative agent model. For this end, we first translate the sequential trading problem into the time 0 problem. More precisely, we convert the sequential trading utility maximization problem to its dual problem or cost minimization in time 0 together with a single promise-keeping constraint, and the solvency constraints, both of which are defined below.

The construction of the time 0 problem greatly simplifies the computation procedure of the constrained equilibrium for the following reasons. First, the value function represented by a promise-keeping constraint can summarize a history of the realized states, and serve as a state variable; consequently, the space of state variables is reduced substantially. Second, Negishi weights can be computed from the cumulation of the Lagrange multipliers associated with solvency constraints. Because the Lagrange multiplier is positive at a default state and zero otherwise, Negishi weights become time-varying depending on whether solvency constraints are binding.

We rewrite a collateral constraint (4) as the following *solvency constraint*. If the net financial asset is zero upon default at state s^t, then consumption from state s^t on has to be financed by only the current and future labor endowment. Accordingly, lifetime consumption is equal to lifetime labor endowment at state s^t:[6]

$$\sum_{j \geq t} \sum_{s^j \succeq s^t} Q(s^t, s^j) c^i \left(s^j\right) = \sum_{j \geq t} \sum_{s^j \succeq s^t} Q(s^t, s^j) e^i \left(y_j\right),$$

where $q\left(s^{t-1}, s^t\right)$ corresponds to a stochastic discount factor between state s^{t-1} and state s^t, and $Q(s_0, s^t) = q\left(s_0, s^1\right) \cdot q\left(s^1, s^2\right) \cdots q\left(s^{t-1}, s^t\right)$. If collateral constraints are not binding and the net financial asset is still positive, then:

$$\sum_{j \geq t} \sum_{s^j \succeq s^t} Q(s^t, s^j) c^i \left(s^j\right) > \sum_{j \geq t} \sum_{s^j \succeq s^t} Q(s^t, s^j) e^i \left(y_j\right).$$

Employing the above solvency constraints, we can translate the sequential trading problem into the time 0 problem:

$$\max_{\{c^i\}} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t | s_0) u\left[c^i(s^t)\right],$$

$$\text{s.t. } \sum_{t \geq 0} \sum_{s^t \in S^t} Q(s_0, s^t)\left[c^i(s^t) - e^i(y_t)\right] \leq w_0^i,$$

$$\sum_{j \geq t} \sum_{s^j \succeq s^t} Q(s^t, s^j) c^i \left(s^j\right) \geq \sum_{j \geq t} \sum_{s^j \succeq s^t} Q(s^t, s^j) e^i \left(y_j\right), \ \forall s^t \in S^t, \ t \geq 0.$$

If a country is in default at a certain state, then the last constraint (solvency constraint) is binding.

The dual problem to the above time 0 problem, that is, the cost minimization problem

[6] See Proposition 3.1 in Lustig (2007).

to attain lifetime utility v_0^i in time 0, is characterized as follows:

$$C^i(s_0) = \inf_{\{c^i\}} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} Q(s_0, s^t) c^i(s^t), \tag{7}$$

$$\text{s.t.} \sum_{t \geq 0} \sum_{s^t \in S^t} \beta^t \pi(s^t|s_0) u\left[c^i(s^t)\right] = v_0^i, \tag{8}$$

$$\sum_{j \geq t} \sum_{s^j \succeq s^t} Q(s^t, s^j) c^i\left(s^j\right) \geq \sum_{j \geq t} \sum_{s^j \succeq s^t} Q(s^t, s^j) e^i(y_j), \ \forall s^t \in S^t, \ t \geq 0. \tag{9}$$

The second equation is called a *promise-keeping constraint* in the sense that the optimal solution allows a consumer to attain at least lifetime utility v_0^i. As mentioned before, v_0^i can summarize the history of realized states, and economize the space of state variables.

2.2.2 Time-varying Negishi weights

In the above cost minimization problem, the Lagrange multiplier μ_0^i is assigned to the promise-keeping condition (8), while the multipliers $\tau^i(s^t)$ are associated with the solvency constraints (9) state by state. The multiplier $\tau^i(s^t)$ may be either zero or positive depending on whether a solvency constraint is binding. Using these multipliers, we rewrite the cost minimization problem (7) as:

$$C^i(s_0) = \inf_{\{c^i\}} \left\{ \begin{array}{c} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} Q(s_0, s^t) c^i(s^t) \\ +\mu_0^i \left[v_0^i - \sum_{t \geq 0} \sum_{s^t \in S^t} \beta^t \pi(s^t|s_0) u\left[c^i(s^t)\right]\right] \\ +\sum_{t=0}^{\infty} \sum_{s^t \in S^t} Q(s_0, s^t) \left[\tau^i(s^t) \left[\sum_{j \geq t} \sum_{s^j \succeq s^t} Q(s^t, s^j) e^i(y_j) - \sum_{j \geq t} \sum_{s^j \succeq s^t} Q(s^t, s^j) c^i\left(s^j\right)\right]\right] \end{array} \right\}.$$

Exploiting a technique presented by Marcet and Marimon (1999),[7] we define the cumulative multiplier $\chi^i(s^t)$ as $\chi^i(s^t) \equiv \chi^i(s^{t-1}) - \tau^i(s^t)$ given $\chi_{-1}^i = 1$,[8] and further rewrite the above cost minimization problem as:

$$C^i(s_0) = \inf_{\{c^i\}} \left\{ \begin{array}{c} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} Q(s_0, s^t) \left[\chi^i(s^t) c^i(s^t) - \tau^i(s^t) \sum_{j \geq t} \sum_{s^j \succeq s^t} Q(s^t, s^j) e^i(y_j)\right] \\ +\mu_0^i \left[v_0^i - \sum_{t \geq 0} \sum_{s^t \in S^t} \beta^t \pi(s^t|s_0) u\left[c^i(s^t)\right]\right] \end{array} \right\}.$$

[7] Marcet and Marimon (1992) used the same technique. Messner and Pavoni (2004) presented some cases in which Marcet and Marimon (1999) may not work.

[8] If Lucas trees are equally endowed in time 0, then $\chi_0^1 = \chi_0^2 = 1$.

The first order condition with respect to $c^i(s^t)$ in the above cost minimization problem is

$$\frac{\mu_0^i}{\chi^i(s^t)} u' \left[c^i(s_t)\right] = \frac{Q(s_0, s^t)}{\beta^t \pi(s^t|s_0)}.$$

Because the right hand side of the above equation is independent of i, we have

$$\frac{\mu_0^1}{\chi^1(s^t)} u' \left[c^1(s_t)\right] = \frac{\mu_0^2}{\chi^2(s^t)} u' \left[c^2(s_t)\right],$$

or

$$\zeta^1(s^t) u' \left[c^1(s_t)\right] = \zeta^2(s^t) u' \left[c^2(s_t)\right], \tag{10}$$

where $\zeta^i(s^t)$ is defined as $\frac{\mu_0^i}{\chi^i(s^t)}$.

In a representative agent framework (a planner's problem), Negishi weights are assigned to each lifetime utility, and correspond to the ratio of period marginal utility between the two agents (countries in our context). Thus, as equation (10) implies, $\zeta^1(s^t)$ and $\zeta^2(s^t)$ can be used as Negishi weights. As Lustig (2007) demonstrates, the time 0 planner's objective (a representative agent model) is formulated as:

$$\max_{\{c^1, c^2\}} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t|s_0) \left[\zeta^1(s^t) u \left[c^1(s^t)\right] + \zeta^2(s^t) u \left[c^2(s^t)\right]\right].$$

Since a period preference is $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, $\left[\frac{c^2(s^t)}{c^1(s^t)}\right]^\gamma = \frac{\zeta^2(s^t)}{\zeta^1(s^t)}$ holds. Given that $h(s^t) \equiv \zeta^1(s^t)^{\frac{1}{\gamma}} + \zeta^2(s^t)^{\frac{1}{\gamma}}$, the consumption of country i is derived as $c^i(s^t) = \frac{\zeta^i(s^t)^{\frac{1}{\gamma}}}{h(s^t)} e(z_t)$. In addition, $\omega^i\left(s^t\right) \equiv \frac{\zeta^i(s^t)^{\frac{1}{\gamma}}}{h(s^t)}$ corresponds to the consumption share of each country.

Without any solvency constraint, Negishi weights are constant over time. Accordingly, the cross-country consumption share does not change dynamically at all. With solvency constraints, however, the consumption share between the two countries may fluctuate. The Negishi weight $\zeta^i(s^t)$ is constant unless country i is in default at state s^t, but otherwise, it is revised upward as a result of positive $\tau^i(s^t)$ (the multiplier associated with a solvency constraint). Therefore, the consumption share of country i increases when country i is in default at state s^t. One country subject to a solvency constraint at state s^t cannot transfer resources from state s^t to any state which is realized earlier. Consequently, the consumption share of the corresponding country at state s^t becomes large relative to the share of a previous state where solvency constraint is not binding. The country in constraint yields higher consumption growth toward a state in which a solvency constraint is binding.

Here, $g(s^{t+1})$ denotes the growth of $h(s^t) = \zeta^1(s^t)^{\frac{1}{\gamma}} + \zeta^2(s^t)^{\frac{1}{\gamma}}$ (the sum of nonlinearly transformed Negishi weights) from state s^t to state s^{t+1}, or

$$g(s^{t+1}) \equiv \frac{h(s^{t+1})}{h(s^t)}.$$

By construction, $g(s^{t+1})$ is one or higher. A higher $g(s^{t+1})$ implies that either of the two countries face severer solvency constraints between time s^t and time s^{t+1}. Lustig (2007) called $g(s^{t+1})$ *liquidity shocks.*

As demonstrated by Lustig (2007),[9] thanks to a complete markets setup, a stochastic discount factor between state s and state s' can be defined as a function of the aggregate endowment and the above liquidity shock, or

$$\pi(s'|s)\left(\frac{e(z')}{e(z)}\right)^{-\gamma} g(s'|s)^{\gamma}. \tag{11}$$

Without any solvency constraint ($g(s'|s) = 1$), a stochastic discount factor reduces to a standard one or $\pi(s'|s)\left(\frac{e(z')}{e(z)}\right)^{-\gamma}$.

2.3 Numerical procedures

The construction of a representative agent model with time-varying Negishi weights helps to substantially simplify the numerical computation procedure. In particular, once the revision rule of Negishi weights is established, it is possible to compute a stochastic discount factor between state s^t and state s_0 ($Q(s_0, s^t)$) by equation (11). Then, we can pin down the equilibrium path of the consumption share of each country and asset pricing without solving any individual optimization problem including optimal portfolio problems.

Thus, the derivation of the revision rule of Negishi weights plays an essential role in

[9] In Lustig (2007), when the aggregate endowment declines (that is, $\left(\frac{e(z')}{e(z)}\right)^{-\gamma}$ is larger), more consumers face solvency constraints as a result of more volatile idiosyncratic shocks (that is, $g(s'|s)^{\gamma}$ is larger). Accordingly, stochastic discount factors tend to correlated heavily negatively with dividends on Lucas trees in a future recession state; this is a source of a larger risk premium in his model.

the numerical procedure. While the appendix reviews the numerical method in detail, a key idea is conceptually simple. To begin with, we compute the consumption share that satisfies a solvency constraint or equation (9) for every one-period ahead state s' for country i, denoted by $\underline{\omega}^i(s')$. If the current consumption share $\omega^i(s)$ is smaller than $\underline{\omega}^i(s')$, then a solvency constraint is regarded as binding at state s', and the Negishi weight for a constrained country is revised upward from state s onto state s'. More concretely, if $\omega^i(s) < \underline{\omega}^i(s')$ at state s' for country i, then the Negishi weight of country i is revised upward as $\zeta^i(s') = \left[\underline{\omega}^i(s')h(s')\right]^{\gamma}$, where $h(s') \equiv \zeta^1(s')^{\frac{1}{\gamma}} + \zeta^2(s')^{\frac{1}{\gamma}}$.[10]

2.4 Derivation of portfolio positions

In standard representative agent models, optimal portfolio problems are implicit in solving equilibrium paths, and are often considered as trivial issues. A major reason for this is that a portfolio problem is reduced to a simple allocation of market portfolios and non-contingent bonds when any constraint other than resource constraints is absent. As mentioned in the previous subsection, portfolio problems are also implicit in solving our planner's problem. However, they are potentially important when solvency constraints are present, because there may emerge complicated financial transactions between a country damaged by catastrophic shocks and a nondamaged country.

Thanks to a two-country setup, it is possible to recover the portfolio positions of country 1 and country 2 as follows. From the budget constraint (3), we obtain the following system of equations to determine portfolio rules together with the market clearing conditions (1) and (2):

$$c^1(s^t)+p(z^t)\theta^1(s^t)+\sum_{s'\in S} q(s^t,s')a^1(s^t,s') = e^1(y_t)+\left[p(z^t)+d(z_t)\right]\theta^1(s^{t-1})+a^1(s^{t-1},s_t),$$

and

$$c^2(s^t)+p(z^t)\theta^2(s^t)+\sum_{s'\in S} q(s^t,s')a^2(s^t,s') = e^2(y_t)+\left[p(z^t)+d(z_t)\right]\theta^2(s^{t-1})+a^2(s^{t-1},s_t).$$

Note that both consumption and asset prices are standardized by the total endowment.

[10] In the numerical procedure described in the appendix, we use as Negishi weights $\omega^i(s)$ instead of $\zeta^i(s)$ after all variables are standardized by the total world endowment $e(z_t)$.

It is possible to identify from simulation results which state and which country faces a solvency constraint. These identified facts simplify the above system of equations. When a solvency constraint is binding on country 1 in state s' in time t, country 1 repays up to:

$$-a^1(s^{t-1}, s') = \left[p(z^t) + d(z_t)\right] \theta^1(s^{t-1}).$$

When a solvency constraint is binding on country 2 in state s' in time t, country 1 is repaid by:

$$a^1(s^{t-1}, s') = \left[p(z^t) + d(z_t)\right] (1 - \theta^1(s^{t-1})).$$

After simplifying the system, we approximate portfolio rules by $\theta^i(s^t) = \nu_0^i + \nu^i c^i(s^t)$ and $a^i(s^t, s') = \alpha_0^i + \alpha^i c^i(s^t)$. Given the simulated series of asset prices and consumption shares, we identify the values of ν_0^i, ν^i, α_0^i, and α^i that minimize the sum of squared residuals of the above system for a certain range of $c^i(s^t)$. In so doing, we classify current states (time t states) into three possible states, including (1) neither country 1 nor country 2 receives adverse shocks, (2) only country 1 receives shocks, and (3) only country 2 receives shocks.

We may have a special and convenient case in which as of time $t-1$, either country would be subject to solvency constraints in any possible state of a one-period ahead period (time t). In this case, binding solvency constraints can identify portfolio positions precisely, and we can obtain exact positions without using any approximation. Indeed, the calibration results presented in Section 3 do not require using any approximation.

3 Calibration Exercises

3.1 Setup

This section explores numerically how country-specific catastrophic shocks are shared between the two countries, and to what extent solvency constraints matter in sharing those shocks *ex ante* and *ex post*. As described in the previous section, we consider *transitory* or *persistent* country-specific catastrophic shocks on the labor endowment; we introduce such shocks with the limited number of states z. For a technical reason, we exclude any *permanent* catastrophic shock.[11]

We determine the size of country-specific catastrophic shocks following the existing empirical literature. Using US data for the period between 1869 and 1985, Cecchetti, Lam, and Mark (1990) identify catastrophic shocks on GDP. In their estimation, total annual output declines by 15.1% in the catastrophic regime, while it grows by 2.5% in the normal regime. The normal regime moves to the catastrophic regime with probability of 1.8% per year. Once the economy enters the catastrophic state, the state repeats itself with probability 51.0%.

On the other hand, Barro (2006) argues that the annual probability of catastrophic states is around 1.7%, and that the loss amounts to 15% through 64% of total output through intensively collecting data of developed and developing countries. These papers find that such catastrophic shocks permanently reduce the level of national output. But, we refer to their findings in determining the size of transitory or persistent catastrophic shocks relative to national outputs.

In addition, we set the degree of persistence of catastrophic states at extremely high levels partly to capture the fact that some types of catastrophic shocks have permanent effects on national output as discussed above, and partly to consider the fact that catastrophically damaged countries often experienced eventual recoveries, as discussed in Gourio (2008).

Following the above findings, we assume that the labor endowment of a country declines by 20% or 40% with probability 1.8% per year. Without the realization of catastrophic shocks, the labor endowment remains at a given level. When we consider the case of extremely persistent catastrophic shocks, we assume that a catastrophic state repeats itself with probability 80%, once a catastrophic state hits one country. That is, a catastrophic state continues for five years ($\frac{1}{1-0.8}$) on average. A catastrophic shock is assumed to be country-specific and uncorrelated between the two countries.

We treat cases where solvency constraints are severely binding by making the ratio of dividends to the world labor endowment (α) rather low. In time 0, both labor endowment and Lucas trees are equally distributed between the two countries. The rate of time

[11] It is possible to introduce permanent shocks with the small number of states for the endowment process of each country, but the number of states that is required to characterize total world outputs or endowment shares between the two countries becomes extremely large.

preference is 5% ($\beta = 0.95$), and the degree of relative risk aversion is five ($\gamma = 5$).

When calibration results are reported below, all variables except for portfolio positions such as $a^i(s, s')$ and $\theta^i(s)$ are standardized by the total world endowment $e(z_t)$. Thus, what is implied by 'share' is the ratio relative to the total endowment.

3.2 Calibration results

3.2.1 Purely transitory case

Almost perfect insurance outcomes We first consider a case with purely transitory catastrophic shocks to understand how risk-sharing works between two countries in the presence of collateral (solvency) constraints. More concretely, a country-specific catastrophic shock reduces labor endowment by 20% with probability 1.8% per year, but without any persistence. That is, the labor endowment share of a damaged country declines from 0.5 to $\frac{1-0.2}{1+(1-0.2)}(1-\alpha)$ upon the realization of catastrophic shocks. When α is close to zero, the labor endowment share declines by about 5.6% ($= 0.5 - \frac{1-0.2}{1+(1-0.2)}$) due to a catastrophic event.

One of the most important observations about this case is that catastrophic shocks are insured almost perfectly between the two countries. With extremely low α ($= 0.1\%$), Figure 1 plots the consumption share between a nondamaged country (country 1) and a damaged country (country 2); a catastrophic state takes place only in time 0. As demonstrated by Figure 1, the consumption share of the damaged country declines only by about 0.2% in time 0, although his labor endowment share declines by 5.6%. It implies that the damaged country can finance 5.4% out of 5.6% losses immediately after a catastrophic event. Even in a long term, the damaged country suffers from only 0.1% permanent losses unless another catastrophic shock hits this country.

However, almost perfect insurance outcomes do not necessarily imply that collateral constraints do not matter in the above case. Due to extremely low α ($= 0.1\%$), the solvency of country 1 as an insurer is indeed crucially limited as follows. Note that all asset prices are standardized by the total endowment. As equation (4) implies, country 1 (insurer) can offer insurance payments to country 2 (insured) only up to $(p(s^0) + \alpha)\theta^1(s^{-1})$ where $\theta^1(s^{-1}) = 0.5$ in time 0. In equilibrium, $(p(s^0) + \alpha) \times 0.5$ amounts to only 0.7% of the total world endowment.

Accordingly, the catastrophe insurance payment from country 1 to country 2 (0.7%) is far short of the catastrophic loss borne by country 2 (5.6%). In other words, *ex ante* insurance arrangements between the two countries are not responsible for almost perfect insurance outcomes. It thus follows that in the constrained competitive equilibrium, most of country 2's uncovered losses are financed from country 1 through *ex post* arrangements.

In which condition can the damaged country finance? Let us first explore in which condition the damaged country can finance a positive amount of resources from the nondamaged country in time 0 without violating solvency constraints at any one-period ahead state. As shown below, the most essential condition is that the price of Lucas trees is slightly expensive relative to the price dictated by arbitrage conditions upon the realization of a catastrophic shock in time 0:

$$p(z^0) = \sum_{s^1 \succeq s^0} q(s^0, s^1) \left[p(z^1) + d(z_1)\right] + \epsilon, \tag{12}$$

where ϵ is positive.

As mentioned in Section 2.1, the above arbitrage opportunity may not be exploited completely in the presence of solvency constraints. More concretely, the damaged country cannot make enough short positions in Lucas trees due to binding solvency constraints. On the other hand, the nondamaged country that intends to spread relative gains over time is not interested in making short positions in Lucas trees at all. Accordingly, the above arbitrage condition (12) may not be exploited to a full extent as of time 0.

How much the damaged country (country 2) can finance by making simultaneously short positions in Lucas trees ($\theta^2(s^0) < 0$) and long positions in relatively safe bonds ($a^2(s^0, s^1) > 0$) is equal to:

$$-p(z^0)\theta^2(s^0) - \sum_{s^1 \succeq s^0} q(s^0, s^1) a^2(s^0, s^1).$$

Substituting equation (12) to the above equation, we obtain:

$$-p(z^0)\theta^2(s^0) - \sum_{s^1 \succeq s^0} q(s^0, s^1) a^2(s^0, s^1) = -\epsilon\theta^2(s^0) - \sum_{s^1 \succeq s^0} q(s^0, s^1) \left[a^2(s^0, s^1) + \left[p(z^1) + d(z_1)\right] \theta^2(s^0)\right]. \quad (13)$$

As long as solvency constraints are satisfied as of time 0, $\left[a^2(s^0, s^1) + \left[p(z^1) + d(z_1)\right] \theta^2(s^0)\right]$ is non-negative for any state in time 1 (s^1). More concretely, as described in footnote 13, $\left[a^2(s^0, s^1) + \left[p(z^1) + d(z_1)\right] \theta^2(s^0)\right]$ is equal to zero for three cases (out of four) where solvency constraints are binding. Thus, the second term of the right hand side of equation (13) is negative, but quite close to zero. If the time-0 price of Lucas trees ($p(z^0)$) is slightly high relative to arbitrage pricing,[12] then there is still a chance for the left hand side of equation (13) to be positive.

What we below demonstrate by simulation is the cases where the equilibrium price of Lucas trees is slightly expensive relative to arbitrage pricing; then, the damaged country effectively finances resources from the nondamaged country. In those cases, the damaged country utilizes the arbitrage opportunity (comparably expensive Lucas trees) to some extent by building up short positions in Lucas trees until solvency constraints are binding.

Expansion of financial balance sheets in two countries Table 1 reports financial transactions between the two countries when a catastrophe hits country 2 in time 0 without any further catastrophe in either country in the case of $\alpha = 0.1\%$. As mentioned above, in the aftermath (time 0), the damaged country (country 2) receives only 0.7% of the total endowment through insurance payments from the nondamaged country (country 1).

There are two means by which country 2 can cover uninsured losses in time 0. First, the damaged country self-finances losses by savings in Lucas trees; the gross return from previously invested Lucas trees ($(p(s^0) + \alpha)\theta^2(s^{-1})$ where $\theta^2(s^{-1}) = 0.5$) amounts to 0.7% of the total endowment in time 0.

Second, more importantly, the damaged country finances uncovered losses by ag-

[12] If ϵ is greater than $-\frac{1}{\theta^2(s^0)} \sum_{s^1 \succeq s^0} q(s^0, s^1) \left[a^2(s^0, s^1) + \left[p(z^1) + d(z_1)\right] \theta^2(s^0)\right]$, then the left hand side of equation (13) turns out to be positive.

gressively expanding his financial balance sheet. With collateral constraints satisfied for every possible future state, country 2 can finance 3.9% of the total endowment by building intensively long positions in contingent-claim portfolios (3.6%) and short positions in Lucas trees (−7.5%) in time 0. Together with insurance receipts (0.7%) and savings in Lucas trees (0.7%), the damaged country can mitigate endowment losses substantially from 5.6% to 0.2% by such *ex post* financing from country 1. Note that because any delivery of Lucas trees is not required in this economy, making short positions in Lucas trees can be regarded as issuing risky bonds whose repayments are proportional to the price of Lucas trees.

On the opposite side, the nondamaged country also expands aggressively his financial balance sheet in time 0. That is, country 1 builds large long positions in Lucas trees (8.8%) and short positions in contingent-claim portfolios (−3.6%), while he satisfies collateral constraints for every possible future state.[13]

Relatively safe assets as collateral As shown above, the contingent contract issued by the nondamaged country (country 1) to the damaged country (country 2) serves as effective collateral for country 2 to finance uninsured losses from country 1. Let us take a close look at the payoff structure of the one-period contingent contract as collateral assets. As shown in Table 2, the contingent contract issued by country 1 has a time-1 claim on 11.8% of the total world endowment in a state without any catastrophic event (Case 1), 9.5% in a case where a catastrophic shock hits on country 2 (Case 2), 8.0% in a case where a catastrophic shock hits on country 1 (Case 3), and 5.1% in a case where catastrophic shocks hit on both countries (Case 4).

This type of contingent contract cannot be regarded as catastrophe insurance because country 1 is obliged to make positive payments in any state of the next period (time

[13]For example, the portfolios that are constructed by both countries in time 0 can satisfy a collateral constraint in each state of time 1 as follows. As to country 2, the time-1 receipt from long positions in contingent bonds ($a^2(s^0, s^1)$) is 11.8% of the total world endowment in a case where no catastrophic shock is realized in either country, 9.5% in a case where a catastrophic shock hits on country 2, 8.0% in a case where a catastrophic shock hits on country 1, 5.1% in a case where catastrophic shocks hit on both countries. On the other hand, the time-1 repayment on short positions in Lucas trees ($-(p(s^1)+\alpha)\theta^2(s^0)$) is 11.8%, 8.0%, 8.0%, and 5.1% respectively. Then, for country 2, the receipt dominates the repayment in any state of time 1. Similarly, as to country 1, the time-1 receipt from long positions in Lucas trees ($p(s^1+\alpha)\theta^1(s^0)$) is 13.9%, 9.5%, 9.5%, and 6.1% respectively. On the other hand, the time-1 repayment on short positions in contingent bonds (-$a^1(s^0, s^1)$) is 11.8%, 9.5%, 8.0%, and 5.1% respectively. Again, for country 1, the receipt dominates the repayment at any state of time 1.

1). In particular, country 1 is forced to make positive payments even if he faces a catastrophic shock in time 1 (Cases 3 and 4). Thus, it may be instead considered as *relatively safe bonds* in the sense that it has larger claims than Lucas trees in Case 2. With such relatively stable payoffs, country 2 can finance enough resources against sufficient receipts from the contingent bonds at any future state. Consequently, the bond issued by country 1 can serve as high quality collateral on behalf of country 2 in the aftermath.

Closing expanded financial balance sheets As reported in Table 1, from time 1 onward, short positions in both Lucas trees disappear completely in terms of international capital flows. In other words, the creation of relatively safe bonds as high quality collateral emerges only immediately after a catastrophic event. The net value of contingent-claim portfolios ($\sum q(s, s')a^i(s, s')$) is close to zero in both countries from time 1 on. In addition, as shown in Table 2, contingent contracts between two countries turn back to conventional insurance contracts with limited coverage. That is, a damaged country receives insurance benefits, though insufficient, from a nondamaged country.

On the other hand, the number of invested Lucas trees ($\theta^i(s^t)$) is between 0 and 1 for both countries during the corresponding period. However, the allocation of Lucas trees tilts heavily to the nondamaged country (country 1); the share of Lucas trees owned by country 1 increases from 50% in time -1 to more than 90% from time 1 on. As a consequence of the above financial transactions, the damaged country is subject to only 0.1% permanent losses.

Note that even the nondamaged country benefits from the above cross-border transactions. By standing on the opposite side of the damaged country, the nondamaged country can smooth consumption by spreading a temporary relative gain (5.6% of the total endowment) in time 0 over time. Alternatively stating, thanks to the above transactions involved with uneven allocation of Lucas trees, the nondamaged country is able to enjoy 0.1% permanent increases in consumption shares.[14]

[14]The nondamaged country can enjoy permanently benefits at an annuity rate equal to 1.8% ($\approx \frac{0.1\%}{5.6\%}$). Since the implied annuity rate is lower than the average equilibrium risk-free rate (5.058%), the consumption smoothing is not achieved to a full extent from the viewpoint of the nondamaged country.

Summary We again ask how the two countries with expanded financial balance sheets can achieve almost perfect insurance outcomes without violating severe collateral constraints. On the one hand, the nondamaged country finances loans to the damaged country by issuing relatively safe bonds (contingent bonds) to the damaged country. On the other hand, with relatively stable payoffs, the bonds issued by the nondamaged country in turn serve as high quality collateral on which the damaged country can issue risky bonds (short positions in Lucas trees) to finance uncovered losses.

The most essential factor that underlies the above financial risk-sharing mechanism is that because of binding solvency constraints, the equilibrium price of risky bonds (Lucas trees) can be slightly expensive relative to arbitrage pricing. Thus, the damaged country is able to utilize such arbitrage opportunities to some extent, thereby successfully financing positive amounts of resources even in the aftermath. In this way, the expansion of financial balance sheets of both countries helps the damaged country to finance *ex post* most of uncovered losses in spite of the presence of severe collateral constraints. At the same time, such cross-border transactions assist the nondamaged country to smooth consumption by spreading temporary relative gains over time. Note that in both countries, the expansion of financial balance sheets is cleared immediately in time 0.

Because such financial transactions between the two countries work to achieve almost perfect insurance outcomes under severe collateral constraints, the asset pricing behavior of the collateral-constrained economy is quite similar to that of the full insurance economy. Table 3 reports the average equity prices and equity premiums for various dividend-to-endowment ratios (α); all pricing variables represent the ratio relative to the total world endowment. For example, when α is 0.1%, the average equity premium (0.994%) is much closer to the perfect insurance premium that emerges when collateral constraints are absent (0.937%) than to the closed economy premium that emerges when cross-border risk-sharing is absent (4.981%). When α is greater than 0.5%, collateral constraints are not binding any longer, and the economy reduces to the full insurance equilibrium.[15]

[15] When α is 0.5%, the equity valuation $((p(s^0) + 0.005) \times 1)$ is equal to 6.3% of the total world endowment (see Table 3), and it can cover completely country 2's losses in labor endowment shares (5.6%). Accordingly, a solvency constraint is no longer binding in *ex ante* arrangements.

3.2.2 Extremely large and persistent case

One-time occurrence of a catastrophic event We next move to a case with extremely large and persistent catastrophic shocks. That is, a country-specific catastrophic state reduces labor endowment by 40% with probability 1.8% per year, while it repeats itself with probability 80%. When α is close to zero, the labor endowment share of the damaged country declines by 12.5% from 50% to 37.5% ($= \frac{1-0.4}{1+(1-0.4)}$) upon the occurrence of a catastrophic event.

As demonstrated below, such huge losses in labor endowment cannot be covered by the insurance transfer from the nondamaged country to the damaged country under severe collateral constraints (extremely small α). Nevertheless, the same *ex post* financing mechanism as in the purely transitory case works effectively to achieve almost perfect insurance outcomes even in this case. Table 4 reports financial transactions between the two countries when a catastrophe hits country 2 in time 0 without any further catastrophe in either country in the case of $\alpha = 0.2\%$. Figure 2 plots the time series of the consumption share in the corresponding case.

In the aftermath (time 0), the damaged country receives only 4.6% of the total endowment as insurance payments from the nondamaged country.[16] However, the damaged country finances 1.5% by expanding his financial balance sheet; country 2 builds intensively long positions in contingent-claim portfolios (46.5%) and short positions in Lucas trees (-48.0%) in time 0. Together with the insurance payment (4.6%) and the gross return from previously invested Lucas trees (4.6%), the damaged country mitigates current endowment losses substantially from 12.5% to 1.9%. At the same time, the nondamaged country expands his financial balance sheet; country 1 builds large net financial positions (10.4%) by having long positions in Lucas trees (56.9%) and short positions in contingent-claim portfolios (-46.5%).

Again, let us take a close look at the payoff structure of the one-period contingent bonds as collateral assets. As shown in Table 5, the contingent bonds issued by country 1 have time-1 claims on 86.5% of the total world endowment in a state without any catastrophic event (Case 1), 54.2% in a case where a catastrophic shock hits on country

[16] The equity valuation is much larger in the case with persistent shocks than in the case with purely transitory shocks as a result of stronger precautionary demand for Lucas trees.

2 (Case 2), 45.7% in a case where a catastrophic shock hits on country 1 (Case 3), and 21.3% in a case where catastrophic shocks hit on both countries (Case 4). As in the previous case, country 1 is obliged to make positive payments in any state of the next period, though the payoff of the contingent contract carries a little less stable proportion to the total endowment than in the previous case. Thus, it may be still regarded as relatively safe bonds in the sense that it has larger claims than Lucas trees in Case 2. Also in this case, such relatively safe bonds serve as high quality collateral for country 2 to finance uncovered losses in the aftermath.

As in the purely transitory case, from time 1 onward, short positions in both Lucas trees disappear completely in terms of international capital flows. Again, the creation of relatively safe bonds as high quality collateral emerges only immediately after a catastrophic event. More precisely, the net value of contingent-claim portfolios is close to zero in both countries from time 1 on. In addition, contingent contracts between two countries become standard insurance contracts with limited coverage. On the other hand, the number of invested Lucas trees ($\theta^i(s^t)$) is between 0 and 1 for both countries during the corresponding period. However, the allocation of Lucas trees tilts heavily to the nondamaged country (country 1); the share of Lucas tress owned by country 1 increases from 50% in time -1 to around 90% from time 1 on. Consequently, the damaged country is subject to only 1.4% permanent losses.

By standing on the opposite side of the damaged country, the nondamaged country can smooth consumption by spreading relative gains over time. More concretely, the nondamaged country enjoys 1.4% increases in consumption shares forever.[17] Consequently, such intensive cross-border transactions help to restore almost perfect insurance outcomes without violating the collateral constraints imposed on either country.

As reported in Table 6, the average equity premium is quite close to the perfect insurance premium because almost full insurance is achieved through the above *ex post*

[17] When catastrophic shocks are persistent, the relative gain acquired by the nondamaged country reflects not only an increase in current labor endowment shares (12.5%), but also a substantial improvement in lifetime income shares. When $\alpha = 0.2\%$, the average equity premium is 7.657% per year. If the risk premium on labor endowment is equal to the equity premium, then the expected gain in lifetime income shares approximately equals $\sum_{t=0}^{\infty}\left[(0.8\,(1+0.07657))^t \times 12.5\%\right] = 90.1\%$. Accordingly, the nondamaged country receives permanently benefits at an annuity rate equal to 1.6% ($\approx \frac{1.4\%}{90.1\%}$). Unlike in the purely transitory case, the implied annuity rate is higher than the average equilibrium risk-free rate (-3.549%).

financial transactions between the two countries. For example, with $\alpha = 0.2\%$, the average equity premium (7.657%) is much closer to the perfect insurance premium (7.397%) than to the closed economy premium (25.076%). When $\alpha = 7.4\%$, the aftermath equity valuation (82.366%) is large enough to cover the required insurance transfer between the two countries,[18] and the average equity premium is accordingly exactly equal to the perfect insurance premium.

Consecutive and alternate occurrence of catastrophic events To demonstrate that the above risk-sharing mechanism is fairly powerful, we present two more examples with extremely large and persistent shocks; one case where a catastrophic state hits on country 2 consecutively for the expected length of time (five periods, $\frac{1}{1-0.8}$) (Table 7 and Figure 3), and the other case where a catastrophic shock in turn hits country 1 in time 1 (Table 8 and Figure 4).

In the former case, the maximum loss of the consumption share of the damaged country is at most 3.2% in time 5 (much lower than 12.5%), while the permanent loss falls to 1.5%. As Table 7 illustrates, the large long (short) position in Lucas trees held by the nondamaged (damaged) country facilitates a large-scale transfer between the two countries, as long as catastrophic shocks hit on country 2; the damaged country finances uncovered losses by building up large-scale long positions in contingent bonds and short positions in Lucas trees; $\theta^2(s^t)$ is negative from time 0 to time 4. In other words, relatively safe bonds are again created as high quality collateral endogenously while catastrophic events take place consecutively.

In the latter case, on the other hand, the newly damaged country (country 1) suffers from 1.1% permanent loss in its consumption share. There, country 1 instead finances uncovered losses by borrowing resources from country 2 on the relatively safe bonds issued by country 1 in time 1. On the opposite side, the previously damaged country (country 2) finances reconstruction loans to country 1 by issuing relatively safe bonds in time 1, and enjoys a long-run benefit from consumption smoothing from time 1 onward.

[18] When catastrophic shocks are persistent, the damaged country has to finance not only current losses, but also a substantial decline in lifetime income shares. As described in footnote 17, the expected loss in lifetime income shares approximately equals around 90%. Hence, the equity valuation (82.4% with $\alpha = 0.5\%$) needs to be comparable to that magnitude to make solvency constraints free in *ex ante* arrangements.

Also in this case, contingent contracts between two countries revert to usual insurance contracts from time 2 on.

4 Conclusion

In this paper, we have examined how catastrophic shocks, possibly persistent, are shared between two countries in the presence of solvency constraints. Given severe collateral constraints, the solvency of an insurer country is extremely limited, and realized catastrophic shocks cannot be covered fully by *ex ante* arrangements such as catastrophe insurance. However, the damaged country can finance *ex post* most uninsured shocks by intensive cross-border financial transactions without violating severe collateral constraints. At the same time, the nondamaged country can smooth consumption by spreading a temporary gain over time.

A key element is that high quality collateral is created endogenously through the simultaneous expansion of financial balance sheets of both countries. More concretely, the nondamaged country finances risky loans (long positions in Lucas trees without any delivery) by issuing relatively safe bonds (short positions in contingent-claim portfolios) to the damaged country. With relatively stable payoffs, the bonds issued by the nondamaged country in turn serve as good collateral assets on which the damaged country can issue risky bonds (short positions in Lucas trees without any delivery). Thanks to the above bilateral transactions, the damaged country can finance effectively uncovered losses from the nondamaged country.

The most essential factor that underlies the above financial risk-sharing mechanism is that because of binding solvency constraints, the equilibrium price of risky bonds (Lucas trees) can be slightly expensive relative to arbitrage pricing upon the realization of catastrophic shocks. Thus, the damaged country is able to utilize such arbitrage opportunities to some extent, thereby successfully financing positive amounts of resources even in the aftermath. In this way, the simultaneous expansion of financial balance sheets in both countries would emerge as a powerful risk-sharing scheme for catastrophic shocks. Such intensive cross-border transactions works effectively to achieve almost perfect insurance outcomes and reduce substantially equity premiums. Surprisingly, almost perfect risk-sharing still emerges even if one country faces severe catastrophic shocks in a consecutive manner. In any case, the expansion of financial balance sheets is closed immediately after catastrophic events go away.

Coming back to the policy question raised in the introduction, we finally inquire into which kinds of economic functions are carried out by public institutions, in particular central banks, in terms of collateral provision during financial crises. We may broadly interpret that in actual financial crises, a central bank assumes the role that is played by the nondamaged country in our model. A central bank generously lends his owned safe assets such as government bonds to private financial institutions in serious need of liquidity, while with fairly positive evaluation of risky bonds such as low-grade corporate bonds, he is willing to swap safe bonds at hand for risky bonds in possession of private agents. To facilitate such operations on a large scale, a central bank is required to expand his own financial balance sheets. Thus, such operations conducted by central banks may be broadly justifiable as long as these operations are closed immediately after financial crises go away.

Appendix: The numerical computation methods

As mentioned in Section 2, it is not possible to directly solve the sequential trading problem characterized by equation (6) because of the presence of solvency constraints. Following Lustig (2007), we instead solve the time 0 cost minimization problem dual to the utility maximization problem. We omit the time subscript t because the problem is formulated in a recursive manner.

We below standardize all endogenous variables except for asset volume by the total world endowment.[19] Accordingly, we transform the stochastic discount factors as follows:

$$\hat{\pi}\left(s'|s\right) = \frac{\pi(s'|s)\left(\frac{e(z')}{e(z)}\right)^{1-\gamma}}{\sum_{s'\in S}\pi(s'|s)\left(\frac{e(z')}{e(z)}\right)^{1-\gamma}}, \tag{14}$$

$$\hat{\beta}\left(s\right) = \beta\sum_{s'\in S}\pi(s'|s)\left(\frac{e(z')}{e(z)}\right)^{1-\gamma}. \tag{15}$$

[19] Alvarez and Jermann (2000) adopted the same transformation to make endogenous variables stationary in the case where the total endowment is growing.

We can use as a state variable the stationary consumption share $(\omega^i\left(s^t\right) = \frac{\zeta^i(s^t)^{\frac{1}{\gamma}}}{h(s^t)} \in [0,1])$ instead of the Negishi weight $\zeta^i(s^t)$. As the Negishi weight is revised upward upon default, the consumption share is revised upward based on a cutoff rule as described below.

There are two steps in finding the equilibrium pricing and allocation. Given the initially guessed liquidity shocks $g^{\text{guess}}(s'|s)$, the first step consists of solving the cost minimization problem given the sequence of prices, and of deriving optimal policy functions. In the second step, the sequence of consumption and asset pricing is computed from the simulation based on the derived policy functions; it is possible to map from liquidity shocks $g(s'|s)$ to stochastic discount factors $\beta\left(\frac{e(z')}{e(z)}\right)^{-\gamma} g(s'|s)^{\gamma}$, and to compute equilibrium asset pricing. We repeat this two-step procedure until the initially guessed liquidity shocks $g^{\text{guess}}(s'|s)$ coincide with the newly generated liquidity shocks $g^{\text{new}}(s'|s)$.

In solving the cost minimization problem, the current history is replaced by a truncated history z^k. Here, the control variable is not current consumption, but a consumption share ω^i in a detrended version of the cost function (7), and it is rewritten in a recursive manner:

$$\hat{C}(\omega^i(s), s, z^k) = \min_{\omega^i}\left[\omega^i + \hat{\beta}\left(s\right)\sum_{s'\in S}\hat{\pi}(s'|s)g(s'|s)^{\gamma}\hat{C}(\omega^{i\prime}, s', z^{k'})\right], \qquad (16)$$

where $\hat{\pi}(s'|s)$ and $\hat{\beta}\left(s\right)$ are defined in equations (14) and (15). Note that $\hat{\pi}(s'|s)g(s'|s)^{\gamma}$ in the cost function corresponds to a stochastic discount factor or a pricing kernel; as a result of detrending, $(\frac{e(z')}{e(z)})$ is always equal to one.

Similarly, a detrended version of the present value of the endowment sequence is written as follows:

$$\hat{C}^e(s, z^k) = \hat{e}^i(s) + \hat{\beta}\left(s\right)\sum_{s'\in S}\hat{\pi}(s'|s)g(s'|s)^{\gamma}\hat{C}^e(s', z^{h'}),$$

where $\hat{e}^i(s)$ is the share of individual labor endowment to the aggregate endowment. Because ω^i is bounded from below upon default, the lower bound of ω^i or $\underline{\omega}(s)$ is determined by:

$$\hat{C}(\underline{\omega}(s), s, z^k) = \hat{C}^e(s, z^k).$$

Lustig (2007) finds that $\omega^i(s')$ is bounded from $\underline{\omega}(s')$ as a result of binding solvency constraints, and constructs the following cutoff rule to revise a state variable ω^i upward: that is, if $\omega^i(s) > \underline{\omega}(s')$, then $\omega^i(s') = \frac{\omega^i(s)}{g(s'|s)}$, and if $\omega^i(s) \leq \underline{\omega}(s')$, then $\omega^i(s') = \frac{\underline{\omega}(s')}{g(s'|s)}$. Given the exogenous endowment process (s, z^k), the social planner solves the above cost minimization problem together with the cutoff rule by adjusting the current consumption share $\omega^i(s)$ and the share allowed in the next period $\omega^i(s')$.

Because equation (16) is a standard dynamic programming problem, we can solve it by a policy function iteration procedure. For this purpose, the cost function is approximated by a cubic spline interpolation with 100 grids for state $\omega^i \in [0, 1]$. In addition, it is assumed that $k = 3$ for the history parameter. It is possible to derive a policy function $\omega' = f(\omega, s, z^k)$ from the computed cost function. It is also possible to obtain the share of consumption of the two countries from the sequence of promised consumption shares ω^i.

In simulation, we first generate the sequence of aggregate and idiosyncratic shocks $\{s_t\}_{t=1}^{31,000}$ for 31,000 periods while omitting the initial 1000 periods. Given this generated sequence, we derive the sequence of consumption shares from the computed policy function, and then compute asset pricing and liquidity shocks. As mentioned above, we repeat this procedure until the generated liquidity shocks converge.

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Table 1: Portfolio transaction behavior with one-time shock (shock size: 20%, persistence: i.i.d., α=0.1%)

	labor endowment share	realized tree value $(p(z)+\alpha)\theta^i(s^{-1})$	insurance receipt $a^i(s^{-1},s)$	consumption share	invested trees $\theta^i(s)$	invested trees value $p(z)\theta^i(s)$	invested contingent claims $\sum q(s,s')a^i(s,s')$
			nondagmaged country				
time -1	0.500	0.011	0.000	0.500	0.500	0.010	0.000
time 0	0.556	0.007	-0.007	0.502	6.540	0.088	**-0.036**
time 1	0.500	0.139	-0.118	0.502	0.920	0.019	0.000
time 2	0.500	0.020	0.002	0.501	0.940	0.019	0.000
time 3	0.500	0.020	0.001	0.501	0.950	0.019	0.000
			damaged country				
time -1	0.500	0.011	0.000	0.500	0.500	0.010	0.000
time 0	0.444	0.007	0.007	0.498	**-5.540**	**-0.075**	0.036
time 1	0.500	-0.118	0.118	0.498	0.080	0.002	0.000
time 2	0.500	0.002	-0.002	0.499	0.060	0.001	0.000
time 3	0.500	0.001	-0.001	0.499	0.050	0.001	0.000

Note: All variables except for the number of labor endowment share and invested Lucas trees (θ^i) represent the ratio relative to the total world endowment. The labor endowment share in the second column represents the ratio relative to the total labor endowment.

Table 2: Receipts from or repayments on contingent contracts at maturity with one-time shock (shock size: 20%, persistence: i.i.d., α=0.1%)

	shares of invested trees $\theta^i(s^{-1})$	no cat. shock realized $a^i(s^{-1},s)$	cat. shock on country 2 $a^i(s^{-1},s)$	cat. shock on country 1 $a^i(s^{-1},s)$	cat. shock on countries 1 and 2 $a^i(s^{-1},s)$
			nondagmaged country		
time 0	0.500	0.000	-0.007	0.007	0.000
time 1	**6.540**	-0.118	-0.095	**-0.080**	**-0.051**
time 2	0.920	0.002	-0.013	0.001	0.001
time 3	0.940	0.001	-0.014	0.001	0.001
time 4	0.950	0.001	-0.014	0.001	0.000
time 5	0.960	0.001	-0.014	0.001	0.000
			dagmaged country		
time 0	0.500	0.000	0.007	-0.007	0.000
time 1	**-5.540**	0.118	0.095	**0.080**	**0.051**
time 2	0.080	-0.002	0.013	-0.001	-0.001
time 3	0.060	-0.001	0.014	-0.001	-0.001
time 4	0.050	-0.001	0.014	-0.001	0.000
time 5	0.040	-0.001	0.014	-0.001	0.000

Note: $a^i(s^{-1},s)$ is standardized by the total world endowment.

Table 3: Averages of equity prices and dividend ratios (shock size: 20%, persistence: i.i.d., unit: %)

	average equity price		average price-dividend ratio		average equity premium
dividend ratio (α)	normal state	shock state	normal state	shock state	
0.1	2.024	1.353	2024.3	1352.7	0.994
0.2	3.931	2.580	1965.7	1290.0	0.984
0.3	5.879	3.857	1959.8	1285.8	0.975
0.4	7.819	5.130	1954.8	1282.5	0.967
0.5	9.680	6.351	1936.1	1270.2	0.937
			perfect insurance equity premium		**0.937**
			closed economy equity premium		**4.981**

Note: All pricing variables represent the ratio relative to the total world endowment.

Table 4: Portfolio transaction behavior with one-time shock (shock size: 40%, persistence: 80%, α=0.2%)

	labor endowment share	realized tree value $(p(z)+\alpha)\theta^i(s^{-1})$	insurance receipt $a^i(s^{-1},s)$	consumption share	invested trees $\theta^i(s)$	invested trees value $p(z)\theta^i(s)$	invested contingent claims $\sum q(s,s')a^i(s,s')$
			nondagmaged country				
time -1	0.500	0.076	0.000	0.500	0.500	0.075	0.000
time 0	0.625	0.046	-0.046	0.519	6.390	0.569	**-0.465**
time 1	0.500	1.025	-0.865	0.516	0.890	0.141	0.003
time 2	0.500	0.135	0.017	0.514	0.900	0.135	0.002
			damaged country				
time -1	0.500	0.076	0.000	0.500	0.500	0.075	0.000
time 0	0.375	0.046	0.046	0.481	**-5.390**	**-0.480**	0.465
time 1	0.500	-0.865	0.865	0.484	0.110	0.017	-0.003
time 2	0.500	0.017	-0.017	0.486	0.100	0.015	-0.002

Note: All variables except for the number of labor endowment share and invested Lucas' trees (θ^i) represent the ratio relative to the total world endowment. The labor endowment share in the second column represents the ratio relative to the total labor endowment.

Table 5: Receipts from or repayments on contingent contracts at maturity with one-time shock (shock size: 40%, persistence: 80%, α=0.2%)

	shares of invested trees $\theta^i(s^{-1})$	no cat. shock realized $a^i(s^{-1},s)$	cat. shock on country 2 $a^i(s^{-1},s)$	cat. shock on country 1 $a^i(s^{-1},s)$	cat. shock on countries 1 and 2 $a^i(s^{-1},s)$
			nondagmaged country		
time 0	0.500	0.000	-0.046	0.046	0.000
time 1	**6.390**	-0.865	-0.542	**-0.457**	**-0.213**
time 2	0.890	0.017	-0.081	0.010	0.004
time 3	0.900	0.015	-0.082	0.009	0.004
			dagmaged country		
time 0	0.500	0.000	0.046	-0.046	0.000
time 1	**-5.390**	0.865	0.542	**0.457**	**0.213**
time 2	0.110	-0.017	0.081	-0.010	-0.004
time 3	0.100	-0.015	0.082	-0.009	-0.004

Note: $a^i(s^{-1},s)$ is standardized by the total world endowment.

Table 6: Averages of equity prices and dividend ratios (shock size: 40%, persistence: 80%, unit: %)

	average equity price		average price-dividend ratio		average equity premium
dividend ratio (α)	normal state	shock state	normal state	shock state	
0.2	14.962	8.904	7481.0	4452.0	7.657
1.0	47.147	26.465	4714.7	2646.5	7.826
2.0	62.651	31.982	3132.5	1599.1	8.134
3.0	85.117	42.552	2837.2	1418.4	8.068
4.0	107.270	53.210	2681.8	1330.3	7.916
5.0	126.450	62.205	2529.1	1244.1	7.740
6.0	147.410	72.314	2456.9	1205.2	7.633
7.0	162.130	78.781	2316.2	1125.4	7.433
7.4	169.760	82.366	2294.1	1113.1	7.397
			perfect insurance equity premium		**7.397**
			closed economy equity premium		**25.076**

Note: All pricing variables represent the ratio relative to the total world endowment.

Table 7: Portfolio transaction behavior with five consecutive shocks (shock size: 40%, persistence: 80%, α=0.2%)

	labor endowment share	realized tree value $(p(z)+\alpha)\theta^i(s^{-1})$	insurance receipt $a^i(s^{-1}, s)$	consumption share	invested trees $\theta^i(s)$	invested trees value $p(z)\theta^i(s)$	invested contingent claims $\sum q(s, s')a^i(s, s')$
			nondagmaged country				
time -1	0.500	0.076	0.000	0.500	0.500	0.075	0.000
time 0	0.625	0.046	-0.046	0.519	6.390	0.569	**-0.465**
time 1	0.625	0.542	-0.542	0.526	10.190	0.844	**-0.745**
time 2	0.625	0.864	-0.864	0.529	9.800	0.811	**-0.717**
time 3	0.625	0.831	-0.831	0.531	9.600	0.795	**-0.702**
time 4	0.625	0.814	-0.814	0.532	9.480	0.785	**-0.693**
time 5	0.500	1.521	-1.361	0.532	0.670	0.106	0.021
time 6	0.500	0.102	0.050	0.522	0.780	0.117	0.012
time 7	0.500	0.118	0.033	0.517	0.850	0.127	0.006
time 8	0.500	0.129	0.023	0.515	0.890	0.133	0.003
			dagmaged country				
time -1	0.500	0.076	0.000	0.500	0.500	0.075	0.000
time 0	0.375	0.046	0.046	0.481	**-5.390**	**-0.480**	0.465
time 1	0.375	-0.457	0.542	0.474	**-9.190**	**-0.761**	0.745
time 2	0.375	-0.779	0.864	0.471	**-8.800**	**-0.728**	0.717
time 3	0.375	-0.746	0.831	0.469	**-8.600**	**-0.712**	0.702
time 4	0.375	-0.729	0.814	0.468	**-8.480**	**-0.702**	0.693
time 5	0.500	-1.361	1.361	0.468	0.330	0.052	-0.021
time 6	0.500	0.050	-0.050	0.478	0.220	0.033	-0.012
time 7	0.500	0.033	-0.033	0.483	0.150	0.022	-0.006
time 8	0.500	0.023	-0.023	0.485	0.110	0.016	-0.003

Note: All variables except for the number of labor endowment share and invested Lucas' trees (θ^i) represent the ratio relative to the total world endowment. The labor endowment share in the second column represents the ratio relative to the total labor endowment.

Table 8: Portfolio transaction behavior with alternate shocks (shock size: 40%, persistence: 80%, α=0.2%)

	labor endowment share	realized tree value $(p(z)+\alpha)\theta^i(s^{-1})$	insurance receipt $a^i(s^{-1}, s)$	consumption share	invested trees $\theta^i(s)$	invested trees value $p(z)\theta^i(s)$	invested contingent claims $\sum q(s,s')a^i(s,s')$
				initially nondagmaged country			
time -1	0.500	0.076	0.000	0.500	0.500	0.075	0.000
time 0	0.625	0.046	-0.046	0.519	6.390	0.569	**-0.465**
time 1	0.375	0.542	-0.457	0.489	**-10.160**	**-0.841**	0.811
time 2	0.500	-1.630	1.630	0.489	0.050	0.008	0.002
				initially damaged country			
time -1	0.500	0.076	0.000	0.500	0.500	0.075	0.000
time 0	0.375	0.046	0.046	0.481	**-5.390**	**-0.480**	0.465
time 1	0.625	-0.457	0.457	0.511	11.160	0.924	**-0.811**
time 2	0.500	1.791	-1.630	0.511	0.950	0.151	-0.002

Note: All variables except for the number of labor endowment share and invested Lucas' trees (θ^i) represent the ratio relative to the total world endowment. The labor endowment share in the second column represents the ratio relative to the total labor endowment.

Figure 1: Consumption shares with one-time shock (shock size: 20%, persistence: i.i.d., α=0.1%)

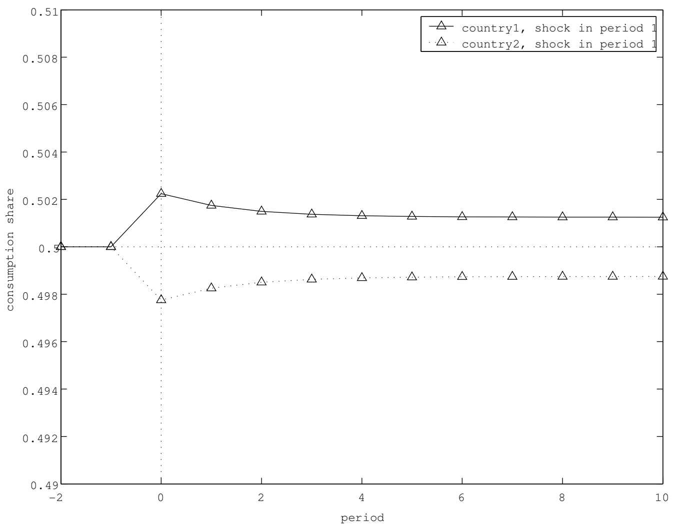

Figure 2: Consumption shares with one-time shock (shock size: 40%, persistence: 80%, α=0.2%)

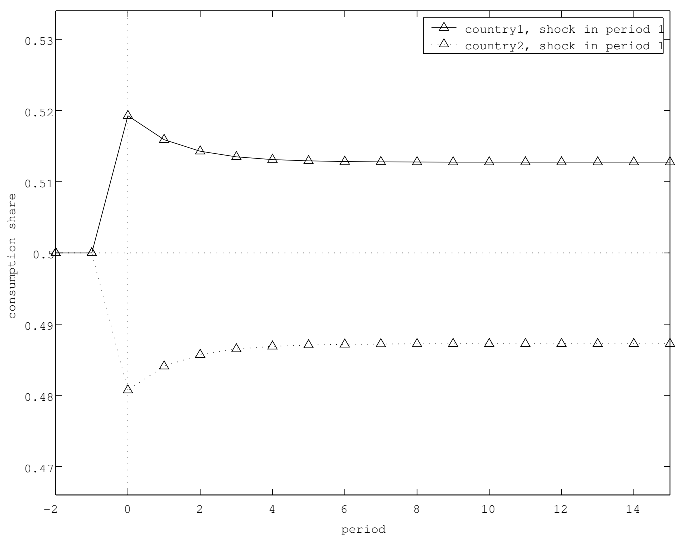

Figure 3: Consumption shares with five consecutive shocks (shock size: 40%, persistence: 80%, α=0.2%)

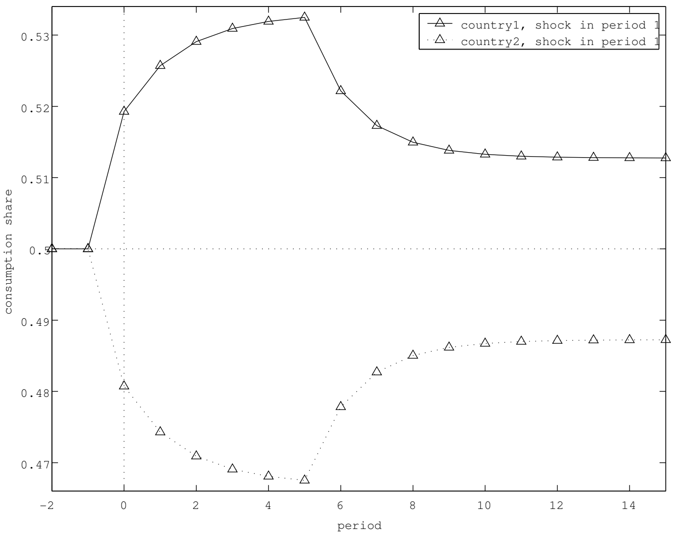

Figure 4: Consumption shares with alternate shocks (shock size: 40%, persistence: 80%, α=0.2%)