On the Quantitative Effects of Unconventional Monetary Policies*

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Abstract

This paper quantitatively evaluates the effects of several unconventional monetary policies. In particular, a new Keynesian model of a small-open economy is extended to include a liquidity premium, deviations from the UIP condition, and a premium in the term structure of interest rates; allowing the Central Bank to set, in addition to the policy rate, the stock of base money, debt of different maturities and foreign assets. The model is calibrated to the case of Chile. We find that policies affecting the liquidity channel can potentially have a big effect, but they depend on expectations about the future policy rate. On the other hand, alternatives working through the term premium have smaller effects, but they are less dependent on the expected path of the reference rate. We also study the possibility of undoing the unconventional policy as a possible exit strategy, with results indicating that this alternative may induce a significant slowdown, particularly if it is anticipated. Finally, we also consider the alternative of driving down and maintaining the policy rate to its lower bound. While this policy can also be greatly expansionary, credibility issues regarding the promise of keeping the rate low for some time can severely undermine these effects.

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1 Introduction

In normal times, many Central Banks use the nominal interest rate as the policy instrument to achieve their inflation goals. However, when the policy rate reaches its lower bound the monetary authority has to look for alternative (or unconventional) policies. What are the different tools that a Central Bank can use in such a situation? What are the quantitative effects of these policies? Although there is a significant branch of the literature that has evaluated the virtues of different options,[1] the analysis of the quantitative impact of different unconventional policies is much less frequent.[2] Moreover, while until recently the experience of Japan (having a near-zero interest rate since the mid 1990's) was almost the only case of study, nowadays the aggressive response of many Central Banks aimed to dampen the effects of the 2008-9 crisis –taking the interest rate to its lower bound– further emphasizes the need for a quantitative assessment of different policy alternatives.

This paper is an attempt in this direction, presenting a quantitative exploration of different alternatives based on a dynamic and stochastic general equilibrium model. In particular, we extend a simple new Keynesian model of a small open economy to include three frictions: a liquidity premium that introduces a gap between the policy rate and that of short-term debt, which depends on the velocity of base money; a portfolio-balance effect for the determination of the nominal exchange rate (implying deviations from the uncovered interest rate parity, UIP); and a premium in the term structure of interest rates that depends on the relative stock of short and long maturity of debt. These frictions, albeit introduced in an *ad-hoc* fashion, allow the Central Bank to choose not only the interest rate as its policy tool, but the composition of its balance sheet as well (the stock of money, short and long term debt and foreign assets).

We use this model to evaluate the dynamics generated by different changes in the Central Bank's balance sheet, taking into account the expected path for the policy rate, alternative "exit strategies", the possibility of (exogenous) lack of credibility in the policy announcements, and the role of the zero lower bound. The model is calibrated to the case of Chile, which is one of the countries that have driven the policy rate down to its lower bound during the 2008-9 crisis. In addition, from a methodological perspective, we present an algorithm to perform these type of exercises in the context of linear rational expectations models, which is based on the idea of backward induction.

The results appears to indicate that polices working through the liquidity channel can potentially have important expansionary effects; which are generated not by the direct effect that a monetary expansion have in reducing this premium, but rather through the expected future inflation that the increase in base money generates. However, these effects greatly depend on the perception about the future path of the interest rate and its credibility. On the other hand, policies aimed to reduce the term premium, while generating smaller expansions, are less dependent on the expected behavior of the policy rate and on credibility issues. The channel associated with the deviations from UIP seems to be of minor importance.

We also consider the possibility of undoing the original change in the balance sheet as a possible exit strategy. This exercise shows that reversing the unconventionally policy can generate important contractions, particularly if these changes are anticipated. Finally, we study the benefits of driving the policy rate down to its lower bound and maintaining it at that value for some time, finding that

[1] See, for instance, Eggertsson and Woodford (2003), Svensson (2001) and Bernanke and Reinhart (2004), among many others.

[2] One exception is the work of Coenen and Wieland (2003) who analyze several alternatives for the case of Japan.

this policy can by itself be highly expansionary, although the issues regarding the credibility of the announcement are also relevant in this case.

In terms of the related literature, several reasons explain the lack of studies quantifying the effect of unconventional policies. First, it is difficult to estimate the impact of these policies directly from the data. For, at least until recently, the lower bound episodes were almost exceptional, limiting the power of direct econometric inference. The alternative is to use a model to tackle these issues, which is the route taken in this paper. However, this is not a trivial task either. On one hand, allowing for the interest rate to have a lower limit entails to consider non-linearities in the model, which are generally difficult to handle computationally. On the other hand, even assuming that we can circumvent the computational issues, most models (particularly in the new-Keynesian tradition) are not rich enough to allow for the monetary authority to use several different instruments. For instance, the role of money and liquidity is generally absent in these model, as well as the role financial frictions.[3] In this work, we tackle the non-linearity by allowing the interest rate to be fixed at an arbitrary value (possibly zero) for some time as a way to capture deviations from the linear Taylor rule,[4] and we introduce frictions, although in an *ad-hoc* way, in order to consider different policy tools.[5]

Before turning to the analysis, a caveat is in order concerning the type of policies analyzed in this paper. The term unconventional policy has been used somehow vaguely, referring to a number of alternative tools that simply depart from the usual management of the monetary policy rate (e.g., in terms of a model, an unconventional policy could be any departure from the Taylor rule). One of such an alternative implemented during the recent crisis was the purchase by Central Banks of risky financial assets that were not part of the usual portfolio of these institutions, a type of policy that is generally labeled as credit policy. The goal of these alternatives was to try to reduce a financial distortion that have widen due to the drop in the price of assets used as collateral (e.g. mortgage-backed securities). Moreover, it is likely the case that these Central Banks would have implemented this same type of strategies regardless of the policy rate reaching its lower bound. Although these alternatives are relevant to discuss as well, we do not analyze them in this paper for at least two reasons. First, these are not the only type of policies that have been evaluated and/or implemented; for many countries did not have to deal with a situation of deep financial distress.[6] This paper is then more representative of the situation faced by countries like Chile, Sweden and Australia, among others, that drove the policy rate to its lower limit and had to look for other alternatives to stimulate the economy. Second, and related to a point previously highlighted, there is in the literature no clear modeling strategy that would allow us to consider these issues; although as we mentioned significant

[3]Some recent studies attempt to expand the basic framework in these lines; for instance, Curdia and Woodford (2009) and Gertler and Karadi (2009).

[4]Although we allow for non-linearities, we are still focusing on a local solution, as most studies in this topic do. A different branch of the literature characterizes global solutions, showing that important differences may arise; most remarkably, the possibility of converging to a liquidity trap (see, for instance, Benhabib et al., 2001, 2002). This alternative is however no explored in this paper.

[5]The fact that frictions are not micro-funded is clearly a drawback of the analysis. However, given that the literature has not yet arrived to a consensus on which are the relevant frictions or how to model them, this analysis can provide some insight on which are the relevant parts of the model (or "wedges") that should be the focus of this research agenda.

[6]For instance, Céspedes et al. (2009) compile a list of 56 policy announcements regarding unconventional policies, in the period from September 2008 to October 2009, for a group of 13 central banks the generally implement (explicit or implicitly) inflation targets, finding that only 12 of these announcements were related with assets purchase programs or direct lending to (non-banks) financial firms, which were implemented by four central banks (Canada, Switzerland, England and the U.S.).

progress is being made in this direction.

The rest of the paper is organized as follows. Section 2 presents the frictions introduced as well as the alternative policy tools that the Central Bank has available. We also show there the estimation exercise using Chilean data. In section 3 we present the quantitative results, where we analyze the role of different expected paths for the policy rate, lack of credibility, possible exit strategies and the role of the zero bound, as well some robustness exercises. Finally, section 4 concludes.

2 Model and Calibration

The structure of the model corresponds to a standard new Keynesian framework for a small open economy with incomplete asset markets, modified to introduce *ad-hoc* frictions (or wedges in the usual equilibrium conditions) that allow the central bank to decide on the different elements in its balance sheet, in addition to the policy rate. In particular, we consider a structure with households having separable preferences in consumption and leisure and that can transfer resources over time using an internationally traded bond, money and bonds of different maturities issued by the central bank. They consume a combination of domestic and foreign goods. There is also an infinite number of monopolistic intermediate good firms that use labor to produce, using a constant returns-to-scale technology, and that are subject to a Calvo-type problem in setting prices, with full indexation to past inflation. Finally, with the goal of simplifying the analysis, we abstract from capital as a productive input (and therefore investment), as well as from fiscal policy.

In this section we first describe the optimality conditions associated with households assets decisions in the standard model and then describe the modifications we introduce.[7] Then, we characterize the central bank and the several policy alternatives we consider. Finally, we discuss the estimation of the parameters describing the frictions we introduce. The appendix contains a description of the other equilibrium conditions of the model.

2.1 Assets and Monetary Policy in the Standard Model

We begin by describing the budget constrain and the optimality conditions in the frictionless model to see how they reduce the central bank policy alternatives to only one instrument (generally, the interest rate associated with short term bonds). Consider the following budget constrain faced by households,

$$P_t C_t + \frac{S_t B_t^*}{i_t^* cp_t} + \frac{D_{1,t}}{i_{1,t}} + \frac{D_{2,t}}{i_{2,t}} + \frac{M_t}{i_t^M} \leq W_t L_t + S_t B_{t-1}^* + D_{1,t-1} + \frac{D_{2,t-1}}{i_{1,t}} + M_{t-1} + T_t.$$

Here, C_t denotes consumption, S_t is the nominal exchange rate, P_t is the price level, W_t stands for nominal wages, L_t are hours worked and T_t are lump-sum transfers from the central bank. In terms of assets, households have access to an internationally traded bond B_t^* that has a price given $1/(i_t^* cp_t)$ (where i_t^* is the world interest rate and cp_t is the country premium that "closes" this small open economy model), bonds issued by the central bank ($D_{k,t}$ is a bond of maturity k with rate $i_{k,t}$),[8] and

[7] As we mentioned in the introduction, these modification are introduced *ad-hoc*, with no micro-foundations. Nevertheless, we discuss alternative mechanisms (introduced in the related literature) that may produce such departures from the standard model.

[8] We only consider two period bonds here to ease the notation, while in the estimation we allow for maturities in line with those available in our empirical case, Chile.

money, M_t, which has an acquisition price equal to $1/i_t^M$ (we discuss the role of this rate below).

In the frictionless model, the first order conditions associated with asset decisions can be written, after a log linear approximation, as[9]

$$\hat{\lambda}_t = E_t\{\hat{\lambda}_{t+1}\} + \hat{i}_{1,t} - E_t\{\hat{\pi}_{t+1}\}, \quad (1)$$

$$\hat{i}_{2,t} = \hat{i}_{1,t} + E_t\{\hat{i}_{1,t+1}\}, \quad (2)$$

$$\hat{i}_{1,t} = \hat{i}_t^* + \hat{cp}_t + E_t\{\hat{s}_{t+1}\} - \hat{s}_t, \quad (3)$$

$$\hat{i}_{1,t} - \hat{i}_t^M = -\varphi_m(\hat{m}_t - \hat{p}_t - \hat{y}_t). \quad (4)$$

where λ_t is the marginal utility of consumption and the parameter φ_m will depend on the specific assumption about the role of money.[10]

In the usual treatment of this model, it is also assumed that $i_t^M = 1$ and that the policy rate, i_t^{MP}, equals the short term rate $i_{1,t}$. Therefore, in the standard model there is only one monetary policy instrument available (usually i_t^{MP}, which is determined by a Taylor-type rule) and the different elements in the central banks balance sheet are irrelevant for the determination of other asset prices: the demand for real balances (4) determines the stock of money for a given i_t^{MP}, the expectations hypothesis (2) determines interest rates at different horizons, and equation (3) (the uncovered interest parity, UIP) pins down the exchange rate. To overcome this impossibility we redefine the monetary policy rate and introduce wedges in these familiar conditions, in such a way that the stock of different assets will show up in these conditions, as we next show.

2.2 Modifying the Standard Framework

The first departure from the standard framework is to modify the assumption $i_t^M = 1$ and to consider this to be the policy rate (i.e. $i_t^{MP} = i_t^M$). The motivation behind this choice is that usually the target for the policy rate it is not that of a short term bond but, instead, that of a monetary market, such as the overnight or the repo rate. These are the type of markets that central banks actually use to introduce money into the economy trough open market operations and, although in those markets treasuries and/or central bank bonds are generally exchanged for money, the interest rate charged for these operations do not need to (and generally they actually do not) coincide with that of these assets, for the later may generally include a liquidity premium.[11]

We are not the first, however, to use this type of assumption. Reynard and Schabert (2009) introduce a framework explicitly modeling the market for money where only short term bonds are eligible for money acquisitions. In their framework monetary policy targets the rate prevailing in that market, which is generally different from that of other assets in the economy. Sims (2009) also consider a model with two policy rates, although his interpretation for the rate i_t^M is rather different;

[9] We use small-caps hatted variables to denote log-deviations from the steady state.

[10] Here we are implicitly assuming that, in the absence of price stickiness, money is neutral; as it is generally assumed by the "cashless limit" modeling strategy. Additionally, it is worth mentioning that in a fully micro-funded model (particularly in one of a small open economy) consumption should appear in equation (4) instead of output. However, because in our estimation we use monthly data to overcome a short available sample we use output because we do not have available a monthly consumption series but we do have a monthly series for aggregate economic activity. Finally, we are implicitly assuming that money demand has a unitary elasticity with respect to output.

[11] Such a premium does not need to be high (as it doesn't seems to be in the data) for this mechanism to work, but as long as it exist this mechanism will generate the results presented below.

for he considers this to be the interest rate paid on reserves as a way to capture the policy recently implemented in the U.S. by the Fed.

This assumption introduces an additional degree of freedom for monetary policy, which can now be implemented by choosing two instruments, in our case i_t^M and the stock of money M_t, leaving the short term rate $i_{1,t}$ to be determined by equation (4). From a technical point of view, this assumption has also be considered in some textbook treatments such as Woodford (2003, Ch. 2). In particular, Woodford shows (in the context of a flexible price model) that a monetary policy specified by exogenous paths for both i_t^M and M_t will deliver a determinate equilibrium.[12] This results is relevant because, although we will consider a familiar Taylor-type rule for i_t^M satisfying the Taylor principle as the "normal times" policy rule (see below), we can also assume that i_t^M is fixed at a given value forever without facing indeterminacy issues; a feature that we will exploit to consider unconventional monetary policies.

From a qualitative point of view, given this friction, a permanent increase in the stock of M_t will have, *ceteris paribus*, two effects. On one hand, the reduction of $i_{1,t}$ will increase desired consumption. On the other hand, the rise in M_t will increase the price level which, in the presence of price rigidities, will generate expectations of future inflation, reducing the real interest rate and further expanding consumption demand.

The second difference with the standard framework is that we also consider deviations from the uncovered interest rate parity (UIP) such that,

$$\hat{s}_t - E_t\{\hat{s}_{t+1}\} - \hat{i}_t^* - \hat{cp}_t + \hat{i}_{1,t} = \varphi_s(\Delta \hat{b}_t^{*BC}), \tag{5}$$

where Δ denotes the first difference operator and B_t^{*BC} denote the holdings of of foreign assets by the central bank (described below). In this way, changes in the central banks holdings of foreign assets will have a direct effect (*ceteris paribus*) on the nominal exchange rate.[13]

Finally, we modify equations (1) and (2) to consider a term premium in the yield curve structure that affect intertemporal allocations. In particular

$$\hat{\lambda}_t = E_t\{\hat{\lambda}_{t+1}\} + \hat{i}_{1,t} - E_t\{\hat{\pi}_{t+1}\} + \Delta \hat{tp}_t,$$

where

$$\hat{tp}_t \equiv \hat{i}_{k,t} - \hat{i}_{1,t} - E_t\{\hat{i}_{1,t+1}\} = \varphi_k \left(\hat{d}_{2,t} - \hat{d}_{1,t}\right). \tag{6}$$

Therefore, different combinations of long and short term debt will have first order effects in consumption decisions.[14] In particular, either increases in $D_{1,t}$ or drops in $D_{2,t}$ will shift consumption demand upwards.[15]

[12]See Proposition 2.11. This result is also present in the model of Reynard and Schabert (2009), which additionally features price rigidities.

[13]A similar deviation from UIP gap can be obtained in a model with costly adjustment of the international portfolio. See, for instance, Sierra (2008).

[14]In principle, we could consider the possibility of $\Delta \hat{tp}_t$ entering the Euler equation with a coefficient different than one. However, to estimate such a coefficient will be difficult, for there is no monthly data on consumption. Nevertheless, in the robustness section we check the robustness with respect to the parameter φ_k which can be seen also as a robustness check in terms of modifying this assumption.

[15]A similar modification of the Euler equation and the expectation hypothesis can be obtained, for instance, in line with Andres et al. (2004), by assuming imperfect assets substitution between long and short term bonds.

2.3 Monetary Policy

In each period t, the Central Bank (CB) decides on four types of assets: base money (M_t), foreign reserves (B_t^{*CB}), and domestic debt of both short and long maturity ($D_{1,t}^{CB}$ and $D_{2,t}^{CB}$).[16] Additionally, it choses the monetary policy rate i_t^{MP} and lump-sum transfers to households (T_t).

The central bank resource constrain in any given period is given by,[17]

$$\frac{S_t B_t^{CB*}}{i_t^* cp_t} + D_{1,t-1}^{CB} + \frac{D_{2,t-1}^{CB}}{i_{1,t}} + M_{t-1} = S_t B_{t-1}^{CB*} + \frac{D_{1,t}^{CB}}{i_{1,t}} + \frac{D_{2,t}^{CB}}{i_{2,t}} + \frac{M_t}{i_t^M} + T_t.$$

It is further assumed that the central bank uses transfers in such a way to rebate any capital gain/loss to households,[18] implying that the bank's total net worth is kept constant in nominal terms, i.e.

$$(S_t B_t^{*CB} - S_{t-1} B_{t-1}^{*CB}) = (M_t - M_{t-1}) + (D_{1,t}^{CB} - D_{1,t-1}^{CB}) + (D_{2,t}^{CB} - D_{2,t-1}^{CB}). \tag{7}$$

In normal times, policy is conducted through a familiar Taylor rule,

$$\hat{i}_t^{MP} = \rho_i \hat{i}_{t-1}^{MP} + (1-\rho_i)(\alpha_\pi \hat{\pi}_t + \alpha_y \hat{y}_t). \tag{8}$$

On the other hand, the unconventional policies that we will consider have two components: first, the policy rate is fixed at some value (either its steady state level or zero) for some time and, second, the CB changes the position of an asset in its balance sheet, compensated by a change in another asset such that the bank's total net worth is kept constant in nominal terms (i.e. equation (7) holds).

2.4 Estimating the Frictions

Given the frictions previously introduced, different changes in the CB balance sheet may have different expansionary effects. To quantify these effects we need to assign values to the parameter describing these frictions (φ_m, φ_s and φ_m) which we will obtain by estimating equations (4) to (6) using monthly Chilean data from 01-2003 to 05-2009. Here we just report the results regarding these parameters, while the Appendix includes details about the estimation and discusses some data limitation issues.[19]

In term of equation (4), we obtain that the relationship between the difference of the short-term rate, $i_{1,t}$, and the policy rate, i_t^{MP}, and velocity is,[20]

$$\hat{i}_{1,t} - \hat{i}_t^{MP} = \underset{(0.0007)}{-0.0014}(\hat{m}_t - \hat{p}_t - \hat{y}_t).$$

For equation (5), the deviations from the standard UIP is related with the stock of central bank's

[16]That a CB can issue its own debt is not a commonly observed characteristic, but it is the case for Chile. More generally, thinking of a CB that does not share this characteristic, we can interpret $D_{1,t}^{CB}$ and $D_{k,t}^{CB}$ as the negative of the CB holdings of treasury bonds.

[17]In the absence of the frictions previously introduced this equation is irrelevant in equilibrium.

[18]Reynard and Schabert (2009), in a closed economy model, make a similar assumption. In principle, this is not the only possibility that one can consider. But allowing for a change in the CB net worth would require to set a rule for the evolution of this variable, and it is not clear how this rule should be specified (nor there is guidance in the literature on these issues). This exercise, on the other hand, is simpler and will allow us to obtain a cleaner intuition.

[19]The calibration of the other parameters in the model is also described in the Appendix.

[20]Newey-West robust standard errors in parenthesis.

foreign assets B_t^{*CB} as,

$$\hat{s}_t - E_t\{\hat{s}_{t+1}\} - \hat{i}_t^* - cp_t + \hat{i}_{1,t} = \underset{(0.036)}{0.2047}(\Delta \hat{b}_t^{*CB}).$$

Finally, in terms of equation (6) relating deviation from the expectations hypothesis with the difference of the stock of bonds at different maturities, $\hat{d}_{k,t}^{CB} - \hat{d}_{1,t}^{CB}$, we obtained,

$$\hat{i}_{k,t} - \sum_{j=0}^{k-1} E_t\{\hat{i}_{1,t+j}\} = \underset{(0.001)}{0.017} \left(\hat{d}_{k,t}^{CB} - \hat{d}_{1,t}^{CB} \right).$$

Finally, it is relevant to highlight that these equations are not meant to represent a full theory explaining these gaps observed in the data (e.g. not all deviations from UIP are due to exchange rate interventions). Actually, the share of the variance of the left-hand-side variables explained by the proposed regressors is generally low. Nevertheless, the fact that the coefficients displayed the correct sign and appear to be statistically different from zero suggests that some explanatory power can be attributed to the proposed regressors.

3 The Effect of Unconventional Policies

We proceed with the numerical exercises in four steps. First, we study the effects of a given change in the CB balance sheet assuming that i_t^{MP} remains fixed in its steady state value forever. Although this is clearly not a realistic assumption, it will be useful to grasp the intuition behind the dynamics implied by the different alternatives. Second, we analyze what happens if the CB promises to keep i_t^{MP} fixed at its steady state value only for a limited period of time, using the Taylor rule afterwards. Additionally, we allow people to assign an exogenous probability to the event that the CB returns to the Taylor rule before it was promised. Third, we consider the possibility of undoing the unconventional policy (either by a surprise or with an anticipated announcement) as a possible exit strategy. Finally, we evaluate the virtues of driving the policy rate down to zero and maintaining it there for some time as another policy alternative.

To keep the analysis as clean as possible we will not assume that the economy is hit by a negative shock that motives the use of an unconventional policy, neither we study what is the optimal response to such a shock (as analyzed, for instance, by Eggertsson and Woodford, 2003, in the context of a much simpler model).[21] Our goal is somehow simpler: to understand what is the effect of a given policy regardless of the shock that is hitting the economy. While the issue of the optimal response to a recessionary shock is clearly a relevant one, we will proceed in this way so we can better understand the different channels through which the unconventional policy alternatives affect the economy. Additionally, if we were to use a shock to represent, for instance, the 2008 crisis, it is not clear either which type of shocks should be considered.

[21] However, the algorithm presented in the appendix can be accommodated to consider the type of shocks analyzed in Eggertsson and Woodford (2003).

3.1 Fixing i_t^{MP} Forever

The exercise we consider first is to fix i_t^{MP} at its steady state value forever and to permanently change two components of the CB balance sheet in such a way that equation (7) is satisfied. In particular, we assume that the change in the balance sheet is equivalent to 10% of nominal GDP in steady state.

Figure 1 shows the effect of a purchase of foreign assets financed with an increase in the stock of money. This policy generates an increase of 6.1% on inflation and a rise on GDP of 3.7% on impact, and these possitive effects last for several quarters. The fact that the response of inflation lies always above zero indicates, in line with the discussion before, that the price level experiences a permanent change in the long run.[22] This change does not fully materialize in the first quarter due to the price rigidities, generating, in particular, an increase in expected inflation. This rise in expectations reduces the ex-ante real interest rate, $r_t \equiv i_{1,t} - E_t\pi_{t+1}$, which has an expansionary effect. In principle, this rate is also reduced by the drop in $i_{1,t}$. However, as can be seen in the figure, $i_{1,t}$ decreases by little compared with the increase in $E_t\pi_{t+1}$.

This alternative also has an effect through the exchange rate channel in equation (5). To see how important this mechanism is, the dashed-dotted line in figure 1 displays the same response assuming $\varphi_s = 0$. As can be seen, the responses are quite similar compared to the benchmark case, which further emphasizes the role of the increase in expected inflation as the relevant channel.

Monetary expansions can also be used to purchase either type of CB debt. In this case, the channel in equation (5) will play no role, but we may have an additional expansionary effect if long debt is acquired, for it reduces the term premium in equation (6). Figure 2 displays the dynamics under both alternatives. We can observe effects on impact on GDP and inflation similar to the previous exercise, with a slightly bigger effect in the case of a purchase of $D_{k,t}^{CB}$. There is a somehow smaller response on the real exchange rate in this case (the impact response in the previous case was a real depreciation of near 5.3%, while in this case it is around 4.7%), but in general the dynamics are similar indicating that this alternative also creates an expansionary effect trough the rise of inflation expectations.

In terms of policies that do not involve monetary expansions, Figure 3 plots the responses to a purchase of long debt financed by new short debt, a policy that has a direct effect through the term premium channel in equation (6). This alternative generates a milder expansion of GDP and inflation on impact, 0.3% and 0.05% respectively. While the nominal exchange rate depreciates in this case, there is a real appreciation on impact which partially explains this smaller response. Moreover, given the shape of the inflation path, the real interest rate actually rise and therefore the expectations channel is not present in this case.

Finally, figure 4 shows the effect of a purchase of foreign assets financed with either long or short new debt. In this case, both the friction affecting the UIP condition, equation (5), and the yield curve, equation (6), are in place. However, we can see that the effect on output is only positive when the operation involves an increase in $D_{1,t}^{CB}$, which appears to indicate that the second channel is relatively more important. Additionally, while the increase in output is smaller compared with the case in figure 3, the response of inflation (at least on impact) brought about by the nominal depreciation is bigger in this case, although this change is reversed in the second quarter.

Overall, these exercises highlight that policies working through the liquidity channel may have significant effects, while other alternatives can also have expansionary effect but of a smaller magnitude.

[22] Actually, given that equation (4) depends on velocity, and given that the stock of money increases by 10% of GDP, the integral of the response of inflation should be 10%, which can be verified numerically.

However, the maintained assumption so far was that the monetary policy rate is expected to remain fixed forever. In what follows, we relax this important assumption.

3.2 A Temporary Fix of i_t^{MP}

We consider the following exercise. At time t_0, the CB announces that the policy rate is going to stay fixed until period $T-1$, and from T onwards it will be determined according to the Taylor Rule in equation (8). Additionally, it announces an unconventional policy, which also takes place at t_0 but that will never be reversed (an alternative that we consider in the next subsection). Everybody believes that, beginning on period T, the Taylor rule will be in place and that it will stay for any $t \geq T$. However, for $t \in [t_0, T-2]$, while the interest rate is fixed, people assign an exogenous probability sequence p_t to the CB breaking its promise and setting the nominal interest rate according to the Taylor rule staring on $t+1$. Moreover, if this is the case, they believe that the Taylor rule will stay forever. Finally, regardless of people not trusting the CB, we assume that (ex-post) it fulfills its promises and actually waits until T to use the Taylor rule. The Appendix presents an algorithm, based on the idea of backward induction applied to linear rational expectation models, that can be used to compute the equilibrium trajectories under these assumptions. The algorithm also constrains the equilibrium path to satisfy the non-negativity constrain on the policy rate.

We begin with the case a purchase of foreign assets financed by a monetary expansion. Figure 5 shows the dynamics for different T's, assuming full credibility (i.e. $p_t = 0$). As can be seen, allowing the Taylor rule to return after some time significantly dampens the expansionary effects of this alternative. For instance, if the policy rate is fixed for eight quarters, the impact response of output is near 0.9%, dropping to close to 0.2% for $T = 2$, while these figures for inflation are, respectively, close to 2% and 0.7%. These are significantly smaller than the rise of 3.7% on GDP and 6.1% of inflation that is generated if i_t^{MP} stays fixed *ad infinitum*.

In part, this smaller effect is due to the qualitatively different response of inflation: while before it used to lie completely above zero, now it increases on impact, decreases to a negative value in the second quarter and then it becomes positive and converges to zero from above.[23] This different path for inflation transaltes into a less expansionary path for the real interest rate, which now actually increases on impact and then becomes negative, converging to zero from below. Although the infinite sum of the real interest rate (i.e. the relevant measure for the determination of consumption according to the IS curve) is still negative (close to -0.6% when $T = 8$), it is significantly smaller that before (almost -4%) which explains why the response of output has dampen in this case.

However, this is only a part of the explanation, for the shape of the inflation path changes in this case because inflation is more affected by the dynamics of the nominal exchange rate than before, which explains why it decreases below zero in the second quarter.[24] In particular, if $\varphi_s = 0$ the response of inflation lies completely above zero. However, it is still the case that the convergence of the price level to its new steady state is much slower.

The difference in terms of the speed of convergence of the price level is clearly due to the expected

[23] The cumulative response of inflation (i.e. the price level) is the same in the long run in both cases (equal to 10% as we dicussed before). However, while in the case of i_t^{MP} fixed forever the convergence was after just a few periods, it takes now more than 3000 quarters to reach this value.

[24] Actually, as can be seen in Figure 1, the response of inflation also displayed a small negative bump in the second quarter even with the policy rate fixed forever. However, the response in inflation in that case was so large that this had a minor impact.

rise in the policy rate. In particular, although this increase is small (less than 3 basis point when $T = 2$), the path of the interest rate also lies above zero for a significant period of time (until the price level converges to its new steady state value).[25] Of course, this description cannot be interprated causally, for both inflation and interest rates are jointly determined in equilibrium. However, it is clear that the anticipation of the return to the Taylor rule significantly limits the expansionary effect that this policy alternative may have.

We consider next the effect of lack of credibility. Figure 6 shows the same unconventional policy, fixing $T = 6$, and setting $p_t = p^t$ for different values of p (this implies that, as times goes by and people see that the CB has actually kept its promise, the assigned probability of an anticipated return to the Taylor rule decreases). This lack of credibility also dampens the expansionary effect that this policy may have. In particular, for $p = 0$ the impact increase in GDP is close to 0.5%, but with $p = 0.9$ it drops to almost 0.3%. The response of inflacion is also milder the higher this probability is, although the difference are relatively smaller. Overall, the patterns are anyway similar to those described for the case of $p_t = 0$.[26]

We can also consider how are the results affected if the unconventional policy implemented is a purchase of long term debt financed with new short term debt. Figure 7 show the dynamics for different T's, keeping $p_t = 0$. Compared with the case of a monetary expansion, here the responses of output and inflation are not that sensitive to changes in T. Actually, the effects seem to be slightly bigger than the case of i_t^{MP} fixed forever, and the responses of both variables lie now completely above zero. This different shapes particularly generates a drop in real interest rate, and although the policy rate rises once the Taylor rule returns, the increase is quite small and the real rate remains below zero for several period.[27] Finally, we analyze the effect of this policy when people assigns a positive but decreasing probability to the early return of the Taylor rule, a case described in 8. It appears that the dynamics are not significantly affected either when we allow for p_t to vary.

3.3 Undoing the Unconventional Policy

Part of the current discussion about unconventional policies refers to possible "exit strategies", which is clearly a broad term. In principle, one can consider to return to the usual Taylor rule being, by itself, an exit strategy. However, the discussion appears to focus on undoing the unconventional policy initially implemented. For instance, one might be worried about the effects that the excess of liquidity introduced. As an example, we described before how a policy that entails increases in M can generate significant persistence in the inflation process once the Taylor rule is reintroduced. In what follows, we present several exercises that help to sharpen this discussion.[28]

A first alternative is to consider the same policies as before (i.e. the CB announces an unconventional policy and that i_t^{MP} will be fixed until $T - 1$, an the Taylor rule will be used for $t > T$), and to additionally assume that the CB also announces that in period $T - k$ (with $k \in [0, T - t_0 + 1]$)

[25] Taking together, while the infinite sum of the real interest rate is more negative if $\varphi_s = 0$ (close to -1.2% when $T = 8$), it is still significantly smaller than what it was in the case of the polict rate fixed forever.

[26] Other policies that entail an increase in M also show similar results as those obtained in this case.

[27] This actually suggest that, for this type of policy, the responses vary non-monotonically with changes in T. In the case of a monetary expansion, the bigger the T the bigger and closer to the case of i_t^{MP} fixed forever the responses were. In this case, on the contrary, there seems to be a value for T (in the numerical exercises it is close to 25) such that the responses are bigger at to that point, becoming closer to the case in Figure 3 afterwards.

[28] To simplify the exposition, we will just show the case of a purchase of foreign currency financed by a monetary expansion. The other alternatives yield qualitatively similar results.

the unconventional policy will be completely reversed (assuming, for the moment, full credibility).[29] For instance, if at t_0 the CB buys foreign assets using base money, at $T-k$ it sells the same amount of dollars in exchange for currency. This alternative is considered in figure 9, which displays the equilibrium paths for different values of k. To clarify the exercise, the figure also displays the paths for M_t and B_t^{*BC}. It is clear that the anticipation of the reversal further dampens the expansion, actually generating a recession in most cases. In particular, the smaller the k, the more negative the responses of output an inflation are. This is because the policy reversal is contractionary and the effect is partially mitigated if the nominal rate remains fixed for more time.

These responses can be rationalized by realizing that the mechanism generating the expansion with a policy that expands M (i.e. the expected rise in the price level) disappears when the reversal is anticipated. Figure 10 actually shows that gradually undoing the unconventional policy is even worst when the change is anticipated, while figure 11 illustrates the if people believes that the change can occur earlier the economy contracts even further.

We also explore the possibility of undoing the unconventional policy unexpectedly. Figure 12 displays the dynamics for different values of k, for the case of full reversal at $T-k$. Evidently, the impact response is the same we described in the previous subsection, and when the policy is undone we have a contraction, with output dropping slightly below zero at that time and inflation also becoming negative. What actually mitigates the negative effect on output is the fact that the shock is unanticipated: although inflation drops on impact at $T-k$, it is expected to be positive in the next period (partly due to the dynamics of the nominal exchange rate), making the real interest rate negative and generating an increase in consumption demand. However, it is relevant to highlight that when the Taylor rule returns, the policy rates displays a minor drop (of less than a basis point) because both output and inflation converge to zero from below slowly. Finally, the case of a gradual reversal starting at $T-k$ is considered in Figure 13. This reversal produces a bigger drop in output and inflation. However, inflation now converges from above zero and, consistently, the policy rate rises after period T.

3.4 Driving i_t^{MP} to the Lower Bound

So far we have assumed the policy rate remaining fixed at its steady state level for some time, as a way to complement the effects of changes in the CB balance sheet. However, we can alternatively consider the possibility of the CB driving the policy rate all the way down to its lower bound, which is also in line with the recent experience. For instance, Céspedes et al. (2009) document that, in a group of 20 countries with (implicit or explicit) inflation targets, 13 of them drove the policy rate down to a lower bound at some point after September 2008 and have maintained it there for an extended period of time. Additionally, while maintaining the rate at its lower bound can be seen as a complementary action to enhance the effects of movements in the balance sheet, driving the policy rate to zero and leaving it there for some time also implies a departure from the usual Taylor rule. Therefore, we can consider this action as an unconventional policy by itself, regardless of its potential role in complementing movements in the balance sheet.

Figure 14 shows the responses associated with a decrease (from its steady state value) of the policy rate down to zero, leaving it at that value for alternative horizons, and assuming that afterwards

[29] As shown in the Appendix, the algorithm can be generalized to consider this alternative.

this rate is determined by the Taylor rule.[30] As can be seen, this policy alternative have important expansionary effects: for instance, if the rate drops to zero and remains at this value for one quarter, output increases by near 5% while inflation rises by almost 7%. In addition, we can see that the key channel for this effect is the reduction in the *ex-ante* real interest rate. To put these numbers in context, in order to obtain a similar response (on impact) for both inflation and output with an expansionary monetary shock to the Taylor rule, we would require a disturbance generating a drop on impact in the policy rate to -17% (in annual terms).

It is relevant to mention at this point an important caveat, particularly in trying to interpret these numbers in the context of recent experiences. If we were to hit this economy with a negative shock in the spirit of current events, the final response will not be equivalent to simply add to the figure presented the impulse response generated by the negative shock; for the presence of the zero bound imposes non linearities in the solution. As we mentioned before, we are reluctant to consider such a shock because it is not clear which one should be considered, nor it is obvious how to calibrate its size. Nevertheless, it is still important to highlight that initial conditions might not only affect the size but the dynamics of the responses as well.

As we did before, we can also analyze the role of (exogenous) credibility in the announcement of the policy rate staying at zero for some time. Figure 15 displays the response for the case in which it is announced that the Taylor rule will return after six quarters, but with people assigning a probability $p_t = p^t$ to the anticipated return of the Taylor rule in $t+1$. In line with the results obtained previously, the effects of lack of credibility can be pervasive in this case as well; significantly dampening the expansionary effects brought about by the zero bound.

Finally, we could also consider the role of driving the policy rate down to zero in complementing unconventional policies that alter the CB balance sheet. However, given that the expansionary effects brought about by the drop in the interest rate are of an order of magnitude larger than what we have obtained in subsection 3.2, it can be verified that adding movements in the balance sheet generate only minor differences compared to the results in figures 14 and 15.

3.5 Robustness

As a final exercise, we check the robustness of the results with respect to some parameter values. In particular, we explore how the analysis presented in subsection 3.2 is affected by alternative values for the parameter describing the estimated frictions (plausible according to their estimated confidence bands), and also in terms of the coefficients in the Taylor rule that is assumed to follow after the temporary fix in i_t^{MP}.

Figure 16 shows the case of a purchase of foreign assets financed by a monetary expansion when the policy rate is fixed for 6 periods, presenting the case of the original value for φ_m (-0.0014), as well as two other alternatives representing plus an minus one standard error of the estimated parameter (-0.0007 and -0.0021). As can be seen, these differences in the coefficient affecting the liquidity channel significantly change the impact of this policy, particularly in terms of the responses of output and the real exchange rate and, to a less extent, in term of inflation and nominal depreciation.

On the other hand, figure 17 presents the alternative of a purchase of long debt financed with new short debt under different values of the parameter φ_k (0.017, the point estimate, as well as 0.019 and

[30] Notice that because the figure plots log-deviations from steady state, the lower bound in the graph corresponds to a value close to -1.6% (given that the steady state of this rate is, in annual, terms is 6.5%).

0.015, i.e. plus and minus two standard errors respectively). In this case, changes in this parameter within its confidence range do not significantly affect the original responses.

We also check the robustness regarding the parameters describing the Taylor that will be in place after the temporary fix in the policy rate. Because agents anticipate the contractionary policy that will follow once the Taylor rule returns, the expansion generated by the unconventional policy will be smaller the more aggressive the response is with respect to inflation and output deviations, and the less important the smoothing part of the rule is. Accordingly, figure 18 displays the result of a purchase of foreign assets financed by a monetary expansion when the policy rate is fixed for 6 periods, considering the original Taylor rule as well as three alternative coefficients: half the response to the previous interest rate, and double the reaction to either inflation or output. As can be seen, it appears that these alternative rules affect mainly the response of output and, to a less extent those of inflation and the real exchange rate.

Similarly, figure 19 shows the impact that considering the same alternative rules has on the evaluation of purchasing long debt financed with new short debt, also fixing the policy rate for 6 periods. In this case, the different specifications of the rule gave little impact on the responses of output and inflation, while affecting more the responses of the real exchange exchange rate and the *ex-ante* real interest rate.

Finally, figure 20 evaluates the role of the Taylor rules parameters on we consider, as in section 3.4, the experiment of driving the policy rate to zero for some time. For this case, what appears to generate significant difference is the smoothing coefficient, particularly for output and the real exchange rate.

4 Conclusions

This paper uses a new Keynesian model of a small-open economy to provide a quantitative evaluation of different policy alternatives that a CB can use when the nominal interest rate is fixed. In particular, we have analyzed the effects of different changes in the CB balance-sheet, such as movements in the holdings of foreign assets, base money and debt at different maturities. To allow for the possibility of the CB changing these assets, the standard model was extended to include several frictions; namely, a liquidity premium, deviations from the UIP condition, and a premium in the term structure of interest rates. Additionally, we introduced an algorithm that allows to take into account issues like the anticipation of future polices, the (exogenous) lack of credibility in the announcements and the role of the zero bound.

The results show that polices working through the liquidity channel can potentially have important expansionary effects, although its impact greatly depends on the perception about the future path of the interest rate and its credibility. On the other hand, policies aimed to reduce the term premium, while generating smaller expansions, are less dependent on the expected behavior of the policy rate and on credibility issues. Additionally, it seems that reversing the unconventionally policy as a possible exit strategy might not be beneficial, particularly if these changes are anticipated. Finally, we described how driving the policy rate down to its lower bound and maintaining it at that value for some time can be highly expansionary as well, although the issues regarding the credibility of the announcement are also relevant in this case.

It is useful to compare these results with those in Eggertsson and Woodford (2003). Using a simpler model (in particular, a closed economy), their analysis suggests that the channel that is relevant to escape from a zero-bound situation is to affect the expectations about future inflation. Moreover, they show that the optimal policy in such a situation is a price level target. In our case, although we are not characterizing the optimal policy, we have described that the potentially big effect generated by policies affecting the liquidity premium are due to the increase in the future price level brought about by the permanent increase in base money, resembling the channel emphasized by these authors. Additionally, they also highlight that the effectiveness of this policy depends on the credibility of the announcement.[31]

It is also worth to mention that, although in our set up is money what helps to reduce the liquidity premium, there is nothing exclusively special about money. Actually, in more general settings where other assets can affect this premium (for instance, it might be the case that short run debt provides liquidity as well), a permanent increase in such an asset can also generate the same effect, and its importance will also be subject to the same caveats about the expected path of interest rates and credibility that we have analyzed.

Additionally, it is worth mentioning that unconventional monetary strategies might not necessarily be the only (neither the most effective) policy alternative available to use in a zero bound situation. In particular, there are several fiscal tools that can be used as well. For instance, Christiano et al. (2009) describe that the government-expenditure multiplier is actually quite large when the nominal interest rate is at its lower bound. Additionally, Eggertsson (2006) highlights that government debt can be used as a commitment device to ameliorate the credibility issues associated with the optimal monetary policy. Our goal, however, was to analyze the virtues of these monetary alternatives, and we left the interaction with fiscal tools for future research.[32]

Finally, an important extension that is left for future research is to rank the different alternative from the point of view of optimal policy. However, as we have argued, for this kind of exercises is where having a more precise microfoundation of the different channels is most relevant, as well as it is to choose which shocks (and of which size) are the relevant to consider for this exercise.

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[31]Eggertsson (2006) formally discusses the issues of credibility arising from this optimal policy.

[32]Moreover, in terms of the country we choose for the application, Chile has a fiscal rule in place since 2001, limiting the fiscal deficits according to the revenues from copper exports. This characteristic places a bound to the type of fiscal tools than can be implemented; which further emphasizes the importance of analyzing the monetary alternatives.

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A Technical Appendix

A.1 The Lower Bound, Anticipating Policy Changes and Credibility in Linear Rational Expectation Models

Here we describe the algorithm to compute the policy exercises in the paper.[33] In a contemporaneous work, Bodenstein et al. (2009) present a similar algorithm, considering how to compute impulses responses when, given an initial shock, the interest rate reaches the lower bound endogenously through the Taylor rule. Here, we additionally consider the possibility that the central bank chooses (exogenously, as a policy) to fix the interest rate, deviating from the Taylor rule, to a given value (possibly zero) for a certain period of time. Moreover, we allow agents to assign an exogenous probability to the central bank breaking its promise an returning to the usual rule before it was announced. We also show how to compute the equilibrium trajectories when the unconventional policy is reversed, either surprisingly or anticipatedly.

The first exercise we want to consider is when the central bank announces, at time t_0, that the interest rate is going to stay at the lower bound until period $t_1 - 1$, and starting at t_1 the rate will be determined by the Taylor Rule. The Taylor rule in "normal times" is constrained by the lower bound, so that the policy rate (measured as deviations from steady state) is

$$i_t = max\{-i, i_t^{target}\}, \tag{9}$$

where i is the steady state value of the policy rate and the targeted policy rate is a linear function of any variable in the model (for instance, $i_t^{target} = \phi_i i_{t-1} + \phi_\pi \pi_t + \phi_y y_t$).[34] In particular, this implies that, even though the central bank can guarantee to keep the interest rate at the lower bound up to period $t_1 - 1$, the state of the economy could be such that the target rate is below $-i$ at t_1. Therefore, because the algorithm works with the idea of backward induction, we first describe how to compute the equilibrium for a given initial value of the state variables and the exogenous process when the policy rate is determined by the rule (9) (i.e. when the zero bound binds endogenously), and then we explore the solution for the period in which the government sets the policy rate at the lower bound (i.e. when the zero bound is imposed exogenously, as a policy choice).

At any point in time in which the $i_t = i_t^{target}$ endogenously, the conditions characterizing a linear rational expectations equilibrium can be written as

$$AE_t\{z_{t+1}\} = Bz_t + Cz_{t-1} + De_t, \tag{10}$$

where z_t is a vector collecting endogenous variables (including i_t and i_t^{target}), e_t are exogenous driving forces following the process $e_t = \rho e_{t-1} + u_t$, u_t is a vector of i.i.d random disturbances with zero mean, and A, B, C, D are conformable matrices describing the linearized equilibrium conditions. In particular, if n_1 denotes the position in the vector z_t of the variable i_t and n_2 is the analogous for the variable i_t^{target}, there is an equation j the system such that (using Matlab notation) $A(j,:) = C(j,:) = D(j,:) = 0$ while $B(j, n_1) = -1$, $B(j, n_2) = 1$ and $B(j, l) = 0$ for $l \neq \{n_1, n_2\}$. On the other hand, when $i_t = -i$,

[33] Although we focus on applying the technique for the case analyzed in the paper, notice that this method is quite general and can accommodate many examples in these lines.

[34] It is however assumed that the constrain doesn't bind at the steady state.

the equilibrium is characterized by the system

$$AE_t\{z_{t+1}\} = \tilde{B}z_t + Cz_{t-1} + De_t + c, \tag{11}$$

where $\tilde{B}$ differs from B only in that $\tilde{B}(j, n_2) = 0$ and c is a column vector full of zeros except in position j where $c(j) = -i$.

The algorithm works under the assumption (which we later discuss) that if, starting at any given period t_1, there is a period $t \geq t_1$ such that $i_t = i_t^{target}$, then $i_{t+j} = i_{t+j}^{target}$ for $j > 0$ (i.e. if at some point the constrain on the interest rate ceases to bind, it will not bind again after that). Let T denote the minimum $t \geq t_1$ such the constrain is no longer binding (i.e. $i_t = i_t^{target}$ for $t \geq T$ and $i_{T-1} = -i$) and assume for the moment that T is known. Therefore, the solution characterizing the equilibrium for $t \geq T$ –obtained using the usual techniques– can written as

$$z_t = F_T z_{t-1} + G_T e_t. \tag{12}$$

At period $T-1$ the policy rate is at the lower bound and the equilibrium is characterized by (11). Moreover, because agents know at T, $i_T = i_T^{target}$, they will use (12) to form expectations, i.e. $E_{T-1}\{z_T\} = F_T z_{T-1} + G_T \rho e_{T-1}$. Therefore, we can write the equilibrium conditions at $T-1$ as

$$A(F_T z_{T-1} + G_T \rho e_{T-1}) = \tilde{B}z_{T-1} + Cz_{T-2} + De_{T-1} + c, \tag{13}$$

which can be rearranged to obtain

$$\begin{aligned} z_{T-1} &= (AF_T - \tilde{B})^{-1}Cz_{T-2} + (AF_T - \tilde{B})^{-1}(D - AG_T\rho)e_{T-1} + (AF_T - \tilde{B})^{-1}c, \\ &\equiv F_{T-1}z_{T-2} + G_{T-1}e_{T-1} + H_{T-1}c, \end{aligned} \tag{14}$$

provided $AF_T - \tilde{B}$ being invertible. Working in this same way backwards until t_1 we can compute the equilibrium for $t \in [t_1, T-1]$, which will have the form

$$z_t = F_t z_{t-1} + G_t e_t + H_t c. \tag{15}$$

where

$$\begin{aligned} J_t &\equiv (AF_{t+1} - \tilde{B})^{-1}, \\ F_t &\equiv J_t C, \\ G_t &\equiv J_t(D - AG_{t+1}\rho), \\ H_t &\equiv J_t(-AH_{t+1} + I) \text{ for } t \in [t_1, T-2],\ H_{T-1} \equiv (AF_T - \tilde{B})^{-1}. \end{aligned} \tag{16}$$

Therefore, using these matrices, for initial values of z_{t_1-1} and e_{t_1-1} we can obtain the perfect foresight path (impulse response) for a given shock u_{t_1}, under the assumption that T is known. To find T, we can just run a progressive search starting on t_1 and find the first period for which $i_T = i_T^{target}$ and $i_{T-1} = -i$.

Before moving to the case in which the central banks imposes the lower bound, two comments are in order. First, notice that once T is found the assumption that $i_t = i_t^{target}$ for $t \geq T$ can be verified numerically by simulating the path for the variables for many periods beyond T (by construction, the assumption will hold in steady state). Second, as noticed also by Bodenstein et al. (2009), the extension for the case in which, given values z_{t_1-1} and u_{t_1}, the lower bound is expected to be reached in a period after t_1 is straight forward.

To consider the case in which the central bank announces, at time t_0, that the interest rate is going to stay at the lower bound until period $t_1 - 1$, and starting at t_1 the rate will be determined by the Taylor Rule, notice that the matrices in (16) characterize any backward induction problem in which the policy rate is expected to be at the lower last for a until a certain period of time. Therefore, given an announced period t_1 and an initial shock u_{t_0}, we can proceed as follows. First, assuming that $T = t_1$ (i.e. that after the period the central has announced the state of the economy is such the constrain on the policy rate is not binding), compute the matrices characterizing the equilibrium according to (16) and numerically verify whether $i_t = i_t^{target}$ for $t \geq t_1$ or not. If that is not the case, increase the candidate value for T until the condition is satisfied.[35]

We can also consider the role of exogenous credibility. In particular, we assume that in any every period $t \in [t_0, t_1 - 2]$ when the interest rate is fixed, people assign a probability sequence p_t to the event that the central bank breaks its promise and sets the nominal interest rate according to the Taylor rule staring on $t + 1$. Moreover, if this is the case, they believe that the Taylor rule will stay forever. Additionally, regardless of people not trusting the central bank, we assume that (ex-post) it fulfills its promises.

This possibility is accommodated by simply modifying the equations in (16) to obtain,

$$
\begin{aligned}
J_t &\equiv [A(p)tF_{t_1} + (1 - p_t)F_{t+1}) - \tilde{B}]^{-1}, \\
F_t &\equiv J_t C, \\
G_t &\equiv J_t[D - A(p_t G_{t_1} + (1 - p_t)G_{t+1})\rho], \\
H_t &\equiv J_t[-A(1 - p_t)H_{t+1} + I] \text{ for } t \in [t_0, t_1 - 2], \; H_{t_1-1} \equiv (AF_{t_1} - \tilde{B})^{-1}.
\end{aligned}
$$

Finally, to consider an unconventional policy in t_0, we just assign values at t_0 for the shocks affecting the different instruments in the central bank's balance sheet. Additionally, the alternative of undoing these policies is also considered by placing appropriate values for the sequence u_t: if the change is not pre-announced we just change the value in the corresponding period, and if the change is pre-announced we can expand the exogenous vector e_t to consider anticipated shocks.

A.2 The Model

Here we present the equilibrium conditions of the model, as well as the calibration of the parameters. It is a standard new Keynesian model for a small open economy with incomplete asset markets. Households have separable preferences in consumption and leisure and can transfer resources over time using an internationally traded bond, as well as the debt created by the central bank. They consume a combination of domestic and foreign goods. There is an infinite number of monopolistic intermediate good firms that use labor to produce using a constant returns-to-scale technology. They are subject to a Calvo-type problem in setting prices, with full indexation to past inflation. The

[35] Notice that if we want to consider to fix the interest rate not at zero but to any arbitrary value, this can be done by setting an appropriate for the constant c.

equilibrium conditions associated with household optimization are,[36]

$$\lambda_t = c_t^{-\sigma}, \tag{E.1}$$

$$w_t = \phi_0 l_t^{\phi_1} c_t^{\sigma}, \tag{E.2}$$

$$\lambda_t = \beta i_{1,t} E_t \left\{ \frac{\lambda_{t+1}}{\pi_{t+1}} \right\} \left(\frac{D_{k,t}^{BC}/D_k^{BC}}{D_{1,t}^{BC}/D_1^{BC}} \right)^{\varphi_k}, \tag{E.3}$$

$$\lambda_t = \beta i_t^* cp_t E_t \left\{ \frac{S_{t+1}\lambda_{t+1}}{S_t \pi_{t+1}} \right\} \left(\frac{B_t^{*BC}}{B_{t-1}^{*BC}} \right)^{\varphi_s}, \tag{E.4}$$

$$c_t = \left[(1-a)^{\frac{1}{\eta}} \left(c_t^H\right)^{\frac{\eta-1}{\eta}} + (a)^{\frac{1}{\eta}} \left(c_t^F\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \tag{E.5}$$

$$c_t^F = a \left(p_t^F\right)^{-\eta} c_t, \tag{E.6}$$

$$c_t^H = (1-a) \left(p_t^H\right)^{-\eta} c_t. \tag{E.7}$$

$$\frac{i_{1,t}}{i_t^{MP}} = \left(\frac{M_t}{P_t y_t} \frac{Py}{M}\right)^{-\varphi_m}. \tag{E.8}$$

Those related to firm's choices are[37]

$$y_t^H = z_t l_t - \kappa. \tag{E.9}$$

$$mc_t = \frac{w_t}{p_t^H z_t}, \tag{E.10}$$

$$f_t^1 = mc_t \left(\tilde{p}_t^H\right)^{-\epsilon} y_t^H + \theta\beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_t}{\pi_{t+1}}\right)^{-\epsilon} \left(\frac{\tilde{p}_t^H}{\tilde{p}_{t+1}^H}\right)^{-\epsilon} \left(\frac{p_t^H}{p_{t+1}^H}\right)^{-1-\epsilon} f_{t+1}^1 \right\}, \tag{E.11}$$

$$f_t^2 = \left(\tilde{p}_t^H\right)^{1-\epsilon} y_t^H \left(\frac{\epsilon-1}{\epsilon}\right) + \theta\beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_t}{\pi_{t+1}}\right)^{1-\epsilon} \left(\frac{\tilde{p}_t^H}{\tilde{p}_{t+1}^H}\right)^{1-\epsilon} \left(\frac{p_t^H}{p_{t+1}^H}\right)^{-\epsilon} f_{t+1}^2 \right\}, \tag{E.12}$$

$$f_t^1 = f_t^2. \tag{E.13}$$

The conditions associated with rest of the world variables are

$$cp_t = \psi_d \left(e^{-p_t^F b_{t+1}^* + \bar{b}} - 1 \right), \tag{E.14}$$

$$\frac{p_t^F}{p_{t-1}^F} = \frac{S_t \pi_t^*}{S_{t-1}\pi_t}. \tag{E.15}$$

[36] Variables without time subscripts denote steady state values. Prices have been rescaled by the price index. Recall that in the case of (E.3), (E.4) and (E.8), which display the deviations from the standard model, these are imposed *ad-hoc* over the usual equilibrium conditions and are not derived from microfoundations.

[37] The assumption of full indexation to past inflation yields that, up to first order, the distortion that in principle price stickiness introduces (i.e. a gap between production and absorption) is not relevant even if steady state inflation is different from zero.

$$c_t^{H*} = \left(\frac{p_t^H}{p_t^f} \right)^{-\eta^*} y_t^*. \tag{E.16}$$

Market clearing conditions are

$$1 = \theta \left(\frac{p_{t-1}^H \pi_{t-1}}{p_t^H \pi_t} \right)^{1-\epsilon} + (1-\theta) \left(\tilde{p}_t^H \right)^{1-\epsilon}. \tag{E.17}$$

$$y_t^H = c_t^H + c_t^{H*}, \tag{E.18}$$

$$y_t = c_t + p_t^H c_t^{H*} - p_t^F c_t^F. \tag{E.19}$$

$$tb_t = p_t^H c_t^{H*} - p_t^F c_t^F. \tag{E.20}$$

$$p_t^F b_{t+1}^* = p_t^F \frac{b_t^*}{\pi_t^*} i_{t-1}^* c p_{t-1} + tb_t. \tag{E.21}$$

Finally, monetary policy is as described in the text.

We calibrate the parameters following the structural estimation for Chile performed by Medina and Soto (2007), and the steady state ratios of the variables in CB balance sheet are taken from the average of the series used for estimation described in the text. The time period is meant to be a quarter. Table 1 presents the parameters and steady state variables that were calibrated. Finally, we assume that the frictions disappear in steady state. This implies that we need to introduce a demand for real balances in steady state, which we choose to be $M/P = y(i)^{-\mu_m}$.

Table 1: Calibration

σ	1	ϕ_1	0.84	a	0.34
η	1.12	ϵ	11	η^*	0.79
θ	0.74	ψ_d	0.01	μ_m	0.2
ρ_i	0.74	α_π	1.67	α_y	0.39
i^*	1.0025	i	1.016	π	1.0074
l	0.2	$\frac{D_1^{CB}+D_k^{CB}}{M}$	1.8	$\frac{D_k^{CB}}{D_1^{CB}}$	0.8

A.3 Estimation

Here we describe the details of the estimation of equations (4) to (6). Before presenting the results, it is relevant to mention several data limitations. First, even though there is data available on the total stock of bonds of the Central Bank of Chile (CBCh) on circulation, the data is not rich enough to obtain a long series for the stock of bonds at different maturities. In addition, although nowadays the CBCh has available a rich maturity structure for its bonds, this has not been always the case, and only the two-years bond in pesos (BCP2) has a long enough series of interest rates.

In term of equation (4), i_t^{MP} is measured by the monetary policy rate set by the CBCh, $i_{1,t}$ is the 90-days deposit rate,[38] M is the monetary base, output is the monthly index of economic

[38] All rates are quarterly to be consistent with the calibration of the model.

activity (IMACEC) and the price level is the core consumer price index (IPCX1).[39] The equation was estimated by GMM, using as instruments lags one to five of the regressor, the J. P. Morgan Emerging Markets Bond Index Global (EMBI) index, the Federal Funds Rate, the rate on a 5- years bond in pesos (BCP5) and the 90-days lending rate, and allowing also for an AR(1) error term.[40] The result obtained was,[41]

$$\begin{aligned} i_{1,t} - i_t^{MP} &= \underset{(0.0017)}{-0.0025} - \underset{(0.0007)}{0.0014}(m_t - p_t - y_t) + u_t, \\ u_t &= \underset{(0.0416)}{0.8056} u_{t-1} + e_t, \\ R^2(Adj) = 0.587&, J - stat = 0.175, Obs = 60. \end{aligned}$$

For equation (5), the foreign interest rate is the Federal Funds Rate and the country premium is measured by the EMBI, the stock of government foreign assets B_t^{*CB} was measured the net foreign asset element in CBCh balance sheet, and the three-month-ahead expectation of the nominal exchange rate comes from the Expectations Survey collected by the CBCh. The equation was estimated by GMM using the same instruments as in the previous equations and allowing also for an AR(1) error term. The results were,

$$\begin{aligned} s_t - E_t\{s_{t+1}\} - i_t^* - cp_t + i_{1,t} &= \underset{(0.001)}{-0.0037} + \underset{(0.036)}{0.2047}(\Delta b_t^{*BC}) + u_t, \\ u_t &= \underset{(0.0307)}{0.2615} u_{t-1} + e_t, \\ R^2(Adj) = 0.014&, J - stat = 0.115, Obs = 76. \end{aligned}$$

Finally, for equation (6) the rate of nominal 2-year bonds was used as a measure for $i_{k,t}$ and the expectations about the future short rate also comes from the Expectations Survey. Because, as we commented before, there is no data on the stock of bonds at different maturities we measure $\hat{d}_{k,t}^{BC} - \hat{d}_{1,t}^{BC}$ as the difference between the total stock of bonds in pesos and the stock of 90-days commercial and consumer deposits. The equation was estimated by GMM using the same instruments as in the previous equations and allowing also for an AR(1) error term. The estimation yields,

$$\begin{aligned} i_{k,t} - \sum_{j=0}^{k-1} E_t\{i_{1,t+j}\} &= \underset{(0.001)}{-0.001} + \underset{(0.001)}{0.017} \left(d_{k,t}^{BC} - d_{1,t}^{BC}\right) u_t, \\ u_t &= \underset{(0.0307)}{0.2615} u_{t-1} + e_t, \\ R^2(Adj) = 0.671&, J - stat = 0.179, Obs = 76. \end{aligned}$$

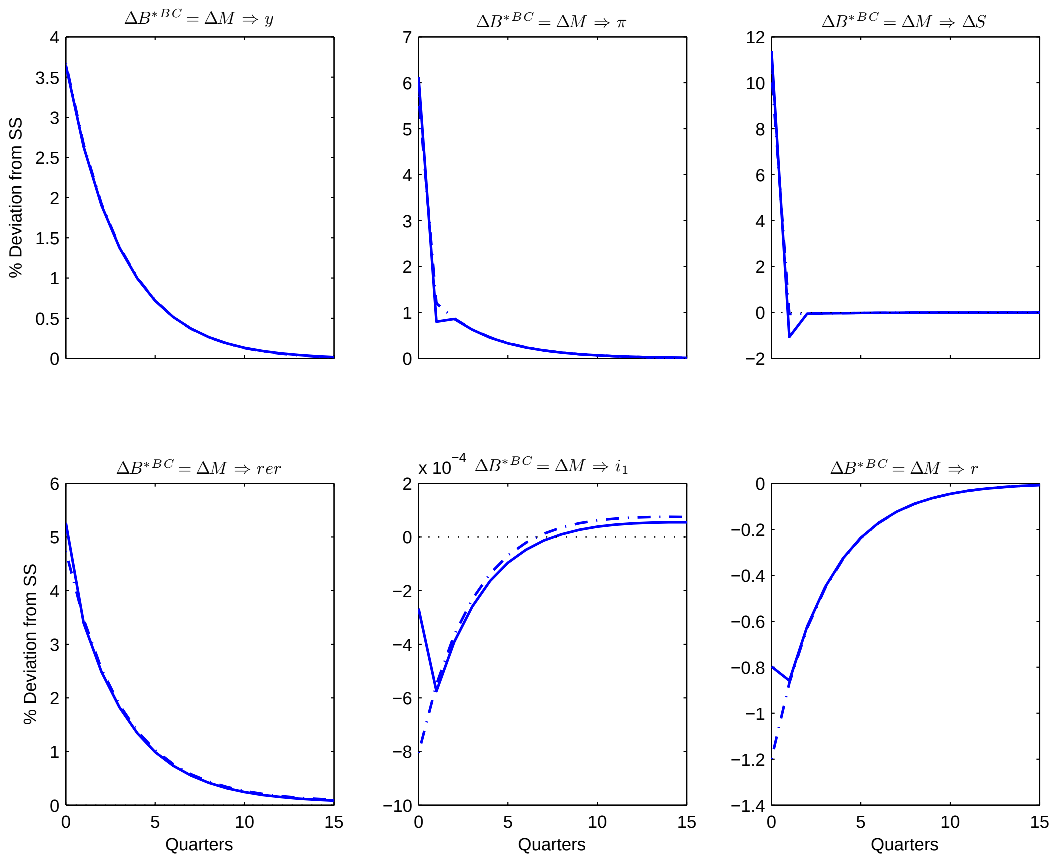
[39] These two series have been seasonally adjusted. The source for all the series is the Central Bank of Chile. Variables are measured in logs.

[40] Adding lags of the dependent variable as regressors did not improve the goodness of fit.

[41] Newey-West robust standard errors in parenthesis

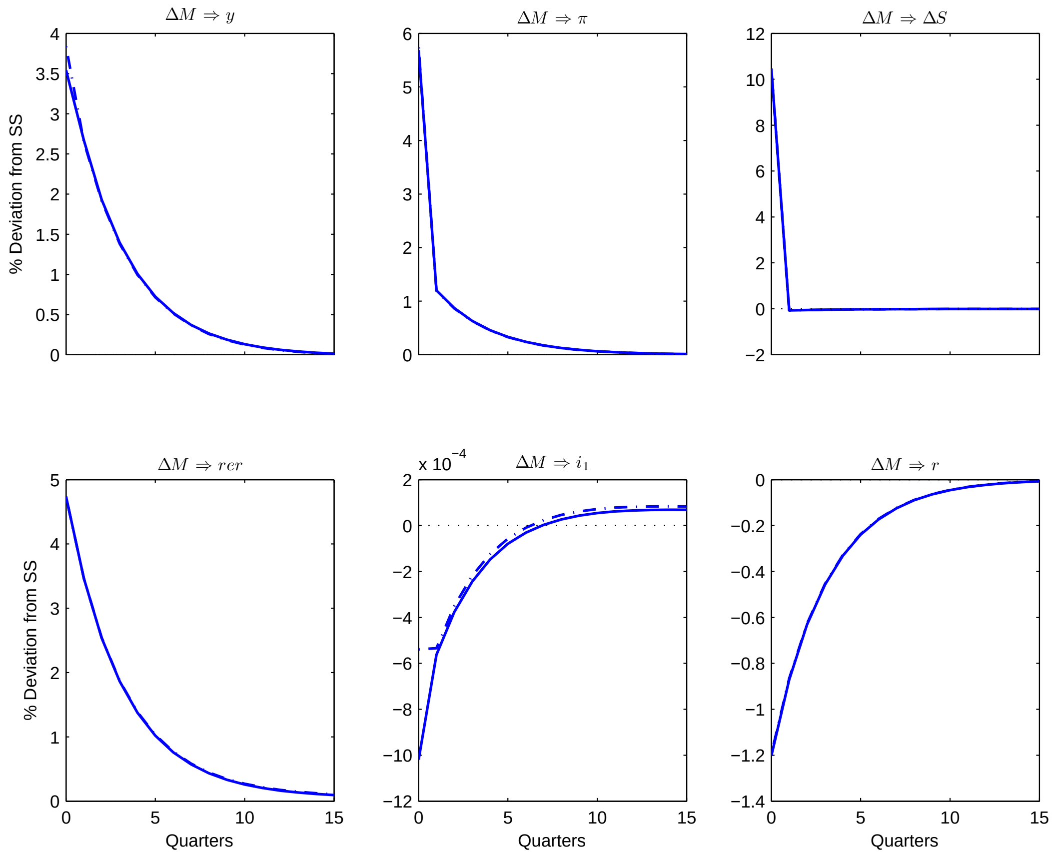
B Figures

Figure 1: Purchase of Foreign Assets Financed with a Monetary Expansion



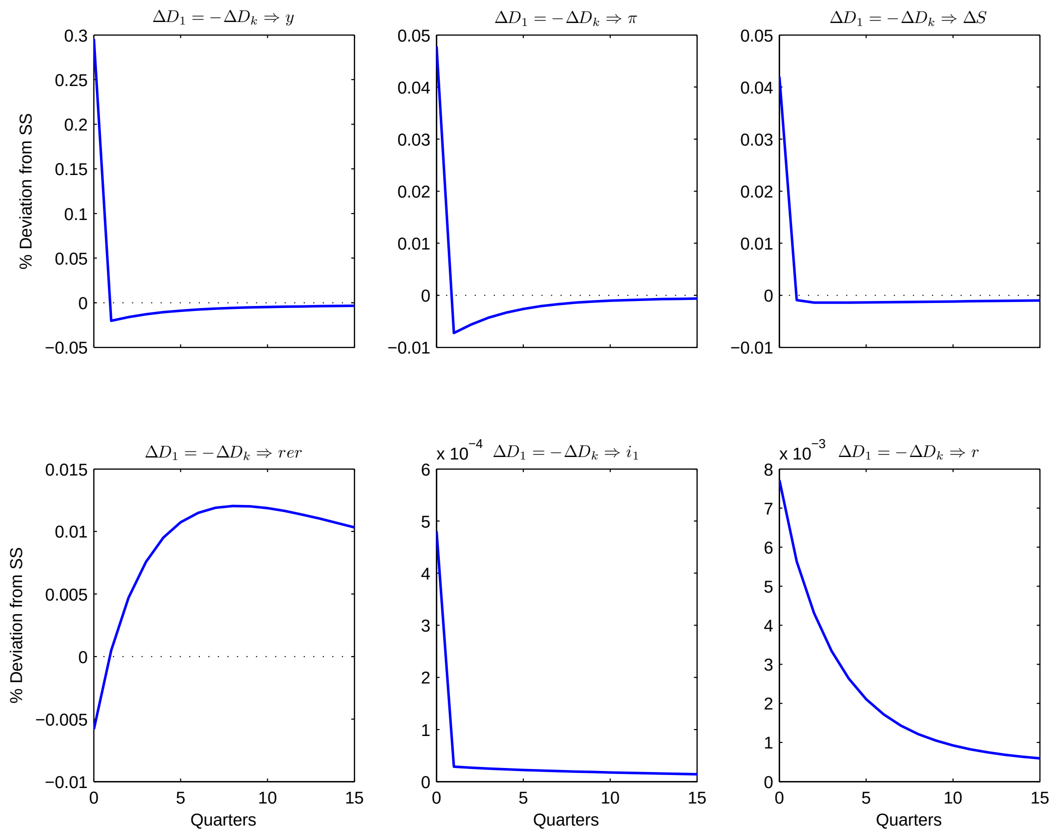
Note: y is real GDP, π is inflation, ΔS is nominal depreciation, rer is the real exchange rate, $r_t \equiv i_{1,t} - E_t \pi_{t+1}$ is the ex-ante real interest rate. All variables are in log-deviations from steady state. The the shock is permanent increase in B_t^{*CB} equivalent to a 10% of nominal GDP in steady state, while i_t^{MP} remains in its steady state value forever. The solid line is the benchmark case, while the dashed-dotted line is the case with $\varphi_d = \varphi_s = 0$.

Figure 2: Purchase of Debt Financed with a Monetary Expansion



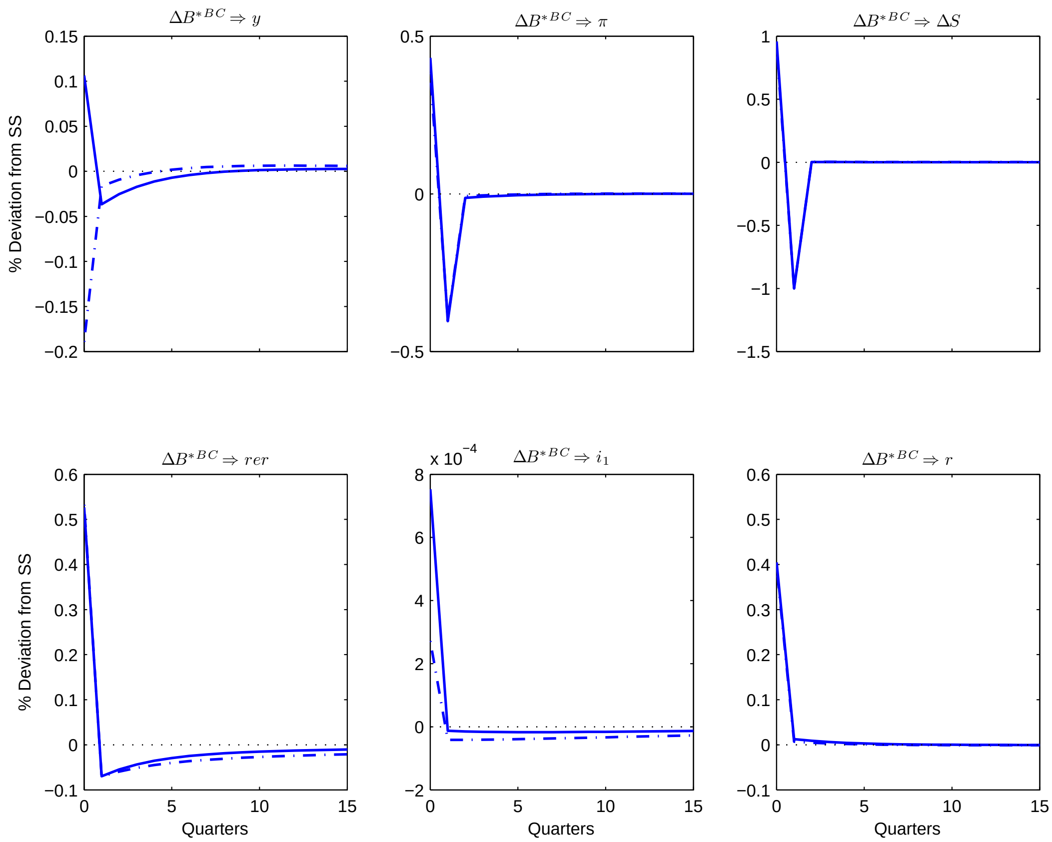
Note: The solid line corresponds to a purchase of short debt, $D_{1,t}$, and the dashed-dotted line is purchase of long debt, $D_{k,t}$. The the shock is permanent increase in M_t equivalent to a 10% of nominal GDP in steady state, while i_t^{MP} remains in its steady state value forever.

Figure 3: Purchase of Long Debt Financed with New Short Debt



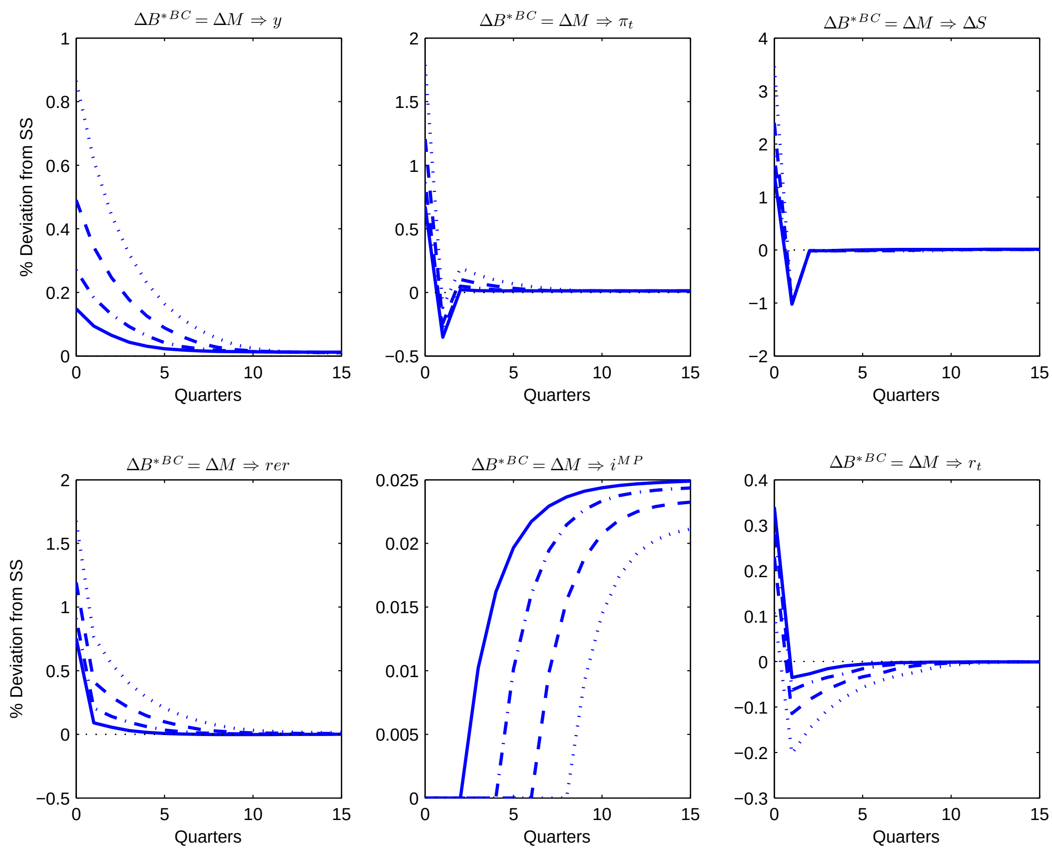
Note: The the shock is permanent increase in $D_{1,t}$ equivalent to a 10% of nominal GDP in steady state, while i_t^{MP} remains in its steady state value forever.

Figure 4: Purchase of Foreign Assets Financed with new Debt



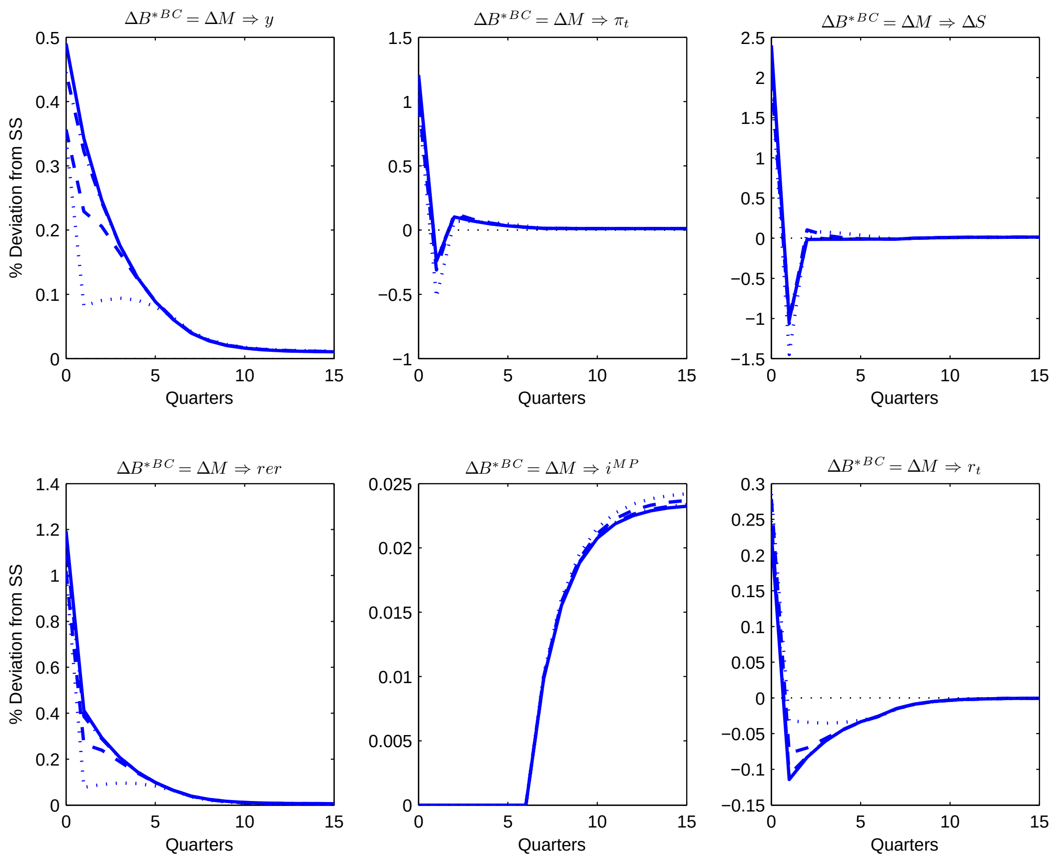
Note: The solid line corresponds to new short debt, $D_{1,t}$, and the dashed-dotted line is new long debt, $D_{k,t}$. The the shock is permanent increase in B_t^{*CB} equivalent to a 10% of nominal GDP in steady state, while i_t^{MP} remains in its steady state value forever.

Figure 5: Purchase of Foreign Assets Financed with a Monetary Expansion. Different T's, $p_t = 0$.



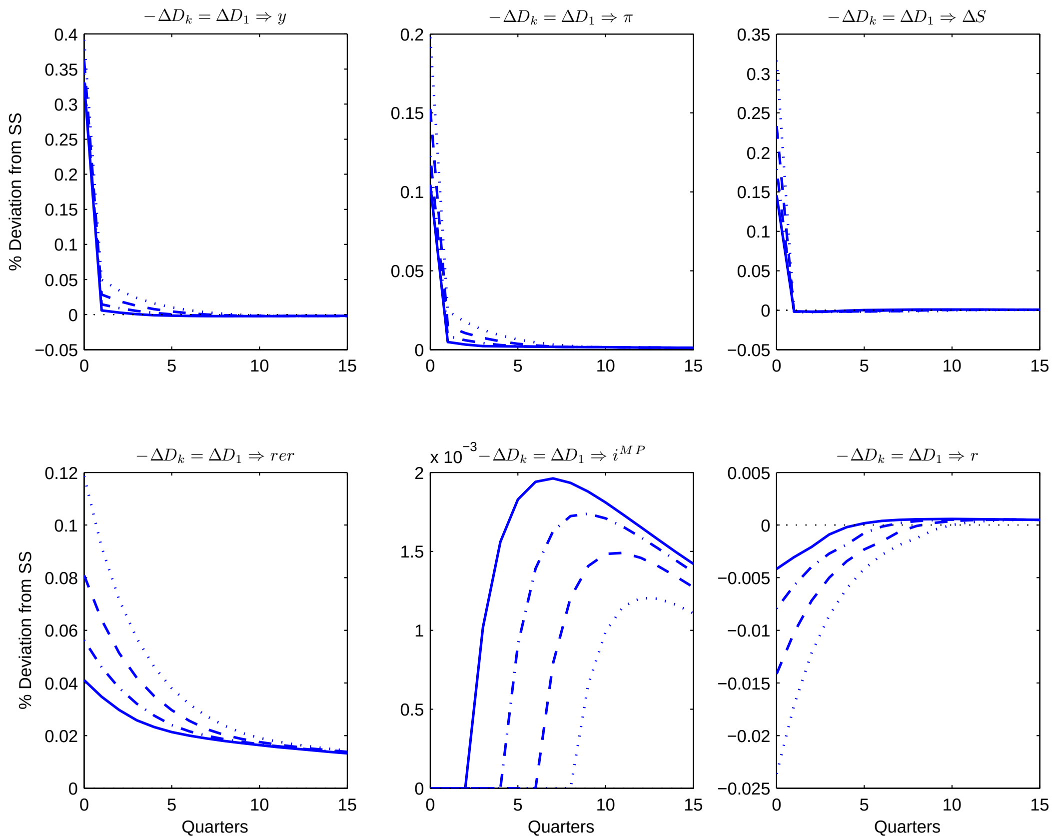
Note: The solid, dashed-dotted, dashed and dotted lines correspond, respectively to $T = 2, 4, 6, 8$. The the shock is permanent increase in B_t^{*CB} equivalent to a 10% of nominal GDP in steady state, while i_t^{MP} remains in its steady state value until T.

Figure 6: Purchase of Foreign Assets Financed with a Monetary Expansion. $T = 6$, $p_t = p^t$, different p's.



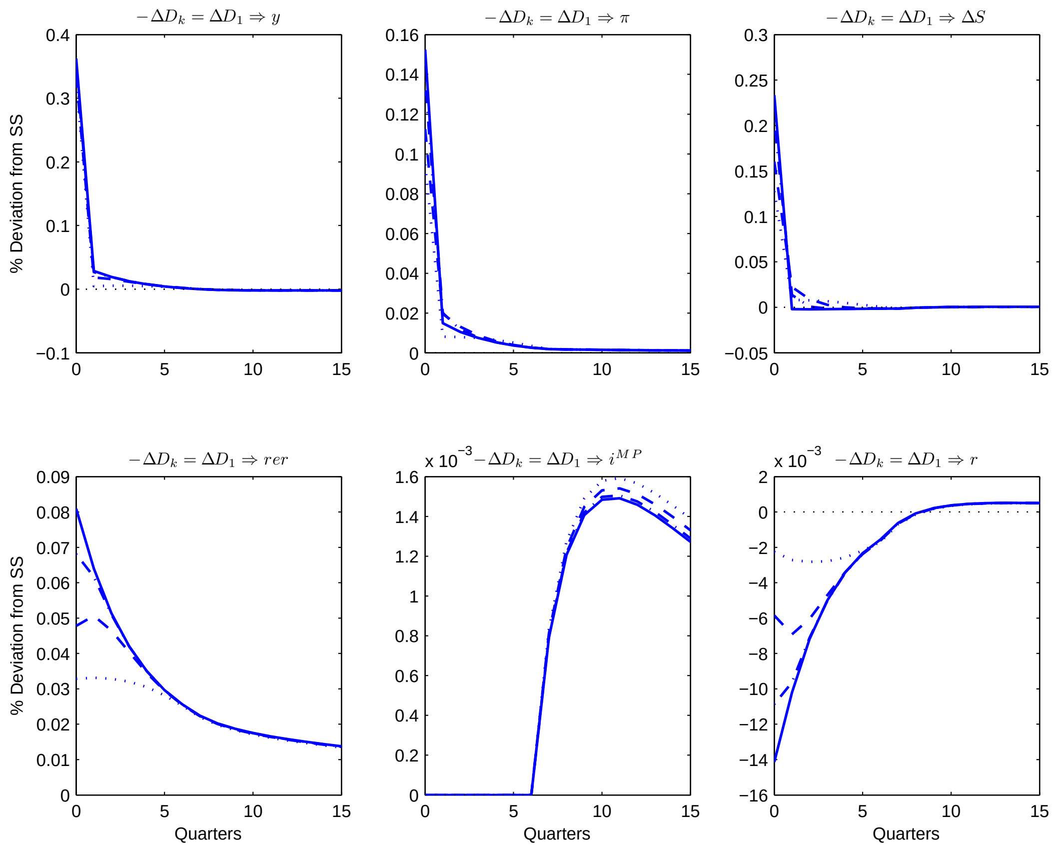
Note: The solid, dashed-dotted, dashed and dotted lines correspond, respectively to $p = 0, 0.3, 0.6, 0.9$. The the shock is permanent increase in B_t^{*CB} equivalent to a 10% of nominal GDP in steady state, while i_t^{MP} remains in its steady state value until T.

Figure 7: Purchase of Long Debt Financed with New Short Debt. Different T's, $p_t = 0$.



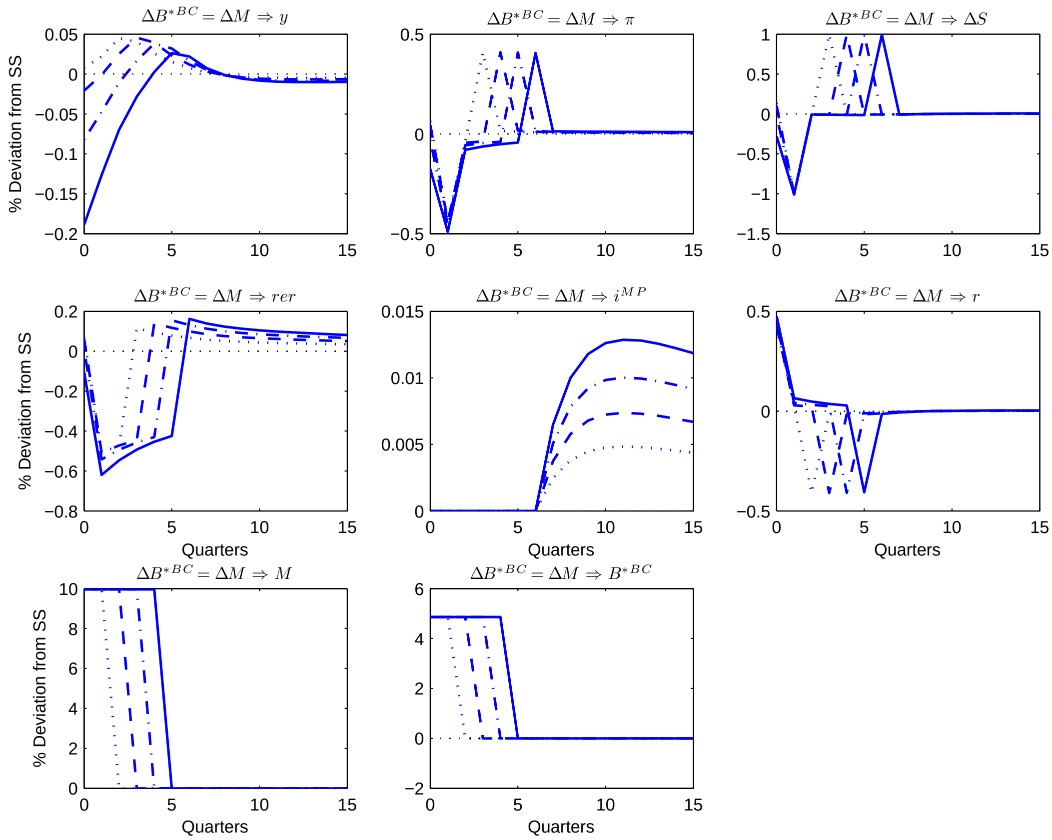
Note: The solid, dashed-dotted, dashed and dotted lines correspond, respectively to $T = 2, 4, 6, 8$. The the shock is permanent increase in B_t^{*CB} equivalent to a 10% of nominal GDP in steady state, while i_t^{MP} remains in its steady state value until T.

Figure 8: Purchase of Long Debt Financed with New Short Debt. $T = 6$, $p_t = p^t$, different p's.



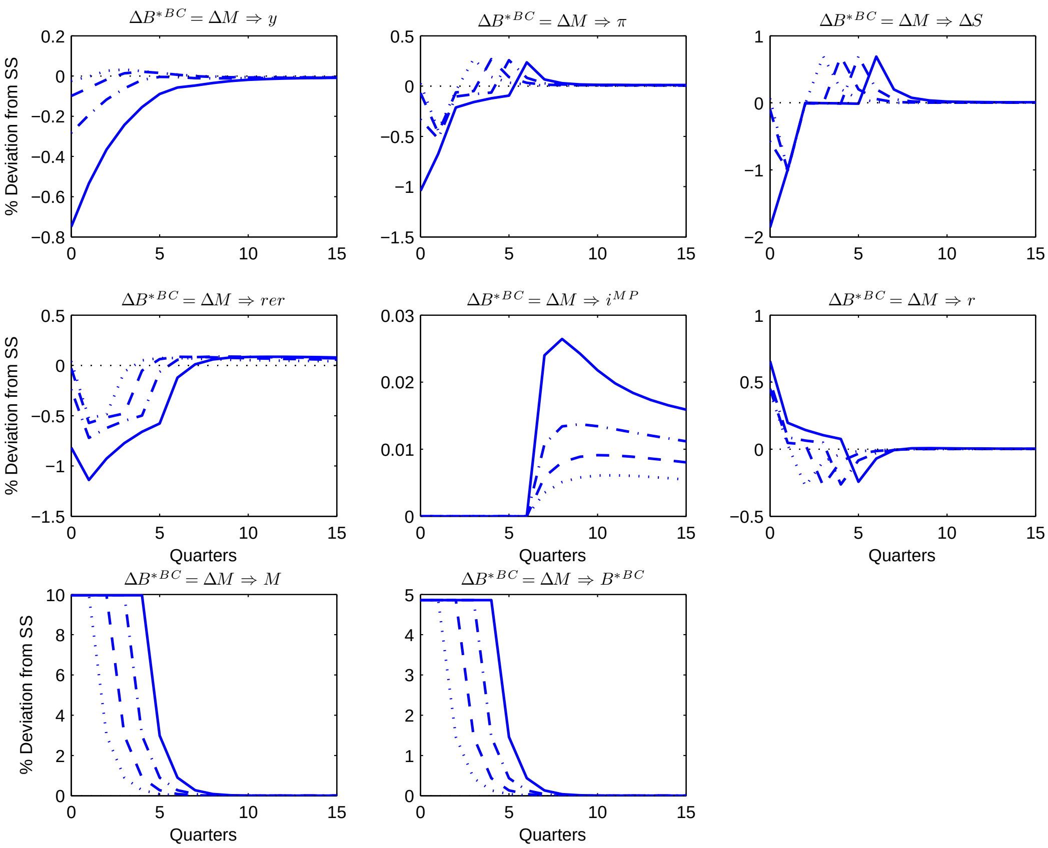
Note: The solid, dashed-dotted, dashed and dotted lines correspond, respectively to $p = 0, 0.3, 0.6, 0.9$. The the shock is permanent increase in B_t^{*CB} equivalent to a 10% of nominal GDP in steady state, while i_t^{MP} remains in its steady state value until T.

Figure 9: Purchase of Foreign Assets Financed with a Monetary Expansion, Pre-announced reversal, different k's



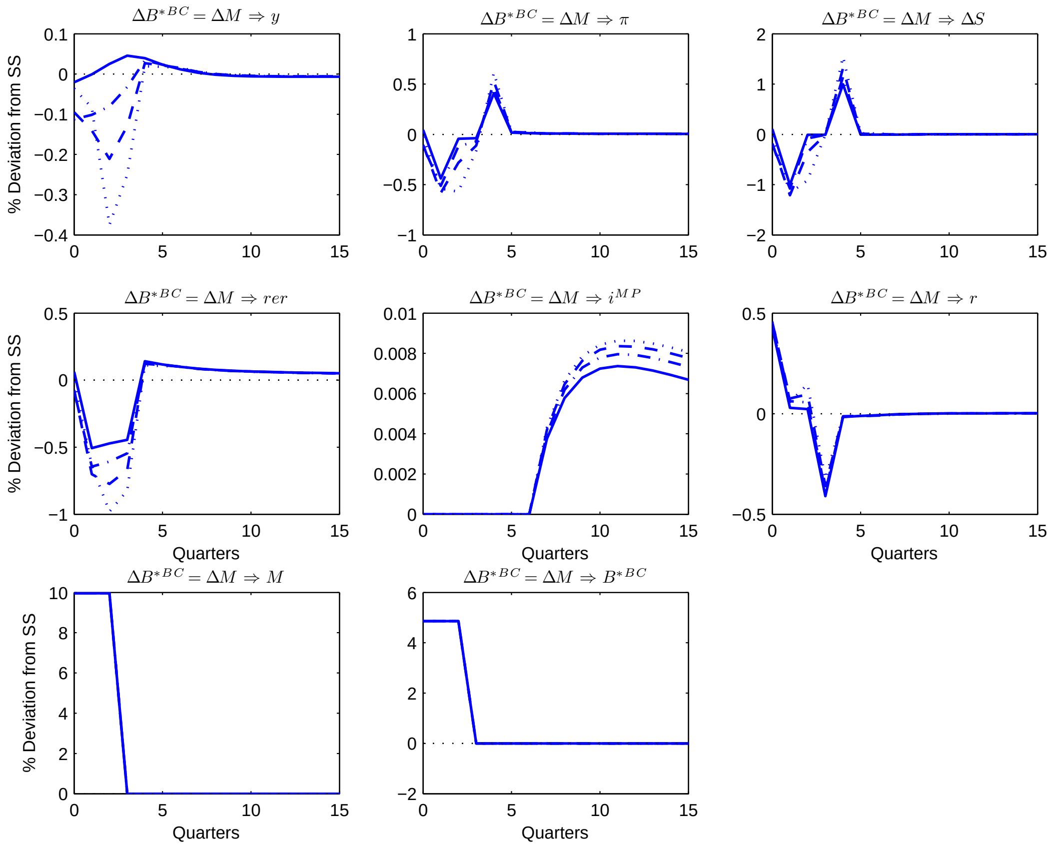
Note: The solid, dashed-dotted, dashed and dotted lines correspond, respectively to $k = 2, 3, 4, 5$, with $T = 6$. The the shock is permanent increase in B_t^{*CB} equivalent to a 10% of nominal GDP in steady state, that is fully reversed in $T - k$, while i_t^{MP} remains in its steady state value until T.

Figure 10: Purchase of Foreign Assets Financed with a Monetary Expansion, Pre-announced gradual reversal, different k's



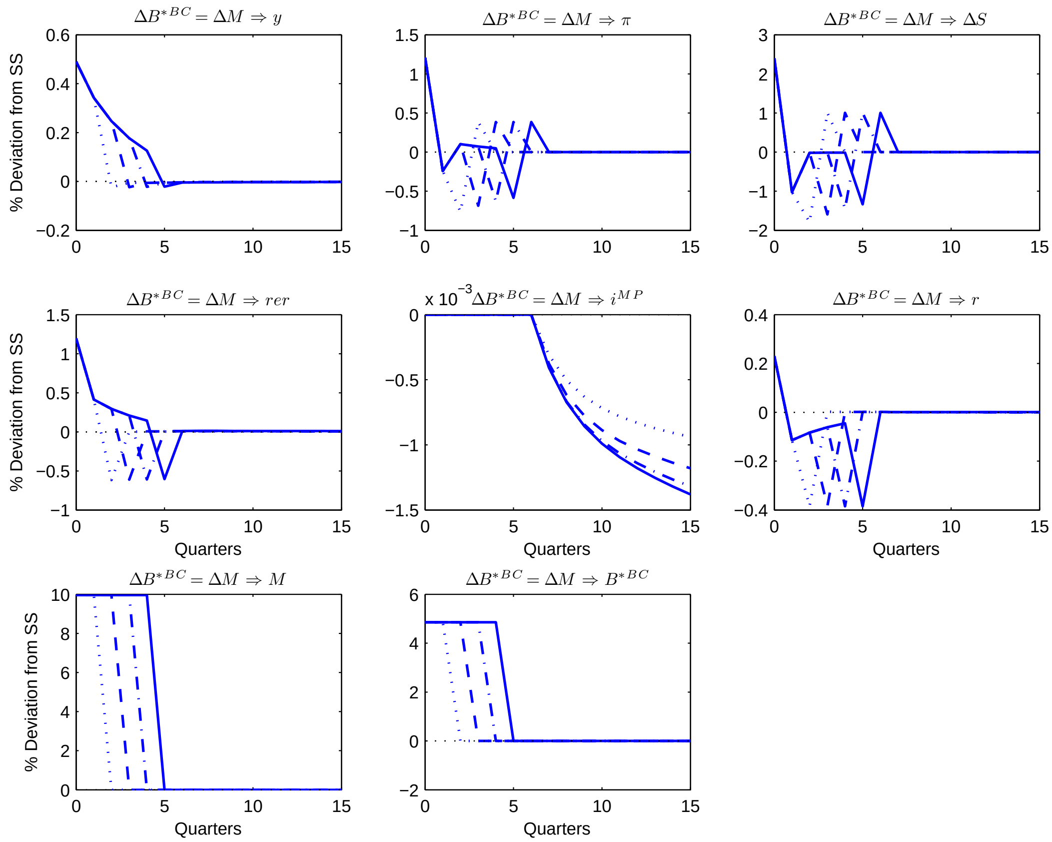
Note: The solid, dashed-dotted, dashed and dotted lines correspond, respectively to $k = 2, 3, 4, 5$, with $T = 6$. The the shock is permanent increase in B_t^{*CB} equivalent to a 10% of nominal GDP in steady state, that is reversed slowly startying at $T - k$, while i_t^{MP} remains in its steady state value until T.

Figure 11: Purchase of Foreign Assets Financed with a Monetary Expansion, Pre-announced reversal, $p_t = p^t$, different p's



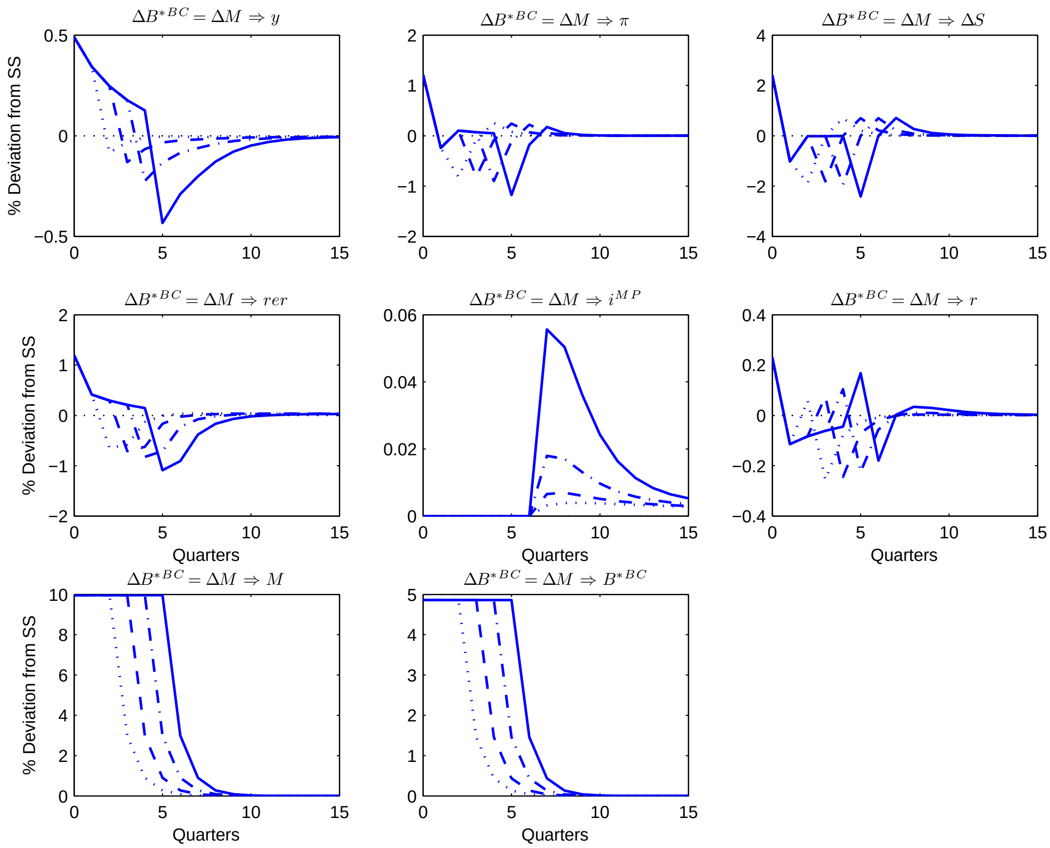
Note: The solid, dashed-dotted, dashed and dotted lines correspond, respectively to $p = 0, 0.3, 0.6, 0.9$. The the shock is permanent increase in B_t^{*CB} equivalent to a 10% of nominal GDP in steady state, that is fully reversed in $T - k$, with $T = 6$ and $k = 4$, while i_t^{MP} remains in its steady state value until T.

Figure 12: Purchase of Foreign Assets Financed with a Monetary Expansion, Unexpected reversal, different k's



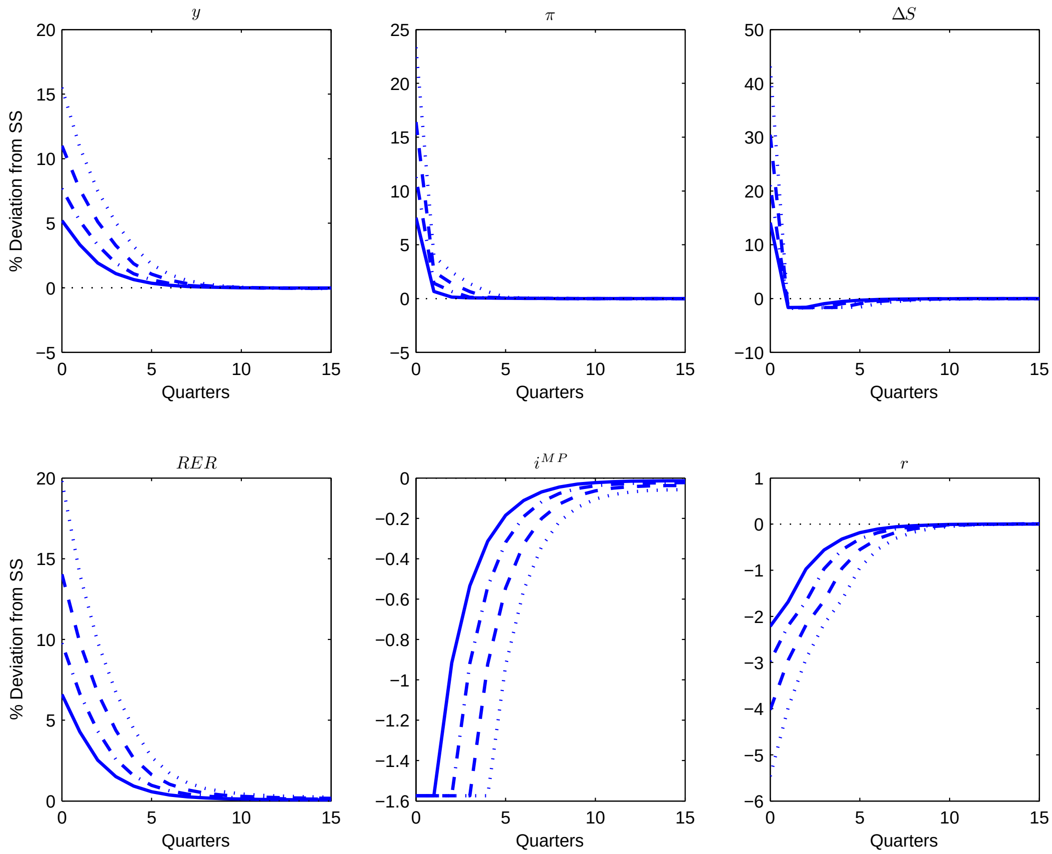
Note: The solid, dashed-dotted, dashed and dotted lines correspond, respectively to $k = 2, 3, 4, 5$, with $T = 6$. The the shock is permanent increase in B_t^{*CB} equivalent to a 10% of nominal GDP in steady state, that is fully reversed in $T - k$ but is not anticipated, while i_t^{MP} remains in its steady state value until T.

Figure 13: Purchase of Foreign Assets Financed with a Monetary Expansion, Unexpected gradual reversal, different k's



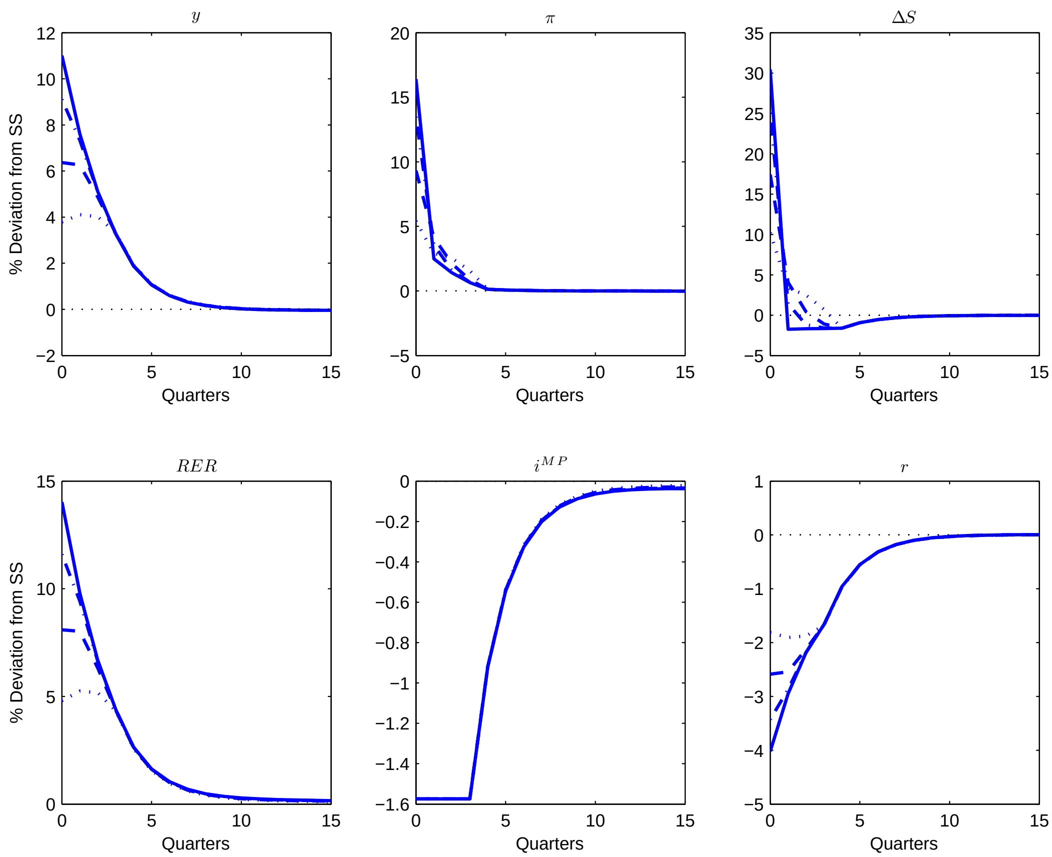
Note: The solid, dashed-dotted, dashed and dotted lines correspond, respectively to $k = 2, 3, 4, 5$, with $T = 6$. The the shock is permanent increase in B_t^{*CB} equivalent to a 10% of nominal GDP in steady state, that is reversed slowly startying at $T - k$ but is not anticipated, while i_t^{MP} remains in its steady state value until T.

Figure 14: Driving i_t^{mp} to zero. Different T's, $p_t = 0$.



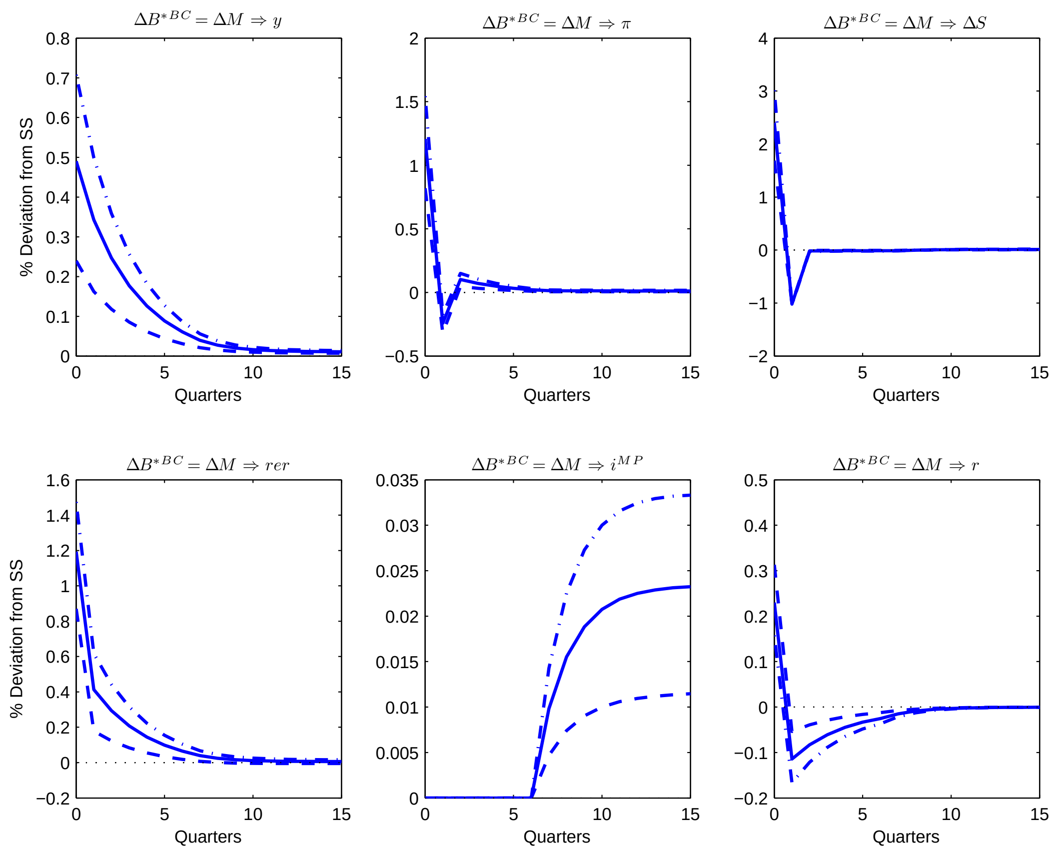
Note: The solid, dashed-dotted, dashed and dotted lines correspond, to the response of driving i_t^{MP} to zero and keeping at this value until, respectively, $T = 1, 2, 3, 4$.

Figure 15: Driving i_t^{mp} to zero. $T = 3$, $p_t = p^t$, different p's.



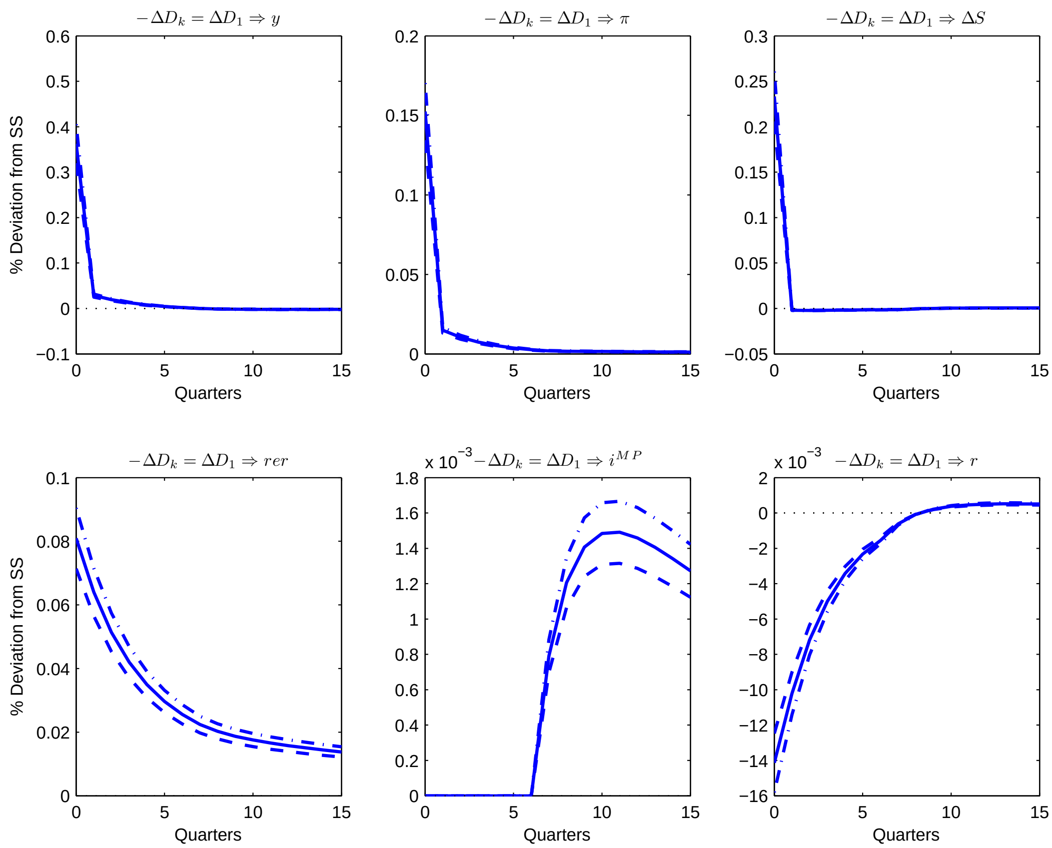
Note: The solid, dashed-dotted, dashed and dotted lines correspond, to the response of driving i_t^{MP} to zero and keeping at this value until $T = 3$, with $p_t = p^t$ and, respectively, $p = 0, 0.3, 0.6, 0.9$.

Figure 16: Purchase of Foreign Assets Financed with a Monetary Expansion. $T = 6$, $p_t = 0$. Robustness with respect to φ_m.



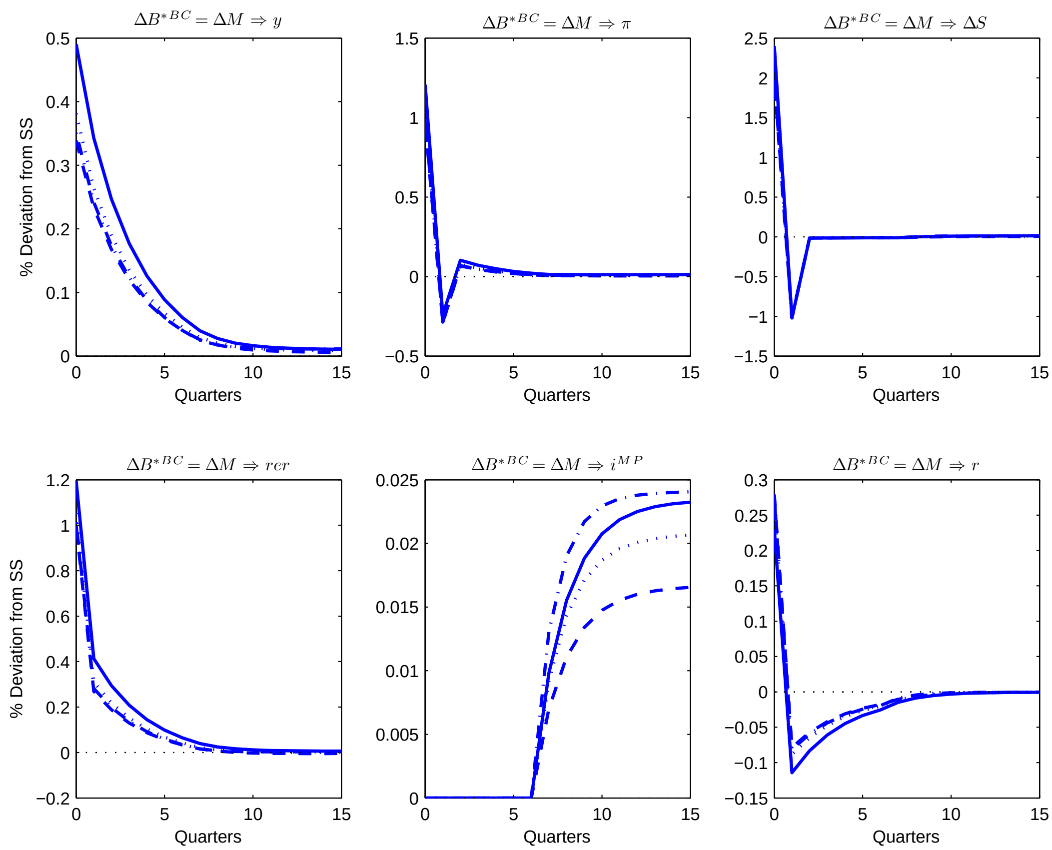
Note: The solid line correspond to the benchmark case ($\varphi_m = -0.0014$), while the dashed-dotted and dashed are the cases with $\varphi_m = -0.0021$ and $\varphi_m = -0.0007$, respectively. The the shock is permanent increase in B_t^{*CB} equivalent to a 10% of nominal GDP in steady state, while i_t^{MP} remains in its steady state value until T.

Figure 17: Purchase of Long Debt Financed with New Short Debt. $T = 6$, $p_t = 0$. Robustness with respect to φ_k.



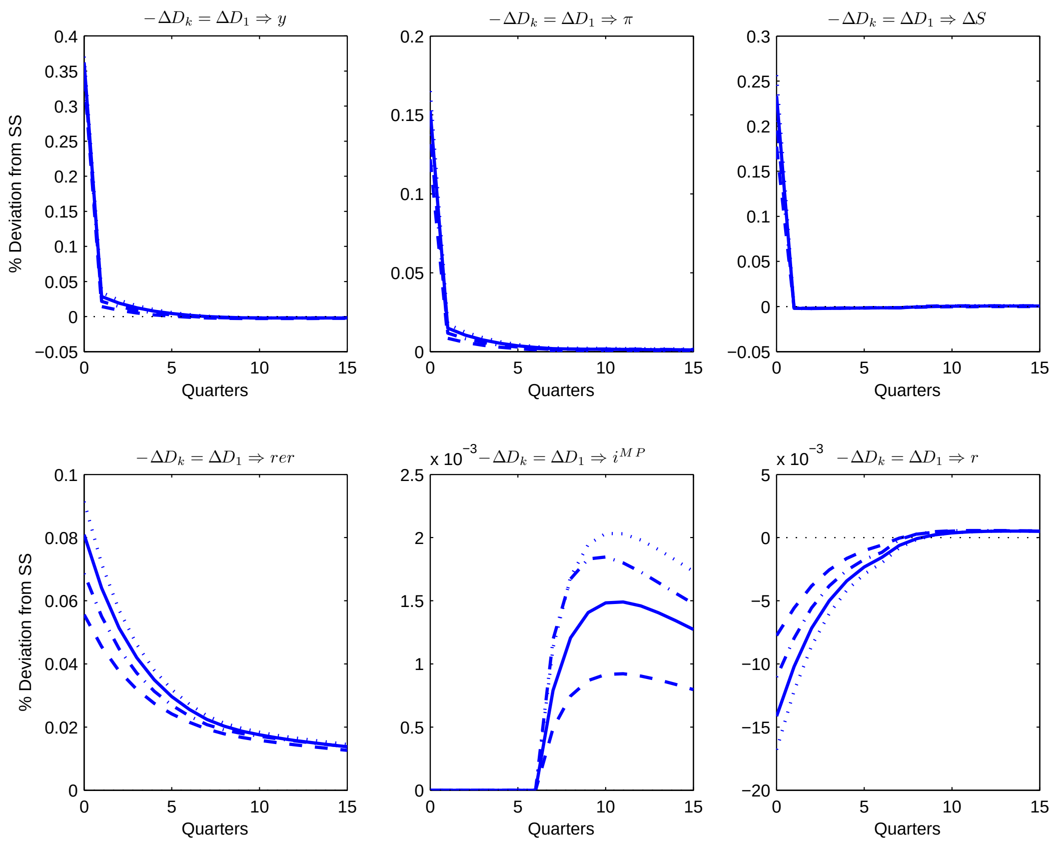
Note: The solid line correspond to the benchmark case ($\varphi_k = 0.017$), while the dashed-dotted and dashed are the cases with $\varphi_m = 0.019$ and $\varphi_m = 0.015$, respectively. The the shock is permanent increase in B_t^{*CB} equivalent to a 10% of nominal GDP in steady state, while i_t^{MP} remains in its steady state value until T.

Figure 18: Purchase of Foreign Assets Financed with a Monetary Expansion. $T = 6$, $p_t = 0$. Robustness with respect to the Taylor rule.



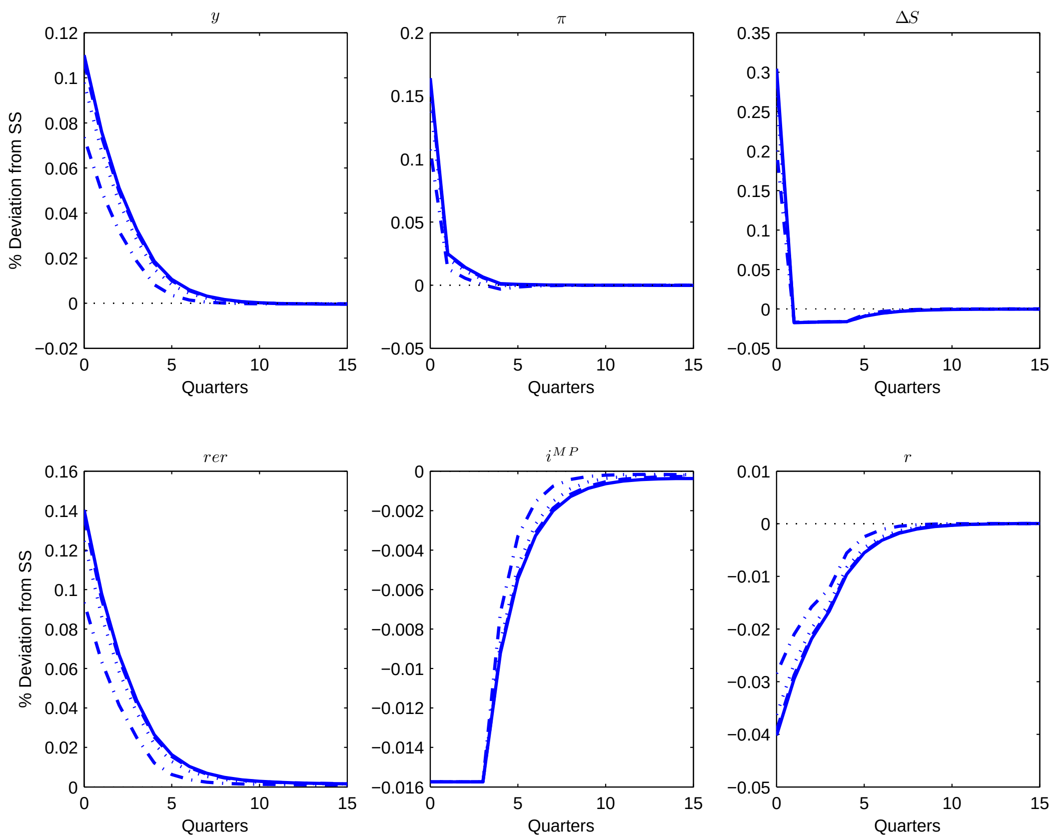
Note: The solid line correspond to the benchmark case ($\rho_i = 0.74$, $\alpha_\pi = 1.67$ and $\alpha_\pi = 0.39$), while the dashed-dotted, dashed and dotted are the cases with $\rho_i = 0.37$, $\alpha_\pi = 3.39$ and $\alpha_\pi = 0.78$, respectively. The the shock is permanent increase in B_t^{*CB} equivalent to a 10% of nominal GDP in steady state, while i_t^{MP} remains in its steady state value until T.

Figure 19: Purchase of Long Debt Financed with New Short Debt. $T = 6$, $p_t = 0$. Robustness with respect to the Taylor rule.



Note: The solid line correspond to the benchmark case ($\rho_i = 0.74$, $\alpha_\pi = 1.67$ and $\alpha_\pi = 0.39$), while the dashed-dotted, dashed and dotted are the cases with $\rho_i = 0.37$, $\alpha_\pi = 3.39$ and $\alpha_\pi = 0.78$, respectively. The the shock is permanent increase in B_t^{*CB} equivalent to a 10% of nominal GDP in steady state, while i_t^{MP} remains in its steady state value until T.

Figure 20: Driving i_t^{mp} to zero. $T = 3$, $p_t = 0$. Robustness with respect to the Taylor rule.



Note: The solid line correspond to the benchmark case ($\rho_i = 0.74$, $\alpha_\pi = 1.67$ and $\alpha_\pi = 0.39$), while the dashed-dotted, dashed and dotted are the cases with $\rho_i = 0.37$, $\alpha_\pi = 3.39$ and $\alpha_\pi = 0.78$, respectively. The experiment is to drive i_t^{MP} to zero and to keep it at this value until $T = 3$, with $p_t = 0$.