

# An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining\*

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October, 2007  
(first draft November, 2006)

## Abstract

We develop and estimate a medium scale macroeconomic model that allows for unemployment and staggered nominal wage contracting. In contrast to most existing quantitative models, employment adjustment is on the extensive margin and the employment of existing workers is efficient. Wage rigidity, however, affects the hiring of new workers. The former is introduced via the staggered Nash bargaining setup of Gertler and Trigari (2006). A robust finding is that the model with wage rigidity provides a better description of the data than does a flexible wage version. Overall, the model fits the data roughly as well as existing quantitative macroeconomic models, such as Smets and Wouters (2007) or Christiano, Eichenbaum and Evans (2005). More work is necessary, however, to ensure a robust identification of the key labor market parameters.

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\*Prepared for the JMCB conference in honor of Ernst Baltensperger, Bern, Switzerland, 2007. Mark Gertler thanks the National Science Foundation and the Guggenheim Foundation for financial support.

# 1 Introduction

This paper develops and estimates a quantitative macroeconomic framework that incorporates labor market frictions. Our starting point is the now conventional monetary DSGE model developed by Christiano, Eichenbaum and Evans (CEE, 2005), Smets and Wouters (SW, 2007) and others. We introduce labor market frictions with a variant of the Mortensen and Pissarides search and matching framework. This variant allows for staggered Nash wage bargaining, as in Gertler and Trigari (GT, 2006).

Our motivation is twofold. First, there are some compelling theoretical considerations. The recent vintage of monetary DSGE models typically has employment adjusting on the intensive margin along with staggered nominal wage contracting. The latter feature, further, is important for the quantitative performance of the model: The wage stickiness helps accounting for the volatility of hours. However, as a consequence, these frameworks are susceptible to Barro's (1977) argument that wages may not be allocational in this kind of environment, given that firm's and workers have an on-going relationship. If wages are not allocational then wage rigidity does not influence model dynamics. By contrast, in the model we present, firms adjust employment along the extensive margin. In this instance, wage rigidity affects employment by influencing the rate at which firms add new workers to their respective labor forces. As emphasized by Hall (2005a), since new workers have yet to form on-going relationships with firms, in this kind of setting the Barro's critique does not apply.

Second, within the search and matching a literature, there is a debate over how well the baseline Mortensen and Pissarides framework can account for labor market volatility, or whether it may be necessary to introduce additional features such as wage rigidity, etc. (e.g. Shimer, 2005, Hall, 2005, Hagedorn and Manovskii, 2006, Mortensen and Nagypal, 2007). A typical approach in the literature has been to develop a calibrated model, subject the model to productivity shocks, and then examine model moments against moments of the data, with various features such as wage rigidity or on-the-job search shut on and then shut off. We instead estimate a complete macroeconomic framework using Bayesian methods. Doing so allows us to formally evaluate the significance of different mechanisms such as wage rigidity to overall model performance. In addition, our full information procedure permits us to account for the complete range of shocks that hit the economy.

In section 2 we develop the model. The basic framework follows CEE and SW closely. The only significant difference involves the treatment of the labor market. As in GT, we incorporate a variation of Mortensen/Pissarides that retains the empirically appealing feature of Nash bargaining, but replace the assumption of period-by-period wage negotiations to allow for staggered multi-period wage contracting. Each period, only a subset of firms and workers negotiate a wage contract. Each wage bargain, further, is between a firm and its existing workforce: Workers hired in-between contract settlements receive the existing wage.<sup>1</sup> In the language of Hall (2005), the existing contract

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<sup>1</sup>A number of authors argue that the wages of new hires are more flexible than those of existing workers. See, for

wage provides a wage norm for the workers hired in-between contracting periods. We restrict the form of the wage contract to call for a fixed wage per period over an exogenously given horizon. Though it surely would be preferable to fully endogenize the contract structure, the payoff from our shortcut is a simple empirically appealing wage equation that is an intuitive generalization of the standard Nash bargaining outcome. In this instance, a key primitive parameter of the model is the average frequency of wage adjustment. Whereas in GT this parameter is calibrated to existing evidence on wage contract length, here we are able to estimate it.

Another significant difference from GT, which is a purely real model, is that wage contracting is in nominal terms. However, as in CEE and SW, we allow for indexing of wages to past inflation and estimate the degree of indexing. This consideration is important for the following reasons. As indexing to past inflation becomes complete, nominal wage rigidity begins to approximate real wage rigidity. As Blanchard and Galí (2006) emphasize, real wage rigidity complicates the short run output/inflation trade-off that the central bank faces, beyond what would arise from simple nominal rigidities.

In section 4 we describe our estimation procedure and then present the model estimates. We use Bayesian methods, following closely SW and Primiceri, Schaumburg and Tambalotti (PST, 2006). We present a variety of diagnostics to evaluate the overall model performance and in particular the role of wage rigidity in our framework.

Before proceeding, we emphasize that there have been a number of papers related to ours. Trigari (2004) and Walsh (2005) were among the first to integrate a search and matching setup within a monetary DSGE model with nominal price rigidities. Blanchard and Galí (2006) develop a qualitative version of this model with a simple form of real wage rigidities. Christoffel et al. (2006) have also estimated a monetary DSGE model with labor market frictions and wage rigidity. They employ a setup with right-to-manage bargaining as in Trigari (2006), where ex post hours may be inefficient. Since part our interest is to address the Barro critique, we employ the setup of GT, which has efficient bargaining along with staggered wage setting. Thus, within our setup the employment of existing workers is fully efficient: wage rigidity affects hiring at the extensive margin. In addition, while Christoffel et al. (2006) model wage rigidity by introducing adjustment costs of wage changes for a representative firm, we do so by having staggered contracting.<sup>2</sup> We also differ in the details of the precise model we estimate, as well as the exact estimation procedure and data.

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example, the survey by Pissarides (2007). In Gertler, Huckfeld and Trigari (2007), however, we show that by more carefully controlling for job tenure and other forms of heterogeneity, the evidence that new workers wages are more flexible disappears. In this study we use data from the CPS and are currently extending the analysis to other data sets.

<sup>2</sup>More recently, Christiano, Motto and Rostagno (2007) have also incorporated the GT variation of the MP model into the CEE/SW framework to study stock market boom bust cycles. The focus of our paper is instead in parameter estimation and model comparison.

## 2 The Model

As we discussed, the model is a variant of the conventional monetary DSGE framework. It has the key features that many have found useful for capturing the data. These include habit formation, costs of adjusting the flow of investment, variable capital utilization, nominal price and wage rigidities, and so on. The key changes involve the labor market. Rather than having hours vary on the intensive margin, we introduce variation on the extensive margin and unemployment. We do so by introducing search and matching in the spirit of Mortensen and Pissarides and others. Further, to introduce nominal wage rigidity, we use the staggered Nash bargaining approach of GT.

We note that in an earlier version of this paper we also allowed for variation in hours on the intensive margin. We drop this feature for two reasons. First, as Figure 1 shows, most of the cyclical variation in hours in the U.S. is on the extensive margin. Second, our earlier estimates confirmed that the intensive margin was unimportant to cyclical variation. We estimated a Frisch elasticity close to zero, which is certainly in line with the microeconomic evidence.

Our model has three types of agents: households, wholesale firms, and retail firms. We use a representative family construct, similar to Merz (1995) in order to introduce complete consumption insurance. Production takes place at wholesale firms. These firms are competitive. They hire workers and negotiate wage contracts with them. Retail firms buy goods from wholesalers and then repackage them as final goods. Retailers are monopolistic competitors and set prices on a staggered basis. We separate retailers from wholesalers to keep the wage bargaining problem tractable.<sup>3</sup>

Finally, following SW, we introduce a number of exogenous shocks that correspond exactly to the number of data series we consider in our estimation.

### 2.1 Households

There is a representative household with a continuum of members of measure unity. The number of family members currently employed is  $n_t$ . Employment is determined through a search and matching process that we describe shortly.

Except for the treatment of labor supply, the household's decision problem is identical to that in CEE and SW. In the latter, the household receives utility from leisure and it varies labor only on the intensive margin. Here there is no utility gain from leisure. Individuals not currently working are searching for jobs.

Accordingly, conditional on  $n_t$ , the household chooses consumption  $c_t$ , government bonds  $B_t$ ,

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<sup>3</sup>To keep the bargaining problem tractable, it is necessary to have constant returns at the firm level. This is to make the average and marginal worker the same, thus avoiding bargaining spillovers among workers. Introducing staggered price setting requires that firms face downward sloping demand curves, implying differences between average and marginal.

capital utilization  $z_t$ , investment  $i_t$ , and physical capital  $k_t^p$  to maximize the utility function

$$E_t \sum_{s=0}^{\infty} \beta^s \varepsilon_{t+s}^b \log(c_{t+s} - hc_{t+s-1}), \quad (1)$$

where  $h$  is the degree of habit persistence in consumption preferences and where  $\varepsilon_t^b$  is a preference shock with mean unity that obeys

$$\log \varepsilon_t^b = \rho^b \log \varepsilon_{t-1}^b + \varsigma_t^b, \quad (2)$$

and where all primitive innovations, including  $\varsigma_t^b$ , are zero-mean i.i.d. random variables.

Let  $\Pi_t$  be lump sum profits,  $T_t$  lump sum transfers,  $p_t$  the nominal price level, and  $r_t$  the one period nominal interest rate (specifically, the central bank policy instrument). Then the household's budget constraint is

$$c_t + i_t + \frac{B_t}{p_t r_t} = w_t n_t + (1 - n_t) b_t + r_t^k z_t k_{t-1}^p + \Pi_t + T_t - a(z_t) k_{t-1}^p + \frac{B_{t-1}}{p_t}. \quad (3)$$

Households own capital and choose the capital utilization rate,  $z_t$ , which transforms physical capital into effective capital according to

$$k_t = z_t k_{t-1}^p. \quad (4)$$

Effective capital is rented to the firms at the rate  $r_t^k$ . The cost of capital utilization per unit of physical capital is  $a(z_t)$ . We assume that  $z_t = 1$  in the steady state and  $a(1) = 0$ .

The physical capital accumulation equation is

$$k_t^p = (1 - \delta) k_{t-1}^p + \varepsilon_t^i \left( 1 - s \left( \frac{i_t}{i_{t-1}} \right) \right) i_t, \quad (5)$$

where we assume  $s(\gamma_a) = s'(\gamma_a) = 0$  and  $s''(\gamma_a) > 0$  where  $\gamma_a$  is the economy's steady state growth rate.

$\varepsilon_t^i$  is an investment specific technological shock with mean unity affecting the efficiency with which consumption goods are transformed into capital. We assume  $\varepsilon_t^i$  follows the exogenous stochastic process

$$\log \varepsilon_t^i = \rho^i \log \varepsilon_{t-1}^i + \varsigma_t^i. \quad (6)$$

The first order necessary conditions yield:

( $c_t$ )

$$\lambda_t = \frac{\varepsilon_t^b}{c_t - hc_{t-1}} - \beta h E_t \frac{\varepsilon_{t+1}^b}{c_{t+1} - hc_t}, \quad (7)$$

( $B_t$ )

$$\lambda_t = r_t \beta E_t \left( \frac{\lambda_{t+1} p_t}{p_{t+1}} \right), \quad (8)$$

$$(z_t) \quad r_t^k = a'(z_t), \quad (9)$$

$$(i_t) \quad q_t^k \varepsilon_t^i \left(1 - s \left(\frac{i_t}{i_{t-1}}\right)\right) = q_t^k \varepsilon_t^i s' \left(\frac{i_t}{i_{t-1}}\right) \frac{i_t}{i_{t-1}} - \beta E_t q_{t+1}^k \varepsilon_{t+1}^i \frac{\lambda_{t+1}}{\lambda_t} s' \left(\frac{i_{t+1}}{i_t}\right) \left(\frac{i_{t+1}}{i_t}\right)^2 + 1, \quad (10)$$

$$(k_t^p) \quad q_t^k = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left( (1 - \delta) q_{t+1}^k + r_{t+1}^k z_{t+1} - a(z_{t+1}) \right), \quad (11)$$

where  $\lambda_t = u'(c_t)$  and  $q_t^k$  is the value of installed capital in consumption units.

Again, except for the treatment of the labor supply decision, the household sector is conventional.

## 2.2 Unemployment, Vacancies and Matching

At time  $t$ , each firm posts  $v_t(i)$  vacancies in order to attract new workers and employs  $n_t(i)$  workers. The total number of vacancies and employed workers are  $v_t = \int_0^1 v_t(i) di$  and  $n_t = \int_0^1 n_t(i) di$ . All unemployed workers at  $t$  look for jobs. Our timing assumptions are such that unemployed workers who find a match go to work immediately within the period. Accordingly, the pool of unemployed workers searching for a job at  $t$ ,  $u_t$ , is given by the difference between unity (the total population of workers) and the number of employed workers at the end of period  $t - 1$ ,  $n_{t-1}$ :

$$u_t = 1 - n_{t-1}. \quad (12)$$

The number of new hires or “matches”,  $m_t$ , is a function of searching workers and vacancies, as follows:

$$m_t = \sigma_m u_t^\sigma v_t^{1-\sigma}. \quad (13)$$

The probability a firm fills a vacancy in period  $t$ ,  $q_t$ , is given by

$$q_t = \frac{m_t}{v_t}. \quad (14)$$

Similarly, the probability a searching worker finds a job,  $s_t$ , is given by

$$s_t = \frac{m_t}{u_t}. \quad (15)$$

Both firms and workers take  $q_t$  and  $s_t$  as given.

Finally, each period, firms exogenously separate from a fraction  $1 - \rho$  of their existing workforce  $n_{t-1}(i)$ . Workers losing their job at time  $t$  are not allowed to search until next period. Accordingly, within our framework fluctuations in unemployment are due to cyclical variation in hiring as opposed to separations. Both Hall (2005) and Shimer (2005, 2007) present evidence in support of this phenomenon.

### 2.3 Wholesale Firms

Each period, wholesale firms produce output  $y_t(i)$  using capital,  $k_t(i)$ , and labor,  $n_t(i)$ :

$$y_t(i) = [k_t(i)]^\alpha [a_t n_t(i)]^{1-\alpha}, \quad (16)$$

where  $a_t$  is a common labor-augmenting productivity factor. We assume  $\varepsilon_t^a = a_t/a_{t-1}$  obeys the following exogenous stochastic process

$$\log \varepsilon_t^a = (1 - \rho^a) \log \varepsilon^a + \rho^a \log \varepsilon_{t-1}^a + \zeta_t^a. \quad (17)$$

Note that the steady state value  $\varepsilon^a$  corresponds to the economy's growth rate  $\gamma_a$ . Thus, following PST, we are allowing technology be non-stationary in levels, though stationary in growth rates.

For simplicity, we assume that capital is perfectly mobile across firms and that there is a competitive rental market in capital. These assumptions ensure constant returns to scale at the firm level, which greatly simplifies the wage bargaining problem (see the discussion in GT).

It is useful to define the hiring rate  $x_t(i)$  as the ratio of new hires  $q_t v_t(i)$  to the existing workforce  $n_{t-1}(i)$ :

$$x_t(i) = \frac{q_t v_t(i)}{n_{t-1}(i)}. \quad (18)$$

Observe that due to the law of large numbers the firm knows  $x_t(i)$  with certainty at time  $t$  since it knows the likelihood  $q_t$  that each vacancy it posts will be filled. The hiring rate is thus effectively the firm's control variable.

The total workforce, in turn, is the sum of the number of surviving workers  $\rho n_{t-1}(i)$  and new hires  $x_t(i) n_{t-1}(i)$ :

$$n_t(i) = \rho n_{t-1}(i) + x_t(i) n_{t-1}(i). \quad (19)$$

Equation (19) reflects the timing assumption that new hires go to work immediately.<sup>4</sup>

Let  $p_t^w$  be the relative price of intermediate goods,  $w_t(i)$  the wage,  $r_t^k$  the rental rate of capital, and  $\beta E_t \Lambda_{t,t+1}$  be the firm's discount rate, where the parameter  $\beta$  is the household's subjective discount factor and where  $\Lambda_{t,t+1} = \lambda_{t+1}/\lambda_t$ . Then, the value of the firm,  $F_t(i)$ , may be expressed as:

$$F_t(i) = p_t^w y_t(i) - w_t(i) n_t(i) - \frac{\kappa_t}{2} x_t^2(i) n_{t-1}(i) - r_t^k k_t(i) + \beta E_t \Lambda_{t,t+1} F_{t+1}(i), \quad (20)$$

with

$$\kappa_t = \kappa a_t. \quad (21)$$

As wage dispersion is present, we replace the standard assumption of fixed costs of posting a vacancy with quadratic labor adjustment costs, given here by  $\frac{\kappa_t}{2} x_t^2(i) n_{t-1}(i)$ . We allow adjustment costs to drift proportionately with productivity in order to maintain a balanced steady state growth path (otherwise adjustment costs become relatively less important as the economy grows.)

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<sup>4</sup>Blanchard and Gali (2006) use a similar timing.

At any time, the firm maximizes its value by choosing the hiring rate (by posting vacancies) and its capital stock, given its existing employment stock, the probability of filling a vacancy, the rental rate on capital and the current and expected path of wages. If it is a firm that is able to renegotiate the wage, it bargains with its workforce over a new contract. If it is not renegotiating, it takes as given the wage at the previous period's level, as well the likelihood it will be renegotiating in the future.

We next consider the firm's hiring and capital rental decisions, and defer a bit the description of the wage bargain. The first order condition for capital is simply:

$$r_t^k = p_t^w \alpha \frac{y_t(i)}{k_t(i)} = p_t^w \alpha \frac{y_t}{k_t}. \quad (22)$$

Given Cobb-Douglas technology and perfect capital mobility, all firms choose the same capital/output ratio and, in turn, the same capital/labor and labor/output ratios

Firms choose  $n_t(i)$  by setting  $x_t(i)$  or, equivalently,  $v_t(i)$ . The firm's hiring decision yields:

$$\kappa_t x_t(i) = p_t^w f_{nt}(i) - w_t(i) + \beta E_t \Lambda_{t,t+1} \partial F_{t+1}(i) / \partial n_t(i), \quad (23)$$

with

$$f_{nt}(i) = (1 - \alpha) \frac{y_t(i)}{n_t(i)} = (1 - \alpha) \frac{y_t}{n_t} = f_{nt}. \quad (24)$$

By making use of the envelope theorem to obtain  $\partial F_t(i) / \partial n_{t-1}(i)$  and combining equations, we obtain

$$\kappa_t x_t(i) = p_t^w f_{nt} - w_t(i) + \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}^2(i) + \rho \beta E_t \Lambda_{t,t+1} \kappa_{t+1} x_{t+1}(i). \quad (25)$$

The hiring rate thus depends on the discounted stream of earnings and savings on adjustment costs.

Finally, for the purpose of the wage bargain it is useful to define  $J_t(i)$ , the value to the firm of having another worker at time  $t$  after new workers have joined the firm, i.e., after adjustment costs are sunk. Differentiating  $F_t(i)$  w.r.t.  $n_t(i)$ , taking  $x_t(i)$  as given yields:

$$J_t(i) = p_t^w f_{nt} - w_t(i) + \beta E_t \Lambda_{t,t+1} \partial F_{t+1}(i) / \partial n_t(i). \quad (26)$$

By making use of the hiring rate condition and the relation for the evolution of the workforce,  $J_t(i)$  may be expressed as expected average profits per worker net of the first period adjustment costs, with the discount factor accounting for future changes in workforce size:

$$J_t(i) = p_t^w f_{nt} - w_t(i) - \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}^2(i) + E_t \frac{n_{t+1}}{n_t}(i) \beta \Lambda_{t,t+1} J_{t+1}(i). \quad (27)$$



## 2.4 Workers

In this sub-section we develop an expression for a worker's surplus from employment, which is a critical determinant of the outcome of the wage bargain.

Let  $V_t(i)$  be the value to a worker of employment at firm  $i$  and let  $U_t$  be the value of unemployment. These values are defined after hiring decisions at time  $t$  have been made and are in units of consumption goods.  $V_t(i)$  is given by

$$V_t(i) = w_t(i) + \beta E_t \Lambda_{t,t+1} [\rho V_{t+1}(i) + (1 - \rho) U_{t+1}]. \quad (28)$$

To construct the value of unemployment, we first define  $V_{x,t}$  as the average value of employment conditional on being a new worker at  $t$ :

$$V_{x,t} = \int_0^1 V_t(i) \frac{x_t(i) n_{t-1}(i)}{x_t n_{t-1}} di, \quad (29)$$

where  $x_t(i) n_{t-1}(i)$  is total new workers at firm  $i$  and  $x_t n_{t-1}$  is total new workers at  $t$ .<sup>5</sup> Next, let  $b_t$  be the flow value from unemployment, including unemployment benefits, as well as other factors that can be measured in units of consumption goods. As before, let  $s_t$  be the probability of finding a job for the subsequent period. Then  $U_t$  may be expressed as

$$U_t = b_t + \beta E_t \Lambda_{t,t+1} [s_{t+1} V_{x,t+1} + (1 - s_{t+1}) U_{t+1}], \quad (30)$$

with

$$b_t = b k_t^p,$$

and where  $k_t^p$  is the economy-wide capital stock. We assume that  $b_t$  grows proportionately to  $k_t^p$  in order to maintain a balanced growth; otherwise  $b_t$  would become a smaller fraction of labor income as the economy grows. The value of unemployment thus depends on the current flow value  $b_t$  and the likelihood of being employed versus unemployed next period. Note that the value of finding a job next period for a worker that is currently unemployed is  $V_{x,t+1}$ , the average value of working next period conditional on being a new worker. That is, unemployed workers do not have a priori knowledge of which firms might be paying higher wages next period. They instead just randomly flock to firms posting vacancies.<sup>6</sup>

$H_t(i)$  and  $H_{x,t}$  are given by:

$$H_t(i) = V_t(i) - U_t, \quad (31)$$

and

$$H_{x,t} = V_{x,t} - U_t. \quad (32)$$

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<sup>5</sup> $V_{x,t}$  is thus distinct from the unconditional average value of employment  $V_t = \int_0^1 V_t(i) \frac{n_t(i)}{n_t} di$ .

<sup>6</sup>There is accordingly no directed search. Note, however, that wage differentials across firms are only due to the differential timing of contracts, which is transitory. Thus, because a worker who arrives at a firm in the midst of an existing contract may expect a new one reasonably soon, the payoff from directed search may not be large.

It follows that:

$$H_t(i) = w_t(i) - b_t + \beta E_t \Lambda_{t,t+1} [\rho H_{t+1}(i) - s_{t+1} H_{x,t+1}]. \quad (33)$$

## 2.5 Nash Bargaining and Wage Dynamics

As we noted earlier, we introduce staggered Nash wage bargaining as in GT, but with the following difference: Because we have a monetary model, we allow for nominal wage contracting, though we also allow for the possibility of indexing to past inflation.

We introduce staggered multi-period wage contracting in a way that simplifies aggregation. In particular, each period a firm has a fixed probability  $1 - \lambda$  that it may re-negotiate the wage.<sup>7</sup> This adjustment probability is independent of its history, making it unnecessary to keep track of individual firms' wage histories. Thus, while how long an individual wage contract lasts is uncertain, the average duration is fixed at  $1/(1 - \lambda)$ . The coefficient  $\lambda$  is thus a measure of the degree of wage stickiness. In GT the parameter  $\lambda$  is calibrated. Here we are able to estimate it.

Since we allow for the possibility of indexing to past inflation,  $\pi_{t-1}$ , the fraction  $\lambda$  of firms that cannot renegotiate their contract set their nominal wages  $w_t^n(i)$  following the indexation rule:

$$w_t^n(i) = \bar{\gamma} w_{t-1}^n(i) \pi_{t-1}^\gamma, \quad (34)$$

where  $\pi_t = p_t/p_{t-1}$ ,  $\bar{\gamma} = \gamma_a \pi^{1-\gamma}$  and where  $\gamma \in [0, 1]$  reflects the degree of indexing to past inflation. We also estimate the parameter  $\gamma$ . The term  $\bar{\gamma}$  in the indexing rule provides an adjustment for trend productivity growth and trend inflation.

Firms that enter a new wage agreement at  $t$  negotiate with the existing workforce, including the recent new hires. Due to constant returns, all workers are the same at the margin. The wage is chosen so that the negotiating firm and the marginal worker share the surplus from the marginal match. Given the symmetry to which we just alluded, all workers employed at the firm receive the same newly-negotiated wage.<sup>8</sup> When firms are not allowed to renegotiate the wage, all existing and newly hired workers employed at the firm receive the existing contracting wage (i.e. last period nominal wage adjusted for possibly indexing.)<sup>9</sup> As we discussed earlier, we appeal to scale economies in bargaining to rule out separate negotiations for worker who arrive in between

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<sup>7</sup>This kind of Poisson adjustment process is widely used in macroeconomic models with staggered price setting, beginning with Calvo (1983).

<sup>8</sup>To be clear, with constant returns, one could either think of the firm bargaining with each marginal worker individually or bargaining with a union that wishes to maximize average worker surplus.

<sup>9</sup>As we noted earlier, in Gertler, Huckfeld and Trigari (2007) we present evidence in support of this assumption. Bewley (1999) also presents survey evidence that the wages of new hires are linked to those of existing workers. Other studies of the cyclical behavior of wages for new hires (e.g. Bils, 1985) do not examine the link with existing workers wages (due to data limitations) and thus do not speak to our hypothesis. We think that explaining the facts in these studies will require introducing heterogeneity into our framework.

contracting periods.<sup>10</sup> Of course, the newly hired workers recognize that they will be able to re-negotiate the wage at the next round of contracting.

Let  $w_t^{n*}$  denote the wage of a firm that renegotiates at  $t$ . Given constant returns, all sets of renegotiating firms and workers at time  $t$  face the same problem, and thus set the same wage. As we noted earlier, the firm negotiates with the marginal worker over the surplus from the marginal match. We assume Nash bargaining, which implies that the contract wage  $w_t^{n*}$  is chosen to solve

$$\max H_t(i)^{\eta_t} J_t(i)^{1-\eta_t}, \quad (35)$$

s.t.

$$w_{t+j}^n(i) = \begin{cases} \bar{\gamma} w_{t+j-1}^n(i) \pi_{t+j-1}^\gamma & \text{with probability } \lambda \\ w_{t+j}^{n*} & \text{with probability } 1 - \lambda \end{cases}$$

$\forall j \geq 1$ , where  $\eta_t \in [0, 1]$  is the worker's relative bargaining power, and where  $J_t(i)$  and  $H_t(i)$  are given by equations (27) and (33).

In the conventional search and matching framework the bargaining power parameter is constant. Here, in order to allow for an error term in the wage equation we allow this parameter to evolve exogenously according to:

$$\eta_t = \eta \varepsilon_t^\eta, \quad (36)$$

where  $\varepsilon_t^\eta$  is a mean-unity bargaining power shock that follows the stochastic process:

$$\log \varepsilon_t^\eta = \rho^\eta \log \varepsilon_{t-1}^\eta + \zeta_t^\eta. \quad (37)$$

The first order necessary condition for the Nash bargaining solution is given by

$$\eta \frac{\partial H_t(i)}{\partial w_t^n(i)} J_t(i) + (1 - \eta) \frac{\partial J_t(i)}{\partial w_t^n(i)} H_t(i) = 0. \quad (38)$$

Letting  $\Delta_t = p_t \partial H_t(i) / \partial w_t^n(i)$  and  $\Sigma_t(i) = -p_t \partial J_t(i) / \partial w_t^n(i)$ , we have:

$$\Delta_t = 1 + E_t \Lambda_{t,t+1} (\rho \lambda \beta) \frac{p_t}{p_{t+1}} \bar{\gamma} \pi_t^\gamma \Delta_{t+1}, \quad (39)$$

and

$$\Sigma_t(i) = 1 + E_t \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(i) (\lambda \beta) \frac{p_t}{p_{t+1}} \bar{\gamma} \pi_t^\gamma \Sigma_{t+1}(i). \quad (40)$$

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<sup>10</sup>In addition to scale economies in bargaining, there are several complementary justifications for why hires in between contracts receive the existing contract wage. First, as we noted earlier, Bewley (1999) argues that internal equity constrains workers of similar productivity to receive similar wages. Second, Menzio and Moen (2006) show how the trade-off between efficient provision of insurance to senior workers and efficient recruitment of junior ones links the wages of new and existing workers in response to small and negative productivity shocks. Third, consistent with Hall (2005a), one might interpret the existing contract wage as the “wage norm” for workers hired in between contracts.

Observe that  $\Delta_t$  and  $-\Sigma_t(i)$  are the derivatives of the worker and the firm surpluses with respect to the real wage.

The first order condition for wages, then, can be rewritten as:

$$\chi_t(i) J_t(i) = (1 - \chi_t(i)) H_t(i), \quad (41)$$

with

$$\chi_t(i) = \frac{\eta_t}{\eta_t + (1 - \eta_t) \Sigma_t(i) / \Delta_t}. \quad (42)$$

One difference between our solution and the period-by-period Nash solution is that the relative weight  $\chi_t(i)$  depends not only on the worker's bargaining power  $\eta_t$ , but also on  $\Sigma_t(i)$  and  $\Delta_t$ , the firm and worker's respective cumulative discount factors. This difference arises because the two agents have different horizons: unlike the worker, the firm cares about the implications of the contract for new workers as well as existing ones, whereas the worker only cares about his or her tenure at the job. Observe that the firm's cumulative discount factor depends on the expectation of the product of five factors: the firm's employment size at time  $t + 1$  relative to time  $t$ ,  $\frac{n_{t+1}}{n_t}(i)$ , the probability the contract survives to  $t + 1$ ,  $\lambda$ , the inverse of inflation at  $t + 1$ ,  $p_t/p_{t+1}$ , a correction for indexing to past inflation and growth,  $\bar{\gamma}\pi_t^\gamma$ , and the households' discount factor,  $\beta\Lambda_{t,t+1}$ . It is similar for the worker, except the survival probability  $\rho$  replaces the relative employment size. Since on average  $\frac{n_{t+1}}{n_t}(i)$  exceeds  $\rho$ , the firm places relatively more weight on the future than does the worker. Since  $\Sigma_t(i) / \Delta_t$  exceeds unity on average,  $\chi_t(i)$  is less than  $\eta_t$  on average. Intuitively, since movements in the contract wage have a larger impact on discounted firm surplus than on worker surplus, the "horizon effect" works to raise the effective bargaining power of firms from  $1 - \eta$  to  $1 - \chi(i)$  on average. On the other hand, while the horizon effect (arising from the difference between  $\eta_t$  and  $\chi_t(i)$ ) may be of interest from a theoretical perspective, it will turn out not to be of quantitative importance in this case.

Following GT and converting nominal to real wages, the solution to the bargaining problem leads to the following difference equation for the real wage  $w_t^* = w_t^{n^*}/p_t$ <sup>11</sup>:

$$\Delta_t w_t^* = w_t^o(i) + \rho\lambda\beta E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^*, \quad (43)$$

where the forcing variable  $w_t^o(i)$  can be thought of as the real "target" wage and is given by

$$\begin{aligned} w_t^o(i) &= \chi \left( p_t^w f_{nt} + \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}^2(i) \right) \\ &+ (1 - \chi) (b_t + s_{t+1} \beta E_t \Lambda_{t,t+1} H_{x,t+1}) + \Phi_t(i), \end{aligned} \quad (44)$$

with

$$\begin{aligned} \Phi_t &= \phi_t(i) - \rho\beta E_t \Lambda_{t,t+1} \phi_{t+1}(i), \\ \phi_t(i) &= (\chi_t(i) - \chi) [J_t(i) + H_t(i)]. \end{aligned}$$

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<sup>11</sup>See the appendix for details and derivations.

In contrast to the standard case of period-by-period Nash bargaining, the wage set in the current period may apply in the future. Hence in this instance the contract wage  $w_t^*$  depends not only on the current target wage, but on the expected sequence of future target wages (as iterating forward equation (43) suggests.)

Observe that the target compensation,  $w_t^o(i)$ , has the same form that would emerge under period-by-period Nash bargaining. In particular, it is a convex combination of what a worker contributes to the match and what the worker loses by accepting a job, where the weights depend (indirectly) on the worker's relative bargaining power,  $\eta$ . The worker's contribution is the marginal product of labor plus the saving on adjustment costs. With our quadratic cost formulation, this saving is measured by  $\frac{\kappa_{t+1}}{2}x_{t+1}^2(i)$ . The foregone benefit from unemployment, in turn, is the flow value of unemployment,  $b_t$ , plus the expected discounted gain of moving from unemployment this period to employment next period,  $s_t\beta\Lambda_{t,t+1}H_{x,t+1}$ . The additional term  $\Phi_t(i)$  reflects the horizon effect, as well as exogenous movements in the bargaining power.

In the limiting case of period-by-period wage contracting,  $\lambda = 0$ , the solution corresponds to the conventional Nash bargaining outcome:  $w_t^* = w_t^o(i)$ , with  $\chi_t(i) = \eta_t$ . With multi-period contracting, however,  $w_t^*$  depends on a discounted stream of future values of  $w_t^o(i)$ , where the latter depend on horizon adjusted bargaining weights.

Finally, the average nominal wage across workers is given by

$$w_t^n = \int_0^1 w_t^n(i) \frac{n_t(i)}{n_t} di. \quad (45)$$

## 2.6 Retailers

There is a continuum of monopolistically competitive retailers indexed by  $j$  on the unit interval. Retailers buy intermediate goods from the wholesale firms described in the previous section. They in turn differentiate them with a technology that transforms one unit of intermediate goods into one unit of retail goods, then re-sell them to the households. In addition, they set prices on a staggered basis.

Following SW, we permit each firm's elasticity to depend inversely on its relative market share, using a formulation due to Kimball (1995). The endogenous elasticity introduces a strategic complementarity in price setting (or real rigidity) that makes it easier for the model to match the micro evidence on price adjustment (see, e.g., the discussion in Woodford, 2003.)

The Kimball formulation is based on a generalization of the Dixit/Stiglitz aggregator of individual goods into a composite that allows for a varying elasticity of substitution between goods. Let  $y_t(j)$  be the quantity of output sold by retailer  $j$  and let  $p_t(j)$  be the nominal sale price. Under the Kimball formulation, accordingly, final goods, denoted with  $y_t$ , are a composite of individual retail goods, defined as follows:

$$\int_0^1 G\left(\frac{y_t(j)}{y_t}, \varepsilon_t^p\right) dj = 1 \quad (46)$$

where the function  $G$  is increasing and strictly concave, with  $G(1) = 1$ .  $\varepsilon_t^p$  is an exogenous non-negative random variable that influences the elasticity of demand and thus the frictionless price markup. We assume that  $\varepsilon_t^p$  evolves as follows:

$$\log \varepsilon_t^p = (1 - \rho^p) \log \varepsilon^p + \rho^p \log \varepsilon_{t-1}^p + \zeta_t^p. \quad (47)$$

Cost minimization then leads to the follow demand curve facing each retailer  $j$ :

$$y_t(j) = G'^{-1} \left[ \frac{p_t(j)}{p_t} \int_0^1 G' \left( \frac{y_t(j)}{y_t} \right) \frac{y_t(j)}{y_t} dj \right] y_t, \quad (48)$$

where  $p_t$  is the aggregate price index.

We assume ‘‘Calvo’’ staggering applies to price setting and we also allow for indexing. Let  $1 - \lambda^p$  be the probability a firm adjusts its price. Firms not adjusting their target price obey the following indexing rule:

$$p_t(j) = \bar{\gamma}^p p_{t-1}(j) \pi_{t-1}^{\gamma^p}, \quad (49)$$

where, in analogy to the case of wage setting, both  $\gamma^p$  and  $\lambda^p$  are parameters that we estimate, and where  $\bar{\gamma}^p = \pi^{1-\gamma^p}$  is an adjustment for trend inflation.

Firms that are able to adjust their price at  $t$ , set the price to maximize expected discounted profits subject to the constraints on the frequency of price adjustment. Since all firms that are resetting price are identical ex ante, they all choose the price,  $p_t^*$ . It follows that by combining equations (46) and (48) and applying the law of large numbers, we can express the price index as

$$p_t = (1 - \lambda^p) p_t^* G'^{-1} \left( \frac{p_t^*}{p_t} \tau_t \right) + \lambda^p (\bar{\gamma}^p p_{t-1} \pi_{t-1}^{\gamma^p}) G'^{-1} \left( \frac{\bar{\gamma}^p p_{t-1} \pi_{t-1}^{\gamma^p}}{p_t} \tau_t \right), \quad (50)$$

where  $\tau_t = \int_0^1 G' \left( \frac{y_t(j)}{y_t} \right) \frac{y_t(j)}{y_t} dj$ .

Re-optimizing retailers choose a target price,  $p_t^*$ , to maximize the following discounted stream of future profits:

$$E_t \sum_{s=0}^{\infty} (\lambda^p \beta)^s \Lambda_{t,t+s} \left[ \frac{p_t^*}{p_{t+s}} \left( \prod_{k=1}^s \bar{\gamma}^p \pi_{t+k-1}^{\gamma^p} \right) - p_{t+s}^w \right] y_{t+s}(j), \quad (51)$$

subject to the demand curve given by equation (48). Note that the sequence of discount factors depend on  $\lambda^p$ , the probability that the price remains fixed in the subsequent period.

The first order condition for the target price is given by

$$E_t \sum_{s=0}^{\infty} (\lambda^p \beta)^s \Lambda_{t,t+s} \left[ (1 + \Theta_t) \frac{p_t^*}{p_{t+s}} \left( \prod_{k=1}^s \bar{\gamma}^p \pi_{t+k-1}^{\gamma^p} \right) - \Theta_t p_{t+s}^w \right] y_{t+s}(j) = 0, \quad (52)$$

with

$$\Theta_t = [G'^{-1}(\tau_t p_t^*/p_{t+s})]^{-1} \{ G' [G'^{-1}(\tau_t p_t^*/p_{t+s})] / G'' [G'^{-1}(\tau_t p_t^*/p_{t+s})] \}.$$

By loglinearizing this condition, one can show that  $p_t^*$  depends on an expected discounted stream of the retailers nominal marginal cost, given by the nominal wholesale price  $p_t p_t^w$ .

By inverting the hiring condition derived earlier, one can obtain an expression for the retailers real marginal cost,  $p_t^w$ :

$$p_t^w = \frac{1}{f_{nt}} \left[ w_t(i) + \kappa_t x_t(i) - \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}^2(i) - \beta E_t \Lambda_{t,t+1} \rho \kappa_{t+1} x_{t+1}(i) \right]. \quad (53)$$

Real marginal costs thus depend on unit labor costs, plus terms that correct for the adjustment costs of hiring workers.

Observe that since we have normalized the relative price of final output at unity, the retailer's markup is given by  $\mu_t^p = 1/p_t^w$ . Since final goods prices are sticky and wholesale prices are flexible, this markup will in general exhibit cyclical behavior, with the direction depending on the nature of the disturbances hitting the economy, as well as other features of the model.

Finally, as we show in the appendix, by loglinearizing expressions for the price index and for the optimal reset price, equations (50) and (52), one can obtain a relation for consumer price inflation that is a variation of the conventional hybrid New Keynesian Phillips curve relating inflation to movements in real marginal cost, expected future inflation and lagged inflation. In this instance real marginal cost is simply the relative price of wholesale goods,  $p_t^w$ . The slope coefficient on real marginal cost, further, depends inversely on the degree of price stickiness, measured by  $\lambda^p$ . It also depends inversely on  $\xi$ , the percent change in the firm's price elasticity with respect to a one percent change in its relative price.<sup>12</sup> Everything else equal, a positive value of  $\xi$  reduces the firm's desired adjustment of its relative price. As discussed in Woodford (2003), this introduces a pricing complementarity that induces price adjusters to keep their relative prices closer to those of non-adjusters. This in turn reduces the sensitivity of inflation to movements in real marginal cost.

## 2.7 Government

Monetary policy obeys the following simple Taylor rule:

$$\frac{r_t}{r} = \left( \frac{r_{t-1}}{r} \right)^{\rho^s} \left[ \left( \frac{\pi_t}{\pi} \right)^{r_\pi} \left( \frac{y_t}{y_{nt}} \right)^{r_y} \right]^{(1-\rho^s)} \varepsilon_t^r, \quad (54)$$

where

$$\log \varepsilon_t^r = \rho^r \log \varepsilon_{t-1}^r + \zeta_t^r. \quad (55)$$

Government spending obeys:

$$g_t = \left( 1 - \frac{1}{\varepsilon_t^g} \right) y_t, \quad (56)$$

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<sup>12</sup>Let  $\eta = -G'(y_t(j)/y_t) / \frac{y_t(j)}{y_t} G''(y_t(j)/y_t)$  be the firm's elasticity of demand with respect to shifts in its market share. Then  $\xi = \frac{\partial \eta}{\partial (p_t(j)/p_t)} \frac{p_t(j)/p_t}{\eta}$ .

where

$$\log \varepsilon_t^g = (1 - \rho^g) \ln \varepsilon^g + \rho^g \log \varepsilon_{t-1}^g + \varsigma_t^g. \quad (57)$$

## 2.8 Resource Constraint

The resource constraint divides output between consumption, investment and adjustment and utilization costs:

$$y_t = c_t + i_t + g_t + (\kappa_t/2) \int_0^1 x_t^2(i) n_{t-1}(i) di + a(z_t) k_{t-1}^p. \quad (58)$$

This completes the description of the model.

## 3 Wage and Hiring Dynamics

The key features of the model that differentiates it from the conventional monetary DSGE model involve wage and hiring dynamics. In this section, accordingly, we derive loglinear relationships for these variables.<sup>13</sup> The analysis closely follows GT.

Loglinearizing the expression for the contract wage, we obtain the following loglinear difference equation for real contract wage,  $\widehat{w}_t^*$ :

$$\widehat{w}_t^* = [(1 - \rho\lambda\beta) \widehat{w}_t^o(i) + \rho\lambda\beta E_t(\widehat{\pi}_{t+1} - \gamma\widehat{\pi}_t + \widehat{\varepsilon}_{t+1}^a)] + \rho\lambda\beta E_t \widehat{w}_{t+1}^*. \quad (59)$$

The contract wage depends on the current and expected future path of the target wage  $\widehat{w}_t^o(i)$  and terms that reflect adjustments for indexing.

Let  $\widehat{w}_t^o$  be the “spillover free” target wage. Absent the horizon effect, it corresponds to the real wage that would arise if all firms were doing period-by-period Nash bargaining, i.e., the case of perfectly flexible wages. It is possible to show that:

$$\widehat{w}_t^o(i) = \widehat{w}_t^o + \frac{\tau_1}{1 - \rho\lambda\beta} E_t(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*) + \frac{\tau_2}{1 - \rho\lambda\beta} (\widehat{w}_t - \widehat{w}_t^*), \quad (60)$$

with

$$\begin{aligned} \widehat{w}_t^o &= \varphi_{fn} (\widehat{p}_t^w + \widehat{f}_{nt}) + (\varphi_x + \varphi_s) E_t \widehat{x}_{t+1} + \varphi_s E_t \widehat{s}_{t+1} + \varphi_b \widehat{b}_t \\ &+ (\varphi_s + \varphi_x/2) (E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t) + \varphi_\chi (\widehat{\chi}_t - \beta(\rho - s) \widehat{\chi}_{t+1}) + \widehat{\varepsilon}_t^w, \end{aligned} \quad (61)$$

with

$$\begin{aligned} \bar{w} &= (w/a) & \epsilon &= (\kappa x)^{-1} \\ \varphi_{fn} &= \chi p^w (f_n/a) \bar{w}^{-1} & \varphi_x &= \chi \beta (1 - \rho) \epsilon^{-1} \bar{w}^{-1} \\ \varphi_b &= (1 - \chi) b (k/a) \bar{w}^{-1} & \varphi_s &= \chi \beta s \epsilon^{-1} \bar{w}^{-1} \\ \varphi_\chi &= \chi (1 - \chi)^{-1} \epsilon^{-1} \bar{w}^{-1} \end{aligned}$$

<sup>13</sup>See the appendix for details and derivations.



and

$$\begin{aligned}\widehat{\varepsilon}_t^w &= \varphi_\eta (1 - \beta \rho^\eta (\rho - s)) \widehat{\varepsilon}_t^\eta \\ \varphi_\eta &= \chi (1 - \eta)^{-1} \epsilon^{-1} \bar{w}^{-1}\end{aligned}$$

and where

$$\begin{aligned}\tau_1 &= \varphi_s \Gamma (1 - \rho \lambda \beta), \\ \tau_2 &= [\varphi_x \lambda - \varphi_\chi (1 - \chi) (1 - \rho) \Psi (1 - \rho \lambda \beta)] \epsilon \Sigma \bar{w} (1 - \rho \lambda \beta).\end{aligned}$$

As in GT, the parameters  $\tau_1$  and  $\tau_2$  in equation (60) reflect the influence of “spillovers” on the bargaining process: The second term on the right is a direct spillover. If, everything else equal,  $E_t \widehat{w}_{t+1}$  exceeds  $E_t \widehat{w}_{t+1}^*$ , opportunities are unusually good for workers expecting to move into employment next period, and vice-versa if  $E_t \widehat{w}_{t+1}$  is below  $E_t \widehat{w}_{t+1}^*$ . By influencing the worker’s outside option in this way, the expected average market wage at  $t + 1$  induces a direct spillover effect on the wage bargain. An indirect spillover emerges in the third term because the hiring rate of the renegotiating firm affects the bargaining outcome. It does so by influencing both the firm’s saving in adjustment costs and the horizon-adjusted bargaining weight (because it affects the firm’s cumulative discount factor.) The difference between the hiring rate  $\widehat{x}_t(i)$  and average hiring rate  $\widehat{x}_t$  depends positively on the difference between the average market wage  $\widehat{w}_t$  and the contract wage  $\widehat{w}_t^*$ . Both spillover effects work to increase the degree of wage rigidity.

The loglinearized real wage index is in turn given by

$$\widehat{w}_t = (1 - \lambda) \widehat{w}_t^* + \lambda (\widehat{w}_{t-1} - \widehat{\pi}_t + \gamma \widehat{\pi}_{t-1} - \widehat{\varepsilon}_t^a). \quad (62)$$

Combining these equations along with the relation for  $\widehat{w}_t^o(i)$  then yields:

$$\widehat{w}_t = \gamma_b (\widehat{w}_{t-1} - \widehat{\pi}_t + \gamma \widehat{\pi}_{t-1} - \widehat{\varepsilon}_t^a) + \gamma_o \widehat{w}_t^o + \gamma_f (\widehat{w}_{t+1} + \widehat{\pi}_{t+1} - \gamma \widehat{\pi}_t + \widehat{\varepsilon}_{t+1}^a), \quad (63)$$

$$\begin{aligned}\gamma_b &= (1 + \tau_2) \phi^{-1}, \\ \gamma_o &= \varsigma \phi^{-1}, \\ \gamma_f &= (\rho \beta - \tau_1) \phi^{-1}, \\ \phi &= 1 + \tau_2 + \varsigma + \rho \beta - \tau_1, \\ \varsigma &= (1 - \lambda) (1 - \rho \lambda \beta) \lambda^{-1}.\end{aligned}$$

with  $\gamma_b + \gamma_o + \gamma_f = 1$ . Note the forcing variable in the difference equation is the “spillover free” target wage  $\widehat{w}_t^o$ .

Due to staggered contracting,  $\widehat{w}_t$  depends on the lagged wage  $\widehat{w}_{t-1}$  as well as the expected future wage  $E_t \widehat{w}_{t+1}$ . Solving out for the reduced form will yield an expression that relates the wage to the lagged wage and a discounted stream of expected future values of  $\widehat{w}_t^o$ . Note that the spillover effects,

measured by  $\tau_1$  and  $\tau_2$  work to raise the relative importance of the lagged wage (by raising  $\gamma_b$ ) and reduce the importance of the expected future wage (by reducing  $\gamma_f$ ). In this way, the spillovers work to raise the inertia in the evolution of the wage. In this respect, the spillover effects work in a similar (though not identical) way as to how real relative price rigidities enhance nominal price stickiness in monetary models with time-dependent pricing (see, for example, Woodford, 2003).

Note also that as we converge to  $\lambda = 0$  (the case of period by period wage bargaining), both  $\gamma_b$  and  $\gamma_f$  go to zero), implying that  $\widehat{w}_t$  simply tracks  $\widehat{w}_t^o$  in this instance. Further, as we noted earlier,  $\widehat{w}_t^o$ , becomes identical to the wage in the flexible case. The model thus nests the conventional period-by-period wage bargaining setup.

Next, loglinearizing the equation for the hiring rate and aggregating yields:

$$\widehat{x}_t = \epsilon p^w (f_n/a) \left( \widehat{p}_t^w + \widehat{f}_{nt} \right) - \epsilon (w/a) \widehat{w}_t + \beta E_t \widehat{x}_{t+1} + (\rho + x/2) \beta \left( E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t \right). \quad (64)$$

The hiring rate thus depends on current and expected movements of the marginal product of labor relative to the wage. The stickiness in the wage due to staggered contracting, everything else equal, implies that current and expected movement in the marginal product of labor will have a greater impact on the hiring rate than would have been the case otherwise.

## 4 Model Estimation

### 4.1 Estimation Procedure

We consider seven variables in our estimation. To facilitate comparison with the literature, we employ the seven quarterly series used in CEE, SW and PST.<sup>14</sup> Thus, the variables we use include: (1) per capita real GDP; (2) per capita real personal consumption expenditures of nondurables; (3) per capita real investment equal to the sum of per capita real private investment plus per capita real personal consumption of durables; (4) hours of all persons in the non-farm business sector divided by the population times the ratio of total employment to employment in non-farm business sector; (5) the real wage (compensation per hour in the nonfarm business sector); (6) inflation as measured by the quarter to quarter growth rate of the GDP deflator; and (7) the Federal Funds rate expressed in quarterly terms. To convert any nominal variable to real terms, we always use the GDP deflator. The sample goes from 1960Q1 to 2005Q1.

We first log-linearize the model around a deterministic balanced growth steady state. The appendix contains the complete log-linear model, as well as the steady state. The coefficients of the log-linear model depend on the primitive parameters of the model, as well as steady state values of variables. We use the steady state conditions of the model to solve out for a number of

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<sup>14</sup>There are some slight differences in the series used in SW versus CEE and PST. As in CEE and PST, we include consumer durables in investment. As in SW, we use an economy-wide measure of hours based on an adjustment of non-farm business hours.

parameters. The model also contains seven exogenous shocks, one corresponding to each variable. In our estimation, further, we allow for the fact that the quantity variables (output, investment, consumption, etc.) are non-stationary due to the presence of a unit root in the technology shock.

There are twenty-two parameters, not including the parameters that characterize the exogenous shocks. Of the twenty-two, there are five new parameters that arise from our modification of the labor market. These include: the steady state flow value of unemployment as fraction of the contribution of the worker to the job,  $\bar{b}$ , the worker's relative bargaining power,  $\eta$ , the elasticity of new matches with respect to labor market tightness,  $\sigma$ , the job survival rate,  $\rho$ , and the steady state job finding rate,  $s$ .<sup>15</sup>

Because we are not adding any new variables but are adding new parameters, in this first pass at the data we calibrate three of the five labor market parameters, for which there exists independent evidence. In particular, we choose the average monthly separation rate  $1 - \rho$  based on the observation that jobs last about two years and a half. Therefore, we set  $\rho = 1 - 0.105$ . We choose the elasticity of matches to unemployment,  $\sigma$ , to be equal to 0.5, the midpoint of the evidence typically cited in the literature.<sup>16</sup> In addition, this choice is within the range of plausible values of 0.5 to 0.7 reported by Petrongolo and Pissarides (2001) in their survey of the literature on the estimation of the matching function. We then set  $s = 0.95$  to match recent estimates of the U.S. average monthly job finding rate (Shimer, 2005).

The two labor market parameters we estimate are  $\bar{b}$  and  $\eta$ . Both these parameters are critical determinants of the effective elasticity of labor supply along the extensive margin in the flexible wage case. As emphasized by Hagedorn and Manovskii (2006) and others, the closer  $\bar{b}$  is to unity, the better able is the period-by-period Nash bargaining framework to capture unemployment and wage dynamics. For example, when  $\bar{b}$  is very close to unity, there is little difference between the value of employment versus unemployment to the worker. Effectively, labor supply along the extensive margin is very elastic in this instance. The response of wages to a shift in the value of a worker to the firm is dampened because in this case a small change in the wage has a large percentage effect on the relative gains to the worker from employment versus unemployment. Indeed, Hagedorn and Manovskii (2006) show that a model calibrated with  $\bar{b}$  close to unity can capture the relative volatilities of labor market variables. This calibration however is quite controversial: Shimer (2005) argues in favor of 0.4 based on the interpretation of  $\bar{b}$  as unemployment insurance, while Hall (2006) suggest 0.7, based on a broader interpretation that permits utility from leisure. Given the critical role of this parameter, it seems to be a prime candidate to estimate. The worker's bargaining parameter  $\eta$  is similarly important. The smaller is  $\eta$ , the less sensitive are wages to movements in the shadow value of labor, and thus the more sensitive is employment. Indeed, the Hagedorn and

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<sup>15</sup>The relative flow benefit from unemployment is given by  $\bar{b} = b(\kappa/a) / [p^w (f_n/a) + (\beta/\gamma_a) (\kappa/2) x^2]$ .

<sup>16</sup>The values for  $\sigma$  used in the literature are: 0.24 in Hall (2005a), 0.4 in Blanchard and Diamond (1989), Andolfatto (1994) and Merz (1995), 0.45 in Mortensen and Nagypal (2006), 0.5 in Hagedorn and Manovskii (2006), 0.5 in Farmer (2004), 0.72 in Shimer (2005a). See also a brief discussion in Mortensen and Nagypal (2006), p. 10, comparing their value of 0.45 to Shimer's one.

Manovskii calculation requires not only a high value of  $\bar{b}$  but also a low value of  $\eta$ .

There are four “conventional” parameters that we calibrate: the discount factor,  $\beta$ , the depreciation rate,  $\delta$ , the “share” parameter on capital in the Cobb-Douglas production function,  $\alpha$ , the steady state ratio of government consumption to output,  $\frac{g}{y}$ , and the sensitivity of firm demand elasticity to market share (the Kimball aggregator),  $\xi$ . We use conventional values for all these parameters:  $\beta = 0.99$ ,  $\delta = 0.025$ ,  $\alpha = 0.33$ ,  $\frac{g}{y} = 0.2$  and  $\xi = 10$ .<sup>17</sup>

$\beta$	$\delta$	$\alpha$	$g/y$	$\xi$	$\sigma$	$\rho$	$s$
0.99	0.025	0.33	0.2	10	0.5	0.895	0.95

The conventional parameters we estimate include: the elasticity of the utilization rate to the rental rate of capital,  $\eta_z$ <sup>18</sup>; the elasticity of the capital adjustment cost function,  $\eta_k$ ; the habit persistence parameter,  $h$ ; the steady state price markup  $\varepsilon^P$ ; the wage and price rigidity parameters,  $\lambda$  and  $\lambda^P$ ; the wage and price indexing parameters,  $\gamma$  and  $\gamma_p$ ; and the Taylor rule parameters,  $r_\pi$ ,  $r_y$  and  $\rho_s$ . In addition, we estimate the first order autocorrelations of all the exogenous disturbances, as well as their respective standard deviations.

We estimate the model with Bayesian methods (see An and Schorfheide, 2007, for a comprehensive survey). We combine the likelihood function of the model,  $L(\theta, Y)$ , with priors for the parameters to be estimated,  $p(\theta)$ , to obtain the posterior distribution,  $L(\theta, Y)p(\theta)$ . Draws from the posterior distribution are generated with the Random-Walk Metropolis Hastings (RWMH) algorithm.

## 4.2 Estimation Results

Table 2 reports the prior and the posterior estimates for each parameter. The table also reports the parameter configuration which maximizes the posterior (called Max), along with the mean and the values at the 5 and 95 percent tails. Similarly, Table 3 presents the estimates of the prior and posterior distribution of the shock processes.

For the conventional parameters, for the most part we use the same priors as in PST, which in turn follow closely those employed by SW. We proceed this way in order to facilitate comparison with the literature. Note that we propose uniforms priors for the two indexing parameters  $\gamma$  and

<sup>17</sup>Note in contrast to the frictionless labor market model, the term  $1 - \alpha$  does not correspond to the labor share and will depend on the outcome of the bargaining process. However, because a wide range of values of the bargaining power imply a labor share just below  $1 - \alpha$ , here we simply follow convention by setting  $1 - \alpha = 2/3$ . In our calculations,  $1 - \alpha$  equals 0.667 and the labor share 0.666.

<sup>18</sup>We follow SW, define  $\psi_z$  such that  $\eta_z = \frac{1 - \psi_z}{\psi_z}$  and estimate  $\psi_z$ . When  $\psi_z = 1$ , it is very costly to change the capital utilization rate and the utilization rate does not vary. When  $\psi_z = 0$ , the marginal cost of changing the capital utilization rate is constant and, as a result, the rental rate of capital does not vary.

$\gamma^p$ , based on the view that existing theory and evidence offer no guidance for the appropriate values of these parameters. Further, a priori, values of zero and unity for these parameters seem equally plausible. Finally, we note that in all instances the priors that we choose are reasonably loose.

Parameter	Prior	Posterior			
	Distribution	Max	Mean	5%	95%
$\psi_z$	Beta (0.5,0.1)	0.695	0.700	0.623	0.775
$\eta_k$	Normal (4,1.5)	2.425	2.375	1.767	3.007
$h$	Beta (0.5,0.1)	0.727	0.708	0.652	0.757
$\eta$	Beta (0.5,0.1)	0.907	0.907	0.862	0.941
$\bar{b}$	Beta (0.5,0.1)	0.726	0.723	0.664	0.786
$\gamma$	Uniform (0,1)	0.816	0.815	0.713	0.918
$\gamma^P$	Uniform (0,1)	0.000	0.018	0.001	0.051
$\lambda$	Beta (0.75,0.1)	0.717	0.725	0.670	0.780
$\lambda^P$	Beta (0.66,0.1)	0.848	0.846	0.803	0.884
$\varepsilon^P$	Normal (1.15,0.05)	1.405	1.408	1.362	1.454
$r_\pi$	Normal (1.7,0.3)	2.015	2.006	1.893	2.125
$r_y$	Gamma (0.125,0.1)	0.333	0.332	0.264	0.405
$\rho_s$	Beta (0.75,0.1)	0.773	0.772	0.731	0.811
$\gamma_a$	Uniform (1,1.5)	1.004	1.004	1.003	1.005

Notes: For the Uniform distribution the two numbers in parenthesis are the lower and the upper bound. Otherwise, the two numbers are the mean and the standard deviation.

As noted, we estimate two new labor market parameters,  $\eta$  and  $\bar{b}$ . There is little direct evidence on the worker's bargaining power parameter  $\eta$ . In their survey paper, Mortensen and Nagypal propose a value of 0.5, which appears to reflect conventional thinking in the literature. Accordingly we set the mean of the prior for this variable at 0.5, with a standard deviation of unity. We choose a similar prior for  $\bar{b}$ . As we noted earlier, Shimer proposes 0.4 as a "generous" value for  $\bar{b}$ , while Hall suggests 0.7 if one permits a broader interpretation of this variable. Both these estimates fit within a ninety-five percent confidence interval of our prior.

Since the estimates of the "conventional" parameters are consistent with other studies, we focus on the new parameters we consider here.<sup>19</sup> In particular, we estimate a very reasonable degree of wage rigidity. The estimate of  $\lambda$  is 0.72, which suggests a mean of just over three quarters between

<sup>19</sup>In Appendix D we present estimates of the SW model, which has been shown to fit the data nearly as well as a VAR. The precise version of the model we estimate is due to PST. Their formulation differs from SW only in some minor details.

wage contracting periods.<sup>20</sup> The evidence from micro-data (Gottshalk, 2006), suggests a modal adjustment time of one year, though is silent (to our knowledge) about medians and means. In addition, the estimates suggest a high degree of indexing of wages to past inflation: The estimate of the indexing parameter is almost 1, which suggests a high degree of effective real wage rigidity.

The estimate of the key parameter  $\bar{b}$ , the flow value of unemployment, is 0.72. What it suggests is that in units of consumption goods, the flow value of unemployment is seventy percent of the worker's marginal flow value to the firm. This percentage is close to the value proposed by Hall (2006), who as we suggested earlier motivates  $\bar{b}$  as reflecting not only unemployment insurance benefits but also utility gains from leisure. We also note that this estimate is well below the near unity value required for the conventional flexible model to account for the data. Thus, the data seem to prefer a combination of highly sticky wages and (effectively) inelastic labor supply along the extensive margin.

Parameter	Prior	Posterior			
	Distribution	Max	Mean	5%	95%
$\rho_a$	Beta (0.5,0.15)	0.140	0.096	0.036	0.167
$\rho_r$	Beta (0.5,0.15)	0.207	0.179	0.096	0.266
$\rho_b$	Beta (0.5,0.15)	0.713	0.724	0.660	0.788
$\rho_i$	Beta (0.5,0.15)	0.605	0.599	0.520	0.675
$\rho_p$	Beta (0.5,0.15)	0.808	0.814	0.762	0.863
$\rho_w$	Beta (0.5,0.15)	0.264	0.261	0.199	0.322
$\rho_g$	Beta (0.5,0.15)	0.991	0.993	0.987	0.998
$\sigma_a$	IGamma (0.15,0.15)	1.039	1.025	0.968	1.084
$\sigma_r$	IGamma (0.15,0.15)	0.224	0.226	0.206	0.248
$\sigma_b$	IGamma (0.15,0.15)	0.362	0.334	0.251	0.436
$\sigma_i$	IGamma (0.15,0.15)	0.166	0.165	0.126	0.207
$\sigma_p$	IGamma (0.15,0.15)	0.062	0.06	0.046	0.076
$\sigma_w$	IGamma (0.15,0.15)	0.578	0.586	0.528	0.650
$\sigma_g$	IGamma (0.15,0.15)	0.357	0.358	0.327	0.391

Notes: The two numbers in parenthesis are the mean and the st. dev. of the distribution.

<sup>20</sup>It is true that our estimate of the degree of price rigidity ( $\lambda^p = 0.85$ ) is somewhat higher than that for wage rigidity. Several points: First, our estimate is similar to what one obtains with the SW model (see Appendix D). Second, our measure of inflation is based on the GDP deflator, which consists of producer prices, which are stickier than consumer prices. In this regard, our estimate of  $\lambda^p$  suggests a median duration of price changes of roughly four quarters, which is not too far above Steinsson and Nakamura (2007) estimates of three quarters for producer prices. Note that the Poisson process for price adjustment leads to the mean exceeding the median, since the constant hazard process suggests that some prices may not be adjusted for a long time.

Finally, the estimate of the worker's bargaining power parameter is 0.9. This value lies above the range considered in the literature, typically 0.5 - 0.7. As noted earlier, however, there is virtually no direct evidence on what an appropriate value of this parameter should be. One possibility, is that within our framework it is very difficult to separately identify  $\bar{b}$  and  $\eta$ . Both parameters enter the loglinear system via their respective impact on the steady state wage (see the Appendix). It may be that to achieve a good identification of these parameters, we may need to introduce additional labor market information.

It is also worth noting that within our framework, the lion's share of the serial correlation in the real wages is accounted for by the wage contracting structure. The exogenous shock to the wage equation (modeled as a shock to bargaining power) has a first order serial coefficient of only 0.26.

We next consider the model without wage rigidity. Table 4 presents the parameter estimates and Table 5 presents the estimates of the shock processes in this case.

Parameter	Prior	Posterior			
	Distribution	Max	Mean	5%	95%
$\psi_z$	Beta (0.5,0.1)	0.861	0.852	0.783	0.911
$\eta_k$	Normal (4,1.5)	1.023	1.179	0.803	1.635
$h$	Beta (0.5,0.1)	0.801	0.803	0.760	0.840
$\eta$	Beta (0.5,0.1)	0.616	0.589	0.451	0.726
$\bar{b}$	Beta (0.5,0.1)	0.983	0.982	0.975	0.987
$\gamma$	Uniform (0,1)	—	—	—	—
$\gamma^P$	Uniform (0,1)	0.000	0.037	0.003	0.090
$\lambda$	Beta (0.75,0.1)	—	—	—	—
$\lambda^P$	Beta (0.66,0.1)	0.574	0.575	0.512	0.630
$\varepsilon^P$	Normal (1.15,0.05)	1.347	1.351	1.298	1.407
$r_\pi$	Normal (1.7,0.3)	1.927	1.999	1.748	2.297
$r_y$	Gamma (0.125,0.1)	0.013	0.019	0.003	0.043
$\rho_s$	Beta (0.75,0.1)	0.685	0.700	0.648	0.746
$\gamma_a$	Uniform (1,1.5)	1.003	1.003	1.001	1.004

Notes: For the Uniform distribution the two numbers in parenthesis are the lower and the upper bound. Otherwise, the two numbers are the mean and the standard deviation.

The estimates of the conventional parameters do not change much. There is however now a large change in the estimates of the two key labor market parameters:  $\bar{b}$  increases to 0.98 and  $\eta$  falls to 0.6. The former is close to the value we described earlier that Hagedorn and Manovskii used to

argue that a flexible wage model could account for labor market volatility.<sup>21</sup> Our estimates confirm that absent wage rigidity, it is necessary to have highly elastic labor supply along the extensive margin to account for the facts.<sup>22</sup>

Parameter	Prior	Posterior			
	Distribution	Max	Mean	5%	95%
$\rho_a$	Beta (0.5,0.15)	0.287	0.282	0.193	0.378
$\rho_r$	Beta (0.5,0.15)	0.249	0.250	0.162	0.334
$\rho_b$	Beta (0.5,0.15)	0.351	0.363	0.231	0.507
$\rho_i$	Beta (0.5,0.15)	0.865	0.852	0.812	0.891
$\rho_p$	Beta (0.5,0.15)	0.916	0.909	0.866	0.946
$\rho_w$	Beta (0.5,0.15)	0.984	0.984	0.977	0.990
$\rho_g$	Beta (0.5,0.15)	0.987	0.987	0.981	0.992
$\sigma_a$	IGamma (0.15,0.15)	1.071	1.083	0.992	1.188
$\sigma_r$	IGamma (0.15,0.15)	0.237	0.241	0.218	0.267
$\sigma_b$	IGamma (0.15,0.15)	0.686	0.738	0.436	1.146
$\sigma_i$	IGamma (0.15,0.15)	0.066	0.074	0.059	0.094
$\sigma_p$	IGamma (0.15,0.15)	0.093	0.093	0.076	0.115
$\sigma_w$	IGamma (0.15,0.15)	0.250	0.274	0.183	0.369
$\sigma_g$	IGamma (0.15,0.15)	0.352	0.355	0.324	0.389

Notes: The two numbers in parenthesis are the mean and the st. dev. of the distribution.

One virtue of the Bayesian approach is that it is straightforward to compare the fit of the baseline model versus the model without wage rigidity. Table 6 reports the log marginal likelihoods for the two models.

Baseline Model	Flex Wage Model
-1191	-1234

<sup>21</sup>Hagedorn and Manvoski employ a slightly smaller value of  $\bar{b}$ , 0.95, as opposed to 0.98, and much smaller value of  $\eta$ , 0.05, as opposed to 0.62. Note that our prior on  $\eta$  is sufficiently loose so as not to exclude values well below 0.5.

<sup>22</sup>The large value of  $\bar{b}$  suggests a huge response of employment to changes in unemployment insurance benefits, as Hornstein, Krusell and Gianluca Violante (2005) and others have noted.



The baseline model clearly is preferred to the flex wage model. The difference in marginal likelihood is forty loglikelihood points, which is a significant difference.

Another way to assess how the model captures the data is to portray the autocovariance functions of the model variables against the data. Figure 2 reports this information. The solid line in each panel reports the autocovariance function of the data. The dashed lines are ninety-five percent posterior intervals of the model autocovariances.<sup>23</sup> Overall, the baseline model does well. For the most part, the empirical autocovariances functions lie within the model standard error bands. In this regard, the model does particularly well in capturing the reduced form dynamics of wages and hours.

As a further check on the model, we explore how well it is able to account for the dynamics of unemployment. As the bottom row shows, the empirical autocovariance function for unemployment lies within the standard error bands of the model generated sample moments. We interpret this as a strong test of the model given that we did not use information on unemployment to fit model parameters.

Figure 3 presents the autocovariance functions for the model without wage rigidity. Overall this model doesn't do as well. For a number of key variables, including output, wages, hours, and unemployment, the empirical autocovariances in places lie well outside the model-generated bands. This is particularly true for wages, hours, output and unemployment.

The sharp differences in the autocovariance functions suggest why the model with wage rigidity appears to outperform the one without. Roughly speaking, the flexible wage model is not sufficiently flexible to permit the framework to capture all the various moments of the data.

We next illustrate the properties of the model economy by simulating the response to several key shocks. We analyze the role of wage rigidity, in particular, by examining both our benchmark model and the same model with staggered contracting replaced by period by period wage negotiations. As Table 7 shows, the estimates suggest that the main driving force is the investment shock which, strictly speaking is interpretable as shock to investment-specific technological change. It accounts for more than half the variation in output growth on impact and more than forty percent at all horizons. This finding is consistent with both SW and PST. Next in importance is the disembodied productivity shock which accounts for roughly seventeen percent of the variation at horizons of a year or greater.

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<sup>23</sup>The posterior intervals are computed as follows. We sample 500 points from the posterior and for each of them we generate 160 observations, which is the length of the data sample, 500 times. Then, for each draw we compute autocovariances and we report the fifth and the ninety-fifth percentile.

Table 7: Variance decomposition for output ( $\Delta \log y$ ) at different horizons (in percentage)

Shocks	Horizons			
	on impact	1 year	4 years	long run
<i>a</i> shock - technology	16.7	32.5	31.0	31.0
<i>r</i> shock - monetary	6.1	5.0	5.4	5.4
<i>b</i> shock - preferences	11.1	9.2	9.5	9.5
<i>i</i> shock - investment	54.8	41.9	42.4	42.4
<i>g</i> shock - government	9.4	8.7	8.2	8.2
<i>p</i> shock - price markup	1.9	2.5	3.2	3.2
<i>w</i> shock - bargaining power	0.0	0.2	0.3	0.3

The recent literature on unemployment fluctuations that we alluded to in the introduction almost uniformly treats productivity shocks as the main driving force. Thus for purposes of comparison, we begin with this disturbance. In particular, Figure 4 illustrates the response of the model economy to a productivity shock. The thick line is the model with wage rigidity. The dotted line has wage rigidity turned off.<sup>24</sup> Notice that the response of output and employment is significantly greater with wage rigidity than without. Conversely, due to the staggered contracting the response of wages is much smoother. The smooth response of wages, of course, implies a larger response of profits to the technology shock than otherwise. This leads to a stronger response of output and employment relative to the flexible wage case. Note also that there is an immediate drop in inflation following the productivity shock, which is in line with the evidence in Altig, Christiano, Eichenbaum and Evans (2004). This does not come as puzzle in this framework: with wage rigidity the rise in productivity reduces marginal costs and hence inflation.

We next turn to the investment shock, as portrayed in Figure 5. In contrast to the case of the productivity shock, in this instance the response of output and employment is very similar across the two models. Note, however, that the responses of wages,  $w$ , and inflation,  $\pi$ , are quite different. In the case with wage rigidity shut off, the responses of these variables appear counterfactually large. In absolute value, the response of wages is three times as large as the response of output and the response of inflation (annualized) is nearly half as large. To our knowledge there do not exist series for wages or inflation that display this kind of volatility relative to output.<sup>25</sup> Intuitively, the investment shock shifts output demand without directly affecting factor productivities. With

<sup>24</sup>To shut off wage rigidity we simply set the probability that wages do not adjust,  $\lambda$ , equal to zero.

<sup>25</sup>It is true that the experiment here is conditional on the investment shock and the evidence to which we are alluding consists of unconditional moments. However, the investment shock accounts for nearly fifty percent of the variation in output growth within the model. If the flex wage model were true then we should observe relatively volatile behavior of real wages and price markups.

nominal price rigidities, markups decline and employment adjusts to meet demand. In the staggered contracting model, because labor costs are sticky, the employment response is associated with only a modest change in real wages and inflation. With wage rigidity shut off, however, there is a sharp increase in wages which boosts up real marginal costs and hence inflation. In sum, in the case of investment demand shocks, the wage rigidity smooths out the response of wages and inflation to the disturbances.

Finally, we consider how well our model fits the data as compared to SW. In appendix D we report the parameter estimates for the SW model. By and large, the estimates for the “non-labor market” parameters are similar across the two models. To explore relative fit, one possibility would be to compare the marginal likelihoods, as we did with the sticky and flex wage versions of our model. In what we did earlier, however, one model (the sticky wage) nested the other (the flex wage) and the priors used in the estimation are identical. Neither of these conditions applies in this instance. While in principle this should not be a problem, in practice it may be, given the sensitivity of the marginal likelihood to the priors. Accordingly, to get a sense of relative fit we simply compare the autocovariance functions of each model. As Figure 6 shows, the model standard error bands of our model overlap closely with those of the SW model. We conclude that from a practical standpoint, the ability of our model to characterize the data is very similar to that of SW.

## 5 Concluding Remarks

We have developed and estimate a medium scale macroeconomic model that allows for unemployment and staggered nominal wage contracting. In contrast to most existing quantitative models, the employment of existing workers is efficient. Thus, the model is immune to the Barro’s (1977) critique that models relying on wage rigidity to have allocative effects in situations where firms and workers have on-going relationships ignore mutual gains from trade. In our model, in contrast, wage rigidity affects the hiring of new workers. The former is introduced via the staggered Nash bargaining setup of Gertler and Trigari (2006). A robust finding is that the model with rigidity provides a better description of the data than does a flexible wage version. Further, our model appears to capture the moments of the data as well as Smets and Wouters (2007). In addition, while the conventional model is silent about the behavior of unemployment, our model generates dynamics for this variable that are in line with the data.

More work is necessary, however, to ensure a robust identification of the key labor market parameters. Our preliminary estimates of the degree of wage rigidity and the flow value of unemployment appear to be quite reasonable. The estimate of worker’s bargaining power lies above conventional wisdom, though there is little direct evidence on what this parameter should be. One possibility is that it may be difficult to separately identify some of the key labor market parameters that influence employment volatility. Accordingly, it may be necessary to introduce additional labor market information.

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## APPENDIX A

### A1. The contract wage

- The worker and the firm surpluses are

$$\begin{aligned} H_t(i) &= w_t(i) - b_t + \rho\beta E_t \Lambda_{t,t+1} H_{t+1}(i) - s_{t+1} \beta E_t \Lambda_{t,t+1} H_{x,t+1} \\ &= W_t(i) - E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} [b_{t+s} + s_{t+s+1} \beta \Lambda_{t+s,t+s+1} H_{x,t+s+1}] \end{aligned}$$

$$\begin{aligned} J_t(i) &= p_t^w f_{nt} - w_t(i) + \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}(i)^2 + \rho\beta E_t \Lambda_{t,t+1} J_{t+1}(i) \\ &= E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left[ p_{t+s}^w f_{nt+s} + \beta \Lambda_{t+s,t+s+1} \frac{\kappa_{t+s+1}}{2} x_{t+s+1}(i)^2 \right] - W_t(i) \end{aligned}$$

where  $W_t(i)$  denotes the discounted sum of expected future wages to be received by a worker over the life of the relationship at the firm:

$$W_t(i) = E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} w_{t+s}(i)$$

- Consider a renegotiating firm. The current and future expected wages are given by:

$$w_t(i) = \frac{w_t^{*n}}{p_t}$$

$$E_t w_{t+1}(i) = E_t \frac{1}{p_{t+1}} [\lambda \bar{\gamma} \pi_t^\gamma w_t^{*n} + (1-\lambda) w_{t+1}^{*n}]$$

$$E_t w_{t+2}(i) = E_t \frac{1}{p_{t+2}} [\lambda^2 \bar{\gamma} \pi_t^\gamma \bar{\gamma} \pi_{t+1}^\gamma w_t^{*n} + \lambda(1-\lambda) \bar{\gamma} \pi_{t+1}^\gamma w_{t+1}^{*n} + (1-\lambda) w_{t+2}^{*n}]$$

*and so on....*

Using these expressions, we can write:

$$\begin{aligned} W_t(i) &= \frac{w_t^{*n}}{p_t} \\ &+ (\rho\beta) E_t \Lambda_{t,t+1} \frac{1}{p_{t+1}} [\lambda w_t^{*n} \bar{\gamma} \pi_t^\gamma + (1-\lambda) w_{t+1}^{*n}] \\ &+ (\rho\beta)^2 E_t \Lambda_{t,t+2} \frac{1}{p_{t+2}} [\lambda^2 w_t^{*n} \bar{\gamma} \pi_t^\gamma \bar{\gamma} \pi_{t+1}^\gamma + \lambda(1-\lambda) w_{t+1}^{*n} \bar{\gamma} \pi_{t+1}^\gamma + (1-\lambda) w_{t+2}^{*n}] \\ &+ \dots \end{aligned}$$



$$\begin{aligned}
W_t(i) &= w_t^* \\
&+ (\rho\beta) E_t \Lambda_{t,t+1} \left[ \lambda w_t^* \frac{p_t}{p_{t+1}} \bar{\gamma} \pi_t^\gamma + (1-\lambda) w_{t+1}^* \right] \\
&+ (\rho\beta)^2 E_t \Lambda_{t,t+2} \left[ \lambda^2 w_t^* \frac{p_t}{p_{t+2}} \bar{\gamma} \pi_t^\gamma \bar{\gamma} \pi_{t+1}^\gamma + \lambda (1-\lambda) w_{t+1}^* \frac{p_{t+1}}{p_{t+2}} \bar{\gamma} \pi_{t+1}^\gamma + (1-\lambda) w_{t+2}^* \right] \\
&+ \dots
\end{aligned}$$

- Collecting terms:

$$\begin{aligned}
W_t(i) &= \left[ 1 + (\rho\lambda\beta) \Lambda_{t,t+1} \frac{p_t}{p_{t+1}} \bar{\gamma} \pi_t^\gamma + (\rho\lambda\beta)^2 \Lambda_{t,t+2} \frac{p_t}{p_{t+2}} \bar{\gamma} \pi_t^\gamma \bar{\gamma} \pi_{t+1}^\gamma + \dots \right] w_t^* \\
&+ (1-\lambda) (\rho\beta) E_t \Lambda_{t,t+1} \left[ 1 + (\rho\lambda\beta) \Lambda_{t+1,t+2} \frac{p_{t+1}}{p_{t+2}} \bar{\gamma} \pi_{t+1}^\gamma + \dots \right] w_{t+1}^* \\
&+ (1-\lambda) (\rho\beta)^2 E_t \Lambda_{t,t+2} \left[ 1 + (\rho\lambda\beta) \Lambda_{t+2,t+3} \frac{p_{t+2}}{p_{t+3}} \bar{\gamma} \pi_{t+2}^\gamma + \dots \right] w_{t+2}^*
\end{aligned}$$

- Given that

$$\Delta_t = E_t \sum_{s=0}^{\infty} (\rho\lambda\beta)^s \Lambda_{t,t+s} \frac{p_t}{p_{t+s}} \left( \prod_{k=1}^s \bar{\gamma} \pi_{t+k-1}^\gamma \right)$$

we have

$$\begin{aligned}
W_t(i) &= \Delta_t w_t^* \\
&+ (1-\lambda) (\rho\beta) E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^* \\
&+ (1-\lambda) (\rho\beta)^2 E_t \Lambda_{t,t+2} \Delta_{t+2} w_{t+2}^* \\
&+ \dots
\end{aligned}$$

- Finally, rearranging:

$$W_t(i) = \Delta_t w_t^* + (1-\lambda) E_t \sum_{s=1}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \Delta_{t+s} w_{t+s}^*$$

- Substituting now the expression for  $W_t(i)$  in the worker and the firm surpluses we get

$$\begin{aligned}
H_t(i) &= \Delta_t w_t^* - E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} [b_{t+s} + s_{t+s+1} \beta \Lambda_{t+s,t+s+1} H_{t+s+1} \\
&- (1-\lambda) (\rho\beta) \Lambda_{t+s,t+s+1} \Delta_{t+s+1} w_{t+s+1}^*]
\end{aligned}$$

$$\begin{aligned}
J_t(i) &= E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left[ p_{t+s}^w f_{nt+s} + \beta \Lambda_{t+s,t+s+1} \frac{\kappa_{t+s+1}}{2} x_{t+s+1} (i)^2 \right. \\
&- \left. (1-\lambda) (\rho\beta) \Lambda_{t+s,t+s+1} \Delta_{t+s+1} w_{t+s+1}^* \right] - \Delta_t w_t^*
\end{aligned}$$

- The Nash first-order condition is

$$\chi_t(i) J_t(i) = (1 - \chi_t(i)) H_t(i)$$

- This can be rewritten as

$$\chi J_t(i) = (1 - \chi) H_t(i) - \phi_t(i)$$

with

$$\phi_t(i) = (\chi_t(i) - \chi) [J_t(i) + H_t(i)]$$

- Substituting the expression of  $J_t(i)$  and  $H_t(i)$  for a renegotiating firm:

$$\begin{aligned} \Delta_t w_t^* - \phi_t(i) &= E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left\{ \left[ \chi \left( p_{t+s}^w f_{nt+s} + \beta \Lambda_{t+s,t+s+1} \frac{\kappa_{t+s+1}}{2} x_{t+s+1}(i)^2 \right) + \right. \right. \\ &\quad \left. \left. (1 - \chi) (b_{t+s} + s_{t+s+1} \beta \Lambda_{t+s,t+s+1} H_{x,t+s+1}) \right] \right. \\ &\quad \left. - (1 - \lambda) \rho \beta \Lambda_{t+s,t+s+1} \Delta_{t+s+1} w_{t+s+1}^* \right\} \end{aligned}$$

- The above equation can be written in a recursive form in the following way:

$$\begin{aligned} \Delta_t w_t^* - \phi_t(i) &= \chi \left( p_t^w f_{nt} + \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}(i)^2 \right) + (1 - \chi) (b_t + s_{t+1} \beta E_t \Lambda_{t,t+1} H_{x,t+1}) \\ &\quad - (1 - \lambda) \rho \beta E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^* + \rho \beta E_t \Lambda_{t,t+1} (\Delta_{t+1} w_{t+1}^* - \phi_{t+1}(i)) \end{aligned}$$

- Simplifying, we obtain

$$\Delta_t w_t^* = w_t^o(i) + \rho \lambda \beta E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^*$$

with  $w_t^o(i)$  denoting the target wage:

$$\begin{aligned} w_t^o(i) &= \chi \left( p_t^w f_{nt} + \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}(i)^2 \right) \\ &\quad + (1 - \chi) (b_t + s_{t+1} \beta E_t \Lambda_{t,t+1} H_{x,t+1}) + \Phi_t \end{aligned}$$

and

$$\Phi_t = \phi_t(i) - \rho \beta E_t \Lambda_{t,t+1} \phi_{t+1}(i)$$

## A2. The loglinearized target wage

- Loglinearizing the target wage then yields

$$\begin{aligned} \widehat{w}_t^o(i) &= \varphi_{fn} \left( \widehat{p}_t^w + \widehat{f}_{nt} \right) + \varphi_x E_t \widehat{x}_{t+1}(i) + (\varphi_x/2) \left( E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t \right) + \\ &\quad \varphi_b \widehat{b}_t + \varphi_s E_t \left( \widehat{s}_{t+1} + \widehat{H}_{t+1} + \widehat{\lambda}_{t+1} - \widehat{\lambda}_t \right) + \varphi_\chi \left[ \widehat{\chi}_t(i) - \rho \beta E_t \widehat{\chi}_{t+1}(i) \right] \end{aligned}$$

where

$$\begin{aligned}\bar{w} &= (w/a) \\ \varphi_{fn} &= \chi p^w (f_n/a) \bar{w}^{-1} & \varphi_x &= \chi \beta \epsilon^{-1} (1 - \rho) \bar{w}^{-1} \\ \varphi_b &= (1 - \chi) b (k/a) \bar{w}^{-1} & \varphi_s &= \chi \beta s \epsilon^{-1} \bar{w}^{-1} \\ \varphi_\chi &= \chi (1 - \chi)^{-1} \epsilon^{-1} \bar{w}^{-1} \\ \epsilon &= (\kappa x)^{-1}\end{aligned}$$

and where

$$\hat{H}_{x,t} = \hat{H}_t$$

- Finally, loglinearizing the weight in the target wage gives

$$\hat{\chi}_t(i) = -(1 - \chi) \left( \hat{\Sigma}_t(i) - \hat{\Delta}_t \right) + (1 - \chi) (1 - \eta)^{-1} \hat{\varepsilon}_t^\eta$$

with

$$\begin{aligned}\hat{\Delta}_t &= \rho \lambda \beta E_t \left( \hat{\lambda}_{t+1} - \hat{\lambda}_t - \hat{\pi}_{t+1} + \gamma \hat{\pi}_t + \hat{\Delta}_{t+1} - \hat{\varepsilon}_{t+1}^a \right) \\ \hat{\Sigma}_t(i) &= x \lambda \beta E_t \hat{x}_{t+1}(i) + \lambda \beta E_t \left( \hat{\lambda}_{t+1} - \hat{\lambda}_t - \hat{\pi}_{t+1} + \gamma \hat{\pi}_t + \hat{\Sigma}_{t+1}(i) - \hat{\varepsilon}_{t+1}^a \right)\end{aligned}$$

### A3. Spillover effects

- In order to illustrate the spillover effects, we derive expressions for  $\hat{x}_{t+1}(i)$ ,  $\hat{\chi}_t(i)$  and  $\hat{\chi}_{t+1}(i)$  as a function of the wage gap  $\hat{w}_t - \hat{w}_t^*$ , as well as an expression for  $\hat{H}_{t+1}$  as a function of the wage gap  $\hat{w}_{t+1} - \hat{w}_{t+1}^*$ .
- The loglinear version of the real wage index is

$$\hat{w}_t = (1 - \lambda) \hat{w}_t^* + \lambda (\hat{w}_{t-1} - \hat{\pi}_t + \gamma \hat{\pi}_{t-1} - \hat{\varepsilon}_t^a)$$

Using the loglinear versions for the expected future wages at a renegotiated firm, we obtain:

$$\begin{aligned}\hat{w}_t - \hat{w}_t(i) &= \hat{w}_t - \hat{w}_t^* \\ E_t(\hat{w}_{t+1} - \hat{w}_{t+1}(i)) &= \lambda (\hat{w}_t - \hat{w}_t^*) \\ E_t(\hat{w}_{t+2} - \hat{w}_{t+2}(i)) &= \lambda^2 (\hat{w}_t - \hat{w}_t^*) \\ &\text{and so on....}\end{aligned}$$

- The loglinear version of the job creation condition, equation (25), is:

$$\widehat{x}_t(i) = \epsilon p^w (f_n/a) \left( \widehat{p}_t^w + \widehat{f}_{nt} \right) - \epsilon \bar{w} \widehat{w}_t(i) + \beta E_t \widehat{x}_{t+1}(i) + (\rho + x/2) \beta E_t \widehat{\Lambda}_{t,t+1}$$

Subtracting the loglinear version of the aggregate job creation condition, we can write:

$$\widehat{x}_t(i) - \widehat{x}_t = -\epsilon \bar{w} (\widehat{w}_t(i) - \widehat{w}_t) + \beta E_t (\widehat{x}_{t+1}(i) - \widehat{x}_{t+1})$$

This can be iterated forward to give:

$$\begin{aligned} \widehat{x}_t(i) - \widehat{x}_t &= \epsilon \bar{w} (\widehat{w}_t - \widehat{w}_t(i)) \\ &\quad + \beta \epsilon \bar{w} E_t (\widehat{w}_{t+1} - \widehat{w}_{t+1}(i)) \\ &\quad + \beta^2 \epsilon \bar{w} E_t (\widehat{w}_{t+2} - \widehat{w}_{t+2}(i)) \\ &\quad + \dots \end{aligned}$$

Substituting the previous expressions and rearranging yields:

$$\widehat{x}_t(i) = \widehat{x}_t + \epsilon \Sigma \bar{w} (\widehat{w}_t - \widehat{w}_t^*)$$

with

$$\Sigma = \frac{1}{1 - \lambda \beta}$$

More generally, we have

$$\begin{aligned} \widehat{x}_t(i) &= \widehat{x}_t + \epsilon \Sigma \bar{w} (\widehat{w}_t - \widehat{w}_t^*) \\ E_t \widehat{x}_{t+1}(i) &= E_t \widehat{x}_{t+1} + \epsilon \Sigma \bar{w} \lambda (\widehat{w}_t - \widehat{w}_t^*) \\ E_t \widehat{x}_{t+2}(i) &= E_t \widehat{x}_{t+2} + \epsilon \Sigma \bar{w} \lambda^2 (\widehat{w}_t - \widehat{w}_t^*) \\ &\text{and so on....} \end{aligned}$$

- The loglinear horizon-adjusted bargaining weight can be written as:

$$\widehat{\chi}_t(i) = \widehat{\chi}_t - (1 - \chi) \left( \widehat{\Sigma}_t(i) - \widehat{\Sigma}_t \right) + (1 - \chi) (1 - \eta)^{-1} \widehat{\varepsilon}_t^\eta$$

with

$$\widehat{\chi}_t = - (1 - \chi) \left( \widehat{\Sigma}_t - \widehat{\Delta}_t \right)$$

and

$$\begin{aligned} \widehat{\Sigma}_t(i) - \widehat{\Sigma}_t &= (x\lambda\beta) E_t (\widehat{x}_{t+1}(i) - \widehat{x}_{t+1}) + \lambda\beta E_t \left( \widehat{\Sigma}_{t+1}(i) - \widehat{\Sigma}_{t+1} \right) \\ &= (x\lambda\beta) E_t (\widehat{x}_{t+1}(i) - \widehat{x}_{t+1}) \\ &\quad + (x\lambda\beta) \lambda\beta E_t (\widehat{x}_{t+2}(i) - \widehat{x}_{t+2}) + \\ &\quad + (x\lambda\beta) (\lambda\beta)^2 E_t (\widehat{x}_{t+3}(i) - \widehat{x}_{t+3}) + \\ &\quad \dots \end{aligned}$$

Substituting the expressions for the future hiring rates and collecting terms, yields:

$$\widehat{\Sigma}_t(i) - \widehat{\Sigma}_t = (1 - \rho) \Psi \epsilon \Sigma \overline{w} (\widehat{w}_t - \widehat{w}_t^*)$$

with

$$\Psi = \frac{\lambda^2 \beta}{1 - \lambda^2 \beta}$$

so that

$$\widehat{\chi}_t(i) = \widehat{\chi}_t - (1 - \chi) (1 - \rho) \Psi \epsilon \Sigma \overline{w} (\widehat{w}_t - \widehat{w}_t^*) + (1 - \chi) (1 - \eta)^{-1} \widehat{\epsilon}_t^\eta$$

and

$$E_t \widehat{\chi}_{t+1}(i) = E_t \widehat{\chi}_{t+1} - (1 - \chi) (1 - \rho) \Psi \epsilon \Sigma \overline{w} \lambda (\widehat{w}_t - \widehat{w}_t^*) + (1 - \chi) (1 - \eta)^{-1} E_t \widehat{\epsilon}_{t+1}^\eta$$

- The loglinear version of the worker's surplus, equation (33), is:

$$\begin{aligned} \widehat{H}_t(i) &= (1 - \chi) \chi^{-1} \epsilon \left( \overline{w} \widehat{w}_t(i) - b(k/a) \widehat{b}_t \right) + \\ &\quad \rho \beta E_t \left( \widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{H}_{t+1}(i) \right) - s \beta E_t \left( \widehat{s}_{t+1} + \widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{H}_{t+1} \right) \end{aligned}$$

Subtracting the loglinear version of the average worker's surplus, we obtain:

$$\widehat{H}_t(i) - \widehat{H}_t = (1 - \chi) \chi^{-1} \epsilon \overline{w} (\widehat{w}_t(i) - \widehat{w}_t) + \rho \beta E_t \left( \widehat{H}_{t+1}(i) - \widehat{H}_{t+1} \right)$$

This can be iterated forward to give:

$$\begin{aligned} \widehat{H}_t(i) - \widehat{H}_t &= (1 - \chi) \chi^{-1} \epsilon \overline{w} (\widehat{w}_t(i) - \widehat{w}_t) \\ &\quad + (1 - \chi) \chi^{-1} \epsilon \overline{w} (\rho \beta) E_t (\widehat{w}_{t+1}(i) - \widehat{w}_{t+1}) \\ &\quad + (1 - \chi) \chi^{-1} \epsilon \overline{w} (\rho \beta)^2 E_t (\widehat{w}_{t+2}(i) - \widehat{w}_{t+2}) \\ &\quad + \dots \end{aligned}$$

Substituting and rearranging

$$\widehat{H}_t(i) = \widehat{H}_t - (1 - \chi) \chi^{-1} \epsilon \Delta \overline{w} (\widehat{w}_t - \widehat{w}_t^*)$$

with

$$\Delta = \frac{1}{1 - \rho \lambda \beta}$$

The loglinear version of the Nash bargaining condition, equation (41), is

$$\widehat{J}_t(i) + (1 - \chi)^{-1} \widehat{\chi}_t(i) = \widehat{H}_t(i)$$

where  $\widehat{J}_t(i) = \widehat{x}_t(i)$ . Substituting the expressions for  $\widehat{x}_t(i)$ ,  $\widehat{\chi}_t(i)$  and  $\widehat{H}_t(i)$  yields:

$$\widehat{H}_t = \widehat{x}_t + \Gamma(\widehat{w}_t - \widehat{w}_t^*) + (1 - \chi)^{-1} \widehat{\chi}_t + (1 - \eta)^{-1} \widehat{\varepsilon}_t^\eta$$

with

$$\Gamma = (1 - \eta(1 - \rho)\Psi)^{-1} \eta^{-1} \epsilon \Sigma \bar{w}$$

We then have:

$$E_t \widehat{H}_{t+1} = E_t \widehat{x}_{t+1} + \Gamma E_t (\widehat{w}_{t+1} - \widehat{w}_{t+1}^*) + (1 - \chi)^{-1} E_t \widehat{\chi}_{t+1} + (1 - \eta)^{-1} E_t \widehat{\varepsilon}_{t+1}^\eta$$

- Substitute now the expressions for  $\widehat{x}_{t+1}(i)$ ,  $\widehat{\chi}_t(i)$ ,  $\widehat{\chi}_{t+1}(i)$  and  $\widehat{H}_{t+1}$  into the loglinear target wage, to obtain

$$\widehat{w}_t^o(i) = \widehat{w}_t^o + \frac{\tau_1}{1 - \rho\lambda\beta} E_t (\widehat{w}_{t+1} - \widehat{w}_{t+1}^*) + \frac{\tau_2}{1 - \rho\lambda\beta} (\widehat{w}_t - \widehat{w}_t^*)$$

with

$$\begin{aligned} \widehat{w}_t^o &= \varphi_{fn} (\widehat{p}_t^w + \widehat{f}_{nt}) + (\varphi_x + \varphi_s) E_t \widehat{x}_{t+1} + \varphi_s E_t \widehat{s}_{t+1} + \varphi_b \widehat{b}_t \\ &\quad + (\varphi_s + \varphi_x/2) (E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t) + \varphi_\chi (\widehat{\chi}_t - \beta(\rho - s) \widehat{\chi}_{t+1}) + \widehat{\varepsilon}_t^w \end{aligned}$$

and

$$\begin{aligned} \widehat{\varepsilon}_t^w &= \varphi_\eta (1 - \beta\rho^\eta(\rho - s)) \widehat{\varepsilon}_t^\eta \\ \varphi_\eta &= \chi(1 - \eta)^{-1} \epsilon^{-1} \bar{w}^{-1} \end{aligned}$$

and

$$\begin{aligned} \tau_1 &= \varphi_s \Gamma (1 - \rho\lambda\beta) \\ \tau_2 &= [\varphi_x \lambda - \varphi_\chi (1 - \chi) (1 - \rho) \Psi (1 - \rho\beta\lambda)] \epsilon \Sigma \bar{w} (1 - \rho\lambda\beta) \end{aligned}$$

## APPENDIX B

### Steady state calculation

- First obtain

$$\begin{aligned} n &= \frac{s}{1 - \rho + s} \\ u &= 1 - n \\ x &= \frac{su}{n} \end{aligned}$$

and

$$\chi = \frac{\eta}{\eta + (1 - \eta) \Sigma / \Delta}$$

- Then get

$$\begin{aligned} r^k &= 1/\bar{\beta} - 1 + \delta \\ \bar{\beta} &= \beta/\gamma_a \\ \frac{k/a}{y/a} &= \frac{\alpha p^w}{r^k} \\ \frac{i/a}{y/a} &= \left(1 - \frac{1 - \delta}{\gamma_a}\right) \gamma_a \frac{k/a}{y/a} \\ \frac{k/a}{n} &= \left(\frac{k/a}{y/a}\right)^{\frac{1}{1-\alpha}} \\ \frac{y/a}{n} &= \left(\frac{k/a}{n}\right)^\alpha \\ f_n/a &= (1 - \alpha) \frac{y/a}{n} \end{aligned}$$

- We also have

$$z = q^k = 1$$

- Then  $\kappa$  and  $w$  solve the following system

$$\begin{cases} \kappa x = p^w (f_n/a) - (w/a) + \beta (\kappa/2) x^2 + \beta \rho \kappa x \\ (w/a) = \chi [p^w (f_n/a) + \beta (\kappa/2) x^2 + \beta s \kappa x] + (1 - \chi) [\bar{b} (p^w (f_n/a) + \beta (\kappa/2) x^2)] \end{cases}$$

where

$$\bar{b} = \frac{b(k/a)}{p^w (f_n/a) + \beta (\kappa/2) x^2}$$

- Finally

$$\frac{c/a}{y/a} = 1 - \frac{g/a}{y/a} - \frac{i/a}{y/a} - \frac{\kappa x^2 n}{2 y/a} = 1 - \frac{g/a}{y/a} - \frac{i/a}{y/a} - \frac{\kappa}{2} \frac{x^2}{(y/a)/n}$$

## APPENDIX C

### The complete loglinear GST model

- Technology

$$\widehat{y}_t = \alpha \widehat{k}_t + (1 - \alpha) \widehat{n}_t \quad (\text{E1})$$

- Resource constraint

$$\widehat{y}_t = cy\widehat{c}_t + iy\widehat{i}_t + gy\widehat{g}_t + zy\widehat{z}_t + xy(2\widehat{x}_t + \widehat{n}_{t-1}) \quad (\text{E2})$$

where  $cy = \frac{c/a}{y/a}$ ,  $iy = \frac{i/a}{y/a}$ ,  $gy = \frac{g/a}{y/a}$ ,  $zy = r^k \frac{k/a}{y/a}$  and  $xy = \frac{\kappa}{2} \frac{x^2 n}{y/a}$ .

- Matching

$$\widehat{m}_t = \sigma \widehat{u}_t + (1 - \sigma) \widehat{v}_t \quad (\text{E3})$$

- Employment dynamics

$$\widehat{n}_t = \rho \widehat{n}_{t-1} + (1 - \rho) \widehat{m}_t \quad (\text{E4})$$

- Transition probabilities

$$\widehat{q}_t = \widehat{m}_t - \widehat{v}_t \quad (\text{E5})$$

$$\widehat{s}_t = \widehat{m}_t - \widehat{u}_t \quad (\text{E6})$$

- Unemployment

$$\widehat{u}_t = -\frac{n}{u} \widehat{n}_{t-1} \quad (\text{E7})$$

- Effective capital

$$\widehat{k}_t + \widehat{\varepsilon}_t^a = \widehat{z}_t + \widehat{k}_{t-1}^p \quad (\text{E8})$$

- Physical capital dynamics

$$\widehat{k}_t^p = \xi \left( \widehat{k}_{t-1}^p - \widehat{\varepsilon}_t^a \right) + (1 - \xi) \left( \widehat{i}_t + \widehat{\varepsilon}_t^i \right) \quad (\text{E9})$$

where  $\xi = \frac{1-\delta}{\gamma_a}$

- Aggregate vacancies

$$\widehat{x}_t = \widehat{q}_t + \widehat{v}_t - \widehat{n}_{t-1} \quad (\text{E10})$$

- Consumption-saving

$$\widehat{\lambda}_t = E_t \widehat{\lambda}_{t+1} + (\widehat{r}_t - E_t \widehat{\pi}_{t+1}) - E_t \widehat{\varepsilon}_{t+1}^a \quad (\text{E11})$$



- Marginal utility

$$(1 - \bar{h})(1 - \beta\bar{h})\widehat{\lambda}_t = \bar{h}(\widehat{c}_{t-1} - \widehat{\varepsilon}_t^a) - (1 + \beta\bar{h}^2)\widehat{c}_t + \beta\bar{h}E_t(\widehat{c}_{t+1} + \widehat{\varepsilon}_{t+1}^a) + (1 - \bar{h})(\widehat{\varepsilon}_t^b - \beta\bar{h}E_t\widehat{\varepsilon}_{t+1}^b) \quad (\text{E12})$$

where  $h$  measures the degree of habit persistence in consumption and where  $\bar{h} = \frac{h}{\gamma_a}$

- Capital utilization

$$\widehat{z}_t = \eta_z \widehat{r}_t^k \quad (\text{E13})$$

where  $\eta_z = a'(z)/a''(z) = \frac{1-\psi_z}{\psi_z}$

- Investment

$$\widehat{i}_t = \frac{1}{1+\beta}(\widehat{i}_{t-1} - \widehat{\varepsilon}_t^a) + \frac{1/(\eta_k\gamma_a^2)}{1+\beta}(\widehat{q}_t^k + \widehat{\varepsilon}_t^i) + \frac{\beta}{1+\beta}E_t(\widehat{i}_{t+1} + \widehat{\varepsilon}_{t+1}^a) \quad (\text{E14})$$

where  $\eta_k = s''$

- Capital renting

$$\widehat{p}_t^w + \widehat{y}_t - \widehat{k}_t = \widehat{r}_t^k \quad (\text{E15})$$

- Tobin's q

$$\widehat{q}_t^k = \bar{\beta}(1 - \delta)E_t\widehat{q}_{t+1}^k + [1 - \bar{\beta}(1 - \delta)]E_t\widehat{r}_{t+1}^k - (\widehat{r}_t - E_t\widehat{\pi}_{t+1}) \quad (\text{E16})$$

- Aggregate hiring rate

$$\widehat{x}_t = \epsilon p^w (f_n/a) (\widehat{p}_t^w + \widehat{f}_{nt}) - \epsilon (w/a) \widehat{w}_t + \beta E_t \widehat{x}_{t+1} + (\rho + x/2) \beta (E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t) \quad (\text{E17})$$

where

$$\epsilon = (\kappa x)^{-1}$$

- Marginal product of labor

$$\widehat{f}_{nt} = \widehat{y}_t - \widehat{n}_t \quad (\text{E18})$$

- Weight in Nash bargaining

$$\widehat{\chi}_t = -(1 - \chi) (\widehat{\Sigma}_t - \widehat{\Delta}_t) \quad (\text{E19})$$

with

$$\widehat{\Delta}_t = \rho\lambda\beta E_t (\widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \widehat{\pi}_{t+1} + \gamma\widehat{\pi}_t + \widehat{\Delta}_{t+1} - \widehat{\varepsilon}_{t+1}^a) \quad (\text{E20})$$

$$\widehat{\Sigma}_t = (1 - \rho) \lambda \beta E_t \widehat{x}_{t+1} + \lambda \beta E_t (\widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \widehat{\pi}_{t+1} + \gamma\widehat{\pi}_t + \widehat{\Sigma}_{t+1} - \widehat{\varepsilon}_{t+1}^a) \quad (\text{E21})$$

- Spillover-free target wage

$$\begin{aligned}\widehat{w}_t^o &= \varphi_{fn} \left( \widehat{p}_t^w + \widehat{f}_{nt} \right) + (\varphi_x + \varphi_s) E_t \widehat{x}_{t+1} + \varphi_s E_t \widehat{s}_{t+1} + \varphi_b \widehat{b}_t \\ &\quad + (\varphi_s + \varphi_x/2) \left( E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t \right) + \varphi_\chi \left( \widehat{\chi}_t - \beta(\rho - s) \widehat{\chi}_{t+1} \right) + \widehat{\varepsilon}_t^w\end{aligned}\quad (\text{E22})$$

where

$$\begin{aligned}\bar{w} &= (w/a) \\ \varphi_{fn} &= \chi p^w (f_n/a) \bar{w}^{-1} \quad \varphi_x = \chi \beta (1 - \rho) \epsilon^{-1} \bar{w}^{-1} \\ \varphi_b &= (1 - \chi) b (k/a) \bar{w}^{-1} \quad \varphi_s = \chi \beta s \epsilon^{-1} \bar{w}^{-1} \\ \varphi_\chi &= \chi (1 - \chi)^{-1} \epsilon^{-1} \bar{w}^{-1}\end{aligned}$$

- Aggregate wage

$$\widehat{w}_t = \gamma_b (\widehat{w}_{t-1} - \widehat{\pi}_t + \gamma \widehat{\pi}_{t-1} - \widehat{\varepsilon}_t^a) + \gamma_o \widehat{w}_t^o + \gamma_f (\widehat{w}_{t+1} + \widehat{\pi}_{t+1} - \gamma \widehat{\pi}_t + \widehat{\varepsilon}_{t+1}^a) \quad (\text{E23})$$

where

$$\begin{aligned}\gamma_b &= (1 + \tau_2) \phi^{-1} \quad \gamma_o = \varsigma \phi^{-1} \quad \gamma_f = (\rho \beta - \tau_1) \phi^{-1} \\ \phi &= 1 + \tau_2 + \varsigma + \rho \beta - \tau_1 \quad \varsigma = (1 - \lambda) (1 - \rho \lambda \beta) \lambda^{-1} \\ \tau_1 &= \varphi_s \Gamma (1 - \rho \lambda \beta) \\ \tau_2 &= [\varphi_x \lambda - \varphi_\chi (1 - \chi) (1 - \rho) \Psi \Delta^{-1}] \epsilon \Sigma \bar{w} (1 - \rho \lambda \beta) \\ \Gamma &= (1 - \eta (1 - \rho) \Psi) \eta^{-1} \epsilon \Sigma \bar{w} \quad \Psi = \beta \lambda^2 / (1 - \beta \lambda^2) \quad \Sigma = (1 - \beta \lambda)^{-1}\end{aligned}$$

- Phillips curve

$$\widehat{\pi}_t = \iota_b \widehat{\pi}_{t-1} + \iota_o (\widehat{p}_t^w + \widehat{\varepsilon}_t^p) + \iota_f E_t \widehat{\pi}_{t+1} \quad (\text{E24})$$

where

$$\begin{aligned}\iota_b &= \gamma^p (\phi^p)^{-1} \quad \iota_o = (\varsigma^p / \tau^p) (\phi^p)^{-1} \quad \iota_f = \beta (\phi^p)^{-1} \\ \phi^p &= 1 + \beta \gamma^p \quad \varsigma^p = (1 - \lambda^p) (1 - \lambda^p \beta) (\lambda^p)^{-1} \quad \tau^p = 1 + (\varepsilon^P - 1) \xi\end{aligned}$$

- Taylor rule

$$\widehat{r}_t = \rho_s \widehat{r}_{t-1} + (1 - \rho_s) [r_\pi \widehat{\pi}_t + r_y (\widehat{y}_t - \widehat{y}_{nt})] + \widehat{\varepsilon}_t^r \quad (\text{E25})$$

- Government spending

$$\widehat{g}_t = \widehat{y}_t + \frac{1 - gy}{gy} \widehat{\varepsilon}_t^g \quad (\text{E26})$$

- Market tightness

$$\widehat{\theta}_t = \widehat{v}_t - \widehat{u}_t \quad (\text{E27})$$

- Benefits

$$\widehat{b}_t = \widehat{k}_t^p \quad (\text{E28})$$

## APPENDIX D

### Estimates of the SW model

Note that the parameters  $\omega$  and  $\varepsilon^w$  are, respectively, the Frisch elasticity of labor supply and the gross steady state wage markup. The parameters  $\rho_w$  and  $\sigma_w$  refer to a wage markup shock.

Table A1: Prior and posterior distribution of structural parameters, SW model					
Parameter	Prior	Max	Posterior		
	Distribution		Mean	5%	95%
$\psi_z$	Beta (0.5,0.1)	0.667	0.657	0.546	0.765
$\eta_k$	Normal (4,1.5)	2.922	3.593	2.286	5.297
$h$	Beta (0.5,0.1)	0.746	0.772	0.703	0.839
$\omega$	Gamma(2,0.75)	3.910	4.041	2.829	5.452
$\gamma$	Uniform (0,1)	0.796	0.763	0.574	0.938
$\gamma^P$	Uniform (0,1)	0.000	0.025	0.001	0.078
$\lambda$	Beta (0.75,0.1)	0.881	0.865	0.783	0.925
$\lambda^P$	Beta (0.66,0.1)	0.856	0.854	0.816	0.890
$\varepsilon^P$	Normal (1.15,0.05)	1.392	1.391	1.336	1.448
$\varepsilon^w$	Normal (1.15,0.05)	1.138	1.127	1.042	1.210
$r_\pi$	Normal (1.7,0.3)	2.057	2.053	1.725	2.413
$r_y$	Gamma (0.125,0.1)	0.307	0.320	0.224	0.437
$\rho_s$	Beta (0.75,0.1)	0.807	0.813	0.765	0.855
$\gamma_a$	Uniform (1,1.5)	1.004	1.004	1.003	1.005

Notes: For the Uniform distribution the two numbers in parenthesis are the lower and the upper bound. Otherwise, the two numbers are the mean and the standard deviation.

Table A2: Prior and posterior distribution of shock processes, SW model					
Parameter	Prior	Max	Posterior		
	Distribution		Mean	5%	95%
$\rho_a$	Beta (0.5,0.15)	0.129	0.143	0.067	0.226
$\rho_r$	Beta (0.5,0.15)	0.241	0.254	0.155	0.359
$\rho_b$	Beta (0.5,0.15)	0.698	0.662	0.527	0.777
$\rho_i$	Beta (0.5,0.15)	0.531	0.508	0.385	0.631
$\rho_p$	Beta (0.5,0.15)	0.810	0.799	0.746	0.851
$\rho_w$	Beta (0.5,0.15)	0.296	0.309	0.195	0.420
$\rho_g$	Beta (0.5,0.15)	0.991	0.989	0.982	0.995
$\sigma_a$	IGamma (0.15,0.15)	1.022	1.041	0.952	1.141
$\sigma_r$	IGamma (0.15,0.15)	0.228	0.231	0.211	0.253
$\sigma_b$	IGamma (0.15,0.15)	0.420	0.583	0.337	0.999
$\sigma_i$	IGamma (0.15,0.15)	0.195	0.241	0.154	0.354
$\sigma_p$	IGamma (0.15,0.15)	0.060	0.063	0.048	0.080
$\sigma_w$	IGamma (0.15,0.15)	0.197	0.197	0.166	0.231
$\sigma_g$	IGamma (0.15,0.15)	0.359	0.362	0.331	0.396

Notes: The two numbers in parenthesis are the mean and the st. dev. of the distribution.

Figure 1: Total hours and employment

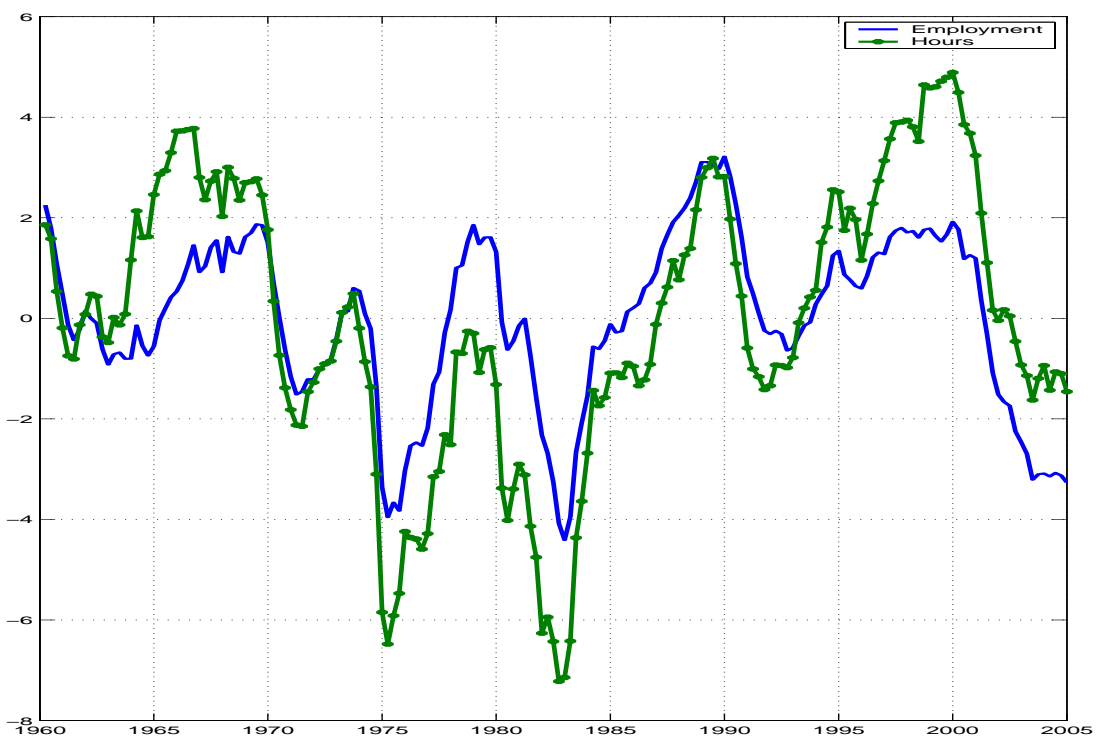


Figure 2: Autocovariance function of GST model vs data

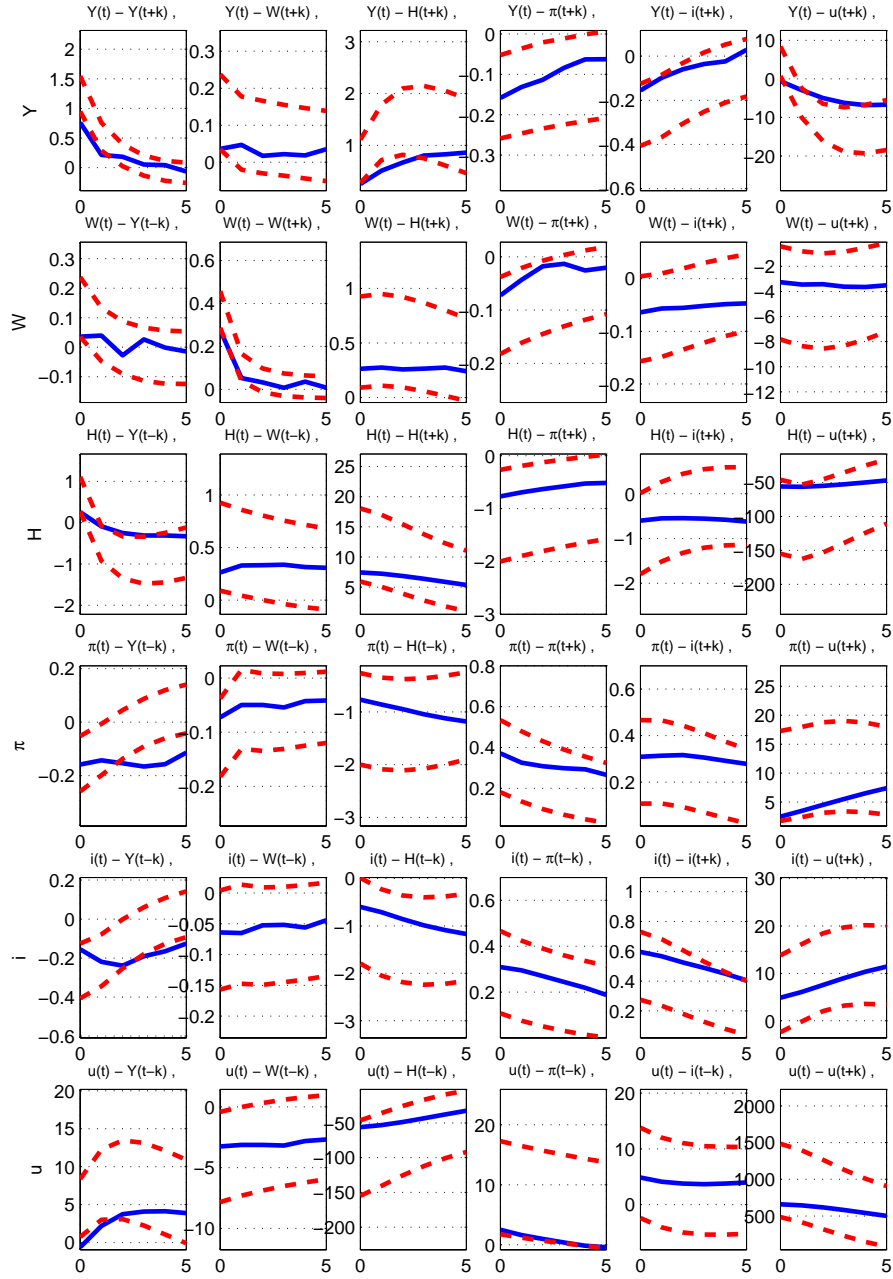


Figure 3: Autocovariance function of flex W model vs data

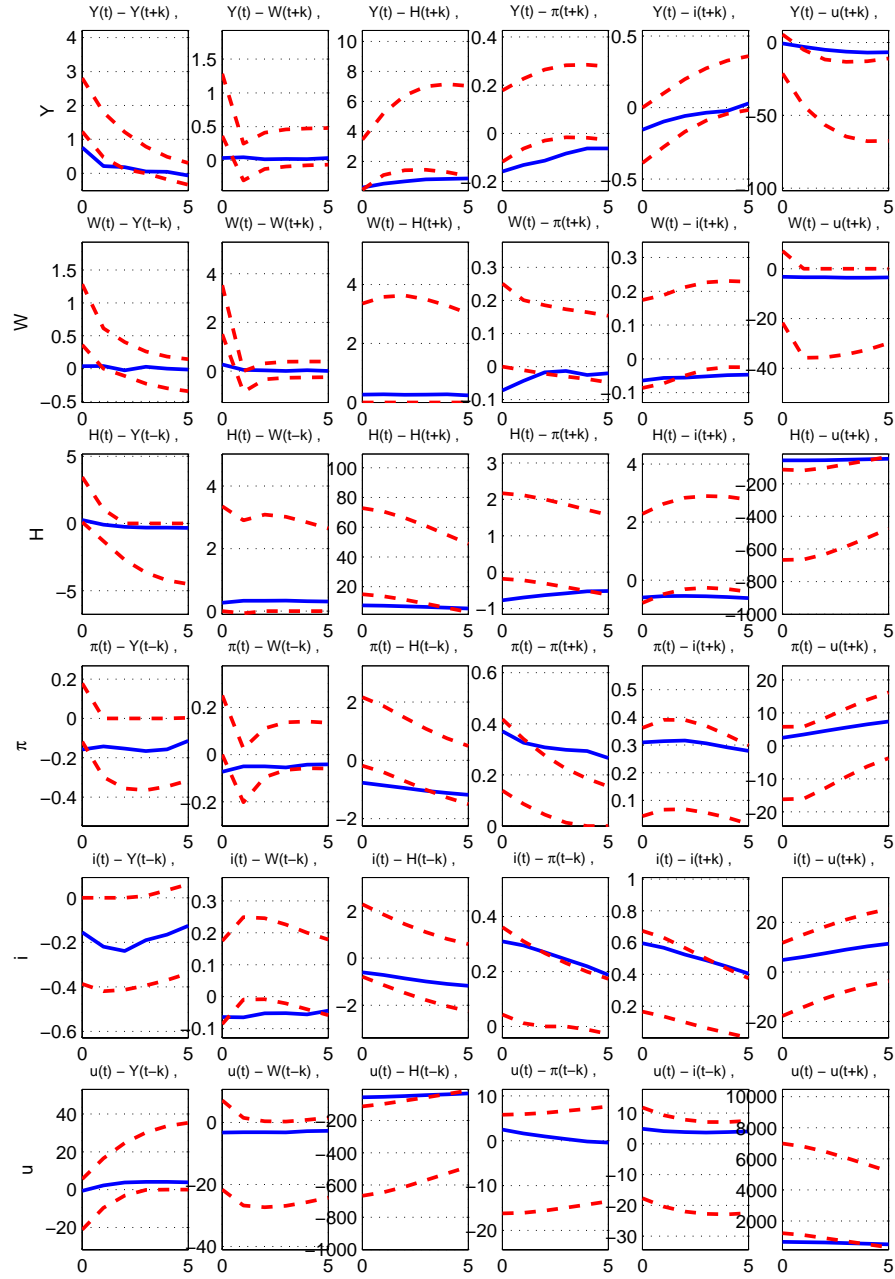


Figure 4: Impulse response functions to a technology shock

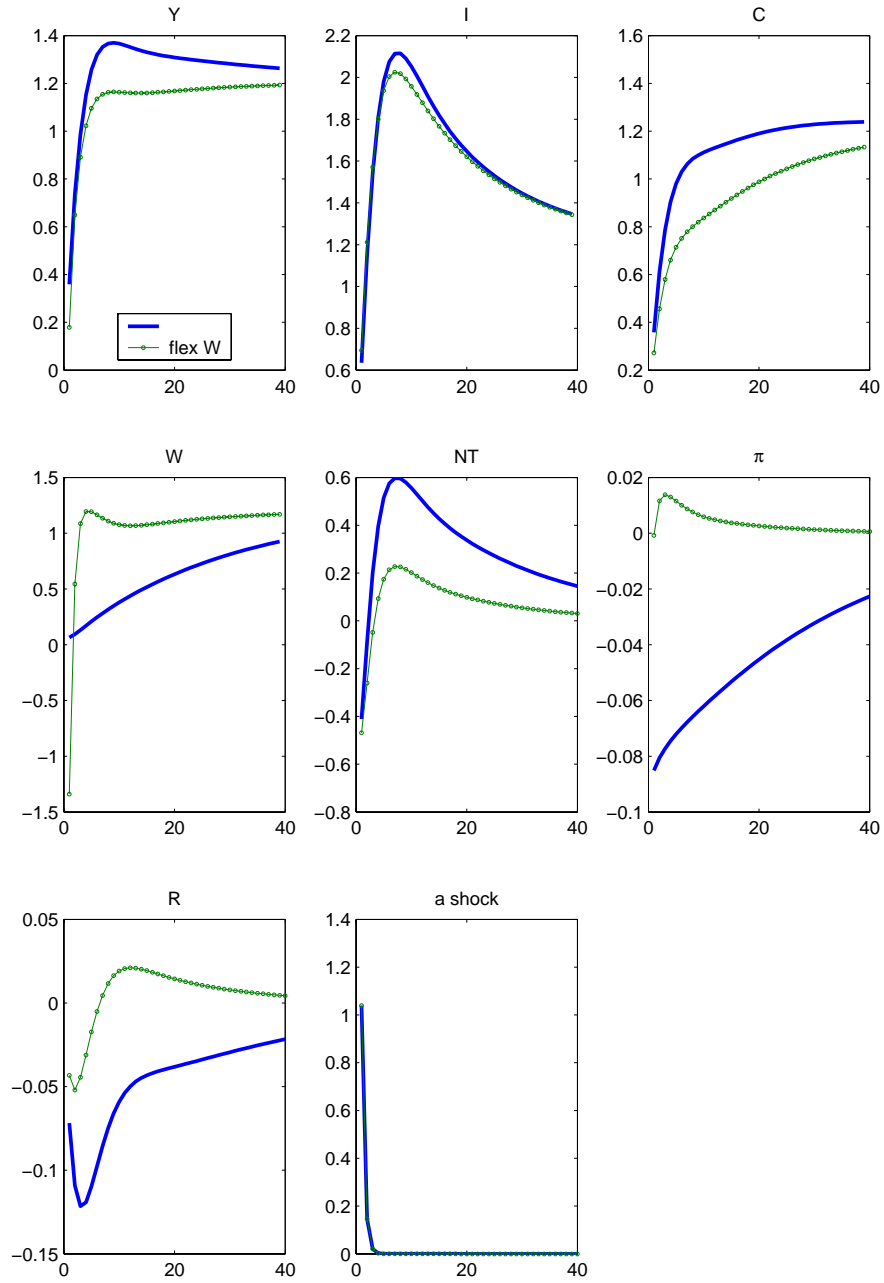




Figure 5: Impulse response functions to an investment specific shock

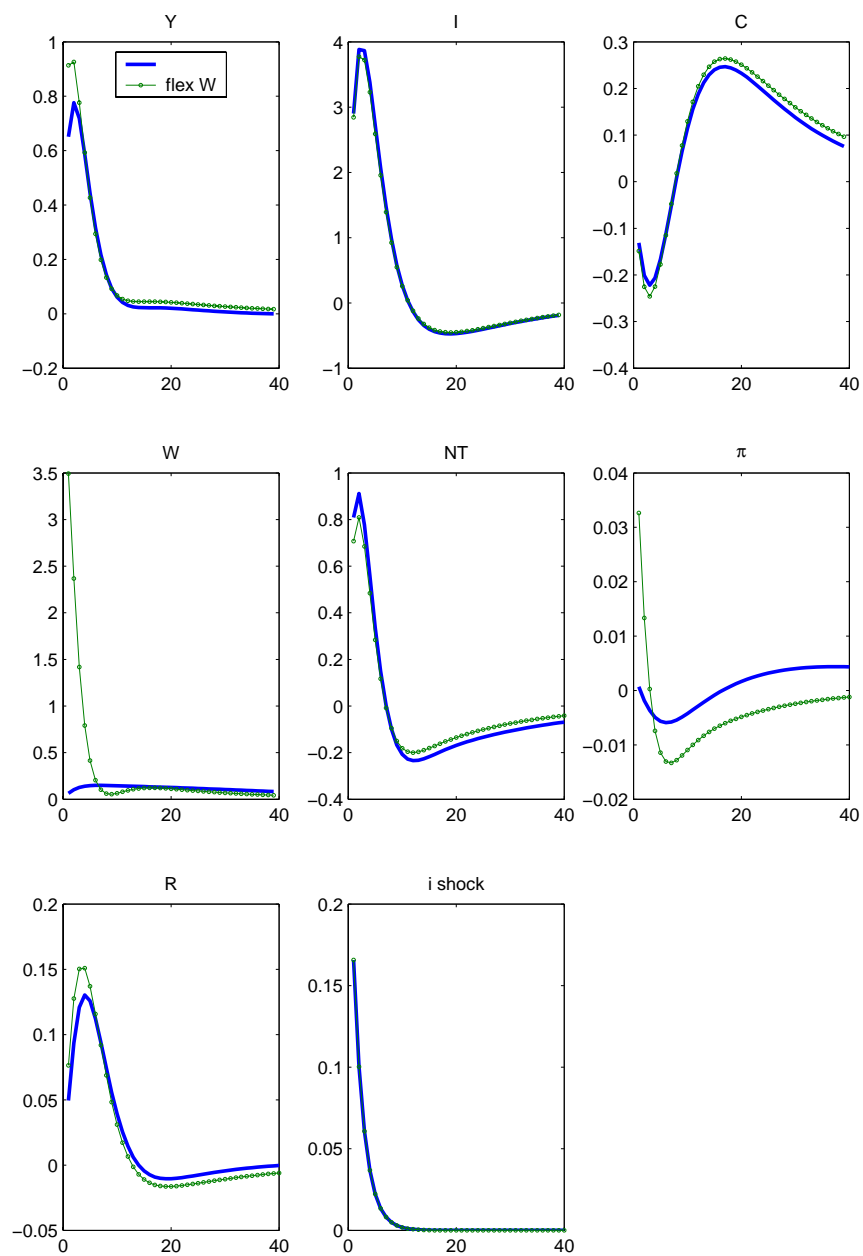


Figure 6: Autocovariance functions of GST and SW vs data

