Application of Akaike's Method to Economic Time Series*

by Yoshiharu Oritani

SUMMARY

Akaike's information criterion, AIC, provides a theoretical justification of "the principle of parsimony" which suggests that overcomplex models yield less accurate forecasts. It is empirically confirmed that the forecasts of economic variables by ARMA models, automatically identified by minimizing AIC, often outperform forecasts by other methods such as large-scale econometric models. It is found that the relative power contribution analysis, also developed by Akaike, is useful for the test of Granger's causality.

1. Introduction

In the time series analysis, the most serious problem is the identification of the model from the observed data ("system identification"). Box and Jenkins [11] suggested "the principle of parsimony" for it. But, as they did not show the formal statistical identification criterion, the procedure has been rather arbitrary and cumbersome.

As a solution to this problem, Akaike proposed the concept of "final prediction error (FPE)[1]" (Akaike [2]) and "an information criterion (AIC)" (Akaike [4]) which is called "Akaike's information criterion" in this paper.

AIC is defined by

AIC = –2(maximum log likelihood) + 2(number of free parameters).

---

* A paper presented at the Econometric Society Summer Meeting, Montreal, Quebec, June 27–30, 1979.

·Views expressed are those of the author and not necessarily related to those of the Bank of Japan.

1 Under the Gaussian assumption, the criterion FPE satisfies the asymptotic equality

$$AIC = N \log FPE.$$

The first term represents the accuracy of the model fitting and the second term the cost of model complication. The more the free parameters are added, the more complex is the model which is chosen, and consequently, the value of the first term would diminish because of the improvement of the fit. But, at the same time, the value of the second term of AIC should increase reflecting the increment of the number of the free parameters to be estimated. Several competing models are fitted by the method of maximum likelihood and the one with the smallest value of AIC is chosen as the best model. This procedure is called the "minimum AIC procedure", or "Akaike's method".

As shown in the above definition of AIC, the prominent feature of Akaike's method lies in the information theoretical treatment of "the principle of parsimony". It is well recognized in the time series analysis that the principle of parsimony is a very important concept for statistical model building. For example, Box and Jenkins [11] wrote "we employ the smallest possible number of parameters for adequate representation". The definition of AIC gives a clear formulation of "the principle of parsimony". Akaike [6] states

> "What has been clarified by the introduction of AIC is the importance of the concept of parsimony in statistical model building. By explicitly taking into account the penalty of introducing the free parameters into a model the definition of AIC clearly shows why we have to be parsimonious in choosing a model".

It is especially important to be parsimonious for economic time series models because of the limited sizes of economic data. Under the assumption of a Gaussian time series AIC takes the form

$$AIC = N \ln(\sigma^2) + 2(\text{number of free parameters}).$$

Here, N denotes the number of data and σ^2 denotes the innovation variance of the model estimated by the maximum likelihood method. Since the value of the first term of AIC tends to be small by over-parameterization in the case of a small number of data as in economic time series, the second term of AIC, which represents the cost of the model complexity, has a stronger meaning for the model selection.

Akaike's method realized several statistical innovations in the field of time series analysis. These are

1) automatic identification and estimation of a univariate ARMA model,
2) almost automatic identification and estimation of a multivariate ARMA model and,
3) a routine procedure for the explicit modeling of feedback systems.

In the field of control engineering, the usefulness of Akaike's method has already been well recognized. In this paper we apply Akaike's method to the Japanese economic time series and demonstrate some of its usefulness.

In section 2, the results of comparison of the forecasting performance between automatically identified univariate ARMA models and other predictions such as the commodity futures price, anticipation survey data and macro-econometric model prediction, etc., are reported. In section 3, the results of forecasting of real GNP by the automatically identified multivariate ARMA model are demonstrated. In section 4, a test of Granger's causality is realized by using the automatically identified multivariate AR model of the variables used in section 3.

The results reported in this paper were obtained by using the computer programs from the TIMSAC (Time Series Analysis and Control) Program Package developed by Akaike and his colleagues. The use of TIMSAC is discussed by Akaike [7]. The description of the program

package is available in the form of a monograph (Akaike [5]) and the computer tape of TIMSAC is available from the Division of Mathematical Sciences of the University of Tulsa, Oklahoma (see, Findley [13]).

2. Forecasting Performance of Automatically Identified Univariate ARMA Models

Since the publication of Box and Jenkins [11] it is well recognized that the univariate ARMA model is useful for forecasting economic time series. The forecasting performance of an ARMA model is dependent on the choice of the order of the model, but no objective procedure has been suggested by these authors for the problem. Thus, except for the special situations where the orders are specified in advance, the choice of the order of an ARMA model requires a kind of artisanship.

Recently, Akaike proposed "Akaike's *information criterion (AIC)*' as a statistical criterion for the choice of a statistical model and applied it to the determination of the order of an ARMA model to develop an automatic procedure for the fitting of an ARMA model. Akaike [6] proposed a procedure for the determination of the initial values by using canonical correlation analysis. By the program AUTARM of TISMAC-74 [5], AIC's are computed for models within some range around the initial choice and the model of the order which gives the minimum of the AIC's is chosen. An example of the computer output of this program is shown in Table 1. In this case the model of No.7 which has AR order 1 and MA order 2 is chosen as the best model.

We estimated the following three models, using the minimum AIC procedure. We made forecasts using those models, and compared them with the alternative prediction data.

	Predicted variable from univariate ARMA model	Date compared
Ex. 1.1	wool market spot price	wool market futures price
Ex. 1.2	industrial production index	anticipation survey data of the index
Ex. 1.3	real GNP growth rate	macro-econometric model prediction

Example 1.1 Forecasting the Wool Market Price
— Comparison with the Future Price

According to the efficient market hypothesis (Fama [12]), the futures price of a commodity at time t is the best predictor of the spot price in the future as long as the commodity market is efficient, and risk premium and transaction cost are negligible in the market. If we denote a 3 month-futures price of the commodity market as $3P_t$, $3P_t$ equals the optimal forecast at time t of the spot price 3 month-ahead, which we denote by $\hat{P}_t(3)$, i.e., we have $3P_t = \hat{P}_t(3) = E(P_{t+3}/I_t)$. Here, $E(P_{t+3}/I_t)$ denotes the expectation at time t of P at time t+3, based on all the available information I_t. The above shows that the market participants do not form the expectation which always errs in the same direction in the efficient market. Plotted in Fig. 1 are the differences between the futures wool prices and the corresponding actual spot prices, $P_{t+3}-3P_t$. The differences, which mean a series of forecasting errors of the futures price, are scattered around zero. It is difficult to identify whether or not the wool market price is formed efficiently, but the above observation suggests that the futures price is at least an unbiased estimator of the actual price. Thus the futures price is used as an observable alter-

(Table 1) Example of the Minimum AIC Procedure

```
INITIAL AUTO REGRESSIVE MODEL FITTING

MINIMUM AIC(M)= 0.18288D+03  ATTAINED AT M=    4
AR-COEFFICIENTS
  1 -0.66064D+00
  2 -0.26364D+00
  3  0.11763D+00
  4  0.14761D+00
MINIMUM DIC(P) =        0.81 ATTAINED AT P=    1
MINIMUM DIC(P) =        1.14 ATTAINED AT P=    1
MINIMUM DIC(P) =       -2.31 ATTAINED AT P=    1
ORDER OF AR-COEFFICIENTS =  3
   I      BETA(I) (AR-COEF)     ALPHA(I) (MA-COEF)
    1                -0.99138                -0.33073
    2                 0.44080                 0.48594
    3                -0.10061                 0.0

AUTOMATIC AR-MA MODEL FITTING; SCALAR CASE

**CASE NO. 1 AR 3 MA 2  AIC= 0.18349D+03

   I              AR(I) STANDARD DEVIATION
    1      -0.10497D+01       0.29389D+00
    2       0.45408D+00       0.41465D+00
    3      -0.70444D-01       0.24425D+00

   I              MA(I) STANDARD DEVIATION
    1      -0.38351D+00       0.26833D+00
    2       0.49759D+00       0.26805D+00

                 INNOVATION VARIANCE
           0.67294D+01

MINIMUM AIC = 0.18349D+03 ATTAINED AT THE BEST CHOICE  AR=  3  MA=  2

                          ⋮

**CASE NO. 7 AR 1 MA 2  AIC= 0.18122D+03

   I              AR(I) STANDARD DEVIATION
    1      -0.64703D+00       0.12138D+00

   I              MA(I) STANDARD DEVIATION
    1      -0.65712D-02       0.13691D+00
    2       0.33320D+00       0.11616D+00

                 INNOVATION VARIANCE
           0.68583D+01

MINIMUM AIC = 0.18122D+03 ATTAINED AT THE BEST CHOICE  AR=  1  MA=  2

                          ⋮

**CASE NO.12 AR 0 MA 3  AIC= 0.18311D+03

   I              MA(I) STANDARD DEVIATION
    1      0.63350D+00       0.96213D-01
    2      0.59035D+00       0.99191D-01
    3      0.39740D+00       0.96213D-01

                 INNOVATION VARIANCE
           0.70026D+01

MINIMUM AIC = 0.18122D+03 ATTAINED AT THE BEST CHOICE  AR=  1  MA=  2

BORDER CHECK COMPLETED
```

native to the forecast by an ARMA model.

First, we transformed the monthly wool stop price P_t to a stationary time series by calculating the deviation from the trend line T_t and obtained $p_t = P_t - T_t$.

Second, we fitted univariate ARMA models to the transformed series P_t and made 3 month-ahead forecasts with the estimated models. The models were estimated every 12 months by fixing the starting point of the period on August 1963, and extending the end of the period by 12 months. An example of the estimated model is shown in Table 2. The 3 month-ahead predictions were tried in the last 12 months of the model estimation period[2]. For instance, the first model estimatation was made with the sample period of August 1963 ~ December 1972. The 3 month-ahead predictions were repeated 12 times by the estimated model with the starting period of the forecast from January 1972 to December 1972.

The predicted values were transformed back to the original base by adding the trend values. The 3 month-ahead predictions with univariate ARMA models are shown by the dotted line in the upper part of Fig. 2, and the percentage deviations of predicted values from the futures prices are shown at the bottom part of the figure. These figures show that the forecasts by the univariate ARMA models are almost as good as the futures prices.

(Table 2) Example of the Estimated ARMA Model of Wool Market Spot Price*

$$\underset{(59.478)}{p_t = 1.915\, p_{t-1}} \underset{(29.907)}{- 0.922\, p_{t-2}} \underset{(28.928)}{- 0.975\, u_{t-1}}$$

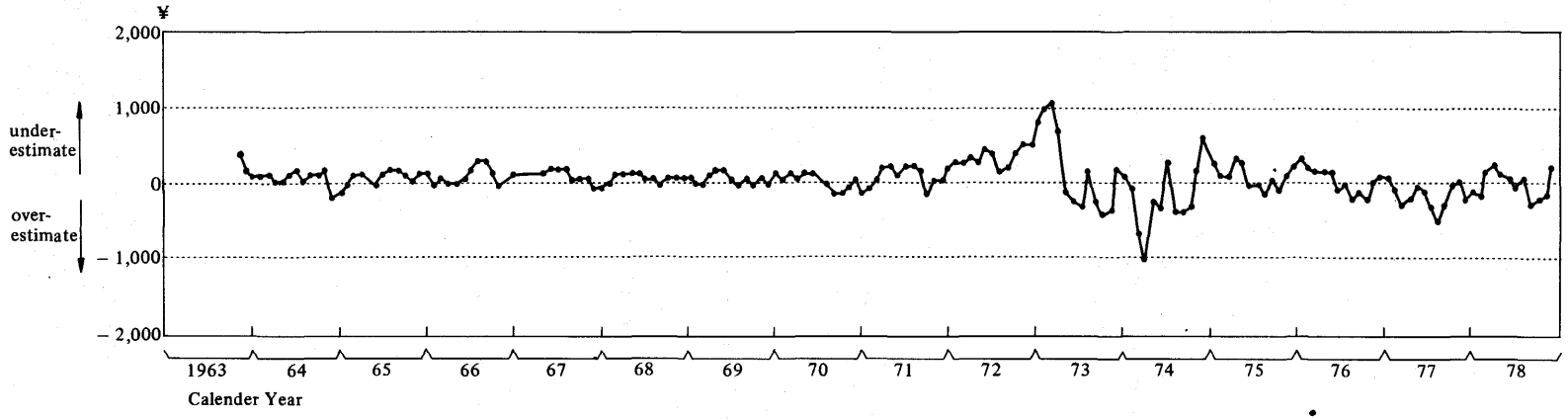
S.E. = 185.20
Sample Period Aug. 1963 ~ Dec. 1978

p_t ; Transformed wool market spot price (deviation from sample average value)
u_t ; Innovation
The numbers in parentheses are t-statistics.

* Seven other models like this were estimated after extending the sample period by every 12 months.

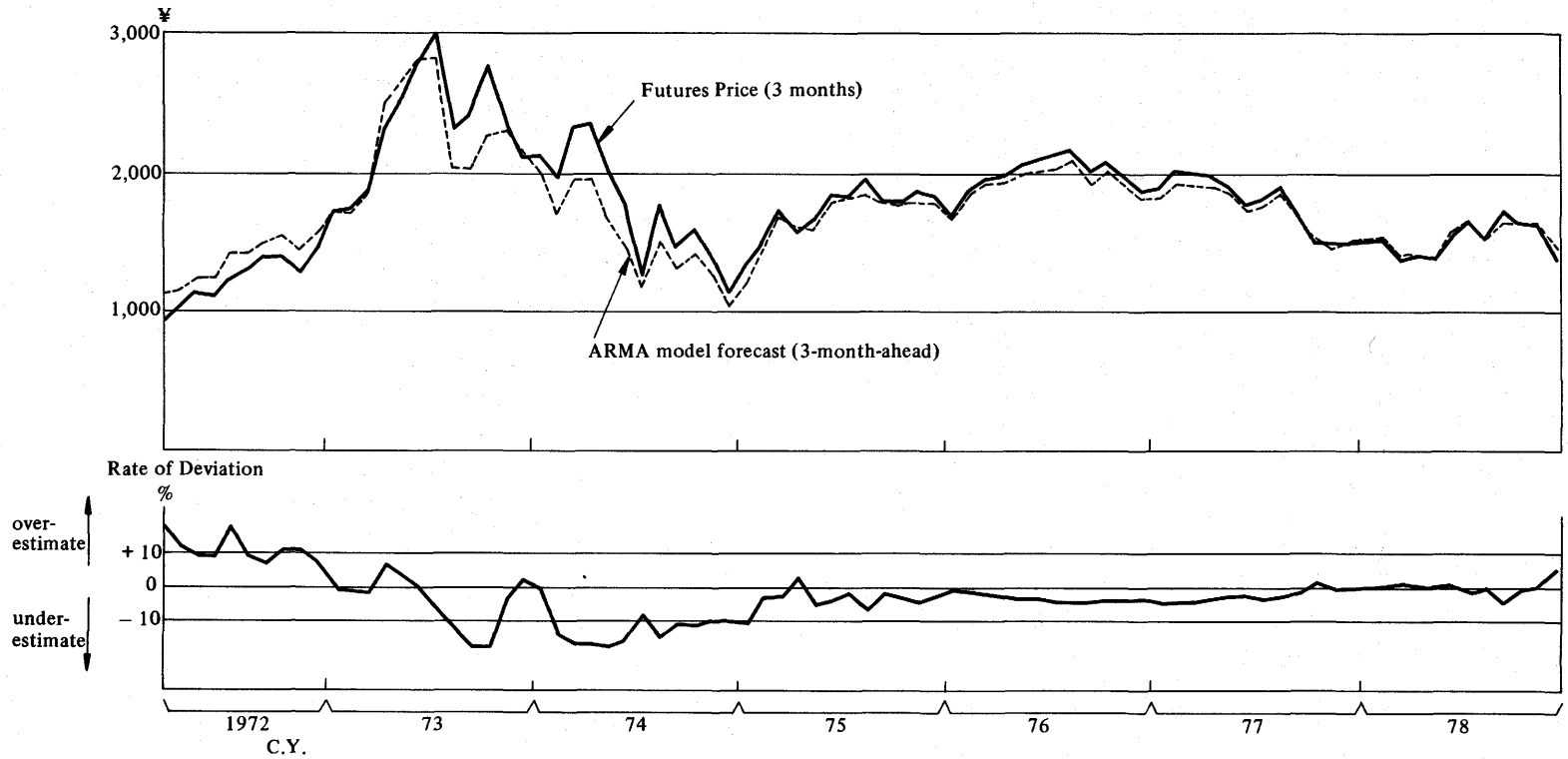
2 We made in-sample forecasts. This is only for computational simplicity. To be more exact, we should have made out-of-sample forecasts using models which are re-estimated every month. However, the effect on the forecasts of these factors seemed to be rather small, because we used only the last 12 months of the all sample period. We made an exact out-of-sample method in the case of real GNP prediction, which are described in the example 1.3 and section 3.

(Fig. 1) Differences between Wool Market Futures Prices and Actual Spot Prices



Data Source; The Nihon Keizai Shinbun

(Fig. 2) Wool Market Futures Prices and ARMA Model Forecasts



Example 1.2 Forecasting Industrial Production Index

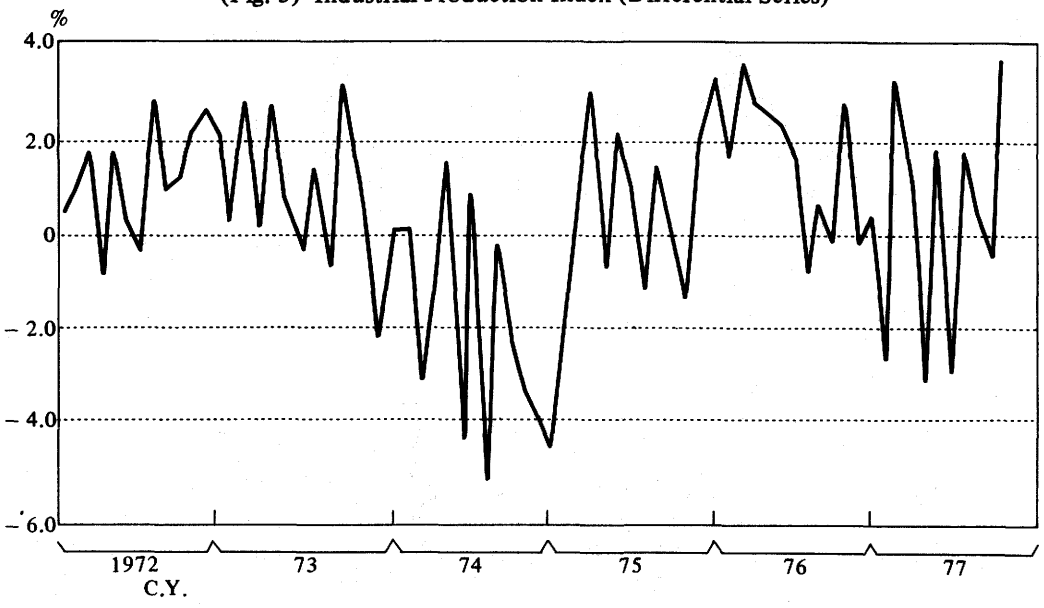
— Comparison with Anticipation Survey Data of the Index

In Japan, the anticipation survey data of the industrial production index are available as well as the actual index. Since the anticipation data are compiled from the direct survey of each industrial firm, the anticipation survey data can be considered to be the figure which summarizes a huge amount of individual information and judgements of experts in each firm. Thus, it is interesting to compare the predictive performance of the automatically estimated univariate ARMA model of the actual index with the anticipation survey data.

First, we transformed the seasonally adjusted actual index series to a stationary series by differencing. The transformed series are shown in Fig. 3. Next, a univariate ARMA model was fitted to the data for the period of January 1953 ~ November 1977 by Akaike's method. The estimated model is shown in Table 3. Third, one month-ahead and 2 month-ahead predictions were made with the estimated model, after January 1976[3]. The predicted values after being transformed back to the original base and converted to the rate of increase, are shown with a dotted line in Fig. 4 and Fig. 5.

The corresponding anticipation survey data are also shown in Fig.s' 4 and 5. Tables 4 and 5, show that the root mean square error (RMSE) of the one month-ahead predictions by the ARMA model is 1.29, compared with 1.24 of the anticipation survey data. The RMSE of 2 month-ahead predictions by the ARMA models is 1.73, compared with 1.63 of the anticipation data. It is interesting that the automatically estimated ARMA models show almost the same predictive performance as the anticipation survey data which are compiled from a huge amount of individual information and experts' judgements.

(Fig. 3) Industrial Production Index (Differential Series)



Data Source; Ministry of International Trade and Industry

3 In this case, too, the predictions are made in the sample period for the same reason as in footnote 2.

(Table 3) Estimated ARMA Model of Industrial Production Index

$$x_t = \sum_{i=1}^{6} \alpha i\, x_{t-i} + \sum_{i=1}^{6} \beta i\, u_{t-i}$$

S.E. = 0.940

Sample Period Jan. 1953 ~ Nov. 1977

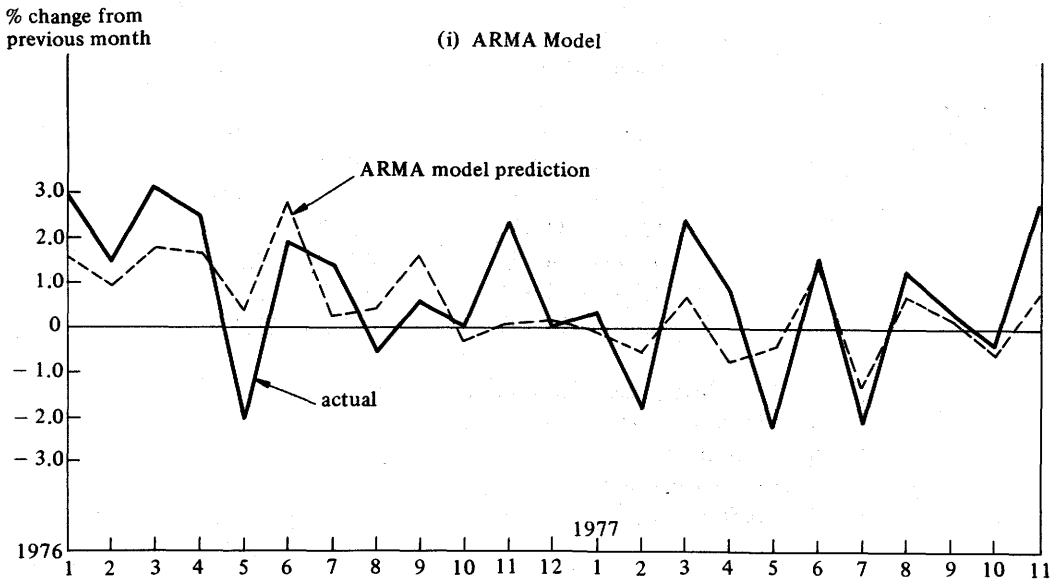
	αi (t-value)	βi (t-value)
i = 1	−1.065 (−22.95)	1.043 (16.25)
2	0.610 (7.94)	−0.520 (− 6.33)
3	1.969 (34.33)	−1.627 (−20.30)
4	0.765 (14.15)	−0.539 (− 7.21)
5	−0.870 (−12.45)	0.807 (11.24)
6	−0.884 (−21.38)	0.738 (13.25)

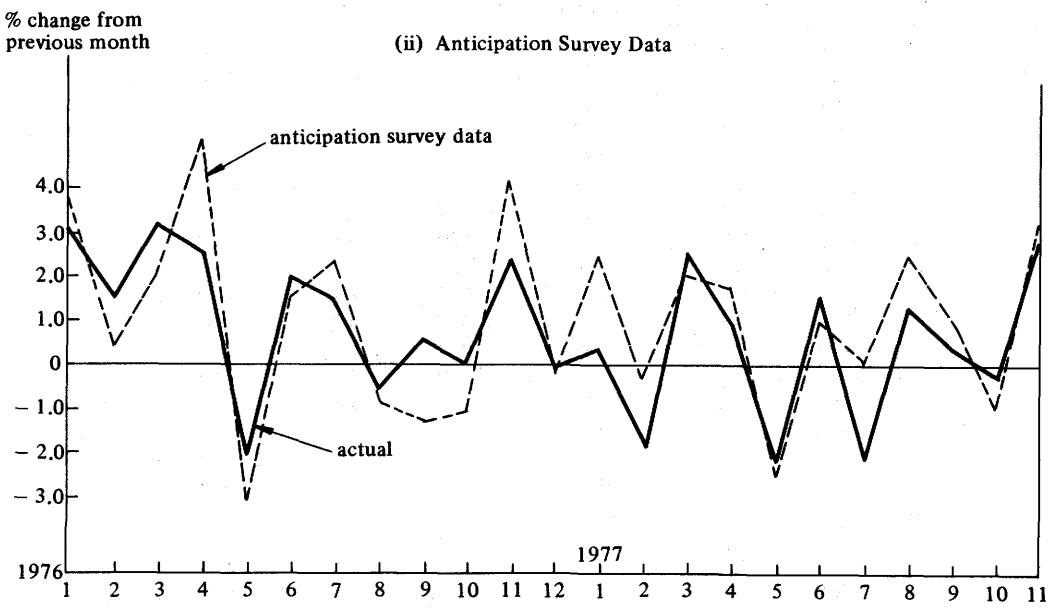
x_t ; First differences of industrial production index (deviation from sample average)
u_t ; Innovation
αi ; Coefficients of x_{t-i}
βi ; Coefficients of u_{t-i}

(Table 4) One Month-Ahead Prediction of Industrial Production Index (Percentage Change from Previous Month)

	actual (A)	ARMA model		anticipation survey data	
		(B) prediction	error (A−B)	(C) prediction	error (A−C)
1976. Jan.	3.00 %	1.53 %	1.47 %	3.7 %	− 0.70 %
Feb.	1.46	0.91	0.55	0.4	1.06
Mar.	3.21	1.79	1.42	2.1	1.11
Apr.	2.45	1.74	0.71	5.1	− 2.65
May	− 2.08	0.34	− 2.42	− 3.1	1.02
June	1.96	2.83	− 0.87	1.5	0.46
July	1.36	0.22	1.14	2.3	− 0.94
Aug.	− 0.63	0.43	− 1.06	− 0.8	0.17
Sept.	0.56	1.60	− 1.04	− 1.3	1.86
Oct.	0.0	− 0.37	0.37	− 1.1	1.10
Nov.	2.37	0.05	2.32	4.2	− 1.83
Dec.	− 0.08	0.17	− 0.25	− 0.2	0.12
1977. Jan.	0.31	− 0.07	0.38	2.4	− 2.09
Feb.	− 1.92	− 0.54	− 1.38	− 0.4	− 1.52
Mar.	2.51	0.63	1.88	2.0	0.51
Apr.	0.92	− 0.84	1.76	1.7	− 0.78
May	− 2.27	− 0.45	− 1.82	− 2.6	0.33
June	1.55	1.37	0.18	1.0	0.55
July	− 2.22	− 1.34	− 0.88	0.1	− 2.32
Aug.	1.33	0.73	0.60	2.4	− 1.07
Sept.	0.39	0.25	0.14	0.9	0.51
Oct.	− 0.31	− 0.51	0.20	− 1.1	0.79
Nov.	2.85	0.72	2.13	3.2	− 0.35
Root Mean Square Error		−	1.29	−	1.24

(Fig. 4) One Month-Ahead Prediction of Industrial Production Index

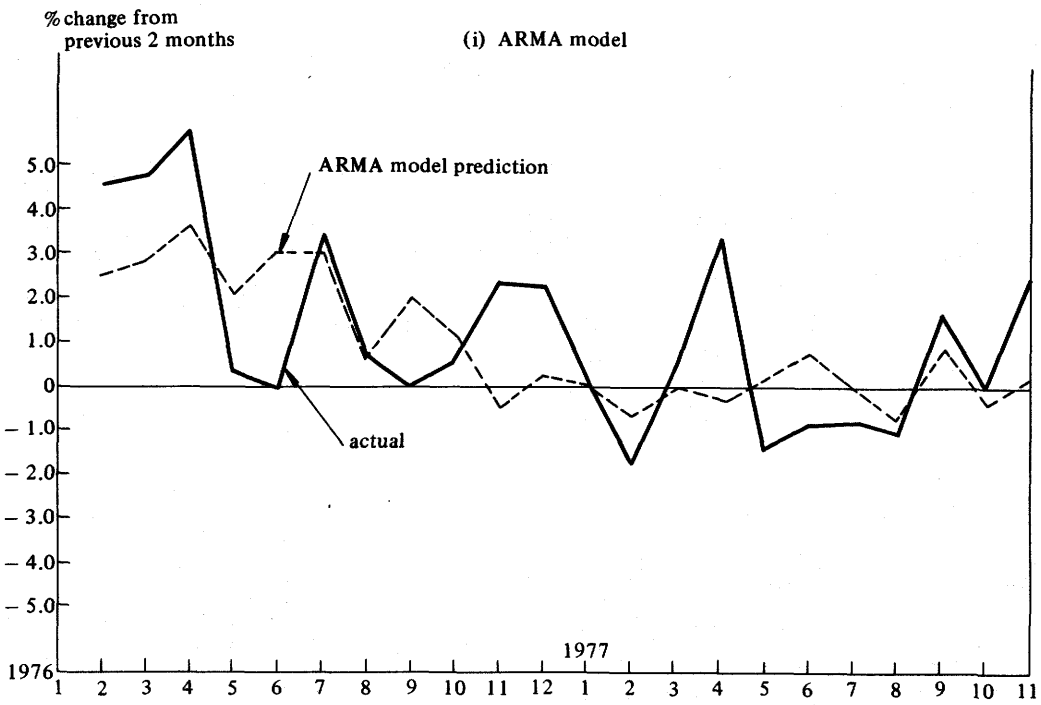


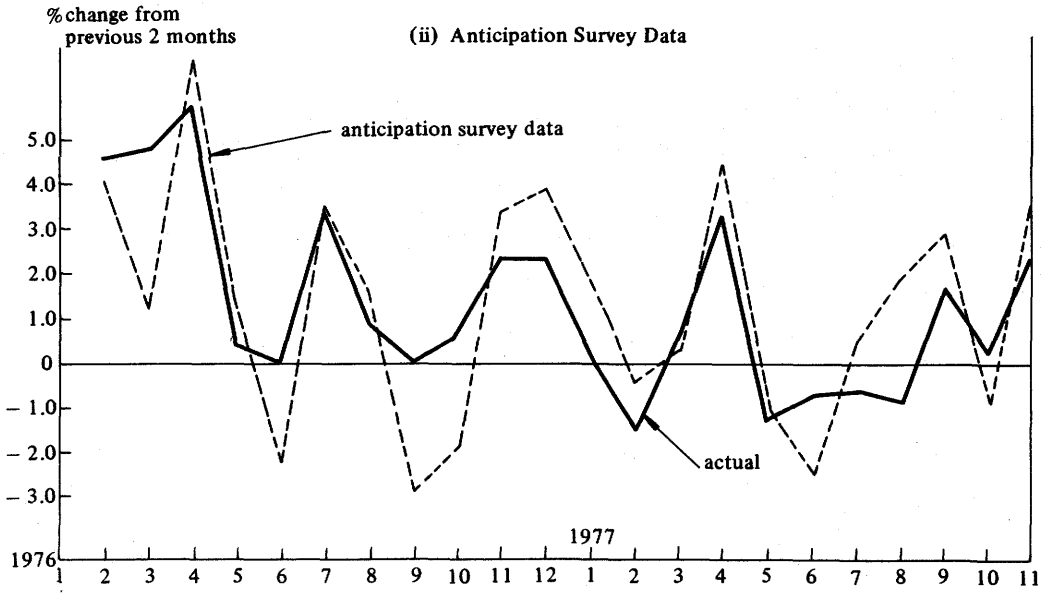


(Table 5) Two Month-Ahead Prediction of Industrial Production Index
(Percentage Change from Previous Month)

	actual (A)	ARMA model		anticipation survey data	
		(B) prediction	error (A–B)	(C) prediction	error (A–C)
1976. Feb.	4.55	2.50	– 2.05	4.01	0.54
Mar.	4.75	2.74	– 2.01	1.20	3.55
Apr.	5.78	3.62	– 2.16	6.80	– 1.02
May	0.35	2.11	1.76	1.32	– 0.97
June	– 0.14	3.06	3.20	– 2.23	2.09
July	3.43	3.03	– 0.40	3.53	– 0.10
Aug.	0.79	0.69	– 0.10	1.58	– 0.79
Spet.	0.	2.00	2.00	– 2.88	2.88
Oct.	0.60	1.21	0.61	– 1.89	2.49
Nov.	2.40	– 0.32	– .272	3.45	– 1.05
Dec.	2.30	0.28	– 2.02	3.89	– 1.59
1977. Jan.	0.20	0.10	– 0.10	1.80	– 1.60
Feb.	– 1.61	– 0.60	1.01	– 0.47	– 1.14
Mar.	0.54	0.05	– 0.49	0.20	0.34
Apr.	3.45	– 0.20	– 3.65	4.55	– 1.10
May	– 1.38	– 1.25	0.13	– 1.15	– 0.23
June	– 0.76	0.85	1.61	– 2.50	1.74
July	– 0.70	0	0.70	0.39	– 1.09
Aug.	– 0.92	– 0.65	0.27	1.80	– 2.72
Sept.	1.72	0.99	0.73	2.81	– 1.09
Oct.	0.08	– 0.26	– 0.34	– 0.92	1.00
Nov.	2.53	0.21	– 2.32	3.65	1.12
Root Mean Square Error		–	1.73	–	1.63

(Fig. 5) Two Month-Ahead Prediction of Industrial Production Index





Example 1.3 Forecating Real GNP Growth Rate

— Comparison with Econometric Model Prediction

Some evidences were given to show that a simple univariate time sereis model could often outperform a large-scale econometric model in short-term forecasting(for example,Ibrahim and Otsuki [17], Maylor, Seaks and Wichern [18], Nelson [19], etc.). The forecasting performance of the time sereis models was thus compared with a large-scale econometric model.

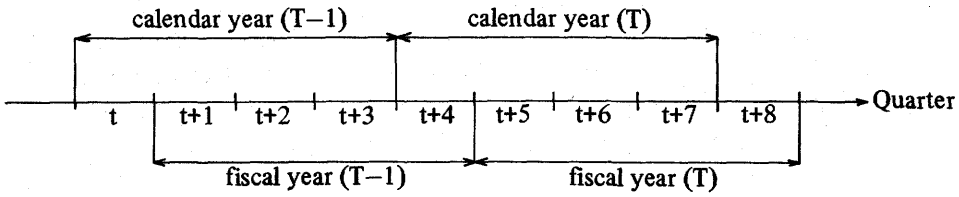
We also compared the forecasting performance of univariate ARMA models with the government judgemental forecasts, because in Japan this shows on the average better predictive performance than that of the econometric model.

For comparison with the time series model forecasts we used the econometric model predictions of the real GNP for the last decade by three well-known forecasting institutions in Japan as well as the Japanese government official forecasts. The predictions were compared in the form of annual percentage changes in real GNP from the previous fiscal year. It is denoted as R and defined by

$$R = \left\{ 1 - \left(\sum_{i=5}^{8} X_{t+i} \Big/ \sum_{i=1}^{4} X_{t+i} \right) \right\} \times 100.$$

Here, X_{t+1} denotes the real GNP in the t+1 quarter. The relation between the fiscal year and the calendar year is as illustrated in Fig. 6.

(Fig. 6) Fiscal Year and Calendar Year in Japan



The fiscal year in Japan covers from the second quarter of the calendar year to the first quarter of the next calendar year. The forecasts for the fiscal year T are made late in the t+3 quarter of the previous calendar year T−1 shown in Fig. 6. When those predictions were made late in the calendar year, only the data of the GNP up to two quarters before the beginning quarter of the target fiscal year were available. We made the out-of-sample, ex ante predictions with the ARMA model based on the almost same data[4] and tried to make them comparable as closely as possible. By using the notations of Figure 6 this can be explained as follows. If the n quarter-ahead forecast which is based on the information including the data of up to the t+2 quarter is denoted as $\hat{X}_{t+2}(n)$, the estimated value of the rate of growth in real GNP $\hat{R}$ is calculated by the following approximation.

$$\hat{R} \doteqdot \left[1 - \frac{\hat{X}_{t+2}(3) + \hat{X}_{t+2}(4) + \hat{X}_{t+2}(5) + \hat{X}_{t+2}(6)}{\left\{ X_{t+1} + X_{t+2} + \hat{X}_{t+2}(1) + \hat{X}_{t+2}(2) \right\}} \right] \times 100$$

4 Although the ARMA model predictions are based on the "revised" data of the GNP of the two quarters before the beginning of the target fiscal year, the comparative predictions are based on the "preliminary" data of the GNP.

First, the orignal real GNP data were transformed to stationary time series using the following formula.

$$x_t = (\ell n X_t - \ell n X_{t-4}) \times 100,$$

where, X_t denotes the original GNP data. We can get from this transformation the approximate growth rate of real GNP compared to the same quarter of the last year. The transformed series X_t of the real GNP for the entire period is illustrated in Fig. 7.

Next, the univariate ARMA models were fitted by Akaike's method. An example of the estimated model used for prediction is shown in Table 6. Every model was estimated automatically by the program AUTARM of TIMSAC-74. Forecasts were obtained by the fitted models over six quarter-ahead and then transformed back to the original data base.

Finally, the predicted rate of growth in real GNP were computed and the out-of-sample, ex ante predictive performance was checked. The relative performances of the ARMA model forecasts are reported in Table 7. The root mean square error (RMSE) of predictions appearing in Table 7 indicates that the univariate ARMA models provided generally more accurate predictions than the econometric predictions of 3 research institutions, which are even less accurate than the naive predictions during the last decade in Japan. Although we should be careful to judge the prediction performance only in terms of the RMSE, these results confirm the following statement of Nelson [20].

> "The fact that econometric models have as yet failed to demonstrate that they can forecast with greater accuracy than extrapolative models – for example, Box-Jenkins model – is perhaps more troublesome ..... We can only speculate that these errors are great enough at the present state of the art to prevent structural models from attaining their potential as tools of prediction."

It is remarkable that we could get these results through Akaike's automatic minimum AIC procedure without any expertise, except for the transformation procedure of the original data.

The government official forecasts, however, outperformed slightly the univariate ARMA model predictions. In figure 8, the actual rate of growth in real GNP, government official forecasts, and the univariate ARMA model predictions are illustrated through the fiscal year 1968 to 1977. It is apparent from this figure that the univariate ARMA model predictions are more accurate than the government official forecasts until the fiscal year 1973. After the fiscal year 1974, the ARMA model predictions have a strong tendency to overestimate compared to the government official forecasts.

(Fig. 7) Transformed Series of Real GNP

Original data source; Economic Planning Agency

(Table 6) Example of the Estimated Univariate ARMA Model of Real GNP*

$$x_t = 0.647\, x_{t-1} - 0.007\, u_{t-1} + 0.333\, u_{t-2}$$
$$(5.331) \qquad (0.048) \qquad (2.868)$$

S.E. = 6.858
Sample Period 1854, 1Q ~ 1976, 3Q

[x_t ; Transformed real GNP (deviation from sample average)
u_t ; Innovation
The numbers in paentheses are t-statistics.]

* Nine other models like this were estimated for forecasting each year.

(Table 7) RMSE* of Real GNP Growth Rate Forecasts (1968F.Y – 1977F.Y)
– Comparison of Univariate ARMA Model Prediction

	RMSE
Univariate ARMA Model	2.84%
Research Institute of National Economy*	3.79
Nomura Research Institute**	3.18
Japan Economic Research Center**	3.32
Government Official Forecast	2.65
Naive Model***	3.05

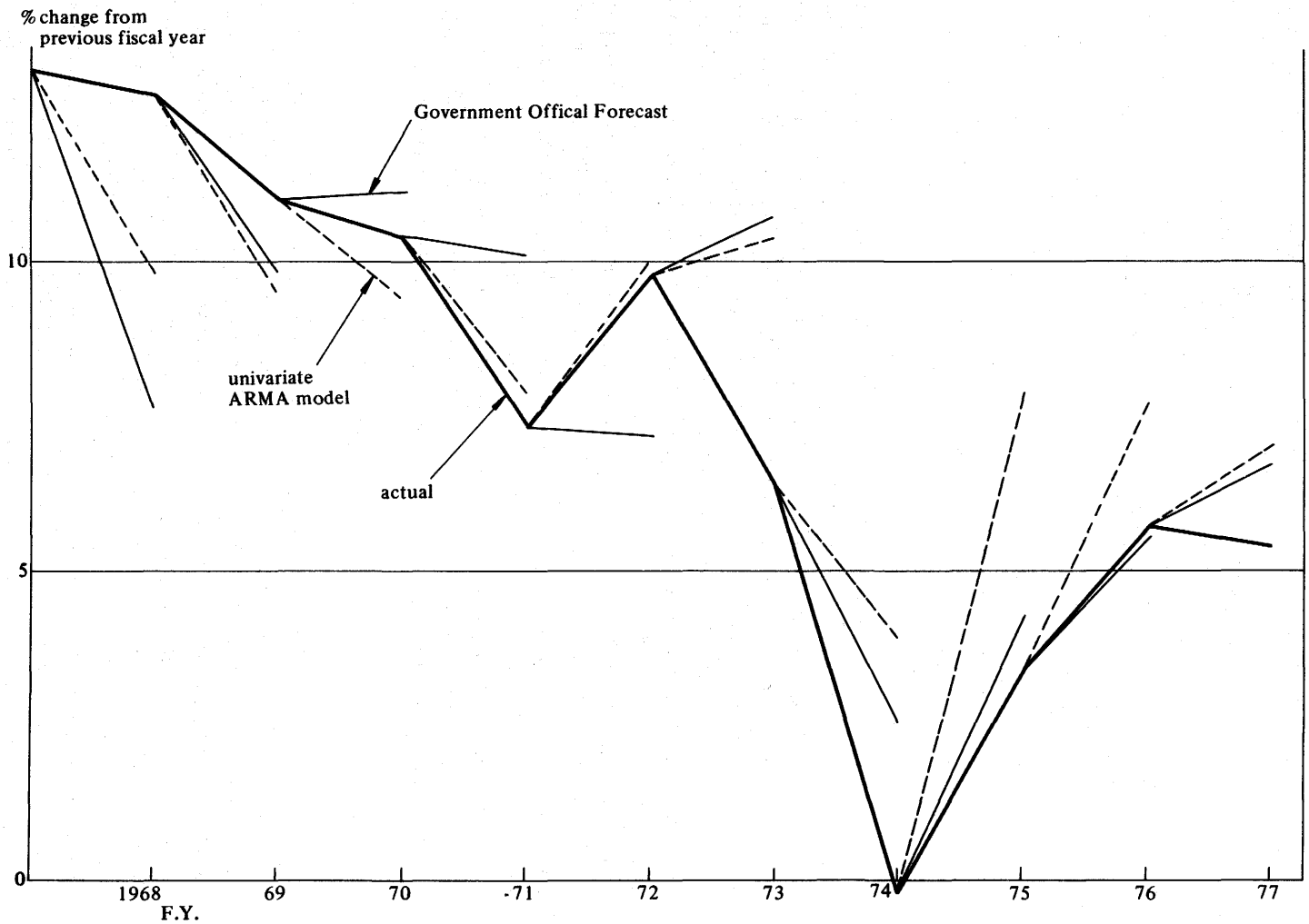
* RMSE (Root Mean Square Error) is defined as

$$RMSE = \sqrt{\sum_{i=1}^{n} (\text{forecast} - \text{actual})_i^2 / n} \quad .$$

** The data were cited from the Nihon Keizai Shinbun, January 20, 1979. The forecasting method of the Japan Economic Research Center is called "Successive Approximation Method".

*** Naive Model is defined as follows.
$X_t = X_{t-1}$
X_t denotes the real GNP growth rate at fiscal year t.

(Fig. 8) Forecast of Real GNP Growth Rate (Government Official Forecast and Univariate ARMA Model Forecast)



3. Forecasting Performance of the Multivariate ARMA Model

A multivariate ARMA model, especially a bivariate ARMA model, is represented as follows:

$$\sum_{i=0}^{p} a_i x_{t-i} + \sum_{i=0}^{q} b_i y_{t-i} = \sum_{i=0}^{\alpha} c_i u_{t-i} + \sum_{i=0}^{\beta} d_i v_{t-i}$$

$$\sum_{i=0}^{r} e_i x_{t-i} + \sum_{i=0}^{s} f_i y_{t-i} = \sum_{i=0}^{\gamma} g_i u_{t-i} + \sum_{i=0}^{\delta} h_i v_{t-i}$$

$$\left[\begin{array}{l} x, y \ : \text{Variables} \\ u, v \ ; \text{Mutually uncorrelated white noise} \\ a, b, c, d, e, f, g, h \ ; \text{Maximum orders} \\ a_0 = f_0 = c_0 = h_0 = 1 \end{array}\right]$$

As is shown by these equations, the multivariate ARMA model can utilize the information contained in the data of related variables which can not be used in the case of the univariate ARMA model. Then, as is shown in Nelson [20], the multivariate ARMA model predictions have in general smaller errors than predictions by the univariate ARMA model. Up to now, however, there were few reports[5] of the economic time series prediction with multivariate ARMA model, presumably because of the difficulties in the identification of the multivariate model. Akaike [6] developed the method of applying the minimum AIC procedure to estimate the multivariate ARMA model almost automatically.

We fitted bivariated ARMA models for the purpose of real GNP forecast using the computer programs of TIMSAC-74 [5] and evaluated the predictive performance. This was then compared with that of the univariate ARMA model prediction and the Government official prediction, and others, all of which are mentioned in Section 2.

First, we picked up the money supply data as the related variable of the real GNP. We have no particular reason for not using other variables. We transformed the data to get stationary series by the following formula, which is the same as used for real GNP data in section 2.

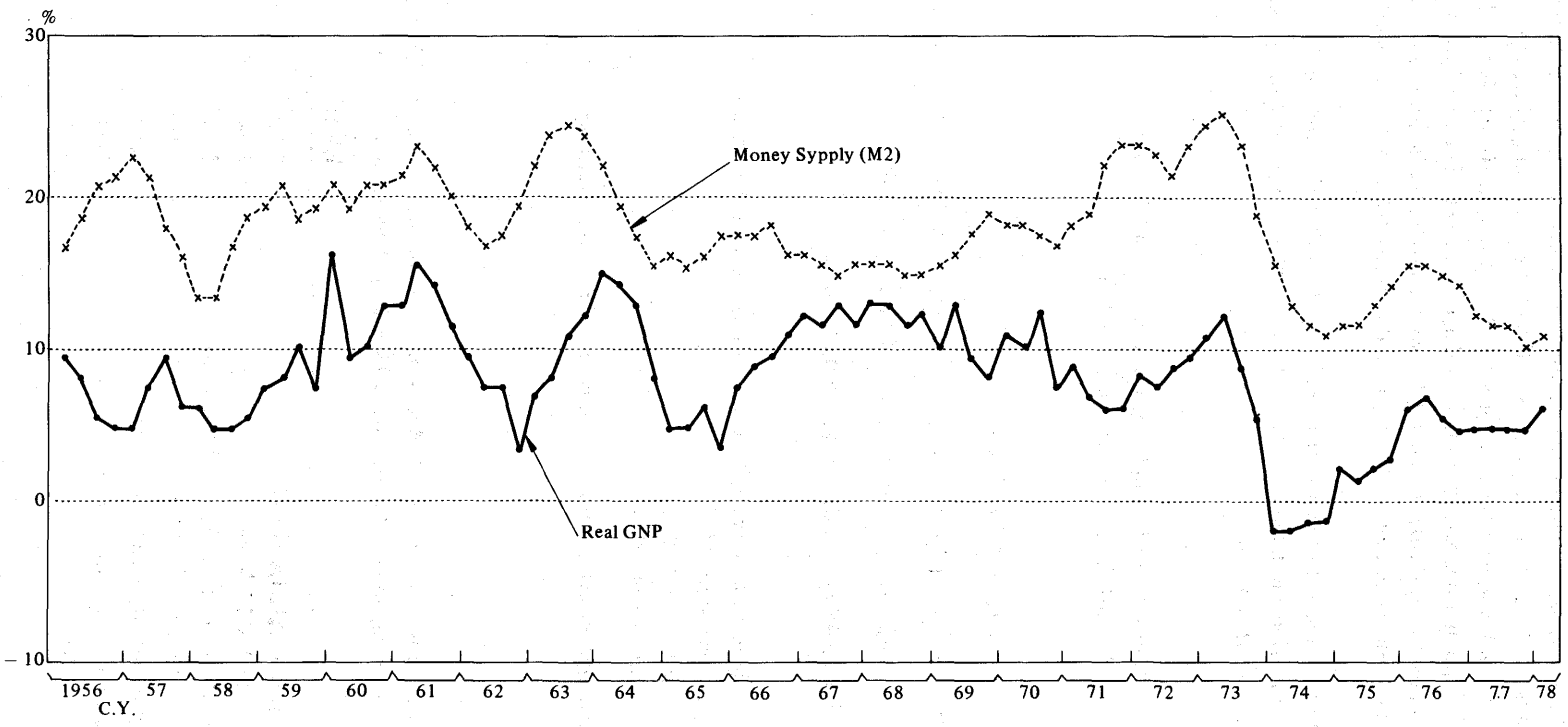
$$y_t = (\ell n Y_t - \ell n Y_{t-4}) \times 100,$$

where Y_t denotes the money supply (M2) data series. We used the same real GNP series as used in section 2. Two series are illustrated in Fig. 9. Bivariate ARMA models of real GNP and Money Supply were obtained by Akaike's method. The example of an estimated bivariate ARMA model is shown in Table 8. We made 10 forecasts with 10 estimated models over a 6 quarter-ahead period and calculated the predicted rate of growth in real GNP by the same procedure used for the univariate ARMA model prediction.

The results of predictions during the last decade are reported in Table 9. The bivariate ARMA models show better predictive performance than the univariate ARMA models and also outperform the government official forecasts in terms of RMSE. As is shown in the model, we used only the past data of two variables, and did not use any a priori information such as

5 For example, Granger and Newbold [15] demonstrated the procedure to estimate bivariate ARMA models.

(Fig. 9) Transformed Series of Real GNP and Money Supply (M2)



Original data source; The Bank of Japan

the assumed exogenous variable used in the econometric forecast, since it is impossible to know the future value of the exogenous variables in the actual forecasting situation.

The predictions by multivariate ARMA models are plotted in Fig. 10 together with the univariate ARMA model predictions and government official forecasts. The problem of the overestimations also observed in the case of univariate ARMA model predictions, is not perfectly resolved, but the problem is somewhat reduced in the case of the bivariate ARMA model predictions.

These results support the above mentioned Nelson's statement that the time series model which uses the information contained in the data of the related variables can predict more accurately than the univariate model.

(Table 8) Example of the Estimated Multivariate ARMA Model*
(Sample Period 1956.1Q ~ 1976.3Q)

$$x_t = 0.747\, x_{t-1} + 0.140\, y_{t-1}$$

$$\begin{aligned} y_t = {} & 0.054\, x_{t-4} + 0.457\, y_{t-1} + 0.184\, y_{t-2} + 0.284\, y_{t-3} \\ & -0.427\, y_{t-4} - 0.058\, u_{t-1} - 0.031\, u_{t-2} - 0.048\, u_{t-3} \\ & + 0.948\, v_{t-1} + 0.820\, v_{t-2} + 0.362\, v_{t-3} \end{aligned}$$

x_t ; Transformed real GNP (deviation from sample average)
y_t , Transformed M2
u_t ; Innovation of x_t
v_t ; Innovation of y_t

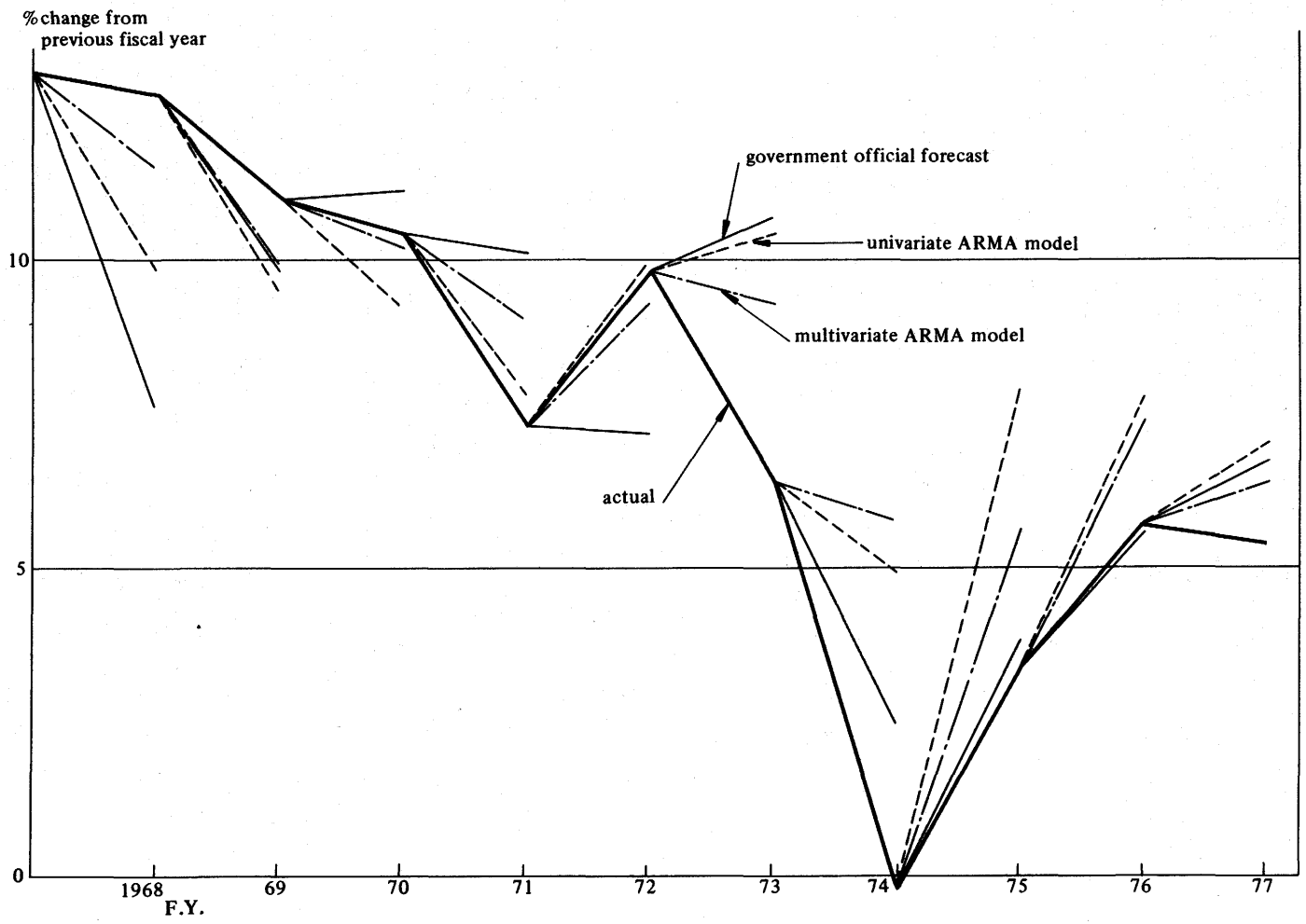
* Nine other models like this were estimated for forecasting each year.

(Table 9) RMSE of Real GNP Growth Rate Forecasts* (1968 F.Y. ~ 1977 F.Y.)
– Comparison of Multivariate ARMA Model Prediction

	RMSE
Multivariate ARMA Model	2.43 %
Univaiate ARMA Model	2.84
Research Institute of National Economy	3.79
Nomura Research Institute	3.18
Japan Economic Research Center	3.32
Government Official Forecast	2.65
Naive Model	3.05

* Reproduced table of Table 8

(Fig. 10) Forecast of Real GNP Growth Rate



4. Test of Granger's Sense of Causality by Time Series Modeling

As was described in section 3, the usefulness of money supply (M2) information for predicting the real GNP strongly suggests the existence of Granger's sense of causality (Granger and Newbold [14]) from M2 to real GNP. This is because the basic idea of Granger's sense of causality can be interpreted as the linear predictability between a group of stochastic processes. We tried to test Granger's sense of causality between M2 and the real GNP using a procedures for feedback system analysis developed by Akaike [8].

Several methodologies such as Sims' test and Haugh's test, etc. for empirical testing for Granger's causality have been proposed. But Pierce and Haugh [22] showed that these methods may underestimate the statistical significance of the empirical causal relationship. They suggested that "the explicitly multivariate approach" is the most promising procedure for detecting Granger's causality if a parsimonious multivariate ARMA model could be identified.

Hsiao [16] estimated the multivate AR model using Akaike's FPE and examined the causality explicitly. Hsiao identified the multivariate AR model comparing the FPE of a univariate process with the FPE of the same variable using the other variable as the system input. He used likelihood ratio tests to check the adequacy of specifications. He inferred the causality by the parameters of b(B) and d(B) in the following identified bivariate AR model.

$$a(B)X_t + b(B)Y_t = \epsilon_t$$

$$c(B)Y_t + d(B)X_t = u_t$$

X_t, Y_t denotes the two stationary series and a, b, c, d are polynominals in the backward operator B, with $a(0) = c(0) = 1$, and ϵ_t, u_t are a pair of uncorrelated white noise series. The unidirectional causality from X_t to Y_t can be inferred if $d(B) \neq 0$ and $b(B) \equiv 0$. When both $b(B) \neq 0$ and $d(B) \neq 0$ are found, feedback causality is inferred.

A more sophisticated method for "the explicitly multivariate approach" to the feedback system analysis is proposed by Akaike [3] [8]. It is the "Relative Power Contribution (RPC)" used as an instrument for the causality detection of dynamic systems. The basic idea of the RPC is to use the innovation of the multivariate ARMA model after the conversion from the time domain to the frequency domain. In other words, if the contribution of the j-th innovation to the power spectrum of the i-th variable (X_i) is low, this means the causality from X_j to X_i is weak.[6]

The prominent usefulness of the RPC exists in the fact that it can provide us with the causal relationship between variables in each frequency band. The former method such as Hsiao's can only detect the causality through the whole frequency band. The cross-spectral method in the spectral approach can provide us the information about the relationship among variables through the frequency band, but Akaike [1] showed that the cross-spectral method can be completely invalidated by the existence of feedback loops. Taking into account that feedback loops usually exist in the economic system, the cross-spectral method suffers from this serious limitation in practical application. The RPC is applicable to evaluate the causal relationship of variables in the frequency domain even under the situation of feedback loops, if only the innovation satisfies a particular condition, i.e., the orthogonality between the components. The RPC has already been applied successfully in the field of control engineering such as a nuclear power plant, a cement kiln plant, etc. (Akaike [3], [8]).

Example 3.1 Causality between real GNP and M2

First, we tested the causality between M2 and real GNP. The estimated bivariate AR model[7] is shown in table 10. Fig. 11 shows the graphs of RPC obtained from the AR model.[8] From the upper figure the real GNP is found to be clearly influenced by M2 in the high frequency band. This means that we can improve the short term prediction performance of the real GNP by making use of the past data of M2. These results correspond to the results obtained in section 2. On the other hand, we found a rather weak causality except in the low frequency band, from real GNP to M2, as shown in the lower figure.

We compared these results with the results of "Haugh's test" using the same data. The univariate ARMA models for generating the innovation series were estimated by Akaike's method described in section 2. The estimated models are shown in Table 11. Cross correlations between the innovation of M2 and the innovation of the real GNP are shown in Table 12. The Q statistics figure on the leading value of the M2 innovations is larger than that of the leading value of the real GNP innovations. This suggests the same direction of causality as detected in the multivariate approach. Also, the large cross correlations in the short term leading value of the M2 innovations and the long term leading value of the real GNP innovations are roughly coincident with the results of the RPC, which show short term causality from M2 to the real GNP and weak but long term causality from real GNP to the M2.

---

6 The more formal definition of the RPC is provided in Akaike [8] as follows. The explanation is for the case of the AR model, but it can also be applied to the ARMA model.

Consider a vector X(n) of variable at time n. A multivariate AR model to time series X(n) is represented as

$$X(n) = \sum_{m=1}^{M} A(m)\, X(n-m) + u(n).$$

By using the autoregressive model, we can get the spectral density matrix

$$P(f) = (A(f))^{-1}\, S(A(f)^*)^{-1},$$

where S denotes the innovation variance matrix $EU(n)U(n)'$, $A(f) = I - \Sigma\, A(m) \exp(-i2\pi fm)$ and $*$ denotes conjugate transpose. The (i, j) element of P(f) and S will be denoted by $P_{ij}(f)$ and σ_{ij}, respectively. If we assume the orthogonality between the components of U(n), we get $S = \mathrm{diag}(\sigma_{11}, \sigma_{22}, \ldots, \sigma_{kk})$, where $\mathrm{diag}(\sigma_{11}, \sigma_{22}, \ldots, \sigma_{kk})$ denotes the diagonal matrix with the i-th diagonal element equal to σ_{ii}. With this assumption we get the decomposition

$$P_{ii}(f) = \sum_{j=i}^{k} |b_{ij}(f)|^2\, \sigma_{jj},$$

where $b_{ij}(f)$ denotes the (i, j) element of $A(f)^{-1}$. The relative power contribution of $U_j(n)$ to $X_i(n)$ is defined by

$$r_{ij}(f) = \frac{|b_{ij}(f)|^2\, \sigma_{jj}}{P_{ii}(f)}.$$

The existence of instantaneous causality can be detected by the cross-covariance between the contemporaneous innovations U(n). When a high value cross-covariance is found, there exists instantaneous causality between the two variables. In that case, the model should be re-estimated containing the contemporaneous lag variable in the model.

7 The reason we did not use the ARMA model, but the AR model, is simply a computational one.

8 In this case, we could not find the existence of instantaneous causality since the value of cross-covariance between contemporaneous innovations is only 0.072.

(Table 10) Bivariate AR Model of Real GNP and M2

Coefficients on lag of \ Dependent Variables		Real GNP	M2
Real GNP	(– 1)	0.689	0.007
	(– 2)	0.268	0.032
	(– 3)	– 0.201	– 0.104
	(– 4)	– 0.044	0.151
	(– 5)	0.057	– 0.035
M2	(– 1)	0.418	1.296
	(– 2)	– 0.361	– 0.322
	(– 3)	0.059	0.004
	(– 4)	0.024	– 0.437
	(– 5)	– 0.139	0.326

* Estimated Period 1956.1Q ~ 1978.1Q

(Table 11) Univariate ARMA Models for Haugh's Test
(Sample Period 1956, 1Q ~ 1978, 1Q)

(Filter for Real GNP)

$$x_t = 0.705\, x_{t-1} + 0.089\, u_{t-1} + 0.311\, u_{t-2}$$

S.E. = 4.775

[x_t ; Real GNP, the same data as used in section 3
ut ; Innovation]

(Filter for M2)

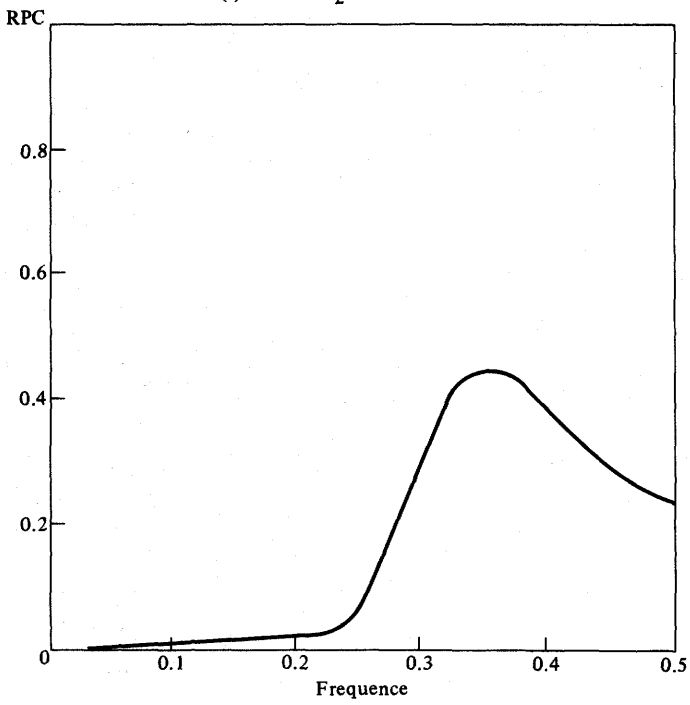
$$y_t = 0.641\, y_{t-1} + 0.688\, u_{t-1} + 0.579\, u_{t-2} + 0.499\, u_{t-3}$$

S.E. = 1.764

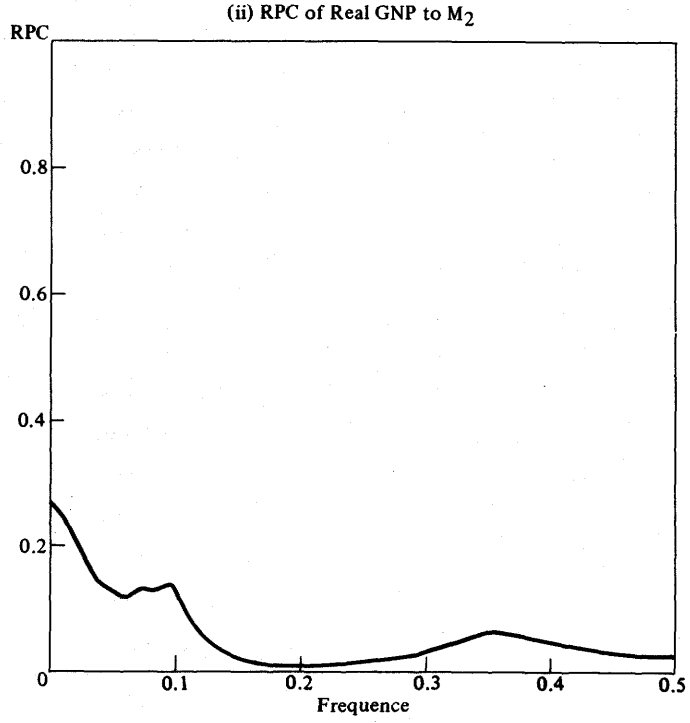
[y_t ; M2, the same data as used in section 3
u_t ; Innovation]

(Fig. 11) Relative Power Contribution between Real GNP and M_2

(i) RPC of M_2 to Real GNP



(ii) RPC of Real GNP to M_2

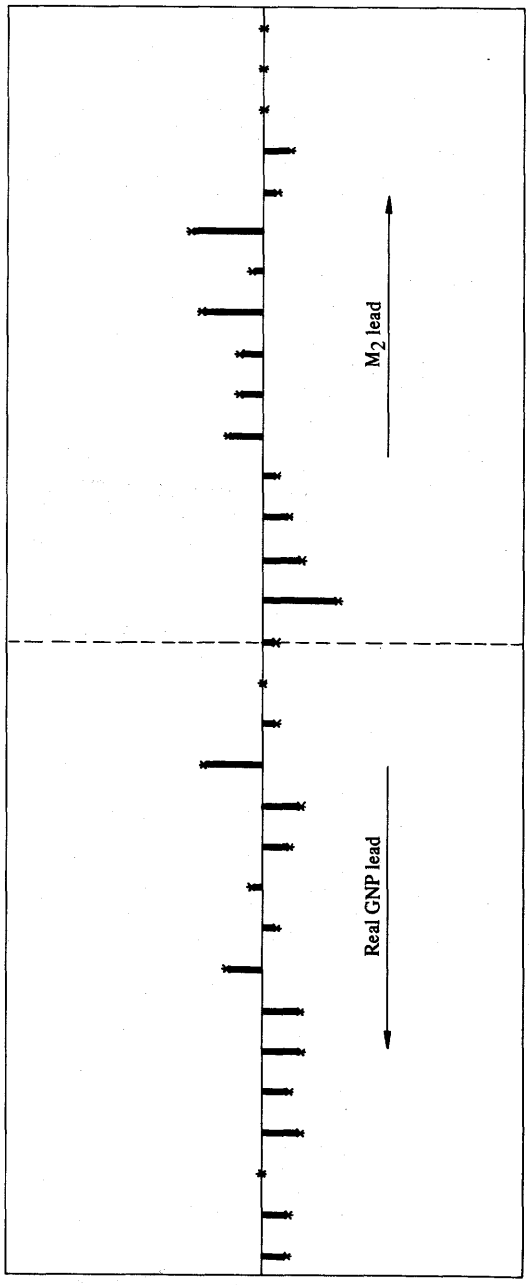


(Table 12) Sample Cross-Correlations between Innovations of Real GNP and Innovations of M_2

Sample period (1956/1–1978/1)

y ..... CODE = 1
NAME ––– R-GNP
x ..... CODE = 2
NAME ––– M2

	LAG	COEFFICIENT	SAMPLE
	y=x (– 15)	0.028	74
	y=x (– 14) MIN	0.003	75
	y=x (– 13)	0.014	76
	y=x (– 12)	0.108	77
	y=x (– 11)	0.057	78
	y=x (– 10)	– 0.257	79
Q Statistics	y=x (– 9)	– 0.047	80
= 25.4	y=x (– 8)	– 0.210	81
	y=x (– 7)	– 0.071	82
	y=x (– 6)	– 0.071	83
	y=x (– 5)	– 0.104	84
	y=x (– 4)	0.058	85
	y=x (– 3)	0.135	86
	y=x (– 2)	0.165	87
	y=x (– 1) MAX	0.338	88
	y=x (0)	0.079	89
	y=x (1)	0.008	88
	y=x (2)	0.071	87
	y=x (3)	– 0.219	86
	y=x (4)	0.169	85
	y=x (5)	0.120	84
	y=x (6)	– 0.042	83
Q Statistics	y=x (7)	0.095	82
= 21.1	y=x (8)	– 0.111	81
	y=x (9)	0.166	80
	y=x (10)	0.193	79
	y=x (11)	0.130	78
	y=x (12)	0.172	77
	y=x (13)	0.024	76
	y=x (14)	0.101	75
	y=x (15)	0.133	74



Example 3.2 Causality from M2 and Government Expenditure to Nominal GNP

Next, we tested the causality from M2 and government expenditure to nominal GNP. The estimated multivariate AR model is shown in table 13. Fig. 12 shows the graph of the RPC obtained from the transformation of the AR model.

It indicates that the M2 has a very strong influence on nominal GNP in the low frequency band, that is, almost half of the power spectrum of the nominal GNP is explained by the innovation of M2. On the other hand, government expenditure has a rather weak influence on nominal GNP in the low frequency band, but the contribution of government expenditure is high in the high frequency band. These results correspond to the monetarists' view that the money supply influences income through long period but government expenditure influences income only in the short run. The same result was confirmed with the application of Sims' test (Oritani [21]).

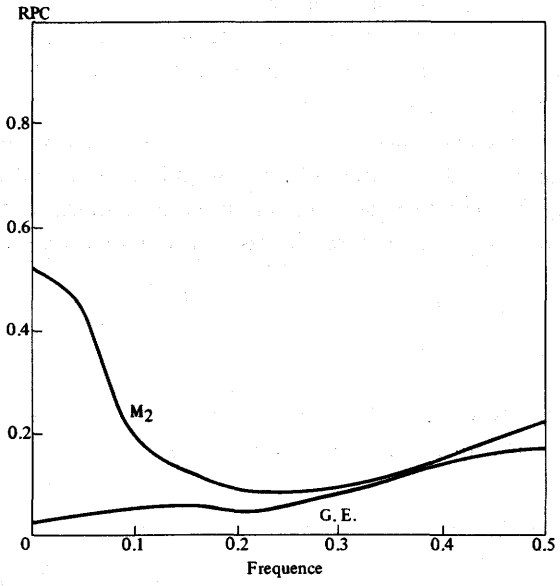
(Table 13) Multivariate AR Model of Nominal GNP, Government Expenditure, and M2
(Sample Period 1956, 1Q ~ 1978, 1Q)

Coefficient on lag of \ Dependent Variable		Nominal GNP *	M2 *	Government * Expenditure
Nominal GNP	(– 1)	0.827	– 0.131	0.019
	(– 2)	0.092	0.009	– 0.131
	(– 3)	0.183	0.016	0.269
M2	(– 1)	0.488	1.210	0.035
	(– 2)	– 0.278	– 0.177	– 0.555
	(– 3)	– 0.023	– 0.135	0.734
G. E.	(– 1)	– 0.148	0.017	0.474
	(– 2)	0.144	0.031	0.182
	(– 3)	– 0.049	– 0.001	– 0.028

* All variables are transformed with the following formula.

$$x_t = (\ell n X_t - \ell n X_{t-4}) \times 100$$

(Fig. 12) Relative Power Contributions of M_2 and Government Expenditure Fluctuation to the Fluctuation of Nominal GNP



Concluding Remarks

It is found that Akaike's method is useful for both the forecasting and the causality test of economic time series. The success of Akaike's method mainly depends on the appropriate treatment of the principle of parsimony in the light of the information criterion AIC. This is especially important in economic time series analysis, as the sizes of economic data are usually limited.

Armstrong [10] made a report based on his survey that 72% of econometricians agreed with the statement, "more complex econometric methods yield more accurate forecasts". Contrary to that belief of econometricians, he found various examples of empirical evidence that there was even a negative correlation between the forecast accuracy and the complexity of the model as measured by the number of the variables used.

Which is more reliable, the belief of econometricians or Armstrong's observation? In the light of AIC, it seems clear that an overcomplex model yields less accurate forecasts.

Of course, we must admit that the statistical method which we used here does not offer a complete answer to the practical needs of economic analysis. One of the most important problems is the nonstationary or nonlinear movement often observed in economic time series. Some encouraging progress has been obtained in this direction with the aid of AIC (Akaike [7], [9]).

Acknowledgements

The present author is grateful to Dr. H. Akaike and Mr. G. Kitagawa of the Institute of Statistical Mathematics in Tokyo, and Professor I. Kuroda of the Tsukuba University for helpful comments. Thanks are also due to Mr. S. Naniwa of the Bank of Japan for comments and the necessary computational work.

References

[1] H. Akaike, "Some Problems in the Application of the Cross-Spectral Method", in *Spectral Analysis of Time Series*, John Wiley, New York, 1967.

[2] H. Akaike, "Statistical Predictor Identification", *Annals of the Institute of Statistical Mathematics*, V. 21, 1969.

[3] H. Akaike, and T. Nakagawa, *Statistical Analysis and Control of Dynamic Systems*, Saiensu-sha, Tokyo, 1972. (In Japanese)

[4] H. Akaike, "A New Look at the Statistical Model Identification", *IEEE Trans. Automat. Contr.*, AC-19, 1974.

[5] H. Akaike, E. Arahata and T. Ozaki, "TIMSAC-74 – A Time Series Analysis and Control Program Package – (1)", *Computer Science Monographs*, No. 5, The Institute of Statistical Mathematics, Tokyo, March 1975.

[6] H. Akaike, "Canonical Correlation Analysis of Time Series and the Use of an Information Criterion", in *System Identification: Advance and Case Studies*, Academic Press, New York, 1976.

[7] H. Akaike, "Time Series Analysis and Control through Parametric Models", in *Applied Time Series Analysis*, Academic Press, New York, 1978.

[8] H. Akaike, "On the Identification of State Space Models and Their Use in Control", presented at the Special Topics Meeting on Time Series Analysis, Ames, Iowa, May, 1978.

[9] H. Akaike, "A Bayesian extension of the minimum AIC procedure of autoregressive model fitting", to be published in *Biometrika* 66, No. 2, 1979.

[10] J. Armstrong, "Forecasting with Econometric Methods: Folklore versus Fact", *Journal of Business*, October 1978.

[11] G. Box, and G. Jenkins, *Time Series Analysis, Forecasting and Control*, Holden-Day, California, 1970.

[12] E. Fama, "Efficient Capital Markets; A Review of Theory and Empirical Work", *Journal of Finance*, May 1970.

[13] D. Findley, "Preface to Applied Time Series Analysis", in *Applied Time Series Analysis*, Academic Press, New York, 1978.

[14] C. Granger, and P. Newbold, *Forecasting Economic Time Series*, Academic Press, 1977.

[15] C. Granger and P. Newbold, "The Time Series Approach to Econometric Model Building", in *New Methods in Business Cycle Research: Proceedings from a Conference*, Federal Reserve Bank of Minneapolis, October 1977.

[16] C. Hsiao, "Autoregressive Modelling and Money-Income Causality Detection", technical report No. 262, Stanford University, June 1978.

[17] I. Ibrahim, and T. Otsuki, "Forecasting GNP Components Using the Method of Box and Jenkins", *Southern Economic Journal*, January 1976.

[18] T. Naylor, T. Seaks, and D. Wichern, "Box-Jenkins Methods: An Alternative to Econometric Models", *International Statistical Review* Vo. 40, No. 2, 1972.

[19] C. Nelson, "The Prediction Performance of the FRB-MIT-PENN Model of the U.S. Economy", *American Economic Review*, December 1972.

[20] C. Nelson, "Rational Expectations and the Predictive Efficiency of Economic Models", *Journal of Business*, July 1975.

[21] Y. Oritani, "Money, Income, Government Expenditure, and Causality in the Japanese Economy: A Test of Monetarist Hypothesis in the Japanese Economy", Research Paper (53)II, the Bank of Japan, March 1978.

[22] D. Pierce, and L. Haugh, "Causality in Temporal Systems: Characterizations and Survey", *Journal of Econometrics*, 5, 1977.