On the Stability of the Japanese Money Demand Function: Estimation Results Using the Error Correction Model

TOMOO YOSHIDA*

Until recently, empirical studies on Japan’s money demand function, like those in other countries, often revealed its instability. In this paper, Japan’s money demand function is reestimated both with the conventional partial adjustment (PA) model and with the up-and-coming error correction model (ECM). The latter results showed that a quite stable money demand function in fact existed for more than two decades. This finding seems to put some weight on the argument that “missing money” phenomena are largely due to specification errors. Some account on the ECM will be also provided in this paper both in the context of British econometrics and recent developments in time series analysis.

I. Introduction

In recent years, management of monetary policies in advanced industrial countries has been geared, to varying degrees, to the movements in the money supply. This tendency grew out of the bitter lessons of the inflation caused by the excessive money supply in the latter half of the 1960s. Proper management of such monetary policy and its effectiveness in the real economy call for a stable relationship between money demand and economic variables such as real GNP, prices and interest rates—that is, stable money demand functions. Recently it is often pointed out, however, that the accelerating deregulation of interest rates and globalization as well as securitization of finance have undermined the stability of the money demand function in advanced industrial countries including Japan. At first glance, such arguments or related empirical studies on the stability of the money demand function may look like purely technical exercises. Nevertheless, if monetary policy is to acknowledge the importance of money supply move-

* Economist, Research Division I, Institute for Monetary and Economic Studies, the Bank of Japan. This work owes a great deal to valuable advice from Professor Charles Nelson of Washington University (formerly a visiting scholar at the Institute for Monetary and Economic Studies). The author is also grateful to Yoshihisa Baba, Hidekazu Eguchi, Takatoshi Ito, Keimei Kaizuka, Takeaki Kariya, Robert Rasche, and Taku Yamamoto for their helpful comments. However, responsibility for the views expressed here as well as for errors is entirely that of the author.
ments, these issues are fundamental, and their practical implications are emphatically serious.

Against this background, this paper brings the perspective of econometrics to a critical review of the traditional empirical studies on the money demand function. It also reestimates Japan's money demand function with the error correction model (ECM), a new estimation method that has rapidly gained currency in recent years, and delves into its stability. Finally, since recent studies are gradually identifying major defects in the conventional approach to regression analysis with time series data, this paper considers these problems at some length as well as other recent developments in econometrics.

Past, empirical studies of a money demand function generally adopted a partial adjustment model which typically treats the real money demand as a dependent variable, with real GNP and interest rates, as well as the first order lag of the dependent variable, as explanatory variables. In the mid-1970s, however, this model began to lose its adequacy in explaining developments in money demand as major divergences emerged between the forecast and actual values. This phenomenon persisted despite efforts to rectify the situation, such as adding new explanatory variables to the conventional function. This led many monetary experts to suggest that financial deregulation and innovation were causing a shift in the money demand functions (the "missing money" phenomenon). Studies over the past several years have, however, revealed a major problem in the conventional methodology of empirical analyses: the dynamic specification, assumed *a priori* for the conventional money demand function, has in fact imposed a severe restriction on the observed data generation processes (DGP). It has been this kind of misspecification that has been the principal cause of erroneous forecasts.

Recently, a new method has been introduced to determine the models' lag structures. This new approach differs from the traditional method of empirical studies by rigorously eliminating prior restrictions on the models' lag structures determining them according to direct information from economic data to the extent possible. With the variables rearranged to reflect current economic theories, the models are then estimated.

This is the thrust of ECM. Simply put, it is a model designed to account for economic realities from the following standpoint: equilibrium relations among economic variables postulated in economic theory hold only in the long-run steady state, while observed economic data reflect behaviour that attempts to compensate for part of people's past errors (that is, the divergences between equilibrium values and actual values). In plain terms, the proposition that "people act to compensate for their past errors" (error correction) means people's everyday behaviour is based on logic such as "I spent a lot last month, so I will cut spending this month" or "Last week I drank too much, so I will be a teetotaler this week." From this perspective, it is fair to say that ECM is an attempt to integrate economic theory useful in characterizing a long-term equilibrium with an observed disequilibrium by building a model that explicitly incorporates behaviour that would restore the equilibrium.
This paper comprises six sections. Following this introduction, Section II surveys empirical studies on the money demand function to establish a context for the issues raised in this paper. Section III introduces the basic premises and practical advantages of ECM, which has been developed mainly by British researchers. Taking a slightly different perspective, Section IV briefly explains how ECM stands among time series analyses—a task that has been pursued mainly in the United States. Building on the discussion, Section V cites our estimated results of the ECM-type money demand function for M₂+CD in Japan and compares its performance with that of a conventional approach. The section also considers the stability of the new money demand function. Section VI concludes this paper with an agenda for further research. In the appendix, a brief overview of previous empirical studies of the ECM-type money demand function for M₁ in both Britain and the United States is given.

Following are summaries of the arguments presented in this paper.

(1) Most traditional empirical studies on money demand functions have typically specified their lag structures as a simple process of partial adjustment. While these models' performances usually appeared satisfactory to an extent in accounting for past money demand developments, out-of-sample forecasts based on these models often yielded inadequate results. It seems that the major source of these problems lies in these models' unwitting imposition of a severe restriction on the models' lag structure prior to estimation. This is often the case with theory-based dynamic specification.

(2) In specifying a model's lag structure, it is preferable to derive information on the lag structure from the data itself (data-based dynamic specification) using an autoregressive distributed-lag model (AD model) instead of imposing an untested restriction in advance. ECM is one type of such an AD model, incorporating formulae adapted for interpretation generally consistent with economic theory. ECM also offers certain practical advantages in estimation exercises, e.g., the "multi-colinearity problem" rarely occurs.

(3) Past regression analyses were often run without a thorough examination of the characteristics of time series economic data. It is, therefore, difficult to refute accusations that some of them are, in fact, "spurious regressions" exhibiting an excellent fit between unrelated variables (particularly when levels of the variables themselves are used in the regression). In general, the estimation of coefficients and inference from them become statistically difficult, if not impossible, when the regression includes nonstationary variables. Moreover, recent empirical studies have overturned earlier assumptions, showing that such major macroeconomic variables as the GNP and money supply may individually be a unit root process (i.e., a non-stationary process such as random walk) rather than a trend-stationary process as has been thought. This implies that the traditional approach in regression has not always yielded reliable results.

(4) Recent studies have shown that ECM is best suited for model estimation when economic variables that function individually as unit root processes demonstrate similar
patterns of movement over the long-term, i.e., when these variables are cointegrated. Bolstered by this finding, studies on ECM, which are most advanced in Britain, are beginning to mesh with studies on cointegration in the United States. In recent years, studies and applications of ECM have grown rapidly. For example, application of ECM to the consumption function is increasing, and the Division of Research and Statistics of FRB recently seems to have incorporated ECM in its money demand function.

(5) The time series property of Japan's macroeconomic variables seems to justify an ECM-type money demand function; both GNP and money supply have individual unit roots but they appear to be cointegrated. The estimation of Japan's money demand function from the first quarter of 1968 to the first quarter of 1989 showed that, over the past 20 years, there had been relatively stable relationships between money demand ($M_2 + CD$) and four relatively simple explanatory variables (real GNP, interest rates, inflation rates and stock price volatility), with a far closer fit than those using a conventional function for the same period. When the estimation period was shortened to 1985 and an out-of-sample forecast was made for the following three years, the result was again relatively satisfactory (with all the disparities between a one-step-ahead forecast and actual values falling well within 95% confidence intervals). A further check with a sequential Chow test to determine whether these functions underwent structural changes at some point produced no evidence of such change, indicating that Japan's money demand function remained relatively stable throughout the period.

(6) Recent studies' findings on the stability of the money demand function in the leading industrial countries, including Japan, would tend to confirm the continued validity of money supply-oriented monetary policy. They also suggest that ECM can be an effective tool for money supply forecasting. It should be noted, however, that these estimations do not explicitly include any explanatory variables that would gauge the impact of financial deregulation. Without such variables, this model is not useful in assessing the impact.

II. A Brief Survey of Empirical Studies on Money Demand Functions

A. The "missing money"

Before launching a detailed discussion of ECM, this chapter briefly surveys earlier studies on money demand functions (Figure 1).\footnote{Among the most noteworthy surveys of studies on the money demand function are Judd and Scadding (1982), Roley (1985), and Cuthberston (1985).}

Empirical studies on money demand functions multiplied from the 1960s through the 1970s, with research in the United States leading the movement. Through these years, attention focused on a) whether stable money demand functions in fact exist, as macroeconomic theory assumes; and b) how sensitive money demand is to interest rates. By
Figure 1. A Brief Survey of Empirical Studies on Money Demand Functions

1. Review and addition of explanatory variables
   → Wealth
      (Goldfeld 1976, B.M. Friedman 1978, etc.)
   → Bank debit (Lieberman 1977)

2. Modeling of the impact of deregulation
   → Ratchet variables
      Use of interest rates at past peak levels
      (Enzler, Johnson and Paulus 1976, Simpson and Porter 1980, etc.)
   → Addition of broker fees
   → Introduction of dummy variables

3. Review of dependent variables
   → Adoption of divisia aggregates
      (Barnet, Offenbacher and Spindt 1981, etc.)

In sum, while performance improved in explaining past data, "only limited success in predicting future money demand" (Roley 1985) has been achieved.

Empirical studies in the United States

- 1950s ~ 1973
  Relatively satisfactory fits from partial adjustment models led to the acceptance of the stability of money demand.

- After 1973
  Fits obtained from conventional money demand functions deteriorated suddenly, with actual values of money supply consistently falling short of forecast values.
  → "Missing Money" episode
     (This triggered intensive discussions on the stability of money demand functions.)

- Buffer-stock money approach
  (Carr and Darby 1981, Lairdler 1984, Goodhart 1984, etc.)

Criticisms by econometricians

   → Criticism of specification searching

   Gordon (1984b)
   → Support for the first difference form, displacing variables such as $y_t$ and $m_t$ with first difference forms such as $\Delta y_t$ and $\Delta m_t$.

3. Hendry (1979)
   → Criticism of imposing severe restrictions on models' lag structures with naive dynamic theory
   Support for AD models and ECM
the early 1970s, researchers had, for the most part, reached a consensus that a) money demand functions were stable in the United States; and b) there was a significant negative correlation between interest rates and money demand. In arriving at these conclusions, the researchers drew on quarterly data as the basis for their "standard" function, a simple specification based on the dynamic economic theory called partial adjustment. Representative of these studies is Goldfeld (1973), which is now regarded as the culmination of the far-ranging research of this period. As (1) below demonstrates, this specification includes first order lags in the real money balance as an explanatory variable as well as income and interest rates. Even today, many researchers still use this specification. (This paper reexamines (1) in view of recent developments in econometrics. The details follow in Section III and succeeding sections.)

\[(m - p)_t = a_1 y_t + a_2 R_t + a_3 (m - p)_{t-1}\]  

(1)

where

\[(m - p) = \ln \frac{M}{P}\]

\[y = \ln Y\]

\[M/P = \text{real money demand}\]

\[Y = \text{real income}\]

\[R = \text{interest rates (or the logarithm of interest rates)}\].

After 1974, the consensus on the stability of the money demand function began to falter. In 1976, Enzler, Johnson and Paulus pointed out that the out-of-sample forecast for the years from 1974 on, which were based on the money demand function estimated in 1973 or before, consistently produced overestimations. Goldfeld (1976) coined the term "missing money" for this phenomenon and argued that it was at least partly attributable to a shift in the money demand function due to financial innovations such as the introduction of the NOW account and the MMMF.

B. Improving conventional money demand functions

In the United States, investigations tracing the "missing money" revitalized studies on the stability of the money demand function. These studies can be divided into three broad categories: a) attempts to improve explanatory power of the conventional function by adding such explanatory variables as wealth and bank debits; b) attempts to incorporate the explicit effects of financial deregulation into the function by introducing the ratchet effect\(^2\) of high interest rates in the past or adding dummy variables; and c) attempts to remove the unsettling effects of financial deregulation on given money supply

---

\(^2\) During a period of high interest rates, the spread between Repo rates and regulated deposit rates becomes so wide that many major firms enter the Repo market. When interest rates subsequently decline, however, these companies generally remain in the Repo market, and their deposits are left intact. Interest rates at past peak levels was incorporated in regressions to reflect such phenomena.
figures by adopting artificial divisia money supply aggregates\(^3\) as a dependent variable. All of these approaches fell, however, within the standard theoretical models of partial adjustment or adaptive expectations. Before long, they soon showed their weaknesses, as Roley (1985) pointed out: "The unprecedented decline in the velocity of \(M_1\) during 1982 and 1983 was not captured fully by any of the previously modified conventional specifications."\(^4\) Moreover, the apparent 'nonshift' in 1981 despite the introduction of nationwide NOWs ... was puzzling to many" and, as a whole, they became again a focus of criticism, since a "vast majority of the specifications presented to explain past episodes of apparent money demand instability achieved only limited success in predicting future money demand."

In Japan, meanwhile, results of empirical studies using the standard money demand function were reported by Tsutsui and Hatanaka (1982), Hamada and Hayashi (1983), Ishida (1984), Furukawa (1985) and Komura (1985), among others. Despite differences in nuance, they generally linked the standard function's apparent deficiencies to the shift in money demand generated by financial deregulation.

C. Development of theory

Prominent in the theory of the money demand function is the Baumol-Tobin’s inventory-theory money demand function. The model focuses on transaction demand for money (Baumol 1952, Tobin 1956), and Friedman’s explanation of portfolio selection money demand functions based on asset holding (Friedman 1956). In empirical studies, money demand function like

\[
(M/P) = M(Y, R),
\]

has often been employed (\(M/P\) is the real money demand, \(Y\) is the real income and \(R\) is interest rates).\(^5\) This function can be corroborated with both theories.

Disputing these mainstream theories, a group of experts recently introduced a new theory of money demand called the buffer-stock money approach. In brief, this theory posits that the primary motivation for money holding is its role as a buffer stock against unanticipated expenditures arising from uncertainties. Building on this premise, the theory holds that people accept changes in their money holdings within certain ranges

\(^3\) See Ishida (1984) for the details of divisia money aggregates.

\(^4\) This phenomenon was named the "Great Velocity Decline" by the Federal Reserve Bank of San Francisco. Roley observed that "previously modified conventional specifications cannot explain such a phenomenon." His conclusion was prompted by out-of-sample estimates of \(M_1\), from 1982 through 1983 using the money demand function for a sample period up to the end of the 1970s, that unanimously fell short of the actual values.

\(^5\) In (2), if \(Y\) is the transaction volume and \(R\) is the opportunity cost for money holdings, they will become an inventory-theoretical money demand function. If \(Y\) is the permanent income and \(R\) is the interest rates on money and other financial assets (in which case, \(R\) becomes the vector for multiple interest rates), they will become a portfolio-selection money demand function. (It should be noted, however, that Friedman's (1956) function also includes expected rates of inflation and the ratio of human assets to non-human assets in its explanatory variables.)
rather than reacting immediately to recover the change in holdings. This approach is unique in that a) it assumes an uncertain world and b) that it defines optimal money holdings as a range rather than a certain level. Due to the inherent difficulties of model construction, however, it appears to have had only limited confirmation in empirical studies.

D. Econometrician’s criticism against conventional money demand functions

As noted earlier, empirical studies on the money demand function have depended principally on (1) or slightly more sophisticated forms with additional explanatory variables. Since the end of the 1970s, however, these approaches to empirical studies have invited various critiques led by the econometricians. Here is a brief review of these criticisms.

Cooley and LeRoy (1981) launched a scathing attack on certain researchers’ approach known as “specification searching,” a term coined by Leamer (1978). The broadside derides their questionable readiness to estimate hundreds of equations, gradually changing explanatory variables or shifting estimation periods until they obtain “reportable” results which conveniently support their conclusion. Cooley and LeRoy argued that the generally accepted view of interest rates’ negative impact on money demand is nothing but a product of “specification searching” and showed that a trial and error approach, if repeated a sufficient number of times, can easily produce a regression showing that interest rates have a positive impact on money demand.

Cooley and LeRoy, moreover, cast doubt on the feasibility of differentiating the money demand and money supply functions. If a money supply function exists, they argued, it would probably be a function of income and interest rates, just as the money demand function is. Thus it is impossible, they continued, to determine whether the regression commonly known as the money demand function represents the demand for money or the supply of money. To this point, there has been no effective solution to this problem of identification. Gordon (1984a) notes that if a regression such as (1) is to stand

6 The concept of buffer-stock stems from inventory control in factories. The following example should make it easier to understand. Let us assume an enterprise manufactures goods according to plan. When demand grows for its products, the enterprise first tries to respond by drawing on inventory. Instead of immediately reviewing its plan for expanded production, the company will wait until its inventory has registered a significant decline. Conversely, when demand weakens, the company’s immediate reaction will be to build up its inventory rather than to cut production. Although carrying inventory entails some costs, this behavior is justified because a) a precise demand projection is impossible in an uncertain world (expanded information-gathering to bolster the reliability of a projection would involve higher costs) and b) administrative costs will inevitably rise if the production plan is revised frequently.


8 The method used by Cooley and LeRoy (1981) is actually a Bayesian approach developed by Leamer, some times called “Extreme Bounds Analysis.” See Leamer (1978, 1983) for details.
as a viable money demand function, we must also assume that interest rates are fixed by the central bank and that there is infinite elasticity in the money supply in the short run. This paper does not pursue the problem beyond this point and, for heuristic purposes, assumes that (1) is a money demand function.\(^9\)

Hafer and Hein (1980) and Gordon (1984b) followed the initial critique, arguing that, to avoid the problem of “spurious regression” resulting from the non-stationarity of economic variables demonstrated by Granger and Newbold (1974) and Plosser and Schwert (1978), it is preferable to estimate a money demand function such as (1) after transforming all the variables into the first difference form. In this case, the specification becomes:

\[
\Delta(m - p)_t = a'_1 \Delta y_t + a'_2 \Delta R_t + a'_3 \Delta(m - p)_{t-1} \tag{3}
\]

In this regard, it should be noted that Hendry, Pagan and Sargan (1984) and Engle and Granger (1987) pointed out that regression using the first difference form of economic variables differs from (1) in that it contains no information concerning the levels of economic variables. Working from this observation, they expressed doubt about the reliability of such regression when level information is of significance—for example when a group of variables is “cointegrated” (see Section IV. E).

Finally, and most importantly, Hendry (1979) challenged conventional approaches to determining the model’s lag structure by relying solely on economic theory and neglecting a thorough examination of the actual data. This tendency, according to Hendry, is rooted in the misplaced assumption that the extremely simplified and abstract lag structures in economic models can invariably account for reality. In so far as researchers have no advance knowledge of the lag structures in true models (or the processes which generate observable data), the a priori assumption that the lag structures based on a certain dynamic theory are correct will leave them satisfied with the estimation of irrelevant models, unless the lag structure happens to coincide with the actual data generation processes (DGP).\(^{10}\) To forestall this deficiency, Hendry and other British econometricians advocated an approach called data-based dynamic specification. This approach discards the conventional method, which relies on naïve dynamic economic theory for lag structure determination, and seeks information on the DGP directly in the data; the model derived from this process is then adapted so it is consistent with economic theory.

The error correction model (ECM) was born out of this ambitious approach. The

---

\(^9\) In Japan, the central bank incorporates interest rates as an operating variable in its monetary policy. To a degree, therefore, the money supply may be viewed as demand-determined in the short run.

\(^{10}\) Hendry and other British econometricians named the mechanism that generates economic data the Data Generation Process (DGP). They argue that, since the DGP is unknown, the role of econometric analysis is to develop a model that mimics the DGP as closely as possible. While DGP’s stability cannot always be guaranteed, it can be confirmed indirectly within a sample period by checking the stability of the coefficients in estimated models.
next section details problems with the conventional method of estimation and introduces the premises of the AD model and ECM.

III. Basic Premises of ECM

A. Theory-based dynamic specification

The preceding section noted that conventional empirical studies on money demand functions have relied principally on (1) as a standard specification including a lagged dependent variable in its explanatory variables. This section deals at some length with the problems inherent in such an approach.

Generally speaking, when researchers attempt to confirm or refute any given economic theory through empirical studies employing time series data, they invariably face the problem of choosing lagged variables for their model.

The problem starts with the fact that economic theory generally rests exclusively on static functions indifferent to time. Such functions cannot, therefore, be used directly in time series analysis and must be modified to introduce a temporal dimension into the model. Using money demand functions as an example, this point is elaborated below.

The money demand function commonly used in macroeconomic textbooks is

\[(M/P) = M(Y,R)\] (4)

where M/P is the real money demand, Y is the real income and R is interest rates. Since this function is too ambiguous for applications in empirical studies, (4) is usually interpreted as showing contemporaneous relations among economic variables. Combined with the assumption of log linearity, it is generally modified into (5).

\[(m - p)_t = b_1y_t + b_2R_t\] (5)

where

\[m = \ln M, \quad p = \ln P, \quad y = \ln Y\]

If a specification such as (5) were used in empirical studies with time series data, however, there would be no dynamic dependence of data in the model due to the lack of lagged explanatory variables. Simply put, in this case the time series data are treated as if they were mutually independent cross-sectional data. This is a major drawback with a static regression such as (5).

The greatest difference between cross-sectional and time series data is this: while changes in the order of data do not lead to any loss of information in the former, the time sequence of the data constitutes a valuable source of information in the latter. For example, estimation of a consumption function from cross-sectional data collected at a given time yields exactly the same result no matter how the samples are ordered since they are, by definition, mutually independent.

This is not the case, however, if the consumption function is estimated from time
series instead of cross-sectional data. Since essential information on the lag structure of the model is embedded in the sequential order of the data, ignoring the order, therefore, imposes difficulties on obtaining a correct model. This is because economic data observed over time are not completely independent of their own history (i.e., the data observed during the period $t-1$ and before). Rather, they evolve through a data generation process, with the data for each period conditional on their past values.\textsuperscript{11} It is only logical, therefore, to think of an economic variable as determined by its own past values and other economic variables as well as other contemporaneous variables. And this recognition automatically suggests that, in empirical modelling using time series data, acknowledging the dynamic dependence among economic variables—or, more simply, the model's lag structure is a prerequisite to obtaining the correct model specification.

Generally speaking, errors in model specification are often reflected in various test statistics, such as residual autocorrelation and heteroscedasticity. For example, unadjusted application of a static regression such as (5) to time series data will probably exhibit a strong residual autocorrelation. This occurs because information concerning dynamic relations across data is not reflected in any way by the static regressors, so such information is inevitably relegated to the unmodelled part of the model, i.e. the residuals. From a statistical viewpoint, therefore, it is only natural for models with time series data to include a certain number of lagged variables.

Based on this observation, what does the standard money demand function look like? The standard money demand function (1) takes the form of adding lagged dependent variable $(m-p)_{t-1}$ to the explanatory variables in the static regression (5). In terms of statistics, then, this is the only variable representing the dynamic interaction among the data. It is worth noting, however, that the lagged variable $(m-p)_{t-1}$ has been explained not from the standpoint of statistics but solely from that of the unsophisticated dynamic theories called partial adjustment or adaptive expectation.

Partial adjustment can be explained briefly as follows: first, assume optimal money holding in period $t$, denoted $(m-p)^*_t$, is a function of $y_t$ and $R_t$. This relationship is shown below and looks similar to that of (5).

\[(m-p)^*_t = b_1y_t + b_2R_t \]  \hspace{1cm} (6)

\textsuperscript{11} Although this paper does not delve into the DGP in detail, the basic concepts can be summarized as follows. The set of observed data on $x_t$ and $y_t$ can be expressed by $w_t = (x_t, y_t)$ so that the sequence of data observed from the period 1 to the period $T$ can be written as $W_T = (w_1, w_2, ..., w_T)$. Then the process generating $W_T$ can be expressed as $D(w_t, w_{t-1}, ..., w_T | W_0, \theta) = D(W_T | W_0, \theta)$ where $W_0$ are the initial conditions and $\theta$ are the parameters of the process. Next, since $P(ab) = P(a|b)P(b)$, the DGP can be transformed into

\[D(W_T | W_0, \theta) = \prod_{t=1}^{T} D(w_t | W_{t-1}, \theta, W_0).\]

This shows that $w_t$ is generated conditionally on its past values and the parameter $\theta$. The AD models introduced in this paper are, in fact, derived from this theory of DGP. For details, see Hendry, Pagan and Sargan (1984).
Next, assume that, due to various adjustment costs, the adjustment from \((m - p)_{t-1}\) — the real money holdings in the preceding period — to \((m - p)_{t}^*\) is only partly achieved (the speed of adjustment \(\lambda\) lies between 0 and 1). This leads to

\[
\Delta(m - p)_t = \lambda \left((m - p)_t^* - (m - p)_{t-1}\right),
\]

(7)

elimination of \((m - p)_t^*\) from (6) and (7) leads to

\[
(m - p)_t = \lambda b_1 y_t + \lambda b_2 R_t + (1 - \lambda)(m - p)_{t-1}.
\]

(8)

Replacing \(\lambda b_1\), \(\lambda b_2\), and \(1 - \lambda\) with \(a_1\), \(a_2\), and \(a_3\), respectively, produces a standard money demand function identical to (1). In effect, assuming adaptive expectation instead of partial adjustment yields a formula similar to (1). This paper does not, however, attempt to explain this process.\(^{12}\)

It should be clear from the foregoing discussion that the money demand function regarded as standard is, in fact, a model structured to include the first order lagged dependent variable in the explanatory variables by combining rudimentary "static" theories and such crude "dynamic" theories as partial adjustment and adaptive expectation. An approach that attempts to account for such lagged variables exclusively from economic theory could be called theory-based dynamic specification. The principal problems with this approach are discussed below.

The first problem lies in the appropriateness of relying solely on economic theory to determine the models' lag structures. To begin with, economic theory aims to grasp the basic framework of an economy by simplifying the actual economic structure, particularly with the lag structures in dynamic theory. It is immediately clear, then, that direct application of its specifications cannot adequately explain the inherently complex developments in the actual economy. Attempts to improve the model's fit by constructing dynamic models with far more complex lag structures than that of simple partial adjustment may not be completely impossible. As theoretical models grow excessively complex, however, they may well compromise the very aim of the theory itself: analyzing and understanding economic realities by simplifying them.\(^{13}\)

---

\(^{12}\) See Kmenta (1986), among others, for the process of deriving a specification similar to (1) from the adaptive expansion model.

\(^{13}\) In this regard, typical textbooks on econometrics have done little to halt the misguided tendency to apply theoretical models directly to empirical studies. In the past, many textbooks on econometrics focused on how to obtain accurate estimates of coefficients, assuming that researchers were already familiar with the specifications of models, i.e., that they had no doubts which explanatory variables (including lagged ones) to include in the models.

Surely, such an assumption may be appropriate in physics or chemistry, where researchers can control their experiments by changing certain data. Unfortunately, such an approach is impossible in a field such as economics, where numerous variables—including some that cannot be observed—are intricately intertwined, and virtually preclude controlled experiments. In empirical studies of the macroeconomy, the authoritative works have not adequately addressed the question of how to obtain correct model specifications, even though the issue is at least as important as how to estimate coefficients accurately. This seems to have led many researchers to
In this connection, Koopmans' (1947) criticism of "measurement without theory" appears to have accelerated the trend toward the direct application of theoretical models to empirical analyses. Koopmans strongly criticized the NBER's business cycle studies based on selected economic indicators as "measurement without theory," an approach devoid of a thorough examination of causal relationships among economic variables. Presumably, Koopmans simply tried to emphasize the futility of empirical studies without a solid theoretical framework. Unfortunately, his comments were lifted from their context and set the tone for research that made "measurement without theory" a taboo—that specification of empirical models must be given \textit{a priori} from economic theory before economic data are considered, or that economic data are nothing more than simple figures necessary to estimate the model's coefficients.

In many cases, the simplification of lag structures of theoretical models is excessive compared with the realities. It is inevitable, then, that researchers who rely solely on theory for their empirical model specification may end up with erroneous results. Such empirical models normally show residual autocorrelation or heteroscedasticity in residuals because vital explanatory variables, including lagged variables, are omitted in these misspecified models.

A second problem with theory-based dynamic specification is whether $\lambda$ in (7) can be interpreted as adjustment costs (or adjustment speed). While economic theorists view (7) as a partial adjustment model, time series analysts may see it as a model with an AR (auto-regressive) process. As such, when it is applied to economic variables with an upward trend, such as the money supply, $(1 - \lambda)$ normally shows high values of approximately 0.9 with the values of $\lambda$ nearing 0.1. As a result, theorists who use this model face questions such as "Why is this $\lambda$ so small?" or "What are the costs of slowing down the adjustment speed so much?" High adjustment costs like this, moreover, undermine the group of researchers' proposition that financial deregulation and technological innovation have helped to lower transaction costs. Unless theoretical clarity is brought to justifying high adjustment costs, it would seem that interpreting (1) as a partial adjustment model will continue to strain logic.

Finally, there is the third problem of multi-collinearity. If real money holding is a function of real income, $y_t$ and $(m - p)_{t-1}$ will naturally show a high correlation. To some extent, then, it is questionable whether precise estimation is possible for the coefficients of these two variables.

The second and the third problems here are not new and have been emphasized from 

underestimate the importance of model specification, which in turn has inevitably contributed to the unfortunate tendency to apply theoretical models directly to empirical studies. Recently, it is encouraging to note, some new texts have started to deal thoroughly with the implications and techniques of model specification. Spanos (1986) and Maddala (1988), for example, adopted a seminal approach that seeks to improve model specification through feedback between economic theory and empirical data. The data-based dynamic specification introduced in this paper represents the essence of such an approach.
time to time over the years. Rather, the standard money demand function has survived its many inherent difficulties due to the lack of a reasonable alternative. In fact, research papers on money demand functions published in Japan and abroad have frequently fallen victim to inconsistency: they inadvertently adopt the standard function in their empirical studies while assiduously pointing out its drawbacks in their theoretical portion. As long as the researchers maintain that models’ lag structures should be derived a priori from economic theory, it would seem to be very difficult to overcome this dilemma.

The foregoing discussion can be summed up in several points.
1) Models dealing with time series data must, in one form or another, have lagged variables to reflect the dynamic dependence of data.
2) The conventional approach to empirical studies of the money demand function seeks to justify incorporating lagged variables in the models solely from the standpoint of economic theory. It may, therefore, be called theory-based dynamic specification.
3) While theory-based dynamic specification does not suffer from “measurement without theory,” it can generate only models with very simple lag structures. When those lag structures do not match the actual data generation process, erroneous results will ensue.

B. Data-based dynamic specification

Data-based dynamic specification is an approach to model building which can alleviate the drawbacks of theory-based dynamic specification while avoiding “measurement without theory.” This approach has been advocated and practiced mainly by British econometricians, notably Sargan and Hendry. The essence of this concept, as the name indicates, is to base models’ lag structures on economic data instead of dynamic theory. In practice, empirical models are constructed with the following steps.\(^\text{14}\)

First, the economic variables necessary for the model are selected according to static theory. When a money demand function such as (4) is assumed, for example, the four variables of \(m, p, y\) and \(R\) are included (\(m = 1nM, p = 1nP, y = 1nY\)).

Second, to extract information on the models’ lag structure from given data, an AD (auto-regressive distributed-lag) model (9) is estimated on the basis of an appropriate order of lagged dependent and explanatory variables.\(^\text{15}\)

\(^\text{14}\) Drawing on the theories of probability and statistics, Sargan, Hendry and Richard have developed the methodology of Data-based Dynamic Specification into a well-defined approach. Representing only part of the framework, this paper does not consider such important issues as weak exogeneity and simultaneous equation bias or encompassing between non-nested models. For the entire basis of this approach, see Harvey (1981), Engle, Hendry and Richard (1983), Hendry and Richard (1982, 1983), Hendry, Sargan and Pagan (1984), Gilbert (1986) and Spanos (1986, 1988).

\(^\text{15}\) (9) uses nominal money supply \(m\) as a dependent variable. It is also possible to work from an AD model with the real money supply \((m−p)\) as a dependent variable:
\[ m_t = a_0 + a_{11}m_{t-1} + \ldots + a_{1n}m_{t-n} \\
+ a_{20}p_t + a_{21}p_{t-1} + \ldots + a_{2n}p_{t-n} \\
+ a_{30}y_t + a_{31}y_{t-1} + \ldots + a_{3n}y_{t-n} \\
+ a_{40}R_t + a_{41}R_{t-1} + \ldots + a_{4n}R_{t-n} \\
+ u_t \] (9)

In this case, the maximum order of lag \( n \) is chosen so that residuals \( u_t \) satisfy conditions such as mutual independence and homoscedasticity. In the case of quarterly data, \( n = 4 \sim 6 \) is said to be sufficient unless important explanatory variables are omitted.

AD models measured this way are usually called general models. From a statistical standpoint, the performance of such general models will, in many cases, prove satisfactory in that they can trace data developments relatively precisely. Unfortunately, they explain little in terms of economic theory, and, parameterisation, such as (9) correct estimation of individual coefficients seems impossible due to the multi-collinearity problem. Consequently, the next step requires changing general model (9) into a form which is both consistent with economic theory and relatively immune to multi-collinearity problems. The error correction model (ECM) to be explained in the next section has been obtained through this transformation.\(^{16}\)

Central to this modification exercise is the process called reparameterisation, to which we shall return later. Efforts have also been made to increase the model’s efficiency by a) eliminating redundant explanatory variables and by b) combining several explanatory variables into one, without serious deterioration in the standard error of regression, and maintaining the serial independence of residuals.

Finally, the validity of the parameterisation of the modified model must be checked. This is usually done by observing a) the results of out-of-sample simulation and b) how the estimated coefficients evolved as new data became available.

This is a brief outline of the data-based dynamic specification. The theory-based approach described earlier is, in effect, a one-way street from theory to empirical work. The data-based approach, on the other hand, a) chooses the dependent and explanatory

\[ (m-p)_t = b_0 + b_{11}(m-p)_{t-1} + \ldots + b_{1n}(m-p)_{t-n} \\
+ b_{20}y_t + b_{21}y_{t-1} + \ldots + b_{2n}y_{t-n} \\
+ b_{30}R_t + b_{31}R_{t-1} + \ldots + b_{3n}R_{t-n} + u_t. \]

Since these AD models are not equivalent, however, the latter is not appropriate as a starting point for model construction. This is because, while a comparison with (9) reveals the set of restrictions on (9) such as a \( b_{20}=1 \), \( a_{ii}=-a_{ii} \) \( (i=1,\ldots,n) \), there is no confirmation of these \emph{a priori} assumptions.

\(^{16}\) ECM is doubtlessly a promising extension of AD models, but it is not the only one. For example, Cuthbertson (1988) attempted to interpret AD models as buffer-stock models.
variables according to static theory, b) determines the lag structure according to data, and c) interprets the model theoretically. In this sense, it is an attempt to utilize both theory information and data information to the fullest. This approach is also called the "general-to-specific" or "general-to-simple" approach since it starts with a large model and gradually reduces it to essentials.

Theory-based dynamic specification, in contrast, often takes the form of a "simple-to-general" approach, starting with a small model and gradually adding explanatory variables to enhance its explanatory power. The criticism by Leamer, Cooley and LeRoy, which was summarized earlier, was directed squarely at the arbitrariness of "specification searching" in increasing the explanatory variables. The "general-to-specific" approach, in contrast, gradually reduces the number of explanatory variables by checking residual autocorrelation and other factors. As such, this process allows for relatively less arbitrariness.

C. Auto-regressive distributed-lag (AD) model and ECM

Here, using the simplest form of the AD model, relations between the AD model and other classes of models often employed in empirical studies are first clarified and then it is shown how ECM is derived from the AD model.

As seen in (9), the AD model normally has more than one explanatory variable as well as high order lags. For the sake of brevity, the following explanation starts with the simplest form of the AD model, which has only one explanatory variable and first order lags (the following explanation sets \((m - p)\) as a dependent variable, \(y\) for an explanatory variable and omits \(R\)).

\[
(m - p)_t = \alpha(m - p)_{t-1} + \beta y_t + \gamma y_{t-1} + u_t
\]  

(10)

Although (10) is the simplest form of the AD model, it is already more "general" than a partial adjustment (or adaptive expectation) model such as (1). The restriction of \(\gamma = 0\) imposed on (10) leads to

\[
(m - p)_t = \alpha(m - p)_{t-1} + \beta y_t + u_t
\]  

(11)

which is basically the same as (1) except that interest rate \(R_t\) is not included in the explanatory variables. The restriction of \(\alpha = 0\) imposed on (10) leads to

\[
(m - p)_t = \beta y_t + \gamma y_{t-1} + u_t
\]  

(12)

which is a second-order distributed lag model. The restriction of \(\beta = \gamma = 0\) imposed on (10) leads to

\[
(m - p)_t = \alpha(m - p)_{t-1} + u_t
\]  

(13)

which is a first-order AR model.

As these examples indicate, the AD model is more "general" than other models used
in empirical studies, such as the partial adjustment model, the adaptive expectation model, the distributed lag model and the auto-regressive model. Conversely, since all of these models are special forms of the AD model, researchers using one of them must constantly check the need to impose restrictions on the general model by examining the residuals' independence and other factors.

Figure 2 illustrates the relationships between the AD model and other models. (This figure has \( x \) as a dependent variable, \( y \) as an explanatory variable and a generalized form for lag structures.) It clearly shows that the AD model is far more "general" than other models.

In this figure, the AD and VAR models share a common feature: they are designed to fully utilize the information contained in economic data by avoiding any significant restrictions on their lag structures. Nevertheless, they differ from each other in that the AD model includes contemporaneous \( y_t \) in its explanatory variables while the VAR model does not, a contrast that leads to certain variations in their characters. According to econometric terminology, the AD model is a "structural form" while the VAR model is a "reduced form" into which the "structural form" model has been incorporated so it can be expressed solely with exogeneous and predetermined variables. Such a difference in form reflects the differing purposes of their development: the AD model is intended to serve as a springboard for model building to corroborate the validity of economic theory while the VAR model disregards the structure of an economic system, focusing instead on simulating how the model reacts to changes in given economic variables.

Let us turn to the procedure for the induction of ECM through the reparameterisation of the AD model. Let \( \alpha = 1 + c_2, \beta = c_1, \gamma = - c_1 - kc_2 \) in (10) and subtract \((m - p)_{t-1}\) from both sides of equation. We obtain the simplest form of ECM (14).

\[
\Delta(m - p)_t = c_1 \Delta y_t + c_2 (m - p - ky)_{t-1} + u_t
\]  
(14)

Since this is merely a linear transformation from the three parameters of \( \alpha, \beta \) and \( \gamma \) into the other three parameters of \( c_1, c_2 \) and \( k \), the generality of the AD model is maintained after this process.

This conversion from the AD model into ECM leads to two major advantages. The first is that, unlike (10), (14) enables us interpret the model meaningfully in terms of economic theory.

The ECM-type money demand function explains behavior in money holdings as follows. First, it assumes

\[
(m - p) = ky \quad (k \text{ is a constant})
\]  
(15)
as the optimal relationship between money holdings \((m - p)\) and real income \(y\) as shown in elementary money demand functions. Due to uncertainty or adjustment costs, however, such a relationship is only rarely achieved in every period. In such a case, deviation from the optimal state in period \(t\), \( EC_t \) (normally called "error") is given as
Figure 2. Relationships between the AD Model and Other Models (examples with two variables)

○: dependent variables  □: explanatory variables

(1) AD model (= ECM)

(2) Partial adjustment model and Adaptive expectation model

(3) Distributed-lag model

(4) AR model

(5) VAR model

(6) Static regression
\[ EC_t = (m - p - ky)_t \]  
(16)

and of course \( EC_t \neq 0 \) in any non-optimal state.

Second, in every period, people try to adjust to changes in their money holdings \( \Delta(m - p)_t \). This adjustment is divided into two parts: the first part \( c_1 \Delta y_t \) is the adjustment in proportion to the changes in real income during the same period, and the second part \( c_2 (m - p - ky)_t \) is the adjustment to correct part of the "error" in the preceding period. This is the interpretation of (14). In this case, correcting the "error" in the preceding term in the pursuit of the optimal state naturally calls for \( c_2 < 0 \). \( EC_{t-1} = (m - p - ky)_{t-1} \) is called the error correction term.

In sum, the ECM-type money demand function assumes that, in the real economy, equilibrium among variables posited by economic theory is by no means achieved in every period. Rather, it sees actual money supply developments as a reflection of the process of people striving to reach an optimal state.

It should be also noted that, consistency between such ECM-type money demand functions and economic theory can also be confirmed as follows. In (14), consideration of a long-run steady state of \( \Delta(m - p)_t = \Delta y_t = u_t = 0 \) leads to

\[ (m - p) = ky \]

which shows that the same money demand function as (15) implicitly exists in (14).

ECM's second advantage is that it greatly alleviates the multicollinearity problem. In (10), high correlations for the three explanatory variables \( (m - p)_{t-1}, y_t \) and \( y_{t-1} \) can be expected. Therefore, accurate estimation of \( \alpha, \beta \) and \( \gamma \) is difficult to obtain by direct estimation of (10). In (14), however, the number of explanatory variables is reduced to two \( (\Delta y_t \text{ and } (m - p - ky)_{t-1} \text{ conditional on known } k) \) and they are, in the normal circumstances, not highly correlated. Accordingly, a relatively precise estimation can be expected for the values of \( c_1 \) and \( c_2 \).

On the other hand, among the three parameters of ECM, \( c_1, c_2 \) and \( k, k \) is placed inside \( EC_t \) (error correction term). This has been a serious disadvantage since a linear model cannot be estimated unless \( k \) is known. For this reason, empirical studies have usually estimated their models under the restriction of \( k = 1 \), with the inevitable result that equivalence is lost between ECM model and AD model.

As will be noted in Section IV, this problem has been resolved by the development of the two-step method in recent years (Engle and Granger 1987). This method first estimates the value of \( k \) alone and then estimates the whole ECM using the estimated value of \( k \).

D. History of ECM

The basic premise of ECM—that people act to correct their past errors—is said to have been introduced first by Phillips (1954, 1957), the well-known originator of the Phillips curve. Working from this proposition, Sargan (1964) specified the first ECM for
the wage determination function in Britain. Since its empirical results were less than satisfactory and his article did not appear in an influential academic journal, Sargan's work went unnoticed for many years.

It was toward the end of the 1970s that ECM gained momentum in Britain. This prominence owed much to the work of Davidson, Hendry, Srba and Yeo (1978), which presented an empirical study of an ECM-type consumption function exhibiting an excellent fit and a theoretical framework for ECM. Subsequently, Hendry (1979) empirically demonstrated a stable money demand function for $M_1$ in Britain. These successes triggered widespread studies on ECM in Britain and some other European countries. According to Patterson, et al (1987), the Economics Division of the Bank of England has already adopted ECM in the consumption function block of its large scale macroeconometric model.

Empirical studies on the ECM-type money demand function have been particularly productive in recent years. In Britain, following the pioneering work of Hendry (1979), Trundle (1982), Hendry and Ericsson (1983), Hendry (1985, 1988) and Patterson (1987) have reported their empirical results. In the United States, on the other hand, Gordon (1984a) expressed some doubts about the effectiveness of ECM in the U.S. economy. More recently, however, empirical results on a stable ECM-type money demand function for $M_1$ in the United States have been reported by Rose (1985) and Baba, Hendry and Starr (1988) for 1952-78 and 1964-84, respectively. ECM also seems to have been recently adopted by the Division of Research and Statistics of FRB. Meanwhile, Arestis (1988), and Gupta and Moazzami (1988) have applied ECM to their studies on money demand functions in developing countries such as the Philippines.

The empirical results of Hendry (1985) and Baba, Hendry and Starr (1988) are briefly reviewed in the Appendix.

IV. Time Series Analysis and ECM

A. Stationary and non-stationary processes

As noted earlier, ECM is a method of dynamic modeling developed mainly by British econometricians. As such, it is still not particularly widespread in the United States, where the mainstream looks to VAR model analysis. Recently, however, Granger and other time series analysts demonstrated the effectiveness of ECM from an entirely different perspective—their study on the properties of time series data. This section briefly explains the relationship between time series analysis and ECM.

---

17 At a FRB conference on “Monetary Aggregates and Financial Sector Behavior in Interdependent Economies” in Washington last year, a paper on the money demand function by the Division of Research and Statistics of the FRB (Moore, Porter and Small 1988) used ECM in its empirical studies. ECM has also been adopted in the supplement to a paper on the developments in the United States’ $M_2$ (Small and Porter 1989) in the April 1989 issue of Federal Reserve Bulletin.
Time series data with AR representation such as (17) is either a stationary process or a non-stationary process. The latter is further subdivided into an explosive process or a unit root process that drifts away from the initial value.\footnote{This classification can be expressed more precisely as follows. Suppose $x_t$ has a finite-order AR representation

$$x_t = \theta_1 x_{t-1} + \theta_2 x_{t-2} + \ldots + \theta_p x_{t-p} + u_t,$$

and $z = z_1 + z_2 i$ solves the root of the characteristic equation

$$1 - \theta_1 z - \theta_2 z^2 - \ldots - \theta_p z^p = 0.$$ 

If every root lies outside the complex unit circle expressed by $\sqrt{z_1^2 + z_2^2} = 1$, $x_t$ is defined as a stationary process. If at least one of the roots lies inside of the circle, $x_t$ is an explosive process. And if at least one of the roots lies on the unit circle, $x_t$ is a unit root process. Consequently, the random walk process

$$x_t = x_{t-1} + \epsilon_t,$$

is the simplest form of unit root process. The trend-stationary process is expressed by

$$x_t = \alpha x_{t-1} + \beta t + u_t, \, \alpha < 1,$$


(17)

$$x_t = \theta_1 x_{t-1} + \theta_2 x_{t-2} + \ldots + \theta_p x_{t-p} + u_t$$

Since it is typical of the unit root process, the following explanation will, for the sake of brevity, focus mainly on the random walk process. (The basic character of random walk process is shared by other unit root processes.)

Stationary process

- Unit root process
  - Random walk, etc.

Non-stationary process

Explosive process

Of the three processes shown above, an explosive process can be identified with relative ease by graphing the given data. Since macroeconomic data such as GNP and money supply look often explosive, their logarithms are often used in empirical studies. (Normally, converted data become either a stationary process or a unit root process.) Distinguishing a stationary process from a unit root process, however, is not so clear-cut in certain instances.

In Figure 3, both an artificially generated “mean-stationary” process (A) and a random walk (C) are shown. While (A) exhibits an irregular fluctuation around the mean, (C) shows a random walk that diverges from the initial value without returning to it even once. In this case, there is little difficulty in distinguishing one from the other. When we compare a trend-stationary process (B) with a random walk with drift (D) in the same figure, however, it is not so simple to differentiate the two simply by examining the two series of data.\footnote{The trend-stationary process is expressed by}

$$x_t = \alpha x_{t-1} + \beta t + u_t, \, \alpha < 1.$$
In time series analysis, the properties of each series of data must be clearly identified since, as will be explained later, we can confidently apply estimation methods such as the least squares method only when all the data series are stationary. It is generally recommended, then, that first difference forms be used if non-stationary variables are to be included in a regression. For example, for a random walk variable $x_t$

$$x_t = x_{t-1} + e_t, \quad e_t \sim N(0, \sigma^2),$$  \hspace{1cm} (18)

the first difference $\Delta x_t$ can be written as

$$\Delta x_t = e_t,$$  \hspace{1cm} (19)

which is, by definition, a stationary process.

Mathematically, it can be shown that a non-stationary process, which can be turned into a stationary process by taking $d$ times of differences, has the same number of unit roots. Utilizing this characteristic, Granger has proposed a new classification for non-explosive processes: variables that are stationary processes are denoted as I(0), those that become stationary processes by taking first differences are designated I(1), those that become stationary by taking second differences are called I(2) and so on.\(^{20}\) According to this definition, a random walk variable is I(1).

Table 1 compares the distinctive features of a stationary process I(0) with those of a non-stationary process I(1). The most important point in this comparison is that, if a regression includes one or more I(1) variables, familiar statistics such as the coefficient of determination ($R^2$) and t-values no longer have simple textbook distributions (except in very limited circumstances). Consequently, they are likely to lead us into incorrect inferences if we apply conventional standards.

The source of this problem lies in the following difficulty. The purpose of a regression, as well as statistics in general, is to make inferences about the shape of an unknown population from small samples, a process that inevitably requires extracting accurate information on the mean, the variance etc. of the population. When the economic variables are I(0), their reliability is corroborated by the fact that the mean and the variance calculated from samples are unbiased estimates of the true mean and variance. When economic variables used are I(1), however, such unbiasedness is unavailable since non-stationary processes such as random walk have neither particular mean nor finite variance, while any sample mean and variance calculated from a finite number of observed data have finite values.

To be on the safe side, then, it seems appropriate to avoid I(1) variables in a

\(^{20}\) I (d) means "integrated of order d."
Table 1.

<table>
<thead>
<tr>
<th>Attributes of I(0)</th>
<th>Attributes of I(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>• I(0) variables fluctuate irregularly around and frequently intersect their mean.</td>
<td>• I(1) variables tend to have wider swings. As sample sizes increase, the probability of their returning to the same value approaches zero.</td>
</tr>
<tr>
<td>• Since the effect of a shock ( u_t ) in each period weakens over time, I(0) variables do not diverge far from their mean or trends.</td>
<td>• At least part of a shock in each quarter has long-lasting effects. ( x_t = x_{t-1} + e_t = x_0 + \sum_{i=1}^{t} e_i )</td>
</tr>
<tr>
<td>• The mean ( \bar{x} ) and variance ( s^2 ) calculated from observed data are unbiased and consistent estimators of the true mean ( \mu_x ) and variance ( \sigma^2 ).</td>
<td>• Average ( \bar{x} ) and variance ( s^2 ) calculated from observed data are often biased. (When ( T \to \infty ), random walk has neither particular mean value nor finite variance)</td>
</tr>
<tr>
<td>• Relatively accurate estimates of coefficients can be obtained by applying an ordinary regression. The estimates are known to follow a t-distribution.</td>
<td>• In small samples, a regression including I(1) variables may well yield erroneous results. Estimates are often biased (cointegration is an exception), and they do not follow a t-distribution.</td>
</tr>
</tbody>
</table>

regression analysis, unless there is a specific reason to do otherwise. In statistics, this point has long been recognized. In empirical studies, however, little attention was paid until Dickey and Fuller (1979) introduced the first statistical test that can help, albeit to a limited extent, to determine whether a given economic variable is I(0) or I(1).

**B. The Dickey-Fuller Test**

Let us consider what happens if I(1) variables are included in a regression, a question that has been explored mainly by the Monte Carlo experimental method. This approach examines the distribution of estimates by repeatedly running a regression with thousands of sets of artificially generated I(1) data. This study, undertaken by Fuller (1976) and Dickey and Fuller (1979), has elucidated several points.

First, when we have an autoregression of a random walk variable, it is clear that if we run a regression such as

\[
x_t = ax_{t-1} + ct + d + u_t,
\]

with the variable \( x_t \) known to be generated by

\[
x_t = x_{t-1} + e_t \quad e_t \sim N(0, \sigma^2),
\]

\[ (20) \]

\[ (21) \]
we obtain the following results.

1) \( \hat{a} \) (the estimates of a) has a distribution around \( 1 - 10/n \) (n is sample size) instead of its true value 1 and \( \hat{a} \) seldom exceeds 1 (Nelson and Kang 1983).
2) The t-value obtained by dividing \( \hat{a} - 1 \) by its standard error lies around \(-2.2\) instead of 0 (Nelson and Plosser 1982).\(^{21}\)
3) The distribution of \( \hat{a} \) and its t-value is largely influenced by the exclusion of trend (ct) or constant (d).

The greater contribution of Dickey and Fuller’s Monte Carlo experiments is that they established a statistical basis for testing whether or not a given economic variable is I(1). This test is a matter of applying a regression such as (20) to the variable and comparing the t-value with Fuller (1976)’s distribution table. For example, when the sample size is 100 and the level of significance is 5%, the critical value is \(-3.45\).\(^{22}\) If the t-value is significantly negative, the variable is regarded as I(0) (trend-stationary process) instead of I(1). Known as the Dickey-Fuller (DF) Test, the approach is gradually gaining ground among empirical researchers, who are recognizing that it is preferable to check the stationarity of economic variables by this or other unit root tests before embarking on empirical studies.

Dickey Fuller have also developed the ADF (Augmented Dickey-Fuller) test. This modified version of the DF test can handle more complex unit root processes with higher order lags. Of course, the ADF test is used more widely, including the empirical studies in Section V of this paper. In the ADF test, we run a regression such as (22), using the t-value of \( \hat{a}' \) as a test statistic. It follows the Dickey-Fuller Test in adopting the t-value of \( \hat{a}' \) as a test statistic.\(^{23}\) While the Dickey-Fuller Test required the somewhat complicated procedure of calculating the t-value by dividing the \( \hat{a} - 1 \) by its standard error, the ADF test simplified this procedure by changing the dependent variable from \( x_t \) to \( \Delta x_t \) and we can now obtain the t-value of \( \hat{a}' \) from ordinary regression package software without any manual calculation.

\[
\Delta x_t = a' x_{t-1} + \sum_{i=1}^{4} b_i \Delta x_{t-i} + ct + d + u_t
\]

Despite its advantages, the ADF test has problems. As its originators have empha-

\(^{21}\) In a strict interpretation, this value would not be called a “t-value” since it does not follow a t-distribution. For example, Fuller (1976) used the term \( \tau \) but, since it has not been widely accepted, this paper follows general usage.

\(^{22}\) When a constant and time trend are included, the major critical values are as follows. For details, see Fuller (1976) or Yamamoto (1988).

<table>
<thead>
<tr>
<th>Sample size</th>
<th>1% level of significance</th>
<th>5% level of significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>-4.15</td>
<td>-3.50</td>
</tr>
<tr>
<td>100</td>
<td>-4.04</td>
<td>-3.45</td>
</tr>
</tbody>
</table>

\(^{23}\) To be precise, the critical values for the ADF test are not identical in the ADF and DF tests, especially in small samples. Nevertheless, many empirical studies, including Nelson and Plosser (1982), rely on Fuller’s (1976) table.
sized, experiments have shown that when the real process is a stationary process such as
\[ x_t = ax_{t-1} + e_t \quad |a| < 1, \]  
(23)
with the a value very close to 1 and a small sample size, the test often fails to distinguish it from a random walk process (in statistics terminology, the power of the test is low). Thus, results of the DF or ADF test should not be accepted as final; rather, they should be treated as an important information on the properties of time series data.

C. Spurious regressions

The problems with the application of an autoregression to a random walk variable were noted earlier. This section deals with a problem that arises from a regression run between two independent random walk variables. Granger and Newbold (1974) named the difficulty “spurious regression.” In their Monte Carlo experiment, Granger and Newbold conducted regressions on the sets of two independent and mutually unrelated random walk variables to see if any significant relations would be identified by normal statistical standards. The experiment produced t-values larger than 2, in 77 out of 100 trials, indicating a causal relationship between the two variables.

Phillips (1986) investigated this phenomenon mathematically and showed that the following results can be obtained from a regression run between two mutually independent random walk variables.\(^{24}\)

1) Since the conventional t-value do not have limiting distributions, there are no asymptotically correct critical values for them. If a normal critical value is used, the probability of finding a significant relationship between the two variables will increase with the sample size.\(^{25}\)

2) The coefficient of determination (R\(^2\)) has a non-degenerate limiting distribution, so moderate values of R\(^2\) are to be expected.

3) As the sample size approaches infinity, the Durbin-Watson (DW) ratio converges toward zero.

The corollary of these observations is that a regression between suspected I(1) variables which have an extremely low DW ratio (or with the DW ratio adjusted upward with the Cochrane-Orcutt procedure) should prompt skepticism about the result, including its spurious nature. In this regard, Granger and Newbold (1974) noted that regressions with DW ratios lower than their coefficient of determination call for close scrutiny.

D. Nelson-Plosser’s findings

Nelson and Plosser (1982) conducted ADF tests on the annual data of the major macroeconomic variables in the United States such as nominal and real GNP, the indust-

---


\(^{25}\) The sample size was 50 in Granger and Newbold’s (1974) Monte Carlo experiment.
rial production index, number of employed, unemployment rate, GNP deflator, consumer price index, nominal and real wage index, money supply, interest rates and stock prices. Surprisingly, they concluded that their tests could not reject the null hypothesis that these data individually followed the random walk pattern with only one exception of unemployment rate. In the wake of this finding, similar tests have been carried out in other countries, and statisticians have worked to develop new types of unit root tests.26 Unfortunately, none of them has proved effective in fully distinguishing I(0) and I(1) variables—no wonder since the empirical results vary so widely according to a) the type of test used, b) differences in the sample period and c) the category (annual, quarterly, monthly etc.) of data.27 In any event, since there is no guarantee that macroeconomic data such as GNP and money supply are trend-stationary, it is obvious that any results from a regression analysis must be interpreted very carefully if the levels of these data have been employed.

E. Cointegration and ECM

Suppose most time series economic variables are random walks, as has been maintained by Nelson and Plosser (1982), it is conceivable that two variables may well show entirely different movements. In fact, however, the evidence suggests the opposite: many macroeconomic variables, such as \( x_t \) and \( y_t \) in Figure 4, often move in the same direction and exhibit similar movements, especially in the middle-and-long-run.

Observing this similarity in the the movements of several economic variables, such as income and consumption, and exports and imports, for example, Granger (1981) hypothesized that economic variables may individually be non-stationary, but are not mutually independent. Rather, there seems to be a mechanism that prevents wide divergence. Granger named this relationship "cointegration."

A formal definition of cointegration was offered by Granger and Weiss (1983)—that is, time series variables \( x_t \) and \( y_t \) are cointegrated if both of them are individually I(1) and there exists a constant \( k \) which makes the linear combination \( y_t - kx_t (=z_t) \) an I(0) process.28 This relationship is depicted in Figure 4 in which \( z_t \sim I(0) \) is plotted along with

27 Stock and Watson (1986) and Perron and Phillips (1987) maintain that a modified version of DF tests shows the United States' GNP is a trend-stationary process. Walton (1988) differs, reporting that the improved version of DF test still shows the GDP in Britain is a random walk process. Yamamoto (1988), who typifies studies in Japan, reports that his Lagrange multiplier test shows the GNP was non-stationary until the oil crisis, then changed to trend-stationary.
28 Cointegration is defined more precisely as follows. If there is a \((n\times 1)\) vector \( x_t = (x_{1t}, j=1, \ldots, n) \) and all \( x_{jt} \) are I(1) and there is a non-zero vector \( \alpha' = (a_1, \ldots, a_n) \) which makes

\[
    z_t = \alpha' x_t = \sum_{j=1}^{n} a_j x_{jt} \sim I(d-b), \quad b>0,
\]

\( x_t \) is said to be cointegrated of order \( d \), \( b \) and expressed as \( x_t \sim C I(d, b) \). For the sake of clarity, the definition in this paper uses the simplest case of \( d=b=1 \).
$x_t \sim I(1)$ and $y_t \sim I(1)$.

Granger (1983) also demonstrated that, when $x_t$ and $y_t$ are cointegrated, there is the following error correction representation among $x_t$, $y_t$ and $z_t$; the opposite always holds as well.\(^{29}\)

\[
\Delta x_t = -\rho_1 z_{t-1} + \alpha_{11} \Delta x_{t-1} + \alpha_{12} \Delta x_{t-2} + \cdots + \alpha_{21} \Delta y_{t-1} + \alpha_{22} \Delta y_{t-2} + \cdots + d(B) \varepsilon_{1t} \tag{24}
\]

\[
\Delta y_t = -\rho_2 z_{t-1} + \beta_{11} \Delta x_{t-1} + \beta_{12} \Delta x_{t-2} + \cdots + \beta_{21} \Delta y_{t-1} + \beta_{22} \Delta y_{t-2} + \cdots + d(B) \varepsilon_{2t} \tag{25}
\]

In (24) and (25), $z_{t-1}$, which shows the size of error in the preceding term, serves as an error correction term; $d(B)$ is a finite polynomial in the lag operator $B$ and is the same in each equation; $\varepsilon_{1t}$, $\varepsilon_{2t}$ are joint white noise. Also, the coefficients of $z_t$, $\rho_1$ and $\rho_2$ are required to satisfy $|\rho_1| + |\rho_2| \neq 0$. This equivalence between cointegration and ECM is called the Granger Representation Theorem. It is worth noting that while (14) in the preceding section is written in a structural form including contemporaneous $\Delta y_t$ in its explanatory variables, (24) and (25) are written in a reduced form, with all of their explanatory variables expressed in terms of predetermined variables. Nevertheless, they both have error correction terms in their explanatory variables as well as other variables in first difference forms. The fact that a similar ECM was obtained from a different line of research in Britain and the United States has spotlighted the model, and an increasing number of researchers, both from the economics and econometrics sides, are working in

\(^{29}\) For proofs of the equivalence between cointegration and ECM, see Granger (1983) or Engle and Granger (1987).
this area.\textsuperscript{30} This development is particularly indebted to Granger, who found that with cointegrated I(1) economic variables ECM is superior to a simple regression in first difference forms, not to mention it's long-recognized advantage of allowing for interpretations consistent with economic theories.

Checking (24) and (25) readily shows that exclusion of error correction term of $z_{t-1}$ reduces them to simple regressions in the first difference form. One of the advantages of using a regression in first difference forms is that, since all the variables have been transformed to I(0), it is immune to the problem of spurious regression. As Granger (1986) and Engle and Granger (1987) pointed out, however, when it is applied to cointegrated variables, the model fails to reflect the underlying mechanism that prevents the variables from drifting apart. Inevitably, such a specification error is likely to reduce the models' explanatory power and forecast performances. On the other hand, ECM's $z_{t-1}$ ensures that the variables do not drift apart, but spuriousness is still prevented since $z_{t-1}$ is I(0) by definition.

F. Testing cointegration

According to Engle and Granger (1987), detecting the cointegration between two economic variables $x_t$ and $y_t$ begins with applying the Dickey-Fuller or other unit root tests individually to the variables. Then the residual

$$\hat{u}_t = y_t - \hat{k}x_t - \hat{e},$$

(26)

from the regression

$$y_t = kx_t + c + u_t \quad (c=\text{constant}),$$

(27)

is examined to see if it is I(0). For this part of the process, Engle and Granger (1987) recommend using the following ADF test without time trend and constant.

$$\Delta \hat{u}_t = \phi \hat{u}_t + \sum_{i=1}^{4} b_i \Delta \hat{u}_{t-i} + \epsilon_t$$

(28)

The critical values for this test are listed in Engle and Granger (1987). For example, with a sample size of 100, the critical value is $-3.17$; if the $t$-value from this test is less than this critical value, then $u_t$ is I(0) and $x_t$ and $y_t$ are considered cointegrated.

\textsuperscript{30} As recent ECM research in Britain and the United States has almost converged, exchanges are growing between the two groups. To trace this development, consult the contributions by Hendry and Granger to the special issue of the \textit{Oxford Bulletin of Economics and Statistics} (1986, No.3). Despite their interchanges, Hendry and Granger do not agree on every point. For example, Hendry's emphasis on the relationship between economic theory and ECM has led him to pursue single-equation models in a structural form such as that of (14). As a time series analyst, on the other hand, Granger stresses the importance of enhancing models' explanatory power and forecast accuracy by utilizing the reduced form ECM, such as (24) and (25), even if the mechanism of cointegration is not theoretically clear (for example, relations between the government's revenues and expenditures and those between exports and imports).
G. Super-consistency and the Engle-Granger two-step method

Since (27) is a regression between I(1) variables, it is only natural to assume that the regression is in fact spurious. On the contrary, Stock (1987) proved that, when \( X_t \) and \( Y_t \) are cointegrated, estimates obtained from (27) (called a cointegrating regression in this case) are far more precise than those obtained in ordinary least squares estimation. In a regression between I(0) variables, the least squares estimators are consistent in their convergence toward the real value as the sample size increases. A cointegrating regression, on the other hand, can produce relatively precise estimates even from a small sample since the variance of \( \hat{k} \) approaches zero more rapidly than with I(0). This characteristic is known as "super-consistency."\(^{31}\)

Exploiting super-consistency, Engle and Granger devised a new method of two-step estimation. First, we estimate \( k \) by (27), then use \( \hat{k} \) to calculate the error correction term, and finally estimate an ECM such as (14). The Engle-Granger two-step method solved the problem of estimating \( k \), a difficulty that had persisted for many years in Britain.\(^{32}\) Engle and Granger (1987) themselves applied this method to the estimation of the consumption function in the United States. Hall (1986) also applied this method to estimating the wage function in Britain.

V. Empirical Studies: Estimating Japan’s Money Demand Functions with ECM

A. Unit root test on the GNP, money supply and other variables

In line with the preceding discussion, this section uses ECM to estimate Japan’s money demand function.\(^{33,34}\) As Section IV indicated, in any empirical analyses venturing beyond the money demand function, it is essential to investigate the time series property of data before any model estimations. The ADF test (29) was therefore, applied to each of the five economic variables in the later estimation: real GNP, money supply \((M_2+CD \text{ average balance})\), GNP deflator, a coupon rate of 5-year bank debentures, and the coefficient of variation for Nikkei Stock Average 225 index.\(^{35}\) The sample period for this ADF test is the more than 20 years from the first quarter of 1968 to the first quarter of 1989.

\[
\Delta x_t = a'X_{t-1} + \sum_{i=1}^{4} b_i \Delta x_{t-i} + ct + d + u_t
\]  

(29)


\(^{32}\) Banerjee, et al (1986) differ, maintaining that their Monte Carlo experiment shows a substantial bias remaining in the estimates of \( k \) even when the sample is relatively large (around 200). We must assume, then, that the Engle-Granger two-step method’s results are not always accurate when the sample size is small.

\(^{33}\) Empirical studies in this section used PC-GIVE, the econometrics software package for time series data analysis developed by Hendry and his associates at Oxford University.

\(^{34}\) Baba (1989) estimated Japan’s money demand function with the ECM. The sample period of his study was, however, limited to the 1980s.

\(^{35}\) The coefficients of variation for the Nikkei Stock Average 225 index are calculated by dividing the standard deviations of monthly data by the mean for the data from each quarter. Their actual values lie between 0 and 4.
The result of this unit-root test is shown in Table 2. Overall, this test could not completely reject the null hypothesis that \(x_t \text{ is I}(1)\) in any of the five variables.

In Table 2, \(y\) represents the real GNP, \(m\) is the \(M_2+CD\) (average balance), \(p\) is the GNP deflator, \(R\) is the coupon of 5-year bank debentures and \(EQV\) is the coefficient of variation for the Nikkei Stock Average 225 index (\(y, m\) and \(p\) are in logarithms). In the same table, \(SE\) represents the standard error of \(\hat{\alpha}'\) in equation (29), \(HCSE\) is the heteroskedasticity-consistent standard error developed by White (1980),\(^{36}\) and \(\hat{\alpha}'/SE\) and \(\hat{\alpha}'/HCSE\) show t-values derived from the two standard errors. The * symbol indicates that the null hypothesis was rejected at the 5% level of significance.

In Table 2, the null hypothesis of the unit root in the real GNP is rejected when \(SE\) is used. If this reflects the heteroskedasticity in residuals, however, the test would prove inconclusive. The interest rate and the coefficients of variation for the stock prices are both regarded as \(I(1)\) processes in Table 2, although it seems fair to treat them as mean-stationary, especially over a long span such as 100 years. Presumably, they appear non-stationary due to the relatively short sample period of this study, as well as the low power of the ADF test itself.

**Table 2.**

<table>
<thead>
<tr>
<th></th>
<th>(\hat{\alpha}')</th>
<th>SE</th>
<th>HCSE</th>
<th>(\hat{\alpha}'/SE)</th>
<th>(\hat{\alpha}'/HCSE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>-0.138</td>
<td>0.031</td>
<td>0.052</td>
<td>-4.51*</td>
<td>-2.65</td>
</tr>
<tr>
<td>(m)</td>
<td>-0.007</td>
<td>0.004</td>
<td>0.004</td>
<td>-1.89</td>
<td>-2.10</td>
</tr>
<tr>
<td>(p)</td>
<td>-0.005</td>
<td>0.008</td>
<td>0.007</td>
<td>-0.65</td>
<td>-0.72</td>
</tr>
<tr>
<td>(R)</td>
<td>-0.090</td>
<td>0.036</td>
<td>0.034</td>
<td>-2.51</td>
<td>-2.68</td>
</tr>
<tr>
<td>(EQV)</td>
<td>-0.421</td>
<td>0.171</td>
<td>0.166</td>
<td>-2.46</td>
<td>-2.54</td>
</tr>
</tbody>
</table>

Critical value at 5% level of significance: -3.45 (n=100)

SE = standard error
HCSE = heteroskedasticity-consistent standard error
n = sample size
* indicates \(H_0: \alpha' = 0\) is rejected at 5% level of significance.

\(^{36}\) Since standard error (SE) calculations usually assume residual homoscedasticity, they tend to be biased when the residuals exhibit heteroscedasticity. For this reason, heteroscedasticity-consistent standard error (HCSE) is a closer approximation when the residuals exhibit heteroscedasticity. Recently, most computer programs for regression analysis calculate the values of both SE and HCSE.
As noted in the preceding section, the results of ADF tests should not be regarded as final. Nevertheless, as long as there is a possibility that the real GNP, money supply or other variables are non-stationary, it is better to exclude their levels from a regression except in special circumstances (the cointegrating regression in the previous section falls in this category).\(^{37}\)

B. Estimating the conventional money demand function

In studying Japan’s money demand functions, we estimated both standard (“conventional” hereafter) and the ECM type.

The results of the conventional money demand function are shown in (30). The sample period is from the first quarter of 1968 to the first quarter of 1989.

\[
(m-p)_t = -0.51 + 0.13y_t - 0.008R_t + 0.89(m-p)_{t-1}
\]

\[
\begin{align*}
(2.63) & \quad (2.41) & \quad (-5.55) & \quad (23.7) \\
R^2 = 0.999 & \quad \sigma = 1.2\% & \quad DW = 0.54
\end{align*}
\]

LM1-5F(5,76)\(^{38}\) = 23.4

In (30), \((m-p)\) is the balance of the real money supply (\(M_2+CD\) divided by the GNP deflator), \(y\) is the real GNP, \(R\) is the coupon rate of 5-year bank debentures. Figure 5, which plots the actual and fitted values, and the coefficient of determination (\(R^2\)) indicates that (30) has an excellent fit. The DW ratio and the LM test, however, both point to a residual auto-correlation in (30), casting serious doubt on the overall credibility of the regression.\(^{39}\)

For ease of comparison with the ECM money demand function, (30) was reestimated with the simple modification of deducting \((m-p)_{t-1}\) from both sides of (30) so that

\(^{37}\) Taking first differences of \(I(0)\) variables turns them into \(I(-1)\) variables. Since the differencing is unnecessary, such a conversion is called “over-differencing.” Yet, even with this conversion, the \(I(-1)\) variable and \(I(0)\) variable are both stationary processes. As such, if the \(I(-1)\) variable is incorporated in a regression, it poses no danger of a spurious regression, unlike the \(I(1)\) variable. Noting this advantage, Flosser and Schwert (1978) advocated using the first difference form whenever a variable cannot be identified as \(I(0)\) or \(I(1)\).

\(^{38}\) LM1-nF is a Lagrange multiplier test to check for a 1st-to-nth-order residual correlation. Developed by Godfrey (1978), this test is superior to the DW test in that 1) it can be used even when lagged dependent variables are included in the explanatory variables, and that 2) it can check for a second or higher order residual correlation. In this test, the statistics follow an \(F\) distribution with \((n, T-k-n)\) degree of freedom under the null hypothesis of no residual correlation (\(T\) is the sample size and \(k\) is the number of explanatory variables). When \(n=5\), LM1-5F yielding a result higher than 2.6 indicates a residual correlation at the 5% level of significance. The statistics for this LM test were originally designed to follow an \(\chi^2\) distribution, but it became clear that an \(F\) distribution was more reliable with a small sample. See Kiviet (1986).

\(^{39}\) In a non-AR regression, with no lagged dependent variables included in the explanatory variables, estimates are generally consistent even if the residuals are serially correlated. With an AR regression such as (30), however, a residual auto-correlation invariably leads to biased estimates. This point has been emphasized by Malinvaud (1980) and Harvey (1981).
Figure 5.

![Graph showing actual and fitted values of (m-p) over a period from 1970 to 1990. The sample period is 1968/1 to 1989/1.]

Figure 6.

![Graph showing the actual and fitted values of Δ(m-p) over a period from 1970 to 1990. The sample period is 1968/1 to 1989/1.]

Sample period: 1968/1 – 89/1
the dependent variable becomes Δ(m−p)_t.  

It is worth noting that Δ(m−p)_t in this set-up expresses the difference between logarithms; hence, it is approximately equal to the quarterly growth rate of the real money demand.

\[
\Delta (m-p)_t = -0.51 + 0.13y_t - 0.008R_t - 0.11 (m-p)_{t-1} \\
(-1.63) (2.41) (-5.55) (-2.85)
\]

\[ R^2=0.297 \quad \hat{\sigma}=1.2\% \quad DW=0.54 \]

LM1-5F(5,76)=23.4

As expected, all the coefficients in (31) are the same as those in (30) except that of (m−p)_{t-1}, which is 1 less than the coefficient in (30). Although the results of diagnostic tests remain unchanged, the coefficient of determination shows a marked decline from 0.999 to 0.298. Moreover, the fit in Figure 6, which plots actual and fitted values from (31), is greatly inferior to that in Figure 5, a change that is consistent with the marked decline in the coefficient of determination. Behind this dramatic change lies the susceptibility of the coefficients of determination and the graphs to changes in parameterisation. To some extent, they are useful in comparing two or more regression results, but are of less help in evaluating single regression results. In these cases, \( \hat{\sigma} \) (standard error of regression), which is immune to parameterisation changes, is more effective.

**C. Estimating the money demand function with ECM**

1. **Cointegrating regression and sub-sample results**

   The next step is estimating the ECM type function with the Engle-Granger two-step method. First, we ran the following cointegrating regression:

\[
(m-p)_t = 1.40y_t - 0.03R_t + 0.003EQV_t - 7.18.
\]

An ADF test was then applied to the residuals from (32) and the null hypothesis that “\( u_t \) is a random walk” was rejected at the 5% level, indicating that the real money balance

---

40 This regression, a mathematically equivalent transformation of (30), is included only for its expediency. While it has no significance in itself, it has dependent variables identical to the ECM money demand function that will be described later. Since (31) expresses the dependent variable as flow and all explanatory variables as stock (or level), such a regression must look unusual at first glance. If (31) has any significance, it is as an indication of the extent to which an increase in real money demand can be forecast using the same data (explanatory variables), as in (30).

41 The extremely high coefficient of determination in (30) is thought to lie in the fact that, although (30) is theoretically called a partial adjustment, it is essentially a model so parameterized that it has an AR process.

42 R_t and \( EQV_t \), both thought to be I(0), are included in this cointegrating regression following the suggestion by Hendry (1986), who recommends the including I(0) or stationary variables in the first-stage cointegrating regression. It may be of interest that a regression without \( R_t \) and \( EQV_t \) yielded a \( \gamma \) coefficient of 1.44, showing no major difference in the result.

43 When variables such as m, p, and y, are not trend-stationary processes, the figures for \( R^2 \) and t-value do not follow their usual distribution. These figures have, therefore, been excluded from (32).
(m−p) and real income (y) are cointegrated. Since this result justified using ECM to estimate Japan’s money demand function, we used EC=m−p−1.4y as the error correction term in the model.\(^4^\)\(^5^\)\(^6^\)

As in the second step, an ECM money demand function for Japan’s M\(_2\)+CD was estimated. Following the hypothesis proposed by Ueda (1988), we assumed a money demand function:

\[
M=M(Y, R, V) \\
\forall M/\forall Y>0, \forall M/\forall R<0, \forall M/\forall V>0
\]

\((Y\ is\ income, \ R\ is\ interest\ rates\ and\ V\ is\ stock\ price\ volatility)\)

The relationship between risky assets and money—in this case, higher stock price volatility increases the demand for risk-free deposit—was factored in explicitly. The sample period is from the first quarter of 1968 to the third quarter of 1985; the data for the remaining three years, from the fourth quarter of 1985 to the first quarter of 1989, were reserved for out-of-sample simulation. The result of this sub-sample estimation is as follows:\(^4^\)

\[
\Delta(m-p)_t = -0.33 + 1.17 \Delta y_{t-1} + 0.79 EC_{t-1} - 0.84 EC_{t-2} - 0.18 \Delta p_t
\]

\((-2.97) \quad (14.6) \quad (13.1) \quad (−13.9) \quad (−2.61)\)

\[
-0.82 \Delta^2 p_t - 0.007 \Delta R_{t-1} + 0.002 \left(\frac{1}{4} \sum_{i=1}^{4} EQV_{t-i}\right)
\]

\((-9.28) \quad (-3.60) \quad (3.04)\)

\[R^2=0.937 \quad \hat{\sigma}=0.41\% \quad DW=2.05\]

\[LM1-5F(5,58)=0.87 \quad Normality \chi^2(2)=29.56\]

\(^4^\) The ADF test is given in (26) in Section IV. This test on residual u\(_t\) in (32) produced \(\hat{\phi}=-0.23\) and \(t=-3.80\). Since the \(t\)-value is lower than the 5% level of significance (−3.17 with a sample of 100) obtained by Engle and Granger’s (1987) Monte Carlo experiment, \(u_t\) is considered stationary.

\(^5^\) It is also possible to use \(u_t\) from (32) as the EC—i.e., including \(R_t\) and \(EQV_t\) in EC instead of making \(EC=m−p−1.4y\). Ueda (1988) adopted this approach in the ECM model in his supplement. This paper assumes, however, that the two variables of \((m−p)\), and \(y\), are cointegrated. The rationale is two-fold. First, when \(R_t\) and \(EQV_t\) are included in EC, a theoretically persuasive explanation becomes increasingly difficult as the number of cointegrated economic variables increases. Second, \(R_t\) and \(EQV_t\) should be seen rather as I(0) regardless of the results of the earlier ADF test.

\(^6^\) The stationarity of EC was not rejected by the ADF test.

\(^4^\) In estimating the model, we followed the general-to-specific modeling approach described in Section III. First, a general model including up to fourth lags of variable

\[m_t = \sum_{i=1}^4 a_i m_{t-i} + \sum_{j=0}^4 (b_i p_{t-j} + c_i y_{t-j} + d_i R_{t-j} + e_i EQV_{t-j}) + \text{constant}\]

was estimated. Then the repeated reparameterization and elimination of insignificant explanatory variables yielded (34). Since \(\hat{\sigma}\) in the general model was 0.39%, the simplification of reducing the explanatory variables from 25 to 8 increased \(\hat{\sigma}\) by only a minuscule 0.02 percentage point.

\(^4^\) Normality \(\chi^2\) is a test developed by Jarque and Bera (1980) to check the normality of residuals. The statistics follow an \(\chi^2\) distribution with two degrees of freedom under the null hypothesis of a normal distribution. If Normality \(\chi^2\) exceeds 6, therefore, the normality of the residuals is rejected. In (34), the normality of the residuals is questioned, casting some doubt on the reliability of the \(t\)-values.
\[ x^2 F(14, 48)^{49} = 1.13 \]
\[ CH(14, 63)^{50} = 0.55 \]

In this result, \( \Delta (m-p)_t \) shows the rate of increase in the real money demand, explanatory variable \( \Delta y_{t-1} \) is the growth rate of the real GNP in the preceding period, \( EC_{t-1} \) and \( EC_{t-2} \) are both errors in the periods \( t-1 \) and \( t-2 \) respectively. \( \Delta p_t \) is the rate of inflation measured by the GNP deflator in the current period, \( \Delta^2 p_t \) is the change (or acceleration) in the rate of inflation during the current period, \( \Delta R_{t-1} \) is the change in the coupon rate for 5-year bank debentures in the preceding period, \( \frac{1}{4} \sum_{t=1}^{4} EQV_{t-1} \) is the 4-quarter moving average of the coefficient of variation for the Nikkei Stock Average 225 index in the preceding period. All data were seasonally adjusted or converted to logarithms except the interest rate and the coefficients of variation in stock prices.

The signs of the coefficients in (34) show that real GNP growth has a positive effect on real money demand; that error correction terms, if taken together, have a small but negative effect\(^{51}\); that the level and the acceleration of inflation rate have a negative effect\(^{52}\), that an increase in interest rates has a negative effect; and that an increase in stock price volatility has a positive effect.

All of these results seem to bear out the suggestion of economic theory. Overall, the

\(^{49}\) \( x^2 F \) is a test statistic for residual heteroscedasticity developed by White (1980). It follows an F distribution with \( (2k-2, T-3k+1) \) degree of freedom under the null hypothesis of homoscedasticity (\( T \) is the sample size and \( k \) is the number of explanatory variables). In (34), since the degree of freedom is \( 14, 48 \), homoscedasticity would have been rejected if \( x^2 > 1.9 \).

\(^{50}\) CH (Chow test for parameter constancy over the forecast period) tests for significant differences between the residual variance during the sample period and the variance of forecast errors in the forecast period. It uses the method developed by Chow (1960). With sample size \( T \), the number of explanatory variables \( k \) and the length of out-of-sample period \( n \), the test statistic CH follows an F distribution with \( (n, T-k) \) degree of freedom under the null hypothesis of no structural change. When \( n=12 \), therefore, the stability of parameters during the out-of-sample period would have been rejected if CH>2.

\(^{51}\) Professor Yoshihisa Baba of Soka University suggested the following transformation for these two error correction terms:

\[ 0.79EC_{t-1} - 0.84EC_{t-2} = 0.79(\Delta (m-p)_{t-1} - 1.4\Delta y_{t-1}) - 0.05EC_{t-2}. \]

This transforms (34) into a specification incorporating the first-order AR process of the dependent variable. Since models including the AR process are difficult to interpret theoretically, this paper uses a specification with past errors corrected by intertwining two ECs as in (34). Models with multiple ECs have been studied theoretically by Hendry, Pagan and Sargan (1984) and Banerjee, Galbraith and Dolado (1988) as well as used in empirical studies by Patterson (1987).

\(^{52}\) The effect of the level and the acceleration of inflation rate can be interpreted in two ways. First, since people cannot foresee the future rate of inflation in the short run, they can only adjust their nominal—not their real—money holdings. In other words, inflation erodes real money holdings. This interpretation is close to the theory of nominal partial adjustment. (For the difference between real adjustment and nominal adjustment, see Cuthbertson (1985), for example.) Second, a more optimistic interpretation holds that, since the rate of inflation is an opportunity cost for money holdings, people try to reduce their money holdings as inflation increases. Unfortunately, the model offers no basis for determining which of these two interpretations is closer to the facts.
ECM money demand function (34) has a better fit than the conventional money demand function (31), both in terms of $R^2$ and $\hat{\sigma}$. Moreover, the LM test shows no sign of residual autocorrelation.

Figure 7, which plots the actual and fitted values in (34), shows that the fit of the ECM function is substantially better than the conventional one shown in Figure 6. Figure 8 shows the result of a 1-step-ahead forecast based on (34) along with the actual values from the fourth quarter of 1985 to the first quarter of 1989.\(^\text{53}\) Throughout the forecast period, actual values fall well within the 95% confidence intervals, showing no indication of the "missing money" in this ECM function even during the forecast period. When a Chow test was also conducted to see if there were any structural changes during the forecast period, the test statistic CH was low enough to indicate none had occurred.

2. Forecast simulation for the increase in the growth rate of the nominal money supply

Since the Bank of Japan releases its quarterly forecasts for the year-on-year increase rate in nominal money supply, we modified Figure 8 to show the out-of-sample forecast from (34) on a nominal and a year-on-year basis. The results in Figure 9 indicate that forecast errors were within $\pm 1\%$.

3. Full-sample estimation with recursive least squares

Next, we extended the sample period to the first quarter of 1989 and reestimated the function with the recursive least squares (RLS) method. The result is shown in (35).

$$
\Delta(m-p)_t = -0.30 + 1.17 \Delta y_{t-1} + 0.80 \Delta c_{t-1} + 0.84 \Delta c_{t-2} - 0.19 \Delta p_t
$$

$$
\begin{align*}
\Delta^2 p_t & = -0.80 \Delta^2 p_t - 0.006 \Delta R_{t-1} + 0.002 \sum_{i=1}^{4} \Delta QV_{t-i} \\
& \text{(AR1)} & \text{DW} = 2.08 \\
& \text{R}_2 = 0.933 & \hat{\sigma} = 0.39\% \\
& \text{LM1-5F(5.72)} = 1.14 & \text{Normality } \chi^2(2) = 38.99 \\
& \text{x}^2 \text{F(14,62)} = 1.20
\end{align*}
$$

While the ordinary least squares (OLS) method would yield the same results, the special algorithm of RLS can be used to check, how the estimated coefficients change each time new data becomes available.\(^\text{54}\) To be precise, the initialization period for RLS was set as the first quarter of 1968 to the fourth quarter of 1975; then the data were added up sequentially to the first quarter of 1989. To detect structural changes, recursive residuals were used in a sequential Chow test.\(^\text{55}\)

\(^{53}\) One-step ahead forecast estimates next quarter's values based on the reported values up to the previous quarter. It therefore has the advantage that no forecast errors accumulate in the process.

\(^{54}\) For the algorithm of the recursive least squares method, see Kmenta (1986).

\(^{55}\) For sequential Chow test in which recursive residuals are used, see Harvey (1976, 1981).
Figure 7.

$\Delta (m-p)$

Actual value ---
Fitted value ---
Forecast ---

Figures for the quarters after 1985/IV are forecast values.

Sample period: 1968/I - 89/I

Figure 8.

$\Delta (m-p)$

Actual value ---
Fitted value ---

95% confidence interval

Forecast period: 1985/IV - 89/I
The estimated coefficients in (35) are only slightly different from those in (34), which corroborates the satisfactory result from the earlier out-of-sample simulation.

Figure 10 plots the change in the coefficient of EC_{t-1} (on the vertical axis) as the end of the sample period (on the horizontal axis) was extended sequentially. A glance shows that the coefficient of EC_{t-1} remains stable between 0.75 and 0.80. When the coefficients of other explanatory variables (35) (not listed here) were plotted in the same way, they showed similar stability.

The result of the sequential Chow test is shown in Figure 11, with all the test statistics rescaled so that 5% critical values become a straight line at unity. In this setup, a test statistic above the horizontal line indicates a structural change.

In Figure 11 no adjusted test statistic exceeds 1.0. Given this evidence, along with the good fit in Figure 7, it is fair to conclude that Japan's money demand function has remained relatively stable since 1976 according to an ECM function.

Generally speaking, the instability of estimated coefficients arises from misspecification in the model or variations in DGP. In either case, unstable coefficients inevitably prompt questions about the validity of the model. In (35), the stability in the coefficients would lead us to conclude that Japan's money demand function can be explained with ECM.

Until recently, empirical studies on Japan's money demand function have been based on the standard function, and structural changes found in the regression have been used to bolster the argument that there was a shift in money demand and that at least part...
Figure 10. Changes in the Value of the Error Correction Term

- $EC_{t-1}$,  
- $EC_{t-1} \pm 2 \times$ standard error

Sample period: 1975/IV – 89/1

Figure 11. Sequential Chow Test Results

- critical value at 5% level of significance

Sample period: 1976/I – 89/I
of it was due to financial deregulation. The results in this section—demonstrating that a stable money demand function can be obtained by making the models' lag structure more flexible—cast serious doubts on this widely accepted view. In other words, it may be that the standard money demand function cannot adequately explain movements only because of misspecification—not because of actual shifts in the money demand function itself as many researchers have maintained. As noted in Section III, the preoccupation with dynamic theory may be responsible for the oversimplification of the models' lag structure in the standard function.

D. **Summary of empirical findings**

The foregoing empirical findings lead to three conclusions.

1. The time series property of data, such as the real GNP money supply and GNP deflator commonly used in estimating the money demand function, is more likely to be non-stationary than trend-stationary as has generally been assumed.

2. According to the coefficient of determination and graphs, the conventional money demand function would seem to indicate an excellent fit. In fact, it is a spurious conclusion arising from the parameterisation incumbent in an autoregression process. This fallacy is confirmed by a slight change in the model's parameterisation. Moreover, the conventional money demand function exhibits a strong residual correlation, pointing to the unreliability of estimated results.

3. A comparison with \( \hat{\sigma} \) (standard error of regression) shows that the ECM money demand function has a far better fit than that of a conventional function. An out-of-sample simulation for three years produced satisfactory results. Moreover, a sequential Chow test revealed that the ECM money demand function has remained stable since 1976 in contrary to the prevailing insistence on shifts in the money demand function or "missing money."

VI. **Suggestions for Future Study**

In an effort to shed light on problems with the conventional money demand function, this paper has drawn on econometric analysis and proposed the alternative of an ECM money demand function. To this purpose, empirical studies here have used only basic explanatory variables such as the real GNP, GNP deflator and interest rates, with the exception of the coefficient of variation in the stock prices. This was necessary to demonstrate that the ECM can produce a stable money demand function with relatively simple explanatory variables.

This model gives only scant attention to the effects of financial liberalization on money demand, currently a subject of intense interest among many researchers. This is because the ECM money demand function here lacks explanatory variables that can measure the impact of financial deregulation. Although the Chow test did not reveal
structural changes, one should not necessarily conclude that financial deregulation has had no impact on money demand in Japan since the primary purpose of the test is to check the stability and reliability of an empirical model. It is simply beyond the scope of the test to access the effects of variables excluded from the model. The question, then, of how the financial deregulation in Japan has affected, or will affect, money demand remains an important and interesting one from both theoretical and empirical perspectives. In this sense, the estimation results of this paper should be seen as a springboard for further empirical studies on Japan's money demand function.

We assume that the value of k is constant in considering cointegration between real money demand and real income. The relatively satisfactory fit of the estimated money demand function in Section V does not seem to pose any serious challenge to this assumption. In some cases—for example, with a very long sample period—a varying-parameter model with a fluid k value may produce better results.

Much is needed to extend the fields and applications of ECM. As Section III indicates, ECM has been used mainly in research on the money demand and consumption functions. Applications to the models for multiple asset pricing or the term structure of interest rates, for example, seem to be promising. For time series analysts, who place little emphasis on consistency with economic theory, ECM seems to have far wider applications (although their research will doubtlessly prompt criticisms of "measurement without theory" by those who stress the importance of a theoretical framework).

In Japan, there have been almost no empirical studies with ECM, and we can only hope that substantial work will be undertaken in the near future. Of particular interest is investigating whether the ECM consumption function, which has achieved considerable success overseas, will prove equally relevant here. As noted earlier, the Bank of England has already adopted ECM in the consumption block of its macroeconomic models. It would be appropriate to follow this example in attempting to incorporate ECM into a portion of large-scale macroeconomic models.

Appendix. Empirical Analysis of ECM-Type Money Demand Functions: Examples in Britain and the U.S.

A. The M₁ money demand function in Britain

This Appendix reviews empirical studies of ECM-type money demand functions for M₁s in Britain and the United States.

Research on the ECM-type money demand function for Britain’s M₁ was initiated by Hendry (1979). His findings were followed by a series of studies by other researchers including Trundle (1982), Hendry and Ericsson (1983). In what follows we review the work of Hendry (1985), a relatively recent example of these studies.

Hendry (1985) reports the reestimated results of his earlier empirical study of the ECM-type money demand function (Hendry 1979) with only the slight difference of
extending the sample period by approximately five years. (A-1) is the result for the initial sample period (from the first quarter of 1963 to the fourth quarter of 1977) and (A-2) is the result for the extended sample period ending in the fourth quarter of 1982. In both regressions, \( \Delta (m-p) \) shows the seasonally adjusted rate of change in the real money balance \((M_1 \text{divided by GNP deflator})\), \( \Delta y \) is the seasonally adjusted growth rate in GNP, \( R \) is the 3-month local authority rate, \( \Delta p \) is the seasonally adjusted rate of change in the GNP deflator, \((m-p-y)\) is the error correction term.\(^{56}\)

(a) Sample period: 1963/I~1977/IV

\[
\frac{\Delta (m-p)_t}{\Delta y_t} = 0.40 \quad \frac{\Delta y_t}{0.52} \quad R_t - 0.86 \quad \frac{\Delta p_t}{0.11} \quad (m-p-y)_{t-2}
\]

\[
(2.50) \quad (4.73) \quad (5.06) \quad (5.50)
\]

\[
- 0.26 \quad \Delta (m-p)_{t-1} + 0.040
\]

\[
(2.89) \quad (6.67)
\]

\[R^2 = 0.62 \quad \hat{\sigma} = 1.5\% \quad \text{LM1-3F(3,65)=0.3}\]

(b) Sample period: 1963/I~1982/IV

\[
\frac{\Delta (m-p)_t}{\Delta y_t} = 0.37 \quad \frac{\Delta y_t}{0.58} \quad R_t - 0.80 \quad \frac{\Delta p_t}{0.10} \quad (m-p-y)_{t-2}
\]

\[
(2.84) \quad (8.28) \quad (6.66) \quad (10.0)
\]

\[
- 0.28 \quad \Delta (m-p)_{t-1} + 0.041
\]

\[
(4.00) \quad (8.20)
\]

\[R^2 = 0.71 \quad \hat{\sigma} = 1.3\% \quad \text{LM1-4F(4,65)=0.3}\]

A comparison of (A-1) and (A-2) shows that the estimated coefficients for each explanatory variable hardly differ even though the sample period was lengthened. Working from this observation, Hendry (1985) concluded that the ECM-type money demand function for Britain’s \( M_1 \) has remained stable for nearly 20 years, little affected by the marked shift in the Thatcher government’s monetary policy of emphasizing management of the money supply.

B. The money demand function in \( M_1 \) in the United States

The United States’ \( M_1 \) is generally thought to lack a stable money demand function since the introduction of the NOW account (started in 1972 and expanded to all the states by 1980) and the introduction of the Super NOW account (1983), which were believed to have produced a wide swing in the demand for money. Baba, Hendry and Starr (1988) challenged this view, arguing that a stable money demand function can be obtained even for the United States’ \( M_1 \) if we use relatively complex ECM equation (A-3), which explicitly incorporates the effect on the demand for \( M_1 \) from other financial assets (long-term bonds, TBs and high interest monetary instruments included in \( M_2 \), etc.).

\(^{56}\) Since Hendry’s (1979, 1985) studies preceded the Engle-Granger two-step method, the ECM was estimated with the restriction of \( k=1 \). In the ECM of Baba, Hendry and Starr (1988), on the other hand, estimation was conducted with the restriction of \( k=0.5 \).
showing that various kinds of Chow Tests on (A-3) revealed no structural changes during the sample period (from the second quarter of 1960 to the second quarter of 1984), Baba, Hendry and Starr (1988) concluded that “For this money demand function, the missing money was never gone.”

Baba, Hendry and Starr (1988) argued, moreover, that the failure to incorporate the effect of the “volatility of long-term bond yields” in the conventional studies on money demand, together with the excessive restrictions imposed on the models’ lag structures, led to the misguided notion of missing money. During the period of the missing money —i.e., from 1973 to 1976—the volatility of long-term bond yields stayed at relatively low levels historically. This, they argued, accounted for the increase in the demand for long-term bonds and the attendant decline in the demand for deposit money. By the same token, they held that the markedly increased volatility after 1980 led to the Great Velocity Decline in 1982 and 1983.

$$\Delta (m-p)_t = -0.337 - 0.243 \Delta (m-p)_{t-4} - 0.141 (m-p - \frac{1}{2} y)_{t-2} - 1.749 AS_t$$

$$\quad (-9.49) \quad (-4.90) \quad (-9.86) \quad (-10.1)$$

$$\quad -0.662 Asz_t - 0.889 \text{rmz}_t - 0.744 \Delta p_t + 0.338 \Delta Ay_t + 0.00439 \text{AVA}_{t-3}$$

$$\quad (-5.81) \quad (-10.3) \quad (-7.8) \quad (6.15) \quad (6.06)$$

$$\quad + 0.155 S* \text{AVA}_{t-1} + 0.276 \text{rmocz}_t + 0.461 \Delta \text{rmoc}_t + 0.013 D_t$$

$$\quad (6.65) \quad (2.72) \quad (3.93) \quad (3.93)$$

$$\text{R}^2 = 0.858 \quad \hat{\sigma} = 0.378\% \quad \text{LM1-4F(4,76)} = 1.57$$

The variables are described below.

\(\Delta (m-p)\): Quarterly change in the real \(M_tB\) (in logarithm and seasonally adjusted)

\(m-p-\frac{1}{2} y\): Error correction term

\(AS\): 3-quarter moving average of the difference between long and short-term interest rates (yield on 20-year T-Bond - yield on 1-year TB)

\(Asz\): Difference between the yield on 1-year TB and \(\text{rmz}\) (2-quarter moving average)

\(\text{rmz}\): The highest interest rate among \(M_z\) instruments. (Since new products need time to gain widespread consumer acceptance, adjustments for new products are made by applying discount rates starting from 100% at the point of introduction and gradually declining to 0% in the following years.)

\(\Delta p\): Quarterly change in the GNP deflator (in logarithm and seasonally adjusted)

\(\Delta Ay\): Changes in the weighted-average of \(\ln(\text{GNP})\)s for the current and the preceding quarter at a ratio of 2:1

\(\text{AVA}\): Variance of long-term government bond yield (6-quarter moving average)

\(S*\text{AVA}\): The differences between short-term and long-term interest rates multiplied by \(\text{AVA}\) when short-term and long-term interest rates are reversed (i.e., yield on 20-year Treasury Bond < yield on 1-year Treasury Bill). Otherwise,
S*AVA is equal to zero

$\text{rmocz}$: The highest interest rate among $M_1$ deposits (the same adjustment was made as the case with $rmz$)

$\Delta \text{rmocz}$: Quarterly changes in the $\text{rmocz}$

$D$: Credit control dummy (for the second quarter of 1980 = $-1$, the third quarter of 1980 = $+1$)

REFERENCES


