A Theory of Loan Rate Determination in Japan*

MASAHIKO TAKEDA**

I. Introduction

This paper attempts to reinterpret the changes in loan rates that have taken place since the 1960s from a new viewpoint regarding the mechanism of loan rate determination.

Many observers have up to now pointed out that Japanese loan rates are strongly correlated with the official discount rate and deposit rates (hereafter called "institutional" rates in the sense that they have been regulated either by law or informal agreements). However, existing theories of loan rate determination, most of which are based on profit-maximizing behavior of banks, have failed to provide a consistent explanation for this phenomenon.

Therefore, in this paper, we introduce an objective function of a bank which includes the maximization of short-run profits as a special case, and which at the same time takes into account business-scale variables (deposit balance, loan balance etc.). It is shown that when the bank attempts to maximize such an objective function, changes in the official discount rate and deposit rates have a direct impact on the supply of bank loans, thus altering the equilibrium loan rates determined in the market. With such a mechanism, it becomes possible to offer an explanation for the

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correlation between loan rates and "institutional" rates from the perspective of equilibrium theory. It also offers a theoretical interpretation to the recent emergence of so-called "spread banking" and to arguments put forth lately by commercial banks that the present prime rate system which fastens loan rates (more precisely, face loan rates as opposed to effective loan rates) to the official discount rate needs to be replaced by a new system which reflects the changes in money market rates.

In Section II below, we look into the actual series of loan rates and describe their features. Then we conduct a brief survey on past theories of loan rate determination to see how they have explained these features. In Section III, we examine the "objective function of the bank," which forms the basis for the model discussed in this paper. Thereafter, in Section IV, the loan supply schedule of a representative bank is derived. In Section V, the loan supply schedule of the banking sector as a whole is put into a simplified framework of market equilibrium, and the nature of the equilibrium effective loan rates determined therein is examined. On the basis of these analyses, Section VI provides a reinterpretation of the developments of loan rates.

The following is a summary of the results obtained in this paper:
(a) Loan rates in Japan, both face rates and effective rates, show a strong correlation with "institutional" rates. Especially since 1973, the range of fluctuations in loan rates expanded in tandem with the expansion of fluctuations in the official discount rate.
(b) This phenomenon has often been interpreted as the result of institutional restrictions, such as the system of voluntary interest rate restrictions (prime rate system) adopted by the Federation of Bankers Associations of Japan. However, such an interpretation is not persuasive since it assumes away the ability of economic agents to adjust effective rates in order to avoid institutional restrictions on face rates. Alternative theories on loan rate determination that have been proposed also fail to explain consistently and comprehensively the actual loan rate series since the 1960s.
(c) In order to interpret the relationship between loan rates and "institutional" rates as a natural outcome of market forces, we assume that banks seek to increase not only their current profits but also their business scale. From this assumption a mechanism is deduced whereby a change in the "institutional" rates exerts direct influence on the loan supply of banks. If this mechanism actually exists, it becomes possible to interpret the expanded fluctuations in loan rates since 1973 as a reflection of the fact that deposit rates started to fluctuate at that time.
(d) This mechanism has recently begun to weaken as a result of the declining ratio of "institutional" liabilities in the bank balance sheet (interest-bearing deposits other than CDs and borrowings from the Bank of Japan) and in its place, the impact of money market rates on effective loan rates has become more important.
(e) This recent development undermines the viability of the existing prime rate system under which face loan rates are tied to "institutional" rates. The proposed
revision of the system and the increased loans in the form of "spread banking" are consistent with our interpretation of this development.

II. Developments in Loan Rates and a Review of Theories of Loan Rate Determination

1. Developments in Loan Rates

   We start our study by looking into the actual series of four types of interest rates during the period between the first quarter of 1960 and the third quarter of 1984 (see Figure 1): loan rates (average short-term loan rates reported by the member banks of the Federation of Bankers Associations of Japan\(^1\)), interbank rates as the representative money market rate (weighted average of call/bills rates; prior to the second quarter of 1971, call rates), official discount rate and deposit rate (one-year time deposit). We can characterize their movements as follows:
   (a) Loan rates fluctuated only slightly in the 1960s, but in the 1970s (especially when the tight money policy commenced just prior to the first oil crisis) the range of fluctuation expanded.
   (b) Interbank rates fluctuated widely throughout the entire period.
   (c) The official discount rate fluctuated only slightly during the 1960s, but significantly following 1973.
   (d) Deposit rates hardly fluctuated in the 1960s, but they, similar to the official discount rate, fluctuated considerably after 1973.

   Thus, it is clear that, except for interbank rates, a change occurred in the fluctuation patterns of various interest rates with the advent of the tight money policy in 1973 as the turning point (see also, the variation coefficients shown in Table 1). The large fluctuations in the official discount rate and deposit rates after 1973 are attributable to a change in the policy stance of the monetary authorities. What is really noteworthy is the fact that the range of fluctuations in loan rates also expanded simultaneously with those rates.

   Let us next examine simple correlation coefficients between interest rates. Taking into account the changes in the fluctuation pattern after 1973, we divide the period concerned into the first half (the first quarter of 1960 – the first quarter of 1973) and the second half (the second quarter of 1973 – the third quarter of 1984). From Table 1, the following three features stand out:

1. In this paper, all references to "loan rates" are to the average short-term loan rates. The interest rate distribution that reflects, among other things, the difference in the degree of riskiness of the borrowers is ignored for simplicity. In other words, the bank loan market here is regarded as homogeneous and characterized by perfect competition.
Figure 1  Movements in Various Interest Rates  
(1960/I ~ 84/III)

- Money market rate (Call rate and bills rate, weighted average)  
- Loan rate (Average short-term loan rate)  
- Official discount rate  
- Deposit rate (1-year time deposit rate)

Date range: 1960 to 1984
### Table 1  Variation and Correlation of Various Interest Rates

<table>
<thead>
<tr>
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<th>60/I ~ 73/I</th>
<th>73/II ~ 84/III</th>
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<tbody>
<tr>
<td></td>
<td>Average Standard deviation Variation coefficient Average Standard deviation Variation coefficient</td>
<td></td>
</tr>
<tr>
<td>$r_l$</td>
<td>7.14 0.44 0.06</td>
<td>7.10 1.42 0.20</td>
</tr>
<tr>
<td>$r_c$</td>
<td>8.35 2.13 0.26</td>
<td>8.02 2.68 0.33</td>
</tr>
<tr>
<td>$r_n$</td>
<td>6.01 0.79 0.13</td>
<td>6.16 1.63 0.26</td>
</tr>
<tr>
<td>$r_d$</td>
<td>5.58 0.18 0.03</td>
<td>6.22 0.95 0.15</td>
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</tbody>
</table>

### (Correlation)

<table>
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<tr>
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<th>60/I ~ 73/I</th>
<th>73/II ~ 84/III</th>
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<tbody>
<tr>
<td></td>
<td>$r_c$</td>
<td>$r_n$</td>
</tr>
<tr>
<td>$r_l$</td>
<td>0.776</td>
<td>0.935</td>
</tr>
<tr>
<td>$r_c$</td>
<td></td>
<td>0.827</td>
</tr>
<tr>
<td>$r_n$</td>
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Notes:  
- $r_l$: Average short-term loan rate (face rate)  
- $r_c$: Call rate and bills rate weighted average  
- $r_n$: Official discount rate  
- $r_d$: One-year time deposit rate

(a) The correlation coefficient between loan rates and the official discount rate is high throughout the period.  
(b) The correlation coefficient between loan rates and deposit rates is low in the first half, reflecting the fact that deposit rates were held almost constant; however, it rises significantly in the second half.  
(c) The correlation coefficient between loan rates and interbank rates is high, but not as high as those between loan rates and the official discount rate and between loan rates and deposit rates (in the second half).

In analyzing loan rates, it is important to pay attention not only to the face rates but also to the effective rates that take into account deposits held by borrowers (hereafter called debtors’ deposits). Unfortunately, however, there are no accurate data concerning debtors’ deposit ratio (debtors’ deposits/loans) and the interest rate composition of the debtors’ deposits. Therefore, we estimate the effective loan rates...
on several assumptions. The estimation result shows that, as was the case with the face loan rates, the following two points can be made for the estimated effective rates (See Table 2, Figure 3, p. 78):

**Table 2 Variation and Correlation of Various Interest Rates**

<table>
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<tr>
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<th>62/I ~ 73/I</th>
<th>73/II ~ 84/III</th>
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<tbody>
<tr>
<td></td>
<td>Average</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>( \tilde{r}_l )</td>
<td>9.63</td>
<td>0.73</td>
</tr>
<tr>
<td>( r_c )</td>
<td>8.11</td>
<td>2.02</td>
</tr>
<tr>
<td>( r_n )</td>
<td>5.83</td>
<td>0.71</td>
</tr>
<tr>
<td>( r_d )</td>
<td>5.53</td>
<td>0.12</td>
</tr>
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</table>

**Correlation**

<table>
<thead>
<tr>
<th></th>
<th>62/I ~ 73/I</th>
<th>73/II ~ 84/III</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>( r_c )</td>
<td>( r_n )</td>
</tr>
<tr>
<td>( \tilde{r}_l )</td>
<td>0.835</td>
<td>0.921</td>
</tr>
<tr>
<td>( r_c )</td>
<td>–</td>
<td>0.919</td>
</tr>
<tr>
<td>( r_n )</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Note: \( \tilde{r}_l \) : Estimated effective loan rate
See Table 1 for other variables.

2. In making the estimates, a survey on the debtors' deposit ratio conducted by the Ministry of Finance is used, while the interest rate on debtors' deposits is assumed to be the three-month time deposit rate. The effective loan rate to be estimated (\( \tilde{r}_l \)) is

\[
\tilde{r}_l = (r_f - b \cdot r_d) / (1 - b),
\]

where

- \( r_f \) : face loan rate
- \( r_d \) : interest rate on deposits
- \( b \) : debtors' deposit ratio

(reserve requirements are ignored).

However, the survey data for the debtors' deposit ratio are on a half-yearly basis (May and November surveys) and available only for the period between May 1964 - May 1982 (since 1982, surveys have been conducted once a year in May). Therefore, by using the "corporate deposit/loan" ratio available on a quarterly basis and the interbank rate as an indicator of money market conditions, the debtors' deposit ratio is regressed to produce a quarterly series of the estimated debtors' deposit ratio from the first quarter of 1962 to the third quarter of 1984 (See Figure 2). Debtors' deposits include both compensating balances, the withdrawal of which is implicitly restricted, and deposits that the borrower holds voluntarily for transactions motives. The estimates here are obtained with no regard to this distinction. Thus, the estimated \( \tilde{r}_l \) may be greater than the true effective rates.
Figure 2  Debtors' Deposit Ratio (original survey data and estimated quarterly data)

<Estimated Equation>

\[ b = 83.7357 \times (DC/L) + 0.2331 \times r_e - 3.2678 \]

(12.67)  
(2.68)  
(-0.8574)

\[ \bar{R^2} = 0.8150 \]

S.E. 1.2114
D.W. 1.49

Notes:  
\( b \) : debtors' deposit ratio (average of city and local banks)  
\( DC \) : corporate deposits  
\( L \) : loans  
\( r_e \) : call rate and bills rate weighted average
(a) The range of loan rate fluctuations increased with 1973 as the turning point.
(b) High correlation coefficients are observed between loan rates and "institutional" rates (official discount rate and deposit rates), though the one between loan rates and deposit rates is high only in the second half.

2. Theories of Loan Rate Determination

In a typical model of bank behavior based on maximization of short-term profits, the interest rates that influence the bank's loan supply schedule are not the "institutional" rates (= regulated rates) such as the official discount rate and deposit rates, but money market rates (= opportunity cost of lending). Thus, if loan rates are determined at a level which equates the supply of and the demand for loans, the following questions must be answered:
(a) Why were fluctuations in the loan rates strongly correlated with fluctuations in the "institutional" rates (institutional nature of loan rates)?
(b) Why did not loan rates (especially in the 1960s) fluctuate as actively as money market rates did (inflexibility of loan rates)?

In this section, we examine how the leading studies on Japan's loan market have responded to these questions.

3. "Short-term profits" here do not include increases in the firm value of the bank that are not realized in the form of "current (net) profits". By contrast, "long-term profits" correspond to the discounted value of net cash flow of the bank (See Wakita 1981 for details).

4. According to the typical bank behavior model based on the profit maximization, (1) changes in the official discount rate do not affect the bank's loan supply and (2) while changes in deposit rates may cause the bank's source of fund to shift between the deposit market and money markets (call, bills, etc.), they do not directly affect the bank's loan supply schedule. As a result, changes in "institutional" rates do not influence equilibrium loan rates, except for a possibility that the bank's shift of fund source (case (2) above) to or away from money markets may lead to a rise or a decline in money market rates, which in turn may cause movements of loan rates in the same direction. However, this is nothing more than rehashing the basic proposition of the profit maximization model that "a rise in money market rates will cause loan rates to rise." Therefore, it cannot provide an adequate explanation for the stylized fact discussed in the previous section that despite the fact that the range of fluctuations in the money market rates did not change significantly during the period after 1960, the range of fluctuations in the loan rates increased with 1973 as a turning point.

5. Since the theoretical proposition that "changes in the money market rates will bring about changes in the loan rates in the same direction" does not have any quantitative implication, as long as the latter is affected to some extent by the former, we cannot necessarily say that the latter is "inflexible." Therefore, in its stead, we rather focus on the fact stated in footnote 4 that the relationship between loan rates and money market rates underwent a change with 1973 as the turning point.
A. "Regulation" Theory of Loan Rate Determination

The idea that loan rates are determined by such regulatory constraints as the Temporary Interest Rate Adjustment Law and the voluntary loan rate agreement among the members of Federation of Bankers Associations of Japan (so-called prime rate system) has been in existence since the 1960s (for example, see Komiya 1964; Suzuki 1966; and more recently, Iwata and Hamada 1980, Chapter 5). The proponents of this theory argue that loan rates are basically fixed artificially at a level lower than the point where the supply and demand curves intersect (credit rationing occurs in the loan market as a result). Their explanation for "institutionality" and "inflexibility" is given by separating the determination of loan rates and market equilibrium.

The problem with this line of thoughts is that their explanation contradicts economic agents' optimizing behavior. It is quite possible that the existence of the above regulations exerts direct influence on the determination of face loan rates. However, it must be possible for effective rates to fluctuate avoiding influences of such institutional factors and adjust themselves to their equilibrium values. This is a natural result of economic agents' efforts to achieve what is best for them.

Nevertheless, as we have seen above, effective rates (estimated rates) were clearly correlated with "institutional" rates, which suggests that some kind of relationship has existed between them originating not from mere restrictions, but from economic agents' rational choice. It is clear that "regulation" theory of loan rate determination fails to provide such an explanation.

B. Adjustment through Lending Conditions Other Than Interest Rate Conditions

Equilibrium in the loan market may not be achieved by interest rate conditions alone, but by a broader range of conditions that include lending periods and collateral requirements. If this way of thinking is adopted, it becomes meaningless to try to explain stylized facts such as "institutional nature" and "inflexibility" of face and effective loan rates. This is because, regardless of how these interest rate conditions behave, the loan market may not be affected at all if other lending conditions move at the same time neutralizing the impact of interest rate conditions. Under these circumstances, meaningful information concerning the loan market cannot be obtained by merely observing face and effective interest rates and it becomes necessary to expand the definition of lending conditions and observe all of them. However, expanding the lending conditions will necessarily cause insufficiency in the availabil-

6. Kaizuka and Onodera (1974) can be cited as an example of a study that expands the range of lending conditions and attempts to examine variables other than interest rate conditions that may have an impact on the loan market.
ity of data, and the reliability of the stylized facts derived from them will be inevitably weakened.

For example, Horiuchi (1980) has constructed a general equilibrium model for Japanese financial asset markets in which he conducts a detailed comparative static analysis on the loan rates determined therein (Chapter 1). However, in interpreting the actual lending market, he concedes that loan demand and supply are influenced by many variables which are not incorporated in his model and asserts, using the difficulty of estimating these variables as a justification, that "given the nature of the problem at hand, it is unwise to take up the price adjustment mechanism in the lending market head on and attempt to evaluate them" (Chapter 3, pp. 101-102). This type of arguments may not in itself contain any logical contradictions, but is not satisfying in that it tends to agnosticism.

C. Theory of "Implicit Contract"

The theory of "implicit contract" offers a good explanation for the "inflexibility" of loan rates. According to the theory, as a result of differences in attitudes toward the risk of profit fluctuations between lenders (banks) and borrowers (firms, etc.), the loan rate fluctuations may be quite small even when money market rates (opportunity cost of lending) fluctuate considerably (see Ikeo 1981; Wakita 1983). It is also possible to use this argument and explain the "institutional" aspect of the loan rates. 7

This approach is certainly an interesting and promising one. We believe that it is important to elaborate this theory further and check its applicability to the Japanese loan market. But at the same time, we also believe that an effort should be made to explain the market using the traditional theory of perfect competition, which may turn out to be complementary to the explanations offered by implicit contract theory.

D. "Equilibrium of Expectations" Hypothesis

Kuroda (1980) explains the "inflexibility" and "institutional nature" of loan rates by a hypothesis which he calls "equilibrium of expectations". First, with regard to "inflexibility," Kuroda argues that since short-term loans in Japan often possess a characteristic of long-term loans as a result of so-called "roll-over lending" whereby loans of the long-term nature are made to firms on a short-term basis, even short-term loan rates essentially possess the characteristics of long-term loan rates. It is

7. Expansion of fluctuations in deposit rates means that the level of profits of the bank is exposed to large swings. If the bank is risk-averse, it has an incentive to write an (implicit) contract that insures itself against such profit fluctuations. Such a contract will be the one in which loan rates are revised in accordance with movements in deposit rates.
contended on the basis of the pure expectations theory for the term structure of interest rates, that current loan rates are affected not only by the current money market rates but also by the expected future money market rates. Therefore, the loan rates will not react very much to current money market rates as long as expected future money market rates remain steady. Meanwhile, with regard to the "institutional nature," an explanation is given focusing on the information content of the official discount rate. Since changes in the official rate suggest a revised policy stance of the monetary authorities and permanent changes in the money market rates, the private sector will change its behavior in anticipation of these changes, thus causing a movement in the loan rates toward a new equilibrium. In the sense that changes in people's expectations concerning money market rates are realized by the subsequent conduct of the monetary authorities, Kuroda asserts that an "equilibrium of expectations" exists in Japan's loan market.

This argument, however, is not quite successful in explaining the stylized facts of the loan rate series. We have seen that money market rates fluctuated significantly since the 1960s; moreover, the range of fluctuations did not widen after 1973. If the private sector had been following the changes in the official rate as information related to the future value of money market rates, it should have found within a short period of time that the expansion of the range of fluctuations in the official rate since 1973 did not mean any expansion of fluctuations in the money market rates. Loan rates in this case might have overreacted for a while immediately following the increase in the official discount rate's fluctuation in 1973, but soon, they should have returned to the "inflexible" movement characteristic of the period before 1973. In reality, however, the movements in loan rates since 1973 are clearly different from what they were before 1973.

Another problem of this theory is that it cannot explain adequately recent developments in the loan market which signify that the relationship between loan rates and money market rates are strengthened. These developments include the emergence of so-called "spread banking", a method of banking in which loan rates are set at the level equal to the sum of marginal fund acquisition costs (money market rates) and a profit margin, and an increasing demand for a new prime rate system which reflects the level of money market rates instead of that of the official discount rate. In order to understand such phenomena using the Kuroda model, it would be necessary, for example, to argue that the official discount rate for one reason or another has lost much of its effectiveness as a vehicle of information and it has ceased to be a reliable predictor of future values of money market rates. Alternatively, one might argue that the information concerning the future values of money market rates has become unimportant as a result of shortening of loan terms. A popular explanation for these phenomena, however, is the change in the liability structure of banks. According to this explanation, because of increased CD issues and abolition of restrictions on
conversion of dollar deposits to yen, the proportion in bank debts of financial instruments which bear money market rates ("market rate" debts) is gradually increasing, and as a result, the average costs of banks, and hence the loan rates charged by them, have begun to respond quickly to money market rates. The argument that the average cost exerts influence on the supply price of loans resembles so-called "mark-up principle" (Iwata and Hamada 1980, Chapter 5), but in this paper, an attempt will be made to interpret this argument from a different perspective.

E. Econometric Approach to the Loan Market

Finally, let us examine, in a slightly different vein from the theoretical considerations presented above, the econometric approach taken to analyze disequilibrium in the loan market. This approach, originated by Fair and Jaffee (1972), is widely applied to Japan's loan market (see Hamada, Iwata and Ishiyama 1977; Furukawa 1979; Kamae 1980; Tsutsui 1982). Under this approach, in measuring the disequilibrium in the loan market, sample data are divided into those for the period of excess demand for loans and those for a period of excess supply of loans in accordance with the direction of the changes in loan interest rates at the period. The former is used to estimate the demand curve, and the latter the supply curve (Fair-Jaffee's directional method I). The estimated results of the two curves thus obtained are compared with the conventional estimation results based on market equilibrium, and the relative validity of the equilibrium and disequilibrium hypotheses is determined comparing the goodness of fit of these estimated results.

This approach is useful in that it provides a method to test quantitatively the common belief that rationing exists in the loan market. But it also has the following weakness; when the Japanese data are divided according to Fair-Jaffee's method noted above, contrary to the common belief, the period of excess supply turns out to be much longer than the period of excess demand. The works cited above contend that this results from the fact that Japanese loan rates are biased by the influence of the official discount rate. Then they divide the sample data again, assuming an ad-hoc interest adjustment equation which supposedly filters the "institutional" bias, to obtain a longer period of excess demand.

As this treatment of the institutional factor clearly shows, they fail to recognize

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8. When the market is in imbalance, it is assumed that the amount of loans is determined on the supply curve if the period in question is that of excess demand, and on the demand curve if the period is that of excess supply. Moreover, at each point in time, when interest rates are declining, the period is defined as that of excess supply, and if they are rising, as that of excess demand. Consequently, the sample is divided into two categories, and each can be used to estimate the supply and demand curves respectively.
the possibility that changes in "institutional" rates shift the loan supply and demand curves. We believe that more attention should be paid to microeconomic backgrounds of loan supply and demand and to the endogenous relationship between loan rates and "institutional" rates.

3. Questions to be Answered

The preceding discussions have brought the importance of the following question to the fore.
(a) Why did the range of fluctuations in the loan rates (face/effective) expand with 1973 as the turning point?

With regard to this phenomenon, the movement of the official discount rate (or deposit rates), whose range of fluctuation also increased at the same time, offers an important clue. This leads us to the second question.
(b) What kind of causal relationship existed between the loan rates and these "institutional" rates (background of the institutional nature of loan rates)?

In addition, a consistent theory should answer the third question.
(c) Why does this relationship between loan rates and "institutional" rates appear to be weakening recently?

In what follows, an attempt will be made to answer these three questions using a general equilibrium framework.

III. Bank's Objective Function—Reconsideration of Business-Scale Variables

Although the "maximization of short-term profits" is the most representative hypothesis concerning the objective of bank behavior, several analyses have been attempted incorporating other factors than simple profit maximization. For example, it has been pointed out that Japanese banks often behave contrary to the maximization of their short-term profits, citing as evidence their tendency to strive for establishment of closer relationship with clients (customer relationship) and to expand their business scale, both of which often seem to be carried so far as to do harm to their current profits (Research and Statistics Department 1966).9

The customer relationship and the hypothesis of maximization of long-term profits behind it are studied in detail by Wakita (1981). An increase in loans beyond

the point of maximum current profits causes a marginal loss to the banks, but it also increases their future revenue by deepening the customer relationship. As long as the discounted value of the latter gain is larger than the former loss, the firm value (long-term profits) of the banks will increase as a result of this increase in loans.

Similarly, it is possible to interpret the tendency of banks to expand their business scale as a rational behavior. For instance, Suzuki (1974) argues that if interest rates are regulated and rationing exists in the deposit and Bank of Japan loan markets, a rise in the market share will immediately lead to increased profits, so that their struggle for a larger market share in these markets should be regarded as a natural result of profit maximizing behavior (pp. 20-21). Furthermore, Iwata and Hamada (1980, Chapter 2) analyze the subjective equilibrium of banks using the hypothesis that the amount of the Bank of Japan loans is dependent on the operating scale (loans, borrowing in the call money market, etc.) of the banks and conclude that they tend to expand their scale of operation in order to increase their share of the loans made by the Bank of Japan. Generally speaking, not only the lending policy of the Bank of Japan, but also various policies under which Japanese banks are operating (window guidance, licensing policy for setting up branches or dealing in bonds) tend to be based on the scale of business operation rather than on the level of profits. Under these circumstances, the possibility of receiving policy benefits in future can be increased by expanding the current scale of business operation even at the cost of reduced current profits. In addition to this, the increase in the scale of business operation as measured by the value of total assets, and the resultant rise in the asset ranking among banks, may bring about visible and invisible benefits such as a higher reputation and improved worker morale which may be turned into future profits. Therefore, a mechanism can exist, without assuming customer relationship, that an expanded business scale leads to increased long-term profits.

The argument above regards the tendency of banks to expand their business scale as maximization of profits defined in one way or another. But there is no need to adhere to this interpretation. Firstly, in order to maintain banks’ profit margin, a policy has been taken by the government to keep their fund acquisition costs at a low level (regulation on deposit rates, control of establishment of branches, etc.) and to restrict new entries into the business of banking. As a result, there has always been a potential for banks to enjoy excess profits, and there has never existed in the banking business perfectly competitive long-run equilibrium where those banks which do not maximize their profits are expelled from the market. Secondly, there is a problem of managerial incentive in an economy characterized by the separation of ownership and management. In a textbook capitalistic economy, the main concern of stockholders is to increase the value of their stocks (maximization of the profits of the firm concerned). They exercise their rights as stockholders and influence decisions related to appointments, dismissals and the wages of managers. But it is difficult to believe
that this is actually taking place in Japan. In other words, a strict institutional framework does not seem to exist in Japan which forces bank managers to pursue maximization of profits as their only goal, and it is more reasonable to look upon them as acting to achieve much broader objectives.\textsuperscript{10} If this is the case, it is possible to regard the expansion of business operation as an objective distinct from their concern for profits. We can list a number of reasons why bank managers prefer a larger operating scale. They include expansion of their own authority, promotion of growth of “Keiretsu” (corporate group) companies, desire to diversify risks, and contribution to the growth of the local economy. In fact, there can be as many potential reasons for expansion as the “preferences” (Williamson 1964) of bank managers.

The foregoing discussion has shown that banks may have a tendency to expand their business operation even at the expense of their current profits if they are trying to maximize their long-term profits or if maximization of profits is not their sole objective. But obviously, such a tendency cannot continue without any consideration to current profit levels. It is more reasonable to assume that bank managers possess yardsticks of one form or another by which they evaluate the current profit levels and the business scale, and select their most desirable combinations.\textsuperscript{11} We express this assumption by the following function:

\[
\Phi = \Phi (\pi, D)
\]  

(1)

where $\pi$: current profits  
$D$: deposit balance (as a variable representative of the business scale).

It is possible to interpret this function as expressing either long-term profits or the preferences of bank managers. It provides a basis for the analysis of bank behav-

\textsuperscript{10} These facts, of course, do not mean that bank managers are oblivious to profits and stock prices. Even though raising the values of these variables may not be their sole objective, this may be the most important among various elements affecting their behavior. If this was the case, however, there would not be a difficulty in explaining stylized facts using the theory based on profit maximization.

\textsuperscript{11} If the highest possible current profit level (regardless of the scale of business operation) was considered the most desirable combination of the two variables for the bank, this would be nothing but the maximization of short-term profits. In this sense, the objective function (1) includes the traditional approach as a special case. The formulation that the bank selects the most appropriate combination of profits and deposits is also employed by Monti (1972) in a framework of imperfect competition.
ior and loan rate determination in the rest of this paper.12

IV. Model Analysis I — Derivation of a Loan-Supply Curve

1. Framework of the Model

The following assumptions are made in deriving the loan-supply curve of a representative bank.
(a) The model is a static one-period model.
(b) The loan market and money markets13 are characterized by perfect competition.
(c) The loans made by the Bank of Japan and the primary deposits are allocated to each bank under the “institutional” rates (official discount rate, deposit rates) and

12. Our objective in the following sections is to deduce a proposition that changes in “institutional” rates affect the level of equilibrium effective loan rates. For this purpose, it is necessary to make an assumption on the shape of the objective function. Namely, it is assumed that the locus obtained by connecting the points where objective functions of different $\Phi$ values become tangent to a set of straight lines having the same angle to the X axis is upward sloping as shown in Figure 4 (the locus corresponds to the income — consumption schedule of price theory). This assumption means that, in the context of the bank manager’s preference, the deposit balance is not an “inferior good”, and that, in the context of long-term profits, the schedule describing the rate at which the current deposit balance increases the bank’s future revenue shifts according to a certain rule, as the value of $\Phi$ (present value of the bank) changes.

13. Money market assets and liabilities here refer to all items in the bank’s balance sheet the yields of which are opportunity costs of loans. Transactions on the call, bills and “gensaki (RP)” markets are typical examples.
their amounts are exogenously determined (existence of rationing).\textsuperscript{14} (d) The derived deposits (debtor's deposits) are generated as a fixed proportion $b$ ($0 < b < 1$) of loans. (e) Bonds, reserves and other items in the bank's balance sheet are ignored.

The representative bank's balance sheet is shown in Table 3. The bank's current profits ($\pi$) can be expressed by the following equation:

$$\pi = r_{l} L - r_{c} C - r_{n} N - r_{d} D - F (L - D_d)$$  \hspace{1cm} (2)

where

- $L$ : loans (face value)
- $r_{l}$ : loan rates (face value)
- $C$ : amount of borrowing in the money markets
  \hspace{1cm} ($C < 0$ indicates lending in the money markets)
- $r_{c}$ : money market rates
- $N$ : borrowing from the Bank of Japan
- $r_{n}$ : official discount rate
- $D$ : deposit balance

\begin{table}[h]
\centering
\begin{tabular}{l|l}
\hline
Loans ($L$) & Money market borrowing ($C$) \\
\hline
Borrowing from the Bank of Japan ($N$) & \\
\hline
Deposits ($D$) & Primary deposits ($D_u$) \\
\hline & Debtors' deposits ($D_d$) \\
\end{tabular}
\caption{Balance Sheet of the Bank}
\end{table}

\textsuperscript{14} The bank in our model aims at increasing its business scale as well as its current profits. Thus, it is more reasonable to assume that this bank turns its efforts into acquisition of primary deposits paying additional costs (implicit interest). However, with regard to the focal argument of this paper, namely that "fluctuations in the 'institutional' rates directly cause changes in bank lending," it does not make substantial differences whether the acts aimed at obtaining primary deposits are incorporated in the model or not (however, within the framework of market equilibrium which is examined in the next section, fluctuations in the "institutional" rates may affect money market rates and cause changes in lending behavior as a result. See footnote 24 for further discussion). Thus, for the sake of simplicity, the discussion below will ignore the bank's activities aimed at obtaining primary deposits, and limit the method of deposit acquisition to those through increased lending (i.e., increased debtors' deposits).
\( r_d \): deposit rates

\( F(\cdot) \): lending cost function \((F'>0, F''>0, F(0)=0)\)^15

\( D_d \): debtors' deposits \(=bL\)

\( L-D_d \): effective loans

\( D_u \): primary deposits \(=D-D_d\).

The following equation is the bank's balance sheet constraint.

\[
L = C + N + D \\
= C + N + D_u + D_d
\]

(3)

Using equations (2) – (3) and \( D_d = bL \), we can express the current profits as follows:

\[
\pi = (\tilde{b} - 1) (\tilde{r}_f - r_e) D_d + (r_e - r_n) N \]

\[
+ (r_e - r_d) D_u - F(\tilde{b} - 1) D_d
\]

(4)

where \( \tilde{b} = 1/b \) \((>1\), reciprocal of debtors' deposit ratio\)

\( \tilde{r}_f = (r_f - b \cdot r_d) / (1-b) \) \(\) (effective loan rate).

Equation (4) indicates the combination of profit level \( \pi \) and the debtors' deposits \( D_d \) (which is proportionate to the loan balance because \( D_d = bL \)) that the bank is able to achieve when the value of exogenous variables (various interest rates, \( N, D_u, \tilde{b} \)) are given.

Next, with regard to the bank's objective function \( \Phi(\pi, D) \) introduced in the previous section, we use a Cobb-Douglas function.\(^{16}\) Namely,

\[
\phi(\pi, D) = \pi^a D^\beta
\]

(5)

15. The lending cost function is defined for effective loans \((L-D_d)\), and not for face loans \((L)\). This does not affect the substance of the discussion below.

16. This specification is not necessary for the discussion below (that is, the function need not be homothetic as is the Cobb-Douglas function). All that is required is to satisfy the conditions indicated in Figure 4 of footnote 12. The advantage of using the Cobb-Douglas function lies in the fact that explanation is made easier because different values of \( \beta \) correspond to different hypotheses of bank behavior.
where $\alpha$, $\beta$ are parameters that take positive or zero values. The behavior of the bank in our model can be described as maximization of the value of equation (5) under the constraint imposed by equation (4). Note that, when $\beta = 0$, the maximization of equation (5) is equivalent to the maximization of current profits. Therefore, in the analysis that follows, a comparison will be made between two cases; one in which $\beta = 0$ and the other in which $\beta \neq 0$.

Figure 5 shows the relationship between current profits and debtors' deposits (equation (4)). First, when the amount of loans is zero, the bank gains profits in $\tilde{\pi}$ units (point A) by investing in money markets the combined amount of its primary deposits in $D_u$ units and the Bank of Japan loans in N units, both of which are allotted to the bank. If the bank allocates part of its funds to loans, its profits increase (as long as $\tilde{r}_l > r_c$, that is, the effective loan rates exceed the money market rates). In this case, debtors' deposits also increase in response to increases in lending, and this process can be described as movements from A toward B in Figure 5 with $D_d$ measured to the right from $D_u$. However, because of increased loan costs, the tempo of increasing profits gradually slows down and then reverses itself; that is, profits begin to shrink after passing point B (the point of maximum profits).

![Figure 5 Feasible Combinations of $\pi$ and D (Equation 4)](image)

17. The values of these parameters can be different for different banks and can change over time even for the same bank. But here, the analysis will be conducted simply treating $\alpha$, $\beta$ as given.
Figure 6 is the result of adding the bank’s objective function \( \Phi \) to Figure 5. When \( \beta = 0 \), that is, when maximization of current profits is the objective of the bank’s behavior, the function \( \Phi \) is a straight line parallel to the horizontal axis \( (\Phi_1) \) and B is the point where its maximum value is given (this is simply the point at which current profits are maximized). When \( \beta \neq 0 \), by contrast, the function \( \Phi \) is shaped like \( \Phi_2 \) on the figure, and the point of subjective equilibrium (the point of tangency with the curve ABC) is given at E. The bank, in this case, expands loans beyond the amount that maximizes its current profits.

![Figure 6 Determination of Subjective Equilibrium of the Bank](image)

2. Effects of Interest Rate Changes

Let us next examine how the subjective equilibrium will be affected when exogenous variables (interest rates, the Bank of Japan’s lending, primary deposits, debtors’ deposit ratio) change, and how endogenous variables (profits, loan and debtors’ deposit balances) will react as a result. The direction of changes in the endogenous variables is shown in Table 4. A diagrammatic exposition is presented below of the effects of changes in money market rates and “institutional” rates (see Appendix 1 for algebraic expressions of comparative statics).

A. Changes in the Subjective Equilibrium when Money Market Rates Increase

\((d\pi/drc, dD/drc [\wedge b \ dL/drc])\)

Figure 7 shows the shift in the equilibrium point accompanying a rise in money market rates \( (r_c) \). Point F in the figure is where the bank’s net borrowing
Table 4  Results of Comparative Statics

<table>
<thead>
<tr>
<th>Exogenous variables</th>
<th>Maximization of $\Phi$ ($\beta \neq 0$)</th>
<th>Maximization of current profits ($\beta = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>d $\pi$</td>
<td>d$L$, d$Dd$</td>
</tr>
<tr>
<td>$d\bar{r}_f$</td>
<td>?</td>
<td>+</td>
</tr>
<tr>
<td>$dr_c$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C $&gt;$ 0</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>C $&lt;$ 0</td>
<td>+</td>
<td>?</td>
</tr>
<tr>
<td>$dr_n$, $dr_d$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$dN$, $dDu$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$db$</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Notes: 1. The signs in the table indicate the directions of changes in endogenous variables ($\pi$, $L$, etc.) when exogenous variables ($\bar{r}_f$, $r_c$, etc.) change. Namely, they represent the signs of derivatives ($d\pi/d\bar{r}_f$, $dL/dr_c$, etc.).
2. $+$, $-$, 0 and ? correspond respectively to positive, negative, zero and uncertain signs.
3. $L$ is the face loan, while $(L-Dd)$ is the effective loan (this distinction is relevant only for the derivatives with respect to b).
4. $C > 0$ corresponds to the money-position bank (borrowing in the money markets), and $C < 0$ to the loan-position bank (lending in the money markets).

Figure 7  Increase in $r_c$

1. loan-position bank

2. money-position bank
in the money markets is zero, and when \( r_c \) increases, the curve ABC shifts clockwise around this point. The bank's money market position (whether it lends or borrows in the money markets) is determined by the position of its subjective equilibrium vis-à-vis F. If \( \beta = 0 \) (the bank's objective is maximization of short-term profits), the equilibrium point shifts from B to B', and thus the directions in which profits and the deposit balance (amount of loans) change are determinate regardless of the bank's money market position. Namely, a rise in money market rates always decreases deposits (loans) of all banks, raises the level of profits of a bank lending in the money market (loan-position bank), and reduces that of a bank borrowing in the money market (money-position bank).

On the other hand, if \( \beta \neq 0 \), the equilibrium point shifts from E to E'. In this case, as the results of comparative statics in Appendix 1 show, the direction of change in deposits (loans) and profits may be indeterminate depending on the money market position of the bank. Namely, if the bank is in a loan position (i.e., if it is lending in the money markets), the amount of deposits (loans) of the bank does not necessarily decline as money market rates rise. Similarly, if the bank is in a money position (i.e., if it is borrowing in the money markets), the level of its profits does not necessarily decrease as money market rates rise. The reason for this can be illustrated as follows. The shift in the subjective equilibrium point (E \( \rightarrow \) E') caused by a rise in \( r_c \) can be divided conceptually into following two components (see Appendix 1); (1) the shift which reflects the increase in the opportunity cost of lending and accompanying reduction of the loan supply of the bank (so-called "cost effect." See Suzuki 1974, p.98); and (2) the shift which reflects the change in the profit level of the bank and the resultant readjustment of its profit-deposit mix (what may be called "profit effect"). Since the direction of the profit effect depends on whether the bank borrows from or lends to the money market, there are cases in which the two effects work in opposite directions making the total effect theoretically indeterminate.

In the case of \( \beta = 0 \) discussed above, the bank seeks only for current profits and is not concerned about the concept of "profit-deposit mix", so that the profit effect does not come into play at all. This is why the directions of changes in endogenous variables are determinate. However, if \( \beta \neq 0 \), the bank is subject to both cost and profit effects, and indeterminate cases emerge. Let us examine these cases in detail.

If the bank is in a loan position, a rise in \( r_c \) increases yields on investment and the profit level. This raises the value of \( \Phi \) even if the bank keeps its amount of loans constant. However, if it expands loans (and debtors' deposits) sacrificing a part of the improved current profits, it can achieve an even higher value of \( \Phi \). The profit effect corresponds to this induced readjustment of the profit-deposit (loan) mix. Meanwhile, the cost effect of an increase in money market
rates always reduces the loan supply, irrespective of the money market position
of the bank. Thus, the cost effect and the profit effect influence the amount of
loans (and deposits) in the opposite directions making the direction of overall
influences of the changes in money market rates ambiguous.

On the other hand, if the bank is in a money position, a rise in \( r_c \) means a
rise in the cost and, the amount of loans being constant, a reduction of profits.
This reduction of profits inevitably reduces the value of \( \Phi \). However, by re-
adjusting the profit-deposit mix, the bank can alleviate the decline of the \( \Phi \).
Namely, since the bank is running a loss on its loans at the margin, it can recover
the profits lost on account of the rise in money market rates to a certain extent
by withdrawing part of its loans and reducing borrowings from money markets.
As a result, while both the cost and profit effects obviously reduce the amount of
lending the bank provides, they exert counteracting influences on the overall
profit level and direction of change cannot be identified from theory.

B. Changes in the Deposits (Loans) when “Institutional” Rates are Raised
\[
(dD_d/dr_n [ = bdL/dr_n], dD_d/dr_d [ = bdL/dr_d])
\]

The official discount rate and deposit rates do not have a cost effect on the
bank’s loan supply as long as their levels are lower than money market rates
(Suzuki 1974, p. 194). In other words, the curve ABC in Figure 8 shifts down-
ward vertically accompanying a rise in “institutional” rates, and the point where
current profits are maximized also shifts downward in a perpendicular fashion (\( B \rightarrow B' \)). Accordingly, as long as the bank’s subjective equilibrium is given at point
\( B \) (when \( \beta = 0 \)), a shift in “institutional” rates has no direct effect on the bank’s

**Figure 8** Increase in \( r_n \) (or \( r_d \))
loan supply.

However, when $\beta \neq 0$, an increase in "institutional" rates clearly has a profit effect, as evidenced by the fact that the equilibrium point shifts from $E$ to $E'$. In other words, a channel exists through which changes in the "institutional" rates can directly cause changes in the bank's loan supply. As it is clear from Figure 8, the extent of this profit effect depends on the amount of the profit reduction accompanying the rise in "institutional" rates. Thus, a 1% change in the deposit rates has a far greater impact than the same 1% change in the official discount rate, because deposits account for a much larger proportion of the balance sheet of the bank.

On the basis of the results of the comparative statics shown in Table 4, the loan supply function of the bank (supply schedule of effective loans $\tilde{L} = L - D_d$) can be expressed as follows\(^{18}\) (the signs over the independent variables indicate positive or negative partial derivatives, where $\oplus$, $\ominus$ indicate that the partial derivative is zero when $\beta = 0$. This notation also applies to the other equations in this paper):

$$\tilde{L} = \tilde{L} ( L, r_f, r_e, r_a, r_d, N, D_u, b ).$$  \hspace{1cm} (6)

Furthermore, we can derive a demand function for borrowings in money markets from equation (6) and the balance sheet constraint imposed by equation (2).\(^{19}\)

$$C = \tilde{L} - N - D_u = C ( L, r_f, r_e, r_a, r_d, N, D_u, b ).$$  \hspace{1cm} (7)

In what follows, equations (6) and (7) will be incorporated into a simplified market equilibrium framework, and the characteristics of effective interest rates determined therein shall be analyzed.

---

18. The representative bank is assumed to be in the money position. This corresponds to the assumption made in the next section that the central bank and private non-bank sector are the suppliers of funds in money markets, and commercial banks borrow these funds.

19. The signs of $dC/dN$, $dC/dD_u$ depend on whether $dL/dN$, $dL/dD_u$ are, respectively, larger or smaller than 1. Here, we assume $dL/dN < 1$, $dL/dD_u < 1$. 
V. Model Analysis II — Determination of Equilibrium Loan Rates

1. Framework of the Model

In analyzing determination of loan rates, the following assumptions are made:

(a) There are three types of economic agents; the central bank, commercial banks and private non-banking sector (hereafter referred to as the private sector).
(b) There are five types of financial assets transacted; cash, money market assets, bank loans, primary deposits and the Bank of Japan loans.
(c) Cash, money market assets and bank loans are transacted on competitive markets.
(d) Primary deposits and Bank of Japan loans are rationed to the commercial banks.
(e) Commercial banks’ reserves are zero (that is, they hold no cash).

The balance sheets of the three types of economic agents are summarized in Table 5.\(^{20}\)

The portfolio selection behavior of each economic agent is formulated as follows. The private sector raises funds either by borrowing from banks or from money

| Table 5 Balance Sheets of the Three Sectors in the Economy |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|
| Central bank | Commercial banks | Private (non-bank) sector | Total |
| Cash | $-X^C$ | $X^P$ | 0 |
| Money market assets | $C^C$ | $-C^B$ | $C^P$ | 0 |
| Effective loans | $L^B$ | $-L^P$ | 0 |
| The Bank of Japan loans | $N^C$ | $-N^B$ | 0 |
| Primary deposits | $D^B$ | $D^P$ | 0 |
| (Real capital) | | | (K) | (K) |
| Net worth | 0 | 0 | $\bar{W}$ | $\bar{W} (=K)$ |

Note: Liabilities are shown with minus signs.

20. The private sector both lends and borrows in money markets. Here, their net balance (excess investment) is expressed as $C^P$. The subscript for the primary deposits ($u$ in Du) will be left out in this section.
markets, and invests its assets in cash, deposits and money market assets. The demand function for each asset and liability of the private sector is expressed as follows (the signs over the independent variables indicate either positive or negative partial derivatives):

(Demand for bank loans)  \[ \tilde{L}^p = \tilde{L}^p \left( \tilde{r}_l, \tilde{r}_e \right) \]  (8)

(Demand for cash)  \[ X^p = X^p \left( \tilde{r}_l, \tilde{r}_e, \tilde{r}_d \right) \]  (9)

(Supply of funds in money markets)  \[ C^p = C^p \left( \tilde{r}_l, \tilde{r}_e, \tilde{r}_d \right) \]  (10)

(Supply of primary deposits)  \[ D^p = D^p \left( \tilde{r}_e, \tilde{r}_d \right) \]  (11)

First, it is assumed that demand for bank loans is correlated negatively with effective loan rates and positively with money market rates. The reason why the latter exerts influence on demand for bank loans is as follows; if the private sector is able to raise funds on the money markets, it will shift its source of funds between borrowing from banks and borrowing from the money markets in response to fluctuations in relative borrowing rates. Meanwhile, gross substitutability is hypothesized among demand for cash, supply of funds in money markets and primary deposit supply. Effective loan rates are included in the supply function \( C^p \) of funds in the money markets and the cash demand function \( X^p \) because they are opportunity costs for holding money market assets and cash.

Commercial banks’ loan supply and fund demand in money markets are given by aggregating individual banks’ loan supply and fund demand (equations (6) and (7)) derived in the previous section.

(Loan supply)  \[ \tilde{L}^b = \tilde{L}^b \left( \tilde{r}_l, \tilde{r}_e, \tilde{r}_n, \tilde{r}_d, N, D, b \right) \]

(Demand for funds in money markets)  \[ C^b = C^b \left( \tilde{r}_l, \tilde{r}_e, \tilde{r}_n, \tilde{r}_d, N, D, b \right) \]

The central bank is assumed to carry out monetary policy through money market operations and adjustments of the Bank of Japan loans with money market rates \( r_e \) as its operating target (that is, \( r_e \) is an exogenous variable). Under this setup, equilibria in the three markets — cash, money market assets and bank loans — are expressed by equations (12) - (14).

(Equilibrium for cash)  \[ X^c = X^p \left( \tilde{r}_l, \tilde{r}_e, \tilde{r}_d \right) \]  (12)
(Equilibrium for money market assets)

\[ C^e + C^p (\tilde{r}_f, r_e, r_d) = C^b (\tilde{r}_f, r_e, r_n, r_d, N^c, D, b) \]  \hspace{1cm} (13)

(Equilibrium for bank loans)

\[ \tilde{L}^B (\tilde{r}_f, r_e, r_n, r_d, N^c, D, b) = \tilde{L}^p (\tilde{r}_f, r_e) \]  \hspace{1cm} (14)

D and \( N^c \) in (13) and (14) represent the amounts of primary deposits and the Bank of Japan loans rationed to the banking sector. According to Walras' law, only two of the three equations are independent. Two endogenous variables (effective loan rates \( \tilde{r}_f \) and the amount of cash in circulation \( X \)) are determined from these two equations.

2. Effects of Changes in Money Market Rates and "Institutional" Rates

The way equilibrium values of the endogenous variables react to changes in the exogenous variables \( (r_c, r_n, r_d, b) \) is shown in Table 6. As the table clearly indicates, different results are obtained depending on whether \( \beta = 0 \) or not. Let us here analyze the influence that changes in the money market rates \( (r_c) \) and "institutional" rates (official discount rate \( r_n \) and deposit rates \( r_d \)) have on effective interest rates.

**Table 6 Results of Comparative Statics**

<table>
<thead>
<tr>
<th>Exogenous Variables</th>
<th>Maximization of ( \Phi ) ( (\beta \neq 0) )</th>
<th>Maximization of current profits ( (\beta = 0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( d \tilde{r}_f )</td>
<td>( dX )</td>
</tr>
<tr>
<td>( dr_c )</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( dr_n )</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( dr_d )</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>( db )</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Note: See footnotes to Table 4.
Changes in \( \bar{r} \) accompanying changes in money market rates are expressed as follows:

\[
d \bar{r}_t / dr_e = \frac{1}{Y} \left( \left( \tilde{L}_e^p - \tilde{L}_e^b \right) - \left( \tilde{L}_e^b - D_e^p \right) \right) > 0
\]

where

\[
\tilde{L}_e^p = \partial \tilde{L}_e^p / \partial r_e \quad \text{,} \quad \tilde{L}_e^b = \partial \tilde{L}_e^b / \partial r_e
\]

\[
\tilde{L}_e^b = \partial \tilde{L}_e^b / \partial \bar{r}_e \quad \text{,} \quad D_e^p = \partial D_e^p / \partial r_e
\]

The value of \( Y \), determinant of the Jacobian matrix, is positive (see Appendix 2).

The three terms in parenthesis in equation (15) correspond to the following three effects respectively: A rise in \( r_e \) (1) causes a shift of the private sector away from money market borrowing toward bank borrowing; (2) weakens the desire of banks to make loans through cost and profit effects (see pp. 91-94); and (3) causes the withdrawal of deposits by the private sector (shift in investment toward money market assets) which weakens the banks' desire to make loans (profit effect)\(^{22}\). All of (1) - (3) tighten the loan market and thus raise \( \bar{r}_t \). As the sign over each term in parenthesis in parenthesis

---

21. In carrying out a comparative statics, it is important to turn our attention to the difference in the means with which the central bank adjusts the money markets (operation or lending). According to the theory based on profit maximization, this difference has no influence whatsoever on the nature of the market equilibrium. However, a slightly different result is obtained if the model in this paper, which takes into account "profit effect," is used. For example, let us assume that the central bank increases the supply of high-powered money with the aim of lowering \( r_e \). If this was achieved through increased Bank of Japan loans, commercial banks would be provided with a subsidy equal to \( (r_e - r_0) \) per unit of additional funds supplied (this subsidy would not be granted in the case of operations in money markets). Increased profits resulting from this subsidy would have the profit effect and stimulate the loan supply of commercial banks. In short, the decline in \( r_e \) caused by increased Bank of Japan loans has a greater expansionary effect than does the same amount of decline in \( r_e \) caused by money market operations (see Appendix 2).

22. As indicated in Appendix 1, the withdrawal of one unit of primary deposits leads to a decline in profits equivalent to \( (r_e - r_0) \). As the level of profits declines, the banking sector will attempt to compensate for this decline in profits by withdrawing a part of the loans (profit effect), thus reducing the loan supply.
shows, the profit effect does not come into play when \( \beta = 0 \) and the effect of (3) becomes zero as a result, but a rise in \( r_c \) still increases \( \tilde{r}_l \) through the effect of (1) and (2).

The changes in \( \tilde{r}_l \) accompanying changes in the official discount rate are expressed as follows:

\[
\frac{d\tilde{r}_l}{dr_n} = -\frac{1}{Y} \frac{\partial L^B_n}{\partial r_n} \geq 0
\]  

(16)

where

\[
\hat{L}_n^B = \frac{\partial L^B_n}{\partial r_n}.
\]

As was discussed in the previous section, a rise in the official rate reduces the supply of loans through the profit effect (see pp. 94-95). Equation (16) demonstrates that this tightens the loan market and causes \( \tilde{r}_l \) to rise (however, when \( \beta = 0 \), the profit effect will not come into play and \( \hat{L}_n = 0 \), thus \( d\tilde{r}_l/dr_n = 0 \)).

Lastly, the movement of \( \tilde{r}_l \) accompanying changes in the deposit rates is expressed as follows:

\[
\frac{d\tilde{r}_l}{dr_d} = -\frac{1}{Y} \left( \frac{\partial L^B_d}{\partial r_d} + \frac{\partial L^B_d}{\partial D^p} \right) \geq 0
\]  

(17)

where

\[
\hat{L}_d^B = \frac{\partial L^B_d}{\partial r_d}, \quad \hat{L}_d^P = \frac{\partial L^P_d}{\partial D^p}, \quad D^p = \frac{\partial D^p}{\partial r_d}.
\]

The two terms within the parenthesis in equation (17) correspond to the following two effects respectively: A rise in deposit rates \( r_d \) (1) weakens the loan supply of banks through the profit effect (see pp. 94-95); and (2) increases the amount of funds deposited by the private sector (a shift away from cash and money market assets), which heightens, through the profit effect, the loan supply of banks. Moreover, since the former tightens the loan market and the latter eases it, the direction of changes in \( \tilde{r}_l \) cannot be determined. The effects of (1) and (2) indicated above do not come into

23. However, the results of the comparative statics presented in Appendix 1 show that the absolute value of \( \hat{L}_d^B \) is considerably larger than \( \hat{L}_n^B (|L^B_d|/L^B_n = D^d/(r_c - r_d)) \). Thus, except when \( D^d \) (the degree of responsiveness to deposit rate changes of demand for deposits) is extremely large, \( d\tilde{r}_l/dr_d \) can be regarded as having a positive value.
play when $\beta = 0$ because both function through the profit effect, thus $d\tilde{r}_f/\partial r_n = 0$ when $\beta = 0$.\textsuperscript{24}

The preceding discussion has demonstrated that changes in the "institutional" rates can have a direct impact on the effective loan rates determined on the competitive market, as long as commercial banks are aiming at both profits and business scale. Let us next examine the differences in the impacts that the "institutional" rates and money market rates have on loan rates; that is, the differences in the effects of monetary policy through these alternative measures. They are expressed in the following equation (we consider the case in which both the official discount rate and deposit rates are changed by the same amount):

$$
\frac{d\tilde{r}_f}{dr_e} - \frac{d\tilde{r}_f}{dr_n} - \frac{d\tilde{r}_f}{dr_d} = \frac{1}{Y} \left[ \frac{1}{L_C^{\ast}} + \frac{D^P}{L_B^{\ast}} (D^P - D^C) + \frac{L_B^{\ast}}{L_C^{\ast}} \right]. \tag{18}
$$

Terms (1) to (3) in parenthesis express respectively: (1) changes in the private sector's borrowing behavior; (2) changes in the loan supply resulting from shifts in the private sector's investment (increases and decreases in primary deposits); and (3) changes in the loan supply caused directly by changes in interest rates. The larger the value of equation (18), the greater the influence of the policy to control money market rates on loan rates; and conversely, the smaller the value of equation (18), the more effective, policy-wise, would the alteration of "institutional" rates be.

Let us examine the changes that have taken place in the value of equation (18) since the 1960s in Japan. Firstly, in order for the effects of (1) and (2) (those exerted on the private sector's borrowing and investment behavior by changes in money market rates and deposit rates) to come into play, the private sector has to be able to enter the money markets and shift its borrowing and investment activities between these markets and the loan and deposit markets. In actuality, the "gensaki (RP)" market has existed as a typical example of a money market into which private investors can freely enter. Borrowing from and investment in this market by private firms

\textsuperscript{24}. It is possible for a rise in the deposit rates to increase money market rates (through changes in the portfolio of the private sector and a shift of the banking sector's source of funds away from deposits toward money market borrowing), and to raise the loan rates. However, as was already mentioned in footnote 4, the stylized facts indicated in Section II cannot be explained by this mechanism. Thus, in this section, we assume that the central bank sets the target for money market rates, and eliminate in advance the influence exerted through the above-mentioned mechanism.
started around mid-1960s, and have expanded steadily since the 1970s hand in hand with the development of the bond circulation market. Meanwhile, with regard to small funds held by households, short-term assets with yields tied to money market rates did not exist at least until the end of the 1970s. However, with the progress of financial liberalization, new instruments (medium-term bond investment funds, etc.) have been introduced which made deregulated money markets more accessible for small savers and their amounts have increased rapidly since then. If these phenomena can be interpreted as increases in the degree of responsiveness of demand for bank loans and supply of primary deposits to interest rate changes (\( \tilde{L}^B_a, D^P, D^B_d \)), it means that the effects of (1) and (2) have become more pronounced since the 1970s, thereby increasing the policy effectiveness of controlling money market rates.

Let us next consider the effect of (3) (that which is exerted directly on banks' supply of loans by changes in money market rates and "institutional" rates). Rewriting this term using the results of the comparative statics presented in Appendix 1, we get the following expression:

\[
\frac{\tilde{L}^B_a + \tilde{L}^B_d - \tilde{L}^B_e}{Z} = \frac{1}{Z} (\tilde{b} - 1) \pi \alpha^{-1} D^g \left[ \alpha D (\tilde{b} - 1) - \beta (N + D_a - C) \right] \tag{19}
\]

where

- \( \tilde{b} = 1/b \) (reciprocal of the debtors' deposit ratio).
- \( Z \) is the determinant of the bordered Hessian matrix (\( Z > 0 \)) obtained in Appendix 1.

The first term \( (\alpha D (\tilde{b} - 1)) \) in parenthesis corresponds to the cost effect brought about by changes in money market rates, while the second term corresponds to the difference in the profit effect caused by changes in money market rates and "institutional" rates (note that \( N + D_u - C \) in the second term corresponds to the difference between \( C \), debts borrowed at money market rates, and \( N + D_u \), debts borrowed at "institutional" rates). In short, equation (19) means that, when \( \beta \) is substantially larger than \( \alpha \), the larger the proportion of liabilities tied to "institutional" rates in the liability composition of banks, the greater the relative influence of changes in "institutional" rates, and conversely, the higher the ratio of funds acquired at money market rates, the larger the loan rates' response to fluctuations in money market rates. As Figure 9 demonstrates clearly, the ratio of bank liabilities acquired at money market rates has been on the rise as a general trend. Furthermore, if interest rates on deposits are liberalized in the future, the ratio will increase significantly. Thus, it can be said that the effect of (3) has raised, and will raise, further the
Figure 9  Changes in Debt Components of Commercial Banks

Notes: 1. The restriction on the rates on non-resident yen deposits held by foreign public organizations was lifted in March 1983.
2. Although the rates on foreign currency deposits are unrestricted, these deposits are classified into "others" since their conversion to yen was restricted till May 1984 and they were separated from the domestic loan market as a result.
effectiveness of policies to control money market rates.

VI. Reinterpretation of the Developments of Loan Rates

On the basis of the foregoing discussions, the following reinterpretation of the actual loan rate series since the 1960s can be advanced (see Figure 9 for the liability composition of banks at various points in time).

Let us first analyze the determination of the effective loan rates. Prior to 1973, the rates on call and bills (the only money market instruments of the day available to the banks) fluctuated actively, exerting a certain influence on loan rates through the cost effect. However, since call and bills accounted for only a small portion of bank liabilities, the profit effect of changes in call and bills rates was small. With regard to the official discount rate, in addition to its small range of fluctuation, the ratio of the Bank of Japan loans to total bank liabilities was small, and so was the impact of the official discount rate on loan rates through the profit effect. Moreover, even with regard to deposits, which accounted for a large portion of bank liabilities, their impact on loan rates was limited since the deposit rates were held almost constant throughout this period. Consequently, the range of fluctuations in effective loan rates was small prior to 1973.25

After 1973, the call and bills rates continued to fluctuate actively influencing the effective loan rates through the mechanism of the cost effect (at least to the same extent as they did prior to 1973). In addition, their profit effect also gradually increased during this period reflecting a rise in the proportion of liabilities which bear money market rates as a result of the introduction of CDs (negotiable certificates of deposits), lifting of restrictions on “gensaki” trading by commercial banks, and so on. Meanwhile, although the range of its fluctuation expanded following 1973, the official discount rate continued to play only a minor role in loan rate determination because the proportion of the Bank of Japan loans in total bank liabilities further declined during this period. By contrast, as we have noted earlier, the range of

25. It is often claimed that it was a fundamental policy of the monetary authorities in the 1960s and early 1970s to keep various interest rates artificially at low levels. Opinions differ as to the true nature of this policy and its contribution to the growth of the Japanese economy (see Royama 1982, 1984). An interpretation of the loan market during this period based on the model in this paper is as follows. Firstly, income was transferred from depositors to banks as a result of interest rates on deposits being held at a level lower than money market rates. Since this transfer caused the supply of loans by banks to increase through the profit effect, part of it was handed to the business sector in the form of “excess” loans (more than appropriate for maximization of bank profits). Thus, the policy of maintaining interest rates on deposits artificially low served to stimulate lending activities, and, in this sense, had a positive effect on economic growth (see Horiuchi 1984, for an opposite interpretation).
fluctuations of interest rates on deposits also expanded after 1973, which had an enormous potential impact on the level of bank profits because of high share of deposits in the liabilities of the banking sector. If the banks were acting, as we have assumed, to increase both their profits and business scale, the changes in deposit rates must have exerted an important influence on loan rates through the profit effect.

Let us next turn to face loan rates. On the basis of the above interpretation, it can be argued that, while there is only a slight direct causal relationship between effective loan rates and the official discount rate reflecting the limited quantitative importance of the Bank of Japan loans, the range and direction of their fluctuations for the most part coincided through the 1960s and 1970s. Therefore, the prime rate system that tied face loan rates to the official discount rate functioned as a leverage expediting adjustment of effective loan rates to their equilibrium levels. Thus, the system economized the process (and cost) through which equilibrium in the loan market was achieved. In this sense, the existing system was a system banks must have found easy to accept.

But in the 1980s, this situation began to change. The share of debts acquired at money market rates has been increasing during this period and is expected to expand dramatically in near future due to liberalization of interest rates on deposits. This means that the profit effect of changes in money market rates will become more pronounced, since the same amount of change in these rates will have much stronger impact on the level of the bank profits than before.26 By the same token, a declining share of the "institutional" debts in the bank balance sheet indicates that the profit level of the bank becomes less sensitive to the changes in "institutional" rates, and that these rates lose their importance as a factor influencing the supply of bank loans. In other words, in the 1980s, the relationship of the effective loan rates with "institutional" rates becomes weaker, and in its stead, their relationship with money market rates becomes stronger. If the existing system of linkage between face loan rates and "institutional" rates continues, unless the latter rates are frequently changed following changes in money market rates, various lending conditions other than face interest rates will have to change to achieve equilibrium in the loan market. If this adjustment fails to operate fully (or if the adjustment cost is high), the existing system will lose its viability.

26. The extent of cost and profit effects is also dependent on parameters $\alpha$ and $\beta$ (elasticity of objective function $\Phi$ with respect to $\pi$ and $D$). Generally speaking, the higher the parameter $\alpha$ relative to $\beta$, the stronger the tendency of banks to seek for current profits and the larger the influence of the cost effect. This fact suggests the existence of another channel through which the relative size of the cost effect and the profit effect (and also the relative impact of changes in money market rates and "institutional" rates on loan rates) may have changed.
The above interpretation of the determination mechanism for face and effective loan rates implies the following points concerning the effectiveness of monetary policy. Firstly, with regard to the alteration of "institutional" rates, policy effectiveness will be maintained through the profit effect as long as liabilities acquired at "institutional" rates remain in banks’ balance sheets. But if interest rates on deposits are deregulated, the influence of "institutional" rates will become weaker. By contrast, the impact of money market rate control is expected to expand in the future and thus to assume a key role in the conduct of monetary policy.

Let us conclude by listing limits and problems of this paper. First of all, since this paper focused on the loan supply schedule of commercial banks, other factors such as changes in firms’ loan demand and influences of banks’ assets other than loans (especially their bond holdings) were ignored. Secondly, the theoretical and actual background of function \( \Phi \), the basis on which the arguments in this paper were derived, still needs to be analyzed in detail. Thirdly, in this paper “cost effect” and “profit effect” were contrasted and the latter’s role was emphasized. But it remains to be tested empirically whether the latter is actually so large relative to the former that the treatment in this paper is justified.\(^{27}\) It is our future task to give answers to these problems.

**Appendix 1 Comparative Statics in Section IV**

The problem that the bank faces is maximization of the function \( V \) shown below:

\[
V = \pi^a D^\beta + \lambda \left\{ \pi - (\bar{b} - 1) (\bar{r}_l - r_e) D_d - (r_e - r_d) D_a - (r_e - r_n) N + F (\bar{b} - 1) D_d \right\}
\]

where \( \lambda \) is a Lagrangean multiplier.

The first order conditions for maximization of \( V \) are as follows:

\[
\frac{\partial V}{\partial \pi} = \alpha \pi^{a-1} D^\beta + \lambda = 0 \tag{I-1}
\]

\[
\frac{\partial V}{\partial D_d} = \beta \pi^a D^{\beta-1} - \lambda (\bar{b} - 1) (\bar{r}_l - r_e - \pi') = 0 \tag{I-2}
\]

27. Noma (1985) conducted an empirical test comparing alternative bank objective functions and obtained a result that suggested that Japanese banks were seeking for a larger business scale rather than maximization of profits.
\[
\frac{\partial V}{\partial \lambda} = \pi - (\bar{b} - 1) (\bar{r}_n - r_n) D_a - (r_e - r_d) D_a \\
- (r_e - r_n) N + F ( (\bar{b} - 1) D_a ) = 0
\]
\[
(1-3)
\]
where \( F' = \frac{dF((\bar{b} - 1) D_a)}{d(\bar{b} - 1) D_a} \).

Eliminating \( \lambda \) from (I-1), (I-2),
\[
\bar{r}_l - r_e - F' = -\frac{\beta \pi}{(\bar{b} - 1) \alpha D} \leq 0
\]
\[
(I-4)
\]
can be obtained. When \( \beta = 0 \), the right hand side of this equation is zero. This equation coincides with the first order condition for the usual profit maximization, namely \( \bar{r}_l = r_e + F' \) (marginal revenue=marginal cost). When \( \beta > 0 \), the right hand side will be negative (that is, \( \bar{r}_l < r_e + F' \)), indicating that lending is at a level where marginal cost exceeds marginal revenue. The determinant of the Hessian matrix can be calculated from equations (I-1~3) as follows:
\[
Z = \pi^{\alpha - 1} D^{\beta - 2} \{ (\alpha + \beta) \frac{\beta}{\alpha + \alpha (\bar{b} - 1)} F'^2 \} > 0
\]
\[
(I-5)
\]
where
\[
F'^2 = \frac{d^2 F((\bar{b} - 1) D_a)}{d \{ (\bar{b} - 1) D_a \}^2}
\]

This implies that the second order condition for maximization is satisfied. Next a comparative static analysis is carried out to determine how the equilibrium point changes when the various interest rates (\( \bar{r}_l, r_e, r_n, r_d \)), loans made by the Bank of Japan (\( N \)), primary deposits (\( D_a \)) and debtors' deposit ratio (\( b \)) change (we examine only those cases in which different results are obtained depending on whether \( \beta = 0 \)).

First, the changes in profits when the effective loan rates change are as follows:
\[
\frac{d\pi}{d\bar{r}_l} = \frac{1}{Z} (\bar{b} - 1) \pi^{\alpha - 1} D^{\beta - 1} \{ \alpha D^2 D_a (\bar{b} - 1) F'^2 - \beta \pi D_a \} \leq 0
\]
\[
(I-6)
\]
When \( \beta = 0 \), the value within the parenthesis in equation (I-6) is positive so that \( d\pi/d\bar{r}_l \) is positive; that is, a rise in loan rates always accompanies a rise in profits. On the other hand, when \( \beta \neq 0 \), the sign of \( d\pi/d\bar{r}_l \) is unclear.\(^{28}\)

Next, the changes in profits and loans when money market rate \( r_e \) changes are as

---

28. An abnormal case \( d\pi/d\bar{r}_l < 0 \) may result when the F function has certain specific attributes and \( \beta \) is sufficiently large.
follows:

\[
\frac{d\pi}{dr_e} = -\frac{1}{Z} \pi^{\alpha - 1} D^{\beta - 1} \left( \alpha D^\beta (\bar{b} - 1)^2 C F^\alpha \right)
+ \beta \pi \left( \frac{C - D (\bar{b} - 1)}{Z} \right) \geq 0
\]

\[
dL/\,dr_e = -\frac{1}{Z} b \pi^{\alpha - 1} D^{\beta - 1} \left[ \alpha D (\bar{b} - 1) + \beta C \right] \geq 0.
\]  

Equation (I-7) shows that when \( \beta = 0 \), the sign of \( d\pi/dr_e \) is the opposite of that of \( C \). Thus, depending on the bank’s money market position, the direction of changes in \( \pi \) is determined without ambiguity. Meanwhile, when \( \beta \neq 0 \), the sign of \( d\pi/dr_e \) is determined as positive only when \( C < 0 \) (lending position bank). Turning to equation (I-8), as in equation (I-7), we find that the sign of the equation is determined when \( \beta = 0 \) (regardless of the bank’s money market position), and a rise in money market rates will always reduce the bank’s loan supply. The effect that comes into play when \( \beta = 0 \) is the cost effect discussed on p. 93 of the main text, so that the first term \( \alpha D(\bar{b} - 1) \) within the parenthesis in equation (I-8) represents this cost effect. Meanwhile, when \( \beta \neq 0 \), the influence of the second term \( \beta C \) within the parenthesis is added to this. The effect that comes into play only when \( \beta \neq 0 \) is the profit effect discussed in the text; it has a restraining effect on loan supply when \( C > 0 \) and a stimulating effect when \( C < 0 \). As a result, the sign of \( dL/dr_e \) is negative when \( C > 0 \), but when \( C < 0 \), it can become either positive, zero or negative depending on the relative sizes of the cost effect and profit effect. And the relative sizes of these two effects are determined by the relative sizes of \( D \) and \( C \), and \( \alpha \) and \( \beta \).

Next, the changes in loans when “institutional” rates \( r_n \), \( r_d \) change are as follows: \(^{29}\)

\[
dL/\,dr_n = -\bar{b} b D_d / dr_n = -\frac{1}{Z} \bar{b} \pi^{\alpha - 1} D^{\beta - 1} \leq 0
\]

\[
dL/\,dr_d = -\bar{b} b D_d / dr_d = -\frac{1}{Z} \bar{b} \beta D_d \pi^{\alpha - 1} D^{\beta - 1} \leq 0.
\]

\( dL/dr_n = dL/dr_d = 0 \) when \( \beta = 0 \), but when \( \beta \neq 0 \), an increase in “institutional” rates reduces the bank’s fund supply (this indicates that the reduction is brought about through the profit effect). Furthermore, the extent of the reduction depends on the

\(^{29}\) Comparative static analysis is carried out assuming that \( \bar{r} \) remains constant. This means that the nominal loan rate \( r_l \) will change in such a way that it offsets any changes in \( \bar{r} \) caused by a change in \( r_d \).
amount of debts the acquisition costs of which are affected by changes in interest rates; that is, it depends on the amount of the Bank of Japan loans and the primary deposits (note that \( N \) and \( D_u \) appear in the equations).

Next, the changes in loans when the amounts of the Bank of Japan loans and the primary deposits rationed to the bank change are as follows:

\[
\frac{dL}{dN} = \frac{b}{Z} \beta (r_e - r_u) \pi^{\alpha - 1} D^{\beta - 1} \geq 0 \quad (I-11)
\]

\[
\frac{dL}{dD_u} = \frac{b}{Z} \beta (r_e - r_d) \pi^{\alpha - 1} D^{\beta - 1} \geq 0 \quad (I-12)
\]

Equations (I-11~12) are the same as equations (I-9~10) except that \( N \) and \( D_u \) in the latter are replaced by \(-(r_e - r_u)\) and \(-(r_e - r_d)\) in the former. In other words, whereas in equations (I-9~10) a change in “institutional” rates causes a change in profits equal to \( \Delta r_n \times N \) and \( \Delta r_d \times D_u \) (\( \Delta \) represents a marginal change), in equations (I-11~12) a change in \( N \) and \( D_u \) causes a change in profits equal to \( \Delta N \times (r_e - r_u) \) and \( \Delta D_u \times (r_e - r_d) \). In both cases, a change in profits affects the supply of loans through the profit effect. Therefore, as with changes in “institutional” rates, \( dL/dN = dL/dD_u = 0 \) when \( \beta = 0 \).

Finally, the changes in profits and effective loans \((L - D_d)\), when the debtors’ deposit ratio \( b \) changes, are as follows:

\[
\frac{d\pi}{db} = -\frac{1}{Z} \frac{b^2}{a(b - 1)} \beta^2 D_u \pi^{\alpha + 1} D^{\beta - 3} \leq 0 \quad (I-13)
\]

\[
\frac{d(L - D_d)}{db} = \frac{1}{Z} b^2 \beta D_u \pi^{\alpha} D^{\beta - 2} \geq 0 \quad (I-14)
\]

An increase in \( b \) will cause changes neither in profits nor in effective loans when \( \beta = 0 \), but when \( \beta \neq 0 \), effective loans increase while profits decline. Meanwhile, the changes in the value of \( \Phi \) are:

\[
\frac{d\Phi(\pi, D)}{db} = \partial \Phi / \partial \pi \cdot d\pi / db + \partial \Phi / \partial D \cdot dD / db
\]

\[
= \frac{b^2}{(b - 1) \beta^2 \pi^{\alpha} D^{\beta - 1} D_u} \geq 0
\]

which indicates that a more desirable combination of \( \pi \) and \( D \) is realized despite the

30. As in the case of footnote 29, a change in \( b \) is offset by a change in \( r_i \), and \( \tilde{r}_i \) is assumed to be held constant.
decline in current profits. It is generally believed that banks try to increase the debtors’ deposit ratio in order to raise effective loan rates, but our model suggests that banks may try to raise the ratio even in a situation where \( \tilde{r}_l \) is determined as a market equilibrium value.

Appendix 2  Comparative Static Analysis in Section V

Of the three equilibrium equations indicated in Section V (pp. 97-98), we exclude equation (13) according to Walras’ law, and differentiate the remaining equations (12) and (14). In doing so, we assume that the monetary policy is conducted by means of money market operations and that the Bank of Japan loans are held constant (that is, \( dX^c = dC^c \) and \( dN^c = 0 \)).

\[
\begin{pmatrix}
1 & -X_l^p \\
0 & \tilde{L}_l^b - \tilde{L}_l^p
\end{pmatrix}
\begin{pmatrix}
dX_l \\
d\tilde{r}_l
\end{pmatrix} =
\begin{pmatrix}
X_c^p \, dr_e + X_d^p \, dr_d \\
(L_e^p - L_e^b - L_d^b \, D_e^p) \, dr_e - \tilde{L}_a^b \, dr_a \\
-(L_d^b + \tilde{L}_d^b \, D_d^p) \, dr_a - \tilde{L}_d^b \, db
\end{pmatrix}
\] (II-1)

The determinant of the Jacobian matrix is:

\[
Y = \tilde{L}_l^b - \tilde{L}_l^p > 0.
\] (II-2)

From (II-1~2), equations (15)~(17) in the text which express the impact of changes in \( r_e \), \( r_n \), \( r_d \) on \( \tilde{r}_l \) can be derived. That of changes in \( b \) is given below:

\[
\frac{d\tilde{r}_l}{db} = -\frac{1}{Y} \tilde{L}_d^b \leq 0.
\] (II-3)

In other words, a rise in the debtors’ deposit ratio raises the supply of loans by banks only when \( \beta \neq 0 \), thus lowering effective loan rates.

Next, we examine a case in which the monetary policy is conducted by means of increases and decreases in the Bank of Japan loans (that is, \( dX^c = dN^c \) and \( dC^c = 0 \)). The Jacobian determinant \( Y' \) has the following value:

\[
Y' = \begin{vmatrix}
1 & -X_l^p \\
\tilde{L}_l^b - \tilde{L}_l^p & \tilde{L}_N^b - \tilde{L}_l^p
\end{vmatrix} = \frac{\tilde{L}_l^b - \tilde{L}_l^p + \tilde{L}_N^b \, X_l^p}{Y} \geq 0.
\] (II-4)
Although the sign of $Y'$ cannot be determined, we assume $Y' > 0$. Comparing equation (II-2) and equation (II-4), we find $Y \geq Y'$ (equality holds when $\beta = 0$). The change in the effective loan rates $\hat{r}_t$ when money market rates are changed (through changes in the Bank of Japan loans) is as follows:

$$
\frac{d\hat{r}_t}{dr_t} = \frac{1}{Y} \left\{ \left( \hat{L}_c^\theta - \hat{L}_e^\beta - \hat{L}_d^\beta \hat{D}_e^\theta \right) - \hat{L}_N^\beta \chi_t^P \right\} > 0.
$$

(II-5)

Comparing equation (II-5) with equation (15) (p. 99 of the main text), we find that the numerator of the former is larger than or equal to the numerator of the latter (equality holds when $\beta = 0$). With regard to the denominator we have seen that $Y \geq Y' > 0$, so that $d\hat{r}_t/dr_c \mid _{dX=dN} \geq d\hat{r}_t/dr_c \mid _{dX=dC}$ (equality holds when $\beta = 0$). In short, the monetary policy may have a greater effect via adjustment of the Bank of Japan loans than via money market operations.
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