

The Role of “Determinacy” in Monetary Policy Analysis

Keynote Speech by Bennett T. McCallum

It is well known that the concept of “determinacy”—a single stable solution—plays a major role in contemporary monetary policy analysis. But while determinacy is desirable, other things equal, it is not necessary for a solution to be plausible and is not sufficient for a solution to be desirable. There is a related but distinct criterion of “learnability” that seems more crucial. This paper argues that recognition of information feasibility requires that a candidate solution must, to be plausible, be quantitatively learnable on the basis of information generated by the economy itself. Since a prominent least-squares (LS) learning process is highly “biased” toward learnability, it is reasonable to regard it as a necessary condition for any specific solution to be relevant. This implies that determinacy is not necessary for policy analysis; there may be more than one stable solution, but only one that is LS learnable. Also, determinacy is not sufficient for satisfactory policy analysis; explosive solutions pertaining to nominal variables will not be eliminated by transversality conditions. For these and other reasons, the role of determinacy in monetary policy analysis should be reconsidered and substantially de-emphasized.

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I. Introduction

The conference topic “Financial System and Monetary Policy Implementation” is extremely important as well as highly topical. It is also quite broad, indeed broad enough to justify my topic, which is the role of determinacy in monetary policy analysis. This may seem somewhat esoteric, but it is very well known to researchers that the concepts of “determinacy” and “indeterminacy” play a fundamental role in contemporary policy analysis—and especially in relation to monetary policy implementation. In terms of statistical evidence regarding research devoted to determinacy issues, a bit of searching shows that explicit references to this general topic appear on about 75 different pages in Michael Woodford’s hugely influential treatise *Interest and Prices* (Woodford [2003a]). In addition, the number of new writings (books, articles, and working papers) with both of the phrases “indeterminacy” and “monetary policy” appearing in their text was 166 over the time span January 1995 through June 2008.¹ In this literature, the meaning of determinacy is that the system being analyzed—a macroeconomic model plus the central bank’s policy rule—has a single rational expectations (RE) solution that is dynamically stable (i.e., not explosive). “Indeterminacy” is usually taken to mean more than one stable solution, so a third possibility is that none of the RE solutions is stable. Then the standard procedure in policy analysis is to treat determinacy as a necessary condition for a recommended (or even potentially recommended) policy rule. In other words, model-plus-rule combinations that imply either indeterminacy or an explosive solution are ruled out as highly undesirable.² If a model is taken as given, then any policy rule that leads to indeterminacy is typically viewed as not worthy of consideration.

For several years, I have taken a minority position based on a belief that the emphasis being given to these particular determinacy/indeterminacy concepts is unwarranted and occasionally misleading, especially in the design of recommended rules for monetary policy. I would agree that, other things equal, determinacy is desirable; but would contend that (assuming a given model) determinacy is not necessary for a solution to be plausible and in any case is not sufficient for a solution to be desirable. There is a related but distinct criterion of “learnability” that is more crucial, and which should be regarded as a *necessary* condition for a solution (resulting from a model-plus-rule combination) to be considered as plausible and thus relevant for policy analysis. This position, which has also been explicitly or implicitly advanced by a few other scholars,³ leads to different conclusions regarding suitable policy rules in a number of cases that have received considerable attention in the literature.

1. This figure comes from use of the EBSCOhost search engine.

2. See, for example, Woodford (2003a, p. 45 and p. 77).

3. These include Evans and Honkapohja (1999, 2001), Bullard (2006), and Bullard and Mitra (2002). It might be said that Woodford, too, supports this position: see Woodford (2003a, pp. 261–276; 2003b, p. 1178).

II. Determinacy and Learnability

Learnability of RE solutions has been discussed for many years (see, e.g., Bray and Krepes [1981] or Bray [1982]), but has come to the forefront of monetary policy analysis fairly recently; in particular, since the publication of the prominent treatise by Evans and Honkapohja (2001).⁴ The basic idea is that individual agents must obtain their quantitative knowledge of the dynamic properties of the system—which is necessary for forming expectations rationally—on the basis of data generated by the economy itself; they are not given such knowledge by magic or divine revelation. Thus, this knowledge must be based on some learning process that depends upon past observations of variables of the system. Then, since random shocks are bound to occur, one needs to determine whether a system that is slightly disturbed from an RE solution path would tend to return to that path, and consequently has to examine the dynamics of learning. If the implied learning process is not stable, the solution under consideration is highly unlikely to prevail.

The particular learning process that is featured most prominently in the work of Evans and Honkapohja (E&H) is least-squares (LS) learnability. For this concept, one imagines that the individual decision-makers in the model economy are continually attempting to learn the quantitative features of a forecasting equation for use in forming expectations about endogenous variables that will prevail (or become known) in the future. The hypothetical process is that in each time period a typical decision-maker will use available data generated by the economy to estimate a forecasting model that includes the relevant endogenous variables. He then makes supply-demand choices based on these forecasts and interacts with other agents on markets. Markets clear and generate new prices and quantities, which then enter datasets for the next period. As time passes, these estimates may become progressively nearer to being correct, since the model economy’s structure is assumed to be unchanging. If this process leads, in the limit as the number of observations increases without bound, to a particular RE solution, then this solution is learnable. Conversely, if the process does not converge to a given RE solution process, after a small displacement from it, then that solution is not learnable. There are, of course, many possible learning procedures that could be considered. The one that is emphasized by E&H, LS learnability, adds to the description above the proviso that the agents’ forecasts are based on an estimated vector autoregression (VAR) model that is correctly specified and estimated by ordinary LS. These features, in the context of an ever-increasing sample size, tend to be highly favorable to the prospects for convergence—that is, “biased” toward a finding of learnability.⁵

The technical apparatus needed for the study of LS learnability has been provided, for linear models, by E&H (2001, pp. 173–263). A brief summary of the essential results is as follows. Almost any linear or linearized model can be written in the form

$$y_t = A E_t y_{t+1} + C y_{t-1} + D z_t, \quad (1)$$

4. The slightly earlier survey article by the same authors—Evans and Honkapohja (1999)—was also influential.

5. Additional discussion on this point will be presented below.

where y_t is an $m \times 1$ vector of endogenous variables and z_t is an $n \times 1$ vector of exogenous variables generated by a stable first-order autoregressive process:

$$z_t = Rz_{t-1} + \varepsilon_t. \quad (2)$$

Specifically, by the use of auxiliary variables, systems with variables lagged any finite number of periods into the past can be expressed in this format, and so can variables expected in period t to prevail any finite number of periods in the future. Also, expectational variables can be lagged.

Consider fundamental solutions of the form

$$y_t = \Omega y_{t-1} + \Gamma z_t. \quad (3)$$

It can easily be shown, by straightforward undetermined-coefficients reasoning, that for any given Ω there is a unique Γ that satisfies (1), but Ω is determined by the matrix quadratic equation

$$A\Omega^2 - \Omega + C = 0, \quad (4)$$

which has many solutions— $(2m)!/(m!)^2$ to be exact. If more than one of these is dynamically stable, we have indeterminacy.

For this very broad class of models, E&H (2001, p. 238) have established that the sufficient and generically necessary conditions for LS learnability are that the eigenvalues of the following three matrices have all their real parts with values less than 1.0:

$$F = (I - A\Omega)^{-1}A, \quad (5a)$$

$$\Omega' \otimes F, \quad (5b)$$

$$R' \otimes F. \quad (5c)$$

It will be noted that, if there are no lagged endogenous variables in the system, then $C = 0$ implying that $\Omega = 0$ and $F = A$. In that special case, the first two conditions amount to the requirement that the eigenvalues of A all have real parts less than one.

III. Determinacy Is Not Necessary

Consideration of the necessity of learnability for an RE solution to be regarded as plausible, as a description of an economy's behavior, indicates clearly that determinacy is not a necessary condition for an RE solution to be satisfactory as the basis of a policy-design exercise. If a situation of indeterminacy—multiple stable solutions—prevails, it is nevertheless possible that all but one of the solutions can be ruled out as implausible on the basis of non-learnability. Here the point is that determinacy requires that all of

the eigenvalues of the matrices Ω and F , denoted λ_Ω and λ_F , must be less than 1.0 in modulus.⁶

Suppose then that for one particular solution of form (3) all $|\lambda_\Omega| < 1$, but $|\lambda_F| < 1$ fails because one $\lambda_F < -1$. Then there are two dynamically stable solutions—indeterminacy prevails—but the particular solution under consideration may be learnable.⁷ Suppose further that there is no other solution—no other grouping of the system eigenvalues—for which all the conditions for learnability are satisfied. Then I would contend that only one of the RE solutions is plausible, since any other dynamically stable solution is not learnable; that is, the relevant learning dynamics needed for the agents to be able to form expectations rationally is expectationally unstable. In this case, there is no need to be concerned by the possibility that the system would be found following the non-learnable solution.

In McCallum (2003), I have argued that this conclusion is relevant for two problematic issues that have been prominent in the monetary policy literature. One of these involves cases in which the central bank is using a policy rule, for setting each period’s one-period nominal interest rate, which involves responses to an expected future inflation rate, rather than the current inflation rate. Several analysts, most prominently Woodford (1994; 2003a, pp. 252–261) and Bernanke and Woodford (1997), have shown that in such cases indeterminacy can be brought about by excessively strong responses (as well as by ones that are too weak to satisfy the Taylor principle) and have suggested that policy should therefore avoid very strong responses to expected future inflation. My 2003 paper argues against the latter conclusion on the ground of a learnability discussion, as suggested above. Woodford (2003b) points out that my argument does not eliminate the possibility of additional (undesirable) solutions of the “resonant frequency sunspot” variety. I believe that these are implausible for reasons that will be hinted at below, but must admit that my case is not theoretically airtight.

A significant extension of the foregoing case has been provided by Kurozumi and Van Zandweghe (2008). These authors consider an extension of the basic type of New Keynesian model so as to explicitly include endogenous investment in physical capital. Previous analysis by Carlstrom and Fuerst (2005) had indicated that the inclusion of investment greatly exacerbates the tendency to indeterminacy described in the previous paragraph, with indeterminacy in fact prevailing for *all values* of the coefficient representing responses to expected future inflation (assuming that there is no policy response to the output gap as a distinct variable). Kurozumi and Van Zandweghe (2008) corroborated that finding, but showed that additional policy response to output would reduce the region of indeterminacy. Moreover, with respect to the matter at hand they showed that adoption of the LS learnability requirement greatly reduces multiplicity problems; they state that “the forward-looking policy (i.e., response to $E_t\pi_{t+1}$) generates a locally unique non-explosive E-stable fundamental rational expectations equilibrium as long as the policy response to expected future inflation is sufficiently strong” (Kurozumi and Van Zandweghe [2008, p. 1489]).⁸

6. This result is established by McCallum (2007).

7. It will be learnable unless there is some eigenvalue of R that is negative and large enough that (5c) is violated.

8. E-stability is an analytical concept that is convenient for establishing LS learnability. It should be recognized that Kurozumi and Van Zandweghe (2008) did not consider sunspot solutions.

A second issue discussed in McCallum (2003) concerns the possibility of zero-lower-bound “inflation traps” of the type discussed extensively by Benhabib, Schmitt-Grohé, and Uribe (2001, 2002) among others. In this context, my 2003 analysis indicates, for the special case with full price flexibility, that the inflation-trap solution is not learnable, whereas the solution that results in the central bank’s target rate of inflation is learnable. There is in this case no disagreement with Woodford (2003b) with respect to the substantive conclusions concerning outcomes under standard policy, although he views the learnability analysis as unnecessary.

IV. Determinacy Is Not Sufficient

The argument that determinacy is not necessary for a single RE solution to be plausible has been developed by several authors over many years.⁹ Rather recently, Cochrane (2007) has argued that determinacy is not sufficient. This argument can be outlined easily and briefly, as follows. In the most standard class of New Keynesian models for monetary policy analysis, there is invariably an RE solution with explosive inflation. Such solutions are usually not considered to be contenders for describing the behavior of the economy in question; they are considered to be ruled out by the requirement of determinacy. But Cochrane asks, what is the justification for that? How does one know that a Taylor-style¹⁰ policy rule will not permit explosive inflation? The usual answer is that explosive solutions typically violate transversality conditions that are necessary for optimality of individuals’ choices. But those conditions pertain to *real* variables, such as the stock of real money balances held, or real bonds, or capital. There is no such rationale for ruling out solutions in which inflation explodes. In Cochrane’s words, “Nothing in economics rules out explosive or ‘non-local’ nominal paths. Transversality conditions can rule out real explosions, but not nominal ones” (2007, p. 2). It is my belief that this position of Cochrane’s is analytically correct. I have disputed, however, his strongly expressed contention that this conclusion warrants a distinctly negative evaluation of current mainstream monetary policy analysis. Instead, I suggest, the latter is in many cases justified by a learnability requirement of the type discussed above in Section III.

V. Information Feasibility

Cochrane’s 2007 discussion mentions but rejects arguments based on learnability. He says:

A wide variety of almost philosophical principles have been advocated to prune equilibria. For example, Evans and Honkapohja (2001) advocate criteria based on least-squares (LS) learnability, and McCallum (2003) advocates a “minimum state variable criterion,” which he relates to learnability. These refinements go

9. In this regard, the work of E&H (1999, 2001), Bullard (2006), and Bullard and Mitra (2002) is quite notable.

10. Introduced in Taylor (1993).

beyond the standard definitions of economic equilibria. One may argue that when a model gives multiple equilibria, we need additional selection criteria. I argue instead that we need a different model.¹¹

I believe that this aspect of Cochrane’s argument is not persuasive. A basic premise of any serious model is that equilibrium outcomes must be feasible—private and public consumption and investment cannot exceed the amount produced and drawn down, and the amount produced is bounded by production possibilities and resource availability. But feasibility applies to information, as well as tangible resources. Thus, in a model in which there is randomness, agents cannot know future values of prices and other variables; they have to form expectations based on (at most) knowledge of past and (perhaps) present values of these variables. Standard formal models accordingly specify information sets assumed to be available to agents in forming expectations, and these invariably include at most current and past values of endogenous and exogenous variables. I think that there should be no dispute over these statements, but would go further by arguing as follows. To form these expectations rationally, agents must have quantitative knowledge of the “laws of motion” of relevant variables. But such knowledge must be based, in reality and therefore in any model to be used to mimic reality, on data generated by the economy itself. Individuals cannot—as mentioned above—obtain such information by magic, or by divine revelation. Accordingly, for a proposed equilibrium to be feasible, it must be the case that that information generated in the past is sufficient to permit individuals to develop forecasting rules that mimic the quantitative properties of the actual laws of motion. In this sense, to be *plausible*, a proposed equilibrium must be “learnable.”

Some analysts object to the presumption that candidate equilibria must be learnable, on the grounds that “there are many possible learning procedures—how can you know that the one mentioned above is correct?” My response to that objection is that it does not distinguish between necessity and sufficiency. In McCallum (2007, p. 1378), the argument is expressed as follows:

The position that learnability (and thus E-stability) should be regarded as a necessary condition for the relevance of an RE equilibrium begins with the presumption that individual agents must somehow learn the magnitudes of parameters describing the economy’s law of motion from observations generated by the economy; they cannot be endowed with such knowledge by magic. Of course any particular learning scheme might be incorrect in its depiction of actual learning behavior. But in this regard it is important to note that the LS learning process in question assumes that (i) agents are collecting an ever-increasing number of observations on all relevant variables while (ii) the structure is remaining unchanged. Furthermore, (iii) the agents are estimating the relevant unknown parameters (iv) with an appropriate estimator (v) in a properly specified model. Thus if a proposed RE solution is not learnable by the process in question—the one to which the E&H results pertain—then it would seem highly implausible that it could prevail in practice.

11. Cochrane (2007, p. 44).

VI. Recent Developments

Additional lines of argument have been developed more recently. In Cho and McCallum (2009) the following argument is put forth. Consider a model in which there are two sectors, one of which is autonomous. That is, this “first” sector determines the values of some of the model’s variables without reference to any variables or activities pertaining to the “second” sector. The latter, by contrast, determines values of the remaining endogenous variables in a manner that has them dependent upon the values generated in the first sector. In such a setting, it is possible that the overall system has the property of determinacy, suggesting that all variables are dynamically stable, whereas analysis reflecting the block-recursive nature of the system yields quite different conclusions. Cho and McCallum (2009) present numerical examples for two-sector models of this type, with each sector having just a single endogenous variable (y_{1t} and y_{2t}) whose value in each period depends upon its own lagged value and its own value expected one period in the future. Sector one is autonomous, but in sector two y_{2t} depends in part on the expected value of y_{1t+1} . In one example, consideration of sector one as autonomous indicates that y_{1t} is dynamically stable, while analysis of sector two, with y_{1t} treated as exogenous, indicates that y_{2t} will explode relative to the path of y_{1t} . In a second example, the variable of the autonomous sector is explosive but the parameters for the other sector suggest that its endogenous variable will behave in a stable manner relative to the explosive path of the autonomous sector.¹² Thus in both cases, standard analysis indicates that the bivariate system is determinate, whereas analysis that recognizes the block-recursive nature of the system indicates that in the first case one variable will be explosive and the other stable, while in the second case both variables will be explosive.

Very recently, I have proposed a distinct and more radical argument. It goes as follows. Consider a linear model with m endogenous variables. In any such model, there are $(2m)!/(m!)^2$ different fundamental RE solutions, that is, expressions relating endogenous to predetermined and exogenous variables, which satisfy all orthogonality conditions for RE. It can be shown, however, that each of these solutions represents a different specification relating to the model’s state variables; there is a one-to-one relationship between solutions and state-variable specifications. But any *structural* model, one that is designed to be policy invariant, is built upon a particular specification regarding state variables. Therefore, any structural model leads to a unique RE solution of the fundamentals type; the other fundamental solutions represent different assumptions pertaining to the relevant state variables. But what about “sunspot” solutions, ones that include random components unrelated to any of the fundamental variables? The answer is that the single fundamental solution that is consistent with the model’s state variable specification will not support sunspot variables; only the other fundamental solutions will do so. Thus sunspot solutions represent, apparently, random variations from solutions that are inconsistent with a crucial aspect of the model’s basic specification. This argument suggests, then, that there is an important sense in which

12. In Cho and McCallum (2009), the notation makes sector two the autonomous sector in this case. Here I have not followed that notation so as to avoid changing the designation of which sector is autonomous.

RE “solution multiplicities” represent a multiplicity of models rather than a multiplicity of solutions to a single model. It should be added that this contention is based on work in progress.

VII. Conclusion

In this paper, I have argued that fundamental recognition of information feasibility requires that a candidate solution must, to be considered plausible, be learnable on the basis of information generated by the economy—model economy or actual economy—itsself. Since the LS learning process is highly “biased” toward a finding of learnability, it is reasonable to regard LS learnability as a necessary condition for any specific solution to be relevant for policy consideration. This implies that determinacy is not necessary for policy analysis; there may be more than one dynamically stable solution but only one that is LS learnable. Determinacy is also not sufficient for satisfactory policy analysis; there needs to be some logical argument for ruling out explosive solutions pertaining to nominal variables; these are not necessarily eliminated by transversality conditions. Furthermore, consideration of models with block-recursive structures suggests that determinacy results pertaining to the overall system may be inconsistent with conclusions based on sectoral analysis that recognizes the block-recursive structure of the system. For these reasons, I believe that the role of determinacy in monetary policy analysis should be reconsidered and substantially deemphasized or replaced.

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