# The Arbitrage Efficiency of the Nikkei 225 Options Market: A Put-Call Parity Analysis

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*This paper is concerned with arbitrage efficiency of the Nikkei index option contracts traded on the Osaka Securities Exchange (OSE) within the put-call parity (PCP) framework. A thorough* ex post *analysis is first carried out. The results reveal a modest number of violations with 2.74 percent of the sample breaching the PCP equation and an average arbitrage profit of 22.61 index points for OSE member firms during the sample period (2003–05).* Ex ante *tests are then conducted whereby* ex post *profitable arbitrage strategies, signified by the matched put and call contracts, are executed with lags of one minute and three minutes. The* ex ante *results reveal that the number of profitable arbitrage opportunities and the average profit are both reduced significantly with an execution lag. In addition, regression analysis is used to provide further evidence about the PCP and arbitrage profitability. Overall, there is no strong evidence found against the efficiency of the Nikkei 225 options market, although arbitrage opportunities do exist occasionally.*

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## I. Introduction

The efficiency of the derivatives markets is important not only to investors for speculation, hedging, and investment purposes, but also to regulators and society as a whole. Growth of the financial markets may also depend on whether the markets are operating efficiently. Thus, the efficiency of the derivatives markets has drawn significant attention from researchers in the past several decades.

There are many papers in the literature investigating the put-call parity (PCP) model since the first study by Stoll (1969). More recently, many studies have focused on index options. Among the most recent, Capelle-Blancard and Chaudhury (2002) test the French index (CAC 40) option market, Mittnik and Rieken (2000) test the German index (DAX) option market, and Cavallo and Mammola (2000) test the Italian index (MIB30) option market. In addition, many studies have examined the joint efficiency of the options and futures markets using put-call-futures parity. These include Fung and Fung (1997), Fung and Mok (2001), Draper and Fung (2002), and Li and Alfay (2005).

This paper focuses on the arbitrage efficiency of the Nikkei 225 index options market in the Osaka Securities Exchange (OSE) and aims to provide evidence on the size and frequency of the arbitrage opportunities in the PCP framework. The Nikkei 225 options market is the largest and the most liquid stock price index option market[1] and ranked among the top 10 in the world in 2004 according to trading volume. Thus, the efficiency of the Nikkei 225 options markets is of great importance to many market participants.

To test for arbitrage opportunities, the option contracts need to be matched within a narrow time interval so that the non-synchronous price problem can be mitigated. This is undertaken mainly because arbitrage is based on the premise of simultaneously buying low and selling high to make a riskless profit. Tick-by-tick data are used in this paper, which permits the option contracts to be matched within a narrow time interval (one minute) for examination. A total of 139,586 matched pairs of put and call contracts over the period from January 2003 to December 2005 are found and used in the PCP tests.

*Ex post* tests are undertaken to provide evidence on the size and frequency of arbitrage profits. We determine whether arbitrage opportunities are available and viable when all costs including the implicit bid-ask spread are taken into account. This is done first for the whole sample. We then further investigate if the arbitrage opportunities are related to calendar years, moneyness, maturity, and whether a long or short arbitrage strategy is observed. *Ex ante* tests are then undertaken to illuminate the dynamic efficiency of the market. These tests allow for an execution lag of up to one minute or three minutes before establishing all arbitrage transactions in the option market and stock market. Finally, regression analysis is carried out to provide further evidence on the PCP and arbitrage profitability.

Research on the Japanese index options markets is very limited in the literature. Nishina and Nabil (1997) consider the return dynamics of Nikkei stock index

1. There are other stock price index options markets in Japan such as Nikkei 300 options on the OSE and the Tokyo Stock Price Index (TOPIX) options traded on the Tokyo Stock Exchange (TSE).

options. They conclude that there is no evidence of detectable intermarket arbitrage opportunities. But their study uses the daily closing prices and thus suffers from the non-synchronous problem. Shiratsuka (2001) considers the information content of implied probability distributions of Japanese price index options. He concludes that the implied probability distribution contains some information regarding future price movements, but its forecasting ability is not superior to that of the historical distribution. This in turn may indicate some degree of efficiency of the Nikkei 225 index options market. But the results are highly dependent on the validity of the option pricing model employed.

To the author's best knowledge, this paper is the first that attempts to investigate the arbitrage efficiency of the Nikkei 225 options market by using both an *ex post* and *ex ante* test. This paper is based on the PCP and thus does not depend on the validity of any option valuation models.

This paper makes a number of contributions to the literature. First, it provides new evidence on the arbitrage efficiency of the Nikkei 225 options market in recent years based on *ex post* analysis. It also contains a detailed breakdown of the arbitrage opportunities across calendar year, moneyness, maturity, and so on. Second, this paper provides *ex ante* evidence of the arbitrage efficiency of the Nikkei 225 options market. Finally, further evidence is also obtained by using regression analysis.

Our results reveal that there are occasionally arbitrage opportunities in the Nikkei 225 options market with significant average profit for OSE members on an *ex post* basis. With a time lag of one minute or three minutes, more than 30 percent of the arbitrage opportunities signified initially by the PCP are not profitable and the average arbitrage profit also decreases. The regression results provide supporting evidence for the *ex post* and *ex ante* results. Overall, there is no strong evidence against the efficiency of the Nikkei 225 options market.

The remainder of this paper is structured as follows. Section II illuminates the methodology. Section III discusses the contracts and data utilized. Section IV presents the empirical results. We summarize and conclude in Section V.

## II. Methodology Issues

In this section, we first recall the PCP that is used in this paper. Then we discuss some technical issues related to the PCP tests, the *ex ante* tests, and the regression analysis.

### A. PCP

Stoll (1969) has shown that the combination of a pair of otherwise equal European call and put options together with a share of the underlying asset form a set of securities, in which the payoff of any one of the instruments can be replicated by a combination of the other two. This gives rise to the PCP condition for European options which states that, at any time $t$, the following relationship holds:

$$C_t + Xe^{-r\tau} = P_t + I_t, \tag{1}$$

where $C_t(P_t)$ is the market price of a call (put) option at time $t$, $X$ is the exercise price for the put and call options, $I_t$ is the level of the underlying index at time $t$, $r$ is the continuously compounded rate of return on a risk-free security, and $\tau$ is the time to maturity of the put and call measured in years.

Note that the put-call parity condition (1) follows from a simple dominance argument and ignores transaction costs and dividends, which will be considered later.

If equation (1) is violated, then an arbitrage opportunity exists. There are two types of strategies that can be undertaken to eliminate an arbitrage opportunity: a conversion strategy when the call is overpriced relative to the put, and a reverse conversion (or reversal) when the put is overpriced relative to the call. A conversion strategy involves writing the overpriced call, buying the underpriced put, buying the index and borrowing $Xe^{-r\tau}$ at the risk-free rate. This leads to an immediate cash inflow of $C_t + Xe^{-r\tau} - P_t - I_t > 0$ and a zero cash flow at terminal time $T$. In contrast, for a reversal conversion strategy, an immediate cash inflow of $P_t + I_t - C_t - Xe^{-r\tau} > 0$ followed by a zero cash flow at terminal time $T$ can be achieved by buying the call, writing the put, shorting the index, and lending $Xe^{-r\tau}$ at the risk-free rate.[2]

A conversion strategy requires taking a long position in the underlying index; thus, it is also known as a long strategy. Similarly, a reversal strategy is also known as a short strategy.

Following Galai (1983) and Jensen (1978), a market is considered to be efficient with respect to a given information set if no trader can consistently make risk-adjusted profits after taxes and transaction costs that exceed the risk-free rate. In the PCP framework, this implies that at any point in time, calls and puts should be efficiently priced relative to each other. In other words, no arbitrage profit can be obtained by exploring (1).

Note that we have ignored the transaction costs and the dividend yield on the underlying index in the above discussion. However, transaction costs and dividend yield must be taken into account in reality. For the dividend adjustments, we can assume a constant dividend yield $\delta$ on the underlying index and replace $I_t$ by $I_t e^{-\delta\tau}$ in the PCP.[3] The dividend yield $\delta$ can be easily estimated, and we use the estimate from Nishina and Nabil (1997) for the Nikkei 225 index. The total transaction cost is much more difficult to estimate, and we shall revisit the issue later. For the time being, let us denote the total transaction cost with an arbitrage by $TC$.

Incorporating the transaction costs and dividend yield, a conversion (long) strategy, where the call is overpriced relative to the put, is profitable when

$$P_t < C_t + Xe^{-r\tau} - I_t e^{-\delta\tau} - TC. \qquad (2)$$

Likewise, a reversal (short) arbitrage strategy, where the put is overpriced relative to the call, is profitable when

2. In practice, it is difficult to "trade" the index. Evnine and Rudd (1985) suggest that index options are expected to exhibit more frequent and larger deviations from rational (equilibrium) prices. A comprehensive discussion with the arbitrage trading of index options can be found in Figlewski (1988).

3. For a proof, we refer to Chance (2004) or Hull (2003). Although dividends are not paid continuously in reality, it is a common practice to use a continuous dividend yield for a stock index.

$$P_t > C_t + Xe^{-r\tau} - I_t e^{-\delta\tau} + TC. \tag{3}$$

In sum, either (2) or (3) results in an arbitrage opportunity that is profitable after transaction costs are taken into account. In the case of (2), a long arbitrage strategy should be executed, whereas in the case of (3) a short arbitrage strategy should be executed.

It should be noted that there are restrictions preventing an arbitrager from short-selling stocks in the Japanese stock market. Consequently, the reversal (short) strategy when the put is overpriced relative to the call can only be implemented by market participants who already own the stocks belonging to the underlying index.

### B. Some Technical Issues

A few issues can cause problems for studies that seek to test the PCP. One important issue is the non-synchronous price problem, which must be accounted for. This problem is mitigated in this paper by matching the option contracts within a one-minute interval. Tick-by-tick data covering the period from January 2003 to December 2005 that are time-stamped to the nearest minute permit the contracts to be matched within a narrow time interval of one minute. Note that a wider interval would have enabled a much larger sample size. However, if a wider interval is chosen, such as a five- or 10-minute interval, then analyzing the *ex post* results would have increased the chances of stale prices. For this reason, it is considered more important to match the option contracts and the index level within a narrow time interval than it is to get a larger sample size by increasing the time interval.

This paper also takes into account the realistic transaction costs that an arbitrager incurs, including the implicit bid-ask spread. Details of estimates of transaction costs are provided in Section III.

### C. *Ex Ante* Tests

*Ex post* tests assume the ability to simultaneously execute all legs of an arbitrage at the prices that indicate the potential arbitrage opportunity. In practice, this seems unrealistic, especially for multi-market arbitrages and small traders. Thus, in addition to the *ex post* tests, it is necessary to undertake the *ex ante* test to see whether traders can profit from orders executed with a time lag after the identification of a violation of the PCP no-arbitrage condition. The *ex ante* test requires considering the time needed to eliminate the arbitrage opportunity. In this paper, a lag of three minutes is regarded as sufficient to account for the execution delay that an arbitrager needs to enter into the positions. The case of a one-minute lag is also considered.

Let us illustrate the procedure of an *ex ante* test with a lag of three minutes. For a given *ex post* profitable matched option pair, a search is carried out to find a matched pair that has traded at three minutes later during the day from the whole sample of matched pairs with the index levels. If no such pair can be found in the sample of matched pairs with index levels, the *ex post* profitable option pair is not included in the *ex ante* sample. If such a pair is found, then it is included in the *ex ante* sample. The realized profit or loss is calculated by using the new option prices and index level of three minutes later, based on the arbitrage strategy signified by the original violation of the PCP condition. As Lee and Nayar (1993) state, the number of observations used in the *ex ante* tests is not necessarily the same as the *ex post* observations because a time-stamped set of matched pairs may not exist for each violation. Note that a key difference between *ex post* and *ex ante* tests is that the *ex ante* tests are not risk free, in other words, the execution of an arbitrage opportunity may result in a loss rather than a profit due to the moving of the market prices.

*Ex post* analysis only indicates the possibility of arbitrage across the stock market and the index options market. On the other hand, *ex ante* tests show to what extent capturing profits from such arbitrage possibilities is possible. Thus, *ex ante* tests should provide more insight about the market efficiency.

**D. Regression Analysis**

In Mittnik and Rieken (2000), a regression analysis is used to test the PCP assuming no transaction costs. A violation of the PCP without transaction costs does not necessarily imply an arbitrage opportunity in reality. Thus, the regression has limited implication to the efficiency of the market. However, the regression analysis may reveal the strength of the relationship between the variables underlying the PCP. This may offer further evidence for the *ex post* and *ex ante* results. Thus, we consider a similar regression analysis based on the following relationship:

$$C_t - P_t = \alpha_0 + \alpha_1(I_t e^{-\delta\tau} - Xe^{-r\tau}) + u_t, \tag{4}$$

where $\alpha_0$, $\alpha_1$ are constants and $u_t$ is the error term. Under the assumption that there is no transaction cost, the coefficients $\alpha_0$, and $\alpha_1$ in equation (4) should be zero and one, respectively. However, given the significant nonconstant transaction costs in reality, we do not expect the hypotheses ($\alpha_0 = 0$ and $\alpha_1 = 1$) to hold. Instead, the focus should be on the overall significance of the PCP relationship.

In addition to the above, we can use a regression analysis to find further evidence on the relationship between the arbitrage profitability and the underlying features such as maturity, moneyness,[4] and arbitrage strategy. Here we consider the arbitrage profits for OSE members. To this end, we consider the following regression:

$$\pi_t = \beta_0 + \beta_1 D_{Maturity} + \beta_2 D_{Moneyness} + \beta_3 D_{Strategy} + \epsilon_t, \tag{5}$$

where $\pi_t$ is the arbitrage profit for OSE members; $\beta_0$, $\beta_1$, and $\beta_2$ are constants; and $\epsilon_t$ is the error term. Furthermore, $D_{Maturity}$, $D_{Moneyness}$, $D_{Strategy}$ are three dummy variables defined as follows: $D_{Maturity}$ is set to be one if the maturity of the pair is short term (less than 30 days) and zero otherwise; $D_{Moneyness}$ is set to be one if the option pair is at the money (ATM) and zero otherwise; and $D_{Strategy}$ is set to be one if a long strategy is needed and zero otherwise.

4. The definition of moneyness for option pairs is given in Section III.C.

## III. Contract Specifications and Data

### A. Contract Specification

The OSE, established on April 1, 1949 as a membership organization under the Japanese Securities and Exchange Law, is one of Japan's oldest and most respected securities exchanges.

The OSE is the largest derivatives market in Japan. Nikkei 225 futures trading is well established as a key product among stock index futures traded on the world's futures exchanges. The Nikkei 225 options, the most actively traded index options in Japan,[5] started listing in June 1989. They are based on the Nikkei stock average index and traded on the OSE.[6] The options are of European type. According to the trading volume in 2004, the Nikkei 225 options ranked among the 10 largest index options in the world.

The underlying Nikkei stock average is a portfolio that equally weights the performance of 225 stocks listed on the First Section of the Tokyo Stock Exchange (TSE) after adjustments for rights issues, stock splits, and so forth.

The contract months are five consecutive months in the March quarterly cycle and three near-term expiration months that do not overlap the March quarterly cycle. Thus, the March quarterly cycle contracts can be traded for 15 months. The contracts have a multiplier of 1,000. The strike price of an option contract is an integer multiple of ¥500 based on the Nikkei 225, with intervals of ¥500.

The last trading day is the business day before the second Friday of each expiration month, and the option can be exercised on the business day following the last trading day. The contracts are cash settled based on the difference between the exercise price and the Special Quotation on the expiration date. Special Quotation calculation is based on the total opening prices of each component issue in the Nikkei stock average on the business day following the last trading day.

The daily trading times for the Nikkei 225 options are from 9:00 a.m. to 11:00 a.m. and from 12:30 p.m. to 3:10 p.m. These trading times are the same as the TSE trading times, except that the TSE closes at 3:00 p.m. instead of 3:10 p.m.

### B. Data

The option data utilized in this study are from a period of nearly three years (January 6, 2003 to December 19, 2005). The data are provided by Nikkei Economic Electronic Databank System (NEEDS),[7] are time-stamped to the nearest minute, and consist of intraday transaction prices for both call and put options contracts. The daily summary data for index option trading and the minute-by-minute Nikkei index levels are also provided by NEEDS.

According to Nishina and Nabil (1997), the expected dividend stream on the Nikkei stock index can be approximated by an annual average dividend yield of

5. For example, the total volume of TOPIX options in 2004 is only 17,643 units, while the volume of Nikkei 225 options is 16,560,874 units (source: TSE and OSE websites).
6. Note that these options are different from the Nikkei Index futures options, which are traded on the Singapore International Monetary Exchange (SIMEX).
7. The data are purchased by the BOJ from Nihon Keizai Shimbun, Inc. and Nikkei Media Marketing, Inc. for the purpose of this research.

0.5 percent, which represents the estimate of dividend yield on the highly correlated but broader TOPIX stock index. Due to the fact that firms tend to have long-run target dividend payout ratios (see, e.g., Lintner [1956]), the dividend yield on the Nikkei index for the sampling period is believed to be close to the estimate for an earlier period as considered in Nishina and Nabil (1997). Thus, the dividend yield on the Nikkei index is assumed to be 0.5 percent throughout the paper.

The mid-rate on three-month certificates of deposit (CDs) whose maturity is closest to the option's expiration date is used as a proxy for the risk-free rate of interest. The interest rate data are obtained from Bloomberg.

Note that the CD rates are used instead of the three-month financing bill (FB) rates, because it is believed that the CD rates are closer to the interest rates that a dealer can borrow or lend.

### C. Matching Option Pairs

Using tick-by-tick data allows call and put contracts to be matched within a narrow time interval for analysis. This will ensure a high level of synchronization between the option prices and the index. Similarly as in Mittnik and Rieken (2000) and Capelle-Blancard and Chaudhury (2002), we require that all prices in a given arbitrage be within one minute of each other. The matching process is as follows.

Call options are first matched with put options that have been traded within a one-minute interval. If there is no match for a call, then the call option is not used and hence disregarded. The matched pair of call and put must have the same exercise price and the same maturity, and are traded within a one-minute interval.

For a given matched pair of call and put, we then look for the index level at the trading time of the option pair. If no such index level can be found, then the option pair is not utilized.

After applying the above selection procedure, a total of 139,586 option pairs are found during the sample period. To see the time trend of the option market efficiency, we also consider three subsamples (2003, 2004, and 2005) based on the calendar years.

A breakdown of the 139,586 matched pairs in each year is presented in Table 1. From this table, it is clear that 2005 has the largest number of matched pairs with 55,067 pairs for the year, although the option data for 2005 are a few days short of a full year. The number of matched pairs is the least in 2004 with 40,189 pairs, accounting for only 28.8 percent of the total sample.

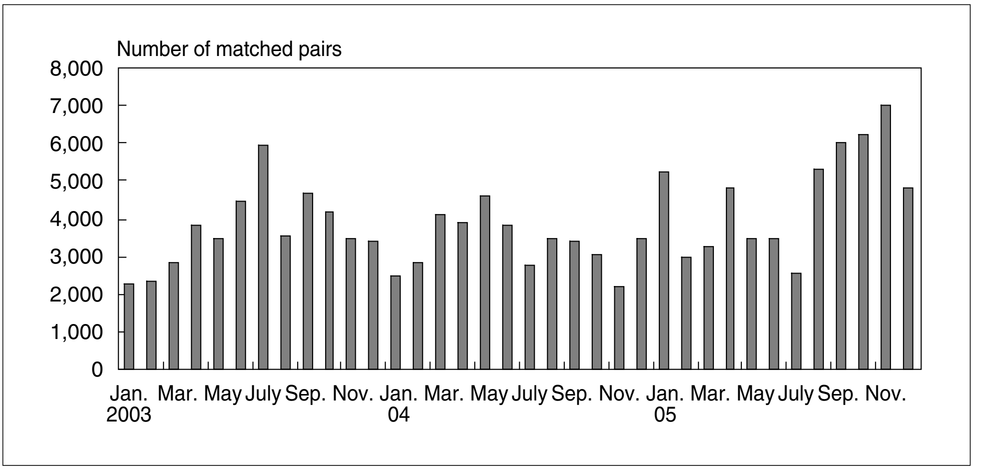
Figure 1 further shows the distribution of the option pairs in our sample according to calendar months. November 2005 has the largest number of pairs occurring in a single month, with a record number of 7,002 (5.02 percent) matched pairs of the

**Table 1 Distribution of the Matched Option Pairs in Each Year**

| Year | Matched pairs |
|---|---|
| 2003 | 44,330 (31.8 percent) |
| 2004 | 40,189 (28.8 percent) |
| 2005 | 55,067 (39.5 percent) |
| Total | 139,586 (100.0 percent) |

Note: The numbers in parentheses are the percentages of the total sample.

**Figure 1 Distribution of the Matched Pairs in the Calendar Month**




total matched pairs. October 2005 also has a high number of matched pairs with 6,201 (4.44 percent), while November 2004 has the least number of matched pairs with 2,202 (1.58 percent).

To investigate if the arbitrage profit is particularly related to certain factors, we will also consider the distribution of arbitrage profits with respect to option maturity and moneyness. The time to maturity (measured in calendar days) for a matched pair is simply the contract maturity for the underlying call and put. However, the moneyness[8] for a pair requires more explanation.

Three distinct classes of moneyness are employed in this paper. For a given matched pair, if the index level is below the exercise price by greater than 3 percent, the pair is defined as out of the money (OTM); if the index level is above the exercise price by greater than 3 percent, the pair is defined as in the money (ITM); otherwise, it is classified as ATM.

Note that the above moneyness definition for option pairs differs from the usual definition for options. According to our definition, for an OTM pair, the call option is out of the money while the put option is in the money. Similarly, for an ITM pair, the call option is in the money and the put option is out of the money.

## D. Transaction Costs

Taking transaction costs into account is vital when empirically investigating PCP. Unfortunately, such costs are difficult to estimate because there are many components (commissions, trading and clearing fees, costs deriving from bid and ask prices, short-selling costs, etc.) and they tend to vary over time, trading strategy, and transaction size.

With our arbitrage strategies, three transaction costs need to be taken into account: the implicit bid-ask spread, the exchange and regulatory fees, and the cost associated with the trading of the stocks that make up the Nikkei 225 index. Of course,

8. Many studies use moneyness as a proxy of liquidity. For example, Cheng, Fung, and Chan (2000) state that futures traders usually hedge their exposure with option contracts. Usually the options that are closest to the futures price have the greatest liquidity and are usually the cheapest.

the transaction costs may differ from trader to trader. However, for the purpose of assessing the option market efficiency, the traders with the lowest costs should be considered. These traders are likely to be the members of the OSE.

As mentioned by Phillips and Smith (1980), the bid-ask spread is an important cost that many studies neglect, and it is important to consider this cost. For OSE member firms, this spread represents an important cost relative to the exchange fees, which are only a fraction of the bid-ask spread cost. However, it is usually quite hard to get an accurate estimate of the bid-ask spread.

In this paper, we estimate the bid-ask spread for option trading based on the daily summary data on options provided by NEEDS. The daily weighted average spread based on the time while both ask and bid quotes are available for each option contract is reported in the daily summary data. We first remove the entries for which no trading has taken place.

In searching for an estimate of the realistic bid-ask spread, we calculate the average of the reported daily average spreads across all the contracts for each year. But these averages do not reflect the number of trades on each contract. Normally, the more liquid contracts should have a lower bid-ask spread. Thus, we also calculated the average spreads across all contracts with weighting based on the number of contracts traded. Furthermore, we investigate the possible difference in average spreads across option moneyness. The results are reported in Table 2.

A few observations are in order. First of all, it is interesting to note that both simple average and weighted average spreads have been decreasing over the past three years. In some sense, this may indicate that the operational efficiency of the Nikkei 225 options market has been improving in the past three years. Another point worth noting is that both the simple average and weighted average spreads for ATM options are actually higher than those of the ITM or OTM options, with 2003 as an exception.[9] This is contrary to the conventional wisdom, which claims that ATM options should be more liquid and thus incur a lower bid-ask spread.[10]

**Table 2 The Average Spread Estimate**

| | 2003 | 2004 | 2005 | Whole |
|---|---|---|---|---|
| Simple average | 19.63 | 18.26 | 17.76 | 18.55 |
| Weighted average | 7.90 | 6.69 | 6.24 | 6.84 |
| ATM | | | | |
| Simple average | 17.42 | 18.60 | 21.71 | 19.25 |
| Weighted average | 8.04 | 7.60 | 7.05 | 7.50 |
| ITM or OTM | | | | |
| Simple average | 19.93 | 18.21 | 17.05 | 18.40 |
| Weighted average | 7.83 | 6.18 | 5.80 | 6.48 |

Note: The simple average is the average of the daily spread over a period. For the weighted averages, the weighting factors are determined by the number of trades for each option contract. The table also provides the average spreads across moneyness. All reported numbers are in index points.

9. The moneyness classification for the spread estimate is based on the index level at 11:00 a.m. on each business day.
10. This is likely due to some measurement errors. For example, the classification of moneyness for each reported daily average entry is based on the index level at a point in time. It would be better to use the average daily index level for the classification. It might also be better to treat OTM and ITM options separately. However, the purpose here is to gauge the scale of the average bid-ask spread to be used in the following empirical study, and the estimates presented are sufficient for the purpose of this study.

The difference between simple averages and weighted averages is large for all the three years in the sample period and different categories of moneyness. However, we believe that the weighted average spreads should be more accurate, as they account for the number of trades of each option contract. Thus, the weighted average spread should be used for the purpose of this research. The weighted average spread of 2003 is about 1.5 index points above that of 2005. Moreover, the difference in the weighted average spreads between the two moneyness categories is about one index point for the whole sample, although the difference is bigger for 2004 and 2005. Given this evidence, we will assume a constant bid-ask spread of 6.84 index points in the forthcoming analysis.[11]

Now let us consider the exchange and regulatory fees associated with the trading of Nikkei 225 options. This cost should be much less than the bid-ask spread cost. According to an Internet broker, the exchange charge and regulatory fees for each option contract are about 0.75 index point.[12] Although it is more likely that the OSE members have a lower exchange and regulatory fee than this, we use 0.75 index point as a reasonable estimate in this paper.

Both the long and short arbitrage strategies involve trading the stocks underlying the Nikkei 225 index.[13] Thus, the transaction costs associated with the stock trading must be taken into account in analyzing the arbitrage efficiency of the option market. In considering the DAX index options, Mittnik and Rieken (2000) assume a transaction cost of 0.1 percent of the index level. Similarly, we assume the cost of trading stocks belonging to the Nikkei 225 index as 0.1 percent of the stock index level.[14] This may be high compared to the low online brokerage fees available in the market. However, given that we ignore the bid-ask spread in the stock trading, we believe that 0.1 percent of the index level should be a reasonable estimate.

To establish a long arbitrage requires writing a call, buying a put, and buying the stocks making up the index initially. At maturity, one needs to sell the stocks bought initially and to close off one option contract,[15] as only one (either the call or the put) can be in the money. Hence the total transaction costs for long arbitrages should consist of three times the bid-ask spread, three times the exchange and regulatory fee, and 0.2 percent of the index level. Hence the total transaction cost for a long arbitrage is estimated to be 22.77 index points plus 0.2 percent of the index level. The total transaction cost for a short arbitrage can be obtained similarly and is the same as for a long arbitrage.

To accommodate other market participants[16] whose cost structure does not correspond with those of the member firms, a sensitivity analysis for the *ex post* arbitrage profitability will be conducted with respect to transaction costs. In this paper, we shall

11. The bid and ask spread generally overestimates transaction costs as trades also occur inside the spread rather than at the quotes, as traders sometimes can bargain for better prices. Thus, our estimate of 6.84 index points is rather conservative for analyzing market efficiency.
12. See www.interactivebrokers.com.
13. To reduce transaction costs, one natural idea is to use the Nikkei 225 index futures instead of trading the stocks. However, the basis risk would need to be considered for the index futures trading. Thus, to avoid the problem of basis risk, we focus on trading the stocks underlying the index.
14. For a given index level of 10,000, this implies a transaction cost of ¥10,000 for one contract.
15. One alternative is to have the option exercised. Then the settlement would be based on the special quotation of the index specified by the OSE. The two ways should give a similar net value to an option.
16. For example, individual investors have to pay an initial margin to enter the contracts.

**Table 3 The Scenarios of Transaction Costs for PCP Arbitrages**

| | Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4 | Scenario 5 |
|---|---|---|---|---|---|
| Option spread | 1.00 | 4.00 | 6.84 | 10.00 | 13.00 |
| Exchange and regulatory fee | 0.10 | 0.40 | 0.75 | 0.80 | 1.00 |
| Index trading cost (percentage of index, percent) | 0.05 | 0.08 | 0.10 | 0.12 | 0.15 |
| Total transaction costs: | | | | | |
| Index point plus | 3.30 | 13.20 | 22.77 | 32.40 | 42.00 |
| Percentage of index (percent) | 0.10 | 0.16 | 0.20 | 0.24 | 0.30 |

consider four other scenarios of total transaction costs as shown in Table 3. Note that Scenario 3 corresponds to the cost structure of OSE members and is mostly considered in this paper.

## IV. Empirical Results

In this section, we analyze the arbitrage profitability based on the PCP. The analysis will cover *ex post* tests and *ex ante* tests that will provide insights into the arbitrage efficiency of the Nikkei 225 options market. In addition, regression analysis is employed to provide further evidence.

As previously discussed, we focus on the OSE member firms that have the lowest total transaction cost. The total transaction cost of 22.77 index points plus 0.2 percent of the index level will be assumed for both the long and short arbitrages.

The *ex post* arbitrage profitability is analyzed below across calendar year, moneyness, and maturity. All the tables below are based on trading one of each contract in a matched pair. Of course, an arbitrager can execute many contracts when an arbitrage opportunity is present in reality.

### A. Calendar Year

Table 4 presents the results for the whole sample and the subsamples. The number of profitable arbitrage opportunities stands at 3,819 (2.74 percent) from the total 139,586 matched pairs over the whole sample period. It should be noted that the percentage of PCP violations is low compared to other studies in the literature. For example, Capelle-Blancard and Chaudhury (2002) find that 4 percent (long) to 8 percent (short) profitable arbitrage opportunities for the French CAC 40 index options market; Cavallo and Mammola (2000) find that the percentage of PCP violations ranges from 5 percent to 6 percent for institution investors for the Italian MIB30 options market.

Over the whole sample period, the mean profit is 22.609 index points. The median value, which mitigates the outlier problem with the mean, is also high with 8.016 index points for OSE members. It appears that the size of arbitrage profits is large on average. The kurtosis and skewness figures show that the arbitrage profit distribution is peaked relative to the normal distribution and right skewed.

The mean values of the arbitrage profits for 2003 and 2005 are similar but much larger than that of 2004. Mittnik and Rieken (2000) state that the mean values should

**Table 4 *Ex Post* Arbitrage Profit and Calendar Year**

| | 2003 | 2004 | 2005 | Whole |
|---|---|---|---|---|
| Total matched pairs | 44,330 | 40,189 | 55,067 | 139,586 |
| Mean | 25.020 | 16.125 | 24.298 | 22.609 |
| Median | 9.938 | 6.270 | 7.775 | 8.016 |
| Standard deviation | 37.198 | 29.395 | 36.867 | 35.536 |
| Kurtosis | 9.395 | 14.775 | 2.866 | 7.035 |
| Skewness | 2.750 | 3.670 | 1.964 | 2.540 |
| Minimum | 0.004 | 0.020 | 0.002 | 0.002 |
| Maximum | 345.679 | 182.310 | 205.946 | 345.679 |
| Profitable matched pairs | 1,300 | 904 | 1,615 | 3,819 |
| Relative profitable pairs[1] (percent) | 34.04 | 23.67 | 42.29 | 100.00 |
| Frequency of total pairs[2] (percent) | 2.93 | 2.25 | 2.93 | 2.74 |

Notes: 1. Profitable matched pairs for each period divided by the total number of profitable matched pairs over the whole sample, for example, 1,300/3,819.
2. Profitable matched pairs divided by the corresponding total matched pairs, for example, 1,300/44,330.

be falling over the sample period, because market participants learn how to price these instruments more efficiently. This pattern is not observed in our case. The year 2003 has the greatest mean value with 25.020 index points. The mean value is 24.298 index points for 2005 and 16.125 points for 2004, respectively. It should be observed that the median values of arbitrage profits over the different calendar years are high with 9.938, 6.270, and 7.775 index points for 2003, 2004, and 2005, respectively. Overall, it appears that the average arbitrage profit for 2004 is significantly less than that for both 2003 and 2005. The distribution of arbitrage profits for 2004 has the biggest kurtosis and skewness among the three calendar years.

Now let us consider the frequency of arbitrage opportunities over the three years. The purpose is to examine whether these opportunities are clustered around a certain year or spread out across the sample period. Table 4 indicates that the number of profitable matched pairs is the lowest for 2004 and the highest for 2005. The total number of profitable matched pairs is 1,300, 904, and 1,615 for 2003, 2004, and 2005, respectively (as mentioned, the total number of profitable matched pairs stands at 3,819 [2.74 percent] over the whole sample period). Table 4 also breaks down the total profitable arbitrage opportunities based on each calendar year. Of the total 3,819 profitable matched pairs observed for the whole period, 34.04 percent, 23.67 percent, and 42.29 percent are from 2003, 2004, and 2005 respectively.

These *ex post* results reveal that arbitrage opportunities are present with only 2.74 percent of breaches of PCP being witnessed for OSE members after considering all transaction costs including the implicit bid-ask spread. However, the average arbitrage profit over the arbitrage opportunities is quite high (22.609 index points). Among the three years of the sample period, 2004 has the lowest frequency of PCP violations and the lowest average arbitrage profit. These figures are much lower than those for 2003 and 2005. Such an up-and-down trend implies that the options market in 2004 appears to be more efficient than the other two years.

The distributions of the arbitrage profits for all the three years as well as the whole sample period are all right skewed and peaked relative to the normal distribution.

### B. Long versus Short Arbitrage Strategy

In this subsection, we consider the relationship between the arbitrage profitability and the arbitrage strategy. The goal here is to find out whether a long or short arbitrage strategy is more profitable and whether one of them is viable more frequently than the other. The results for the whole sample and subsamples are presented in Table 5.

Let us first consider the whole sample. The total of 3,819 breaches of PCP is broken down into 1,032 (27.02 percent) long arbitrage opportunities and 2,787 (72.98 percent) short arbitrage opportunities. Thus, short arbitrage opportunities are much more frequent than long ones. This implies that the put contracts are more frequently overpriced relative to the call contracts.[17] However, the long arbitrage strategy is more profitable on average with a mean of 53.092 index points relative to 11.322 index points for the short arbitrage strategy. The standard deviation of the arbitrage profits for the long arbitrage strategy is 47.670 index points, which is much higher than that of the short arbitrage strategy, which has a value of only 20.446 index points. Thus, it appears that the long arbitrage strategy is also more risky than the short arbitrage strategy.

**Table 5 *Ex Post* Arbitrage Profit and Arbitrage Strategy**

| | 2003 | 2004 | 2005 | Whole |
|---|---|---|---|---|
| Total profitable pairs | 1,300 | 904 | 1,615 | 3,819 |
| Total matched pairs | 44,330 | 40,189 | 55,067 | 139,586 |
| | Long arbitrage strategy | | | |
| Mean | 44.657 | 21.625 | 76.626 | 53.092 |
| Standard deviation | 48.925 | 22.403 | 42.565 | 47.670 |
| Minimum | 0.198 | 0.104 | 0.835 | 0.104 |
| Maximum | 345.679 | 94.347 | 205.946 | 345.679 |
| Profitable matched pairs | 481 | 162 | 389 | 1,032 |
| Frequency of profitable pairs[1] (percent) | 37.00 | 17.92 | 24.09 | 27.02 |
| Relative profitable pairs[2] (percent) | 46.61 | 15.70 | 37.69 | 100.00 |
| Frequency of total pairs[3] (percent) | 1.09 | 0.40 | 0.71 | 0.74 |
| | Short arbitrage strategy | | | |
| Mean | 13.487 | 14.924 | 7.695 | 11.322 |
| Standard deviation | 20.804 | 30.592 | 8.454 | 20.446 |
| Minimum | 0.004 | 0.020 | 0.002 | 0.104 |
| Maximum | 119.234 | 182.310 | 77.644 | 182.310 |
| Profitable matched pairs | 819 | 742 | 1,226 | 2,787 |
| Frequency of profitable pairs[1] (percent) | 63.00 | 82.08 | 75.91 | 72.98 |
| Relative profitable pairs[2] (percent) | 29.39 | 26.62 | 43.99 | 100.00 |
| Frequency of total pairs[3] (percent) | 1.85 | 1.85 | 2.23 | 2.00 |

Notes: 1. Profitable matched pairs divided by the corresponding total profitable pairs, for example, 481/1,300, 819/1,300.
2. Profitable matched pairs for each period divided by the total number of profitable matched pairs of that strategy over all the years, for example, 481/1,032, 819/2,787.
3. Profitable matched pairs divided by the corresponding total matched pairs, for example, 481/44,330, 819/44,330.

17. Note that longing index put option is a convenient and relatively cheap method for hedging. Therefore, buying pressure on index put options is larger than that on index call options. This could result in put contracts being overpriced.

Having discussed the results for the full sample period, let us turn to the subsamples based on the calendar years. For the long arbitrage strategy, 2005 has the highest mean profit of 76.626 index points, while 2004 has the lowest mean arbitrage profit of 21.625 index points. The standard deviations of the arbitrage profits for 2003 and 2005 are also much higher than those for 2004. Turning to the short arbitrage strategy, the trend for the mean profit is opposite to the long arbitrage strategy. In contrast to the long arbitrage strategy, 2004 has the highest mean profit and highest standard deviation among the three sample years.

Among the 1,300 profitable matched pairs in 2003, 481 (37.00 percent) are profitable with the long arbitrage strategy, while 819 (63.00 percent) are profitable with the short arbitrage strategy. The same pattern holds for 2004 and 2005. This implies that the put contracts are more frequently overpriced relative to call contracts over each of the three years.

It is interesting to see that the total number of profitable matched pairs in each annual period relative to the total matched pairs for each period is quite small, in particular for the long arbitrage strategy. For the long arbitrage strategy, there are 0.40 percent to 1.09 percent profitable pairs among the total matched pairs for each year. Turning to the short arbitrage strategy, there are 1.85 percent to 2.23 percent profitable pairs from the total matched pairs in each year.

In sum, our results indicate that short profitable arbitrage opportunities are more frequent and less profitable compared to long ones. This pattern holds true for the whole sample as well as for the subsamples. Given the restriction on short-selling of stocks in Japan, the results are more supportive of the arbitrage efficiency of the Nikkei 225 options market.

**C. Moneyness**

This subsection presents the empirical results on the relationship between arbitrage profitability and the moneyness of the option pairs. The question addressed here is whether the size and frequency of arbitrage opportunities are related to the moneyness of the option pairs.

Table 6 shows that of the 3,819 profitable pairs, 2,814 (73.68 percent) pairs are ATM, 366 (9.58 percent) are OTM, and 639 (16.73 percent) are ITM pairs. Thus, the majority of profitable arbitrage pairs are ATM. This is mainly due to the large number of ATM pairs in the sample.

Table 6 also reports the total number of profitable matched pairs relative to the total number of matched pairs for each type of moneyness category. Here, 2,814 (2.51 percent) breaches of PCP are observed from the total 112,022 matched pairs that are ATM. The total figures for OTM and ITM matched pairs are 10,253 and 17,311 pairs, respectively, of which only 366 (3.57 percent) OTM pairs and 639 (3.69 percent) ITM pairs are profitable. Clearly, ATM pairs have the lowest frequency of profitable arbitrage opportunities, while ITM pairs have the highest frequency of profitable arbitrage opportunities.

ATM pairs report a mean value of 22.418 index points. OTM and ITM pairs provide similar mean values of arbitrage profit. Thus, the average size of arbitrage profit is similar across all moneyness categories.

**Table 6 *Ex Post* Arbitrage Profit and Moneyness**

| | ATM | OTM | ITM |
|---|---|---|---|
| Total matched pairs | 112,022 | 10,253 | 17,311 |
| Mean | 22.418 | 23.839 | 22.749 |
| Standard deviation | 35.350 | 37.540 | 35.216 |
| Minimum | 0.002 | 0.004 | 0.007 |
| Maximum | 215.835 | 345.679 | 205.946 |
| Profitable matched pairs | 2,814 | 366 | 639 |
| Relative profitable pairs[1] (percent) | 73.68 | 9.58 | 16.73 |
| Frequency of total pairs[2] (percent) | 2.51 | 3.57 | 3.69 |

Notes: 1. Profitable matched pairs for each class divided by the total number of profitable matched pairs, for example, 2,814/3,819.
2. Profitable matched pairs divided by the corresponding total matched pairs, for example, 2,814/112,022.

In sum, though the most profitable arbitrage opportunities are ATM pairs, ITM pairs have the highest frequency of profitable arbitrage opportunities. Furthermore, the average arbitrage profits are similar across the three categories of moneyness.

### D. Maturity

This subsection addresses the maturity (in days) of the profitable arbitrage opportunities for the OSE members. The goal here is to see whether the arbitrage opportunities concentrate on contracts with certain maturities. The results are reported in Table 7.

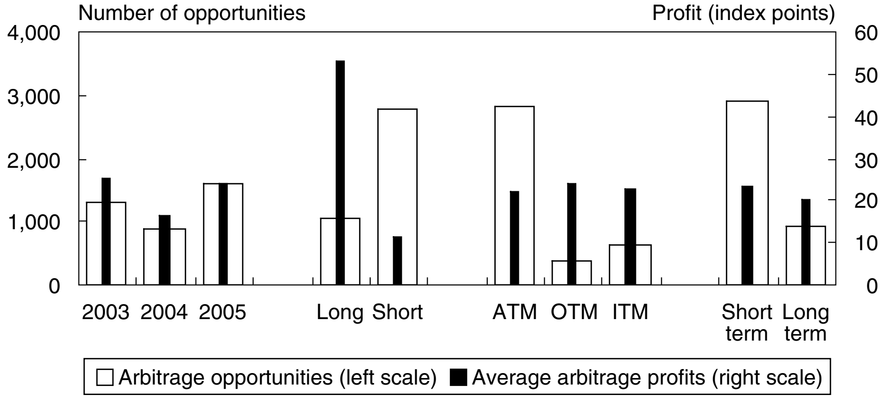
It is interesting to see that the longest maturity is 444 days for the profitable pairs. This is very close to 15 months, which is the maximum possible maturity for the Nikkei 225 options. Furthermore, the median value for the whole profitable pairs is 21 days. The mode observations are reported here because they signify the most frequent maturity. The mode value is 10 days for the whole profitable pairs. It should also be noted that the profitable opportunities with the short arbitrage strategy have a longer average maturity and mode than the profitable opportunities with the long arbitrage strategy.

Before proceeding to the sensitivity analysis, let us briefly summarize the *ex post* results on the arbitrage opportunities for OSE members. Figure 2 presents a brief summary that clearly shows that the 3,819 arbitrage opportunities concentrate mainly in the short arbitrage strategy, ATM pairs and short-term pairs, although they are rather evenly spread over the three years. In terms of average arbitrage profit, ATM,

**Table 7 Maturities of the *Ex Post* Arbitrage Opportunities**

| | Whole | Long | Short |
|---|---|---|---|
| Mean | 24.501 | 18.680 | 26.656 |
| Median | 21 | 10 | 22 |
| Mode | 10 | 10 | 15 |
| Standard deviation | 23.049 | 16.922 | 24.595 |
| Minimum | 1 | 1 | 1 |
| Maximum | 444 | 183 | 444 |
| Profitable matched pairs | 3,819 | 1,032 | 2,787 |

**Figure 2 Summary of the Profitable Arbitrage Opportunities**




Note: A brief summary of the distribution of the profitable arbitrage opportunities and average arbitrage profits during the sample period for OSE members across calendar year, arbitrage strategy, moneyness, and maturity. Short term refers to a maturity of less than 30 days, and long term refers to a maturity of more than or equal to 30 days.

OTM, and ITM pairs have similar figures. The average profit for short-term pairs is slightly higher than that for the long-term pairs. The average profit is also slightly lower for 2004 than the other two years. However, the long arbitrage strategy gives a much higher average profit than the short arbitrage strategy.

## E. Sensitivity Analysis

This subsection presents a sensitivity analysis of the size and frequency of arbitrage opportunities when transaction costs vary. The five scenarios given in Table 3 are considered. Table 8 presents the results of the sensitivity analysis for the whole sample only.

**Table 8 Sensitivity Analysis with Respect to Transaction Costs**

| | Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4 | Scenario 5 |
|---|---|---|---|---|---|
| Total matched pairs | 139,586 | 139,586 | 139,586 | 139,586 | 139,586 |
| Mean | 17.588 | 17.266 | 22.609 | 43.124 | 49.177 |
| Median | 12.268 | 10.239 | 8.016 | 27.358 | 37.289 |
| Standard deviation | 22.097 | 25.824 | 35.536 | 44.009 | 41.648 |
| Kurtosis | 24.706 | 19.355 | 7.035 | 1.586 | 1.679 |
| Skewness | 4.011 | 3.976 | 2.540 | 1.272 | 1.082 |
| Minimum | 0.000 | 0.000 | 0.002 | 0.026 | 0.104 |
| Maximum | 373.107 | 358.432 | 345.679 | 332.865 | 318.490 |
| Profitable matched pairs | 25,631 | 10,762 | 3,819 | 1,274 | 791 |
| Frequency of total pairs (percent) | 18.36 | 7.71 | 2.74 | 0.91 | 0.57 |

Note: Frequency of total pairs is profitable matched pairs divided by the corresponding total matched pairs, for example, 25,631/139,586.

For the whole sample, the frequency of profitable arbitrage opportunities decreases as transaction costs increase. For Scenario 1, 18.36 percent of the whole sample is profitable. This figure falls to 0.57 percent for Scenario 5. The mean profit figures are high and increasing as transaction costs increase. Thus, as transaction costs increase, the arbitrage opportunities are becoming less frequent but more profitable on average.

Table 8 also illustrates that even if transaction costs are assumed to be very low (Scenario 2), only 7.71 percent of the sample would be profitable with a mean profit of 17.266 index points. This implies that violations in PCP are not frequent even for participants with a very low cost structure. Thus, the sensitivity analysis provides further supporting evidence for the arbitrage efficiency of the Nikkei 225 options market.

### F. *Ex Ante* Results

Table 9 presents the results on the *ex ante* arbitrage profitability for members of the OSE. It should be noted that as opposed to the *ex post* tests where the mispricing signal can be exploited without any risk, the *ex ante* profit is affected by possible price movements during the execution lag, so it can be negative. This risk is described as immediacy risk by Kamara and Miller (1995).

**Table 9 *Ex Ante* Arbitrage Profit**

| | 2003 | 2004 | 2005 | Whole | Long | Short |
|---|---|---|---|---|---|---|
| Panel I: One-minute lag | | | | | | |
| Total valid pairs | 486 | 370 | 580 | 1,436 | 535 | 901 |
| Mean | 18.580 | 9.840 | 27.883 | 20.085 | 39.920 | 8.308 |
| Median | 9.293 | 1.395 | 9.467 | 7.685 | 28.883 | 4.716 |
| Standard deviation | 37.308 | 36.219 | 43.953 | 40.492 | 51.040 | 26.379 |
| Kurtosis | 3.699 | 8.213 | 0.844 | 2.760 | –0.429 | 15.414 |
| Skewness | 1.758 | 2.514 | 1.293 | 1.676 | 0.617 | 3.180 |
| Minimum | –47.088 | –55.187 | –70.278 | –70.278 | –70.278 | –54.655 |
| Maximum | 163.164 | 182.310 | 149.628 | 182.310 | 163.164 | 182.310 |
| Profitable pairs | 337 | 196 | 442 | 1,003 | 401 | 602 |
| Frequency of profitable pairs[1] (percent) | 69.34 | 52.97 | 76.21 | 69.85 | 74.95 | 66.81 |
| Panel II: Three-minute lag | | | | | | |
| Total valid pairs | 481 | 332 | 500 | 1,313 | 528 | 785 |
| Mean | 3.375 | –1.598 | 24.686 | 10.233 | 23.942 | 1.013 |
| Median | –1.354 | –5.038 | 7.434 | 2.111 | 14.696 | 0.552 |
| Standard deviation | 35.908 | 33.266 | 46.475 | 41.288 | 55.382 | 24.069 |
| Kurtosis | 5.037 | 10.658 | 0.605 | 2.997 | –0.381 | 17.531 |
| Skewness | 1.745 | 2.555 | 1.150 | 1.613 | 0.720 | 2.737 |
| Minimum | –62.015 | –59.051 | –77.756 | –77.756 | –77.756 | –63.637 |
| Maximum | 169.591 | 173.565 | 145.126 | 173.565 | 169.591 | 173.565 |
| Profitable pairs | 233 | 115 | 358 | 706 | 293 | 413 |
| Frequency of profitable pairs (percent) | 48.44 | 34.64 | 71.60 | 53.77 | 55.49 | 52.61 |

Note: Frequency of total pairs is profitable pairs divided by the corresponding total valid pairs, for example, 337/486, 233/481.

Let us first focus on the case of a three-minute lag, which is probably closer to the execution time required for an arbitrage. With a lag of three minutes, the size of arbitrage profits is much smaller compared to the *ex post* results. The *ex post* tests report a mean value of 22.609 index points while the *ex ante* tests show a mean value of 10.233 index points over the whole sample. The long and short arbitrage strategies also report a much lower *ex ante* profit relative to the *ex post* profit. In addition, there are more than 40 percent arbitrage opportunities signified in the *ex post* analysis that are not profitable with an execution lag of three minutes. The mean *ex ante* profit for 2004 with three-minute lag is actually negative.

Now let us turn to the case of a one-minute lag. Compared to the *ex post* results, the average profit has decreased significantly for the whole sample, each subsample, and each strategy. However, the average arbitrage profit figures are much higher than the corresponding figures in the case of a three-minute lag. There are more than 30 percent arbitrage opportunities signified in the *ex post* analysis that are not profitable with an execution lag of one minute. A contrast of Panel I and Panel II reveals that it is important to execute an arbitrage quickly to capture the arbitrage profit.

In sum, it is observed that a large proportion (more than 30 percent) of arbitrage opportunities that are detected *ex post* are not profitable for the arbitrager to execute the transaction with a one-minute or three-minute lag. The average size of the arbitrage profit is also significantly less than the corresponding average profit reported in the *ex post* results. Therefore, the *ex ante* results further enhance the existing supportive evidence for the efficiency of the Nikkei 225 option market.

## G. The Regression Results

Table 10 reports the regression results on equation (4). As expected, both the null hypotheses $\alpha_0 = 0$ and $\alpha_1 = 1$ are rejected for each of the three years in the sample period.[18] However, this does not have much implication against the efficiency of the Nikkei 225 options market due to the transaction costs in reality.

The $R^2$ values for all the three years are above 0.995. Furthermore, all the *F*-statistics for the three sample years are highly significant, indicating that the relationship (4) holds fairly strongly. Thus, the relationship between the dependent and independent

**Table 10 Regression Evidence on the PCP**

| | 2003 | 2004 | 2005 |
|---|---|---|---|
| $\alpha_0$ | 1.0026<br>(0.0000) | –0.6226<br>(0.0000) | –0.8800<br>(0.0000) |
| $\alpha_1$ | 0.9983<br>(0.0000) | 0.9982<br>(0.0000) | 0.9955<br>(0.0000) |
| Observations | 44,330 | 40,189 | 55,067 |
| Adjusted $R^2$ | 0.9971 | 0.9969 | 0.9960 |
| *F*-statistic ($\times 10^7$) | 1.5505<br>(0.0000) | 1.2798<br>(0.0000) | 1.3623<br>(0.0000) |

Note: The regression results on equation (4) for each sample year. The numbers in parentheses below the coefficient estimates are *p*-values associated with *t*-test of $\alpha_0$ against zero and $\alpha_1$ against one. The numbers in parentheses below the *F*-statistic values are their significance levels.

18. Due to the extreme large number of observations for the whole sample period, the regression results for the whole sample period are not reported.

variables is quite strong. This may be some weak evidence for the efficiency for the options market.

Table 11 presents the regression results on equation (5). The results for the whole sample reveal that both maturity (short or long term) and moneyness (ATM or not) are not significant to the arbitrage profitability. However, strategy (long or short) is highly significant to the arbitrage profitability.

Turning to the subsamples, strategy is always highly significant to the arbitrage profitability, while the results for maturity and moneyness are mixed. Maturity is not significant for 2003 at the 10 percent level, is significant for 2004 at the 5 percent level, and highly significant for 2005. Moneyness is only significant for 2004 and 2005 at the 10 percent level.

Furthermore, the relationship between arbitrage profits and strategy is positive for all three years and the whole sample period. This implies that option pairs profitable with a long arbitrage strategy on average give rise to more arbitrage profit. The arbitrage profit does not have a fixed relationship with maturity or moneyness over the three years, although both relationships are positive for the whole samples.

Overall, we can conclude that strategy is most significant for the arbitrage profitability and maturity is the least significant for the arbitrage profitability. The regression observations are consistent with and supportive of our previous *ex post* and *ex ante* findings.

**Table 11 Regression Evidence on Arbitrage Profitability**

| | 2003 | 2004 | 2005 | Whole |
|---|---|---|---|---|
| Intercept | 11.9456<br>(0.0000) | 26.6733<br>(0.0000) | 0.9437<br>(0.5233) | 9.2779<br>(0.0000) |
| Maturity | 2.4179<br>(0.2867) | –8.1372<br>(0.0047) | 5.2728<br>(0.0000) | 1.2934<br>(0.2633) |
| Moneyness | –0.3307<br>(0.8671) | –6.2815<br>(0.0051) | 3.8518<br>(0.0072) | 1.4320<br>(0.2015) |
| Strategy | 30.8706<br>(0.0000) | 5.7693<br>(0.0224) | 68.6258<br>(0.0000) | 41.7896<br>(0.0000) |
| Observations | 1,300 | 904 | 1,615 | 3,819 |
| Adjusted $R^2$ | 0.1626 | 0.0268 | 0.6456 | 0.2726 |
| *F*-statistic | 85.0796<br>(0.0000) | 9.2916<br>(0.0000) | 980.8691<br>(0.0000) | 477.8637<br>(0.0000) |

Note: The regression results on equation (5). The numbers in parentheses below the coefficient estimates and the *F*-statistic values are their significance levels.

## V. Summary and Conclusion

In this paper, we have examined the size and frequency of arbitrage profits utilizing Nikkei 225 option contracts traded on the OSE. The results reveal that arbitrage opportunities are present but infrequent for OSE members whose cost structure is much lower relative to other market participants.

To allow for synchronous prices, the call and put pairs are matched within a one-minute interval. This issue is important, as non-synchronous prices are a major problem for studies that test PCP. Transaction costs including the implicit bid-ask spread, which is a large cost facing arbitragers, are considered. Furthermore, this paper provides evidence on the factors that contribute to mispricing and also considers the *ex ante* size and frequency of arbitrage profits.

It has been observed that 3,819 (2.74 percent) of the total 139,586 matched pairs are profitable over the complete sample period for OSE members. The mean profit is 22.609 index points. The frequency of arbitrage opportunity is lower than that observed in the literature. For example, Lee and Nayar (1993) find that only 9.5 percent of the sample is profitable after considering transaction costs, while Fung and Mok (2001) find violations with 4.34 percent and 1.83 percent of the sample for members and non-members, respectively.

It appears that the long arbitrage strategy is more profitable relative to the short arbitrage strategy on average. However, the short arbitrage strategy occurs more frequently relative to the long arbitrage strategy. This indicates that the puts are more frequently overpriced while the calls are more frequently underpriced relative to the PCP equation. Cavallo and Mammola (2000) also provide evidence that the short arbitrage strategy is more profitable relative to the long arbitrage strategy.

A large part of the profitable pairs are ATM. This result is similar to Draper and Fung (2002), who note that a large part of the profitable pairs clusters around ATM options. However, ITM pairs have the highest frequency of profitable arbitrage opportunities. The average arbitrage profit is similar across all three moneyness categories.

The *ex post* analysis also reveals that the average maturity for arbitrage profitable opportunities is about 25 days for the whole sample. The most frequent maturity among the arbitrage opportunities during the sample period is 10 days.

A sensitivity analysis is also conducted to investigate the arbitrage opportunities with various transaction costs. It is found that, assuming low transactions costs, only a low percentage of the sample is profitable.

The *ex ante* results reveal that arbitrage opportunities are significantly reduced with a one-minute or three-minute execution period. It is shown that the *ex ante* profits are also largely reduced on average for the whole sample compared to the *ex post* results.

Regression analysis has been conducted to further assess the PCP and arbitrage profitability. The regression results provide some supporting evidence to the *ex post* and *ex ante* results.

In conclusion, this paper has presented an accurate and detailed analysis of arbitrage profitability using tick-by-tick transaction data on Nikkei 225 options. Although infrequent violations of PCP are observed, the average size of arbitrage opportunities is large after accounting for realistic transaction costs. Therefore, the conclusion reached in this paper is that there is no strong evidence against the efficiency of the Nikkei 225 options market.

Finally, it should be noted that the efficiency analysis performed in this paper can be extended in many directions. For example, the PCP is a cross-market relationship and other strategies such as various types of spreads and convexity conditions can be used to check the internal option market efficiency. These issues will be left for future research.

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