Effects of External Debt on Domestic Resource Allocation in a Small Open Economy with Limited Access to the World Capital Market

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This paper studies a small open economy with a large external debt. It begins by considering the long-term effects of shocks in the international capital market and domestic fiscal policy on the amount of outstanding external debt and the domestic reallocation of resources between the tradable and non-tradable goods sectors theoretically. Then, numerical examples on various shocks are illustrated to help understand those theoretical predictions. For example, an exogenous increase in government expenditures, together with an increase in the external borrowing rate, requires resource reallocation between the tradable goods sector and the non-tradable goods sector as well as a small amount of external borrowings to shift toward the new stationary equilibrium, whose welfare level worsens.

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I. Introduction

Discussions of the sound macroeconomic policy management and optimal exchange rate regimes under the free flow of capital have been intensifying since global capital flows became increasingly active from the 1990s.[1] Amid these developments, evaluations of the Argentine currency board system, which attracted a great deal of attention from the early 1990s as a hard-peg experiment under the free flow of capital, seem to have been greatly revised.[2]

Specifically, Argentina overcame the effects of the Mexican currency crisis to achieve high growth rates of 5.5 percent in 1996, 8.1 percent in 1997, and 3.9 percent in 1998, and through that time the belief that this economic success was partially thanks to the currency board system became widespread. During the high economic growth years from 1993 through 1998, however, Argentina's public-sector government debt to GDP ratio shot up by 12 percentage points to 40 percent in 1998. Moreover, Argentina's unemployment rate has remained close to 20 percent ever since 1995, with few signs of any lasting improvement.[3] Ultimately, in 2002 Argentina was forced to abandon the currency board system. From this Argentine experience, it is often understood that when sound fiscal policy is not maintained, even hard pegs fail to function under the free flow of capital.[4]

For example, Mussa (2002) says the essence of the Argentine crisis was that loose fiscal policy led to the default on the nation's external debt and to the collapse of its financial markets and economy. A growing cumulative government deficit will inevitably lead to tragic consequences eventually, regardless of which exchange rate system is followed. Corden (2002) argues that Argentina's fatal error was the failure to slash government expenditures while the economy was still favorable, which he considers as a prerequisite to a viable currency board system.

Based on these lessons, discussions have begun appearing in academic circles regarding the type of fiscal policy that would make a fixed exchange rate system sustainable. In particular, these have included reexaminations of sudden stops of international capital inflows,[5] and of the types of fiscal policies that lead to them.

1. The debate on whether the liberalization of capital accounts during the 1990s itself was beneficial or not is also continuing (see, for example, Rodrik [1998]). (For the most recent research developments, see Eichengreen [2001].)
2. Discussions on the choice of exchange rate system, centered around the merits and demerits of dollarization in Argentina and elsewhere in the Americas, lie beyond the scope of this paper. For an excellent review of this field, see Corbo (2002). Also, see Fujiki and Otani (2002) for the recent developments regarding exchange rate systems and regional currency blocs in the Americas, Europe, and East Asia.
3. For example, Mussa (2002), who was then serving as the International Monetary Fund (IMF) Chief Economist, reviewed Argentina's economic performance following the Mexican currency crisis and concluded (1) that the currency board system, which he and other IMF staff were somewhat skeptical about when it was introduced, was in fact effective, but (2) that the failure to significantly reduce the annual fiscal deficit despite the vigorous economic growth following the Mexican currency crisis and the subsequent ongoing increase in the consolidated government deficit were cause for concern.
4. Some stress that the diminished competitiveness of Argentine export goods following the Brazilian crisis was one cause of the Argentine crisis (see, for example, Feldstein [2002] and Perry and Servén [2002]). These authors conclude that nations should consider the exchange rate systems of their key trading partners when selecting their own exchange rate systems.
5. Some believe that it was the substantial reductions in external capital inflows more than the loose fiscal policy that had the gravest effects on the Argentine economy. For example, Calvo *et al.* (2002) state that as a result of the continuous rapid decrease of external capital inflows into Argentina from 1998: (1) because tradable goods account for only a small percentage of the Argentine economy, the decrease in external capital inflows itself

For example, Calvo (2002) explicates the relationship between fiscal policy and the sudden stop of capital inflows in an open economy utilizing an endogenous economic growth model. This model posits an economy where capital is the only factor of production, where production follows a linear production function, and where the government levies distortional taxes on tradable goods and pays its external debt. This economy has "good" and "bad" equilibriums—positive economic growth accompanied by the inflow of foreign capital, and zero economic growth with no inflow of foreign capital. The coordination among foreign investors determines which of these two equilibriums is chosen. Normally, the first "good" equilibrium with positive economic growth and influxes of foreign capital is chosen. Under this model, when a shock occurs, for example, because hidden government debts are discovered or because the government debt balloons for some other reason, the real exchange rate appreciates, capital inflows stop, and the economy flips over to the zero economic growth "bad" equilibrium.

Similarly, while the leading analyses of the East Asian currency crisis have generally focused on the moral hazard issue (see, for example, Corsetti *et al.* [1999], Krugman [1998], and Schneider and Tornell [2003]), or on the relationship between liquidity provision, the exchange rate system, and the banking crisis (see, for example, Chang and Velasco [2000, 2001]),[6] papers emphasizing the effects of fiscal policy have recently appeared in various contexts.

For example, Burnside *et al.* (2001) state that the recognition of an implicit government debt guarantee for financial institutions may lead to an upward revision of private-sector expectations concerning the amount of future government deficits. They argue that to cover these future deficits, the government will have to (1) devalue its own currency to reduce the nominal debt, (2) garner seigniorage from issuing currency, or (3) have inflation tax, and that once expectations regarding the future tax collection spread among agents, this will result in an attack on the currency, regardless of which option is selected.[7]

Calvo (2002) and Burnside *et al.* (2001) use single-good models to analyze these types of capital inflows and the relationship between the current account balance and fiscal policy. When analyzing the effects of fiscal policy on the domestic economy and on capital inflows, however, it would be preferable and more useful to make a distinction between tradable and non-tradable goods. This is because it is appropriate to assume that the goods that may be used as "international collateral" as defined by Caballero and Krishnamurthy (2001a)—that is to say, the goods that may be allocated for external borrowings and interest payments—are limited to tradable items that are

demanded a major change in the real foreign exchange rate to reduce imports; (2) because the debt was dollar denominated, the changes in the real foreign exchange rate amplified the balance-sheet effect at enterprises and financial institutions; and (3) because the dollar-denominated debt was massive, the decrease in the exchange rate effectively expanded the external debt measured in domestic currency, to the point that repayment became impossible. They therefore conclude that the cessation of external capital inflows may have been more important than the government budget deficit.

6. Hattori (2002) combines these two viewpoints, considers two different cases—the one when a nation that has no problems with its long-term external debt repayment capacity temporarily falls into a liquidity crisis, and the other where the extent of a currency crisis is more severe than what may be explained by the economic fundamentals—and discusses the effects and necessity of emergency liquidity provision when currency crises occur.

7. Here, the cause of the crisis is the future government debt, and there is no increase in the present government debt as under the first-generation currency crisis models pioneered by Krugman (1979).

convertible assets on the international market. And if that is the case, then the ability to maintain the production of tradable goods should be viewed as the most important factor when evaluating the capacity to service external debt.

Moreover, from the perspective of crisis prevention, it would also be beneficial to deepen our understanding of what kinds of changes in the resource allocations between the tradable and non-tradable goods sectors occur in nations where crises have yet to emerge, but which have vast external debts and where there are concerns regarding a possible currency crisis accompanying changes in capital market conditions.[8]

Many analyses using two-sector economic models have already been conducted on the reduction of external debt in a small open economy and the accompanying reallocation of resources between the tradable and non-tradable goods sectors, and on the role of government expenditures, as summarized in Turnovsky (1997), chapter 4.1. In our paper, we use this type of model to investigate the relationship between the currency crises and fiscal policy in an attempt to apply the model to more present-day issues.

Specifically, we utilize an expanded version of the model presented in Turnovsky (1997), chapter 4.2. Following Bhandari *et al.* (1990), we add the following assumptions to make the Turnovsky (1997) model more realistic. First, because the sovereign risk premium increases along with increases in the outstanding external debt, the borrowing from international capital markets does not ensure consumption smoothing over time. Second, nations with external debts must produce tradable goods to repay these debts with interest.[9] We then consider the reallocation of domestic economy resources demanded by changes in the international capital market, such as increases in world interest rates and in the sovereign premium. In addition, we consider the cases where government expenditures increase, and when the productivity of the tradable goods production sector declines.

8. In fact, two-sector models are being used in some recent analyses of currency crises. For example, Schneider and Tornell (2003) use a tradable goods/non-tradable goods two-sector model to consider how the balance-sheet effect from borrowings from overseas investors influences the real economy. Under their model, a tradable goods production enterprise is in a perfect capital market with fixed tradable goods prices. This enterprise uses tradable goods and labor to produce tradable goods, and fulfills the terms of its loan agreements without fail. Meanwhile, a non-tradable goods production company produces non-tradable goods by borrowing in domestic currency (for non-tradable goods) and in foreign currency (for tradable goods). This model assumes that non-tradable goods are needed for the production of non-tradable goods. The economy randomly experiences booms and recessions, and the real foreign exchange rate fluctuates. The entrepreneur running the non-tradable goods production company increases borrowings in expectation of good times, the boom arrives as expected, and the production volumes and the price of the non-tradable goods (which is the reciprocal of the real foreign-exchange rate) both rise. Suppose the government provides a debt guarantee to the non-tradable goods production company. If this company expects a recession and a decline in the real foreign exchange rate, the company increases its foreign currency-denominated (tradable goods) borrowings and intentionally fails to reduce its foreign currency debt burden despite a decline in its foreign currency income in an effort to increase the likelihood of bankruptcy. Since the government is providing debt guarantees that cover the foreign investors, the firm's best option is to continue promoting investment from overseas. When the recession actually arrives and the real exchange rate declines, the large overseas borrowings are amplified by the balance sheet effect, the firm goes bankrupt, and the economy falls into an equilibrium from which it is difficult to recover. Schneider and Tornell (2003) emphasize the influence of implicit government debt guarantees in the non-tradable goods boom and bust cycle via the balance-sheet effect from external borrowings and the production of non-tradable goods using non-tradable goods. In contrast, our paper focuses on the changes in resource allocation in the domestic economy in the long run following an external shock, rather than explicating the boom and bust cycle and the reasons why crises emerge.
9. Turnovsky (1997) himself states that analyses based on the additional Bhandari *et al.* (1990) assumptions are possible, but does not provide any numerical examples.

Based on the theoretical model, we numerically illustrate the changes in the resource allocation in nations with large external debts hit by shocks. For example, consider an exogenous increase in wasteful government expenditures, such as those under inefficient government spending characterized by loose expenditures on non-tradable goods, resulting in a higher risk premium and cost of funds. The long-term equilibrium before and after the shocks demonstrates that unlimited funds at a constant world interest rate are no longer provided from the international capital market and that a reallocation of resources between the tradable goods and non-tradable goods sectors becomes necessary.[10]

Obviously, the appropriateness of the description "inefficient government spending characterized by loose expenditures on non-tradable goods" in countries where currency crises emerge, assumed for the analyses herein, varies from country to country. Regardless, at the very least, as summarized by Tommasi *et al.* (2001), this description does approximate the characteristics of Argentina's fiscal policy during the period under consideration. Specifically, Argentina's constitution only gives the national authorities exclusive competence over areas associated with defense and foreign affairs. The areas of economics and social infrastructures lie under the joint authority of the central and provincial governments, but the provincial governments are solely responsible for primary education and local services. As a result, the provincial governments bear about half of consolidated government expenses. Meanwhile, the provincial governments, which historically do not have strong tax-collection abilities, delegate much of this work to the central government, and about 80 percent of all tax revenues are collected by the central government. Because of this, the central government has to transfer tax revenues to the provincial governments, in accordance with their expenditures. Under this system, even when economic conditions are favorable, the provincial governments tend to increase their expenditures out of fear that transfers from the central government may decline in the future. For example, an increase in public servant personnel expenses was reportedly responsible for the dramatic increase in provincial government expenditures during the economic boom of the 1990s.[11]

10. This paper takes the position that given the actual conditions whereby the insurance function of the free capital market is limited, the type of policy that the debtor nations themselves should follow over the middle to long term to avoid a crisis is an important issue. On the other hand, Caballero (2003) asserts that the debt restructuring process led by the IMF is only directly beneficial to countries that are expected to fall into bankruptcy, and that for the many other nations it is far more important for the private and public sectors to diversify the class of assets circulated in the world debt market and enhance the risk reallocation function of the international capital market. This type of analysis on the insurance function of the international capital market is certainly important, but it lies beyond the scope of this paper.
11. It is well known that as a result of the Argentine government reforms under the Menem Administration (1990–95), the number of central government public servants declined from 870,000 in 1989 to 180,000 in 1994. Over this same period, however, the number of provincial government public servants remained essentially unchanged at around 1.1 million. Thus, it seems that the Menem Administration reforms were limited to the central government. Additionally, under the Menem Administration, central government expenditures on transfers to provincial governments doubled, and those funds that the government transfers on a discretionary basis were allocated not so much to urban areas with high unemployment but more to relatively poor areas where the ruling party enjoyed strong support. Central government transfers covered 43 percent of the provincial government budgets in urbanized regions, compared with 78 percent in rural regions (Gibson and Calvo [1997]).

Our analyses proceed as follows. Section II presents the theoretical model, and derives the stationary equilibrium. Section III confirms the effects on the stationary equilibrium from changes in exogenous variables. Section IV summarizes the analytical conclusions, and the detailed mathematical explications are presented in the Appendix.

II. Open Economy Model

This section introduces our open economy model. We follow Bhandari *et al.* (1990) in expanding the two-sector model presented in chapter 4.2 of Turnovsky (1997) to explicitly assume that the risk premium rises along with increases in the amount of external debt outstanding.

In this economy, there is a representative household that produces and consumes tradable and non-tradable goods (this representative household is referred to as "the consumer" hereinafter), the government, and foreign investors. At each point in time, the consumer makes decisions on his/her production volumes, consumption volumes, and capital accumulation (investment) based on given relative goods prices and interest rates for borrowings from the foreign investors. The non-tradable goods are consumed domestically, but are utilized as investment goods. The tradable goods are consumed domestically or exported.[12]

By trading bonds on international capital markets at each point in time, the consumer can conclude borrowing contracts with foreign investors. The bonds are issued based on tradable goods, and non-tradable goods cannot be used as backing for international capital borrowings. This assumption that only tradable goods can be used captures the concept of "international collateral" defined by Caballero and Krishnamurthy (2001a), which means the goods that may be used for international borrowings. The foreign investors who provide loans to this country's consumers form a group, and offer funds at a common interest rate, which is the world interest rate plus a risk premium set in accordance with the conditions of the country's external debt. Finally, the government collects a lump-sum tax from the consumers, and uses this tax revenue to purchase both tradable and non-tradable goods.

The following analyses assume that the country is a net debtor to focus on the overseas debt adjustment process. The prices of tradable goods are determined by the international goods market, and are given for this country. By selecting the volume units, the prices of the tradable goods are normalized at one, and the prices of the non-tradable goods at q. The outline of the model is explained mathematically here, and a more detailed discussion is presented in the Appendix.

12. One could also posit the investment goods as tradable goods, but in that case, unless the adjustment cost of investment is introduced, since tradable goods prices are fixed on overseas markets, the capital stock would be instantly adjusted and the dynamics we are interested in would not emerge. Alternatively, when non-tradable goods are used as tradable goods, since costs are incurred in increasing the production of non-tradable goods, the capital accumulation dynamics do emerge even without the investment adjustment costs. See chapter 4.1 of Turnovsky (1997) for a detailed review of the relevant literature.

A. Consumer

The consumer's utility function over different points in time is defined as follows.

$$\int_{t=0}^{\infty} U(C_t^T,\ C_t^N)e^{-\beta t}dt.$$

Here, β indicates the subjective discount rate, C is the goods consumption volume, and the superscripts [($i = T, N$); the same hereafter] indicate tradable goods and non-tradable goods, respectively. The subscript letter t expresses the time.

The production functions for tradable goods and non-tradable goods are as follows:

$$Y^T = F(K^T, L^T),$$
$$Y^N = H(K^N, L^N).$$

Here Y is the production volume, K is the amount of capital, and L shows the labor input. The production functions for both sectors are homogeneous of degree 1.

There is no depreciation in capital stock, so all investment represents an increase in capital stock,[13] which is to say that the following equation holds.

$$\dot{K}_t = I_t.$$

Here K is the total amount of capital in this economy, $\dot{K}$ is the time differential of capital K (hereafter the time differential is expressed by the symbol $\cdot$), and I is the investment. The capital is allocated to the two sectors in accordance with the following equation.

$$K = K^T + K^N.$$

There is no population growth, and the working population $\bar{L}$ is allocated to the two sectors in accordance with the following equation.

$$\bar{L} = L^T + L^N.$$

B. Government

The government collects a lump-sum tax T from the consumer, and uses this tax revenue to purchase both tradable and non-tradable goods. For simplification, assuming a balanced-budget fiscal policy, the government budget constraint can be written as follows.

$$G^T + q \cdot G^N = T.$$

13. In models that assume capital depreciation, the long-term equilibrium investment only corresponds to the depreciation of capital stock. Therefore, the assumption of no capital depreciation adopted here does not have a large qualitative influence on the following analytical results.

Here G^T and G^N express the government expenditures in the tradable goods and non-tradable goods sectors, respectively. The government expenditures in these two sectors are not included in either the utility function or the production function. In other words, the government consumes a portion of the production without increasing either productivity or utility, and in this sense the government spending may be said to be purely wasteful. On the other hand, because the government expenditures are covered by a lump-sum tax and not a proportional tax on market prices, the taxation does not result in any price distortion.

C. Overseas Investors

In the international capital market, there is a foreign investors' group that makes loans to the consumer in the debtor country at a common interest rate, which is set considering the consumer's external debt repayment capacity (creditworthiness), collateral, and other risk factors.

Here, for simplification, we assume the foreign investors predict that the default risk will increase as the cumulative balance of the country's private-sector external debt rises, and that their lending rate rises in line with this. Specifically, following Bhandari *et al.* (1990), the outstanding external debt is expressed as b (≥ 0) and the lending rate as r_s, which is defined as follows.

$$r_s = r_s(b),\ r' > 0,\ r'' > 0,\quad (b \geq 0). \tag{1}$$

The following model also assumes that all of the borrowings are private-sector debt. Incidentally, according to Perry and Servén (2002), as shown on the second column of Table 1, in Argentina the ratio of external debt to GDP shot up from 27.7 percent in 1993 to 58.3 percent in 2001. And as shown in the third and fourth columns of the table, the ratio for the private sector rose from 5.6 percent to 25.1 percent during this same period. Also, while the consolidated government debt remained basically flat over 1999–2001, which was the term just prior to the crisis,

Table 1 External Debt of Argentina

Percentage of GDP

Year	Total	Consolidated government debt	Nonfinancial private and financial debt
1993	27.7	22.1	5.6
1994	29.6	23.5	6.1
1995	39.0	26.8	12.1
1996	41.8	27.3	14.5
1997	44.8	28.2	16.6
1998	48.6	30.5	18.0
1999	53.6	33.2	20.4
2000	54.0	33.9	20.1
2001	58.3	33.2	25.1

Source: Perry and Servén (2002), p. 47, table 4.5.

the weight of the private-sector borrowings increased. Accordingly, we can easily adopt the assumption that focusing only on the private-sector debt is appropriate as an approximation for the period just prior to the crisis.[14]

D. Consumer Optimization

The consumer owns enterprises, provides them with capital and labor, receives wages from them, and has claims on their profits. He also purchases the investment goods required for future capital accumulation in future periods.

Each period, the consumer repays the principal and interest on the previous period's borrowings and arranges the current period's borrowings from the international capital market. Because the outstanding balance of external debt is positive, the budget constraint equation has the opposite sign to the usual, as follows.

$$\dot{b}_t = r_{dt} \bullet b_t - (Y_t^T - C_t^T) - q_t\,(Y_t^N - C_t^N) + q_t \bullet I_t + T_t.$$

Here, r_d is the borrowing rate on the international money market, which is given for the consumer. The consumer determines his investment, production, and consumption plans over time to maximize his utility function, so the dynamic optimization can be expressed as follows.

$$\underset{\{C_t^T, C_t^N, K_t^T, K_t^N, L_t^T, L_t^N, I_t, b_t\}_{t\in[0,\infty]}}{\mathrm{Max}} \int_{t=0}^{\infty} U(C_t^T,\, C_t^N) e^{-\beta t} dt$$

$$\begin{aligned} \text{subject to} \quad & \dot{b}_t = r_{dt} \bullet b_t + C_t^T + q_t \bullet C_t^N + q_t \bullet I_t + T_t - Y_t^T - q_t \bullet Y_t^N, \\ & K_0 = \overline{K}, \\ & b_0 = \overline{b} > 0, \\ & I_t = \dot{K}_t, \\ & K_t = K_t^T + K_t^N, \\ & \overline{L} = L_t^T + L_t^N. \end{aligned}$$

The current value Hamiltonian for this equation is defined as follows.

$$\begin{aligned} \mathrm{H} = {} & U(C^T, C^N) + \lambda_1\{Y^T + q \bullet Y^N - C^T - q \bullet C^N - q \bullet I - T - r_d \bullet b\} \\ & + \lambda_2 \bullet I + \lambda_3(K - K^T - K^N) + \lambda_4(\overline{L} - L^T - L^N), \end{aligned}$$

where λ_1, λ_2, λ_3, and λ_4 are Lagrangean multipliers.

The first-order conditions for this optimization problem are

$$\frac{\partial U}{\partial C^T} = \frac{1}{q} \bullet \frac{\partial U}{\partial C^N} = \lambda_1,$$

14. Perry and Servén (2002) point out that from 1998 the volume of domestic bonds issued by the government of Argentina increased while the volume of external bonds issued remained flat, and that a large percentage of the domestic bonds were held by domestic financial institutions. This implies that the external borrowings by the private sector during this period were not so much to maintain the level of private-sector consumption but rather were used for investment in domestic bonds, and thus indirectly financed Argentina's government expenditures.

$$\frac{\partial F}{\partial K^T} = q\frac{\partial H}{\partial K^N}, \frac{\partial F}{\partial L^T} = q\frac{\partial H}{\partial L^N},$$

$$\frac{\dot{q}}{q} = r_d - \frac{1}{q}f'(k^T) = r_d - h'(k^N),$$

$$\dot{\lambda}_1 = \lambda_1\{\beta - r_d\},$$

$$\lim_{t\to\infty}\lambda_{1t} \bullet b_t \bullet e^{-rt} = 0 \quad \text{(non-Ponzi game condition)},$$

$$\lim_{t\to\infty}\lambda_{3t} \bullet K_t \bullet e^{-rt} = 0 \quad \text{(transversality condition)},$$

where k^i $(i = T, N)$ expresses that capital-labor ratio.

E. Stationary Equilibrium

The following equality must hold when this economy is in a stationary equilibrium.

$$\dot{\lambda}_1 = \dot{b} = \dot{K} = \dot{q} = 0.$$

In other words, the instantaneous marginal utility of wealth at each period, which is the Lagrangean multiplier for each period's budget constraint equation, the outstanding external debt, capital, and the price of non-tradable goods all remain constant over time.

The equilibrium condition in the international capital market should be the following equation.

$$r_d = r_s(b) \equiv r(b).$$

From the first-order conditions of the consumer's optimization and the equilibrium conditions of the capital and labor markets, under a stationary equilibrium the following equations hold.

$$\frac{\partial U(C^T*, C^N*)}{\partial C^T} = \lambda_1^*, \frac{\partial U(C^T*, C^N*)}{\partial C^N} = \lambda_1^* \bullet q^*, \quad (C^T*, C^N* > 0), \tag{2}$$

$$\beta = r(b^*) = h'(k^N*) = \frac{1}{q^*}f'(k^T*), \tag{3}$$

$$f(k^T*) - k^T*f'(k^T*) = q^*(h(k^N*) - k^N*h'(k^N*)), \tag{4}$$

$$K^* = K^T* + K^N*, \tag{5}$$

$$\bar{L} = L^T* + L^N*, \tag{6}$$

$$F(K^T*, L^T*) = C^T* + G^T + r(b^*) \bullet b^*, \tag{7}$$

$$H(K^N*, L^N*) = C^N* + G^N, \tag{8}$$

$$G^T + q^* \bullet G^N = T, \tag{9}$$

where the superscript asterisks denote the values taken at the stationary equilibrium.

Equation (2) is the Euler equation for the consumption of tradable and non-tradable goods. Equation (3) shows the equalization of the subjective discount rate and the interest rate on external borrowings as well as the equalization of the interest rate on external borrowings and the marginal productivity of capital in the tradable and non-tradable goods sectors. Equation (4) is the equalization for the marginal productivity of labor in both production sectors. Equations (5) and (6) are market clearing conditions for the capital and labor markets. Equations (7) and (8) are market clearing conditions for the tradable goods and non-tradable goods markets. Finally, equation (9) is the government budget constraint.

From the first equal sign of equation (3), in this economy the subjective discount rate is equal to the external borrowings interest rate, which determines the amount of external debt. From the next equal sign, the marginal productivity of capital is also equal to the external borrowing rate, which also determines a common capital-labor ratio in both sectors. Combining the third equal sign in equation (3) with equation (4) determines the capital-labor ratio in the tradable goods sector as well as the non-tradable goods price q^* (the long-run equilibrium price for non-tradable goods, namely, the real exchange rate).

Equations (7)–(9) indicate that the effect of government expenditures on the production volume of each sector varies depending on the amounts directed to the tradable goods and non-tradable goods sectors. Finally, because capital accumulation has finished under this stationary equilibrium, the investment is zero.

F. Discussion of the Stationary Equilibrium and of the Effect from Expanding Government Expenditures

From equation (3), the distinctive characteristics of this stationary equilibrium are that the external borrowings interest rate determines the outstanding debt, and that this interest rate is also equal to the marginal productivity of capital. Moreover, the external borrowings interest rate in turn ensures that the capital-labor ratios in the tradable and non-tradable goods sectors remain equal. Additionally, the non-tradable goods prices are set by equation (4).

In this stationary equilibrium, increases in exogenous demand from an expansion of government expenditures should necessitate adjustments such as a reduction in private-sector consumption or an increase in the production volume at a constant price.

In a stationary equilibrium, because the capital-labor ratios remain equal, adjustments to the production volumes of tradable and non-tradable goods occur via inter-sectoral adjustments in labor and capital inputs. Specifically, since adjustments in production volumes in reaction to an increase (or decrease) in the demand for goods occur via the transfer of labor and capital between the two production sectors, in cases where there are structural or systematic hindrances to the reallocation of labor and/or capital, this should imply a growing risk of failure to produce a sufficient volume of tradable goods to repay the external debt.

And during this period, the outstanding external debt remains unchanged because it is fixed by the external borrowings interest rate.

It may seem strange that under a long-run equilibrium an exogenous increase in government expenditures does not result in a rise in the outstanding external debt.

This is a result of the assumption of equation (1) following Bhandari *et al.* (1990), and may be explained as follows.

If the representative consumer could borrow at a fixed interest rate from the international capital market regardless of the outstanding external debt, r_s would no longer be dependent on b in equation (1). From equations (2)–(6), the fixed world interest rate is equal to the subjective discount rate in this stationary equilibrium. This determines the consumption of tradable and non-tradable goods and the aggregate capital stock of the economy, so the influence on the outstanding debt disappears. Conversely, if government expenditures increase the amount of tradable goods, the only variable that may change in equations (7)–(9) is the external borrowings, so while the domestic distribution of resources remains unchanged, the private sector uses the increased external debt to finance the government expenditures, which are used for imports. Thus, when access to a free capital market is assumed, the outstanding external debt inevitably rises. However, this does not have any effect on domestic private-sector consumption.

Yet because this paper assumes that the outstanding external debt is adjusted to equal the subjective discount rate as well as the interest rate paid to foreign investors (including the risk premium), the route with an increase in the external debt and an increase in government expenditures is cut off. In other words, the market discipline apparently prevents covering wasteful government expenditures through an increase in external borrowings. Conversely, if government expenditures increase without this type of limitation, this will have an adverse effect on domestic production and on the consumption schedule.

The argument here is also consistent with that in Caballero and Krishnamurthy (2001b), who note that the theoretical assumption of a free international capital market where an unlimited volume of funds may be borrowed at a fixed interest rate, albeit at a somewhat high interest rate, is sometimes inappropriate. It may be more important to focus the analysis on the domestic adjustments that occur assuming a fixed amount for the inflow of foreign capital.

III. Influence on the Long-Term Equilibrium from Changes in Exogenous Variables: Numerical Example

In this section, we examine how the values of the endogenous variables under the stationary equilibrium derived in the previous chapter are influenced by changes in exogenous variables. Those changes include changes in the world interest rate, the foreign investors' risk premium (marginal increase rate), increases in government expenditures, and declines in productivity.

As shown in the Appendix, if tradable goods are gross substitutes for non-tradable goods, a saddle path exists nearby the equilibrium. Accordingly, we now conduct the analysis using a numerical example. We first specify the parameters of the production and utility functions to obtain our benchmark long-run stationary equilibrium. We then add shocks to the relevant exogenous variables to see how this stationary equilibrium shifts to a new equilibrium over the long term. The main features of transitional paths between two stationary equilibria are summarized at the end of this chapter.

First, we suppose the following addi-log utility function.

$$U(C_t^T, C_t^N) = \frac{(C_t^T)^{1-\sigma_T}}{1-\sigma_T} + \frac{(C_t^N)^{1-\sigma_N}}{1-\sigma_N}.$$

Following Nishiyama (2002), we set the parameters at $\sigma_T = 1/1.4$, $\sigma_N = 1/4$.

Next, we specify the production functions for tradable and non-tradable goods as the following Cobb-Douglas functions.

$$F(K, L) = A \cdot K^{\alpha} \cdot L^{1-\alpha},$$
$$H(K, L) = K^{\phi} \cdot L^{1-\phi}.$$

Because the analytical results are sensitive to the relative size of the factor intensity of both sectors, we separate the examinations into a capital-intensive tradable goods sector case ($\alpha = 0.3$, $\phi = 0.25$) and a capital-intensive non-tradable goods sector case ($\alpha = 0.25$, $\phi = 0.3$). Finally, we approximate the foreign investors' lending function as follows by separating the fixed world interest rate $r0$ from the risk premium.

$$r_s(b) = r0 + r1 \cdot b^2.$$

The numerical analysis takes the following order.

First, we set the initial period values as subjective discount rate = 0.03; $r0$ (world interest rate) = 0.02; $r1$ (risk premium parameter) = 0.004; government expenditures on tradable goods = 0.3; government expenditures on non-tradable goods = 0.3; and A (productivity parameter) = 1.

Next we calculate the endogenous variables: external debt, interest rate, interest payment, non-tradable goods price, capital-labor ratio in the tradable goods sector, capital-labor ratio in the non-tradable goods sector, labor employment in the tradable goods sector, labor employment in the non-tradable goods sector, aggregate capital stock, production volume of tradable goods, production volume of non-tradable goods, consumption volumes of tradable and non-tradable goods, instantaneous utility, and the marginal utility of wealth. Then, to examine the external debt burden, we calculate the ratio of interest payments to the production volume of tradable goods, and the ratio of interest plus capital payments to the production volume of tradable goods.

Finally we show the characteristic roots of the dynamic equation coefficient matrix, and the existence of a saddle-point path is confirmed by the presence of two (real) negative characteristic roots. The stationary equilibrium derived in this manner serves as a benchmark for the simulations.

Next, we confirm the effects on this equilibrium from changing four exogenous variables, as follows: (1) increasing the world interest rate from 0.02 to 0.025; (2) increasing the risk premium from 0.004 to 0.007; (3) increasing the government expenditures on tradable goods from 0.3 to 0.5; and (4) increasing the government expenditures on non-tradable goods from 0.3 to 0.5. We then examine the effects from (5) simultaneously increasing the risk premium and increasing the government expenditures on non-tradable goods. Finally, we examine the effects on the benchmark case and on the results of cases (1) through (5) above from decreasing the productivity of the tradable goods sector (a decrease in A from 1 to 0.95). So we conduct calculations for 12 different cases under the two scenarios whereby the tradable goods sector is capital intensive ($\alpha = 0.3$, $\phi = 0.25$, Table 2) and the non-tradable goods sector is capital intensive ($\alpha = 0.25$, $\phi = 0.3$, Table 3). In all cases, we assume that the shock is permanent. The results may be explained as follows.

A. Increase in the World Interest Rate

When the world interest rate rises and all other conditions remain unchanged, the interest rate on external borrowings rises and thus consumers decrease the outstanding external debt until the interest rate on external borrowings comes to equal the subjective discount rate. Whether or not the amount of interest payments declines along with this adjustment in the outstanding external debt depends upon the shape of the lending rate coefficient. If the outstanding external debt and the interest payments both decline, the amount of tradable goods that need to be produced to cover the interest payments declines along with the decline in the interest payments, and this is the case presented in Table 2, column 1. In this example, the tradable goods sector is capital intensive. Labor moves from the tradable goods sector to the non-tradable goods sector, and the aggregate capital stock, which was being used intensively in the tradable goods sector, also declines. Moreover, consumption increases and the utility level rises. On the contrary, if the non-tradable goods sector is capital intensive, Table 3, column 2 shows that labor moves from the tradable goods sector to the non-tradable goods sector, and the aggregate capital stock increases.

B. Increase in the Risk Premium

Like the effects from an increase in the world interest rate, when the risk premium rises and all other conditions remain unchanged, the interest rate on external borrowings rises and this prompts the consumers to decrease the outstanding external debt.[15] Here again, any increase or decrease in the amount of interest payments depends on the size of $r1$ in the external investors' lending function. If the tradable goods sector is capital intensive, the results are presented in Table 2, column 2. When the amount of interest payments does decline, the amount of tradable goods that need to be produced to cover the interest payments decreases, and labor moves from the tradable goods sector to the non-tradable goods sector. If the non-tradable goods sector is capital intensive, Table 3, column 2 shows that labor moves from the tradable goods sector to the non-tradable goods sector, and the aggregate capital stock increases.

15. In fact, as noted by Calvo *et al.* (2002), from 2001 Argentina's sovereign spread rose sharply, the influx of external capital shrank, and the ratio of current account deficit to imports worsened to 14 percent.

Table 2 Shock Tests: Capital-Intensive Tradable Goods Sector ($\alpha = 0.3$, $\phi = 0.25$)

	Benchmark	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
A	1.0000	0	0	0	0	0	−0.0500	−0.0500	−0.0500	−0.0500	−0.0500	−0.0500
$r0$	0.0200	+0.0050	0	0	0	0	0	+0.0050	0	0	0	0
$r1$	0.0040	0	+0.0030	0	0	+0.0030	0	0	+0.0030	0	0	+0.0030
G^T	0.3000	0	0	+0.2000	0	0	0	0	0	+0.2000	0	0
G^N	0.3000	0	0	0	+0.1726	+0.1726	0	0	0	0	+0.1816	+0.1816
$G^T + qG^N$	0.6478	0	0	+0.2000	+0.2000	+0.2000	−0.0174	−0.0174	−0.0174	+0.1826	+0.1826	+0.1826
b^*	1.5811	−0.4631	−0.3859	0	0	−0.3859	0	−0.4631	−0.3859	0	0	−0.3859
r^*	0.0300	0	0	0	0	0	0	0	0	0	0	0
r^*b^*	0.0474	−0.0139	−0.0116	0	0	−0.0116	0	−0.0139	−0.0116	0	0	−0.0116
q^*	1.1592	0	0	0	0	0	−0.0580	−0.0580	−0.0580	−0.0580	−0.0580	−0.0580
k^{T*}	21.7221	0	0	0	0	0	0	0	0	0	0	0
k^{N*}	16.8950	0	0	0	0	0	0	0	0	0	0	0
K^*	19.5477	−0.0180	−0.0150	+0.2546	−0.1385	−0.1529	−0.0036	−0.0227	−0.0195	+0.2660	−0.1476	−0.1628
L^{T*}	0.5495	−0.0037	−0.0031	+0.0527	−0.0287	−0.0317	−0.0007	−0.0047	−0.0041	+0.0551	−0.0306	−0.0337
L^{N*}	0.4505	+0.0037	+0.0031	−0.0527	+0.0287	+0.0317	+0.0007	+0.0047	+0.0041	−0.0551	+0.0306	+0.0337
$F(K^{T*}, L^{T*})$	1.3838	−0.0094	−0.0079	+0.1328	−0.0722	−0.0798	−0.0019	−0.0119	−0.0102	+0.1388	−0.0770	−0.0849
$G(K^{N*}, L^{N*})$	0.9133	+0.0076	+0.0063	−0.1069	+0.0582	+0.0642	+0.0015	+0.0096	+0.0082	−0.1117	+0.0620	+0.0684
$r^*b^*/F(K^{T*}, L^{T*})$	0.0343	−0.0099	−0.0082	−0.0030	+0.0019	−0.0068	+0.0001	−0.0098	−0.0082	−0.0031	+0.0020	−0.0067
$(1 + r^*)b^*/F(K^{T*}, L^{T*})$	1.1769	−0.3390	−0.2822	−0.1031	+0.0648	−0.2328	+0.0016	−0.3375	−0.2806	−0.1073	+0.0694	−0.2291
C^{T*}	1.0364	+0.0045	+0.0037	−0.0672	−0.0722	−0.0682	−0.0710	−0.0666	−0.0673	−0.1374	−0.1423	−0.1383
C^{N*}	0.6133	+0.0076	+0.0063	−0.1069	−0.1144	−0.1084	+0.0015	+0.0096	+0.0082	−0.1117	−0.1196	−0.1132
$U(C^{T*}, C^{N*})$	4.4599	+0.0129	+0.0108	−0.1908	−0.2048	−0.1935	−0.0692	−0.0557	−0.0579	−0.2701	−0.2849	−0.2729
λ_1^*	0.9748	−0.0030	−0.0025	+0.0478	+0.0516	+0.0485	+0.0507	+0.0473	+0.0479	+0.1042	+0.1085	+0.1050
Characteristic roots	−1.6123	−1.3759	−1.8415	−1.6193	−1.6082	−1.8367	−1.5862	−1.3552	−1.8111	−1.5872	−1.5854	−1.8101
	−0.0496	−0.0492	−0.0505	−0.0449	−0.0450	−0.0459	−0.0439	−0.0436	−0.0447	−0.0392	−0.0389	−0.0397
	0.0810	0.0802	0.0816	0.0763	0.0764	0.0769	0.0973	0.0959	0.0982	0.0926	0.0923	0.0932
	1.6610	1.4149	1.8904	1.6680	1.6568	1.8856	1.6365	1.3966	1.8613	1.6375	1.6357	1.8603

Note: The shaded variables are the exogenous variables that are hit by shocks. Columns (1)–(11) show the deviations from the benchmark values. The characteristic roots of the Jacobian matrix in equation (A.10) are evaluated around the new stationary equilibrium after the shocks are given.

Table 3 Shock Tests: Capital-Intensive Non-Tradable Goods Sector (α = 0.25, ϕ = 0.3)

	Benchmark	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
A	1.0000	0	0	0	0	0	–0.0500	–0.0500	–0.0500	–0.0500	–0.0500	–0.0500
$r0$	0.0200	+0.0050	0	0	0	0	0	+0.0050	0	0	0	0
$r1$	0.0040	0	+0.0030	0	0	+0.0030	0	0	+0.0030	0	0	+0.0030
G^T	0.3000	0	0	+0.2000	0	0	0	0	0	+0.2000	0	0
G^N	0.3000	0	0	0	+0.2343	+0.2343	0	0	0	0	+0.2466	+0.2466
$G^T + qG^N$	0.5561	0	0	+0.2000	+0.2000	+0.2000	–0.0128	–0.0128	–0.0128	+0.1872	+0.1872	+0.1872
b^*	1.5811	–0.4631	–0.3859	0	0	–0.3859	0	–0.4631	–0.3859	0	0	–0.3859
r^*	0.0300	0	0	0	0	0	0	0	0	0	0	0
r^*b^*	0.0474	–0.0139	–0.0116	0	0	–0.0116	0	–0.0139	–0.0116	0	0	–0.0116
q^*	0.8536	0	0	0	0	0	–0.0427	–0.0427	–0.0427	–0.0427	–0.0427	–0.0427
k^{T*}	20.8654	0	0	0	0	0	0	0	0	0	0	0
k^{N*}	26.8270	0	0	0	0	0	0	0	0	0	0	0
K^*	23.6575	+0.0285	+0.0238	–0.4021	+0.1449	+0.1678	+0.1145	+0.1438	+0.1389	–0.2994	+0.2558	+0.2794
L^{T*}	0.5316	–0.0048	–0.0040	+0.0675	–0.0243	–0.0281	–0.0192	–0.0241	–0.0233	+0.0502	–0.0429	–0.0469
L^{N*}	0.4684	+0.0048	+0.0040	–0.0675	+0.0243	+0.0281	+0.0192	+0.0241	+0.0233	–0.0502	+0.0429	+0.0469
$F(K^{T*}, L^{T*})$	1.1363	–0.0102	–0.0085	+0.1442	–0.0519	–0.0602	–0.0411	–0.0515	–0.0498	+0.1073	–0.0917	–0.1002
$G(K^{N*}, L^{N*})$	1.2565	+0.0128	+0.0107	–0.1810	+0.0652	+0.0755	+0.0515	+0.0647	+0.0625	–0.1347	+0.1151	+0.1257
$r^*b^*/F(K^{T*}, L^{T*})$	0.0417	–0.0120	–0.0100	–0.0047	+0.0020	–0.0084	+0.0016	–0.0108	–0.0087	–0.0036	+0.0037	–0.0071
$(1 + r^*)b^*/F(K^{T*}, L^{T*})$	1.4333	–0.4106	–0.3417	–0.1614	+0.0686	–0.2893	+0.0537	–0.3717	–0.3002	–0.1237	+0.1258	–0.2451
C^{T*}	0.7888	+0.0037	+0.0031	–0.0558	–0.0519	–0.0486	–0.0411	–0.0376	–0.0382	–0.0927	–0.0917	–0.0886
C^{N*}	0.9565	+0.0128	+0.0107	–0.1810	–0.1691	–0.1588	+0.0515	+0.0647	+0.0625	–0.1347	–0.1315	–0.1209
$U(C^{T*}, C^{N*})$	4.5602	+0.0173	+0.0144	–0.2556	–0.2382	–0.2230	+0.0022	+0.0195	+0.0166	–0.2535	–0.2489	–0.2338
λ_1^*	1.1846	–0.0039	–0.0033	+0.0638	+0.0591	+0.0550	+0.0461	+0.0421	+0.0428	+0.1106	+0.1093	+0.1052
Characteristic roots	–1.5304	–1.3089	–1.7475	–1.5215	–1.5335	–1.7510	–1.5242	–1.3058	–1.7395	–1.5077	–1.5298	–1.7459
	–0.0502	–0.0498	–0.0512	–0.0455	–0.0454	–0.0463	–0.0615	–0.0606	–0.0628	–0.0567	–0.0559	–0.0572
	0.0818	0.0808	0.0824	0.0770	0.0769	0.0775	0.0728	0.0721	0.0733	0.0681	0.0672	0.0677
	1.5789	1.3479	1.7963	1.5699	1.5820	1.7998	1.5708	1.3422	1.7869	1.5542	1.5763	1.7932

Note: The shaded variables are the exogenous variables that are hit by shocks. Columns (1)–(11) show the deviations from the benchmark values. The characteristic roots of the Jacobian matrix in equation (A.10) are evaluated around the new stationary equilibrium after the shocks are given.

C. Increases in Government Expenditures

The effects from an increase in government expenditures on a given production sector should depend on the capital-labor ratio for that sector. Accordingly, the following examinations cover the cases where the tradable goods sector is capital intensive (Table 2) and where the non-tradable goods sector is capital intensive (Table 3).

First, the outstanding external debt is not changed from the initial stationary equilibrium condition because the outstanding external debt is determined so that the subjective discount rate and the interest rate on external borrowings are equal. Accordingly the repayment amount remains unchanged and all increases in government expenditures must be covered by increases in the production volume and/or decreases in consumption.

When the tradable goods sector is capital intensive, as shown in Table 2, column 3, an increase in government expenditures on tradable goods increases the production of tradable goods and decreases the ratio of external debt to tradable goods production. Conversely, as shown in Table 2, column 4, an increase in government expenditures on non-tradable goods increases the production of non-tradable goods, necessitating a shift of labor into the non-tradable goods sector, and thus works to decrease aggregate capital stock.

Next, when the non-tradable goods sector is capital intensive, as shown in Table 3, column 4, an increase in government expenditures on tradable goods works to decrease aggregate capital stock. However, because the production of tradable goods increases, the ratio of external debt to tradable goods production declines. In contrast, as shown in Table 3, column 4, an increase in government expenditures on non-tradable goods works to increase aggregate capital stock.

Suppose that the tradable goods sector is capital intensive. Suppose further that an increase in government expenditures on non-tradable goods occurs at the same time as an exogenous increase in the risk premium. As shown in Table 2, column 5, this leads to economic adjustments whereby aggregate capital stock, the production of tradable goods and the external debt all contract, and economic welfare is worsened.

D. Decreases in Productivity

Here we examine the case where the tradable goods sector is capital intensive and undergoes a negative productivity shock.

In this situation there is a transfer of labor to the non-tradable goods sector, whose productivity has risen on a relative basis, and the production volume of non-tradable goods increases. Conversely the production volume of the tradable goods sector decreases, and along with this the tradable goods production volume ratio of capital plus interest payments rises (as shown in Table 2, column 6).

As for the causes of the hardships that afflicted Argentina's economy, some have noted the worsening terms of trade, especially the decrease in the export competitiveness of Argentine goods following the Brazilian crisis (Calvo *et al.* [2002]). Calvo *et al.* find that the external debt problem may be exacerbated under such conditions. As shown in Table 2, column 10, the analyses in this paper indicate that when the productivity of the export sector declines while government expenditures on non-tradable goods increase, the ratio of interest payments to tradable goods production and the ratio of outstanding external debt to tradable goods production both rise even though the outstanding external debt remains unchanged.

Table 2, column 11 shows the case where government expenditures on non-tradable goods increase under these conditions while the risk premium also rises. In this case, the tradable goods sector contracts, the price of non-tradable goods drops, and the outstanding external debt decreases. The instantaneous utility level also declines.

The case where the non-tradable goods sector is capital intensive is summarized in Table 3, columns 6–11. Labor moves into the non-tradable goods sector because its relative productivity has risen, and the production volume of non-tradable goods rises. The aggregate capital stock increases when the government expenditure on non-tradable goods increases (Table 3, column 6). Another possibility (Table 3, column 11) is the case where the government expenditure on non-tradable goods increases while the risk premium also rises. In this case the tradable goods sector contracts, the prices of non-tradable goods decline, the outstanding external debt drops, and the instantaneous utility level decreases, but the aggregate capital stock increases.

E. Notes Regarding the Short-Term Effects

This model has four variables; however, as can be seen in the Appendix, following Bhandari *et al.* (1990), we can obtain the analytical solutions for the variables that we are interested in. Thus, this subsection illustrates some of the dynamic transition path.

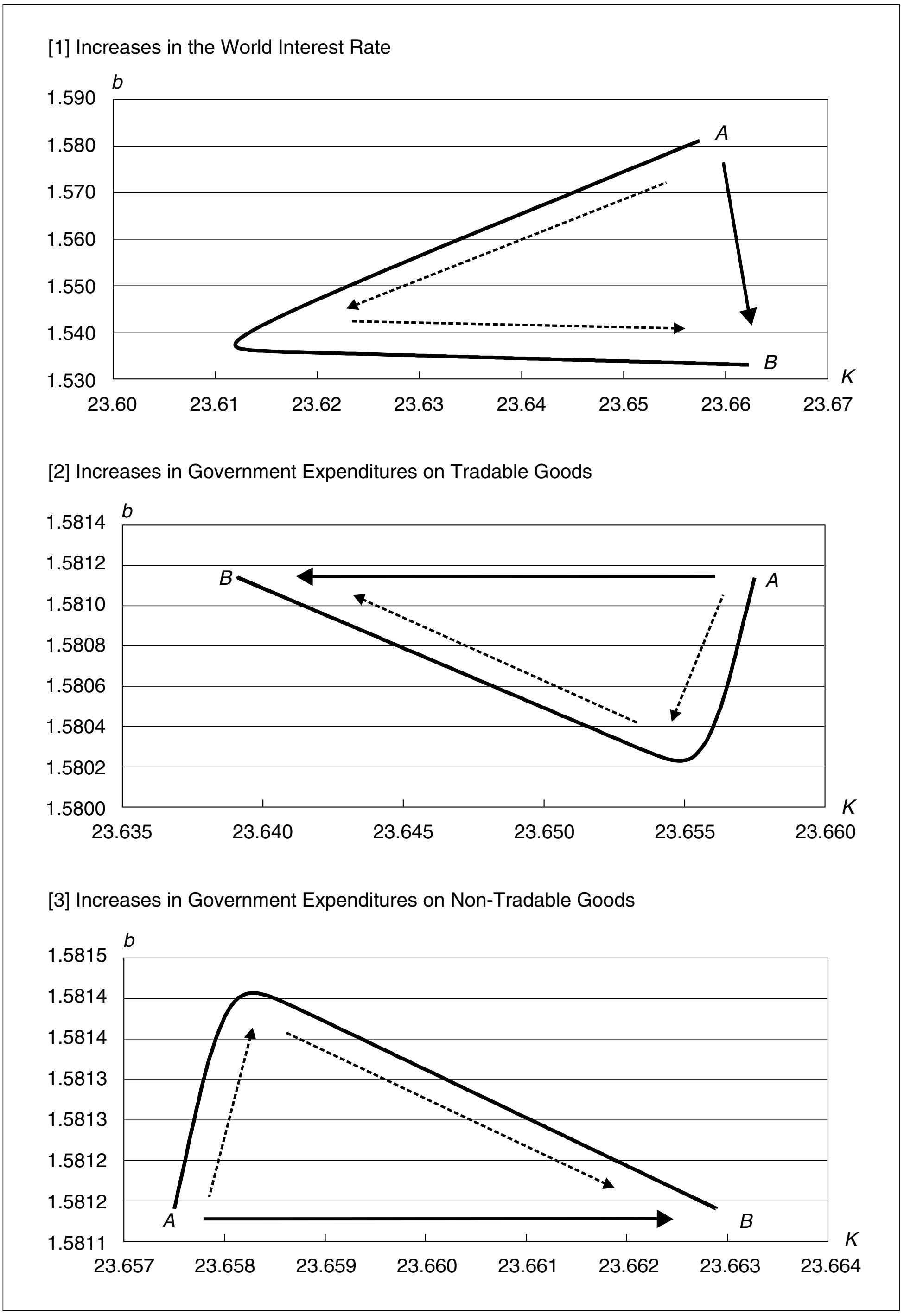
As explained in detail in the Appendix, the dynamic behavior of this model is determined by four variables: the marginal utility of wealth, the prices of non-tradable goods, capital, and the outstanding external debt. The above shock tests assumed that the shocks are all permanent and perfectly anticipated. Under these assumptions, it would be reasonable to assume that the marginal utility of wealth and the prices of non-tradable goods are jump-variables, while the two other variables—capital and the outstanding external debt—gradually move toward the new equilibrium.

Hence, let us check the dynamic properties of capital and the outstanding external debt first, and then examine the transition path of other variables.

For example, the effects from a rise in the world interest rate (Table 3, column 1, the non-tradable goods sector is capital intensive) is illustrated in Figure 1 [1]. In the adjustment process in moving from the initial equilibrium A to the new equilibrium B, aggregate capital stock declines substantially and then gradually increases to the new equilibrium level. The outstanding external debt monotonically declines to the new stationary state.

The effects from an increase in government expenditures on tradable goods and an increase in government expenditures on non-tradable goods increases (summarized in Table 3, columns 3 and 4), are illustrated in Figure 1 [2] and [3]. The long-run levels of outstanding external debt remain constant between the initial equilibrium A and the new equilibrium B, since the world interest rate is constant. However, an increase in government expenditures on tradable goods requires a long-run decrease in aggregate capital stock and a temporary decline in the outstanding external debt (Figure 1 [2]). To the contrary, an increase in government expenditures on non-tradable goods leads

Figure 1 An Example of Short-Run Dynamics (The Case Where the Non-Tradable Goods Sector Is Capital Intensive)



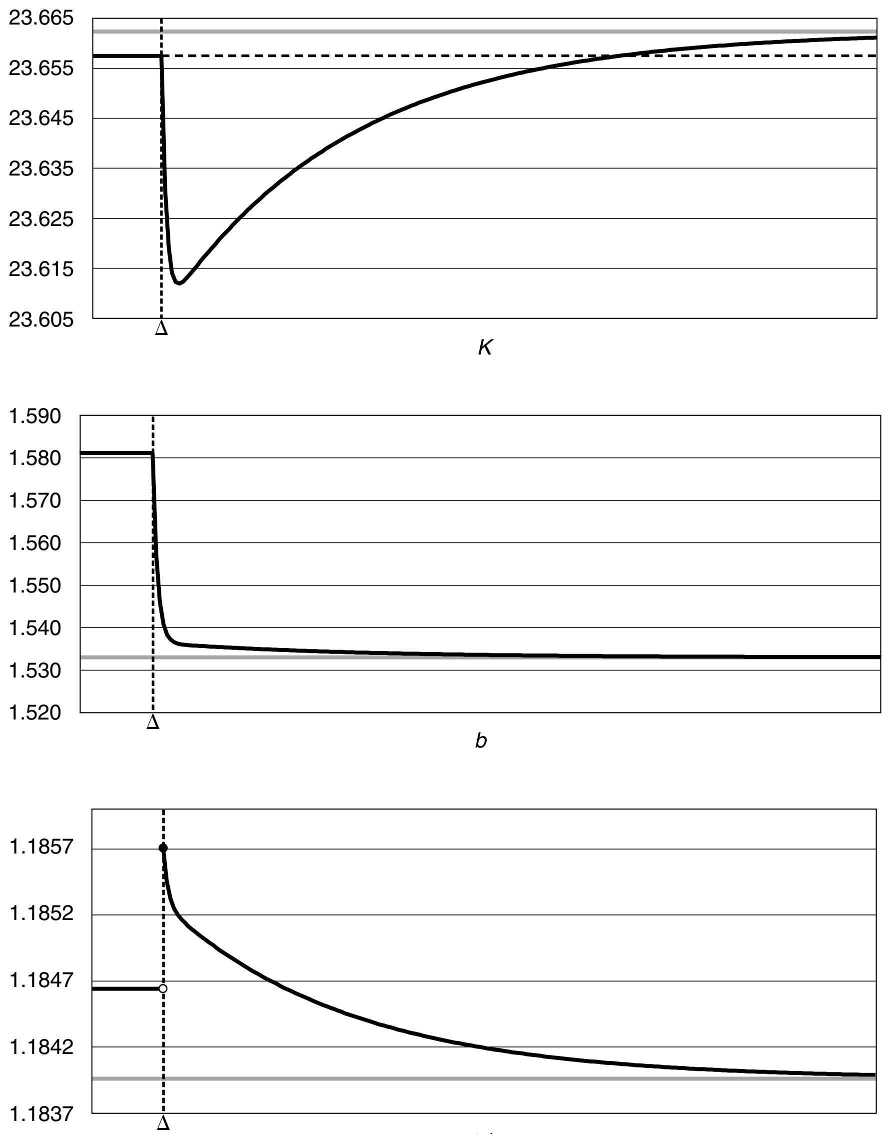
to the long-run increase in aggregate capital stock and a temporary increase in the outstanding external debt (Figure 1 [3]).

Let us see the changes in the other variables. For example, the effects from a rise in the world interest rate (Table 3, column 1, the non-tradable goods sector is capital intensive) on the other variables are illustrated in Figure 2. As we have seen in Figure 1, aggregate capital stock and the outstanding external debt change only gradually move to the new equilibrium. However, one of the two jump-variables, marginal utility of wealth, jumps up immediately after the shock, and gradually declines to a new equilibrium. The price of non-tradable goods falls immediately after the shock, increases above the level of new equilibrium, and then gradually declines to the new equilibrium level. The amount of employment in the non-tradable goods sector rises right after the shock, due to the change of the capital-labor ratios in both production sectors that reflects the increase of the interest rate and the fall of price of non-tradable goods, and then gradually converges to the new equilibrium level. Finally, the ratio of interest payments to the production volume of tradable goods initially increases and declines gradually, because the interest rate increases immediately after the shock while the outstanding external debt only changes gradually.

Figure 3 shows the effects of an increase in government expenditures on non-tradable goods on the important variables we are interested in (Table 3, column 3, and Figure 1 [3], the non-tradable goods sector is capital intensive). Aggregate capital stock increases gradually, while the outstanding external debt temporarily increases and returns to its initial level. In the meantime, one of the two jump-variables, marginal utility of wealth, overshoots immediately after the shock, and gradually declines to a new equilibrium. The price of non-tradable goods increases immediately after the shock, and returns to its initial level. The amount of employment in the non-tradable goods sector and the ratio of interest payments to the production volume of tradable goods increase gradually.

For the sake of completeness, Figure 4 examines the effects from a rise in the world interest rate, from an increase in government expenditures on tradable goods, and from an increase in government expenditures on non-tradable goods increases on aggregate capital stock and the outstanding external debt under the alternative assumption that the tradable goods sector is capital intensive. Long-run changes between the initial equilibrium and new equilibrium are summarized in Table 2, columns 1, 4, and 5. In the adjustment process in moving from the initial equilibrium A to the new equilibrium B, aggregate capital stock declines substantially, and then arrives at the new equilibrium level, as Figure 4 shows. The outstanding external debt monotonically declines to the new stationary state. The effects from an increase in government expenditures on tradable goods (or an increase in government expenditures on non-tradable goods) on aggregate capital stock and the outstanding external debt are symmetric to the results shown in Figure 1. This is because we reversed the factor intensities between the non-tradable goods sector and tradable goods sector.

Figure 2 An Adjustment Process of the Variables (Non-Tradable Goods Sector; Capital-Intensive, the World Interest Rate; Increase, the Case of Table 3, Column 1)



Note: The symbol Δ denotes the point when the shock occurs, and the dark solid lines indicate the trajectories of endogenous variables in the transition. The light solid lines correspond to the levels at the stationary equilibrium after the shock to which the variables converge.

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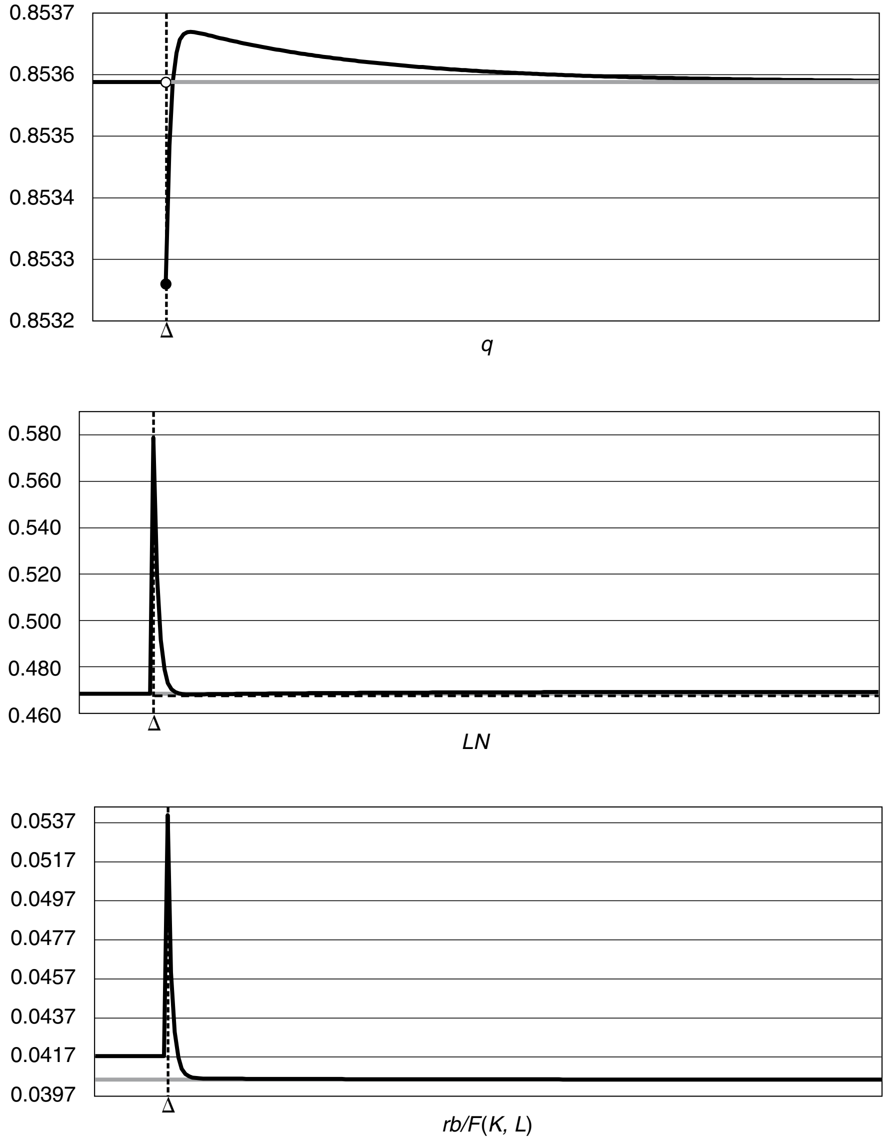
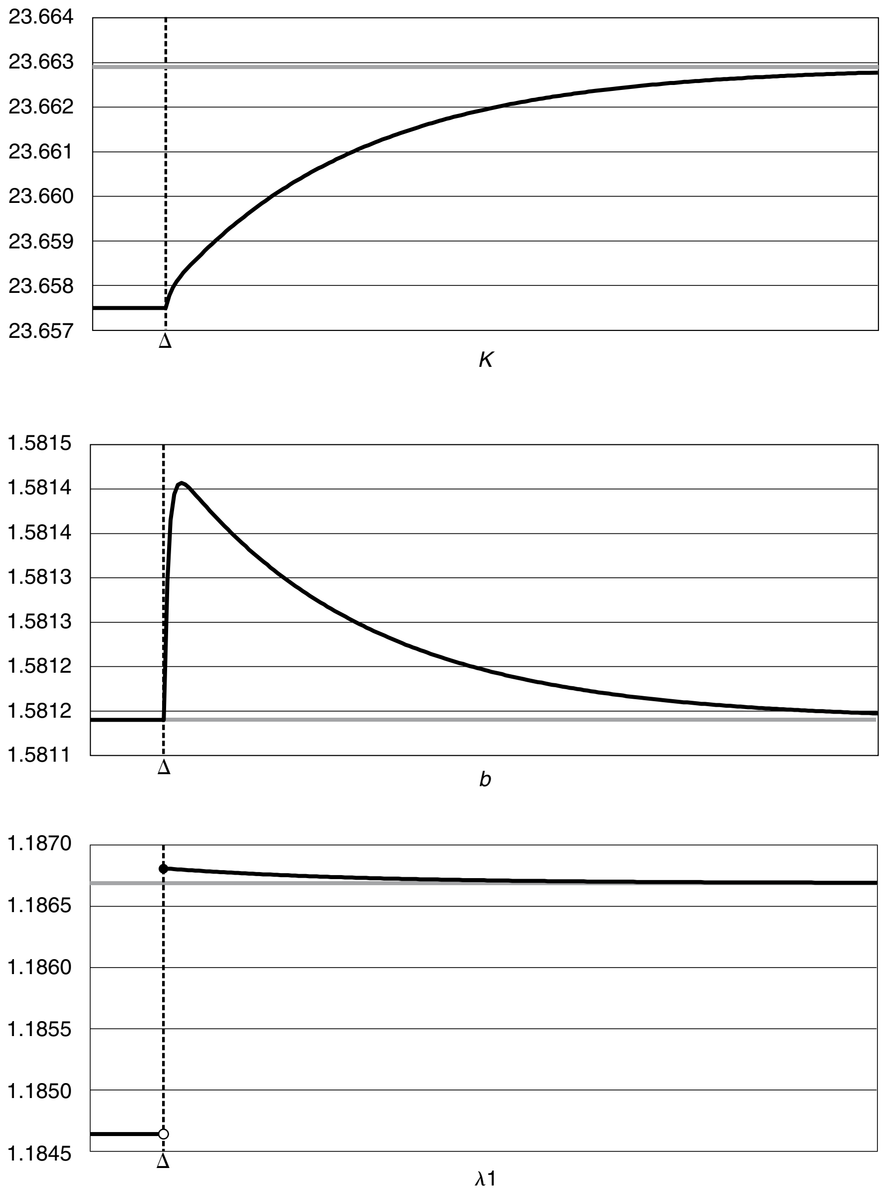
Figure 2 (continued)



Note: The symbol Δ denotes the point when the shock occurs, and the dark solid lines indicate the trajectories of endogenous variables in the transition. The light solid lines correspond to the levels at the stationary equilibrium after the shock to which the variables converge.

Figure 3 An Adjustment Process of the Variables (Non-Tradable Goods Sector; Capital-Intensive, Government Expenditures on Non-Tradable Goods; Increase, the Case of Table 3, Column 4)



Note: The symbol Δ denotes the point when the shock occurs, and the dark solid lines indicate the trajectories of endogenous variables in the transition. The light solid lines correspond to the levels at the stationary equilibrium after the shock to which the variables converge.

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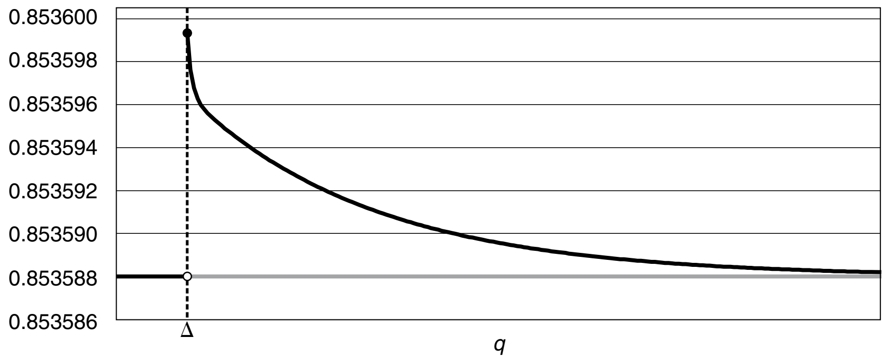
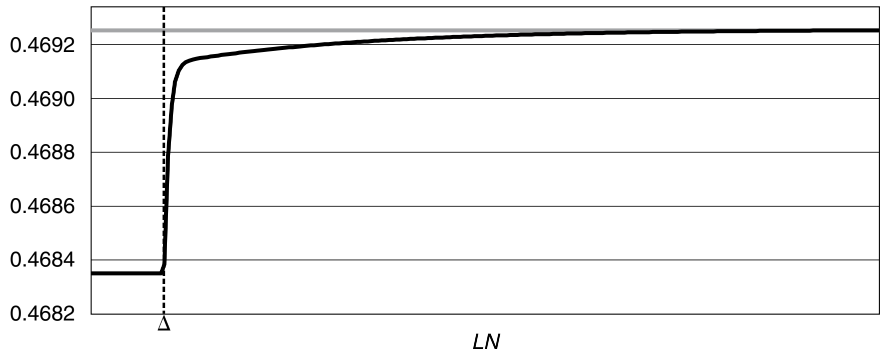
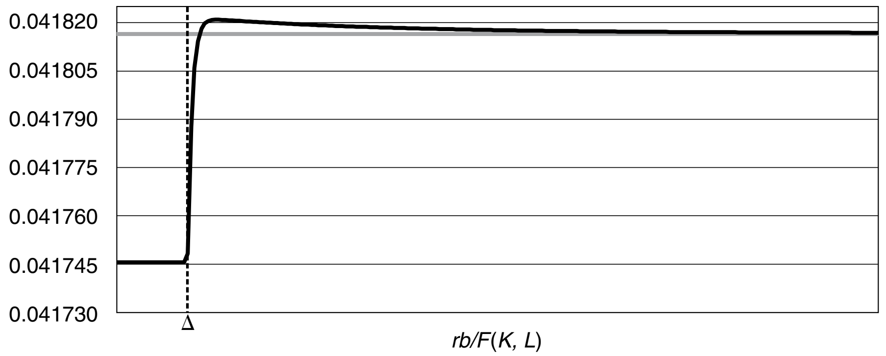
Figure 3 (continued)

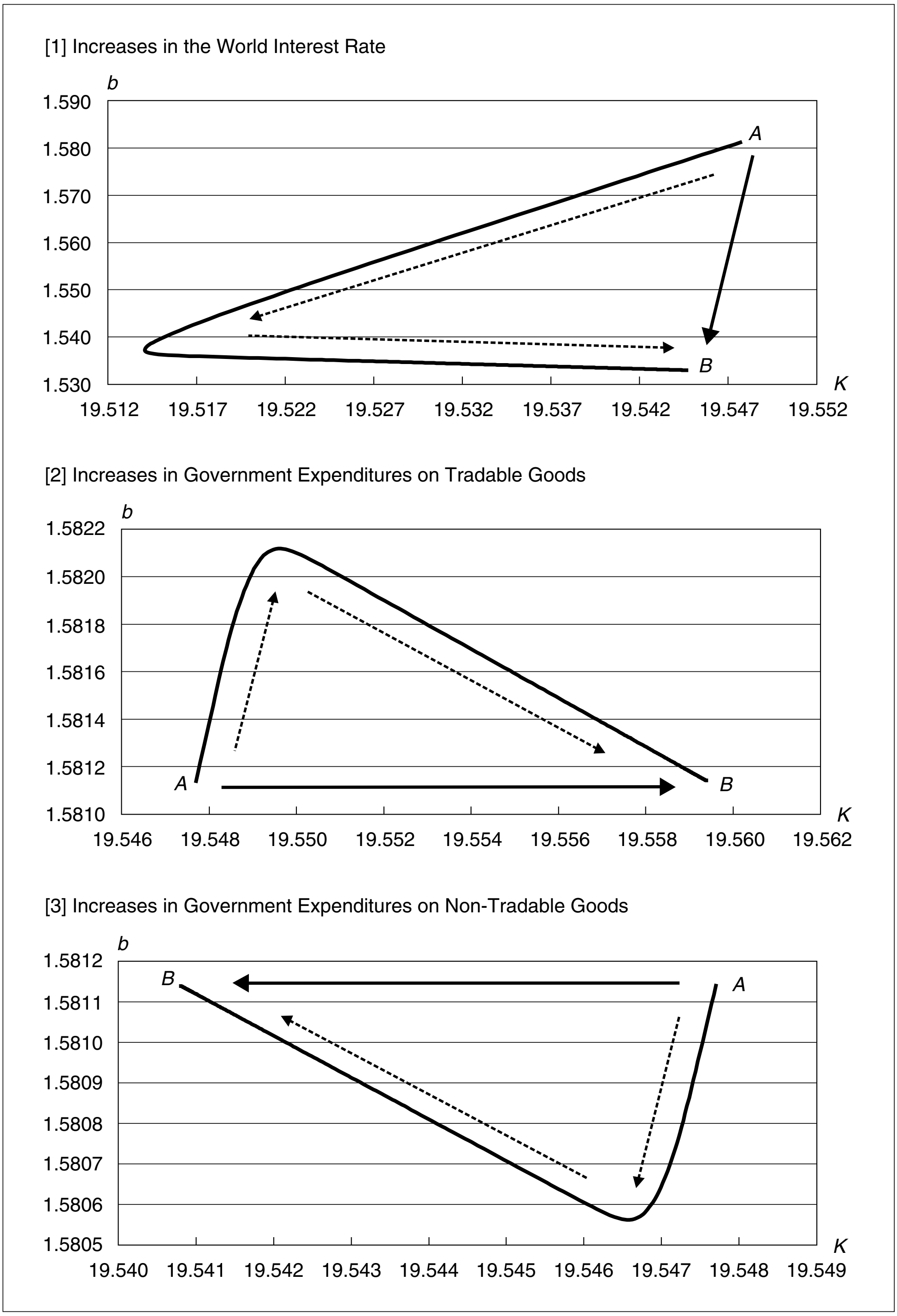






Note: The symbol Δ denotes the point when the shock occurs, and the dark solid lines indicate the trajectories of endogenous variables in the transition. The light solid lines correspond to the levels at the stationary equilibrium after the shock to which the variables converge.

Figure 4 An Example of Short-Run Dynamics
(The Case Where the Tradable Goods Sector Is Capital Intensive)



VI. Summary and Outlook

The analyses in this paper can be summarized as follows. When a nation has a large external debt, and all other conditions are constant, a spendthrift fiscal policy might cause the risk premium to rise, which might boost the interest rate on overseas borrowings simultaneously. In this case, even under a free international capital market, unlimited external borrowing at a constant world interest rate is not guaranteed, and so an active movement of resources between the domestic tradable goods and non-tradable goods sectors becomes necessary.

Especially, if tradable goods alone serve as "international collateral" in the sense of Caballero and Krishnamurthy (2001b), and when the tradable goods sector is capital intensive and a decline in productivity leads to diminished export competitiveness, then the domestic economy is forced into a major reallocation of resources.

Furthermore, the analyses in this paper suggest that if there were factors that hindered this structural adjustment, the domestic economy could not smoothly move to a new stationary equilibrium following an exogenous shock, and this could imply that the economy would then face additional difficulties such as rising unemployment and idle capital.[16]

In this paper, keeping in mind the assertions made in Tomassi *et al.* (2001), we have summarized the effects of wasteful government expenditures on economic structural adjustments via numerical examples. However, the model here does not explicitly incorporate items that would make this problem even harsher, such as downward wage rigidity or taxation that distorts the price mechanism because the government has only weak tax-collection abilities. Additionally, if Tomassi *et al.* (2001) have correctly identified the essence of the Argentine crisis as the growing overall government dependence on external borrowings because the provincial governments pursued their own short-term interests and did not cooperate with central government efforts to reduce debt, such crises will not disappear, even with a greater understanding of the movement of resources among sectors, unless a framework is created whereby the citizens themselves voluntarily cooperate with efforts to reduce fiscal expenditures. In this sense, the discussion is about the income distribution, and needs to transcend conventional macroeconomics and move into the political realm.

According to Tommasi *et al.* (2001), the prescription for Argentina emphasized by economists, the decentralization of microeconomic management combined with the centralization of fiscal rules for macroeconomic management, was too simplistic due to the following political context. First, under the Argentine electoral system, individuals running for the National Congress must be included on party lists to be reelected, but only about one in four incumbents seeking reelection in the 1989–99 period were renominated by their parties. Since provincial power-brokers have a decisive influence in deciding who gets listed, provincial governments have a great deal of power in the congressional elections. Under this electoral system, the provincial governments often

16. In relation with this, Calvo *et al.* (2002) state that in economies with rigid nominal domestic prices, the real exchange rate will decline greatly until the fixed foreign exchange rate system collapses. Their hypothesis is that it is politically difficult to decrease the government budget deficit because the private sector does not understand this economic consequence, and thus nations put off the required structural reforms.

fight over central government fiscal sources and repeated short-sighted decisions are possible. For example, in seven separate instances from 1992 to 1994, provincial governments went bankrupt, were given central government bonds by the central government, and sold these bonds on the market to procure additional funds. Tommasi *et al.* (2001) also report cases where regional pension systems went bankrupt because disbursements were maintained despite difficulties in collecting insurance premiums, and the shortfalls were covered by the central government. Tommasi *et al.* (2001) say that the effectiveness of fiscal discipline is clearly not guaranteed under this type of political framework, and that the prescription in this case is electoral reforms to reduce the dependency of national legislators on the local party elite, and reform of the instruments that legislate the interaction between the president and Congress, which could strengthen the role of the Congress and prepare a medium- to long-term framework for the fiscal policies of the entire government (both central and regional).

Our paper does not delve into this political background. We focus on the types of domestic structural adjustments required over the long term following external economic shocks, and leave the more political aspects as topics for future research.

APPENDIX

This appendix presents the details of the derivation of the results presented in Section II.

A. Consumer's Problem

A representative household (hereafter consumer) maximizes his/her lifetime utility function by solving the following optimal control problem:

$$\underset{\{C_t^T, C_t^N, K_t^T, K_t^N, L_t^T, L_t^N, I_t, b_t\}_{t\in[0,\infty]}}{\text{Max}} \int_{t=0}^{\infty} U(C_t^T,\ C_t^N) e^{-\beta t} dt$$

$$\begin{aligned}
\text{subject to} \quad & \dot{b}_t = r_{dt} \bullet b_t + C_t^T + q_t \bullet C_t^N + q_t \bullet I_t + T_t - Y_t^T - q_t \bullet Y_t^N, \\
& K_0 = \overline{K}, \\
& b_0 = \overline{b} > 0, \\
& I_t = \dot{K}_t, \\
& K_t = K_t^T + K_t^N, \\
& \overline{L} = L_t^T + L_t^N.
\end{aligned}$$

To analyze this problem, it is convenient to define the current value Hamiltonian, H:

$$\begin{aligned}
\mathrm{H} = {} & U(C^T,\ C^N) + \lambda_1\{Y^T + q \bullet Y^N - C^T - q \bullet C^N - q \bullet I - T - r_d \bullet b\} \\
& + \lambda_2 \bullet I + \lambda_3(K - K^T - K^N) + \lambda_4(L - L^T - L^N).
\end{aligned}$$

The first-order conditions for optimization are as follows.

$$\frac{\partial \mathrm{H}}{\partial C^T} = \frac{\partial U}{\partial C^T} - \lambda_1 = 0,$$

$$\frac{\partial \mathrm{H}}{\partial C^N} = \frac{\partial U}{\partial C^N} - \lambda_1 \bullet q = 0,$$

$$\frac{\partial \mathrm{H}}{\partial K^T} = \lambda_1 \frac{\partial F}{\partial K^T} - \lambda_3 = 0,$$

$$\frac{\partial \mathrm{H}}{\partial K^N} = \lambda_1 \bullet q \frac{\partial H}{\partial K^N} - \lambda_3 = 0,$$

$$\frac{\partial \mathrm{H}}{\partial K} = \lambda_3 = \beta \bullet \lambda_2 - \dot{\lambda}_2,$$

$$\frac{\partial \mathrm{H}}{\partial L^T} = \lambda_1 \frac{\partial F}{\partial L^T} - \lambda_4 = 0,$$

$$\frac{\partial \mathrm{H}}{\partial L^N} = \lambda_1 \bullet q \frac{\partial H}{\partial L^N} - \lambda_4 = 0,$$

$$\frac{\partial \mathrm{H}}{\partial I} = -\lambda_1 \bullet q + \lambda_2 = 0,$$

$$\frac{\partial \mathrm{H}}{\partial b} = -\lambda_1 \cdot r_d = -\beta \cdot \lambda_1 + \dot{\lambda}_1,$$

$$\lim_{t\to\infty} \lambda_{1t} \cdot b_t \cdot e^{-rt} = 0 \quad \text{(non-Ponzi game condition)},$$

$$\lim_{t\to\infty} \lambda_{3t} \cdot K_t \cdot e^{-rt} = 0 \quad \text{(transversality condition)}.$$

Combining the market clearing conditions, the first-order conditions, and the terminal conditions, we obtain the following equations that characterize the equilibrium path:

$$\frac{\partial U}{\partial C^T} = \lambda_1, \frac{\partial U}{\partial C^N} = \lambda_1 \cdot q, \quad (C^T, C^N > 0),$$

$$\frac{\dot{\lambda}_1}{\lambda_1} = \beta - r_d,$$

$$f'(k^T) = q \cdot h'(k^N),$$

$$(f(k^T) - k^T f'(k^T)) = q(h(k^N) - k^N h'(k^N)),$$

$$K = K^T + K^N,$$

$$\overline{L} = L^T + L^N,$$

$$\frac{\dot{q}}{q} = r_d - \frac{1}{q} f'(k^T) = r_d - h'(k^N),$$

$$\dot{K} = H(K^N, L^N) - C^N - G^N,$$

$$\dot{b} = r_d \cdot b + C^T + G^T - F(K^T, L^T),$$

$$G^T + q \cdot G^N = T,$$

$$r_s(b) = r_d.$$

B. Stationary Equilibrium

Under the stationary equilibrium, equations (A.1) through (A.9) must hold (the superscript asterisks indicate the stationary-equilibrium values):

$$\frac{\partial U(C^T*, C^N*)}{\partial C^T} = \lambda_1^*, \quad \frac{\partial U(C^T*, C^N*)}{\partial C^N} = \lambda_1^* \cdot q^*, \quad (C^T*, C^N* > 0), \quad \text{(A.1)}$$

$$\beta = r(b^*) \equiv r_d = r_s(b^*), \quad \text{(A.2)}$$

$$\frac{1}{q^*} f'(k^{T*}) = h'(k^{N*}) = r(b^*), \quad \text{(A.3)}$$

$$f(k^{T*}) - k^{T*} f'(k^{T*}) = q^*(h(k^{N*}) - k^{N*} h'(k^{N*})), \quad \text{(A.4)}$$

$$K^* = K^{T*} + K^{N*}, \quad \text{(A.5)}$$

$$\overline{L} = L^{T*} + L^{N*}, \quad \text{(A.6)}$$

$$F(K^{T*}, L^{T*}) = C^{T*} + G^T + r(b^*) \cdot b^*, \quad \text{(A.7)}$$

$$H(K^{N*}, L^{N*}) = C^{N*} + G^N, \tag{A.8}$$

$$G^T + q^* \cdot G^N = T. \tag{A.9}$$

Equation (A.1) shows that at the stationary equilibrium, the consumption for both goods will be a function of λ_1 and q:

$$C^{T*} = C^{T*}(\lambda_1, q),$$
$$C^{N*} = C^{N*}(\lambda_1, q).$$

Equations (A.3) and (A.4) show that k^T, k^N are functions of q:

$$k^{T*} = k^{T*}(q),$$
$$k^{N*} = k^{N*}(q).$$

We rearrange equation (A.5) as

$$K = k^T \cdot L^T + k^N \cdot L^N.$$

Combining this expression with equation (A.6) yields

$$L^{T*} = L^{T*}(q, K),$$
$$L^{N*} = L^{N*}(q, K).$$

These results show that the traded goods and non-tradable goods outputs will be functions of q and K:

$$Y^{T*} = L^{T*}(q, K) \cdot f(k^{T*}(q)) = Y^{T*}(q, K),$$
$$Y^{N*} = L^{N*}(q, K) \cdot h(k^{N*}(q)) = Y^{N*}(q, K).$$

Moreover, equations (A.1)–(A.9) provide us with the following useful relationships for further analysis:

$$\left.\frac{\partial k^{T*}}{\partial q}\right|_{q=q^*} = \frac{h(q^*)}{f''(q^*)(k^{N*}(q^*) - k^{T*}(q^*))},$$

$$\left.\frac{\partial k^{N*}}{\partial q}\right|_{q=q^*} = \frac{f(q^*)}{q^{*2} \cdot h''(q^*)(k^{N*}(q^*) - k^{T*}(q^*))},$$

$$\left.\frac{\partial L^{T*}}{\partial K}\right|_{q=q^*, K=K^*} = \frac{1}{k^{T*}(q^*) - k^{N*}(q^*)},$$

$$\left.\frac{\partial L^{N*}}{\partial K}\right|_{q=q^*, K=K^*} = \frac{1}{k^{N*}(q^*) - k^{T*}(q^*)},$$

$$L^{T*}(q^*, K^*) = \frac{K^* - k^{N*}(q^*)}{k^{T*}(q^*) - k^{N*}(q^*)},$$

$$L^{N*}(q^*, K^*) = \frac{k^{T*}(q^*) - K^*}{k^{T*}(q^*) - k^{N*}(q^*)},$$

$$\left.\frac{\partial L^{T*}}{\partial q}\right|_{q=q^*, K=K^*} = \left[\frac{h(q^*) \bullet L^{T*}(q^*, K^*)}{f''(q^*)} + \frac{f(q^*) \bullet L^{N*}(q^*, K^*)}{q^{*2} \bullet h''(q^*)}\right]\frac{1}{(k^{N*}(q^*) - k^{T*}(q^*))^2},$$

$$\left.\frac{\partial L^{N*}}{\partial q}\right|_{q=q^*, K=K^*} = -\left[\frac{h(q^*) \bullet L^{T*}(q^*, K^*)}{f''(q^*)} + \frac{f(q^*) \bullet L^{N*}(q^*, K^*)}{q^{*2} \bullet h''(q^*)}\right]\frac{1}{(k^{N*}(q^*) - k^{T*}(q^*))^2},$$

$$\left.\frac{\partial Y^{T*}}{\partial K}\right|_{q=q^*, K=K^*} = f(q^*)\left.\frac{\partial L^{T*}}{\partial K}\right|_{q=q^*, K=K^*} = \frac{f(q^*)}{k^{T*}(q^*) - k^{N*}(q^*)},$$

$$\left.\frac{\partial Y^{N*}}{\partial K}\right|_{q=q^*, K=K^*} = h(q^*)\left.\frac{\partial L^{N*}}{\partial K}\right|_{q=q^*, K=K^*} = \frac{h(q^*)}{k^{N*}(q^*) - k^{T*}(q^*)},$$

$$\left.\frac{\partial Y^{T*}}{\partial q}\right|_{q=q^*, K=K^*}$$

$$= \left[\frac{q^*(h(q^*))^2 L^{T*}(q^*, K^*)}{f''(q^*)} + \frac{(f(q^*))^2 L^{N*}(q^*, K^*)}{q^{*2} \bullet h''(q^*)}\right]\frac{1}{(k^{T*}(q^*) - k^{N*}(q^*))^2} < 0,$$

$$\left.\frac{\partial Y^{N*}}{\partial q}\right|_{q=q^*, K=K^*}$$

$$= -\left[\frac{(h(q^*))^2 L^{T*}(q^*, K^*)}{f''(q^*)} + \frac{(f(q^*))^2 L^{N*}(q^*, K^*)}{q^{*3} \bullet h''(q^*)}\right]\frac{1}{(k^{T*}(q^*) - k^{N*}(q^*))^2} > 0.$$

C. Stability of the Equilibrium

The following five equations summarize the dynamic properties of the equilibrium path:

$$\frac{\partial U}{\partial C^T} = \lambda_1, \ \frac{\partial U}{\partial C^N} = \lambda_1 \bullet q,$$

$$\dot{\lambda}_1 = \{\beta - r(b)\}\lambda_1,$$

$$\frac{\dot{q}}{q} = r(b) - \frac{1}{q}f'(k^T) = r(b) - h'(k^N),$$

$$f(k^T) - k^T f'(k^T) = q(h(k^N) - k^N h'(k^N)),$$

$$\dot{K} = H(K^N, L^N) - C^N - G^N,$$

$$\dot{b} = C^T + G^T + r(b) \bullet b - F(K^T, L^T).$$

We linearize the above five equations to obtain four linear differential equations around the stationary equilibrium:

$$\begin{aligned}\dot{\lambda}_1 &= \{\beta - r(b^*)\}\lambda_1^* - \lambda_1^* \bullet r'(b^*)(b - b^*) \\ &\quad + \{\beta - r(b^*)\}(\lambda_1 - \lambda_1^*) \\ &= -\lambda_1^* \bullet r'(b^*)(b - b^*),\end{aligned}$$

$$\dot{q} = \{r(b^*) - h'(k^{N*}(q^*))\}q^* + \left\{-q^* h''(k^{N*}(q^*)) \cdot \frac{\partial k^{N*}}{\partial q}\bigg|_{q=q^*}\right\}(q - q^*)$$

$$+ q^* \cdot r'(b^*)(b - b^*)$$

$$= \left\{\frac{f(k^T(q^*))}{(k^{T*}(q^*) - k^{N*}(q^*))q^*}\right\}(q - q^*) + q^* \cdot r'(b^*)(b - b^*),$$

$$\dot{K} = Y^{N*}(q^*, K^*) - C^{N*}(\lambda_1^*, q^*) - G^N - \frac{\partial C^{N*}}{\partial \lambda_1}\bigg|_{\lambda_1=\lambda_1^*, q=q^*}(\lambda_1 - \lambda_1^*)$$

$$+ \left\{\frac{\partial Y^{N*}}{\partial q}\bigg|_{q=q^*, K=K^*} - \frac{\partial C^{N*}}{\partial q}\bigg|_{\lambda_1=\lambda_1^*, q=q^*}\right\}(q - q^*) + \frac{\partial Y^{N*}}{\partial K}\bigg|_{q=q^*, K=K^*}(K - K^*)$$

$$= -\frac{\partial C^{N*}}{\partial \lambda_1}\bigg|_{\lambda_1=\lambda_1^*, q=q^*}(\lambda_1 - \lambda_1^*) + \left\{\frac{\partial Y^{N*}}{\partial q}\bigg|_{q=q^*, K=K^*} - \frac{\partial C^{N*}}{\partial q}\bigg|_{\lambda_1=\lambda_1^*, q=q^*}\right\}(q - q^*)$$

$$+ \frac{\partial Y^{N*}}{\partial K}\bigg|_{q=q^*, K=K^*}(K - K^*),$$

$$\dot{b} = \frac{\partial C^{T*}}{\partial \lambda_1}\bigg|_{\lambda_1=\lambda_1^*, q=q^*}(\lambda_1 - \lambda_1^*)$$

$$+ \left\{\frac{\partial C^{T*}}{\partial q}\bigg|_{\lambda_1=\lambda_1^*, q=q^*} - \frac{\partial Y^{T*}}{\partial q}\bigg|_{q=q^*, K=K^*}\right\}(q - q^*)$$

$$- \frac{\partial Y^{T*}}{\partial K}\bigg|_{q=q^*, K=K^*}(K - K^*) + \{r(b^*) + r'(b^*) \cdot b^*\}(b - b^*).$$

These equations can be compactly written in matrix notation, as follows:

$$\begin{bmatrix} \dot{\lambda}_1 \\ \dot{q} \\ \dot{K} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -\lambda_1^* \cdot r'(b^*) \\ 0 & \dfrac{f^*}{q^*(k^{T*} - k^{N*})} & 0 & q^* \cdot r'(b^*) \\ -\dfrac{\partial C^{N*}}{\partial \lambda_1} & \dfrac{\partial Y^{N*}}{\partial q} - \dfrac{\partial C^{N*}}{\partial q} & \dfrac{h^*}{(k^{N*} - k^{T*})} & 0 \\ \dfrac{\partial C^{T*}}{\partial \lambda_1} & \dfrac{\partial C^{T*}}{\partial q} - \dfrac{\partial Y^{T*}}{\partial q} & -\dfrac{\partial Y^{T*}}{\partial K} & r(b^*) + b^* \cdot r'(b^*) \end{bmatrix} \begin{bmatrix} \lambda_1 - \lambda_1^* \\ q - q^* \\ K - K^* \\ b - b^* \end{bmatrix} \tag{A.10}$$

The four characteristic roots of the Jacobian matrix of equation (A.10), hereafter x, y, z, w, must satisfy the following equations (A.11) through (A.14):

$$x + y + z + w$$

$$= \frac{f(k^{T*}(q^*))}{(k^{T*}(q^*) - k^{N*}(q^*))q^*} + \frac{\partial Y^{N*}}{\partial K}\bigg|_{q=q^*, K=K^*} + r(b^*) + b^* \cdot r'(b^*) \tag{A.11}$$

$$= 2\beta + b^* \cdot r'(b^*) > 0,$$

$$\begin{aligned}
& xy + xz + xw + yz + yw + zw \\
&= \frac{h(k^N(q^*))}{(k^{T*} - k^{N*})} \cdot \frac{\partial Y^{N*}}{\partial K}\bigg|_{q=q^*,K=K^*} + \lambda_1^* \cdot r'(b^*)\frac{\partial C^{T*}}{\partial \lambda_1}\bigg|_{\lambda_1=\lambda_1^*,\,q=q^*} \\
&\quad - q^* \cdot r'(b^*)\left(\frac{\partial C^{T*}}{\partial q}\bigg|_{\lambda_1=\lambda_1^*,\,q=q^*} - \frac{\partial Y^{T*}}{\partial q}\bigg|_{q=q^*,K=K^*}\right) + \beta(r(b^*) + b^* \cdot r'(b^*)),
\end{aligned} \tag{A.12}$$

$$\begin{aligned}
& xyz + xyw + xzw + yzw \\
&= q^* \cdot r'(b^*)\frac{\partial Y^{N*}}{\partial K}\left\{\frac{\partial C^{T*}}{\partial q}\bigg|_{\lambda_1=\lambda_1^*,\,q=q^*} - \frac{\partial Y^{T*}}{\partial K}\bigg|_{q=q^*,K=K^*}\right\} \\
&\quad + \lambda_1^* \cdot r'(b^*) \cdot \frac{\partial C^{N*}}{\partial \lambda_1}\bigg|_{\lambda_1=\lambda_1^*,\,q=q^*} \cdot \frac{\partial Y^{T*}}{\partial K}\bigg|_{q=q^*,K=K^*} \\
&\quad + q^* \cdot r'(b^*)\left\{\frac{\partial Y^{N*}}{\partial q}\bigg|_{q=q^*,K=K^*} - \frac{\partial C^{N*}}{\partial q}\bigg|_{\lambda_1=\lambda_1^*,\,q=q^*}\right\}\frac{\partial Y^{T*}}{\partial K}\bigg|_{q=q^*,K=K^*} \\
&\quad - \frac{h^*}{(k^{T*} - k^{N*})} \cdot r'(b^*)\frac{\partial Y^{N*}}{\partial K}\bigg|_{q=q^*,K=K^*},
\end{aligned} \tag{A.13}$$

$$\begin{aligned}
& xyzw \\
&= \lambda_1^* \cdot r'(b^*)\frac{h^*}{(k^{T*} - k^{N*})}\left\{\frac{\partial Y^{N*}}{\partial K}\bigg|_{q=q^*,K=K^*} \cdot \frac{\partial C^{T*}}{\partial \lambda_1}\bigg|_{\lambda_1=\lambda_1^*,\,q=q^*} \right. \\
&\quad \left. - \frac{\partial Y^{T*}}{\partial K}\bigg|_{q=q^*,K=K^*} \cdot \frac{\partial C^{N*}}{\partial \lambda_1}\bigg|_{\lambda_1=\lambda_1^*,\,q=q^*}\right\} > 0.
\end{aligned} \tag{A.14}$$

The signs of equations (A.11) and (A.14) are both positive. Equation (A.12) shows that if the following inequality holds, we get $xy + xz + xw + yz + yw + zw < 0$.

$$\begin{aligned}
\beta(r(b^*) + b^* \cdot r'(b^*)) < \frac{h^*}{(k^{N*} - k^{T*})} \cdot \frac{\partial Y^{N*}}{\partial K}\bigg|_{q=q^*,K=K^*} - \lambda_1^* \cdot r'(b^*)\frac{\partial C^{T*}}{\partial \lambda_1}\bigg|_{\lambda_1=\lambda_1^*,\,q=q^*} \\
+ q^* \cdot r'(b^*)\left(\frac{\partial C^{T*}}{\partial q}\bigg|_{\lambda_1=\lambda_1^*,\,q=q^*} - \frac{\partial Y^{T*}}{\partial q}\bigg|_{q=q^*,K=K^*}\right).
\end{aligned}$$

If this inequality holds, equation (A.10) shows that the characteristic roots of the Jacobian matrix will have two roots with positive real parts and two roots whose real parts are negative. Note that we have two jump variables λ_1 and q, and two predetermined variables K and b. The number of jump variables is equal to the number of negative roots, thus, we verify that the system equation (A.10) has saddle-point equilibrium. Under the assumption of the existence of saddle-point equilibrium, one can obtain similar solutions from the equation (A.10) following Bhandari *et al.* (1990).

$$\begin{aligned}
\lambda_1(t) = \lambda^{**} + \phi(\mu_1)\frac{a_{14}}{\mu_1} \cdot \frac{\phi(\mu_2) \cdot \Delta K - \Delta b}{\phi(\mu_1) - \phi(\mu_2)} \exp(\mu_1 \cdot t) \\
+ \phi(\mu_2)\frac{a_{14}}{\mu_2} \cdot \frac{\Delta b - \phi(\mu_1) \cdot \Delta K}{\phi(\mu_1) - \phi(\mu_2)} \exp(\mu_2 \cdot t),
\end{aligned} \tag{A.15}$$

$$
q(t) = q^{**} + \phi(\mu_1)\frac{a_{24}}{\mu_1 - a_{22}} \cdot \frac{\phi(\mu_2)\cdot\Delta K - \Delta b}{\phi(\mu_1) - \phi(\mu_2)}\exp(\mu_1 \cdot t)
$$

$$
+ \phi(\mu_2)\frac{a_{24}}{\mu_2 - a_{22}} \cdot \frac{\Delta b - \phi(\mu_1)\cdot\Delta K}{\phi(\mu_2) - \phi(\mu_1)}\exp(\mu_2 \cdot t), \tag{A.16}
$$

$$
K(t) = K^{**} + \frac{\phi(\mu_2)\cdot\Delta K - \Delta b}{\phi(\mu_1) - \phi(\mu_2)}\exp(\mu_1 \cdot t)
$$

$$
+ \frac{\Delta b - \phi(\mu_1)\cdot\Delta K}{\phi(\mu_1) - \phi(\mu_2)}\exp(\mu_2 \cdot t), \tag{A.17}
$$

$$
b(t) = b^{**} + \phi(\mu_1)\cdot\frac{\phi(\mu_2)\cdot\Delta K - \Delta b}{\phi(\mu_1) - \phi(\mu_2)}\exp(\mu_1 \cdot t)
$$

$$
+ \phi(\mu_2)\cdot\frac{\Delta b - \phi(\mu_1)\cdot\Delta K}{\phi(\mu_1) - \phi(\mu_2)}\exp(\mu_2 \cdot t), \tag{A.18}
$$

where μ_1, μ_2 are two negative characteristic roots of the Jacobian matrix of equation (A.10) (where $\mu_1 < \mu_2 < 0$). Let the capital stock and the outstanding external debt at the initial stationary equilibrium and those at the new equilibrium be (K^*, b^*), (K^{**}, b^{**}), respectively, and define $\Delta K := K^{**} - K^*$, $\Delta b := b^{**} - b^*$. Let

$$
\begin{aligned}
\phi(\mu) := -((\mu^2(-\mu + a_{22})a_{43}(\mu^3 a_{33} - \mu^2 a_{22}a_{33} - \mu a_{14}a_{33}a_{41} + a_{14}a_{22}a_{33}a_{41} \\
- \mu a_{24}a_{33}a_{42} + \mu a_{14}a_{31}a_{43} - a_{14}a_{22}a_{31}a_{43} + \mu a_{24}a_{32}a_{43} - \mu^2 a_{33}a_{44} \\
+ \mu a_{22}a_{33}a_{44}) \\
((-\mu^4 + \mu^3 a_{22} + \mu^2 a_{14}a_{41} - \mu a_{14}a_{22}a_{41} + \mu^2 a_{24}a_{42} + \mu^3 a_{44} - \mu^2 a_{22}a_{44}) \\
(\mu a_{24}(a_{33}a_{42} - a_{32}a_{43}) - (\mu - a_{22})(a_{14}(a_{33}a_{41} - a_{31}a_{43}) + \mu a_{33}(\mu - a_{44}))))^{-1}
\end{aligned}
$$

where a_{ij} is (i, j) components of the Jacobian matrix of equation (A.10).

Numerical examples shown in Figures 1 through 4 are simulated based on equations (A.15) through (A.18).

References

Bhandari, Jagdeep S., Nadeem Ul Haque, and Stephen J. Turnovsky, "Growth, External Debt, and Sovereign Risk in a Small Open Economy," *IMF Staff Paper*, 37 (2), 1990, pp. 388–417.

Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo, "Perspective Deficits and the Asian Currency Crisis," *Journal of Political Economy* 109, 2001, pp. 1155–1197.

Caballero, Ricardo J., "On the International Financial Architecture: Insuring Emerging Markets," NBER Working Paper No. 9570, National Bureau of Economic Research, 2003.

———, and Arvind Krishnamurthy, "International and Domestic Collateral Constraints in a Model of Emerging Market Crises," *Journal of Monetary Economics*, 48, 2001a, pp. 513–548.

———, and ———, "A 'Vertical' Analysis of Crises and Intervention: Fear of Floating and Ex-Ante Problems," NBER Working Paper No. 8428, National Bureau of Economic Research, 2001b.

Calvo, Guillermo A., "Explaining Sudden Stop, Growth Collapse and BOP Crisis: The Case of Distortionary Output Tax," paper presented at the Third IMF Research Conference, 2002.

———, Alejandro Izquierdo, and Ernesto Talvi, "Sudden Stops, the Real Exchange Rate and Fiscal Sustainability: Argentina's Lessons," mimeo, 2002.

Chang, Roberto, and Andrés Velasco, "Financial Fragility and the Exchange Rate Regime," *Journal of Economic Theory*, 92, 2000, pp. 1–34.

———, and ———, "A Model of Financial Crisis in Emerging Markets," *Quarterly Journal of Economics*, CXVI, 2001, pp. 489–517.

Corbo, Vittorio, "Exchange Rate Regimes in the Americas: Is Dollarization the Solution?" *Monetary and Economic Studies*, 20 (S-1), Institute for Monetary and Economic Studies, Bank of Japan, 2002, pp. 91–101.

Corden, Max, *Too Sensational: On the Choice of Exchange Rate Regimes*, MIT Press, 2002.

Corsetti, Giancarlo, Paolo Pesenti, and Nouriel Roubini, "What Caused the Asian Currency and Financial Crisis?" *Japan and the World Economy*, 11, 1999, pp. 305–373.

Eichengreen, Barry, "Capital Account Liberalization: What Do Cross-Country Studies Tell Us?" *World Bank Economic Review*, 15 (3), 2001, pp. 341–365.

Feldstein, Martin, "Lessons from Argentina," mimeo, 2002, (http://www.nber.org/~confer/2002/argentina02/feldstein.pdf).

Fujiki, Hiroshi, and Akira Otani, "Do Currency Regimes Matter in the 21st Century? An Overview," *Monetary and Economic Studies*, 20 (S-1), Institute for Monetary and Economic Studies, Bank of Japan, 2002, pp. 47–90.

Gibson, Edward L., and Ernesto Calvo, "Electoral Coalitions and Market Reforms: Evidence from Argentina," paper presented at XX International Congress, Latin American Studies Association, April 17–19, 1997.

Hattori, Masazumi, "Tsuka Kiki eno Taiousaku toshiteno Ryudosei Kyokyu no Igi ni tsuite (A Survey on Recent Theories and Empirical Analyses Regarding Currency Crises—The Role of Liquidity Provision as a Policy Measure in Currency Crisis Management)," *Kin'yu Kenkyu* (Monetary and Economic Studies), 21 (2), Institute for Monetary and Economic Studies, Bank of Japan, 2002, pp. 179–211 (in Japanese).

Krugman, Paul, "A Model of Balance-of-Payment Crisis," *Journal of Money, Credit and Banking*, 11 (3), 1979, pp. 311–325.

———, "What Happened to Asia?" unpublished manuscript, 1998.

Mussa, Michael, *Argentina and the Fund: From Triumph to Tragedy*, Institute for International Economics, March 25, 2002.

Nishiyama, Shinichi, "The Cross-Euler Equation Approach to Intertemporal Substitution in Import Demand," IMES Discussion Paper 2002-E-21, Institute for Monetary and Economic Studies, Bank of Japan, 2002.

Perry, Guillermo, and Luis Servén, "The Anatomy of a Multiple Crisis: Why Was Argentina Special and What Can We Learn from It?" background paper, Project on Exchange Rate Crisis in Emerging Markets: The Argentina Case, 2002, http://www.nber.org/~confer/2002/argentina02/perry.pdf.

Rodrik, Dani, "Who Needs Capital-Account Convertibility?" *Princeton Essays in International Finance*, No. 207, International Finance Section, Department of Economics, Princeton University, 1998.
Schneider, Martin, and Aaron Tornell, "Balance Sheet Effects, Bailout Guarantees, and Financial Crises," *Review of Economic Studies*, 2003 (forthcoming).
Tommasi, Mariano, Sevastián Saiegh, and Pablo Sanguinetti, "Fiscal Federalism in Argentina: Policies, Politics and Institutional Reform," *Economia*, 1 (2), 2001, pp. 147–201.
Turnovsky, Stephen J., *International Macroeconomic Dynamics*, MIT Press, 1997.