On the Risk Capital Framework of Financial Institutions

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In this paper, we consider the risk capital framework adopted by financial institutions. Specifically, we review the recent literature on this issue, and clarify the economic assumptions behind this framework. Based on these observations, we then develop a simple model for analyzing the economic implications of this framework.

The main implications are as follows. First, risk capital allocations are theoretically unnecessary without deadweight costs for raising capital, which are not usually assumed in the business practices of financial institutions. Second, the risk-adjusted rate of return is redundant as it provides no additional information beyond the net present value. Third, risk capital allocation is intrinsically difficult because it is hard to incorporate the correlations among asset returns.

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I. Introduction

In recent years, an increasing number of financial institutions have adopted risk capital frameworks.[1] Under these frameworks, banks determine the amount of capital needed to cover their risk, and measure their risk-adjusted performance. According to Zaik *et al.* (1996), James (1996), and Matten (2000), the elements of the risk capital frameworks adopted by financial institutions are as follows.

(1) Holding sufficient capital to cover risk

Financial institutions hold a sufficient amount of capital to cover the risk of their business activities (this capital is called "risk capital"). They determine the amount of risk capital as the unexpected losses of their operations. They measure the unexpected losses using value-at-risk and other risk measures.

(2) Allocating risk capital to each operating division

Financial institutions allocate risk capital to individual lines of business according to their respective risks.

(3) Evaluating profitability based on risk-adjusted rates of return

Financial institutions evaluate the performance of individual lines of business using their respective "risk-adjusted rates of return" (profit divided by allocated risk capital).

In this paper, we call this framework the "standard framework" of risk capital and consider its economic implications. We consider this framework as the representative example of risk capital frameworks, since it is both typical and common to the business practices of financial institutions.[2]

This standard framework assumes that financial institutions are risk-averse decision-makers and make investment decisions based on risk-return trade-offs. However, finance theory does not assume, *a priori*, that financial institutions are risk-averse decision-makers. Rather, finance theory says that a firm is primarily a nexus of contracts, and that a firm is risk-neutral in a perfect market (Modigliani and Miller [1958]). Therefore, to justify the risk capital framework from the theoretical viewpoint, we must establish the reason why financial institutions are risk-averse.

Since the 1990s, a number of research papers have addressed this issue[3] from the viewpoint of finance theory. They establish how and why financial institutions are risk-averse and propose risk capital frameworks that are well aligned with their economic analyses.

In this paper, we consider the risk capital framework generally adopted at financial institutions. Specifically, we review the recent literature and clarify the economic assumptions behind this framework. Based on these observations, we then develop a simple model for analyzing the economic implications of this framework.

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1. The "risk capital framework" characterized here is also called "integrated risk management" or the "risk-adjusted return on capital (RAROC) system."

2. There are several variations to this framework. See Zaik *et al.* (1996), James (1996), and Matten (2000).

3. Froot and Stein (1998) examine the capital of financial institutions, explicitly incorporating the concavity of the payoff under the existence of economic friction. Merton and Perold (1993) argue that risk capital is viewed as insurance purchased as required against bankruptcy. Crouhy *et al.* (1999) define risk capital as the risk-free assets required to restrain the probability of bankruptcy to within a certain level, and primarily address capital budgeting decision-making problems.

The main findings are as follows.

- The allocation of risk capital is theoretically unnecessary when we do not assume the deadweight costs of raising capital. As the deadweight costs of raising capital are not usually assumed in the business practices of financial institutions, the allocation of risk capital seems unnecessary for practical use.
- The risk-adjusted rate of return in the standard framework is equivalent to the net present value (NPV). The risk-adjusted rate of return is inferior to the NPV, since the former requires the calculation of risk capital (i.e., value-at-risk) while the latter does not.
- The allocation of risk capital is intrinsically difficult, as we must consider the correlations among asset returns.

In the first half of this paper (through Section III), we describe the theories presented in the prior research on risk capital. In Section II, we consider a firm's financing decision-making in a perfect market. In a perfect market, the Modigliani-Miller proposition (hereafter the "MM proposition") holds and risk capital plays no role in financing decision-making. In Section III, we introduce economic friction such as bankruptcy costs and information asymmetry. With economic friction, a firm becomes risk-averse and risk capital plays some roles. We also note that economic friction is more serious for financial institutions than for nonfinancial firms. Thus, risk capital plays greater roles at financial institutions than at other firms.

In the second half of this paper (from Section IV onward), we develop a simple model for analyzing the standard framework. From this model, we derive the economic implications of the risk capital framework in Section IV, and then consider the difficulty of risk capital allocation in Section V. Section VI presents the conclusions.

II. Financing Decisions in a Perfect Market

In this section, we describe a firm's[4] capital structure, risk management, and capital budgeting under a perfect market.[5] This description prepares for the discussions of risk capital in subsequent sections.

A. The MM Proposition

The MM proposition, stated in Modigliani and Miller (1958), is the most important factor in considering a firm's capital structure in a perfect market. Based on the MM proposition, Fama (1978) demonstrates that in a perfect market that satisfies the following conditions, how firms raise funds has no influence on their value.

(1) Frictionless market (there are no transaction costs or taxes, and all assets are perfectly divisible and marketable).

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4. We make no distinctions between financial institutions and nonfinancial firms in this section, as we only consider the decision-making in a perfect market.
5. See Brealey and Myers (2000), McKinsey & Company Inc. *et al.* (2000), and Damodaran (1999) for firms' financing decision-making in a perfect market.

(2) Information efficiency (all investors, firms, and firms' managers have the same information, and hold homogeneous expectations regarding how this information will affect market prices).

(3) Given investment decisions (the investment decisions are given conditions, and are completely independent from the decisions on raising new external funds).

(4) Equal access (all market participants can issue securities under the same terms).

The essence of the MM proposition is that a firm's capital structure does not matter, because investors can arrange their own desired payoffs by adjusting their portfolios by themselves.

The MM proposition also says that a firm's risk management does not matter in a perfect market. A firm has no need to hedge the tradable risks it is exposed to, because the investors who invest in its stocks or bonds can hedge the firm's risks by trading those risks by themselves. So, in a perfect market where the MM proposition holds, one cannot assume, *a priori*, that a firm is risk-averse.

In the following subsections, we explicate the firm's decision-making in a perfect market, where the MM proposition holds, using the asset pricing model.

B. Asset Pricing

To examine how risk management and capital structure affect the firm's value, we need to calculate the present value of the firm's future payoff (hereafter "the firm's value"). In this paper, we adopt the Capital Asset Pricing Model (CAPM[6]) to determine the present value of the future uncertain payoff.

We adopt a two-period model (period 0 and period 1). Under CAPM, equation (1) shows the relationship between the rate of return r_i for a given asset i and the rate of return for the market portfolio (the market return).[7]

$$E[r_i] = r_f + \beta_i(E[r_M] - r_f), \tag{1}$$

where[8] $\beta_i = \text{cov}(r_i, r_M)/\text{var}(r_M)$, r_M is market return, r_f is the risk-free rate, and β_i is a constant for each asset.

Here, the random variable X_i indicates the asset's payoff at period 1, and $P(X_i)$ indicates the asset's present value at period 0. By definition of r_i,

$$1 + E[r_i] = \frac{E[X_i]}{P(X_i)}. \tag{2}$$

Substituting equation (2) into equation (1), and using $\gamma \equiv (E[r_M] - r_f)/\text{var}(r_M)$ which is not dependent on the characteristics of each asset, the value of $P(X_i)$ is then determined as shown in equation (3).

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6. The argument in this paper could also be developed using a wider class of pricing models that can be nested in the stochastic discount factor model. See Cochrane (2001) for the details of the stochastic discount factor.

7. $E[\cdot]$ represents the expectation operator with the given information at period 0.

8. $\text{cov}(X, Y)$ expresses the covariance of the random variables X and Y, and $\text{var}(X)$ expresses the variance of the random variable X.

$$P(X_i) = \frac{E[X_i] - \gamma \operatorname{cov}(X_i, r_M)}{1 + r_f}. \tag{3}$$

C. Capital Budgeting, Capital Structure, and Risk Management in a Perfect Market

Next, we explain a firm's financing decision-making (capital budgeting, capital structure, and risk management) in a perfect market, where the MM proposition holds. In this paper, capital budgeting, capital structure, and risk management decision-making are defined as follows.

(1) Capital budgeting

To decide how much to invest in an investment opportunity.

(2) Capital structure

To select how the investment is financed. For simplicity, we assume that a firm can finance only with common stocks and straight bonds.

(3) Risk management

To decide whether or not to hedge the tradable risks that a firm is exposed to.

In this paper, those three decisions are collectively referred to as the firm's "financing decision-making."

The firm's NPV is expressed by equation (4), where $P(X)$ represents the firm's present value and I represents the amount of capital.

$$NPV = P(X) - I. \tag{4}$$

One of the most important characteristics of a firm's financing decision-making in a perfect market is "value additivity."[9] The concept of value additivity is as follows. Suppose there are two investment opportunities A and B with payoffs X_A and X_B, respectively, and their present values are $P(X_A)$ and $P(X_B)$. Value additivity holds when the sum of these present values is the same as the present value of the two investment opportunities taken together, that is, where $P(X_A + X_B) = P(X_A) + P(X_B)$. From equation (3), value additivity holds when the following holds.

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\begin{aligned}
P(X_A + X_B) &= \frac{E[X_A + X_B] - \gamma \operatorname{cov}(X_A + X_B, r_M)}{1 + r_f} \\
&= \frac{E[X_A] - \gamma \operatorname{cov}(X_A, r_M)}{1 + r_f} + \frac{E[X_B] - \gamma \operatorname{cov}(X_B, r_M)}{1 + r_f} \\
&= P(X_A) + P(X_B).
\end{aligned}
\tag{5}
$$

Value additivity results from the linearity of the present value $P(X)$ to the payoff X.

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9. See Brealey and Myers (2000) for a detailed introduction to value additivity.

Additionally, because $I_{A+B} = I_A + I_B$, the NPV is also value additive as shown in equation (6).

$$NPV_A + NPV_B = \{P(X_A) - I_A\} + \{P(X_B) - I_B\}$$
$$= \{P(X_A) + P(X_B)\} - \{I_A + I_B\}$$
$$= P(X_A + X_B) - \{I_A + I_B\}$$
$$= NPV_{A+B}. \tag{6}$$

This value additivity has the following important implications for the firm's financing decision-making.

(1) Capital budgeting

The capital budgeting decisions on multiple investment opportunities are independent of each other when value additivity holds. As is clear from equation (6),[10] the NPV of one investment opportunity is independent of the NPV of another investment opportunity. For example, suppose that a firm has two operating divisions, and they both implement capital budgeting. If each division independently maximizes its NPV without any consideration for the other, the NPV of the firm will also be maximized.

(2) Capital structure

Value additivity is equivalent to the MM proposition that capital structure does not matter. This can be shown as follows.[11] Let a random variable X indicate a firm's payoff at period 1. When the firm's only means of fund-raising is the issuance of stocks, the firm's value becomes $P_1 = P(X)$. Next, consider the firm's value when the firm can raise funds by issuing stocks and/or bonds. When the stock and bond payoffs at period 1 are S and D, respectively, the firm's present value P_2 is the sum of those of the stocks and bonds, so $P_2 = P(S) + P(D)$. As the firm's overall payoff at period 1 can be divided into those of stocks and bonds, $X = S + D$. Because of the value additivity, it can then be demonstrated that $P_1 = P_2$, as in equation (7).

$$P_1 = P(X) = P(S + D) = P(S) + P(D) = P_2. \tag{7}$$

(3) Risk management

Value additivity is equivalent to the irrelevance of risk management.[12] This can be shown by the following example. Let a random variable X indicate a firm's payoff at period 1. Suppose that the variability of the firm's payoff X can all be hedged by market trading, specifically by a forward contract of the

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10. Rather than utilizing equation (3), practitioners commonly calculate the present value using a modified version of equation (2): $P(X_i) = E(X_i)/(1 + r_i)$. In this case, although it is necessary to calculate the value of r_i (or β_i) for each investment opportunity, a single value is often adopted for an entire firm. If the individual investment opportunity's β were always equal to (or nearly equal to) the firm's β, this would not present many problems. Since this is not the case, the use of the $P(X_i) = E(X_i)/(1 + r_i)$ equation is generally inappropriate, as detailed below.
11. See Ross (1978) for this proof of the MM proposition.
12. See Culp (2001) for a more detailed explanation.

payoff's underlying assets. In a perfect market, the NPV of the forward contract ($F - X$, where the forward delivery price is F) is zero.[13] In other words, equation (3) can be restated as follows.

$$0 = \frac{E[F - X] - \gamma \operatorname{cov}(F - X, r_M)}{1 + r_f}. \tag{8}$$

Thus, the forward delivery price F can be expressed as follows.[14]

$$F = E[X] - \gamma \operatorname{cov}(X, r_M). \tag{9}$$

Here, after the risk on the payoff X is hedged and the payoff becomes the forward delivery price F, the firm's value after the hedge P_1' becomes identical to the firm's value before the hedge $P(X)$.

$$P_1' = P(F) = \frac{E[F] - \gamma \operatorname{cov}(F, r_M)}{1 + r_f} = \frac{F}{1 + r_f} = \frac{E[X] - \gamma \operatorname{cov}(X, r_M)}{1 + r_f}$$
$$= P(X). \tag{10}$$

Therefore, because of the value additivity, the firm's hedge operation does not change its value. Thus, risk management is irrelevant when value additivity holds.

III. Financing Decisions in a Real Market

A. Existence of Economic Friction
In the previous section, we assumed a perfect market. In a perfect market, value additivity holds and capital structure and risk management are irrelevant. As shown in the previous section, value additivity is based on the linear relationship between the NPV and the payoff. Thus, the irrelevance of capital structure and risk management depends on this linear relationship.

In the real world, the firm's NPV may not be a linear function of the payoff. This is usually due to the existence of "economic friction"[15] such as bankruptcy costs and progressive corporate tax rates. This friction places costs on the firm that have a convex function with the payoff.

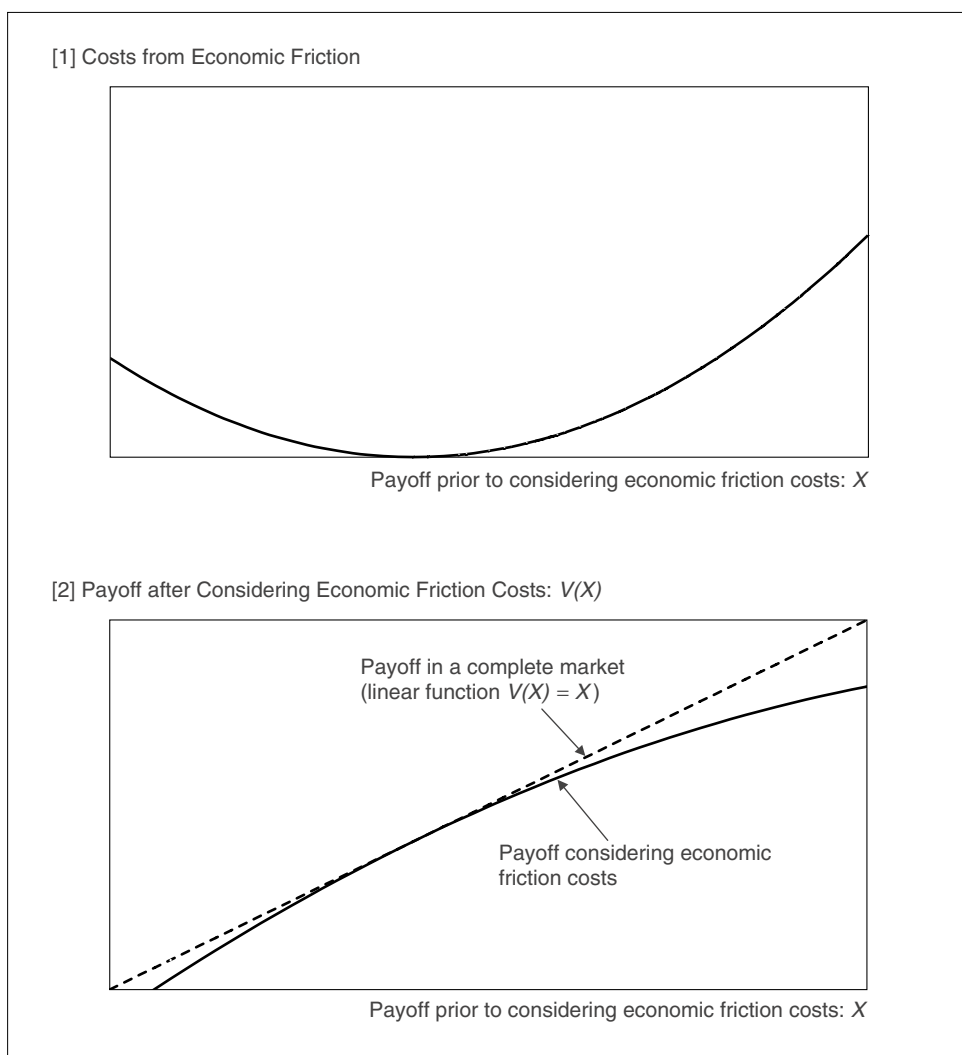..

13. In a perfect market, the NPV of market trading is always zero. This can be explained as follows. Assume a given trading opportunity with a positive NPV in a perfect market. Because all the participants can freely engage in trading, they would all take advantage of this trading opportunity. This would result in a surplus demand for this trading opportunity, so the market price would then move downward, reducing the NPV. This market price adjustment would continue until the NPV declines to zero, and reach equilibrium when the NPV equals zero. In a perfect market, this price adjustment would be done instantaneously. Thus, market trading with a positive NPV cannot exist in a perfect market.

14. From equation (3), $E[X] = (1 + r_f)P(X) + \gamma \operatorname{cov}(X, r_M)$, so $F = (1 + r_f)P(X)$. This is the forward delivery price derived from the non-arbitrage conditions whereby there are no risk-free profits.

15. Although the agency problem is also an important reason for the imperfection of financial markets, it is not addressed in this paper. See Barnea *et al.* (1985) for the details of the agency problem.

When the payoff before considering the costs of economic friction is X and the payoff after considering the costs is $V(X)$, there is a concave function between $V(X)$ and X, as shown in Figure 1. The firm's NPV is no longer linear with its payoff, and the MM proposition and value additivity no longer hold.

Figure 1 Relation between Costs from Economic Friction and Payoff

[1] Costs from Economic Friction

Payoff prior to considering economic friction costs: X

[2] Payoff after Considering Economic Friction Costs: $V(X)$

Payoff in a complete market
(linear function $V(X) = X$)

Payoff considering economic
friction costs

Payoff prior to considering economic friction costs: X

The following three costs have been noted as types of economic friction that affect the linearity of NPVs (Smith and Stulz [1985] and Froot *et al.* [1993]).
(1) Corporate taxes
   If effective marginal tax rates on a firm are progressive, or an increasing function of the firm's pretax value, the after-tax value of the firm is a concave function of its pretax value.

(2) Bankruptcy costs

A firm must pay bankruptcy costs,[16] such as legal and administrative costs, when the firm's value is below its debt level. Thus, the bankruptcy costs are high when the firm's value is low. This results in a concavity between the payoff before the bankruptcy costs and the payoff after the bankruptcy costs.

(3) Higher cost of raising external capital

Due to agency and information problems, new external capital is more expensive than internal capital. Froot *et al.* (1993) demonstrate that the payoff function becomes increasingly convex from raising external capital. They state that declines in the firm's payoff deplete internal reserves, and then lead to a higher level of dependence on external capital. When external capital is more expensive than internal capital, this growing dependence on external capital imposes a higher cost on the firm.

B. Payoff Concavity and Value Non-Additivity

We explained how the costs of economic friction result in a concave function between the payoffs before the costs are considered and the payoffs after the costs are considered.

We now demonstrate that the firm's value is not value additive when the payoff function is concave. Suppose the firm's payoff before considering the costs of economic friction is X and the payoff after considering the costs is V. Also suppose V is a concave function of X, as follows.

$$V = V(X), \quad \text{where } V'(\cdot) > 0, \ V''(\cdot) < 0. \tag{11}$$

Using equation (3), the firm's value can now be expressed as equation (12).

$$P(V(X)) = \frac{E[V(X)] - \gamma \operatorname{cov}(V(X), r_M)}{1 + r_f}. \tag{12}$$

Since $V(\cdot)$ is a concave function, as shown in equation (13), there is no value additivity.

$$P(V(X_A + X_B)) = \frac{E[V(X_A + X_B)] - \gamma \operatorname{cov}(V(X_A + X_B), r_M)}{1 + r_f}$$

$$\neq \frac{E[V(X_A)] - \gamma \operatorname{cov}(V(X_A), r_M)}{1 + r_f} + \frac{E[V(X_B)] - \gamma \operatorname{cov}(V(X_B), r_M)}{1 + r_f}$$

$$= P(V(X_A)) + P(V(X_B)). \tag{13}$$

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16. There are two types of bankruptcy costs: direct and indirect. The direct costs of bankruptcy are the costs of processing bankruptcy procedures such as legal and administrative costs. The indirect costs of bankruptcy include the costs of deterioration of the firm's business from the bankruptcy procedure. See Brealey and Myers (2000) for details of bankruptcy costs.

Because there is no value additivity, the conclusions reached for the firm's financing decision-making in a perfect market no longer hold. In other words, the firm's financing decision-making in a real market is far more complex compared with that in a perfect market.

C. Economic Friction at Financial Institutions

Thus far, we have made no distinction between financial institutions and non-financial firms. However, economic friction has a greater influence on the former than the latter. Merton and Perold (1993) note that bankruptcy costs and information costs are more remarkable at financial institutions than at nonfinancial firms.

(1) Bankruptcy costs

The major customers of financial institutions can be major liability holders; for example, policyholders, depositors, and swap counterparties are all liability holders as well as customers. When the bankruptcy risk at a financial institution rises, customers typically move to other financial institutions. This exodus of customers imposes another indirect cost on the struggling financial institution, because it loses the profits that would otherwise have been derived.

An exodus of customers may also occur at nonfinancial firms, but the impact is far greater at financial institutions because their customers, especially those who are outside of the deposit insurance safety net, are extremely sensitive to bankruptcy risk. Thus, the bankruptcy costs at financial institutions[17] are far greater than those at nonfinancial firms.

(2) Information costs

In general, financial institutions do not disclose their assets or activities in great detail, and thus their business appears *opaque* to customers and investors.[18] That is, the detailed asset holdings and business activities of the firm are not publicly disclosed (or, if disclosed, only with a considerable lag in time). Furthermore, principal financial firms typically have relatively liquid balance sheets that, in the course of just weeks, can and often do undergo substantial changes in size and risk. The information asymmetry between the management of financial institutions and outsiders leads to the problem of higher costs of raising external capital (Myers and Majluf [1984]). As a result, the problem of higher costs of raising external capital is more serious at financial institutions than at nonfinancial firms.

Froot and Stein (1998) develop a framework for analyzing the capital allocation and capital structure decisions of financial institutions incorporating economic friction. In their framework, they propose a two-factor model that can be used for capital budgeting problems at financial institutions. With the model, they clarify the roles of economic friction on the financing decision-making of financial institutions.

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17. See Merton (1997) for a discussion of the scale of bankruptcy costs at financial institutions.
18. The "opaqueness" of financial institutions is first noted in Ross (1989).

However, the framework of Froot and Stein (1998) is not widely adopted in the practices of financial institutions. As they note, their framework requires calculation of the concavity of the payoff function ($V(\cdot)$ in equation [11]), which is hard to measure in practice. Instead, financial institutions adopt risk capital frameworks, which are not consistent with the framework of Froot and Stein (1998).

IV. Standard Framework

In this section, we analyze the risk capital frameworks adopted by financial institutions. As noted in Section I, we use the "standard framework" as a generalized example of the risk capital frameworks. The elements of the standard framework are as follows.

(1) Holding sufficient capital to cover risk

Financial institutions hold a sufficient amount of capital to cover the risk of their business activities (this capital is called "risk capital"). They determine the amount of risk capital as the unexpected losses of their operations. They measure the unexpected losses using value-at-risk and other risk measures.

(2) Allocating risk capital to each operating division

Financial institutions allocate risk capital to individual lines of business according to their respective risks.

(3) Evaluating profitability based on risk-adjusted rates of return

Financial institutions evaluate the performance of individual lines of business using their respective "risk-adjusted rates of return" (profit divided by allocated risk capital).

In this section, we first demonstrate how the standard framework can be developed into a model (hereafter, the "standard model") by positing assumptions that simplify the firm's payoff. We then examine the economic implications of the standard framework of risk capital adopted by financial institutions.

Also, we limit our considerations of the standard framework in this section to elements (1) "holding sufficient capital to cover risk" and (3) "evaluating profitability based on risk-adjusted rates of return." We proceed with considerations of element (2) "allocating risk capital to each operating division" in the subsequent section.

A. Basic Concept of the Standard Framework

The basic concept of the standard framework is "holding sufficient capital to cover risk." More precisely, the risk is quantified using value-at-risk or other risk measures and capital equal to or greater than the calculated risk is held as a buffer against it.

B. Standard Model

We develop the standard model based on the setup presented by Froot and Stein (1998), as follows. The model has two time periods, period 0 and period 1. We assume that a financial institution invests in an investment opportunity that will generate a per unit payoff of X at period 1. At period 0, X is a random variable with a

mean of μ. Note that we do not have to assume that X obeys a normal distribution. We also assume that this investment opportunity is available for an infinite number of units and that the investment can be implemented without cost.

Meanwhile, this financial institution raises an amount of capital K at period 0, and invests the proceeds in a risk-free asset. The following discussions through Section IV.E all assume that this amount of capital K is exogenously given.

The financial institution makes an investment of α units in the investment opportunity. Now, we introduce the standard framework whereby the institution retains sufficient capital to cover the risk. In this paper, we assume the linear homogeneity presented in equation (14) for the amount of risk $\rho(\cdot)$.

$$\rho(\alpha X) = \alpha\rho(X). \tag{14}$$

The financing decision-making of the institution takes place under the restrictions expressed by equation (15).

$$\rho(\alpha X) \leq K. \tag{15}$$

Equation (15) may be viewed as a function expressing the financial institution's "behavioral principle" under the standard framework of risk capital whereby the risk may not exceed the risk capital. We assume that the payoff for this financial institution is as expressed in equations (16) and (17).

$$V(w) = w, \quad \text{when } \rho(\alpha X) \leq K. \tag{16}$$

$$w = \alpha X + (1 + r_f)K. \tag{17}$$

Equation (16) says that economic friction is nonexistent as long as the capital covers the risk of the financial institution. This is the basic principle of the standard framework.[19] Given the linear homogeneity, equation (15) can be rewritten as follows.

$$\alpha \leq \frac{K}{\rho(X)}. \tag{18}$$

The institution's NPV can now be expressed as follows based on equations (16) and (17).

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19. Some may argue that the standard framework of risk capital assumes a convex function for a firm's value. If we assume a convex function, however, the firm's financing decision-making would be the same as that in Froot and Stein (1998), which is not widely adopted by financial institutions.

$$NPV = P(V(w)) - K = P(w) - K = \frac{E[w] - \gamma \operatorname{cov}(w, r_M)}{1 + r_f} - K$$

$$= \frac{E[\alpha X + (1 + r_f)K] - \gamma \operatorname{cov}(\alpha X + (1 + r_f)K, r_M)}{1 + r_f} - K$$

$$= \frac{\alpha E[X] + (1 + r_f)K - \alpha \gamma \operatorname{cov}(X, r_M)}{1 + r_f} - K$$

$$= \alpha \frac{E[X] - \gamma \operatorname{cov}(X, r_M)}{1 + r_f} = \alpha P(X). \tag{19}$$

As long as the inequality of equation (15) holds, the NPV has nothing to do with the capital K, and is equal to the present value of the investment in α units. Thus, the financial institution's NPV has value additivity.

C. Capital Budgeting under a Single Investment Opportunity
Under the standard model, the capital budgeting is very simple. When the institution is holding capital K, the objective is to maximize the NPV as calculated by equation (19) under the restrictions imposed by equation (15). This can be expressed as equation (20).

$$\max_{\alpha} \alpha P(X), \quad \text{subject to} \quad \alpha \le K/\rho(X). \tag{20}$$

The solution to this maximization problem is clearly $\alpha = K/\rho(X)$, so the maximized NPV becomes $NPV = KP(X)/\rho(X)$.

Moreover, the decision on whether or not to invest in any given investment opportunity is simply determined by whether or not its NPV is positive. In other words, as long as $P(X) > 0$, the investment will increase the financial institution's NPV (see equation [19]) and should therefore be implemented. Meanwhile the investment amount α is determined by equation (18).

The expression $P(X) > 0$ can be reformulated into the following expression.

$$P(X) > 0 \Leftrightarrow E[X] > \frac{\operatorname{cov}(X, r_M)}{\operatorname{var}(r_M)}(E[r_M] - r_f), \tag{21}$$

(Multiplying both sides by α/K and adding r_f)

$$\Leftrightarrow \frac{E[w] - K}{K} > r_f + \beta_{CAPM, \alpha X/K}(E[r_M] - r_f), \tag{22}$$

where $\beta_{CAPM, \alpha X/K} = \operatorname{cov}(\alpha X/K, r_M)/\operatorname{var}(r_M)$.

The left-hand side of equation (22) now expresses the expected rate of return on the capital K, and the right-hand side expresses the shareholders' expected rate of return calculated using the CAPM. Thus, equation (22) is an expression of the risk-adjusted rates of return approach often adopted by financial institutions. The

right-hand side of equation (22) is often called the "hurdle rate." Equation (22) shows that this standard model well describes the standard risk capital framework of financial institutions.

However, notably, the capital budgeting determined by equation (22) is not dependent on the capital K. This is because equation (22) is equivalent to equation (21), which is not dependent on the capital K. It indicates that an investment should be made whenever the NPV of the investment opportunity is positive. Therefore, the risk-adjusted rate of return derived from equation (22), which is widely used for capital budgeting, provides no additional information to the capital budgeting based solely on present value.

D. Risk Management
The influence of risk management on a firm's value can also easily be analyzed.

From the conclusions of Section IV.C, the maximized NPV of the financial institution can be expressed as follows.

$$NPV = \frac{KP(X)}{\rho(X)}. \tag{23}$$

Let us examine how hedge trading with zero NPV influences the value of equation (23). To begin with, when we assume trading with zero NPV to ensure value additivity for the present value of investment opportunities, this does not affect the numerator of equation (23). On the other hand, such hedge trading decreases the denominator $\rho(X)$. Therefore, it is optimal to completely hedge all tradable risks.

E. Capital Budgeting under Multiple Investment Opportunities
Next, we consider the case when a financial institution has already invested in a given investment opportunity, and now has to consider its capital budgeting for another investment opportunity.

We assume that the financial institution has already made a one-unit investment in the prior investment opportunity without cost, and define X_P as a random variable expressing its payoff. Similarly, we assume that the new investment opportunity can also be made without cost, and define X_N as a random variable expressing its payoff.

If the amount of investment in the new investment opportunity is α, the payoff that will be gained at period 1 is expressed as follows.

$$w = X_P + \alpha X_N + (1 + r_f)K. \tag{24}$$

In this case, the financial institution's NPV is expressed as equation (25).

$$NPV = P(w) - K = \frac{E[w] - \gamma \operatorname{cov}(w, r_M)}{1 + r_f} - K$$

$$= \frac{E[X_P + \alpha X_N + (1 + r_f)K] - \gamma \operatorname{cov}(X_P + \alpha X_N + (1 + r_f)K, r_M)}{1 + r_f} - K$$

$$= \frac{E[X_P] + \alpha E[X_N] + (1 + r_f)K - \gamma \operatorname{cov}(X_P, r_M) - \alpha \gamma \operatorname{cov}(X_N, r_M)}{1 + r_f} - K$$

$$= \frac{E[X_P] - \gamma \operatorname{cov}(X_P, r_M)}{1 + r_f} + \alpha \frac{E[X_N] - \gamma \operatorname{cov}(X_N, r_M)}{1 + r_f}$$

$$= P(X_P) + \alpha P(X_N). \tag{25}$$

This demonstrates that the financial institution's NPV equals the sum of the present values of the two investment opportunities. The constraint condition is expressed in equation (26).

$$\rho(X_P + \alpha X_N) \le K. \tag{26}$$

Therefore, the maximization problem is

$$\max_{\alpha} [P(X_P) + \alpha P(X_N)], \quad \text{subject to} \quad \rho(X_P + \alpha X_N) \le K. \tag{27}$$

Although equation (27) is simple, it is not easily solved because the constraint is nonlinear in general.

As a calculation example, we assume that the risk is proportional to the standard deviation of the payoff. That is to say, we assume $\rho(X) = \kappa \sigma(X)$ (where κ is a constant).

In this case, we arrive at the following expression.

$$\max_{\alpha} [P(X_P) + \alpha P(X_N)], \quad \text{subject to} \quad \kappa \sigma(X_P + \alpha X_N) \le K. \tag{28}$$

The value of α can then be derived as follows.

$$\alpha \le \bar{\alpha} = \frac{\sqrt{\operatorname{cov}(X_N, X_P)^2 + (K^2/\kappa^2 - \sigma^2(X_P))\sigma^2(X_N)} - \operatorname{cov}(X_N, X_P)}{\sigma^2(X_N)}. \tag{29}$$

In other words, the optimal solution that maximizes the NPV is $\alpha = \bar{\alpha}$.

As demonstrated by this example, the amount of investment in the new investment opportunity is determined depending on the correlation $\operatorname{cov}(X_N, X_P)$.

Just as under the case with a single investment opportunity, when there are multiple investment opportunities, it is still optimal to make all investments that have positive NPV. It can be shown that estimating risk-adjusted rates of return does not provide any additional information for capital budgeting. We show this below in Section V.

F. Capital Structure

For practitioners, the capital is exogenously given. Theoretically, however, the amount of capital K should be determined so that it maximizes the financial institutions' NPV.

Section IV.C demonstrated that when there is only one investment opportunity and the capital K is given, the maximized NPV becomes $KP(X)/\rho(X)$. Because this is proportional to K, the optimal amount of capital is infinite, that is, $K = \infty$.

However, this conclusion that the optimal amount of capital is limitless is based on an implicit assumption that there are an infinite number of investment opportunities with positive NPV. As this assumption is unrealistic, now we assume that the investment opportunities that have a positive NPV are limited. In this case, the financial institution should retain just enough capital to take advantage of these limited investment opportunities. In other words, we arrive at the following equation.

$$K = \rho(\alpha X). \tag{30}$$

V. Capital Allocation

In this section, we consider the capital allocation. To do so, we apply the standard model to financial institutions with multiple operating divisions.[20]

A. Rationale of Capital Allocations

Here the rationale to allocating capital to different operating divisions is examined in light of the standard framework.

1. The standard framework

As explained in Section IV, in the standard framework, a financial institution's payoff is presented in equation (17) as long as the institution holds risk capital that is greater than the risk. Here we assume that capital of K_i is allocated to each operating division i. Each operating division invests this capital in risk-free assets and, at the same time, costlessly invests in an investment opportunity that generates a payoff of X_i per unit at period 1. Following the same approach as that adopted for equation (17), assuming that each division invests in one unit of the investment opportunity, the payoff at each operating division then becomes as shown in equation (31).

$$w_i = X_i + (1 + r_f)K_i. \tag{31}$$

The NPV of each operating division can then be expressed as follows.

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20. Zaik *et al.* (1996) and Culp (2001) introduce specific capital allocation methods that practitioners actually utilize.

$$NPV_i = P(w_i) - K_i$$

$$= \frac{E[X_i + (1 + r_f)K_i] - \gamma \operatorname{cov}(X_i + (1 + r_f)K_i, r_M)}{1 + r_f} - K_i$$

$$= \frac{E[X_i] - \gamma \operatorname{cov}(X_i, r_M)}{1 + r_f} = P(X_i). \tag{32}$$

While equation (32) shows the NPV of each operating division, this equation does not include the capital K_i. Moreover, this NPV is equal to the present value of the investment, and is not influenced by the capital allocation or by the investments made by other operating divisions.

Following the same approach adopted in Section IV.C, equation (32) can be restated to express the risk-adjusted rate of return for each operating division i as follows.

$$NPV_i = \frac{E[X_i] - \gamma \operatorname{cov}(X_i, r_M)}{1 + r_f} > 0$$

$$\Leftrightarrow \frac{E[w_i] - K_i}{K_i} > r_f + \beta_{CAPM, \alpha X_i/K_i}(E[r_M] - r_f), \tag{33}$$

where $w_i = X_i + (1 + r_f)K_i$ and $K_i > 0$.

While equation (33) shows the capital budgeting based on the risk-adjusted rate of return, it has the same value as the capital budgeting based on NPV, and thus the amount of capital K_i allocated to each operating division i has absolutely no influence on the results. Therefore, just as in Section IV.C, this demonstrates that calculating the risk-adjusted rate of return provides no additional information.

For practical use, it is necessary to calculate $\beta_{CAPM, \alpha X_i/K_i}$ for each investment opportunity, but in actual practice it seems that financial institutions use a fixed value of β, regardless of the investment opportunity (Zaik *et al.* [1996]). This would not present many problems if the individual investment opportunity's β were always equal to (or nearly equal to) the firm's β. Since this is not the case, the use of a fixed value is generally inappropriate.[21]

Furthermore, as shown in the following equation, the sum of the NPVs of each operating division is equal to the NPV of the financial institution.

$$\sum_{i=1}^{n} NPV_i = \sum_{i=1}^{n} \frac{E[X_i] - \gamma \operatorname{cov}(X_i, r_M)}{1 + r_f}$$

$$= \frac{1}{1 + r_f} \left(E\left[\sum_{i=1}^{n} X_i\right] - \gamma \operatorname{cov}\left(\sum_{i=1}^{n} X_i, r_M\right) \right) = NPV_T. \tag{34}$$

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21. When an oil refining company makes an investment to expand its existing facilities, using the firm's β for this new investment may be appropriate. However, if this same firm decides to advance into the convenience store business, using the firm's β to evaluate this convenience store investment is certainly inappropriate.

Here, the term NPV_T represents the total NPV of the financial institution. This equation holds regardless of the methodology adopted to determine the capital allocation to each division.

Our conclusions regarding the standard framework can be summarized as follows. First, for capital budgeting, the capital allocation to each operating division should be based on equation (32), which shows the NPV of each division. Because equation (32) is not influenced by other investment opportunities, the capital budgeting works for the individual operating divisions are independent of one other. Additionally, the sum of the NPVs of each division is equal to the total NPV of the financial institution, that is to say, it has value additivity. Thus, as long as each operating division maximizes its own NPV, the financial institution's NPV will automatically be maximized.

The next conclusion is that the level of capital should be determined centrally at headquarters. The headquarters should monitor the risks at all operating divisions, and raise sufficient capital to cover all of these risks. The capital allocation does not matter, since how capital is allocated does not affect equation (34), which ensures that the NPV maximization at individual divisions leads automatically to the NPV maximization of the entire financial institution.

2. Introduction of deadweight costs

Section V.A.1 concluded that all positive-NPV investment should be implemented. It also concluded that capital allocations are irrelevant. However, the discussion in Section V.A.1 assumes that financial institutions are able to raise an infinite amount of capital without any cost. We now expand the argument to encompass a world in which holding capital incurs deadweight costs.[22] We then show how the existence of deadweight costs may provide the basis for the capital allocations that are actually implemented by financial institutions.

The basic model parameters are the same as those in Section V.A.1. Here the deadweight cost of holding an amount of capital K is given by τK. In this case, the payoff of each operating division becomes as follows.

$$w_i = X_i + (1 + r_f - \tau)K_i. \qquad (35)$$

The NPV can now be expressed as in equation (36).[23]

$$NPV_i = P(w_i) - K_i = \frac{E[X_i] - \gamma \operatorname{cov}(X_i, r_M)}{1 + r_f} - \frac{\tau}{1 + r_f} K_i. \tag{36}$$

Unlike the NPV in equation (32), the NPV of each operating division in equation (36) depends on the capital K_i. This is because costs are incurred in holding this capital, which becomes necessary when risks are taken by implementing investments. Such deadweight costs must be borne by financial institutions, and the institutions must also devise some sort of rules for the allocation of capital to the individual operating divisions. In other words, once deadweight costs are introduced, capital allocation becomes necessary. This means the financial institutions themselves have implicitly assumed the existence of deadweight costs as the basis for their capital allocations.

B. Methodologies for Capital Allocation

We now consider specific methodologies for capital allocation, which presumes the existence of deadweight costs. First we introduce the approach to capital allocation in Merton and Perold (1993), which focuses on the additional risk capital required for implementing investments. We then point out the problems with this approach, and explain the intrinsic difficulty of determining an appropriate allocation method.

1. Capital allocation under Merton and Perold (1993)

In Merton and Perold (1993), "marginal risk capital" is obtained by calculating the risk capital required for the firm without a new business and subtracting it from the risk capital required for the full portfolio of businesses. They claim that management decisions on whether or not to invest into a new business must be based on the cost of marginal risk capital. This argument can be expressed using the standard model developed in the previous section as follows.

Assume that a financial institution has n operating divisions, and that the total NPV of the institution excluding a new operating division s is NPV_T^s. In this case, NPV_T and NPV_T^s may be calculated as shown in equations (37) and (38), respectively.

$$NPV_T = \frac{E[w_T] - \gamma \operatorname{cov}(w_T, r_M)}{1 + r_f} - \rho(X_T)$$

$$= \frac{E\left[\sum_{i=1}^{n} X_i\right] - \gamma \operatorname{cov}\left(\sum_{i=1}^{n} X_i, r_M\right)}{1 + r_f} - \frac{\tau}{1 + r_f} \rho(X_T), \tag{37}$$

$$NPV_T^s = \frac{E[w_T - w_s] - \gamma \operatorname{cov}(w_T - w_s, r_M)}{1 + r_f} - \rho(X_T - X_s)$$

$$= \frac{E\left[\sum_{i=1}^{n} X_i - X_s\right] - \gamma \operatorname{cov}\left(\sum_{i=1}^{n} X_i - X_s, r_M\right)}{1 + r_f} - \frac{\tau}{1 + r_f} \rho(X_T - X_s). \tag{38}$$

Here, w_T represents the financial institution's portfolio payoff, and $\rho(\cdot)$ is a risk measure.

The differential between NPV_T and NPV_T^s is shown by equation (39).

$$NPV_T - NPV_T^s$$
$$= \frac{E[X_s] - \gamma \operatorname{cov}(X_s, r_M)}{1 + r_f} - \frac{\tau}{1 + r_f}(\rho(X_T) - \rho(X_T - X_s)). \tag{39}$$

So equation (39) shows the difference in the financial institution's NPV with and without operating division s, and can therefore be used to determine whether or not the institution should invest in this new business s. As long as the solution to equation (39) is positive, operating division s contributes to increasing the financial institution's total NPV.

Now if the capital allocation rule is defined by equation (40), the NPV calculated using equation (39) becomes equal to the NPV calculated using equation (36), which is the standard for capital budgeting among divisions when deadweight costs exist.

$$K_i = \rho(X_T) - \rho(X_T - X_i). \tag{40}$$

The capital allocation rule stated by equation (40) may be viewed as indicating that when a new operating division is added to a financial institution, the risk capital that should be allocated to this new division should equal the increase in the total risk resulting from it. This means that implementing capital budgeting based on the marginal risk capital can also be used to measure the extent to which each division contributes to the financial institution's total NPV.

2. Intrinsic difficulty of appropriate capital allocation

Conversely, after risk capital is allocated to each operating division by the rule expressed by equation (40), can equation (36) then be used to evaluate the relative contributions of each of the divisions to the financial institution's total NPV? Unfortunately, this is generally impossible under equation (40). This is because risk measures normally have a risk diversification effect (i.e., $\rho(X_i + X_j) \leq \rho(X_i) + \rho(X_j)$), and thus under this allocation rule the sum of the risk capital allocated to each operating division does not equal K_T.

$$\sum_{i=1}^{n} K_i = \sum_{i=1}^{n}(\rho(X_T) - \rho(X_T - X_i))$$
$$\neq \rho(X_T) = K_T. \tag{41}$$

In this case, value additivity does not hold for the sum of the NPVs of the individual operating divisions and the financial institution's total NPV. So, for example, even if the NPVs of the individual divisions are all positive, this does not necessarily guarantee that the institution's total NPV will be positive.

This can be explained as follows. From equation (39), the sum of the NPVs of the individual divisions can be calculated using equation (42).

$$\sum_{s=1}^{n}(NPV_T - NPV_T^s)$$

$$= \frac{E\left[\sum_{s=1}^{n}X_s\right] - \gamma\,\mathrm{cov}\left(\sum_{s=1}^{n}X_s,\, r_M\right)}{1 + r_f} - \frac{\tau}{1 + r_f}\sum_{s=1}^{n}(\rho(X_T) - \rho(X_T - X_s)). \qquad (42)$$

Comparing equation (42) with equation (37), whether or not NPV_T is larger than $\sum_{s=1}^{n}(NPV_T - NPV_T^s)$ depends on the relative magnitude of $\rho(X_T)$ and $\sum_{s=1}^{n}(\rho(X_T) - \rho(X_T - X_s))$.

When $n = 2$,

$$\sum_{s=1}^{2}(\rho(X_T) - \rho(X_T - X_s))$$
$$= \rho(X_T) + \rho(X_T) - \rho(X_T - X_1) - \rho(X_T - X_2)$$
$$= \rho(X_T) + \rho(X_T) - \rho(X_2) - \rho(X_1)$$
$$\le \rho(X_T). \quad (\because \rho(X_T) \le \rho(X_1) + \rho(X_2)) \qquad (43)$$

Thus, when $n = 2$, if the NPV of each individual division is positive, the financial institution's total NPV is not necessarily positive.

When $n = 3$,

$$\sum_{s=1}^{3}(\rho(X_T) - \rho(X_T - X_s))$$
$$= 3\rho(X_T) - \rho(X_T - X_1) - \rho(X_T - X_2) - \rho(X_T - X_3)$$
$$= 3\rho(X_T) - \rho(X_2 + X_3) - \rho(X_1 + X_3) - \rho(X_1 + X_2)$$
$$\ge 3\rho(X_T) - 2(\rho(X_1) + \rho(X_2) + \rho(X_3)), \qquad (44)$$

and at the same time,

$$3\rho(X_T) - 2(\rho(X_1) + \rho(X_2) + \rho(X_3))$$
$$\le \rho(X_T). \quad (\because \rho(X_T) \le \rho(X_1) + \rho(X_2) + \rho(X_3)) \qquad (45)$$

Thus, when $n = 3$, no *a priori* conclusions can be reached regarding the relative magnitude of $\rho(X_T)$ and $\sum_{s=1}^{3}(\rho(X_T) - \rho(X_T - X_s))$.

These two cases ($n = 2, 3$) are sufficient to demonstrate that even when the NPVs of the individual operating divisions calculated using equation (36) are all positive (negative), this does not necessarily guarantee that the institution's total NPV will be positive (negative).

In other words, even though equation (39) can serve as a capital budgeting standard to determine the risk capital that should be allocated to any given division, this equation cannot be used to compare the relative NPV contributions of all the

individual divisions. This means that intrinsically any capital allocation method whereby $\rho(X_T) = K_T$ must be based on some principle other than the level of contribution to the firm's value.

From this perspective, Denault (2001) proposes "fairness" as one principle for the allocation of capital. He utilizes game theory to demonstrate that a "fair" capital allocation methodology exists that fulfills the condition $\rho(X_T) = K_T$.

VI. Conclusions

In this paper, we first considered the risk capital framework generally adopted by financial institutions. We noted that risk capital allocations are theoretically irrelevant under this framework. However, when deadweight costs are introduced for raising capital, risk capital allocations become relevant. We then argued that the risk-adjusted rate of return is theoretically unnecessary for evaluating the profitability of investment opportunities, as it provides no additional information beyond simple NPV calculations. Finally, we pointed out the intrinsic difficulty of capital allocation because of the risk diversification effect.

However, it is important to note that we have set aside several important issues in developing the model in this paper. The most important of these issues are summarized below, and we would like to leave these as topics for future research.

(1) Agency problem

At financial institutions, an agency problem[24] exists between corporate management (at headquarters) and the individual operating divisions. In general, the individual operating divisions have more detailed information regarding the investment opportunities available to them, compared with the information available to corporate management. If the objectives of the operating divisions diverge from those of corporate management, the operating divisions may take advantage of their superior information to maximize their own interests.[25]

(2) Appropriateness of the assumptions regarding economic friction

Although it is difficult to actually measure economic friction, the issue of how economic friction influences the payoff of financial institutions needs to be further considered.

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24. For the details of this type of in-house agency problem, see, for example, Brealey and Myers (2000) and Stein (2001).
25. See Krishnan (2000) and Stoughton and Zechner (1999).

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