A New Technique for Simultaneous Estimation of Potential Output and the Phillips Curve

Yasuo Hirose and Koichiro Kamada

A new technique is demonstrated for the simultaneous estimation of potential output and the Phillips curve. In this paper, we define potential output as the non-accelerating inflation level of output (NAILO). The NAILO is not a simple trend of actual output. Instead, it is the critical level of output such that, were actual output at this level, the inflation rate would be neither accelerating nor decelerating. Our application is the case of Japan, for which we estimate both the NAILO and the Phillips curve and investigate their properties. It is shown that during the 1980s and 1990s, the Japanese output gap, as measured using the NAILO, was negative on average, reflecting the global trend of disinflation. We also point out that this NAILO-based output gap has displayed a tendency to move in line with corporate sentiment and is thus a useful indicator of business conditions. However, being subject to re-estimation due to the revision of source data and the arrival of new data, the NAILO estimate is surrounded by uncertainty. This uncertainty needs to be kept in mind in real-time analysis, and the NAILO estimate should be interpreted with care, particularly in the process of policymaking.

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I. Introduction

The Phillips curve describes an empirically observed trade-off between the inflation rate and real economic activity. Since the seminal paper by Phillips (1958), economists have devoted substantial efforts to establishing a firm theoretical background and to finding new empirical evidence for this phenomenon.[1] The Phillips curve has strong policy implications and is of special interest to central bankers, whose ultimate mission is to stabilize price movements.

Many recent discussions related to the Phillips curve involve the estimation of potential output and the output gap. Denote the inflation rate by π, the logarithm of actual output by y, and the logarithm of potential output by y^N. Then the simplest Phillips curve is written as

$$\pi_t = \pi_{t-1} + \beta(y_t - y_t^N) + \epsilon_t. \tag{1}$$

There exist various definitions for potential output. Equation (1) defines potential output, y^N, as the *non-accelerating inflation level of output (NAILO)*. If actual output is at the NAILO, the inflation rate will be neither accelerating nor decelerating. Given y^N, it is easy to estimate β in equation (1), using some standard econometric method (e.g., ordinary least squares [OLS]). The problem that arises in estimating equation (1), however, is the fact that y^N is unobservable.

Various methods are proposed for estimating potential output. There are two strands in the literature: the *production-function approach* and the *time-series approach*. In the former, we first estimate an aggregate production function and then plug in the "normal" amounts of inputs to obtain potential output.[2] In the time-series approach, we take potential output to be a specific moving average of actual output. The Hodrick-Prescott (HP) filter is an example of this approach and defines potential output as the series, y^{HP}, that minimizes the following objective function:

$$\sum_{t=1}^{T}(y_t - y_t^{HP})^2 + \theta\sum_{t=2}^{T-1}(\Delta y_{t+1}^{HP} - \Delta y_t^{HP})^2 \tag{2}$$

That is, y^{HP} is defined so as to cling to actual output (the first term) and to move smoothly (the second term). Parameter θ in equation (2) determines the degree of smoothness in the movement of y^{HP}. As θ increases, y^{HP} moves more smoothly.[3]

1. Turner (1995) and Watanabe (1997) compare Phillips curves across countries. Higo and Kuroda Nakada (1999) point out that the properties of Phillips curves may change over time. Although interesting, the comparison of Phillips curves across countries and over time is outside the focus of this paper.
2. The "normal" amount of a production input is defined in various ways. The following three definitions are used frequently: (1) full utilization of production factors (Kamada and Masuda [2001]); (2) utilization of production factors at their historically average rates (Economic Planning Agency [2000] in Japan); and (3) utilization of production factors at rates consistent with stable factor price inflation rates (Congressional Budget Office [1995] in the United States). Relatively speaking, our method is closest to the third alternative.
3. The following values are frequently used for θ: 14,400 for monthly data; 1,600 for quarterly data; and 100 for annual data.

Since y^{HP} is nothing but a simple moving average of actual output, its relationship with prices is obscure. Moreover, as is obvious from the calculation procedure, under the HP filter, y^{HP} is not allowed to depart from actual output for long. Hence, replacing y^N by y^{HP} in equation (1) results in a biased estimate of β. As a clue to resolving this problem, suppose that β is known and define a new series, z, as follows:

$$z_t = y_t - (\pi_t - \pi_{t-1})/\beta = y_t^N - \epsilon_t/\beta. \tag{3}$$

(We use equation [1] to derive the last equality.) Equation (3) suggests that we can obtain a more accurate estimate of y^N by HP-filtering the z series, which is adjusted for the development of the inflation rate, rather than simply filtering y. An important point is that, by taking the Phillips curve relationship into consideration, this technique gives a clear meaning to potential output y^N. That is, y^N is the output level such that, when actual output is at this level, the inflation rate is neither accelerating nor decelerating. Our estimation technique developed in the later section hinges upon this fundamental insight.

The idea of adjusting y^{HP} by the residuals of Phillips curve regression is not new in the literature. Laxton and Tetlow's (1992) multivariate (MV) filter is one of the earliest examples (see also Butler [1996] for an extension of the MV filter). More recently, Ball and Mankiw (2002) estimate the time-varying non-accelerating inflation rate of unemployment (NAIRU) by applying the HP filter to the z series (in their treatment, y is the unemployment rate). They face the same problem that we have seen above: the value of β must be known before constructing z. A shortcoming of their method is that estimation of the time-varying NAIRU is carried out, using a value of β previously estimated under the assumption that the NAIRU is time-invariant. Our technique resolves this obvious inconsistency by simultaneously estimating y^N and β, instead of estimating them separately.[4]

The remainder of this paper is constructed as follows. In Section II, we provide a formal description of our technique for the simultaneous estimation of potential output and the Phillips curve. In Section III, we apply our method to Japanese data to estimate potential output (i.e., the NAILO) and the associated output gap (the rate of deviation of actual from potential output) for Japan during the 1980s and 1990s. We calculate a simple HP-filtered trend of actual output and show quantitatively to what extent it diverges from the NAILO. In the course of this analysis, the contribution of our method to the accurate estimation of the Phillips curve coefficients is underlined. We also compare the NAILO-based output gap with representative business-cycle indices to check its robustness as an indicator of the aggregate balance between supply and demand. Section IV draws attention to some caveats that should be borne in mind when making use of our technique. In Section V, we discuss possible extensions of our method. In Section VI, we conclude our discussion by summing up the main results of the paper.

4. In the Phillips curve regression under the assumption that the NAIRU is time-invariant, a negative estimate may be obtained for β. In this case, Ball and Mankiw's (2002) method is no longer applicable for the estimation of the time-varying NAIRU.

II. Method

In general, the expectations-augmented Phillips curve is written as follows:

$$\pi_t = \pi_t^e + \beta(y_t - y_t^N) + \epsilon_t, \tag{4}$$

where π is the inflation rate, π^e is the expected inflation rate, y is the logarithm of actual output, and y^N is the logarithm of potential output. The derivation of equation (1) depends on the assumption that the expected inflation rate is static ($\pi_t^e = \pi_{t-1}$). According to this Phillips curve, the inflation rate will accelerate when y rises above y^N, while it will decelerate when y falls below y^N. In this paper, we call y^N the non-accelerating inflation level of output (NAILO). Below, we relax the static-expectations assumption slightly and estimate the following Phillips curve instead.

$$\pi_t = \alpha\, \pi_{t-1} + (1 - \alpha)\pi_{t-2} + \beta(y_t - y_t^N) + \epsilon_t. \qquad (5)^5$$

In this paper, we estimate potential output (i.e., the NAILO) and the Phillips curve simultaneously under the assumption that potential output moves smoothly. The principle of our technique is similar to that of OLS, which minimizes the residuals in the Phillips curve regression. We, however, minimize the following objective function, which is a bit more complicated than that of the OLS estimators.

$$\begin{aligned} V(\alpha, \beta, y_1^N, \ldots, y_t^N) \equiv & \sum_{t=1}^{T}\{\pi_t - \alpha\, \pi_{t-1} - (1 - \alpha)\pi_{t-2} - \beta(y_t - y_t^N)\}^2 \\ & + \lambda \sum_{t=2}^{T-1}(\Delta y_{t+1}^N - \Delta y_t^N)^2. \end{aligned} \tag{6}$$

The one and only difference between equation (6) and the OLS objective function is the existence of the sum of squares at the end of the equation. This term captures the total amount of penalty incurred for any abrupt changes in the potential rate of growth (Δy^N). As λ becomes large, y^N moves more smoothly. In one limit, when λ is infinitely large, y^N moves on a linear trend. In the other limit, when λ is zero, y^N is determined so as to achieve a perfect fit for the Phillips curve. We discuss the determination of λ shortly.

Our objective is to determine $T + 2$ unknowns, α, β, $y_1^N, \ldots, y_T^N$, so as to minimize V, taking λ as given. It is inefficient, however, to solve for all of these unknowns at once. Rather, we take the following two-step approach. In the first step, we fix α and β at arbitrary values and, given these, solve for the optimal $y_1^N, \ldots, y_T^N$.[6]

5. The appropriate Phillips curve specification may vary across countries. For instance, in some countries the current inflation rate may depend on the output gap in the previous quarter rather than the output gap in the current quarter. In addition, there may be a country-specific inflation rate lag-structure that approximates the expected rate of inflation. Furthermore, supply-side factors other than the NAILO may have substantial effects on our method. For example, we investigate the effects of an appreciation of the yen on the estimates of the Japanese NAILO and the Phillips curve in Section IV.

6. Initial values for α, β, and y^N are entirely arbitrary. One choice is to use y^{HP} as the initial y^N and estimate Phillips curve coefficients to obtain initial values for α and β.

In doing so, it is convenient to use the fact that the values that minimize V are the same as those that minimize $W \equiv V/\beta^2$, defined below.

$$W(\alpha, \beta, y_1^N, \ldots, y_T^N) \equiv \sum_{t=1}^{T}(z_t - y_t^N)^2 + \mu \sum_{t=2}^{T-1}(\Delta y_{t+1}^N - \Delta y_t^N)^2, \tag{7}$$

where $z_t \equiv y_t - \{\pi_t - \alpha\ \pi_{t-1} - (1-\alpha)\ \pi_{t-2}\}/\beta$ and $\mu \equiv \lambda/\beta^2$. Note that equation (7) is similar to equation (2), which defines the HP filter. The sole difference is that in equation (7), the y in equation (2) is replaced with z, which is the inflation-adjusted output. This similarity suggests that we can readily obtain the T unknowns, $y_1^N, \ldots, y_T^N$, that minimize W by HP-filtering the z series.

In the second step, we take y^N as given and minimize V with respect to α and β. Given y^N, the second term in equation (6) becomes constant. Hence, our task is reduced to minimizing the first term in the equation. Notice that this is exactly the same as the derivation of the OLS estimators. Note that the OLS estimates of α and β thus obtained will in general differ from their initial values. We, therefore, return to the first step and iterate the above process until α and β converge. Once convergence occurs, the minimization process is complete.

Finally, we discuss how to choose a value for λ. We have two options. First, we can treat λ as literally exogenous. Note that, as is clear from equation (7), it is μ rather than λ that directly governs the smoothness in the behavior of y^N. Armed with an exogenous choice of λ and the β obtained from the minimization procedure developed above, we can calculate the smoothness in the NAILO as $\mu = \lambda/\beta^2$. In the second option, which we exploit throughout this paper, we determine λ endogenously to force a certain degree of smoothness in the NAILO. Since we use quarterly data in this paper, it is reasonable to choose λ so that $\mu = 1{,}600$, as recommended by Hodrick and Prescott (1997). To do so, it is enough to assume $\mu = 1{,}600$ for each iteration of the first step above, rather than resetting $\mu = \lambda/\beta^2$ every time. When α and β converge, we can obtain λ as $1{,}600\beta^2$.[7]

7. Laxton and Tetlow (1992) develop a similar method for measuring potential output, and call it the multivariate (MV) filter. They start with y^{HP} and adjust it using the information obtained from Phillips curve regression. Haltmaier (1996) provides a useful reference for clarifying the differences between the MV filter and ours. The first step of the MV filter is to calculate y^{HP} by HP-filtering actual output. The second step is to estimate the Phillips curve, using the y^{HP} obtained in the first step as a "provisional" measure of potential output. Denote the estimated parameters by $\hat{\alpha}$ and $\hat{\beta}$. The third step is to construct an objective function, $\tilde{V}$, as follows.

$$\tilde{V} = \sum_{t=1}^{T}(y_t - y_t^{LT})^2 + \lambda \sum_{t=2}^{T-1}(\Delta y_{t+1}^{LT} - \Delta y_t^{LT})^2 + \psi \sum_{t=1}^{T}\{\pi_t - \hat{\alpha}\ \pi_{t-1} - (1-\hat{\alpha})\pi_{t-2} - \hat{\beta}(y_t - y_t^{LT})\}^2.$$

The "final" potential output is the series, y^{LT}, that minimizes the above equation. Note that by ignoring the first term and letting $\psi = 1$, $\tilde{V}$ is reduced to our objective function, V. One drawback of the MV filter is that the Phillips curve and potential output are not estimated simultaneously. As a result, when we estimate a new Phillips curve, given this y^{LT}, the new estimates of the coefficients do not coincide with $\hat{\alpha}$ and $\hat{\beta}$, unless by accident. Another drawback is that there are no obvious criteria for choosing a value of ψ. Our simultaneous-estimation technique addresses these drawbacks and takes Laxton and Tetlow's method one step further.

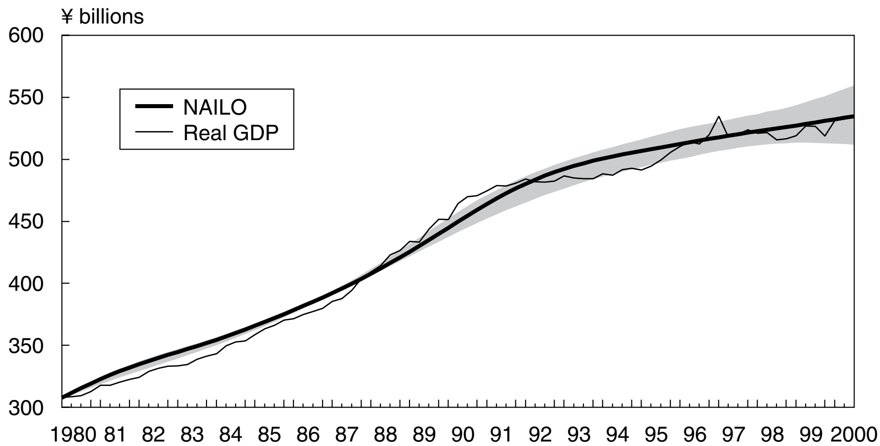
III. Results

In this section, we estimate the Japanese NAILO, based on the consumer price index (CPI), excluding fresh food and adjusted for the effects of the consumption tax, and real GDP. The sample takes the form of seasonally adjusted quarterly data, ranging from the first quarter of 1980 to the third quarter of 2000. To examine the properties of the NAILO, we compare it with a simple HP-filtered real GDP trend and also with some business-cycle indicators.

A. The Japanese NAILO

The estimate of the Japanese NAILO is shown as the thick line in Figure 1. For comparison, we also present real GDP (the thin line). It is estimated that the NAILO grew over 4 percent per year around the peak of the Japanese asset bubble in 1989 and 1990; thereafter, its growth slowed and in the late 1990s was only 1 percent. While the NAILO and real GDP move together in the long run, they may diverge in the short run due to business-cycle effects or some supply shock.[8]

Figure 1 Japanese NAILO



Note: The 95 percent confidence interval for the NAILO is shown as the shaded area.

Sources: Cabinet Office, *National Accounts*; Ministry of Public Management, Home Affairs, Posts and Telecommunications, *Consumer Price Index*.

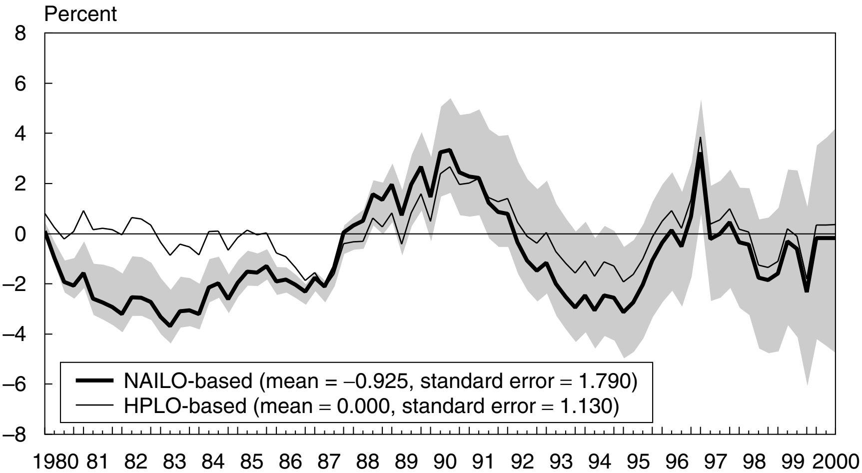
8. The NAILO is estimated to have grown faster than actual GDP in the early 1980s. As the effects of the second oil shock in the late 1970s faded, the inflation rate decelerated from its historical high. Such a movement in inflation shifts the z series and thus the estimated NAILO. In contrast, the inflation rate was stable from the late 1990s on, with the result that the estimated growth rate of the NAILO has recently converged to the actual rate of growth.

The NAILO estimate is a stochastic quantity, and its 95 percent confidence interval is shown as the shaded area in Figure 1 (see the appendix for the bootstrap construction of this confidence interval).[9] The estimate of the NAILO during the 1990s is attended by so much uncertainty that actual output falls safely within the confidence interval, while the Japanese economy slowed apparently for more than 10 years after the asset bubble burst at the beginning of the 1990s. Clearly, when making use of the NAILO estimate and especially in targeting the NAILO as a goal of monetary policy, due attention should be paid to this uncertainty. (See Staiger *et al.* [1997] for a similar discussion on the U.S. time-varying NAIRU and Hirose and Kamada [2002] for the Japanese case.)[10]

B. Comparison with a Simple HP-Filtered Trend

For comparison, we simply HP-filter real GDP and call the resulting series, y^{HP}, the *HP-filtered level of output (HPLO)*. The degree of smoothness in the HPLO is 1,600, as assumed for the NAILO. We then calculate the two different output gaps, as shown in Figure 2: the *NAILO-based output gap* ($y - y^N$, the thick line) and the *HPLO-based output gap* ($y - y^{HP}$, the thin line).[11] The sample mean of the

Figure 2 Comparison between NAILO-Based and HPLO-Based Output Gaps



Note: The 95 percent confidence interval for the NAILO-based output gap is shown as the shaded area.

Sources: Cabinet Office, *National Accounts*; Ministry of Public Management, Home Affairs, Posts and Telecommunications, *Consumer Price Index*.

9. The inflation rate was extremely volatile in 1980 because of the second oil shock. For this reason, we take the innovation terms during this year as deterministic and ignore them in constructing the confidence interval.
10. See Gordon (1997, 1998) for the foundations of the time-varying NAIRU.
11. The sharp rise in the output gap during the first quarter of 1997 reflects the front-loaded demand that occurred before the rate of consumption tax rose in the following quarter. The recent fluctuations observed in both output gaps reflect GDP instability directly.

NAILO-based output gap is −0.9 percent, reflecting the global trend of disinflation experienced since the 1980s. In contrast, the HPLO-based output gap averages zero, failing to catch this trend. The HPLO in the early 1980s and the mid-1990s suffers an especially strong upward bias in comparison to the NAILO. In particular, the HPLO-based output gap during the 1980s falls outside the 95 percent confidence interval for the NAILO-based output gap.

Furthermore, the standard error of the NAILO-based output gap (1.790) is larger than that of the HPLO-based output gap (1.130). This means that the HPLO-based output gap tends to undervalue the amplitude of the business cycle. Note that these phenomena arise for the technical reason that the HPLO cannot depart from real GDP for a long time, since the former is nothing but a moving average of the latter.

C. The Phillips Curve

Clearly, the extent of any difference between the NAILO and the HPLO will affect the respective Phillips curve estimates produced under each method. Table 1 presents the estimates of the Phillips curve coefficients and the related diagnostic statistics, with y^N chosen optimally.[12] The R^2 is above 60 percent, indicating that the Phillips curve relationship fits the Japanese inflation data well.[13] The estimate of β is 0.026, meaning that a 1 percent increase in the output gap raises the CPI inflation rate by about 0.1 percent annually. This is quite reasonable for Japan, where the price trend has been relatively stable.

Table 1 NAILO-Based and HPLO-Based Phillips Curves

Potential output	α	β	R^2	D.W.
NAILO	0.677 (6.139)	0.026 (1.356)	0.635	2.108
HPLO	0.693 (6.186)	0.012 (0.355)	0.632	2.083

Notes: 1. Sample: 1980/I–2000/III.
2. t-values in parentheses.
3. R^2 is the quasi-coefficient of determination.
4. Smoothness: $\mu = 1{,}600$.

We replace the NAILO by the HPLO and carry out a similar regression. The results are also shown in Table 1. The excellent fit is almost untouched. However, the estimate of β drops to 0.012, which is half as large as the β estimate obtained with

12. Since our Phillips curve includes lags of a dependent variable as explanatory variables, the Durbin-Watson test is likely to conclude that there is no auto-correlation in the regression residuals. Durbin's h is proposed to remedy the problem. But when the sample size is as small as in this analysis, Durbin's h is not necessarily an effective alternative.

13. Since our Phillips curve includes no intercept, we cannot define the usual R^2. Instead, we calculate the quasi-R^2, which is defined as follows:

$$\text{quasi-}R^2 = \{\textstyle\sum(\pi_t - \bar{\pi})(\hat{\pi}_t - \bar{\hat{\pi}})\}^2 / \{\textstyle\sum(\pi_t - \bar{\pi})^2 \textstyle\sum(\hat{\pi}_t - \bar{\hat{\pi}})^2\},$$

where $\bar{\pi}$ is the sample mean of π_t, $\hat{\pi}_t$ is the estimated inflation rate, and $\bar{\hat{\pi}}$ is its mean value. It should be noted that the quasi-R^2 encompasses the usual R^2 as a special case, and the two coincide with each other if the model includes an intercept.

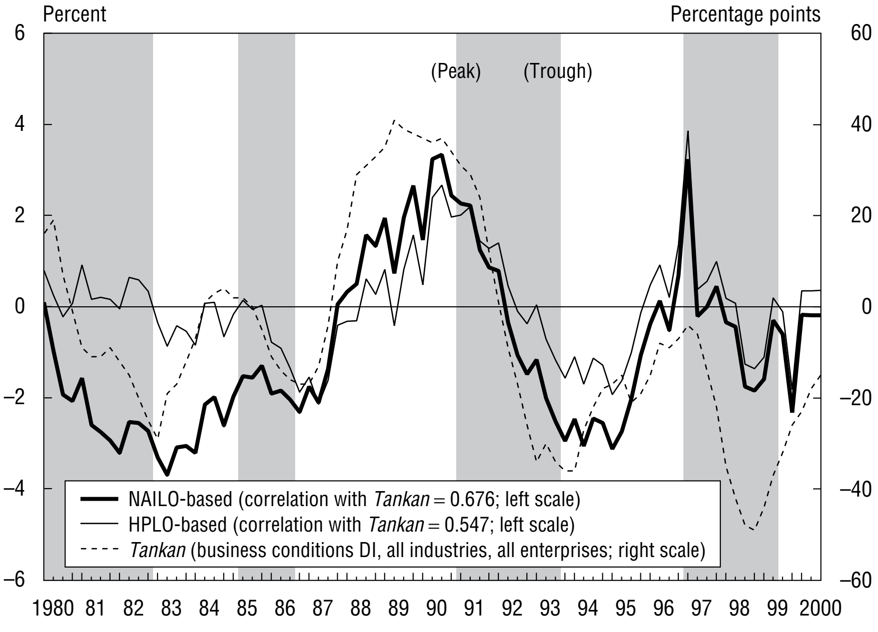
the NAILO and falls outside the 95 percent confidence interval (0.018 to 0.058).[14] This substantial bias casts doubt on the effectiveness of the HPLO as a reference point for inflationary pressure.

D. Output Gap as a Business-Cycle Indicator

Since the NAILO represents a country's production capacity, the NAILO-based output gap measures how efficiently economic resources are utilized in Japan. Hence, we can check the robustness of the NAILO estimate, based on its consistency with other business-cycle indicators.

The *Reference Dates of the Business Cycle (RDBC)* contains official business-cycle data published by the Cabinet Office in Japan. In Figure 3, the shaded areas indicate downturn phases of the Japanese business cycle, i.e., the peak-to-trough periods defined by the RDBC. Looking at developments after the mid-1980s, both the NAILO-based output gap (the thick line) and the HPLO-based output gap (the thin line) behave consistently with the RDBC. Turning to the early-1980s recession and

Figure 3 Consistency of NAILO-Based and HPLO-Based Output Gaps with Business-Cycle Indicators



Sources: Cabinet Office, *National Accounts, Reference Dates of the Business Cycle*; Ministry of Public Management, Home Affairs, Posts and Telecommunications, *Consumer Price Index*; Bank of Japan, *Short-Term Economic Survey of Enterprises in Japan*.

14. The HPLO-based estimate of α (0.693) falls inside the 95 percent confidence interval of the estimate of the NAILO-based α (0.63 to 0.77).

the following recovery phases, however, we may observe a contrast between the poor performance of the HPLO-based output gap and the rather better performance of the NAILO-based output gap.

The *Short-Term Economic Survey of Enterprises in Japan (Tankan)* is a business survey conducted by the Bank of Japan. Among many diffusion indices, we focus on the *business conditions diffusion index (DI)*, which is the share of firms that reported "favorable" minus the share of firms that reported "unfavorable," shown by the broken line in the figure.[15] The correlation coefficient of the NAILO-based output gap with the *Tankan* (0.676) is larger than the corresponding figure for the HPLO-based output gap (0.547).[16] These results suggest that the NAILO-based output gap captures business sentiment more accurately than the HPLO-based output gap.

IV. Caveats

Caveats are in order. Here, we focus on three issues, each of which is important in the practical use of the NAILO: (1) the selection of an appropriate value for the NAILO smoothing parameter; (2) difficulties in the real-time assessment of the NAILO; and (3) disturbances to the NAILO caused by currency appreciation.[17]

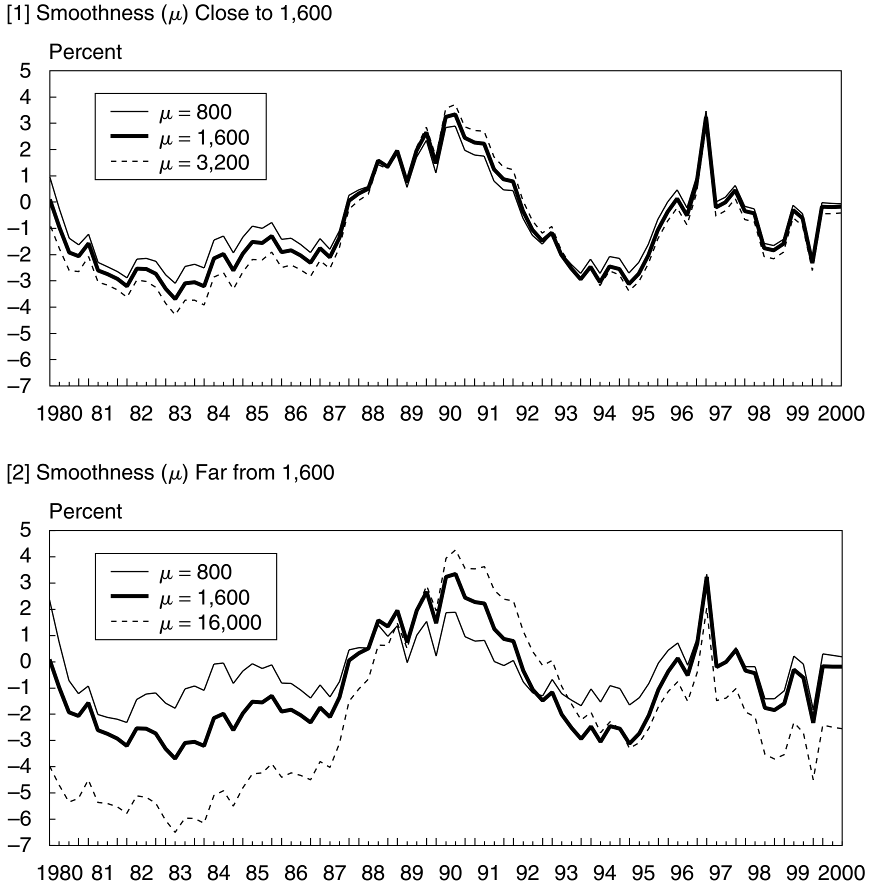
A. Selection of Smoothness

As often recommended for quarterly data, we choose λ so that the degree of smoothness in the NAILO, captured by the smoothing parameter μ, is equal to 1,600. In general, an appropriate value for μ varies both over time and across countries. Our practical concern is to what extent a change in μ affects the estimates of the NAILO and Phillips curve coefficients. Figure 4 shows that small deviations of μ from 1,600 leave the NAILO-based output gap virtually untouched. The estimates of Phillips curve coefficients are also unaffected, as shown in Table 2.

A substantial rise in μ, however, amplifies the volatility of the NAILO-based output gap, with the result that the estimate of β is halved (from 0.026 to 0.011). Similarly, a considerable reduction in μ lessens the volatility of the NAILO-based output gap, more than doubling the estimate of β (from 0.026 to 0.067), which becomes significant at the 5 percent level. The last result suggests that the Japanese NAILO may have behaved less smoothly during the 1980s and 1990s than is implied by $\mu = 1{,}600$.

15. The *Tankan*'s business conditions DI reflects entrepreneurs' overall evaluation of business conditions, including their projections for current profits and other related indicators.
16. In the above, we compare the contemporaneous correlation coefficients of the NAILO-based and HPLO-based output gaps with the business conditions DI. It may be more sensible to choose an optimal lead or lag for each output gap. We can show, however, that even when leads and lags are optimally chosen, the NAILO-based output gap has a higher correlation with the business conditions DI than is possessed by the HPLO-based output gap.
17. Our estimation method makes use of the HP filter as one of its components and thus inherits the filter's drawbacks directly. There is a large body of literature on the drawbacks of the HP filter (see European Central Bank [2000], for instance). In particular, following the criticism of Harvey and Jeager (1993), we should be aware of the possibility that the HP filter may produce a spurious cycle in a series. Additionally, when the economic structure is changing rapidly, the assumption that the NAILO moves smoothly is inappropriate. These are obviously important caveats, but they are deemed beyond the scope of this paper.

Figure 4 NAILO-Based Output Gaps of Varying Degrees of Smoothness



Sources: Cabinet Office, *National Accounts*; Ministry of Public Management, Home Affairs, Posts and Telecommunications, *Consumer Price Index*.

Table 2 NAILO-Based Phillips Curves with Varying Degrees of Smoothness

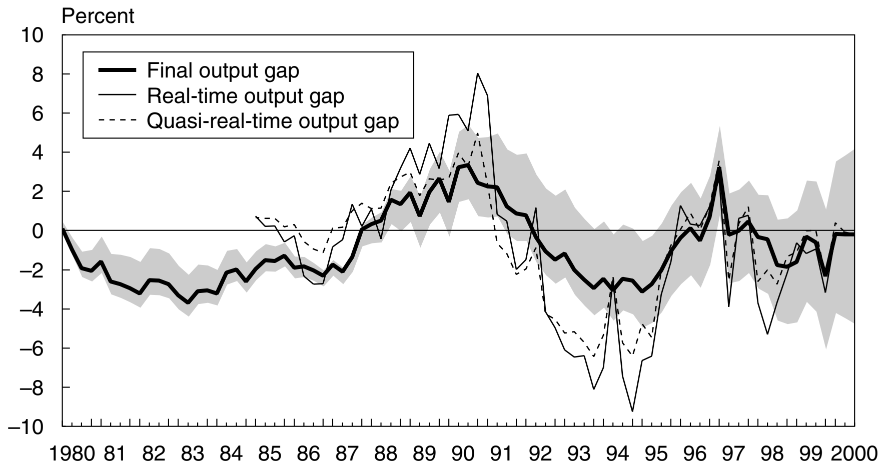
Smoothness (μ)	α	β	R^2	D.W.
160	0.656 (6.034)	0.067 (2.158)	0.648	2.159
800	0.672 (6.119)	0.034 (1.539)	0.638	2.116
1,600	0.677 (6.139)	0.026 (1.356)	0.635	2.108
3,200	0.680 (6.152)	0.020 (1.209)	0.634	2.102
16,000	0.684 (6.157)	0.011 (0.947)	0.630	2.095

Notes: 1. Sample: 1980/I–2000/III.
2. *t*-values in parentheses.
3. R^2 is the quasi-coefficient of determination.

B. Real-Time Assessment

Policymakers are required to provide a timely assessment of economic conditions, and thus the reliability of the real-time estimate of the NAILO is their primary concern (see Orphanides and van Norden [1999] for the United States and Kamada and Masuda [2001] for Japan). Insight into this issue is gained by distinguishing three output gaps, as shown in Figure 5. First, the *final output gap* (the thick line) is the NAILO-based output gap calculated from the GDP series available in the third quarter of 2000 (the same as in Figure 2). Second, to construct the *real-time output gap* (the thin line), we first specify the "real-time GDP series," which was the most precise GDP data available at some given point in time; we then calculate the corresponding output-gap series, retaining the endpoint measurement. By repeating this procedure for every point in time, we obtain the full series for the real-time output gap. Finally, to obtain the *quasi-real-time output gap* (the broken line), we replace the real-time GDP series, at every point in time, with the appropriate subsample of the GDP series available in the third quarter of 2000.

Figure 5 NAILO-Based Output Gaps in Real Time



Note: The 95 percent confidence interval for the final output gap is shown as the shaded area.

Sources: Cabinet Office, *National Accounts*; Ministry of Public Management, Home Affairs, Posts and Telecommunications, *Consumer Price Index*.

The NAILO is subject to re-estimation for two reasons: (1) revision of the real GDP data and (2) the arrival of new data. The differences between the real-time and quasi-real-time output gaps measure the pure effects of GDP revisions. They reflect the accuracy of the preliminary estimates of GDP, and the figure shows the vulnerability of our method to such revisions. In a similar way, discrepancies between the quasi-real-time and final output gaps show the pure effects of the arrival of new data. A large discrepancy is likely to occur at a turning point in the business cycle, as is observed, for example, around 1993–94 in the case of Japan. The data-arrival effects shrink toward the end of the sample, but the final output gap becomes more uncertain, as shown in the widening confidence interval.

The imprecise estimate of the NAILO results in inaccurate estimation of the Phillips curve and may affect the optimal design of monetary policy rules. Table 3 shows the regression results for two real-time samples in addition to those obtained for the most recent sample. The first real-time sample is the one that was available in the fourth quarter of 1990, and the second is that available in the fourth quarter of 1993. The estimates of β (0.021 and 0.018) are smaller than the one obtained with the most recent sample (0.026). Armed with a real-time sample, policymakers estimate β and take an *ex ante* best action, based on this preliminary estimate, while taking the parameter's ambiguity into consideration. This action taken is often found to be inefficient *ex post* (Orphanides [2001]) and is likely to be too conservative (Brainard [1967]). Data-revision effects will be reduced in line with future enhancements of the accuracy of preliminary GDP estimates. Data-arrival effects, however, are intrinsic for a family of two-sided filters like ours, and it is difficult to see how they could be removed from our method.

Table 3 NAILO-Based Phillips Curves with Real-Time Data

Sample	α	β	R^2	D.W.
1980/I–1990/IV	0.725 (4.608)	0.021 (1.205)	0.572	2.062
1980/I–1993/IV	0.707 (5.179)	0.018 (1.011)	0.555	2.086
1980/I–2000/III	0.677 (6.139)	0.026 (1.356)	0.635	2.108

Notes: 1. *t*-values in parentheses.
2. R^2 is the quasi-coefficient of determination.
3. Smoothness: μ =1,600.

C. Effects of Exchange Rates on Phillips Curves

Although basically the NAILO represents a country's production capacity, it is also affected by other supply-side factors, including changes in import prices (particularly, oil prices and exchange rates). For instance, currency appreciation lowers import prices and pushes the inflation rate downward, resulting in an upward shift of the NAILO.

The simple way to remove the effects of currency appreciation is to include it as an explanatory variable in the Phillips curve, as follows:

$$\pi_t = \alpha\ \pi_{t-1} + (1-\alpha)\pi_{t-2} + \beta(y_t - y_t^N) + \gamma\ x_t + \epsilon_t, \qquad (8)$$

where x denotes the rate of appreciation of the nominal effective yen exchange rate. Table 4 gives the estimation results, and it can be seen that the coefficient on the yen appreciation rate is statistically insignificant; Figure 6 shows that the resulting NAILO shift is small, and so is the shift in the NAILO-based output gap.

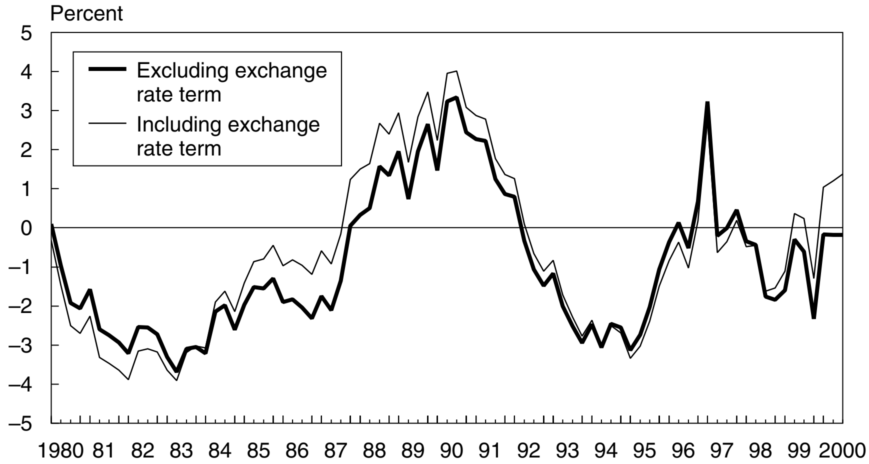
These results may be taken to suggest that most of the effects of the appreciation of the yen are short-lived and are filtered out in the residuals of the Phillips curve

Table 4 NAILO-Based Phillips Curves with and without Exchange Rate Effects

Exchange rate term	α	β	γ	R^2	D.W.
Excluded	0.677 (6.139)	0.026 (1.356)	—	0.635	2.108
Included	0.658 (5.988)	0.019 (1.108)	–0.011 (–1.456)	0.643	2.196

Notes: 1. Sample: 1980/I–2000/III.
2. *t*-values in parentheses.
3. R^2 is the quasi-coefficient of determination.
4. Smoothness: μ =1,600.

Figure 6 Exchange Rate Effects on the NAILO-Based Output Gap



Sources: Cabinet Office, *National Accounts*; Ministry of Public Management, Home Affairs, Posts and Telecommunications, *Consumer Price Index*.

regression. Nonetheless, interpretation of these results requires care, since the results themselves may be sensitive to how the exchange rate is incorporated into the Phillips curve specification. We discuss this issue further in the next section.

V. Extensions

In this section, we suggest two promising extensions of our method: (1) a system of equations and (2) various filters for the NAILO.

A. System of Equations

So far, our treatment has been confined to the case of a single equation with a single unobservable variable (i.e., y^N). Here, we generalize our method to deal with the case of multiple equations with multiple unobservable variables. Consider the following system of equations:

$$A(L)\Pi_t = B(Y_t - Y_t^N) + C(L)X_t + E_t, \tag{9}$$

where Π, Y, Y^N, and X are $k \times 1$ vectors, which are analogous to π, y, y^N, and x in Section IV; E is a $k \times 1$ vector of innovations; $A(L)$ and $C(L)$ are $k \times k$ lag-polynomial matrices; and B is a $k \times k$ scalar matrix.

In the first stage, with initial values for $A(L)$, B, and $C(L)$ given arbitrarily, we define Z analogously to z as follows.

$$Z_t \equiv Y_t^N - B^{-1}E_t = Y_t - B^{-1}\{A(L)\Pi_t - C(L)X_t\}. \tag{10}$$

We extract Y^N from Z by HP-filtering the right-hand side of system (10) row by row. In the second stage, we plug the Y^N thus obtained into system (9) and carry out a joint estimation of the system. In general, the estimates of $A(L)$, B, and $C(L)$ differ from their initial values. We return to the first stage and repeat the above procedure until they converge.

It may be wrong for us to use the first difference of the nominal effective yen exchange rate as an explanatory variable in the Phillips curve in Section IV, where we investigate the effects of the exchange rate on the NAILO. At least, in the long run, the exchange rate is governed by the purchasing power parity (PPP) condition and, should it deviate, it will be pushed back toward the long-run level consistent with PPP. In other words, the current level of the exchange rate contains important information about which direction the exchange rate will take in the future. Thus, by taking a log-difference of the exchange rate, we lose this useful information. For our current purposes, therefore, a natural setup is a two-equation system with two unobservable variables, i.e., conditions for both PPP and the NAILO.

B. Alternative Filters

Basically, our method is a two-step procedure: the first step involves estimation of the NAILO, given Phillips curve coefficients, while in the second step, Phillips curve coefficients are estimated, given the NAILO. Remember that we utilize the HP filter in the second step. It is not necessary, however, to confine ourselves to this particular filter.

According to Baxter and King's (1999) list of filters, candidates are first differences, deviations from five-year moving averages, approximate high-pass filters, and approximate band-pass filters (the Baxter-King filter).[18] In particular, the Baxter-King filter is designed so as to extract business-cycle components by removing the trend and noise components from the original time series, and is thus a promising alternative to the HP filter.

The business-cycle components extracted by the Baxter-King filter are periodic within 6–32 quarters, as defined by the National Bureau of Economic Research (NBER) for the U.S. economy. Unquestionably, tailoring a new Baxter-King filter specifically for the Japanese business cycle would be desirable. We leave this interesting extension for future research.

18. See also Higo and Kuroda Nakada (1998) for various trend-extraction methods.

VI. Conclusion

In this paper, we introduced a new technique for the simultaneous estimation of the Phillips curve and potential output, where the latter was called the NAILO. Our method was designed so as to possess two desirable properties of particular note: it achieved the best possible fit for the Phillips curve, and it allowed us to specify the degree of smoothness in the NAILO. Applying our method to the Japanese economy during the 1980s and 1990s, we attained the following results. First, the growth rate of the Japanese NAILO declined from 4 percent per year during the 1980s to 1 percent during the 1990s. Second, the Japanese NAILO behaved differently from the simple HP-filtered real GDP trend (which we called here the HPLO). For instance, the sample mean of the NAILO-based output gap during the 1980s and 1990s was negative, while that of the HPLO-based gap was zero, failing to catch the global trend of disinflation. Third, use of the NAILO-based output gap implied a CPI inflation rate twice as sensitive to real activity as that estimated using the HPLO-based output gap (i.e., the sacrifice ratio was smaller). Fourth, it was also shown that the NAILO-based output gap was superior to the HPLO-based output gap as a business-cycle indicator.

We presented a few caveats that should be borne in mind to make best use of our technique. First, the choice of the NAILO smoothing parameter affects the estimates of the Phillips curve coefficients, and our experiments suggest that the Japanese NAILO might in fact be less smooth than is generally assumed. Second, the real-time estimates of the NAILO suffer from uncertainty stemming from the revision of source data and the arrival of new data. This uncertainty affects the estimates of Phillips curve coefficients and consequently alters the optimal design of monetary policy rules. Third, the NAILO, which basically represents a country's production capacity, is also affected by other supply-side factors. We investigated, in particular, the effects of currency appreciation on the NAILO and found that these were tiny, although further research is required before we can dispose of this problem conclusively.

We suggested two possible extensions of our method as subjects for further research. First, our method may be easily extended to encompass a system of equations within which it would be possible to take multiple unobservable variables into consideration. Second, in implementing the second step of the NAILO estimation, we could replace the HP filter with one of a number of possible alternative filters, including the approximate band-pass filter developed by Baxter and King (1999). We could also choose to design a new filter so that the NAILO-based output gap captures the Japanese business cycle more precisely.

Some final remarks are appropriate. First, our method may be easily applied to any countries that have readily available data on prices and production. In practice, however, the most difficult task is to decide what data to use. For price data, we must choose among the CPI, the consumption deflator, the GDP deflator, and so on; we also must decide whether to exclude the prices of food and energy and whether to adopt some concept of core inflation. For production data, candidates are real GDP, industrial production, and so forth. It may be necessary to control for some supply-side factors, such as currency appreciation, so that the short-run Phillips curve relationship holds between the inflation rate and real activity. Second, the current model does not make clear what is happening to the growth of capital, labor, and productivity on which the NAILO depends. To provide an answer for this, we would have to construct a model that takes explicit account of these production factors (Haltmaier [1996] is one such attempt).

APPENDIX: BOOTSTRAP PROCEDURE FOR THE NAILO CONFIDENCE INTERVAL

This appendix presents the procedure used for constructing the confidence interval for the NAILO. Of many possible procedures for constructing confidence intervals, we select Efron and Tibshirani's (1993) *bootstrap percentiles* (see also MacKinnon [2002] for a recent review of bootstrap inference). The following is a step-by-step description of this procedure as applied to our model.

(1) Estimate Phillips curve coefficients and the NAILO by our method and denote the results by $\hat{\alpha}$, $\hat{\beta}$, and $\hat{y}_t^N$ ($t = 1, \ldots, T$). The regression errors $\hat{\epsilon}_t$ ($t = 1, \ldots, T$), termed the original errors, are also retained.

(2) Generate bootstrap errors $\tilde{\epsilon}_t$ ($t = 1, \ldots, T$) by sampling randomly with replacement from the original errors.

(3) Generate bootstrap data for the inflation rate $\tilde{\pi}_t$ ($t = 1, \ldots, T$) as follows:

$$\tilde{\pi}_t = \hat{\alpha}\ \tilde{\pi}_{t-1} + (1 - \hat{\alpha})\tilde{\pi}_{t-2} + \hat{\beta}(y_t - \hat{y}_t^N) + \tilde{\epsilon}_t.$$

(4) Estimate the following equation by OLS with $\hat{y}_t^N$ renamed $\tilde{y}_t^N$.

$$\tilde{\pi}_t = \alpha\ \tilde{\pi}_{t-1} + (1 - \alpha)\tilde{\pi}_{t-2} + \beta(y_t - \tilde{y}_t^N) + \epsilon_t.$$

Denote the parameter estimates by $\hat{\alpha}^*$ and $\hat{\beta}^*$.

(5) Calculate $\tilde{z}_t$ as follows:

$$\tilde{z}_t = y_t - \{\tilde{\pi}_t - \hat{\alpha}^*\ \tilde{\pi}_{t-1} - (1 - \hat{\alpha}^*)\ \tilde{\pi}_{t-2}\}/\hat{\beta}^*.$$

HP-filter the results to obtain a new estimate of the NAILO, $\tilde{y}_t^{N*}$.

(6) Return to (4) and substitute $\tilde{y}_t^{N*}$ for $\tilde{y}_t^N$. Iterate this process until $\hat{\alpha}^*$ and $\hat{\beta}^*$ converge. Retain the resulting $\tilde{y}_t^{N*}$ as one realization of the NAILO.

(7) Repeat the process (2) to (6) many times, thus obtaining a set of realizations of $\tilde{y}_t^{N*}$. Arrange realizations in ascending (or descending) order and select those that determine the boundaries of the confidence interval.

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