Divisia Monetary Aggregates and Demand for Money: A Japanese Case

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I. Introduction

Financial innovation since the 1970s has been accompanied by active substitution among financial assets in general and portfolio shifts to financial assets with higher interest rates in particular (from settlement financial assets to investment financial assets, from financial assets with regulated interest rates to financial assets with unregulated interest rates). This has given rise to many problems related to the interpretation of the existing monetary aggregates, and the role of these aggregates as targets or indicators of monetary policy.

Thus for example, in the United States, since 1973-74, as the process of portfolio shift from components of existing monetary aggregates (especially settlement assets, M_1) to other financial assets continues, the quantity of money as measured by the narrowly defined monetary aggregates (M_1) has been declining. This phenomenon has been widely discussed as the case of "missing money".[1]

Furthermore, the recent appearance of new assets such as investment assets possessing settlement functions (MMMF, etc.) and high interest rate settlement

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1. Missing money is usually viewed as the result of shifts (structural changes) in the money demand function. That is, missing money is said to occur when the predicted values, derived by extrapolating a sufficiently stable money demand fuction, exceed actual values (over-prediction) successively. For details, see Goldfeld [12], Boughton [6].

assets (Super-NOW, etc.) has led to large fluctuations in M_1 and this casts doubt over the appropriateness of using M_1 as an intermediate target in implementing monetary policy.

In the case of Japan, as financial innovation up until now has only progressed at a moderate pace compared with the case of the United States, using the existing simple-sum monetary aggregates as targets or indicators would not have resulted in serious problems.[2] However, even in the Japanese case, episodes such as the introduction of the deposit combined account in 1972 and the large portfolio shift from bank deposits to postal savings in the second half of 1980 have created an ambiguity concerning the interpretation of the developments of the monetary aggregates. Furthermore, new financial assets such as "medium-term government bond funds," "Big," "Wide," "Jumbo," etc.[3] have been introduced one after another recently, and if this trend continues, it might be probable that Japan will also face the same problem as one the United States is confronting with.

One way to deal with this situation is to extend the range of aggregation to include new financial assets and to use more broadly defined monetary aggregates as targets or indicators (for example, extension from M_1 to M_2 and then to M_3 and L). However, simple-sum monetary aggregates now commonly used have some theoretical problems as both pure money (currency and checking accounts) and investment assets that have low substitutability with pure money are assigned the same weight. In practical use, extension of the range of aggregation also renders the meaning of such aggregates ambiguous.

To cope with this situation, attempts have been made by the FRB of the United States to estimate theoretically meaningful measures of the quantity of money (the total quantity of "moneyness" or "monetary service") by applying economic aggregation theory and statistical index number theory (an approximation of economic aggregation theory).[4] Divisia monetary aggregates discussed in this paper

2. In fact, when the issuance of CDs was approved in 1979, it was expected that the portfolio shift from existing time deposits (particularly corporate time deposits) to CDs would lead to a sharp decline in M_2. The concept of M_2 was then extended to include CDs and the more broadly defined M_2+CDs came to be stressed. Worth noting is the point that, being an asset with unregulated interest rates and being negotiable, CDs are quite distinct from other components of M_2.

3. "Big": new type of loan trusts, "Wide": new type of bank debentures, "Jumbo": new type of unit bonds trust. These three types of instruments offer higher yields through compound interest.

4. While economic aggregation theory seeks to derive an economically meaningful aggregator function (that reflects specific utility function or production function), statistical index number theory takes the optimizing behavior of economic agents as a starting point and tries to

are one of such attempts.[5]

Divisia monetary aggregates are designed to approximate the quantity of money in a manner that is consistent with the underlying utility function and production function, by computing statistical indices called Divisia indices. As these aggregates have a micro-economic foundation, they are superior to simple-sum aggregates and weighted-sum aggregates that take into consideration the arbitrary degree of moneyness.

In this paper we shall compute such aggregates using Japanese data. In addition, we shall also compare the velocities of money and the money demand functions using these Divisa monetary aggregates with those derived from the ordinary simple-sum monetary aggregates.[6]

Our main findings are summarized as follows:

(i) A comparison between Divisia monetary aggregates and ordinary simple-sum monetary aggregates reveals that, while the degree of substitutability among components of M_1 is high (there is almost no difference in the degree of moneyness among various components of M_1), the components of M_2+CDs and M_3+CDs that are excluded from M_1 show low degree of substitutability with M_1 (components of M_1 have higher degree of moneyness).
(ii) The downward trends in the velocities (the upward trends in Marshallian k's) of Divisia M_2+CDs and M_3+CDs are much weaker than those of ordinary M_2+CDs and M_3+CDs. That is to say, the relations between Divisia monetary aggregates and GNP are more stable than those between ordinary monetary aggregates and GNP.
(iii) The results of estimating the money demand functions using Divisia monetary aggregates also support our contention that the relations between Divisia monetary aggregates and GNP are stable. Furthermore, the money demand functions using Divisia monetary aggregates are superior to those using ordinary simple-sum

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approximate the aggregator function using statistical indices based on price information. As statistical index numbers do not assume any utility function of a specific form and there is no need to estimate unknown parameters in it, they are suitable aggregates for official use by the central banks and government agencies. Furthermore, in the case of monetary aggregates, financial innovation has made it necessary to include newly introduced assets into these aggregates. As the estimation of the parameters of the utility function and production function that include these new assets requires the accumulation of data over a long period of time, it is more convenient to use statistical index numbers.

5. For empirical studies of the US case, see Barnett [2] [3], Barnett and Spindt [4]. Also, Cockerline and Murray [8] computed Divisia monetary aggregates for Canada and used them to analyze the stability of the money multiplier and their causality with GNP.

6. For empirical studies in Japan that use Divisia monetary aggregates, see Suzuki [15]. Its major concern is to study the structural changes in the money demand.

monetary aggregates in the sense that they are more stable.

(iv) The above conclusions suggest that when the traditional simple-sum monetary aggregates such as M_2+CDs, M_3+CDs or some other aggregates more broadly defined are used as indicators or targets for the conduct of monetary policy, aggregates with a micro-economic foundation such as Divisia monetary aggregates should also be considered.

II. Computation of Divisia Monetary Aggregates

We compute the Divisia monetary aggregates in Japan, using the same method proposed by Barnett [2] [3]. (For the details of its theoretical background, see Appendix 1.) For computation, we first classify monetary data in Japan into 11 categories: (1) currency; (2) current deposits; (3) ordinary deposits; (4) deposits at notice; (5) corporate time deposits; (6) personal time deposits; (7) CDs; (8) postal savings; (9) money trusts; (10) loan trusts; (11) other deposits included in M_3 (such as deposits at agricultural cooperatives). We assume that current deposits, deposits at notice, corporate time deposits and CDs are exclusively held by firms while the remaining categories of assets are held only by households, since this is considered to be a good approximation for many financial assets held actually and also since portfolio behavior concerning money holding is quite different between firms and households.[7] Secondly we compute the user cost of money, that is, the opportunity cost of money holding, for each category. By the user cost of money we mean the difference between its own rate and an interest rate on bond (the benchmark rate), which is supposed to be a substitute for money. Using this user cost of money as a price of money, Divisia monetary aggregates are computed by aggregating over the categories of (1) through (4) for M_1, (1) through (7) for M_2+CDs, and (1) through (11) for M_3+CDs.[8] In addition, Divisia price indices are also computed as the

7. Theoretically, if we accept this view, Divisia monetary aggregates should first be computed separately for firms and households and then aggregated to produce the monetary aggregates for the whole economy. However, as aggregation across different economic agents also has problems of its own, for the sake of convenience, we shall simply compute Divisia indices for firms and households together and take them as Divisia monetary aggregates for the whole economy, although we distinguish between firms and households in calculating user costs.

8. As to the incorporation of new instruments in the index, especially CDs in 1979/II, the rate of increase in the Fisher-ideal index is used as a proxy for the rate of increase in Divisia index for the period 1979/I-1979/II alone, and we obtain an unbroken time series for Divisia M_2+CDs and Divisia M_3+CDs. (For the details, see Appendix 1).
As the difference between Divisia index and Fisher-ideal index is very small (their Taylor expansions only differ in terms higher than the second order), such an approximation is satisfactory in practice, and the same method can be used when the need to incorporate new financial assets into the aggregates arises in the future.

Table 1 Data Used in Computing Divisia Monetary Aggregates

Components of money	Quantity	Interest rate
(1) Currency	q_1 : cash currency in money supply statistics	r_1 : no interest (0)
(2) Current deposits	q_2 : (deposit money in money supply statistics) x (share of current deposits)* *(share of current deposits) = (total of current deposits with all banks, mutual loan and savings banks, credit associations)/(total of demand deposits with these institutions)	r_2 : no interest (0)
(3) Ordinary deposits	q_3 : (deposit money in money supply statistics) x (share of ordinary deposits)* *(share of ordinary deposits) = (total of ordinary deposits with all banks, mutual loan and saving banks, credit associations)/(total of demand deposits with these institutions)	r_3 : interest rate on ordinary deposits (upper limit of guide-line rate)
(4) Deposits at notice	q_4 : (deposit money in money supply statistics) x (share of deposits at notice)* *(share of deposits at notice) = (total of deposits at notice with all banks, mutual loan and saving banks, credit associations)/(total of demand deposits with these institutions)	r_4 : interest rate on deposits at notice (as above)
(5) Corporate time deposits	q_5 : corporate quasi-money in money supply statistics	r_5 : interest rate on time deposits (3 months, as above)
(6) Personal time deposits	q_6 : personal quasi-money in money supply statistics	r_6 : interest rate on time deposits (2 years, as above) as r'_6 $r_6 = [(1+r'_6/2)^4-1]/2$ – (yield curve adjustment)
(7) CDs	q_7 : negotiable deposits in money supply statistics	r_7 : interest rate on CDs (less thatn 120 days)
(8) Postal savings	q_8 : balance of postal savings in flow of funds accounts	r_8 : interest rate on time deposits (3 years or more) as r'_8 $r_8 = [(1+r'_8/2)^{10}-1]/5$ – (yield curve adjustment)
(9) Money trusts	q_9 : outstanding principal of money trusts in trust accounts of all banks	r_9 : interest rate on money trusts (5 years, upper limit of guide-line rate) as r'_9 $r_9 = [(1+r'_9/2)^{10}-1]/5$ – (yield curve adjustment)
(10) Loan trusts	q_{10} : outstanding principal of loan trusts in trust accounts of all banks	r_{10} : interest rate on loan trusts (5 years, as above) as r'_{10} $r_{10} = [(1+r'_{10}/2)^{10}-1]/5$ – (yield curve adjustment)
(11) Other components of M3	q_{11} : (M3 in money supply statistics) $- \sum_{i=1}^{10} q_i$	r_{11} : $r_{11} = r_6$
Benchmark rate R_t		R_t^f (R_t for firms) = max $[r_b, r_2, r_4, r_5, r_7]$ R_t^h (R_t for households) = max $[r_b, r_1, r_3, r_6, r_8, r_9, r_{10}, r_{11}]$ r_b = yield in secondary market of Electric Power Bonds of the longest maturity

1) all interest rates are annual rates 2) all data refer to end of quarter

corresponding duals. (For the details about the data, see Table 1.)

Some might disagree on the choice of the data, especially when it comes to interest rates. However the difference among Divisia monetary aggregates computed with interest rates adopted here and those with some alternative interest rates is very small; in this sense, Divisia monetary aggregates are relatively robust to the choice of interest rates. As regards the selection of interest rates, let us briefly explain the following points:

(1) Treatment of interest rates on such assets as postal savings and loan trusts

Household-oriented financial assets such as postal savings and loan trusts are usually held for a long period of time; households take long-term holding as a premise when deciding their portfolio. It is therefore insufficient to use nominal regulated interest rates as their rate of return. In the case financial assets are held to maturity, we first calculate yields to maturity using compound rates and then translate them into annual simple rates. In order to compare their interest rates with those of other financial assets with shorter maturity, we further adjust these rates using a then-prevailing yield curve to obtain rates of return on these assets.[9]

(2) The benchmark rate

Theoretically, the benchmark rate R_t of the substitute for money is defined as the expected period holding rate of return (including capital gain or loss) of a pure store of value (usually a bond). However, as there are no data corresponding to such a concept, we need some proxy measures.

Barnett and Spindt [4] has shown that max $\{r_b, r_i(i=1, \cdots, 11)\}$ (r_b denotes the yield on standard-risk long-term bonds, r_i denotes the interest rate on the corresponding component of money) serves as a good approximation to R_t. However, in the case of Japan, as some components of money are held either by firms or by households alone and as portfolio behavior of firms and households differ, it is somewhat inappropriate to group these assets together and to use the maximum value above as the benchmark rate in computing user costs. Thus, in this paper, in order to calculate the user costs of various financial assets, we classify components of money into those held by firms alone and those held by households alone as we did above, and compute the benchmark rate R_t for these two groups of

9. The rate of return obtained by the yield curve adjustment is defined as;

$$r_t^* = r_t - (r_{st} - r_{1t})$$

where r_t^* : yield curve adjusted rate of return

r_t : rate of return with a term of maturity of s years calculated at compound rates

r_{st} : yield on bank debentures with remaining maturity of s years.

r_{1t} : yield on bank debentures with remaining maturity of 1 year.

assets separately; that is, the benchmark rate for firms is

$$R_t^f = \max\{r_b, r_2, r_4, r_5, r_7\}$$

and the benchmark rate for households is

$$R_t^h = \max\{r_b, r_1, r_3, r_6, r_8, r_9, r_{10}, r_{11}\}$$

Here, r_b refers to the yield on Electric Power Bonds of the longest maturity in the secondary market and r_i's ($i = 1, ..., 11$) are the corresponding interest rates on the 11 components of money depicted above.

Chart 1 through 3 show the developments in Divisia monetary aggregates of M_1, M_2+CDs and M_3+CDs we have computed with 1974/I as the base period. Comparing them with those of the corresponding ordinary aggregates (hereafter to be called simple-sum M_1, M_2+CDs and M_3+CDs), we have found interesting relations between the range of monetary aggregation and their degree of moneyness as follows:

(i) When the range of aggregation covers only M_1, developments in Divisia M_1 and simple-sum M_1 are not significantly different (see Chart 1). This suggests that substitutability between currency and deposits composing M_1 is high. That is, there is almost no difference in their degree of moneyness.
(ii) When the range of aggregation is extended, deviation of Divisia M_2+CDs and M_3+CDs from the corresponding simple-sum M_2+CDs and M_3+CDs becomes fairly large (see Charts 2 and 3). That is, the degree of substitutability is low between the components of M_1, and the components of M_2+CDs and M_3+CDs that are excluded from M_1. Furthermore, since Divisia M_2+CDs and M_3+CDs are smaller than the corresponding simple-sum M_2+CDs and M_3+CDs, components of M_2+ CDs and M_3+CDs excluded from M_1 have lower degree of moneyness than components of M_1.

III. Developments in the Velocities of Divisia Monetary Aggregates

In this section we explore the relations between Divisia monetary aggregates and nominal GNP. Chart 4 through 6 show the developments in the velocities of money, which are calculated by converting nominal GNP into an index with the same base period of 1974/I. Comparing them with those of the corresponding simple-sum aggregates, we have found that the relations between money and nominal GNP are more stable when we use Divisia monetary aggregates. The main findings are as follows:

Chart 1 Developments in Divisia and Simple-sum M_1

(1974/I = 1.0)

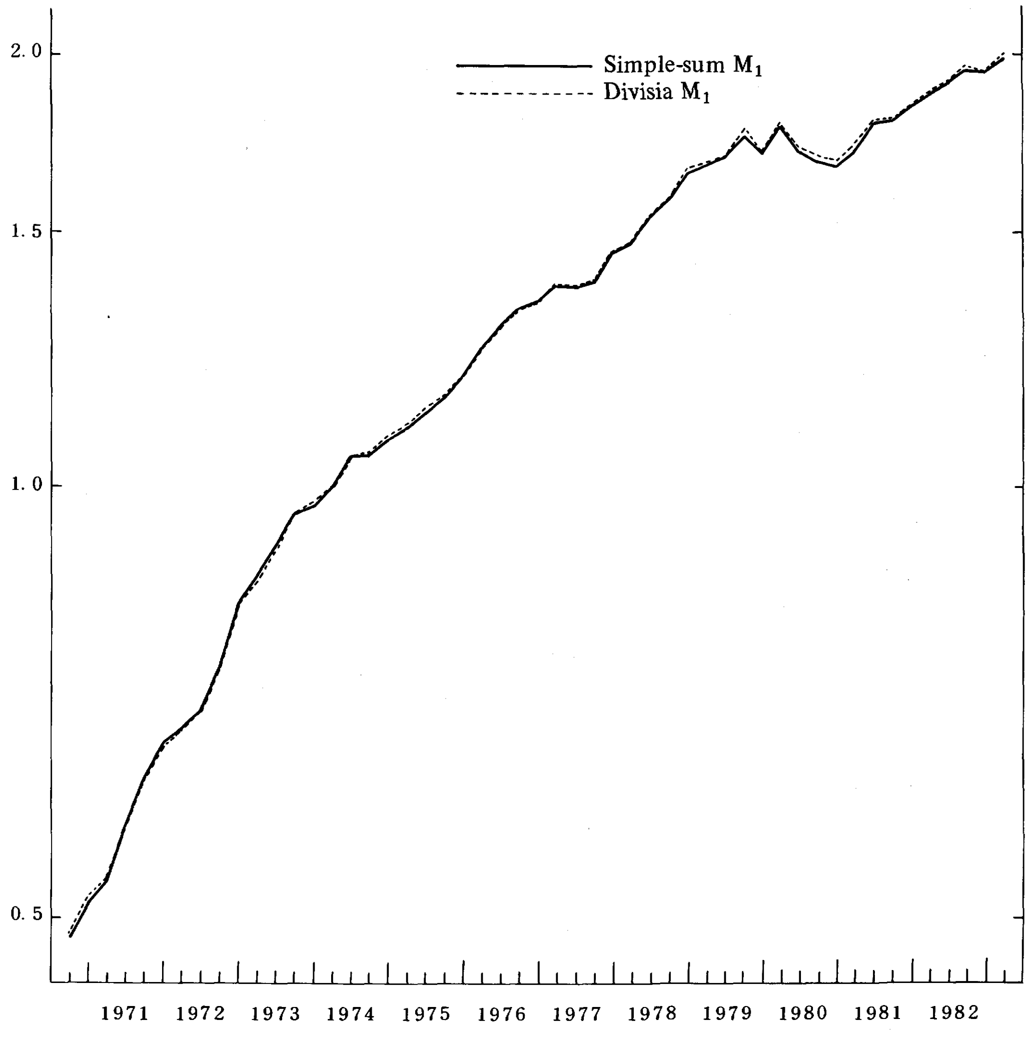


Chart 2 Developments in Divisia and Simple-sum M_2+CDs

(1974/I = 1.0)

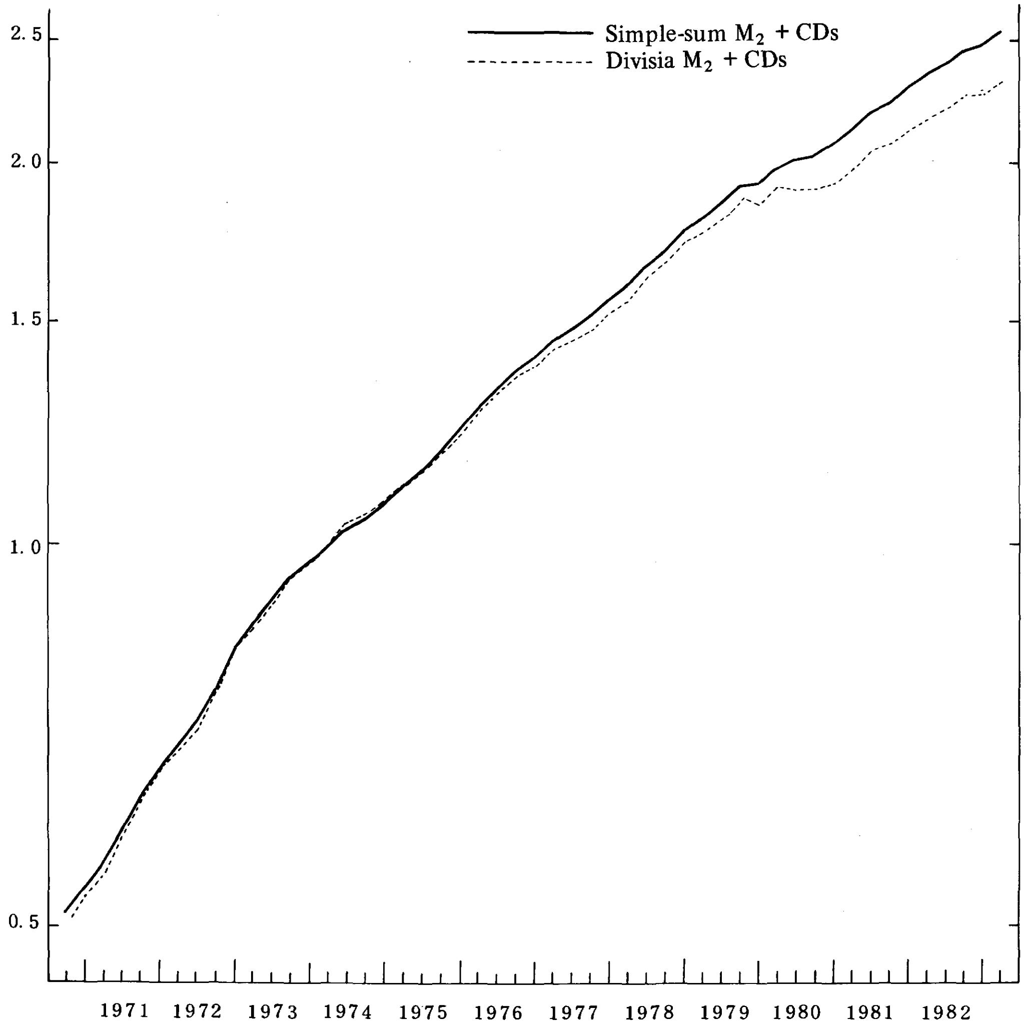
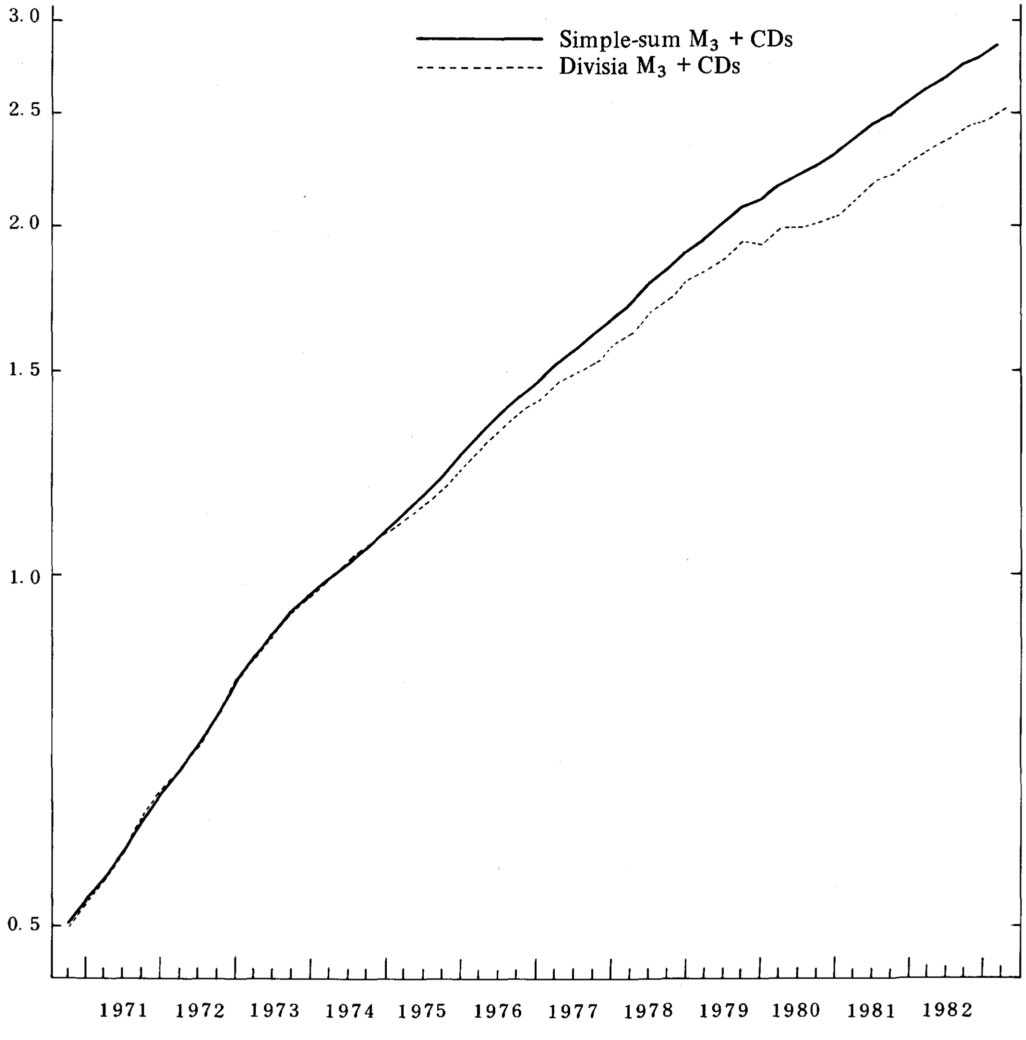


Chart 3 Developments in Divisia and Simple-sum M_3+CDs

(1974/I = 1.0)



(i) The velocities of M_1, measured both in terms of Divisia M_1 and simple-sum M_1, have been increasing consistently since 1974 (see Chart 4). This suggests that the economization of M_1 started in 1974 and continued thereafter.
(ii) While the velocity of simple-sum M_2+CDs has clearly been declining, this trend can barely be observed in the case of Divisia M_2+CDs. Such a declining trend in simple-sum M_2+CDs has often been referred to as the case of increasing Marshallian k and many reasons have been pointed out to explain this phenomenon. Our results suggest that a large part of this phenomenon is in fact due to the increase in the share of assets with low degree of moneyness.
(iii) While the velocity of simple-sum M_3+CDs has shown a strong downward trend, the declining trend in Divisia M_3+CDs has been less evident.[10]

We now take a closer look at the difference in the trends between Divisia monetary aggregates and simple-sum monetary aggregates from the viewpoint of the difference in the degree of moneyness among various components of money.

Suppose that there is a portfolio shift from components of M_1 to components of M_2+CDs or M_3+CDs not included in M_1 due to the shift in the preference for higher interest rate assets, or due to technological innovation by financial institutions. Then a shift of one unit of M_1 to other components of M_2+CDs or M_3+CDs would lower the total quantity of moneyness, since assets included in M_2+CDs or M_3+CDs but not in M_1 carry lower degree of moneyness. Therefore, in order to keep the total quantity of moneyness constant, components of M_2+CDs or M_3+CDs other than those included in M_1 have to increase by more than one unit.[11] In this case, although the total quantity of moneyness remained unchanged, ordinary simple-sum M_2+CDs or M_3+CDs increases and this tends to lower the velocity. The same can be said when there is a portfolio shift from assets included in M_2+CDs to other components of M_3+CDs not included in M_2+CDs.

As can be seen from Chart 4 through 6, the patterns of cyclical fluctuations in the

10. In their empirical studies on the velocities in the United States using Divisia monetary aggregates, Barnett [2] [3], and Barnett and Spindt [4] found that the velocities of M_2 and M_3 had shown no declining trend (rather the velocity of M_2 had shown an upward trend). On the other hand, in the Japanese case, using Divisia monetary aggregates does not eliminate the downward trend in the velocities altogether. As is frequently pointed out, this may be due to the fact that the household sector is still accumulating financial assets at a faster rate than GNP growth rate (which also suggests that GNP may not be a good proxy for total wealth). However, our results show that such downward trends are not as sharp as has been generally thought.

11. In fact, as portfolio shifts occur, user costs of money, and thus the total quantity of moneyness also changes (a kind of income effect). In order to see the effects of portfolio shifts while taking their impact on user costs into consideration, it is necessary to estimate a money demand function that takes user costs into account. We turn to this in the next section.

Chart 4 Developments in the Velocities of Divisia and Simple-sum M_1

(1974/I = 1.0)

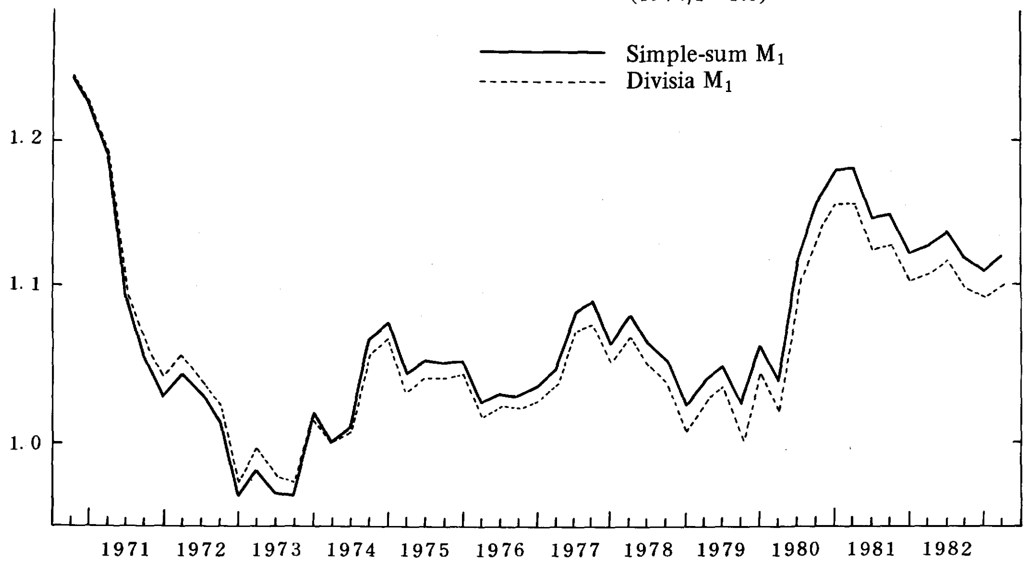


Chart 5 Developments in the Velocities of Divisia and Simple-sum M_2+CDs

(1974/I = 1.0)

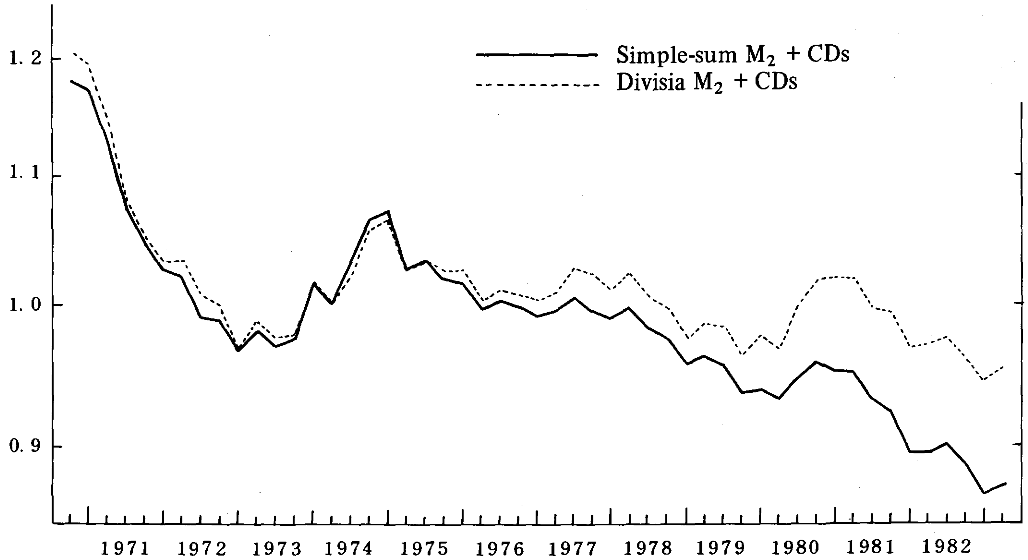
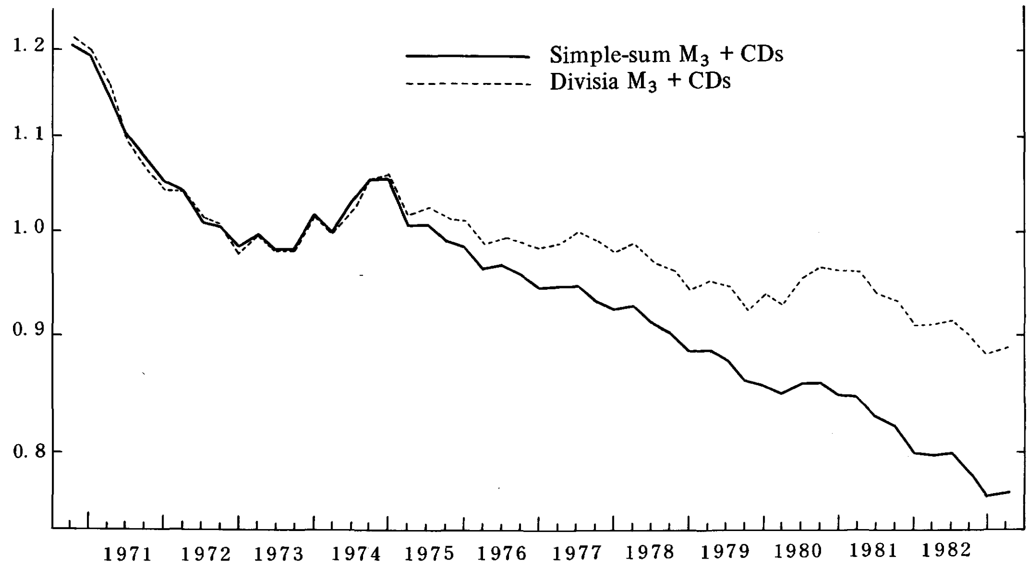


Chart 6 Developments in the Velocities of Divisia and Simple-sum M_3+CDs

(1974/I = 1.0)



velocity of money along the trend are similar for both Divisia and simple-sum monetary aggregates. Therefore, using simple-sum monetary aggregates as indicators for the conduct of monetary policy in the short run would not cause too much of a problem. However, in view of the long-run stability of price, it is more desirable to stabilize the rate of increase in Divisia monetary aggregares whose relations with nominal GNP are more stable.

IV. Estimation of the Money Demand Functions for Divisia Monetary Aggregates

In order to examine the problem related to the determination of the total quantity of moneyness, we formulate the money demand functions using Divisia monetary aggregates, taking into account the effects of changes in interest rates or user costs which we have ignored in the last section. We also estimate these functions and use them for extrapolation tests. These results are then compared with those of simple-sum monetary aggregates.

(a) Formulation and the results of estimation

The money demand functions to be estimated here take the following two forms. (For the theoretical basis of such formulations, see Appendix 2.)

(i) Ordinary type of money demand function used by Goldfeld [12].

$$\log(M/P) = \alpha_0 + \alpha_1 \log(M/P)_{-1} + \alpha_2 \log(GNP/P) + \alpha_3 \log R + \alpha_4 \log r \quad (1)$$

where,

- M: Divisia or simple-sum M_1, M_2+CDs, and M_3+CDs
- GNP: Nominal GNP
- P: GNP deflator
- R: The interest rate on the substitute for money. Instead of the commonly used call rate or Gensaki (RPs) rate, the benchmark rate used in computing the Divisia index is adopted here in conformity with the theory of Divisia monetary aggregates. For the sake of convenience, it is taken as the average of the firms' benchmark rate R^f and the households' benchmark rate R^h .
- r: "Own rate," that is, the interest rate on money. For M_1, it is taken to be 0 (excluded from the formulation) and for M_2 +CDs and M_3+CDs, it is taken as the interest rate on time deposits and fixed-amount postal savings respectively.

(ii) Money demand function formulated using Divisia price (user cost) index.

$$\log(M/P) = \alpha_0 + \alpha_1 \log(M/P)_{-1} + \alpha_2 \log(GNP/P) + \alpha_3 \log(PM) \quad (2)$$

where,

- M: Divisia M_1, M_2+CDs, and M_3+CDs
- PM: Divisia price (user cost) index obtained as the dual of the corresponding Divisia monetary aggregate.

Table 2 shows the results of estimating Divisia and simple-sum money demand functions using equation (1) and Divisia money demand functions using equation (2) for M_1, M_2+CDs, and M_3+CDs. The Beach and Mackinnon [5] method which assumes first-order serial correlation in error terms is used and the period of estimation corresponds to the whole period for which Divisia monetary aggregates have been computed (1970/IV - 1983/I).

When comparing the results of estimating the money demand functions using Divisia monetary aggregates with those using simple-sum aggregates, we note that the money demand functions using Divisia monetary aggregates are superior to those using simple-sum aggregates with respect to the stability of relations with GNP.

(i) Although they do not differ significantly in terms of the goodness of fit, the fit is somewhat better when Divisia monetary aggregates are used. (That is, the coefficients of determination of the M_2+CDs demand function are 0.9908 for simple-sum monetary aggregates and 0.9929 and 0.9931 for Divisia monetary aggregates using equations (1) and (2) respectively.)
(ii) In the sense that long-run GNP elasticities $\alpha_2/(1-\alpha_1)$of the demand for Divisia M_1, M_2+CDs, and M_3+CDs come closer to unity than those of the corresponding simple-sum monetary aggregates, the theoretical meaning of the Divisia money demand functins can be interpreted more easily. (That is, in the case of M_2+CDs, long-run elasticities are 1.216 for simple-sum monetary aggregates and 1.048 and 1.058 respectively for Divisia monetary aggregates using equation (1) and (2).)
(iii) In the case of M_2+CDs and M_3+CDs, the parameters of the GNP term are evidently more significant when Divisia monetary aggregates are used. (That is, the t-values of the parameter of the GNP term in the M_3+CDs demand function are 1.4188 in the simple-sum case, while 2.5128 and 3.3655 respectively in the Divisia case when equation (1) and (2) are used.)

These results show that the observations regarding the velocities of money still hold when the effects of interest rates or user costs are taken into consideration. Furthermore, the difference between these two sets of monetary aggregates becomes more and more evident when the range of aggregation is extended from M_1 to M_2+CDs and then to M_3+CDs. Thus when the need to use more broadly defined monetary aggregates arises in the future as financial innovation progresses, it is more desirable to use aggregates with a micro-economic foundation such as Divisia monetary aggregates than simple-sum monetary aggregates.

(b) The results of the extrapolation tests and the stability of the money demand functions

In order to examine the stability of the above money demand functions, we divide the period of estimation into 1970/IV-1977/I and 1977/II-1983/I and estimate money demand functions for these subperiods separately.[12] The results are shown in Tables 3 and 4.

12. We have tried extrapolation tests for different estimation periods using various Divisia money demand functions and we note that in all cases, predicted values roughly follow actual values up to around 1977/I and deviate from them thereafter. Therefore we divide the estimation period into two using 1977/I as the dividing point.

Table 2 Results of Estimating the Divisia and Simple-sum Money Demand Functions (1)

(Estimation Period: 1970/IV – 1983/I)

Dependent variable	Explanatory variables						R^2 (S.E.)	D.W. (ρ)	Long-run GNP elasticity	Long-run interest elasticity
	CONSTANT	LAG(-1)	GNP	R	r	user cost				
Divisia M_1	0.1620 (4.7718)	0.8359 (17.661)	0.1052 (2.1542)	−0.0702 (−4.5884)			0.9874 (0.0207)	2.0001 (−0.1910)	0.641	−0.428
Divisia M_1	−0.0106 (−1.8082)	0.8469 (17.934)	0.0955 (1.9650)			−0.0727 (−4.6719)	0.9877 (0.0206)	2.0084 (−0.2066)	0.624	−0.475
Simple-sum M_1	0.1709 (4.6724)	0.8250 (16.813)	0.1051 (2.1711)	−0.0748 (−4.5210)			0.9833 (0.0213)	1.9859 (−0.1223)	0.600	−0.427
Divisia M_2 + CDs	0.1356 (4.4676)	0.7855 (12.219)	0.2247 (2.8275)	−0.0822 (−2.7311)	0.0284 (0.8626)		0.9929 (0.0148)	2.0031 (0.0912)	1.048	−0.383
Divisia M_2 + CDs	−0.0063 (−1.1317)	0.7814 (12.309)	0.2313 (2.9487)			−0.0757 (−4.3401)	0.9931 (0.0148)	1.9997 (0.0744)	1.058	−0.254
Simple-sum M_2 + CDs	0.1244 (3.5127)	0.8171 (10.174)	0.2244 (2.0010)	−0.0830 (−2.9384)	0.0372 (1.1729)		0.9908 (0.0126)	2.1867 (0.4122)	1.216	−0.453
Divisia M_3 + CDs	0.1233 (4.0888)	0.7819 (10.453)	0.2627 (2.5128)	−0.0347 (−1.2233)	−0.0187 (−0.8669)		0.9949 (0.0137)	2.0043 (0.1178)	1.204	−0.159
Divisia M_3 + CDs	−0.0056 (−1.0044)	0.7476 (10.811)	0.3211 (3.3655)			−0.0516 (−4.3919)	0.9950 (0.0137)	2.0168 (0.1037)	1.257	−0.202
Simple-sum M_3 + CDs	0.1170 (3.4478)	0.8626 (10.261)	0.2015 (1.4188)	−0.0265 (−0.9133)	−0.0208 (−0.9168)		0.9942 (0.0113)	2.2242 (0.4566)	1.467	−0.193

Table 3 Results of Estimating the Divisia and Simple-sum Money Demand Functions (2)

(Estimation Period: 1970/IV – 1977/I)

Dependent variable	Explanatory variables						R^2 (S.E.)	D.W. (ρ)	Long-run GNP elasticity	Long-run interest elasticity
	CONSTANT	LAG(-1)	GNP	R	r	user cost				
Divisia M_1	0.2355 (4.1368)	0.7681 (8.9360)	0.2736 (1.9161)	−0.1043 (−4.0502)			0.9780 (0.0187)	1.9500 (0.0015)	1.180	−0.450
Divisia M_1	−0.0178 (−2.0567)	0.7805 (9.0261)	0.2458 (1.7234)			−0.0929 (−3.9977)	0.9777 (0.0187)	1.9524 (0.0018)	1.120	−0.423
Simple-sum M_1	0.2675 (4.3021)	0.7663 (9.0995)	0.2635 (1.8853)	−0.1191 (−4.2048)			0.9726 (0.0366)	1.8797 (0.0732)	1.128	−0.510
Divisia M_2 + CDs	0.2200 (4.4920)	0.7628 (9.9042)	0.2761 (2.1778)	−0.1478 (−4.1892)	0.0611 (1.3137)		0.9862 (0.0133)	1.8834 (0.0806)	1.164	−0.623
Divisia M_2 + CDs	−0.0183 (−2.9445)	0.7155 (10.008)	0.3680 (3.2160)			−0.0980 (−6.2018)	0.9884 (0.0127)	1.9066 (0.0344)	1.276	−0.340
Simple-sum M_2 + CDs	0.2274 (5.0714)	0.7489 (10.211)	0.3008 (2.5332)	−0.1682 (−5.5825)	0.0816 (2.0384)		0.9989 (0.0111)	1.8667 (0.1420)	1.198	−0.607
Divisia M_3 + CDs	0.4366 (4.9279)	0.5339 (5.1300)	0.6801 (4.0592)	−0.3318 −3.5200)	0.1421 (2.4375)		0.9915 (0.0118)	1.9552 (0.0047)	1.459	−0.712
Divisia M_3 + CDs	−0.0173 (−2.8890)	0.6772 (9.0060)	0.4484 (3.57728)			−0.0934 (−6.4438)	0.9910 (0.0121)	1.9280 (0.0141)	1.389	−0.728
Simple-sum M_3 + CDs	0.3676 (3.8064)	0.6332 (4.8368)	0.5729 (2.5664)	−0.2496 (−2.5792)	0.0918 (1.5618)		0.9860 (0.0115)	2.1460 (0.3578)	1.562	−0.680

Table 4 Results of Estimating the Divisia and Simple-sum Money Demand Functions (3)

(Estimation Period: 1977/II - 1983/I)

Dependent variable	Explanatory variables						R^2 (S.E.)	D.W. (ρ)	Long-run GNP elasticity	Long-run interest elasticity
	CONSTANT	LAG(-1)	GNP	R	r	user cost				
Divisia M_1	0.1862 (5.0670)	0.9635 (11.255)	0.1173 (1.7622)	−0.0957 (−4.7998)			0.9633 (0.0194)	2.2017 (−0.5814)	3.213	−2.622
Divisia M_1	−0.0570 (−3.2280)	1.0207 (11.845)	0.0831 (1.3033)			−0.1166	0.9662 (0.0193)	2.3334 (−0.6320)	−4.014	5.633
Simple-sum M_1	0.1871 (4.9824)	0.9497 (11.066)	0.1308 (1.9956)	−0.0971 (−4.7928)			0.9581 (0.0196)	2.1634 (−0.5615)	2.600	−1.930
Divisia M_2 + CDs	0.0419 (3.5660)	0.7369 (5.8924)	0.3694 (2.3821)	0.0016 (0.3774)	−0.0123 (−1.8453)		0.9943 (0.0110)	2.0727 (−0.3803)	1.404	0.001
Divisia M_2 + CDs	0.0510 (3.1333)	1.0072 (9.1012)	0.0248 (0.1892)			−0.0618 (−3.1669)	0.9926 (0.0132)	1.9796 (−0.4807)	−3.444	8.583
Simple-sum M_2 + CDs	0.0243 (2.3118)	0.7836 (8.4164)	0.3680 (2.5301)	−0.0001 (−0.0487)	−0.0067 (−1.6034)		0.9978 (0.0080)	2.1094 (−0.3140)	1.701	0.001
Divisia M_3 + CDs	0.0886 (5.7333)	0.8180 (11.727)	0.3231 (3.1532)	−0.0775 (−5.3023)	0.0310 (2.4900)		0.9979 (0.0084)	2.5154 (−0.6315)	1.775	−0.425
Divisia M_3 + CDs	−0.0157 (−1.6424)	0.9637 (11.888)	0.1033 (0.9227)			−0.0487 (−4.4604)	0.9968 (0.0100)	2.2421 (−0.5995)	2.862	−1.342
Simple-sum M_3 + CDs	0.0576 (5.1716)	0.8742 (15.558)	0.2665 (2.5578)	−0.0491 (−4.8852)	0.0230 (2.7130)		0.9993 (0.0057)	2.2940 (−0.5243)	2.118	−0.390

As can be seen from these results, although we fail to obtain stable results in some cases because the estimation period is too short (for example, Divisia M_1 and M_2+CDs demand functions taking the form of equation (2) in Table 4), as a whole, the superiority of Divisia money demand functions in their relation to GNP still holds for both estimation periods.

However, a comparison between the results of estimation for these two periods shows that the parameters and long-run elasticities have changed significantly. This suggests that both Divisia and simple-sum money demand functions have probably changed structurally around 1977.

In order to test explictly whether the money demand functions have changed and if so, in what way they have changed, Divisia M_1, M_2+CDs and M_3+CDs demand functions estimated for the period 1970/IV - 1977/I (see Table 3) are extrapolated to yield predicted values for the period 1977/II - 1983/I.[13] Chart 7 through 9 show the predicted values so obtained as well as the actual values. The results of similar extrapolation for simple-sum aggregates are also included for comparison. The following points are worth noting;

(i) In the case of M_1 (Chart 7), since 1977/II predicted values (extrapolation by dynamic simulation is used here, as in what follows) greatly exceed actual values for both Divisia and simple-sum money demand functions (in the terminology of Goldfeld [12], the missing money has occurred) and this suggests that both functions have changed around 1977.

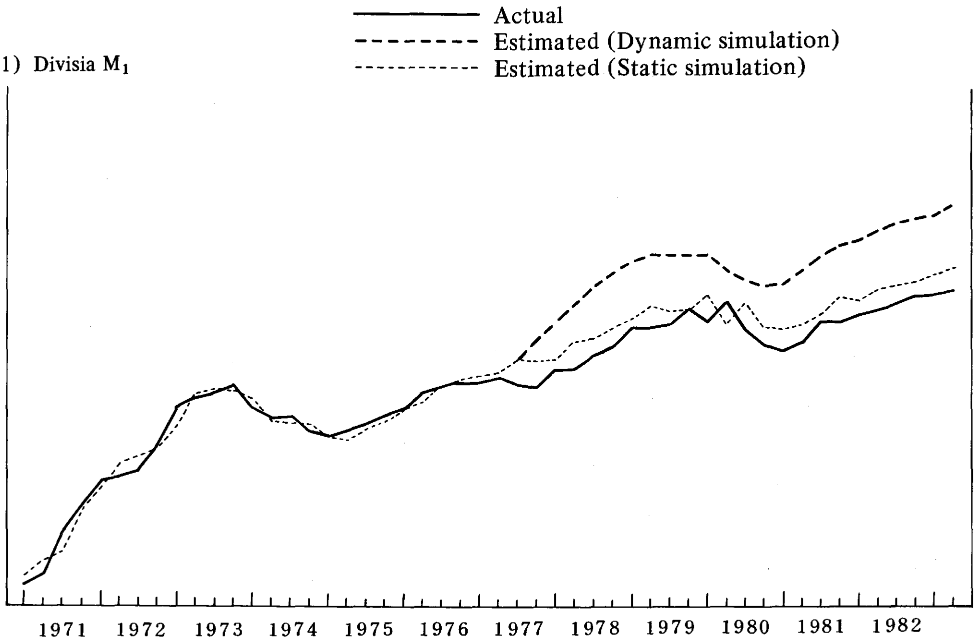
(ii) In the case of M_2+CDs (Chart 8), predicted values obtained by extrapolating the simple-sum money demand function exceed actual values in 1978-79 and this relation has been reversed since 1980. Thus the simple-sum M_2+CDs function has become unstable since 1977. On the other hand, the results of extrapolating the Divisia M_2+CDs demand function show that from 1978 to 1979, over-prediction of a large magnitude occurred as in the case of the simple-sum demand function. Thereafter predicted values followed actual values relatively well, although over-prediction (missing money) of almost constant magnitude remained.[14] It may be

13. When applying the single-equation extrapolation test, it is desirable that the explanatory variables are exogenous to the dependent variable (that is, there is no feedback from the dependent variable). It would be inconsistent to use the price index, which is determined simultaneously as the dual of Divisia quantity index, as an explanatory variable in the money demand functions that take the form of equation (2). Therefore, extrapolation tests here are carried out using a function in the form of equation (1).

14. We note that this phenomenon is consistent with the fact that the large portfolio shift from bank deposits to postal savings since 1980 has lowered the demand for M_2+CDs. This is also evident from the fact that in the case of Divisia M_3+CDs, over-prediction since 1980 is smaller than that of Divisia M_2+CDs.

Chart 7 Results of Extrapolating the M_1 Demand Functions

(1) Divisia M_1



(2) Simple-sum M_1

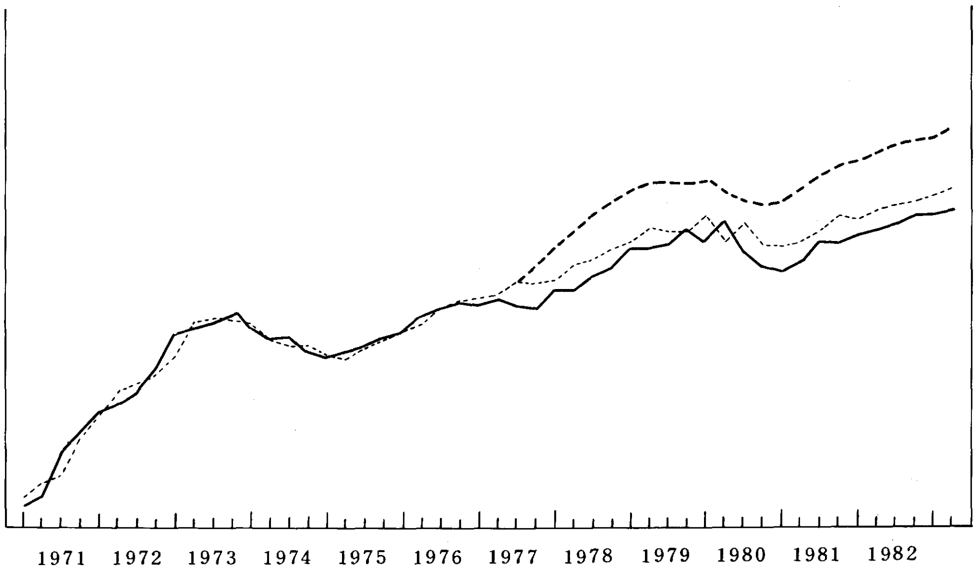
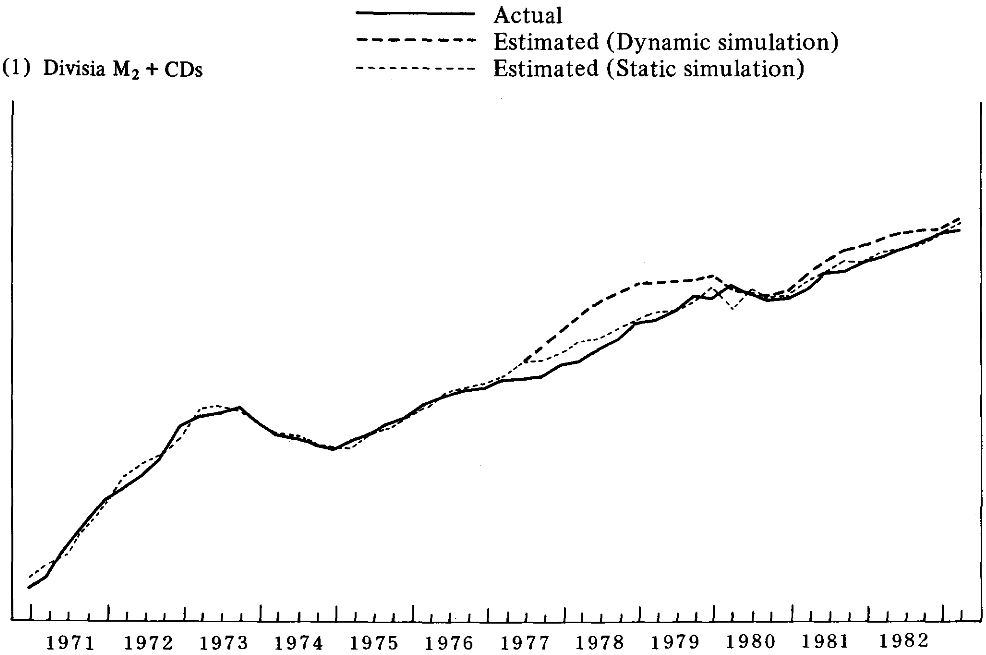


Chart 8 Results of Extrapolating the M_2+CDs Demand Functions

(1) Divisia M_2 + CDs



(2) Simple-sum M_2 + CDs

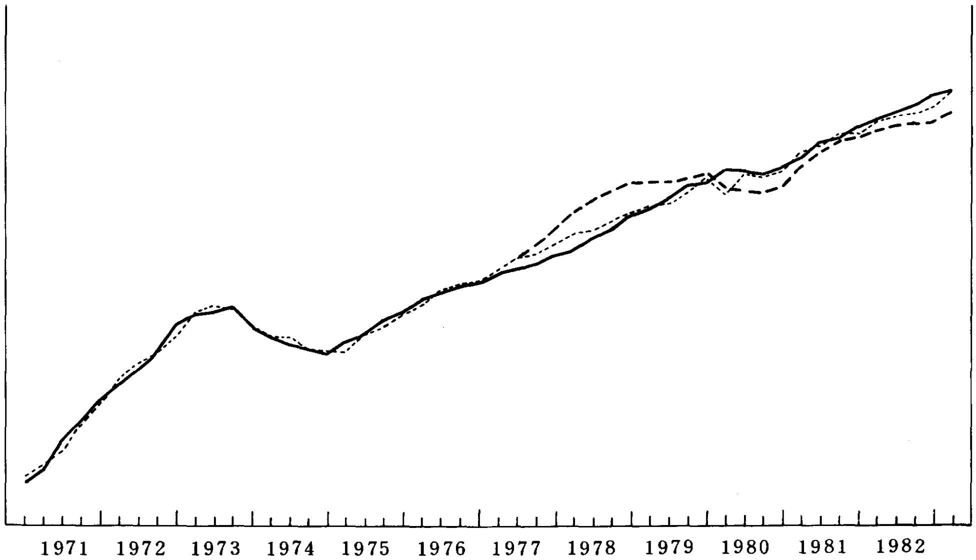
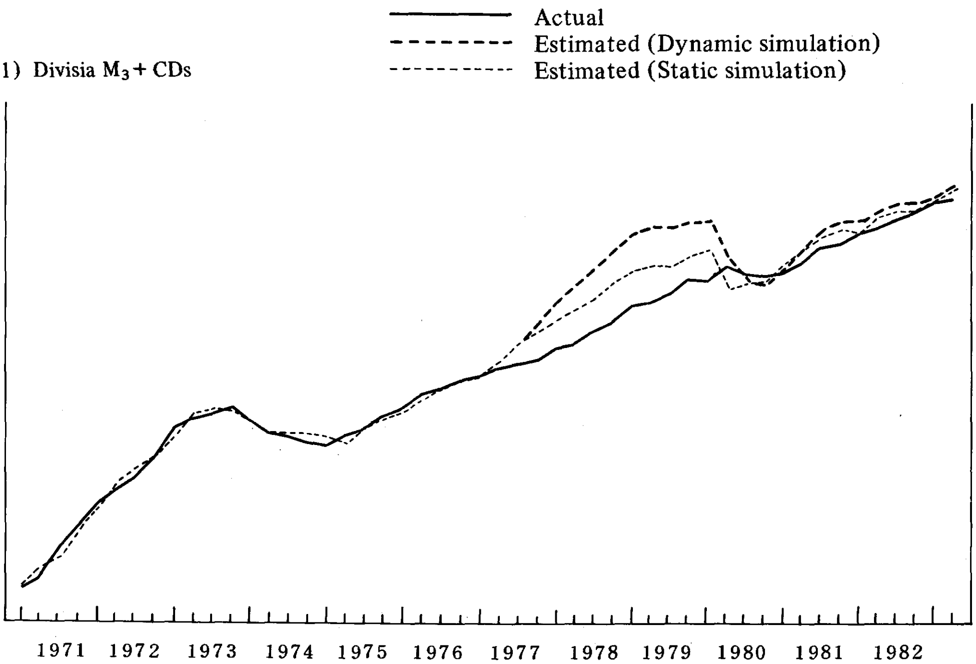
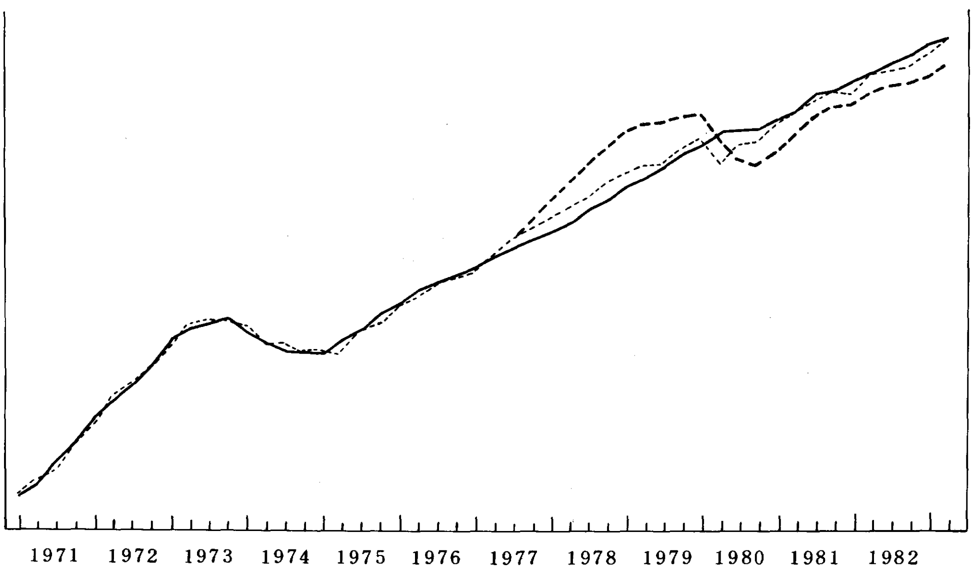


Chart 9 Results of Extrapolating the M_3+CDs Demand Functions

(1) Divisia M_3 + CDs



(2) Simple-sum M_3 + CDs



said that Divisia M_2+CDs demand function is more stable than simple-sum M_2+CDs.

(iii) The results of extrapolating the M_3+CDs demand function (Chart 9) are similar to those of M_2+CDs. While the simple-sum M_3+CDs demand function is more unstable than that of simple-sum M_2+CDs and the magnitude of under-prediction after 1980 is larger, over-prediction after 1980 in Divisia M_3+CDs is smaller in magnitude than in Divisia M_2+CDs.

As depicted in the last section, these differences between Divisia money demand functions and simple-sum money demand functions result from the fact that, when portfolio shifts among financial assets occur (especially when they involve shifts to financial assets that have lower substitutability with pure money), ordinary simple-sum monetary aggregates broadly defined (M_2+CDs, M_3+CDs, etc.) tend to be biased upward when they are used to measure the total quantity of moneyness. The results of the extrapolation tests support our contention that Divisia monetary aggregates are more suitable ones to use, especially when we consider broadly defined monetary aggregates.

V. Conclusions

In this paper, we have applied the idea of Divisia monetary aggregates to the case of Japan. As the results of empirical analysis on the velocities of money and money demand functions suggest, many phenomena concerning the existing monetary aggregates which are difficult to explain are the result of computing monetary aggregates using simple-sum methods. Therefore, aggregates with a micro-economic foundation such as Divisia monetary aggregates discussed in this paper are useful in grasping accurately the total quantity of moneyness for the economy as a whole and they supplement traditional simple-sum monetary aggregates as indicators or intermediate targets for the conduct of monetary policy.

In Japan, as financial innovation progresses, it is expected that portfolio shifts among existing financial assets and shifts to new financial assets will become more and more active in the future and this will make it necessary to enlarge the range of financial assets to be included in money. Under such a situation, if existing simple-sum monetary aggregates, especially broadly defined monetary aggregates, which include financial assets that have only lower substitutability with pure money, continue to be used, problems discussed so far will probably become more and more serious and the meaning of these aggregates will also become more ambiguous. There is even a possibility that simple-sum aggregates become incapable of performing their role as indicators or intermediate targets for the conduct of monetary policy.

On the other hand, in the case of Divisia monetary aggregates, as the degree of moneyness of different financial assets including new ones is reflected in these aggregates, the broader the range of aggregation is, the more accurately they reflect the total quantity of moneyness for the whole economy and the more stable these aggregates become. In the course of financial innovation described above, the usefulness of these aggregates will be increased.

Of course, in order for Divisia monetary aggregates to be used not only as indicators but also as intermediate targets, it is necessary to examine in detail, both theoretically and empirically, problems such as the controllability of these aggregates and their causality with ultimate targets (such as GNP), in addition to analyzing the velocities of money and the money demand functions as we have done in this paper. These remain as topics of further research.

APPENDIX 1

Theoretical Background of Divisia Monetary Aggregates

Statistical Index Number Theory

Given the quantity vectors and price vectors of the goods to be aggregated in the previous and current period (hereafter referred to as period 0 and period 1 respectively) $\mathbf{q}_0$, $\mathbf{q}_1$, $\mathbf{p}_0$ and $\mathbf{p}_1$, statistical index numbers express the aggregates of these quantities Q_1 and prices P_1 in the current period in the form of indices as functions of $\mathbf{q}_0$, $\mathbf{q}_1$, $\mathbf{p}_0$ and $\mathbf{p}_1$. That is

$$Q_1 = Q(\mathbf{q}_0, \mathbf{q}_1; \mathbf{p}_0, \mathbf{p}_1) : \text{statistical quantity index} \tag{A-1}$$

$$P_1 = P(\mathbf{p}_0, \mathbf{p}_1; \mathbf{q}_0, \mathbf{q}_1) : \text{statistical price index} \tag{A-2}$$

Statistical index number theory seeks to provide a theoretical basis for these statistical index numbers by relating them to utility and production functions, and conversely to approximate the utility and production functions implicit in the optimizing behavior of agents using such statistical index numbers. For an aggregator function $f(\mathbf{q}_t)$, a statistical quantity index Q_t that satisfies

$$f(\mathbf{q}_t)/f(\mathbf{q}_0) = Q_t = Q(\mathbf{q}_0, \mathbf{q}_t; \mathbf{p}_0, \mathbf{p}_t) \tag{A-3}$$

for all t is said to be *exact* for the aggregator function.

For any arbitrary linear homogeneous aggregator function, if there exists a statistical index that is exact for it, then aggregates reflecting the underlying utility (production) function can be derived from price and quantity information alone, even when information about the aggregator function is totally unknown.

In reality, when q_t、p_t are discrete, such a statistical index does not exist,[1] and in this case a statistical index known as *Diewert superlative* can be used as an approximation for this purpose. A statistical index is said to be Diewert superlative if it is exact for a function $g(q_t)$ that is capable of providing a second order approximation to an arbitrary linear homogeneous aggregator function $f(q_t)$ (that is, their respective Taylor expansions only differ in terms higher than the second order), then aggregates reflecting the utility function or production function approximately can be derived, even if information about the form of the aggregator function and parameters in it is unknown.

Diewert [9] showed that if producers and consumers behave optimally as ordinary price theory postulates, the following two kinds of statistical indices are superlative.

(i) Fisher-ideal index

$$\begin{aligned} Q_1^F &= Q^F(q_0, q_1; p_0, p_1) \\ &= (Q_1^{La} \cdot Q_1^{Pa})^{1/2} \\ &= \{ (q_1' p_0 / q_0' p_0) \cdot (q_1' p_1 / q_0' p_1) \}^{1/2} \end{aligned} \tag{A-4}$$

This index was proposed by Bowley [7] and Fisher [10]. It takes the form of a geometric mean of the Laspeyres index and Paasche index(equation (A-4)). Frisch [11] and Wald [18] showed that, given the optimizing behavior of economic agents, the Fisher-ideal index is exact for aggregator functions that are represented by the square root of a quadratic form

$$f(q_t) = \left(\sum_{j=1}^{N} \sum_{k=1}^{N} q_t^j \, r_{jk} \, q_t^k \right)^{\frac{1}{2}} \qquad (r_{jk} = r_{kj}) \tag{A-5}$$

As the quadratic aggregator function can provide a second-order approximation to an arbitrary linear homogeneous aggregator function, the Fisher-ideal index is Diewert superlative.

1. When q_t, p_t are continuous, continuous Divisia index depicted in Footnote 2 satisfies this property.

(ii) Törnqvist-Theil Divisia index

$$Q_1^D = Q^D(q_0, q_1; p_0, p_1) = \prod_{n=1}^{N} (q_1^n/q_0^n)^{\frac{1}{2}(s_1^n+s_0^n)} \tag{A-6}$$

or

$$\log Q_1^D = \sum_{n=1}^{N} \frac{1}{2}(s_1^n + s_0^n)(\log q_1^n - \log q_0^n) \tag{A-6)'$$

Here

$$s_t^n = q_t^n \cdot p_t^n / \sum_{j=1}^{N} q_t^j \cdot p_t^j \quad (t=0,1)$$

This index was proposed by Törnqvist [17], Theil [16], Kloek [14]. It takes the form of a weighted average of the rate of increase in each component, using the corresponding expenditure share as weights.

Diewert [9] showed that under the same assumptions, this index is exact for the translog aggregator function.

$$\log f(q_t) = \alpha_0 + \sum_{n=1}^{N} \alpha_n \log q_t^n + \frac{1}{2}\sum_{j=1}^{N}\sum_{k=1}^{N} r_{jk} \log q_t^j \log q_t^k \tag{A-7}$$

As the translog aggregator function also provides a second-order approximation to an arbitrary linear homogeneous aggregator function, the Törnqvist-Theil Divisia index is also Diewert superlative.

As the Törnqvist-Theil Divisia index uses expenditure shares as weights, its economic meaning can be interpreted intuitively. In addition, it has the advantage of furnishing a discrete approximation of the Divisia index in the continuous case.[2] This explains why it is more widely used than the Fisher-ideal index.

For convenience, we have focused on the quantity index in the above discussion. The same analysis also applies to the price index (the dual of quantity index). For example, Divisia price index P^D is given by

$$\begin{aligned} P_1^D &= P^D(p_0, p_1; q_0, q_1) \\ &= \prod_{n=1}^{N} (p_1^n/p_0^n)^{\frac{1}{2}(s_1^n+s_0^n)} \end{aligned} \tag{A-8}$$

2. When q_t, p_t are continuous, a Divisia index

$$d \log Q_t = \sum_{n=1}^{N} S_t^n d \log q_t^n, \quad S_t^n = p_t^n \cdot q_t^n / \sum_{k=1}^{N} p_t^k \cdot q_t^k$$

is exact for an arbitrary continuous linear homogeneous aggregator function and the above Divisia index is the best statistical index.

Divisia Monetary Aggregates

Divisia monetary aggregates are obtained by applying the idea of the Törnqvist-Theil Divisia index (hereafter referred to as the Divisia index). Divisia index is a kind of Diewert superlative statistical index. In this sense, Divisia monetary aggregates can be thought of as approximate measures of the theoretically meaningful aggregate of money (the total quantity of moneyness or monetary service) that reflects the underlying utility or production function.

Likewise, Fisher-ideal monetary aggregates can be computed using the Fisher-ideal index. As Fisher-ideal index is also Dierwert superlative, Fisher-ideal monetary aggregates also give approximations to theoretically meaningful aggregates of money. Although, when compared with the Divisia index, the Fisher-ideal index has the advantage that new goods can be incorporated more easily, the Divisia index is more widely used for the reasons stated above. Therefore, in the empirical analysis in sections II to IV we have been mainly concerned with monetary aggregates based on the Divisia index, although we have also made use of the Fisher-ideal index when incorporating new goods into the aggregates.

In order to calculate the statistical quantity index, we need, in addition to the quantity vector q_t of the components, the price vector p_t as well. Therefore, when applying this concept of statistical indices to aggregating money, we have to measure the prices of money, which cannot be observed directly.

In this context, Barnett [2] viewed the components of money as sorts of durable goods and took their corresponding user costs as the prices of money. Here user costs are defined as the costs incurred in order to obtain one unit of the flow of monetary service per unit of time, which are simply the opportunity costs of holding different components of money. Specifically, the user cost of each component of money is taken as the difference between its own rate and interest rate on bond which is supposed to be a substitute for money and at the same time a pure store of value theoretically. In order to give theoretical basis to the user costs of money Barnett [1] [2] modelled the optimal behavior of a consumer as related to money holding as follows:

Let x_s : consumption vector of goods and services in period s

p_s : price vector of goods and services in period s

m_s^i : real balance of the i-th component of money held in period s

r_s^i : expected rate of return on the component of money held in period s

A_s : real bond holding in period s

R_s : expected rate of return on bond in period s

L_s : quantity of labor supplied by a consumer in period s

w_s : wage rate in period s

$p_s^* = p_s^*(p_s)$: price index of goods and services and let the planning horizon of the consumer's inter-temporal optimization problem be T periods. The optimization problem can then be written as

$$\max\ u_t(m_t, \cdots, m_{t+T}, x_t, \cdots, x_{t+T}, A_{t+T}) \qquad \text{(A-9)}$$

subject to

$$p_s' x_s = w_s L_s + \sum_{i=1}^{n} [(1+r_{s-1}^i) p_{s-1}^* m_{s-1}^i - p_s^* m_s^i]$$

$$+ [(1+R_{s-1}) p_{s-1}^* A_{s-1} - p_s^* A_s] \text{ for } \forall_s \qquad \text{(A-10)}$$

$$(m_t = (m_t^1, \cdots, m_t^n)')$$

Here the discount factor ρ_s is taken to be

$$\rho_s = \begin{cases} 1 & s = t \\ \prod_{u=t}^{s-1} (1+R_u) & t+1 \leqq s \leqq t+T \end{cases}$$

Solving (A-9) for all As and then substituting back into (A-10) recursively starting from A_{t+T}, budget constraints in (A-10) can be grouped in a single budget constraint. That is,

$$\sum_{s=t}^{t+T} (p_s'/\rho_s) x_s + \sum_{s=t}^{t+T} \sum_{i=1}^{n} [\frac{p_s^*}{\rho_s} - \frac{p_s^*(1+r_s^i)}{\rho_{s+1}}] m_s^i$$

$$+ \sum_{i=1}^{n} \frac{p_{t+T}^*(1+r_{t+T}^i)}{\rho_{t+T+1}} m_{t+T}^i + \frac{p_{t+T}^*}{\rho_{t+T}} A_{t+T} \qquad \text{(A-10)'}$$

$$= \sum_{s=t}^{t+T} (w_s/\rho_s) L_s + \sum_{i=1}^{n} (1+r_{t-1}^i) p_{t-1}^* m_{t-1}^i$$

$$+ (1+R_{t-1}) A_{t-1} p_{t-1}^*$$

As can be seen from equation (A-10)', the user cost of the i-th component of money in period s is given by

$$\pi_s^i = \frac{p_s^*}{\rho_s} - \frac{p_s^*(1+r_s^i)}{\rho_{s+1}} \qquad \text{(A-11)}$$

Therefore, the user costs of money in the current period (s=t) are

$$\pi_t^i = \frac{p_t^*(R_t - r_t^i)}{1+R_t} \tag{A-12}$$

Furthermore, when the effect of taxing interest income with tax rate τ is taken into consideration, the user costs of money become

$$\pi_t^i = \frac{p_t^*(R_t - r_t^i)(1-\tau)}{1+R_t(1-\tau)} \tag{A-12'}$$

Therefore, theoretically, when computing Divisia monetary aggregates, (A-12) or (A-12)' should be used to derive the user costs of money. However, in (A-12) and (A-12)', terms other than $(R_t - r_t^i)$ are common for all *i* and are eliminated from the numerators and denominators in equation (A-5) and equation (A-5)' which are used to compute the Divisia indices. Therefore, in computing such indices, it is sufficient to use $(R_t - r_t^i)$ as the user cost of the i-th component of money.

APPENDIX 2

The Idea of Divisia Money Demand Function

If we assume that the consumer's utility function depicted in Appendix 1 is (1) time additive $(u_t = \sum_{j=0}^{T} [U(x_{t+j}, v(m_{t+j}))\beta^j])$ and (2) linear homogeneous, then the two types of Divisia money demand function used in the text can be given the following theoretical interpretation. (Here, for the sake of simplicity, we assume that there is only one good x_t.)

Let the solution of the optimization problem represented by equations (A-9) and (A-10) in Appendix 1 be

$$m_t^*, \cdots, m_{t+T}^*, x_t^*, \cdots, x_{t+T}^*$$

Then, the following propositions concerning the optimal solution hold:

(i) For $\boldsymbol{\pi}_t = (\pi_t^1, \cdots, \pi_t^n)'$, there exists a linear homogeneous price (user cost) index $\Pi(\boldsymbol{\pi}_t)$ such that

$$\Pi(\boldsymbol{\pi}_t)\, v(m_t) = \sum_{i=1}^{n} \pi_t^i m_t^i$$

(ii) Total expenditure on goods, services and money

$$F_t^* = p_t^* x_t^* + \sum_{i=1}^{n} \pi_t^i m_t^{i*}$$

is determined as a function of $R_t, \cdots, R_{t+T}, p_t^*, \cdots, p_{t+T}^*, \Pi_t, \cdots, \Pi_{t+T}, W_t$

That is

$$F_t^* = f(R_t, \cdots, R_{t+T}, p_t^*, \cdots, p_{t+T}^*, \boldsymbol{\pi}_t, \cdots, \boldsymbol{\pi}_{t+T}, W_t)$$

(Here, p_t^* denotes the price of x_t, $\Pi_t = \Pi(\boldsymbol{\pi}_t)$, and W_t is so-called total wealth that appeared in the right-hand-side of equation (A-10) in Appendix 1.)

(iii) x_t^*, $v_t^* = v(m_t^*)$ is the solution of

$$\max_{x_t, v_t} U(x_t, v_t) \quad \text{s.t. } p_t^* x_t + \Pi_t v_t = F_t^*$$

(iv) m_t^* is the solution of

$$\max_{m_t} v(m_t) \quad \text{s.t. } \sum_{i=1}^{n} \pi_t^i m_t^i = \sum_{i=1}^{n} \pi_t^i m_t^{i*}$$

From (iii), $v_t^* = v(m_t^*)$ is expressed as a function of Π_t / p_t^* and F_t^*/p_t^*. Furthermore, if F_t^*/p_t^* is roughly determined by W_t^*/p_t^* and if we assume that GNP is a comparatively better approximation of W_t, we obtain the Divisia money demand function in the form of equation (2). Also, since Π is linearly homogeneous, we can use the relation

$$\Pi_t / p_t^* = \Pi(\pi_t^1 / p_t^*, \cdots, \pi_t^n / p_t^*)$$

to derive the ordinary form of the money demand function such as equation (1).

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