Optimal Timing in Banks' Write-Off Decisions under the Possible Implementation of a Subsidy Scheme: A Real Options Approach

Naohiko Baba

This paper provides a formal model that investigates optimal timing in banks' writing off their nonperforming loans. The motivation comes from the recent episodes of Japanese banks, which have been slow to clean up their nonperforming loans after the collapse of the "bubble" economy in the early 1990s. A real options approach is employed to measure the value of the rationality of the "forbearance policy." It is assumed that uncertainty will arise from the following sources: (1) the reinvestment return from freeing up funds through write-offs; (2) liquidation losses; (3) the possible implementation of a subsidy scheme; and (4) the reputational repercussions from not immediately writing off their nonperforming loans. This paper attaches particular importance to the uncertainty from the possible implementation of the subsidy scheme to explore its desirable features. Also, this paper examines the possible role of monetary policy in boosting the banks' incentive to write off.

Key words: Write-off; Nonperforming loans; Dynamic programming; Real options; Reputation; Forbearance policy

Financial Markets Department, Bank of Japan (E-mail: naohiko.baba@boj.or.jp)

I am grateful to Takeshi Amemiya, Joseph Haubrich, Takeo Hoshi, Yukinobu Kitamura, Kotaro Tsuru, and staff members of the Bank of Japan for their helpful comments and suggestions on the earlier version of this paper. I would also like to thank seminar participants at the Institute of Economic Research of Hitotsubashi University, particularly Naoto Abe, Tokuo Iwaisako, and Satoru Kanoh, for their helpful comments. Any remaining errors are entirely mine. The views expressed in this paper are those of the author and do not necessarily reflect those of the Bank of Japan.

I. Introduction

The Japanese economy has been in a prolonged recession, with many banks weighed down by large-scale nonperforming loans. The primary cause has been a sharp fall in land prices that began in late 1991. The Ministry of Finance (MOF), once Japan's primary regulatory agency,[1] was slow in reacting to this problem.[2] Recently, harsh criticism has been directed at the MOF's so-called "forbearance" or "buy-time" policy, which allowed banks to keep nonperforming loans on their balance sheets in the hope that the economy and real estate market would recover in the not-too-distant future.[3]

As argued by many authors including Cargill (2000), the failure to promptly solve the nonperforming-loan problem generated a credit crunch. It has contributed to stagnant or declining real GDP growth for almost a decade and has interfered with the efforts by the Bank of Japan (BOJ) to stimulate the economy.

Note, however, that purely from the banks' perspective, the forbearance policy itself can be a rational choice. This is because under the stochastic circumstances with potentially large losses associated with write-offs, the option to wait (delay write-offs) should have some value. Hence, in deciding whether to write off their nonperforming loans immediately the banks should weigh the value of the option to wait against the (net) value of carrying out the write-off immediately.

Hoshi (2000) pointed out that if banks are not required by the authorities to disclose the true magnitude of their nonperforming loans, there is no incentive to dispose of such loans. Rather, the banks tend to increase their lending to riskier projects. The true problem caused by nonperforming loans, he argues, is that banks lose their incentive to lend to manufacturing corporations (among others) with prospective projects, which might damage the intermediation function of banks.[4] If the social costs caused by the damage of banks' financial intermediary function outweigh the subsidy costs, then it might be justified to use public funds to push self-help efforts on banks to clean up their nonperforming loans.[5] At last, in March 1998 and 1999, the Japanese government injected public funds into some banks as capital support.[6] The use of public funds was justified with the judgment that a prompt

1. In June 1998, the Financial Supervisory Agency (FSA) was established, directly under the prime minister and independent of the MOF. The functions of monitoring financial markets and supervising financial institutions were transferred from the MOF to the FSA. In July 2000, the FSA was upgraded to the Financial Services Agency responsible for wide-ranging matters related to the financial system. The MOF became mainly responsible for budgetary and taxation matters.
2. Ito (2000) pointed out that bank analysts at brokerage firms began to discuss a potential nonperforming-loan problem in 1992–93, but the MOF was reluctant to force banks to disclose the specific amounts of their nonperforming loans.
3. As explained by Ueda (2000), with the exception of a brief period around 1975, postwar Japan had never experienced a decline in land prices. Thus, no time-series analysis of Japanese land prices through the late 1980s would have given a high probability of a sharp fall in land prices in 1990.
4. Hoshi (2000) also argues that even if the economy is in the state of a liquidity trap, the credit channel works to ensure the effectiveness of monetary policy.
5. It should be noted, however, that bank regulators may pursue self-interests rather than social welfare. Boot and Thakor (1993) examine this possibility by introducing uncertainty in regulators' ability to monitor banks' asset choices.
6. In March 1998, an initial wave of less than ¥2 trillion in capital injection was provided to 21 banks. The size of the injection, in the form of subordinated loans and preferred stocks, was almost uniform across the banks, yet insufficient. In April 1998, the "prompt corrective action (PCA) rule" took effect, requiring the banks with capital ratios below certain levels to restructure or even cease operations. Also in October 1998, an agreement to appropriate ¥60 trillion of public funds to strengthen the financial system and recapitalize the banks was enacted. Recapitalization took place in March 1999. See Ito (2000) for further details.

resolution of the nonperforming-loan problem benefits the economy as a whole in the long run.

As pointed out by Corbett and Mitchell (2000), however, one of the most puzzling and interesting facts regarding Japan's recent bank rescue package is that the government's offer was not welcomed by the rescued banks. The key to understanding this seemingly puzzling fact lies in the existence of asymmetric information and a possible reputation problem.

Typically, asymmetric information exists between banks and the public regarding the true magnitude of nonperforming loans on their balance sheets.[7] More specifically, "the public" means shareholders, depositors, and other participants in financial asset markets. This asymmetric information creates an incentive for banks to roll over their nonperforming loans to disguise their true financial standing. The government, which is generally in a position to grasp the true standing of the banks' balance sheets through bank examinations and monitoring on a regular basis, is able to rescue the banks by providing capital supports.

Corbett and Mitchell (2000) further argue that banks may decline rescue offers due to reputation concerns. This is because accepting such offers may force banks to write off nonperforming loans, revealing the hidden information to the public.

More generally, however, a bank's reputation depends on what market participants infer from its write-off decision. In fact, on many occasions, stock prices rose for Japanese banks[8] that announced they would increase write-offs. Such episodes suggest there could be reputational repercussions from *not* writing off nonperforming loans. This paper models reputational concerns as a fear of increasing fund-raising (outside finance) costs.[9] And based on these episodes, I introduce the fear only in specifying the value of the option to wait (delay write-offs), not in specifying the value of immediate write-offs.

Motivated by the discussion above, this paper attempts to evaluate (1) optimal timing in banks' write-off decisions;[10] and (2) how much compensation or subsidization is needed for banks to carry out write-offs as self-help efforts. This paper uses the so-called real options approach to measure the value of the option to wait. More specifically, the paper treats banks' optimal write-off decisions as continuous-time problems within an infinite horizon.[11]

This paper recognizes that the capital injections from the Japanese government in 1998 and 1999 were not literally a subsidy for write-offs. Nevertheless, it is of some benefit to regulatory authorities to conduct this kind of theoretical experiment insofar

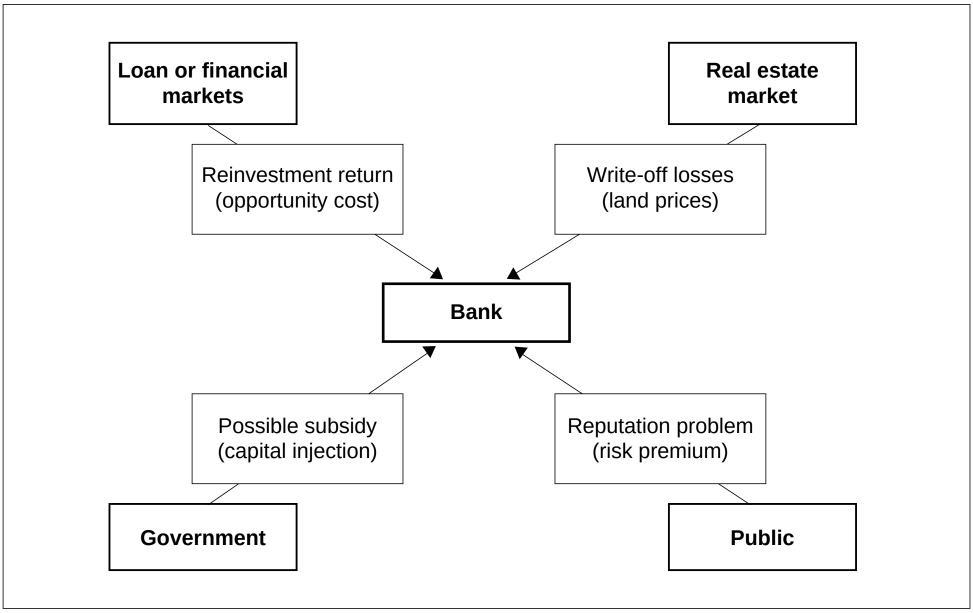
7. In fact, attempts to identify the scale of the problem have been hampered by lack of disclosure and frequent changes in the definition of "nonperforming loans."
8. For example, recall a surge in the stock prices of Sanwa, Tokai, and Asahi banks after they released news of large-scale write-offs for fiscal 2000.
9. In this regard, see Boot and Greenbaum (1993) as an example.
10. In reality, the write-off procedure consists of two steps. The first step is known as an indirect write-off. In this step, the bank only reports an estimated loss, but does not actually liquidate collateral. The second step is a direct write-off, which makes the bank liquidate collateral and the actual amount of the associated loss is fixed. This paper's approach skips the first step of indirect write-offs for simplicity.
11. A simpler class of models with only two or three discrete decision points might suffice for our qualitative analysis needs. It should be noted, however, that such a class of models is based on the unrealistic assumption that there is no uncertainty after two or three units of time. In most markets, future returns are always uncertain, and the degree of uncertainty increases with the time horizon.

as they believe that cleaning up the nonperforming-loan problem is a prerequisite to stabilizing the financial system as a whole.

The sources of uncertainty in this paper are as follows (Figure 1):

(1) The reinvestment return from freeing up funds[12] (collected by liquidating collateral): the banks can lend (invest) the funds to prospective projects (in financial assets).
(2) The loss caused by carrying out write-offs: this is closely linked to land prices, since real estate collateral has been extensively used when bank loans were contracted.
(3) The future implementation of a government subsidy scheme.
(4) The reputational repercussions from not writing off immediately, which takes the form of an upward jump in fund-raising costs.

Figure 1 Sources of Uncertainty Influencing Bank Decisions



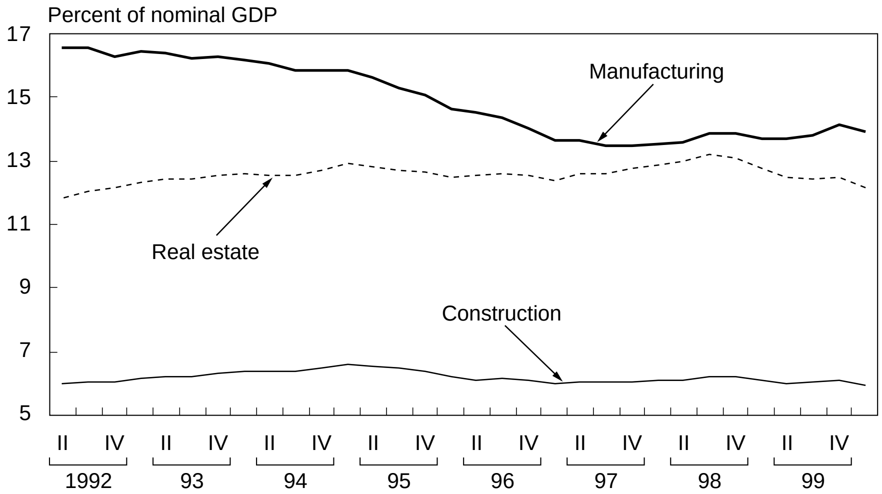
Another (secondary) aim of this paper is to provide a solid microeconomic foundation to the question of why loans to manufacturing corporations have stagnated in Japan. As explained by Hoshi (2001), during the "bubble" period, banks eagerly shifted into collateralized lending. For them, lending to real estate and construction corporations was particularly promising, because they owned real estate collateral. Thus, when land prices collapsed in the early 1990s, a non-negligible portion of the loans to such industries became nonperforming.

Figure 2 shows that bank loans to real estate and construction industries relative to the size of real economic activity have remained almost unchanged even after the bursting of the "bubble" economy, while loans to the manufacturing industry have

12. In other words, this is an opportunity cost arising from keeping nonperforming loans on the balance sheet.

declined significantly. At the same time, Figure 3 shows that until recently profitability measured by the ratio of current profits to sales had been markedly lower in the real estate industry than in other industries. These facts, viewed in conjunction, imply that Japanese banks have rolled over nonperforming loans to the real estate industry and thus have not had any incentive to explore prospective projects in the manufacturing industry.

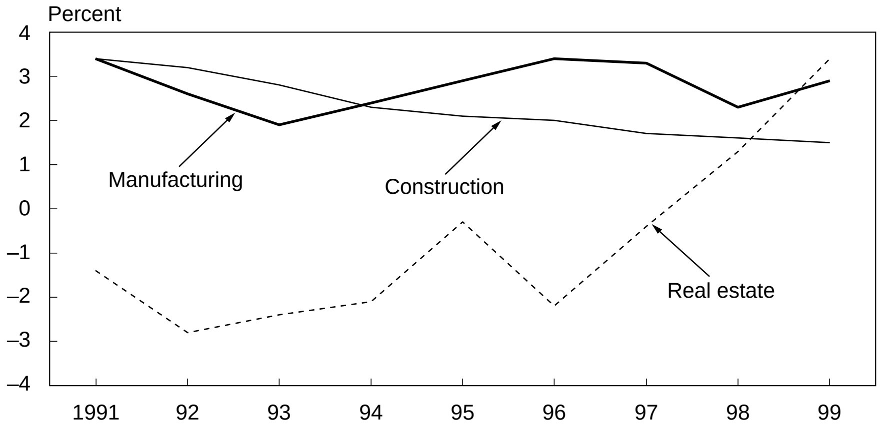
Figure 2 Loans and Discounts Outstanding by Industry



Note: The definition of the domestic banks changed in 1992/I. Thus, I used data after 1992/II.

Source: Bank of Japan, *Financial and Economic Data CD-ROM*.

Figure 3 Ratio of Current Profits to Sales



Source: Ministry of Finance, *Corporate Business Statistics*.

The rest of the paper is organized as follows. Section II describes the theoretical foundation using the dynamic programming technique. Section III numerically analyzes the model. Section IV discusses some policy implications. Section V concludes the study.

II. Theoretical Foundation

A. Assumptions

The problem that a typical bank faces is whether to exercise the option to write off its nonperforming loans. Figure 4 illustrates a simplified bank's balance sheet on the premise that the bank lends all collected money to good projects.[13] It shows that once the bank writes off its nonperforming loans, it can lend collected money, denoted $L_B + S - L$, to good projects.[14] Here, L denotes the amount of losses associated with the write-off defined as the difference between the value of nonperforming loans (L_B) and the current value of the collateral. S is the government subsidy. At the same time, net worth changes from N to $N + S - L$.

There are informational asymmetries regarding the true magnitude of nonperforming loans between bank managers and the public including its shareholders.[15]

Figure 4 A Simplified Balance Sheet

[1] Before Write-Off

Assets	Liabilities
Loans ($L_G + L_B$)	Funds (F_U)
Good loans (L_G)	
Bad loans (L_B)	
Other assets (OA)	Net worth (N)

[2] After Write-Off

Assets	Liabilities
Loans ($L_G + L_B + S - L$)	Funds (F_U)
Good loans ($L_G + L_B + S - L$)	
Other assets (OA)	Net worth ($N + S - L$)

Notes: 1. S: Government subsidy, L: Liquidation losses.
2. The figures are based on the assumption that collected money is lent to good projects.

13. Alternatively, the bank can reinvest collected money in financial assets markets.
14. If one assumes that the bank lends collected money to other projects, then the loss L should include monitoring costs.
15. Some recent studies in corporate governance suggest that bank managers in Japan probably do not act to maximize banks' share prices. Instead, they engage in activities to entrench themselves. Particularly, Claessens,

The bank's incentive to write off lies in the fact that the bank can lend money collected from liquidating collateral to other borrowers with prospective projects (or reinvest it in the financial markets). The return from the lending (reinvestment) after netting out a possible rise in fund-raising cost from not writing off immediately is denoted R.

This paper assumes that the reinvestment return itself follows the standard geometric Brownian motion. However, due to possible reputation problems when the bank delays the write-off, there is a possibility that the net reinvestment return will fall as a result of rising fund-raising costs in the future. Hence, this paper assumes that there is a probability, denoted λ, that the reinvestment return net of fund-raising costs exhibits a downward jump.

Also, the bank suffers losses denoted L in carrying out write-offs, particularly due to a fall in the value of collateral. It should be noted, however, that L itself moves stochastically because it mainly reflects land prices.[16] L also follows the standard geometric Brownian motion.

The stochastic processes of the reinvestment return and the write-off losses can be summarized as

$$dR = \alpha_R R dt + \sigma_R R dz_R - R dq, \tag{1}$$

and $$dL = \alpha_L L dt + \sigma_L L dz_L, \tag{2}$$

where $\alpha_R(\alpha_L)$ denotes the expected growth rate of $R(L)$, $\sigma_R(\sigma_L)$ the standard deviation parameter of $R(L)$, $dz_R(dz_L)$ the increment of a Wiener process of $R(L)$, and dq the increment of a Poisson (jump) process with the probability λdt. The paper assumes $E[(dz_R)(dq)] = 0$, that is, dz_R and dq are independent of each other. Also, it is assumed $E[(dz_R)^2] = E[(dz_L)^2] = dt$ and $E[(dz_R)(dz_L)] = \rho dt$, implying that the correlation ρ between R and L is considered in the following analysis. Lastly, equation (1) states that when a jump occurs, R falls by the fixed ratio $\phi (0 \leq \phi \leq 1)$. For computational facility, I assume $\phi = 1$ throughout the paper.

Further, the bank faces another source of uncertainty from the future implementation of the government's rescue scheme under which the government subsidizes the bank's write-off. The paper's strategy is to express the possible policy implementation by a Poisson jump.[17] Also, for simplicity, the subsidy amount is proportional to the loss from the write-off.

Djankov, and Lang (2000), and Morck and Nakamura (1999) documented that Japanese banks engage in substantial cross-shareholdings with other companies as an explicit defense against competitive changes in corporate control. Through crossholdings, companies can acquire control rights in banks without getting cash flow rights. Such controlling shareholders could have an incentive to encourage bank management to do things that maximize their own wealth, such as continue to finance their company's unprofitable operations, which does not maximize the bank's stock price.

16. The majority of nonperforming loans are in the real estate and construction industries.

17. This strategy basically follows Hassett and Metcalf (1999). They examine the case where there is one underlying stochastic variable and a possible Poisson jump-type policy intervention, and analyze the impact of uncertain tax policy on investment decisions. The model in this paper augments their model by using two underlying stochastic variables, one entailing a possible downward jump risk, as well as a Poisson jump-type policy intervention.

When the subsidy scheme is not in effect, the probability that the government will implement it in the next short period dt is denoted $\lambda_1 dt$. In such cases, the amount of the subsidy is θL. On the other hand, when the scheme is in effect, the probability that it will be removed in the next short period dt is $\lambda_0 dt$. In sum, the subsidy process is given by the following equation of motion:

$$d\theta = \begin{cases} \begin{cases} \theta & \text{with probability} \quad \lambda_1 dt \\ 0 & \text{with probability } 1-\lambda_1 dt \end{cases} \\ \begin{cases} -\theta & \text{with probability} \quad \lambda_0 dt \\ 0 & \text{with probability } 1-\lambda_0 dt \end{cases} \end{cases}. \tag{3}$$

In what follows, first I will examine the case without the possible implementation of the subsidy scheme, and then consider the government subsidy.

B. The Case without the Subsidy Scheme

1. Basic setup

First, the value of (immediate) write-offs is given by considering only the part of the standard geometric Brownian motion of equation (1) such that[18]

$$V(R) = E\left[\int_0^\infty R(t)e^{-\mu_R t}dt\right] = \int_0^\infty Re^{-(\mu_R-\alpha_R)t}dt = \frac{R}{\mu_R-\alpha_R} = \frac{R}{\delta_R}, \tag{4}$$

where the relationship $\mu_R \equiv \alpha_R + \delta_R = r + \upsilon\rho(R, M)\sigma_R$ is assumed to hold[19] as in Dixit and Pindyck (1994).[20] Here, μ_R denotes the risk-adjusted discount rate, δ_R the rate of return shortfall in R (hereafter, shortfall rate), r the risk-free interest rate, υ the market price of risk, and $\rho(R, M)$ the coefficient of correlation between R and the market return M. For the value of write-off $V(R)$ to be bounded, the condition $\delta_R > 0$ must hold.[21] Otherwise, the bank would never carry out write-offs irrespective of uncertainty and sunk costs.

Second, let $F(R, L)$ denote the value of keeping the option to write off alive in the future (hereafter, the value of waiting). The Bellman equation can be written as[22]

$$\mu F(R, L)dt = E[dF(R, L)]. \tag{5}$$

18. Notice that once write-offs are carried out, further uncertainty associated with losses is irrelevant, hence the value of write-offs depends only on the return from reinvesting the collected money.
19. The relationship $\mu_R = r + \upsilon\rho(R, M)\sigma_R$ is derived from the Capital Asset Pricing Model (CAPM). To get this relationship, one needs the stochastic fluctuations in R to be spanned by financial markets. Also, implicitly, I assume that the jump risk is non-systematic, that is, uncorrelated with the market portfolio.
20. See chapter 4 for details.
21. In understanding the role of δ_R, it is helpful to draw upon the analogy with a financial call option in which R corresponds to the price of a share of common stock, and δ_R the dividend rate. Thus, the total expected rate on the stock is written as $\mu_R = \alpha_R + \delta_R$. In such a case, if the dividend rate δ_R were zero, the call option would always be held to maturity, and never exercised since the opportunity cost to keep the option alive is zero.
22. In what follows, for simplicity, I drop the subscript R for μ_R.

Expanding $dF(R, L)$ in equation (5) by Ito's Lemma for the combined geometric Brownian motion and Poisson jump[23] yields

$$\mu F(R, L) = \frac{1}{2}\left(\sigma_R^2 R^2 F_{RR} + 2\rho\sigma_R\sigma_L RLF_{RL} + \sigma_L^2 L^2 F_{LL}\right) + \alpha_R RF_R$$
$$+ \alpha_L LF_L + \lambda\{F(0, L) - F(R, L)\}, \tag{6}$$

where $F_R \equiv \partial F/\partial R$, $F_L \equiv \partial F/\partial L$, $F_{RR} \equiv \partial^2 F/\partial R^2$, $F_{LL} \equiv \partial^2 F/\partial L^2$, and $F_{RL} \equiv \partial^2 F/\partial R\partial L$. Now, boundary conditions can be written as

$$F(\hat{R}, \hat{L}) = V(\hat{R}) - \hat{L} = \frac{\hat{R}}{\delta_R} - \hat{L}, \tag{7}$$

$$F_R(\hat{R}, \hat{L}) = V'(\hat{R}) = \frac{1}{\delta_R}, \tag{8}$$

and $$F_L(\hat{R}, \hat{L}) = -1, \tag{9}$$

where condition (7) is the value-matching condition and conditions (8) and (9) are both the smooth-pasting conditions. Also, $\hat{R}$ and $\hat{L}$ indicate the threshold values of R and L at which the bank becomes indifferent between writing off and waiting.

2. The free boundary problem

The problem in the last section is called a "free boundary" problem.[24] In such a case, it is very difficult to obtain clear-cut analytical solutions. Nevertheless, the property of homogeneity of the net value function $V(R) - L$[25] allows one to reduce the problem to one dimension.

Thus, the optimal decision only depends on the ratio $r_R \equiv R/L$, which implies that the value of waiting $F(R, L)$ should also be homogeneous of degree one with respect to R and L. That is, the following set of relationships holds:

23. In general, if the stochastic process is

$$dx = a(x, t)dt + b(x, t)dz + g(x, t)dq,$$

then the expected value of the change in any function $H(x, t)$ can be given by

$$E[dH] = \left[\frac{\partial H}{\partial t} + a(x, t)\frac{\partial H}{\partial x} + \frac{1}{2}b^2(x, t)\frac{\partial^2 H}{\partial x^2}\right]dt + E_\phi\{\lambda[H(x + g(x, t)\phi, t) - H(x, t)]\}dt.$$

For more details, see Dixit and Pindyck (1994), chapter 3, p. 86.

24. For more details, see Dixit and Pindyck (1994), chapter 6, p. 209.

25. Note that if the current values of both R and L are doubled, it will double the value V and the cost L simultaneously.

$$F(R,L) = Lf\left(\frac{R}{L}\right) \equiv Lf(r_R), \tag{10}$$

$$F_R(R,L) = f'(r_R), \tag{11}$$

$$F_L(R,L) = f(r_R) - r_R f'(r_R), \tag{12}$$

$$F_{RR}(R,L) = f''(r_R)/L, \tag{13}$$

$$F_{RL}(R,L) = -r_R f''(r_R)/L, \tag{14}$$

and

$$F_{LL}(R,L) = (r_R)^2 f''(r_R)/L. \tag{15}$$

Using equations (10)–(15), equation (6) can be rewritten as

$$\frac{1}{2}(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2)(r_R)^2 f''(r_R)$$
$$+ (\alpha_R - \alpha_L) r_R f'(r_R) + (\alpha_L - \mu - \lambda) f(r_R) = 0. \tag{16}$$

The solution to the second-order differential equation (16) takes the form

$$f(r_R) = A(r_R)^{\beta}, \tag{17}$$

where A and β are coefficients to be determined.

Direct substitution of solution (17) into equation (16) yields

$$\frac{1}{2}\left(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2\right)\beta(\beta - 1) + (\alpha_R - \alpha_L)\beta + (\alpha_L - \mu - \lambda) = 0. \tag{18}$$

Thus, β can be solved analytically as

$$\beta = \frac{1}{2} - \frac{(\alpha_R - \alpha_L)}{(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2)}$$
$$+ \sqrt{\left[\frac{(\alpha_R - \alpha_L)}{(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2)} - \frac{1}{2}\right]^2 + \frac{2(\mu + \lambda - \alpha_L)}{(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2)}}. \tag{19}$$

Now boundary conditions (7)–(9) can be rewritten as

$$f(\hat{r}_R) = A(\hat{r}_R)^{\beta} = \frac{\hat{r}_R}{\delta_R} - 1, \tag{20}$$

$$f'(\hat{r}_R) = A\beta(\hat{r}_R)^{\beta-1} = \frac{1}{\delta_R}, \tag{21}$$

and $$f(\hat{r}_R) - \hat{r}_R f'(\hat{r}_R) = A(\hat{r}_R)^{\beta}(1-\beta) = -1, \tag{22}$$

where $\hat{r}_R$ denotes the threshold ratio. Note that of these three boundary conditions, no single one is independent of the other two.

Equations (20) and (21) jointly imply

$$\hat{r}_R = \frac{\beta}{\beta - 1}\delta_R. \tag{23}$$

Figure 5 illustrates a free boundary $\hat{r}_R$ of the ratio of reinvestment return to write-off losses. In regime I, the current value of r_R is below the threshold value $\hat{r}_R$ so that the bank prefers waiting to writing off now. Also, Figure 6 depicts boundary conditions for $f(r_R)$ and the determination of $\hat{r}_R$. At the threshold ratio $\hat{r}_R$, the value from write-offs meets the value of waiting tangentially.

Figure 5 Free Boundary of $\hat{r}_R$ without a Subsidy Scheme

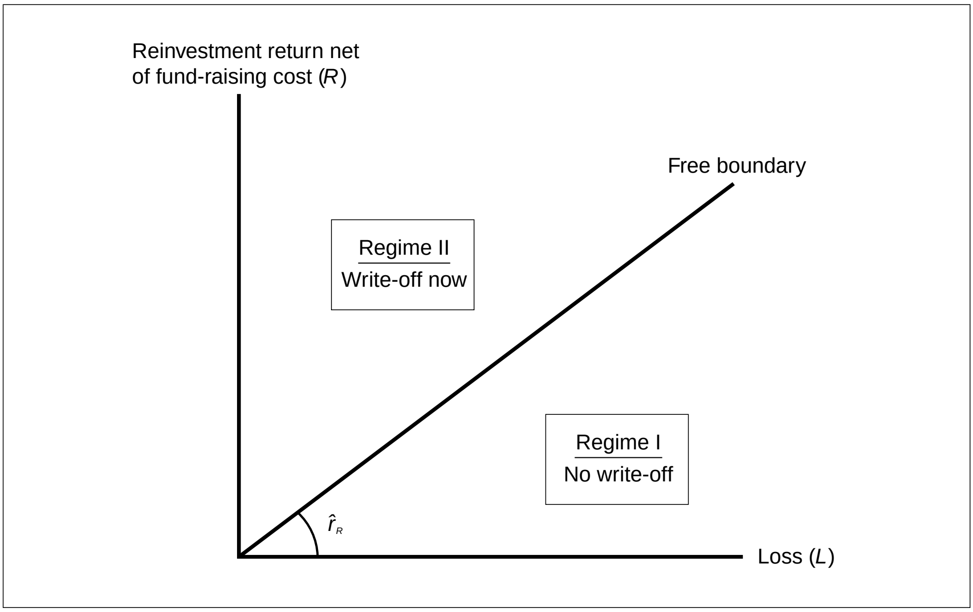
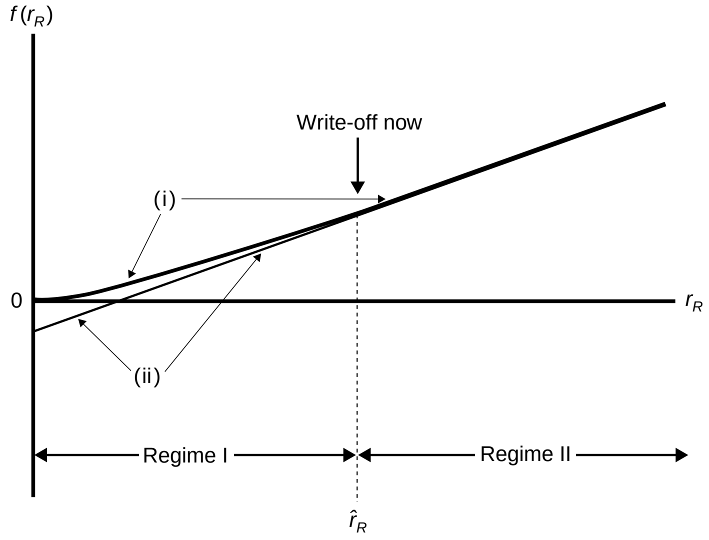


Figure 6 Boundary Conditions for $f(r_R)$ and the Determination of $\hat{r}_R$



(i): $f(r_R) = A(r_R)^\beta$ (equation [17])

(ii): $f(r_R) = \frac{r_R}{\delta_R} - 1$ (right-hand side of equation [20])

3. The relationship between $\hat{r}_R$ and the required reinvestment rate of return $\overline{rr}$

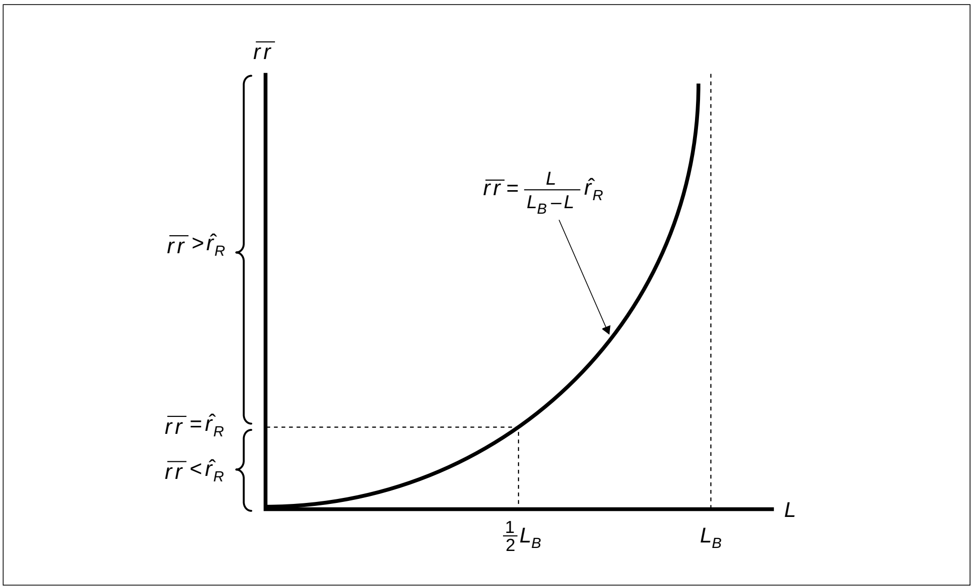
Note that the threshold ratio $\hat{r}_R$ is in terms of write-off losses, not in terms of the amount of reinvested funds. Thus, in evaluating $\hat{r}_R$ in line with a realistic economic situation, it is helpful to translate $\hat{r}_R$ into a usual rate of return form.

Figure 7 shows the relationship between $\hat{r}_R$ and the required reinvestment rate of return $\overline{rr}$. The following relationship is evident:

$$\begin{cases} \text{if } L \leq \frac{1}{2} L_B, \text{ then } \overline{rr} = \frac{L}{L_B - L}\, \hat{r}_R \leq \hat{r}_R \\ \text{otherwise,} \qquad \overline{rr} > \hat{r}_R \end{cases} . \tag{24}$$

For example, when a loss amounts to a quarter of the nonperforming loan, $\overline{rr}$ is equal to $1/3\hat{r}_R$. And when the loss is three-quarters, $\overline{rr}$ is $3\hat{r}_R$.

Figure 7 The Relationship between $\hat{r}_R$ and the Required Reinvestment Rate of Return $\overline{rr}$



C. The Case with the Possible Implementation of a Subsidy Scheme

Now let me consider the case in which the bank expects the implementation of a subsidy by the government with some probability. To begin, let $F_0(R, L) = Lf_0(r_R)$ denote the value of waiting in the absence of a subsidy scheme and let $F_1(R, L) = Lf_1(r_R)$ denote the value in the presence of the scheme, respectively. In this setting, one can divide the decision rule of the bank into the following three regimes, implying the existence of two threshold ratios. See Figures 8 and 9 for the illustration of the three regimes.

Figure 8 Three Regimes of a Bank's Optimal Decisions

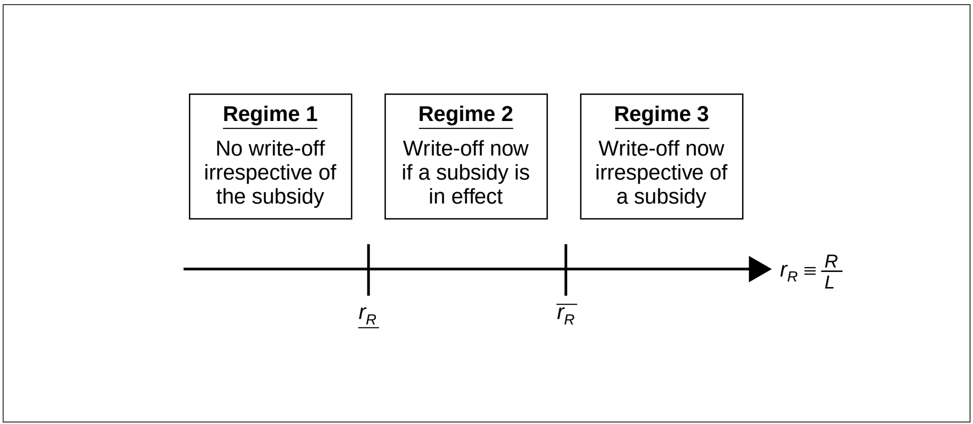
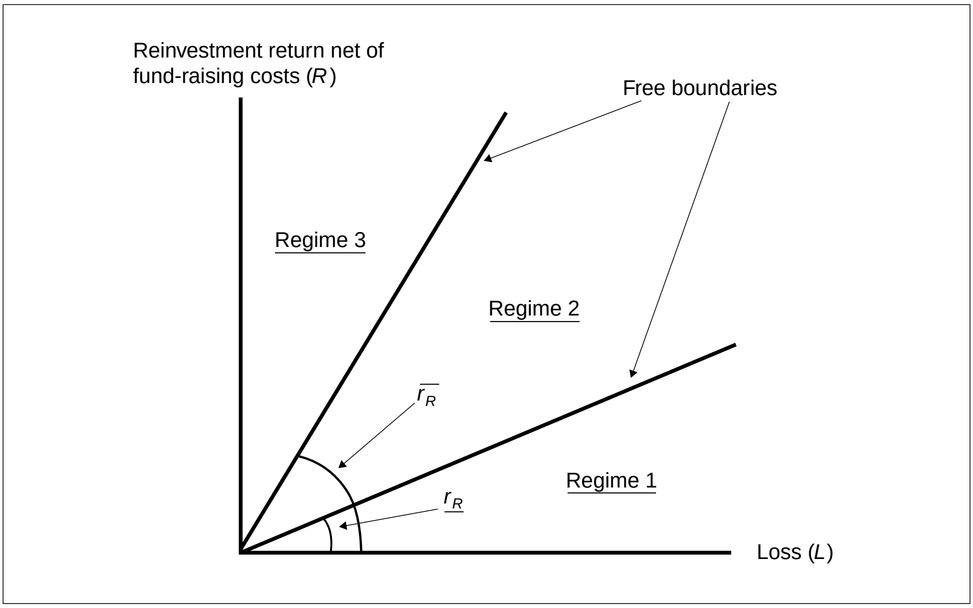


Figure 9 Free Boundaries of the Ratio of the Reinvestment Return to Write Off Losses



First, over the interval of low values of r_R, denoted $(0, \underline{r_R})$, the bank will not write off irrespective of whether or not the subsidy scheme is in effect.

Second, over the interval denoted $(\underline{r_R}, \overline{r_R})$, the bank will write off if the subsidy scheme is in effect. Otherwise, the bank will prefer to wait in the hope that the subsidy scheme will be implemented and/or land prices will recover so that liquidation losses will decrease in the future.

Third, over the interval denoted $(\overline{r_R}, \infty)$, the bank is willing to write off irrespective of a subsidy scheme. Referring to Hassett and Metcalf (1999), let me find the two thresholds, $\overline{r_R}$ and $\underline{r_R}$ below.

1. Regime 1 $(0, \underline{r_R})$: No write-off irrespective of the subsidy scheme

Over the interval $(0, \underline{r_R})$, the bank prefers to wait irrespective of a subsidy scheme, and each regime can switch to the other. Thus, the following pair of equations holds[26]:

$$\frac{1}{2}(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2)(r_R)^2 f_0''(r_R) + (\alpha_R - \alpha_L)r_R f_0'(r_R)$$

$$+ (\alpha_L - \mu - \lambda)f_0(r_R) + \lambda_1[f_1(r_R) - f_0(r_R)] = 0. \qquad (25)$$

26. The derivation of equation (25) is made in the following way. When the subsidy is not in effect, over the next short interval of time dt, the probability that the subsidy will be implemented is $\lambda_1 dt$. In this case, the value of the option to write off is $F_1(R + dR, L + dL)$. Otherwise, it is $F_0(R + dR, L + dL)$. Hence,

$$F_0(R + dR, L + dL) = e^{-\mu dt}\{(\lambda_1 dt)E[F_1(R + dR, L + dL)] + (1 - \lambda_1 dt)E[F_0(R + dR, L + dL)]\}$$

follows. Expanding the preceding equation by Ito's Lemma and using the assumption of homogeneity yields equation (25). Equation (26) can be derived in the same way.

$$\frac{1}{2}(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2)(r_R)^2 f_1''(r_R) + (\alpha_R - \alpha_L)r_R f_1'(r_R)$$

$$+ (\alpha_L - \mu - \lambda)f_1(r_R) + \lambda_0[f_0(r_R) - f_1(r_R)] = 0. \quad (26)$$

Since both value functions $f_0(r_R)$ and $f_1(r_R)$ appear in each equation, one needs to consider the two linear combinations that can be solved easily. For example, consider new value functions $f_a(r_R)$ and $f_b(r_R)$ such that

$$f_a(r_R) = \frac{f_0(r_R)}{\lambda_1} + \frac{f_1(r_R)}{\lambda_0}, \quad (27)$$

and $f_b(r_R) = f_1(r_R) - f_0(r_R). \quad (28)$

Then, equations (25) and (26) can be rewritten in terms of $f_a(r_R)$ and $f_b(r_R)$ as

$$\frac{1}{2}(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2)(r_R)^2 f_a''(r_R) + (\alpha_R - \alpha_L)r_R f_a'(r_R)$$

$$+ (\alpha_L - \mu - \lambda)f_a(r_R) = 0, \quad (29)$$

$$\frac{1}{2}(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2)(r_R)^2 f_b''(r_R) + (\alpha_R - \alpha_L)r_R f_b'(r_R)$$

$$+ (\alpha_L - \mu - \lambda - \lambda_0 - \lambda_1)f_b(r_R) = 0. \quad (30)$$

The solutions to the second-order differential equations take the forms[27]

$$f_a(r_R) = B(r_R)^{\beta_1}, \quad (31)$$

and $f_b(r_R) = C(r_R)^{\beta_2}, \quad (32)$

where B, C, β_1, and β_2 are coefficients to be determined. Here note that β_1 is the positive root of

$$\frac{1}{2}(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2)\beta(\beta - 1) + (\alpha_R - \alpha_L)\beta + (\alpha_L - \mu - \lambda) = 0, \quad (33)$$

and β_2 is the positive root of

$$\frac{1}{2}(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2)\beta(\beta - 1) + (\alpha_R - \alpha_L)\beta$$

$$+ (\alpha_L - \mu - \lambda - \lambda_0 - \lambda_1) = 0. \quad (34)$$

27. Note that since the interval of r_R extends to zero, only the positive root of a quadratic equation matters.

With this background information, the solutions for $f_0(r_R)$ and $f_1(r_R)$ over the interval $(0, \underline{r_R})$ are given by

$$f_0(r_R) = \frac{\lambda_0\lambda_1 B(r_R)^{\beta_1} - \lambda_1 C(r_R)^{\beta_2}}{\lambda_0 + \lambda_1}, \tag{35}$$

and $$f_1(r_R) = \frac{\lambda_0\lambda_1 B(r_R)^{\beta_1} + \lambda_0 C(r_R)^{\beta_2}}{\lambda_0 + \lambda_1}. \tag{36}$$

2. Regime 2 ($\underline{r_R}$, $\overline{r_R}$): Write-off now if the subsidy scheme is in effect

Over the interval $(\underline{r_R}, \overline{r_R})$, the bank will write off nonperforming loans immediately if the subsidy scheme is in effect. Otherwise, the bank will not. Thus, $f_1(r_R)$ is given by[28]

$$f_1(r_R) = \frac{r_R}{\delta_R} - (1-\theta), \tag{37}$$

where θ denotes the portion of the subsidy[29] in the loss.

On the other hand, $f_0(r_R)$ is found in the same way as equations (25) and (26).

$$\frac{1}{2}(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2)(r_R)^2 f_0''(r_R) + (\alpha_R - \alpha_L)r_R f_0'(r_R)$$
$$+ (\alpha_L - \mu - \lambda) f_0(r_R) + \lambda_1[f_1(r_R) - f_0(r_R)] = 0. \tag{38}$$

Using equation (37), equation (38) can be rewritten as

$$\frac{1}{2}(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2)(r_R)^2 f_0''(r_R) + (\alpha_R - \alpha_L)r_R f_0'(r_R)$$
$$+ (\alpha_L - \mu - \lambda - \lambda_1) f_0(r_R) + \frac{\lambda_1}{\delta_R} r_R - \lambda_1(1-\theta) = 0. \tag{39}$$

The general solution takes the form

$$f_0(r_R) = D(r_R)^{\beta_3} + E(r_R)^{\beta_4} + \frac{\lambda_1 r_R}{\delta_R(\mu + \lambda + \lambda_1 - \alpha_R)} - \frac{\lambda_1(1-\theta)}{\mu + \lambda + \lambda_1 - \alpha_L}, \tag{40}$$

where D and E are constants to be determined and β_3 and β_4 are the positive and negative roots of the quadratic function of the form:

$$\frac{1}{2}(\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2)\beta(\beta - 1) + (\alpha_R - \alpha_L)\beta + (\alpha_L - \mu - \lambda - \lambda_1) = 0. \tag{41}$$

28. Notice that equation (37) is equivalent to $F_1(R, L) = R/\delta_R - (1-\theta)L$.
29. See equation (3) for its definition.

3. Regime 3 ($\bar{r}_R$, ∞): Write-off now irrespective of the subsidy scheme

Over the interval ($\bar{r}_R$, ∞), the bank always writes off nonperforming loans, irrespective of the subsidy scheme. Thus, the following relationships hold:

$$f_0(r_R) = \frac{r_R}{\delta_R} - 1, \tag{42}$$

and
$$f_1(r_R) = \frac{r_R}{\delta_R} - (1-\theta). \tag{43}$$

4. Boundary conditions linking each regime[30]

Now that each solution form has been found, the next step is to find boundary conditions, which relate to each of the above-derived value functions.

First, at the threshold $\underline{r}_R$, the bank will write off if the subsidy scheme is in effect. Thus, for the expressions for $f_1(r_R)$, equations (36) and (37) yield

$$\frac{\lambda_0\lambda_1 B(\underline{r}_R)^{\beta_1} + \lambda_0 C(\underline{r}_R)^{\beta_2}}{\lambda_0+\lambda_1} = \frac{\underline{r}_R}{\delta_R} - (1-\theta), \tag{44}$$

and
$$\frac{\lambda_0\lambda_1 B\beta_1(\underline{r}_R)^{\beta_1-1} + \lambda_0 C\beta_2(\underline{r}_R)^{\beta_2-1}}{\lambda_0+\lambda_1} = \frac{1}{\delta_R}, \tag{45}$$

where equations (44) and (45) denote value-matching and smooth-pasting conditions.

Second, for $f_0(r_R)$, although this is not actually associated with a decision threshold, the function has to be continuously differentiable across it. Thus, equations (35) and (40) yield

$$\frac{\lambda_0\lambda_1 B(\underline{r}_R)^{\beta_1} - \lambda_1 C(\underline{r}_R)^{\beta_2}}{\lambda_0+\lambda_1} = D(\underline{r}_R)^{\beta_3} + E(\underline{r}_R)^{\beta_4}$$
$$+ \frac{\lambda_1 \underline{r}_R}{\delta_R(\mu+\lambda+\lambda_1-\alpha_R)} - \frac{\lambda_1(1-\theta)}{\mu+\lambda+\lambda_1-\alpha_L}, \tag{46}$$

$$\frac{\lambda_0\lambda_1 B\beta_1(\underline{r}_R)^{\beta_1-1} - \lambda_1 C\beta_2(\underline{r}_R)^{\beta_2-1}}{\lambda_0+\lambda_1} = D\beta_3(\underline{r}_R)^{\beta_3-1}$$
$$+ E\beta_4(\underline{r}_R)^{\beta_4-1} + \frac{\lambda_1}{\delta_R(\mu+\lambda+\lambda_1-\alpha_R)}. \tag{47}$$

Third, at the threshold $\bar{r}_R$, the expressions for $f_0(r_R)$ should satisfy the value-matching and smooth-pasting conditions. Hence, equations (40) and (42) yield

30. Notice that the case without the government subsidy corresponds to a special case where $\lambda_0 = \lambda_1 = \theta = 0$ in this model.

$$D(\bar{r}_R)^{\beta_3} + E(\bar{r}_R)^{\beta_4} + \frac{\lambda_1 \bar{r}_R}{\delta_R(\mu + \lambda + \lambda_1 - \alpha_R)} - \frac{\lambda_1(1-\theta)}{\mu + \lambda + \lambda_1 - \alpha_L} = \frac{\bar{r}_R}{\delta_R} - 1, \quad (48)$$

and $$D\beta_3(\bar{r}_R)^{\beta_3 - 1} + E\beta_4(\bar{r}_R)^{\beta_4 - 1} + \frac{\lambda_1}{\delta_R(\mu + \lambda + \lambda_1 - \alpha_R)} = \frac{1}{\delta_R}, \quad (49)$$

where one can analytically find βs from equations (33), (34), and (41) as follows:

$$\begin{cases} \beta_1 = \dfrac{1}{2} - \dfrac{F}{G} + \sqrt{\left(\dfrac{F}{G} - \dfrac{1}{2}\right)^2 + \dfrac{2(\mu + \lambda - \alpha_L)}{G}} \\ \beta_2 = \dfrac{1}{2} - \dfrac{F}{G} + \sqrt{\left(\dfrac{F}{G} - \dfrac{1}{2}\right)^2 + \dfrac{2(\mu + \lambda + \lambda_0 + \lambda_1 - \alpha_L)}{G}} \\ \beta_3 = \dfrac{1}{2} - \dfrac{F}{G} + \sqrt{\left(\dfrac{F}{G} - \dfrac{1}{2}\right)^2 + \dfrac{2(\mu + \lambda + \lambda_1 - \alpha_L)}{G}} \\ \beta_4 = \dfrac{1}{2} - \dfrac{F}{G} - \sqrt{\left(\dfrac{F}{G} - \dfrac{1}{2}\right)^2 + \dfrac{2(\mu + \lambda + \lambda_1 - \alpha_L)}{G}} \end{cases}, \quad (50)$$

$$F = \alpha_R - \alpha_L, \text{ and } G = \sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2. \quad (51)$$

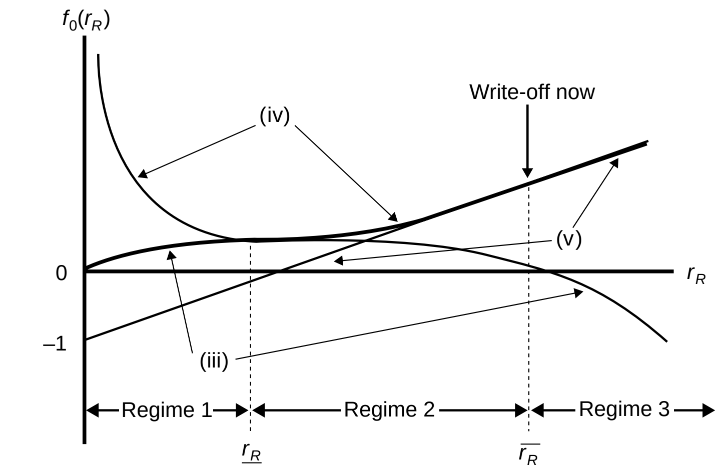
It is evident that the relation $\beta_4 < 0 < 1 < \beta_1 \leq \beta_3 \leq \beta_2$ holds. In sum, there are six equations to determine two thresholds $\underline{r_R}$ and $\bar{r}_R$ and four constants B, C, D, and E.[31]

Figure 10 [1] and [2] illustrates these boundary conditions and the determination of the threshold ratios $\underline{r_R}$ and $\bar{r}_R$. They show that at the threshold ratio $\underline{r_R}$, two expressions of $f_0(r_R)$ representing regimes 1 and 2 (equations [35] and [40]) meet tangentially, and at the same time, two expressions of $f_1(r_R)$ representing regimes 1 and 2 (equations [36] and [37]) meet in the same manner. And at the threshold ratio $\bar{r}_R$, only the two expressions of $f_0(r_R)$ representing regimes 2 and 3 (equations [40] and [42]) meet tangentially to ensure continuity.

31. This paper uses a Levenberg-Marquardt method included in Mathcad 2000 Professional as a solving algorithm. It is a variant of the usual quasi-Newton method. To make the Levenberg-Marquardt method more efficient, Mathcad modifies the following points:
   (1) The first time the solver stops at a point that is not a solution, Mathcad adds a small random amount to all the variables and makes another attempt. This helps avoid dead ends in local minima and other points from which there is no preferred direction.
   (2) If inequality constraints are included as in the case of mine, Mathcad first solves the subsystem consisting of only the inequalities before adding the equality constraints and attempting a full solution.

Figure 10 The Determination of $\underline{r_R}$ and $\overline{r_R}$

[1] Boundary Conditions for $f_0(r_R)$



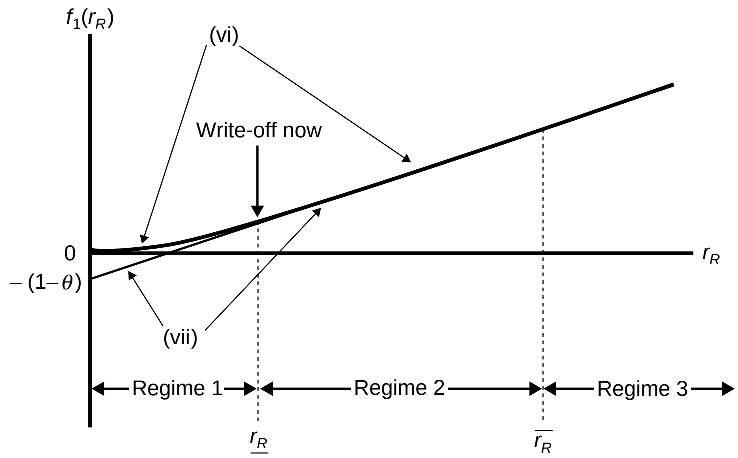
(iii): $f_0(r_R) = \dfrac{\lambda_0\lambda_1 B(r_R)^{\beta_1} - \lambda_1 C(r_R)^{\beta_2}}{\lambda_0 + \lambda_1}$ (equation [35])

(iv): $f_0(r_R) = D(r_R)^{\beta_3} + E(r_R)^{\beta_4}$

$+\dfrac{\lambda_1 r_R}{\delta_R(\mu + \lambda + \lambda_1 - \alpha_R)} - \dfrac{\lambda_1(1-\theta)}{\mu + \lambda + \lambda_1 - \alpha_L}$ (equation [40])

(v): $f_0(r_R) = \dfrac{r_R}{\delta_R} - 1$ (equation [42])

[2] Boundary Conditions for $f_1(r_R)$



(vi): $f_1(r_R) = \dfrac{\lambda_0\lambda_1 B(r_R)^{\beta_1} + \lambda_0 C(r_R)^{\beta_2}}{\lambda_0 + \lambda_1}$ (equation [36])

(vii): $f_1(r_R) = \dfrac{r_R}{\delta_R} - (1-\theta)$ (equations [37] and [43])

III. Numerical Analysis

This section reports the results of numerical analysis based on the theoretical model described in the last section. The baseline parameters are set in annual terms as follows:

$$\alpha_R = 0.02,\ \alpha_L = -0.02,\ \delta_R = 0.02,\ \sigma_R = 0.2,\ \sigma_L = 0.3,\ \rho = 0.0, \text{ and } \lambda = 0.0.$$

Here, the negative value of the expected growth rate of the write-off loss α_L reflects the banks' optimistic expectations about future conditions in the real estate market,[32] and the relative magnitude between σ_R and σ_L reflects larger volatility in the real estate market. As for the correlation term ρ and the probability of a jump in fund-raising costs λ, it is difficult to find "plausible" values. Thus, this paper tentatively set both values at zero in the baseline case and changed them over a wide range. Table 1 summarizes the qualitative results of numerical analysis, and I will examine the resulting details.

Table 1 Summary of Numerical Analysis

Exogenous variables			Endogenous variables	
			$\hat{r}_R$	$(\underline{r_R}, \overline{r_R})$
Underlying parameters				
Volatility	σ_R		+	+ +
	σ_L		+	+ +
Correlation	ρ		–	– –
Expected growth	α_R	Case (i)	+	+ +
		Case (ii)	–	– –
	α_L		–	– –
Shortfall rate	δ_R		+	+ +
Reputation problem				
Risk premium	λ		–	– –
Policy uncertainty				
Probability about the subsidy scheme	λ_1		····	+ +
	λ_0		····	– –
Portion of the subsidy	θ		····	– ±

Notes: 1. + indicates that the endogenous variable rises when the exogenous variable rises.
– indicates vice versa. ± denotes that the direction depends on other parameter values.
···· indicates that $\hat{r}_R$ does not depend on these parameters.
2. Case (i) denotes the case in which δ_R is held constant, while letting μ adjust freely.
Case (ii) denotes the case in which μ is held constant, while letting δ_R adjust freely.

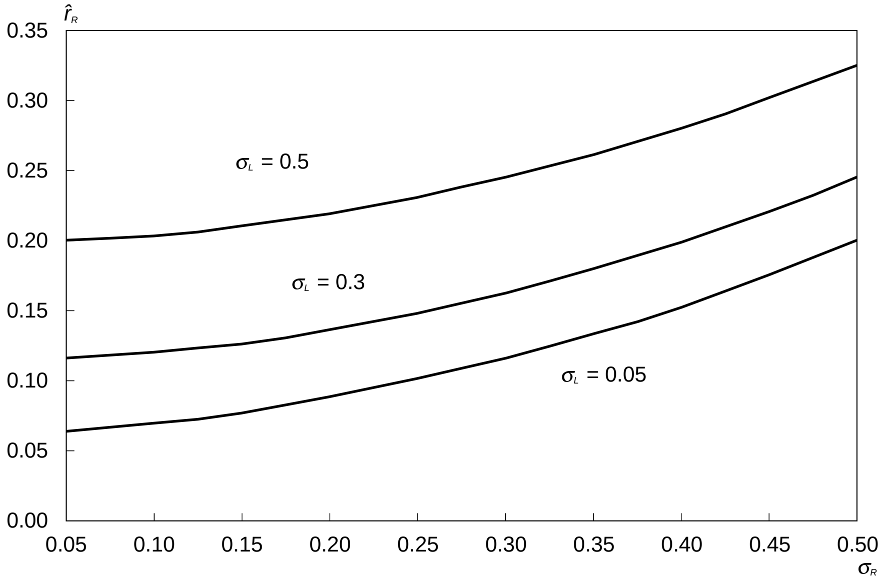
A. The Case without a Subsidy Scheme

Figure 11 [1] to [4] shows the dependence of the threshold ratio $\hat{r}_R = \hat{R}/\hat{L}$ on various parameter values. Before examining the detailed numerical results, note that a very large rate of return is required for the banks to immediately write off their nonperforming

32. With the benefit of hindsight, we know that this expectation was not realized, but as Cargill, Hutchison, and Ito (1997) argue, the MOF also had such an optimistic view of the real estate market, not to mention the Japanese banks.

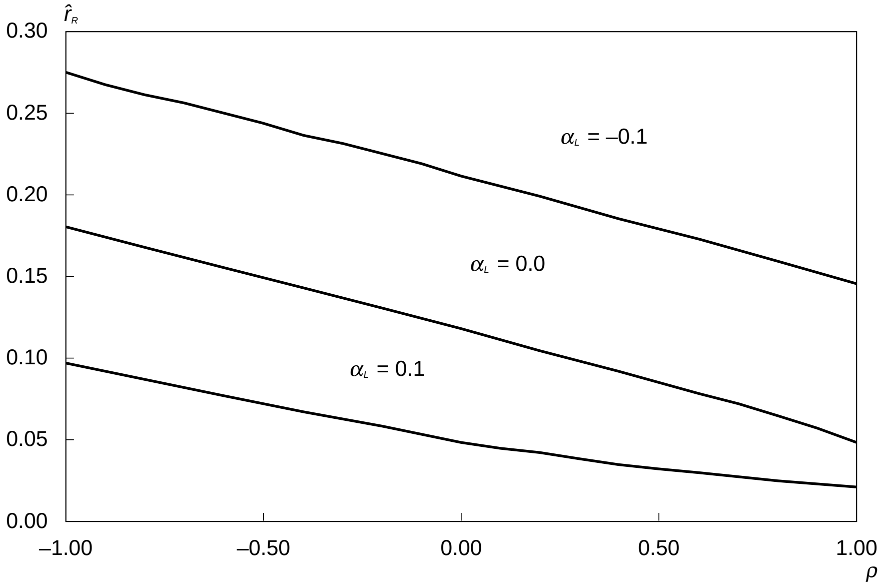
Figure 11 Threshold Ratio $\hat{r}_R$ as a Function of Parameters

[1] Dependence of $\hat{r}_R$ on σ_R and σ_L



Note: α_R = 0.02, α_L = –0.02, δ_R = 0.02, ρ = 0.0, and λ = 0.0.

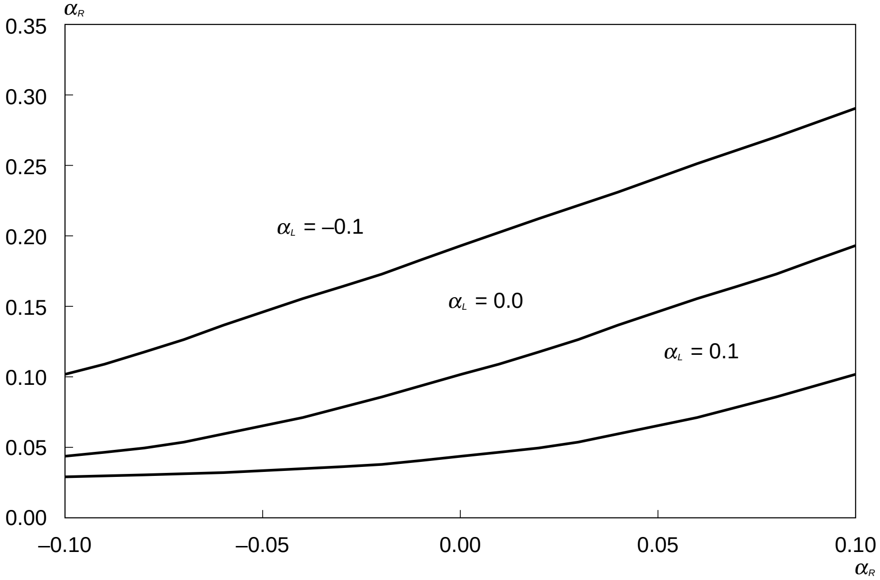
[2] Dependence of $\hat{r}_R$ on ρ and α_L



Note: α_R = 0.02, δ_R = 0.02, σ_R = 0.2, σ_L = 0.3, and λ = 0.0.

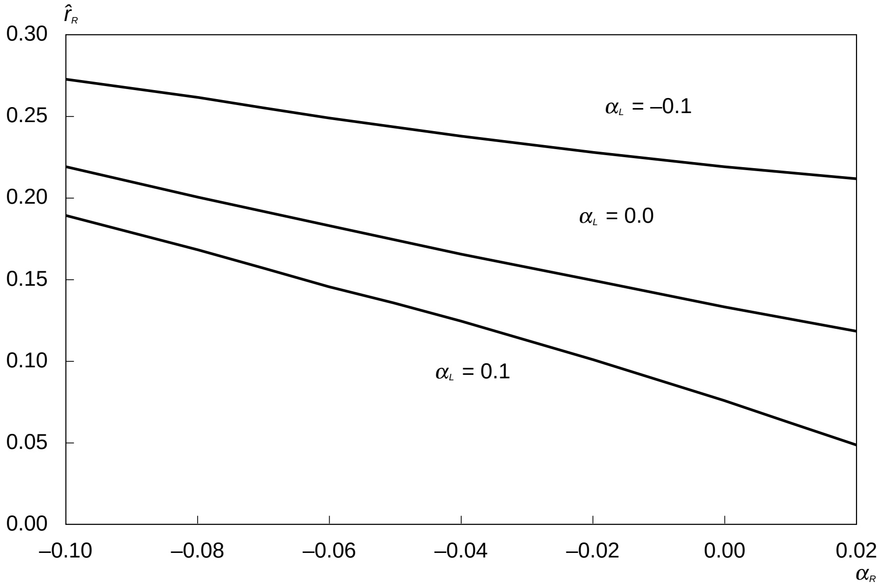
[3] Dependence of $\hat{r}_R$ on α_R and α_L

A. In the case of adjustable μ and fixed δ_R



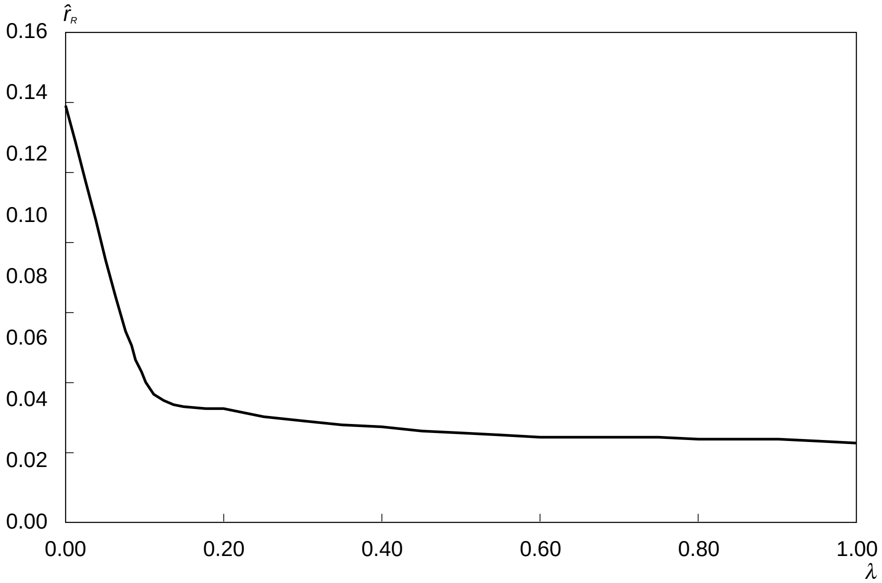
Note: δ_R = 0.02, σ_R = 0.2, σ_L = 0.3, ρ = 0.0, and λ = 0.0.

B. In the case of adjustable δ_R and fixed μ



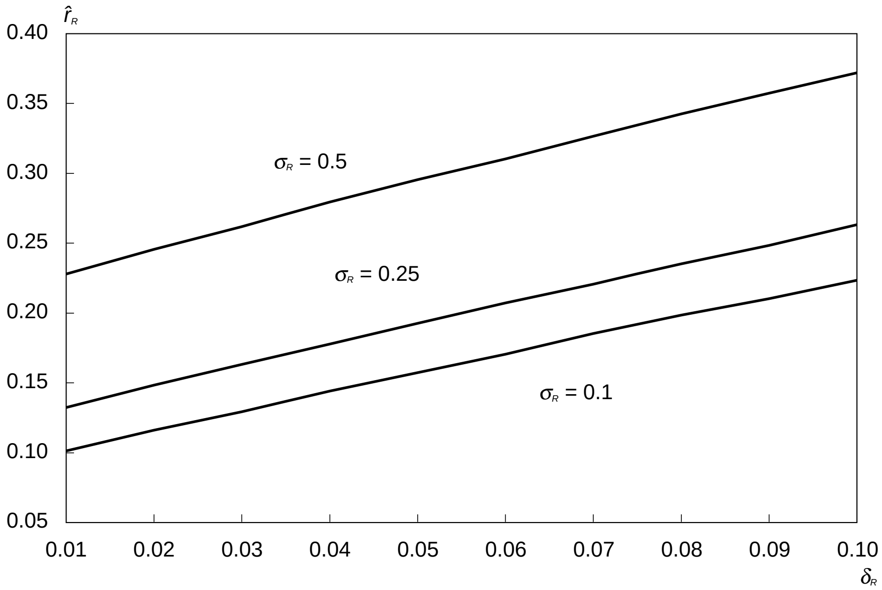
Notes: 1. μ = 0.04, σ_R = 0.2, σ_L = 0.3, ρ = 0.0, and λ = 0.0.
2. The range of α_R must satisfy the constraint $\delta_R \geq 0$.

[4] Dependence of $\hat{r}_R$ on λ



Note: Calculations are under the assumption that α_R and μ are fixed irrespective of the level of λ. Parameters are set as follows: μ = 0.04, α_R = 0.02, α_L = –0.02, δ_R = 0.02, σ_R = 0.2, σ_L = 0.3, and ρ = 0.0.

[5] Dependence of $\hat{r}_R$ on δ_R and σ_R: The Case of Adjusting μ and Fixed α_R



Note: α_R = 0.02, α_L = –0.02, σ_L = 0.3, ρ = 0.0, and λ = 0.0.

loans under various settings. For example, a value of $\hat{r}_R = 0.05$ means more than a 5 percent annual reinvestment rate of return if the loss is more than half of the nonperforming loans. Judging from the low current investment rate of return in the Japanese financial markets,[33] obtaining such a high return seems almost impossible in reality.

Now, let me check how the basic model works. First, Figure 11 [1] shows that the more uncertain the bank's economic environment becomes (larger values of σ_R and σ_L), the larger the threshold ratio $\hat{r}_R$ becomes.[34] This result holds for uncertainty in both underlying stochastic variables, reinvestment return R and the loss from the write-off L.

Second, consider the effect of a rise in the coefficient of correlation ρ between R and L. As shown in Figure 11 [2], a greater ρ results in a lower $\hat{r}_R$. This result directly follows the fact that the variance of r_R under the assumption of homogeneity of degree one can be expressed as $\sigma_R^2 - 2\rho\sigma_R\sigma_L + \sigma_L^2$ so that a larger value of ρ implies a smaller volatility, which increases the incentive to immediately write off.

Third, look at the effects of a change in expected growth parameters, α_R and α_L. It turns out that the effect of α_L is more straightforward than that of α_R. As shown in Figure 11 [2] and [3], the larger (smaller) the expected losses from future write-offs, the greater the (weaker) banks' incentive to immediately write off. This result is closely related to the familiar story about the forbearance policy taken by the government regarding write-offs of nonperforming loans in the Japanese banking industry.

For the effect of α_R, one should be careful about the results obtained under alternative assumptions regarding which parameter is adjustable, μ or δ_R. Under the assumption that μ changes exactly as much as α_R[35] (Figure 11 [3]A), the threshold ratio $\hat{r}_R$ rises as a result of a rise in α_R. Intuitively, the underlying reason is that the present value of write-off losses carried out in the future is discounted by the risk-adjusted discount rate μ, while the present value of the reinvestment of collected money is discounted by δ_R,[36] which is assumed to be constant. Hence, an increase in α_R (thus μ) reduces the present value of future write-off losses, but does not reduce its payoff.

On the other hand, under the alternative assumption that μ is constant while adjusting δ_R in response to changes in α_R (Figure 11 [3]B), the direction of the effect of rises in α_R (thus falls in δ_R) does the opposite of the preceding case. The reason can be found using the preceding logic; while the present value of the future write-off losses is invariant, reinvestment payoff will rise.

Fourth, how does an increase in the downward jump risk of the reinvestment return influence the optimal decision-making of rational banks? Generally, the effects of a positive value of the probability of the downward jump risk λ can be stated as follows. First, it reduces the expected rate of capital gain on R, which decreases the

33. For example, in December 1999 the average contracted interest rate on new loans and discounts was 1.82 percent and the yield of 10-year government bonds, which occupy a non-negligible part of Japanese banks' working assets, was 1.84 percent (both in annual terms).

34. As pointed out by Dixit and Pindyck ([1994]; see chapter 5, p. 153), an interesting point here is that write-off (investment) decisions are highly sensitive to volatility in write-off (project) values, irrespective of investors' or managers' risk preferences. Thus, if one assumes that a bank is risk-neutral, the same result will follow.

35. Recall the relationship $\mu \equiv \alpha_R + \delta_R$.

36. See equation (4).

value of waiting. On the other hand, it increases the variance of changes in R and thus raises the value of waiting. It turns out that under normal circumstances, the former effect is more dominant. Figure 11 [4] lays out the result that the former effect is much larger than the latter, thereby reducing the threshold ratio $\hat{r}_R$. Further, notice that a small increase in λ leads to a substantial decline in $\hat{r}_R$, prompting the bank to immediately write off.

Fifth, look at the effect of an increase in the shortfall rate δ_R on the threshold ratio of $\hat{r}_R$. Figure 11 [5] shows the result under the assumption that the risk-adjusted discount rate μ moves in response to a change in δ_R. In the present model, although the effect via μ has an influence through β (equation [19]), the direct effect of δ_R has a greater influence as shown by equation (23). Thus, the net effect increases the threshold ratio $\hat{r}_R$.

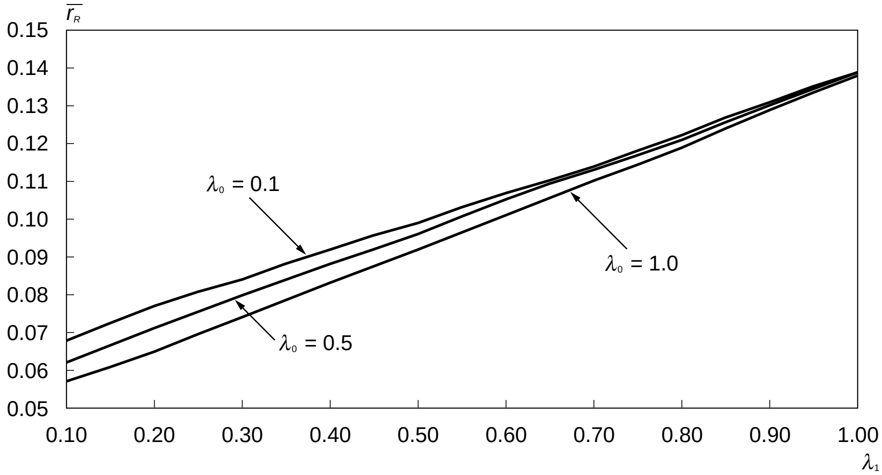
B. The Case with the Possible Implementation of a Subsidy Scheme

Now let me examine the case with the possible implementation of a subsidy scheme by the government.[37] As mentioned earlier, this case generalizes the last case in that λ_0, λ_1, and θ take on positive values. In fact, the threshold ratios $\overline{r_R}$ and $\underline{r_R}$ should converge to $\hat{r}_R$ as the values of λ_0, λ_1, and θ approach zero. Thus, in this subsection, I focus on the numerical results when one changes the values of λ_0, λ_1, and θ.

First, consider the situation where the scheme is not currently in effect. Figure 12 [1] shows that as the probability of the implementation λ_1 increases, the threshold ratio $\overline{r_R}$ increases. This result is very intuitive. The prospect of reduced write-off losses

Figure 12 Threshold Ratios $\overline{r_R}$ and $\underline{r_R}$ as a Function of Parameters

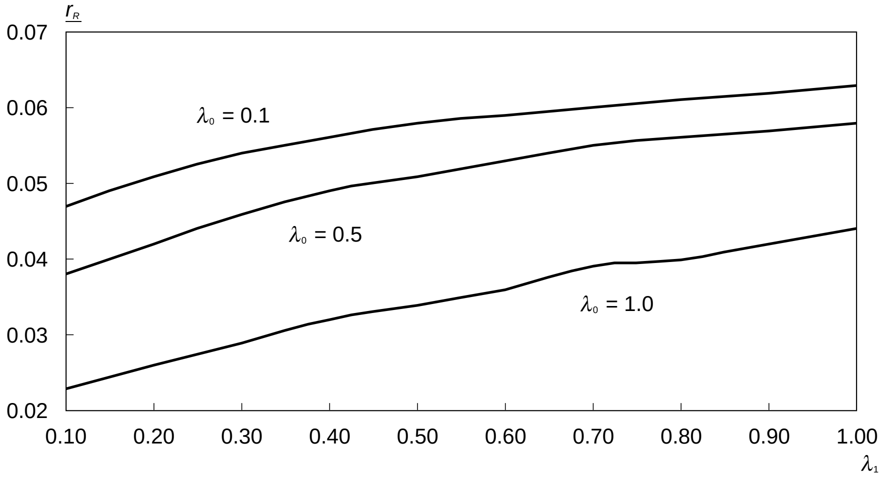
[1] Dependence of $\overline{r_R}$ on λ_1 and λ_0



Note: α_R = 0.02, α_L = –0.02, δ_R = 0.02, σ_R = 0.2, σ_L = 0.3, ρ = 0.0, λ = 0.1, and θ = 0.5.

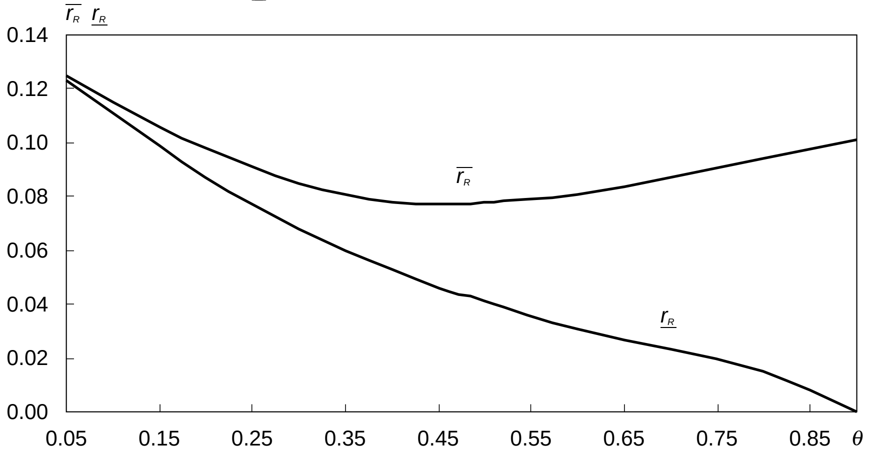
37. The results of the effects of changes in parameters other than those of policy uncertain on $\underline{r_R}$ and $\overline{r_R}$ are the same as in the case without the possible subsidy.

[2] Dependence of $\underline{r_R}$ on λ_1 and λ_0



Note: $\alpha_R = 0.02$, $\alpha_L = -0.02$, $\delta_R = 0.02$, $\sigma_R = 0.2$, $\sigma_L = 0.3$, $\rho = 0.0$, $\lambda = 0.1$, and $\theta = 0.5$.

[3] Dependence of $\overline{r_R}$ and $\underline{r_R}$ on θ



Note: $\alpha_R = 0.02$, $\alpha_L = -0.02$, $\delta_R = 0.02$, $\sigma_R = 0.2$, $\sigma_L = 0.3$, $\rho = 0.0$, $\lambda = 0.1$, $\lambda_0 = 0.3$, and $\lambda_1 = 0.3$.

inevitably increases the value of waiting. One of the most impressive aspects to note here is the magnitude of the effect of an increase in λ_1. When the bank is 100 percent certain[38] of the implementation of the subsidy scheme[39] in the next period, the threshold ratio $\overline{r_R}$ approximately doubles from when $\lambda_1 = 0.1$. Also, notice that even when the implementation of the scheme is being discussed and is still uncertain,

38. Note that the 100 percent probability of a subsidy enactment is just in the bank's expectation and does not imply that the policy will really be enacted within the next year.
39. One can rephrase this condition as saying "if the regulatory authorities can make a fully credible commitment to implement the subsidy scheme in the near future."

the effect is to strongly discourage the incentive to immediately write off. Another important point is that even in the absence of the scheme, the threshold ratio $\overline{r_R}$ is influenced by the probability λ_0 of the scheme's removal.

Next consider the situation in the presence of the scheme. Figure 12 [2] shows that the threshold ratio $\underline{r_R}$ decreases as λ_0 increases. This result is also intuitive because it is natural that the prospect of losing the scheme should induce the bank to immediately write off. Further, this figure shows that an increase in λ_1 also increases $\underline{r_R}$.

Now let me examine the effect of an increase in the ratio of the subsidy to the loss θ. Figure 12 [3] shows the dependence of $\overline{r_R}$ and $\underline{r_R}$ on the value of θ. This figure shows that both threshold ratios $\overline{r_R}$ and $\underline{r_R}$ are inversely related to θ for low values of θ. The effect is also much stronger on $\underline{r_R}$ than on $\overline{r_R}$.

An interesting point is that the threshold ratio $\overline{r_R}$ in the absence of a scheme is also influenced by θ. Numerical analysis suggests that there are two competing channels through which θ can influence $\overline{r_R}$. One channel increases the incentive to wait by lowering the last term of $f_0(r_R)$ (equation [40]). The other channel works in the opposite direction through a fall in D in the same equation. Which force is stronger depends on the range of the parameter θ. Generally, as shown in Figure 12 [3], when θ is small, the latter effect is greater than the former effect, but at some value of θ the net effect reverses the direction, and a rise in θ raises the threshold ratio $\overline{r_R}$.

IV. Some Policy Discussions

A. Implications for the Implementation of a Subsidy Scheme

Uncertainty about the implementation of a subsidy scheme will give banks an incentive to delay write-offs. In other words, if the government aims to accelerate banks' self-help efforts toward reducing their nonperforming loans, sound policy should have properties of low λ_1, high λ_0, and large θ. The government should immediately implement the subsidy scheme, giving banks a credible threat to immediately abolish it and pledging never to restore it, although we have difficulty imagining such a threat in reality.

B. Possible Implications for Monetary Policy

Until quite recently, the BOJ has controlled short-term interest rates such as the overnight call rate for purposes of influencing the real economy.[40] The overnight call rate is considered to be riskless. Under this paper's assumption of $\mu \equiv \alpha_R + \delta_R = r + \upsilon\rho(R, M)\sigma_R$, a rise in the call rate by the BOJ leads to a rise in the risk-adjusted discount rate μ of banks either via a rise in α_R or δ_R, other things being equal.

The analysis shows that a rise in the risk-adjusted discount rate μ via either α_R or δ_R makes banks more hesitant to immediately dispose of their nonperforming loans. Hence, if the central bank would like to accelerate banks' efforts to clean up their balance sheets, it should lower the interest rate to a point where banks regard it as

40. In March 2001, the BOJ changed the main operating target for money market operations from the overnight call rate to the outstanding balance of the current accounts at the BOJ.

long-lasting and so revise downward their perceived risk-adjusted discount rate. In this regard, recent monetary policy conducted by the BOJ is worthy of attention. To stimulate a depressed economy, the BOJ lowered short-term interest rates, including the call rate, to almost zero from 1995,[41] which might have some effect in raising the incentive for the banks to write off.[42]

V. Concluding Remarks

This paper has investigated how rational banks' optimal timing of write-offs is influenced by uncertainty stemming from various sources. A real options approach was employed to evaluate the value of the option to delay write-offs, that is, the value of forbearance policy.

Numerical analysis shows that, under normal circumstances, a very large rate of reinvestment return is required for the banks to immediately write off their nonperforming loans. Another important result is that uncertainty about the implementation of a subsidy scheme gives banks incentive to wait. This is contrary to the government's intention. If the government aims to encourage banks' self-help efforts toward reducing nonperforming loans, it should immediately enact a subsidy scheme, giving them a legitimate threat to immediately abolish it and pledging never to restore it in the future. This kind of policy is theoretically possible, but seems difficult to implement in reality.

41. Monetary easing of this nature can be characterized as a policy to "buy time," that is, to buy time until the structural policy bears fruit. In fact, when it lowered the official discount rate to 0.5 percent in September 1995, the BOJ Policy Board issued a statement stressing that such monetary easing would only be effective if it were accompanied by structural policies.

42. However, it is also true that the enlargement of profit margin resulting from the so-called zero interest rate policy gave banks room to retain their nonperforming loans. This paper pays no attention to the monetary policy effect on the interaction between the cost structure and the incentive to dispose of nonperforming bank loans. Thus, we should be careful when evaluating implications for monetary policy.

References

Boot, A., and S. Greenbaum, "Bank Regulation, Reputation, and Rents: Theory and Policy Implications," in C. Mayer and X. Vives, eds. *Capital Markets and Financial Intermediation*, Cambridge, United Kingdom: Cambridge University Press, 1993.

———, and A. Thakor, "Self-Interested Bank Regulation," *American Economic Review*, 83, 1993, pp. 206–212.

Cargill, T., "What Caused Japan's Banking Crisis?" in T. Hoshi and H. Patrick, eds. *Crisis and Change in the Japanese Financial System*, Boston/Dordrecht/London: Kluwer Academic Publishers, 2000, pp. 37–58.

———, M. Hutchison, and T. Ito, *The Political Economy of Japanese Monetary Policy*, Cambridge, Massachusetts: The MIT Press, 1997.

Claessens, S., S. Djankov, and L. H. P. Lang, "The Separation of Ownership and Control in East Asian Corporations," *Journal of Financial Economics*, 58, 2000, pp. 81–112.

Corbett, J., and J. Mitchell, "Banking Crises and Bank Rescues: The Role of Reputation," CEPR Discussion Paper, No. 2453, 2000.

Dixit, A., and R. Pindyck, *Investment under Uncertainty*, Princeton, New Jersey: Princeton University Press, 1994.

Hassett, K., and G. E. Metcalf, "Investment with Uncertain Tax Policy: Does Random Tax Policy Discourage Investment?" *Economic Journal*, 109, 1999, pp. 372–393.

Hoshi, T., "Naze Nihon ha Ryudousei no Wana kara Nogarerarenainoka? (Why Is the Japanese Economy Unable to Get Out of a Liquidity Trap?)," in M. Fukao and H. Yoshikawa, eds. *Zero Kinri to Nihon Keizai* (Zero Interest Rate and the Japanese Economy), Nihon Keizai Shimbunsha, 2000 (in Japanese).

———, "What Happened to Japanese Banks?" *Monetary and Economic Studies*, 19 (1), Institute for Monetary and Economic Studies, Bank of Japan, 2001, pp. 1–30.

Ito, T., "The Stagnant Japanese Economy in the 1990s: The Need for Financial Supervision to Restore Sustained Growth," in T. Hoshi and H. Patrick, eds. *Crisis and Change in the Japanese Financial System*, Boston/Dordrecht/London: Kluwer Academic Publishers, 2000, pp. 85–108.

Morck, R., and M. Nakamura, "Banks and Corporate Control in Japan," *Journal of Finance*, 54, 1999, pp. 319–339.

Ueda, K., "Causes of Japan's Banking Problems in the 1990s," in T. Hoshi and H. Patrick, eds. *Crisis and Change in the Japanese Financial System*, Boston/Dordrecht/London: Kluwer Academic Publishers, 2000, pp. 59–81.