Uncertainty, Monitoring Costs, and Private Banks' Lending Decisions in a Duopolistic Loan Market: A Game-Theoretic Real Options Approach

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This paper explores banks' entry decisions into a duopolistic loan market to shed light on the prolonged slump in the Japanese loan market in the 1990s. The game-theoretic real options approach is employed to analyze the effects of uncertainty on lending decisions. Special emphasis is given to the differences resulting from the alternative assumptions regarding whether the roles of leader and follower are interchangeable or predetermined. The theoretical model shows that when the roles are predetermined as in the case of the Japanese main bank system, both leader and follower banks have a greater incentive to wait until the loan demand condition improves sufficiently than when the roles are interchangeable. The numerical analysis shows that a rise in the demand volatility raises threshold values of current demand, which raises the incentive to wait for both leader and follower banks. In contrast, the direction of the effect of a change in the expected growth rate of demand depends on the assumptions regarding which parameter is adjustable, the risk-adjusted discount rate, or the dividend rate. The effects of a change in the probability of bankruptcy of a borrowing firm and the interest rate elasticity of loan demand are also examined.

Key words: Bank lending; Uncertainty; Entry decision; Monitoring costs; Duopolistic market; Real options; Stochastic game

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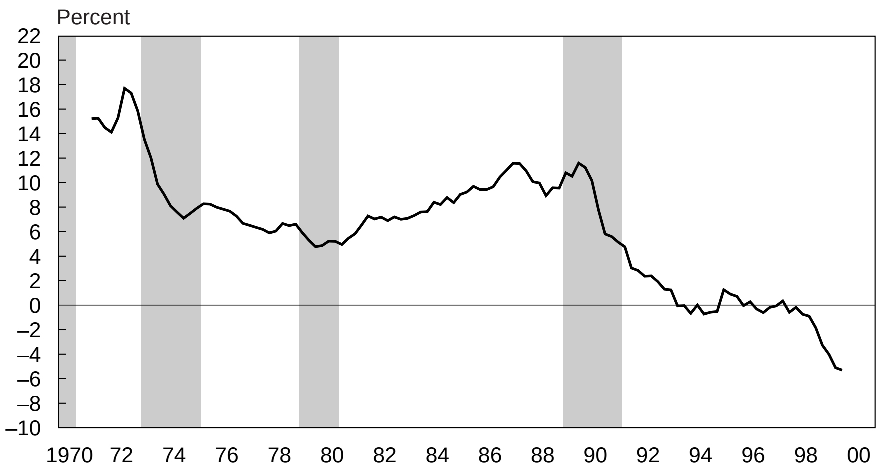
I. Introduction

The Japanese economy has experienced unprecedented economic fluctuations since 1985. In particular, a significant surge in private bank[1] loans during the so-called bubble period, a subsequent sharp fall starting in 1991, and a declining trend that continues up to today are widely recognized as noteworthy phenomena.[2] Figure 1[3] shows that the change in loans and discounts as a percentage of nominal GDP has been fluctuating below 0 percent since around 1993.

Figure 1 Increase in Loans and Discounts by Domestically Licensed Banks



Notes: 1. The statistic is in terms of the percent ratio of an increase in loans and discounts by domestically licensed banks to nominal GDP. It is seasonally adjusted by taking the three-quarter moving average.
2. Shaded intervals indicate the period during which the official discount rate was raised.
3. The statistic includes both banking accounts and trust accounts of domestically licensed banks.

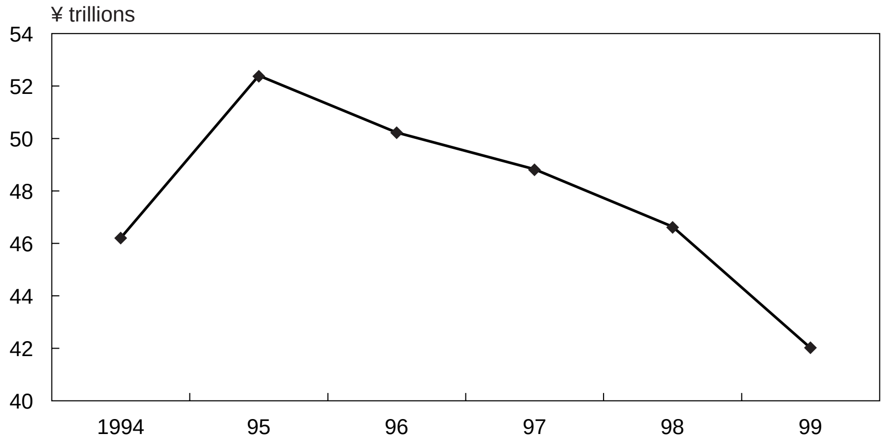
Source: Bank of Japan, *Financial and Economic Data CD-ROM.*

There are two aspects to the recent slump in bank lending. One is the acceleration of loan repayments by corporations aiming at reducing interest-bearing liabilities as part of restructuring plans. The other is a decrease in new bank loans reflecting both a decline in loan demand by corporations and a prudent attitude by banks toward

1. I use the word "private" to distinguish between domestically licensed (private) banks and government-related (public) financial organizations such as Development Bank of Japan, Housing Loan Corporation, and Japan Finance Cooperation for Small Business. The financial organizations in the latter category are expected to play the role of funding the fields that are not profitable enough in the perspective of private banks, but provide benefits from a social point of view.
2. For more details, see Ogawa and Kitasaka (2000), for example.
3. One interesting point in Figure 1 is that although previously a decrease in bank loans relative to the size of nominal GDP was typically seen during periods of tight monetary policy, the recent slump occurred despite unprecedented easy monetary policy.

extending new loans. Figure 2 shows a steady downward trend in new loans for equipment funds after a temporary pickup in 1995.[4] This paper focuses on the latter aspect, that is, the decline in new loans.

Figure 2 New Loans for Equipment Funds by Domestically Licensed Banks



Notes: 1. The statistic is new loans for equipment funds extended by domestically licensed banks.
2. The statistic includes both banking accounts and trust accounts of domestically licensed banks.

Source: Bank of Japan, *Financial and Economic Data CD-ROM.*

Many plausible causes have been pointed out for the slump in loan demand, besides the direct impact of the prolonged recession. For example, many analysts cite a downward shift in investment planning by nonfinancial corporations. Also, it is widely believed that a series of liberalization measures in the Japanese capital markets have prompted a switch from indirect financing to direct financing such as equity financing and the issue of corporate bonds, particularly among leading corporations.

Turning to the supply side of bank loans, some argue that one of the reasons for the slump in bank lending lies in the fact that real estate has been extensively used as collateral, which is particularly the case with new borrowers.[5] Related to this point, it is often pointed out that, during the bubble period, banks did not have to make rigorous monitoring efforts due to an almost religious faith that potential losses in the future could be sufficiently covered by the real estate that borrowers put up as collateral.

In some sense, monitoring ability can be viewed as fixed capital that takes a long time to accumulate. Hence, if accumulation of monitoring ability is neglected for a prolonged period, then its recovery (re-accumulation) cannot be done in a short period and thus banks will suffer larger costs in judging the creditworthiness of

4. This statistic is available only from 1994.
5. See Higano (1987) and Ogawa and Kitasaka (2000).

potential borrowers.[6] Thus, a delay in accumulating monitoring ability might have something to do with the recent overly prudent attitude of banks toward extending loans to borrowers.

As argued by Aoki (1994), transactions involving funds between firms planning to undertake projects and intermediaries like banks entail a high degree of informational asymmetry.[7] To overcome this problem,[8] there need to be some mechanisms to assess the creditworthiness of the projects. Monitoring is one of the mechanisms and, from the perspective of banks, it incurs sunk costs in the sense that they cannot be retrieved once they are actually paid.[9]

To be more specific, monitoring has three categories. The first one is *ex ante* monitoring that aims to assess borrowers' creditworthiness regarding projects and to screen them. The second one is interim monitoring, the purpose of which is to closely observe management in order to alleviate the problem of moral hazard. The last one is *ex post* monitoring, which tries to verify borrowers' financial condition and apply appropriate punitive and corrective actions if necessary. Among these categories, this paper focuses on the first, *ex ante* monitoring. Thus, this paper concentrates on the entry decisions by banks as to whether they extend a new loan or not.

As emphasized by Sheard (1994), among the most salient features in Japanese banks' monitoring activities is that most large firms have maintained a close relationship with a bank. Such close bank-corporation ties are often termed the "main bank system." The main bank is, in most cases, a principal shareholder in the firm and plays a decisive role in monitoring it.

Monitoring is sometimes delegated to the main bank.[10] In the words of Higano (1987), the main bank plays a role of "bell cow" or "bellwether" in that other banks follow its decisions, because information regarding the screening process effected by the main bank is revealed (or sent as a signal) via its actual lending decisions.[11]

Another important aspect of actual lending in Japan is that the main bank has the largest loan share, but often it is not the sole lender. Thus, the loan market for a specific potential borrower can be reasonably approximated to be an oligopolistic market.[12] Also, it is widely known that the loan syndicate led by the main bank is hierarchical in terms of proportionate loan shares. In that scheme, the main

6. In this regard, the credit guarantee system is considered to have facilitated lending to small and medium-sized firms by reducing monitoring costs. But the possibility should not be overlooked that the system itself weakened the banks' incentive to accumulate monitoring ability.
7. See Akerlof (1970) for the original discussion on informational asymmetry.
8. It should be noted here that even if the problem of informational asymmetry is completely eliminated, uncertainty inherent in projects themselves remains.
9. In other words, monitoring efforts are irreversible. Dixit and Pindyck (1994) define the term "sunk costs" or "irreversible" as follows: *investment expenditures are sunk costs or irreversible when they are firm or industry specific*. For typical examples, they cite most investments in marketing and advertising.
10. The delegated monitoring theory was first developed by Diamond (1984). It states that monitoring typically involves increasing returns to scale, implying that specialized banks are more efficient in handling it. Therefore, individual lenders tend to delegate monitoring activity instead of performing it themselves.
11. It should be noted, however, that this information activity entails the problem of free-riding by the follower bank. One possible solution is to impose fees on the follower bank to internalize the externality. Although in the Japanese case this kind of information fee has not been explicit, it is often said that the monopolization of some profitable businesses such as domestic and foreign exchange operations by the leader bank has played the role.
12. In words of Sheard (1994), there is *exclusivity in monitoring with non-exclusivity in lending.*

bank decides loan shares in advance, and then the follower banks judge whether participation in the loan syndicate is really beneficial to them. Hence, if a researcher takes a perfectly competitive or monopolist market structure as given in analyzing bank lending decisions, he or she might miss some important aspects.

Further, under the assumption of uncertainty and irreversibility, it is natural to think that banks[13] should consider the option to wait until economic conditions improve sufficiently. This is a typical setting of a so-called real options approach first applied by McDonald and Siegel (1984) and later extensively reviewed by Dixit and Pindyck (1994).

As emphasized by Trigeorgis (1993) and Dixit and Pindyck (1994), among others, the options approach helps explain why actual investment decisions by the business sector cannot be explained by conventional wisdom such as the net present value (NPV) approach. In reality, firms invest only in projects that are expected to yield a return well in excess of the required rate of return.[14]

The adoption of the real options framework is likely to provide an important insight into the role of uncertainty and sunk costs in the recent slump in the Japanese bank loan market. Specifically, within the framework, one can see the change in the value of the option to wait as one changes the values of such parameters as sunk costs (monitoring costs), the risk-adjusted discount rate, uncertainty in the future demand conditions (volatility), the expected growth rate of demand, and the subjective probability of bankruptcy of the borrowing firm. The last three parameters characterize the stochastic process.[15] Thus, one can numerically assess the optimal lending decisions directly in terms of uncertainty and monitoring ability.

Motivated by the above discussion, this paper attempts to analyze lending (entry) decisions employing the real options approach. The market structure I assume is a duopolistic market in which the leader bank makes entry decisions taking the reaction of followers into consideration and then, given the leader bank's action, the follower bank determines whether to enter the loan market or not. Note that although this paper is motivated by the literature on the main bank system, the aim is not necessarily to directly analyze the main bank system itself, but to examine the role of uncertainty in extending a new loan in a duopolistic loan market setting.

The rest of the paper is organized as follows. Section II describes the basic theoretical framework of the game-theoretic real options approach. Section III numerically analyzes lending decisions in a duopolistic loan market. Section IV links the insights derived from the real options approach to episodes of the recent bank lending situation in Japan. Section V concludes the study.

13. Throughout the paper, uncertainty indicates that the best one can do is to assess the subjective probabilities of the alternative outcomes that entail greater or smaller profit (or loss) for a project.
14. On the downside, firms prefer to stay in business for a long time despite the situation that operating profit is well below operating costs so that they lose.
15. Actually, I regard the shift parameter of loan demand as a stochastic variable instead of the return itself because I analyze a duopolistic loan market. To be specific, I adopt a combined geometric Brownian motion and (Poisson) jump process as an underlying stochastic process.

II. Theoretical Framework

A. Basic Setup

Introduction of an oligopolistic market structure into a stochastic dynamic model usually causes many practical difficulties. In fact, applications of the game-theoretic option theory are among the most recent ones. Smets (1993) developed a very simplified version of this kind of model in which there are a leader and a follower[16] to analyze the decision-making between exporting and foreign direct investment.

Despite the modeling difficulties, in its easiest form, the essence of the model is actually not too difficult to state. The existence of uncertainty and irreversibility implies that there is some value to an option to wait, and the higher the degree of uncertainty, the greater the hesitancy both players have. The fear of preemption by a rival, however, prompts the leader to make decisions without delay. Which of these considerations is more relevant depends on the underlying parameters and the current state of the stochastic variable.

Also, whether the roles of leader and follower are interchangeable or predetermined is of particular importance to modeling. This is because if the roles are predetermined, the follower bank cannot enter the loan market before the leader has done so, and the leader recognizes the value of the option to wait from this source. I describe each case in turn.

The theoretical framework basically follows Smets (1993) and Dixit and Pindyck (1994), but the following modifications are made to enrich implications for bank lending behavior in a duopolistic market:

(1) The demand curve is specified as downward-sloping and its demand elasticity is constant in any region.
(2) *Ex ante* loan shares can be arbitrarily changed to investigate the relationship between *ex ante* loan shares and threshold values of current demand for entry.
(3) Sunk (monitoring) costs of both banks can be separately specified to explicitly take the leader bank's informational cost advantage into consideration.
(4) A combined geometric Brownian motion and Poisson downward jump process is adopted to the demand shift parameter instead of the standard geometric Brownian motion to take the possibility of bankruptcy of the borrowing firm[17] into consideration.

Now, let me consider the value of the follower bank contemplating entry to the loan market.[18] Let $v_f(\Pi_f)$ denote the present discounted value of the follower bank's future cash flow net of operating cost from actual lending $\Pi_f \equiv r_L L_f$, where r_L is the interest rate (net of operating costs) that is common to both leader and follower banks and L_f is the amount of a new loan extended by the follower bank.

16. For other works on the trade-off between the strategic incentive to invest early in an oligopoly and the value of flexibility under uncertainty, see Appelbaum and Lim (1985) and Spencer and Brander (1992), for example.
17. In fact, after the bursting of the bubble economy, the liabilities of bankrupt corporations as a proportion of total financial liabilities held by nonfinancial corporations rose from 0.25 percent in 1990 to 2.5 percent in 1998 according to a survey by Teikoku Databank, Ltd. In 1999, the figure dropped to about 0.6 percent due to the adoption of stabilization measures under the credit guarantee system.
18. This "backward solution" is a familiar method in analyzing the dynamic duopolistic strategy.

I assume that r_L is specified as

$$r_L = Y(L_l + L_f)^{-\varepsilon}, \tag{1}$$

where Y denotes the shift parameter of loan demand, L_l the amount of a loan extended by the leader bank, and ε the inverse of the interest rate elasticity of loan demand.[19] Here, by loan demand I mean demand by a specific potential borrower.[20] Demand uncertainty is assumed to follow the following combined geometric Brownian motion and Poisson downward jump process:

$$dY = \alpha Ydt + \sigma Ydz - Ydq, \tag{2}$$

where α denotes the expected growth rate parameter that is relevant only in the Brownian motion part, σ the volatility parameter, dz the increment of the standard Wiener process, and dq the increment of a Poisson process with mean arrival time rate λ. For computational facility, I assume that $E\,[(dz)(dq)] = 0$ holds. Equation (2) states that if an event occurs, Y falls by some fixed ratio $\phi(0 \leq \phi \leq 1)$.[21]

Equation (2) implies

$$d\Pi_f = \alpha\Pi_f dt + \sigma\Pi_f dz - \Pi_f dq, \tag{3}$$

since L_l, L_f, and ε are assumed to be fixed. Here, note that it does not make sense unless $v_f(0) = 0$ holds because if profits are zero in the geometric Brownian motion, they will remain zero forever.

It is important to note that the expected rate of change in Π_f is not α as in the case of the standard geometric Brownian motion, but

$$\frac{E\,[d\Pi_f/\Pi_f]}{dt} = \alpha - \lambda\phi. \tag{4}$$

Hence, given the value of ϕ, an increase in λ decreases the expected rate of capital gains on Π_f by increasing the chance of a sudden downward jump in Π_f. Also, note that since a Poisson event occurs infrequently, most of the time the variance of $d\Pi_f/\Pi_f$ over a short interval of time dt is just that of the part governed by the Brownian motion $\sigma^2 dt$. If the jump happens, however, it causes a large deviation, thus its contribution to the variance cannot be neglected.

19. In the original model by Smets (1993), demand is assumed to be sufficiently elastic to ensure capacity production, implying that total output is either zero, one, or two depending on the number of active firms.
20. In this paper, for simplicity, I assume that the borrowing firm is passive in that it does not have any bargaining power in making loan contracts. Introducing the game-theoretic interaction between borrowers and lenders is one of my future tasks.
21. Formally, one can write

$$dY = \begin{cases} \alpha Ydt + \sigma Y\sqrt{dt} & \text{with probability } (1-\lambda dt)/2 \\ \alpha Ydt - \sigma Y\sqrt{dt} & \text{with probability } (1-\lambda dt)/2 \\ -\phi Y & \text{with probability } \lambda dt \end{cases}.$$

As will be discussed later, the case of $\phi = 1$ can be regarded as the case of bankruptcy of the borrower.

B. The Case of Interchangeable Roles of Leader and Follower

The first step to solve this kind of oligopolistic model is to find a decision-making rule for the follower, assuming that the leader has already entered the market. The second step is to consider the entry decision by the leader taking account of the follower's response.

Now let $F_f(\Pi_f)$ denote the follower's value of the option to lend. For simplicity, I assume that there is no fixed finite time horizon. The Bellman equation for the optimal lending rule can be written as

$$\rho F_f(\Pi_f) = \max_{\theta} E\left[\frac{1}{dt} dF_f(\Pi_f)\right], \tag{5}$$

where ρ denotes the risk-adjusted discount rate and θ the control (decision) variable of the follower.[22] Applying Ito's Lemma for the combined geometric Brownian motion and jump process[23] yields

$$\begin{aligned}\rho F_f(\Pi_f)dt = \alpha\Pi_f F_f'(\Pi_f)dt + \frac{1}{2}\sigma^2\Pi_f^2 F_f''(\Pi_f)dt \\ - \lambda\{F_f(\Pi_f) - F_f[(1-\phi)\Pi_f]\}dt,\end{aligned} \tag{6}$$

22. Formally, derivation of equation (5) goes as follows. First, the original form of the Bellman equation can be expressed as

$$F_f(\Pi_f, t) = \max_{\theta}\left\{\frac{1}{1+\rho\Delta t} E[F_f(\Pi_f', t+\Delta t)|\Pi_f, \theta]\right\},$$

where Π_f' denotes the value of Π_f a time Δt later.

Multiplying this equation by $(1 + \rho\Delta t)$ yields

$$\rho F_f(\Pi_f, t) = \max_{\theta} E\left[\frac{1}{dt} dF_f(\Pi_f)\right],$$

where I let Δt approach zero and $E[(1/dt)dF_f]$ denotes the limit of $E[\Delta F_f/\Delta t]$. If one assumes that the time horizon is infinite, then the preceding equation becomes an ordinary differential equation with Π_f as its only independent variable. Thus, equation (5) follows.

23. If the stochastic process is

$$dx = a(x,t)dt + b(x,t)dz + g(x,t)dq,$$

then the expected value of the change in any function $H(x, t)$ can be given by

$$E[dH] = \left[\frac{\partial H}{\partial t} + a(x,t)\frac{\partial H}{\partial x} + \frac{1}{2}b^2(x,t)\frac{\partial^2 H}{\partial x^2}\right]dt + E_\phi\{\lambda[H(x+g(x,t)\phi, t) - H(x,t)]\}dt,$$

where ϕ is the size of the jump when the event happens. For more technical details, see Dixit and Pindyck (1994). In general, inclusion of a jump process is advantageous because it enables one to describe a more realistic situation, but there are some practical problems. The most important problem is that the adoption of a jump process makes building a perfect hedge impossible. Thus, it is not possible to build a riskless portfolio, which is used in Black-Scholes-Merton type contingent claims analysis. This is why I use dynamic programming with an exogenous risk-adjusted discount rate ρ instead of contingent claims analysis. To avoid such a disadvantage, one sometimes assumes that the jump-risk is nonsystematic, that is, uncorrelated with the market portfolio. That enables one to construct a risk-free portfolio. In such a case, equation (6) can also be derived by contingent claims analysis.

where $F'_f(\Pi) \equiv \partial F_f / \partial \Pi_f$ and $F''_f(\Pi) \equiv \partial^2 F_f / \partial \Pi_f^2$. Equation (6) can be rewritten as

$$\begin{aligned} &\frac{1}{2}\sigma^2 \Pi_f^2 F''_f(\Pi_f) + (\rho - \delta)\Pi_f F'_f(\Pi_f) \\ &- (\rho + \lambda) F_f(\Pi_f) + \lambda F_f[(1 - \phi)\Pi_f] = 0. \end{aligned} \tag{7}$$

Note that in deriving equation (7), I use the relationship of $\rho \equiv \alpha + \delta$, where δ denotes the dividend rate. As suggested by Dixit and Pindyck (1994), in such a case, the solution[24] has a form

$$F_f(\Pi_f) = A(\Pi_f)^{\beta_1}, \tag{8}$$

where A and β_1 are constants to be determined. The expression for $\beta_1 > 1$ can be found by solving the following fundamental quadratic equation:

$$\frac{1}{2}\sigma^2 \beta(\beta - 1) + (\rho - \delta)\beta - (\rho + \lambda) + \lambda(1 - \phi)^\beta = 0. \tag{9}$$

Unfortunately, however, one cannot find any closed-form solutions for equation (9). Hence, I consider a special case of $\phi = 1$, which means that once the jump happens, it removes the full value of Π_f and remains at zero forever. That is, one can think of the event as an abrupt bankruptcy. In such a special case, the two roots β_1 and β_2 of equation (9) are given by

$$\begin{cases} \beta_1 \equiv \left[\dfrac{1}{2} - \dfrac{\rho - \delta}{\sigma^2}\right] + \sqrt{\left[\dfrac{\rho - \delta}{\sigma^2} - \dfrac{1}{2}\right]^2 + \dfrac{2(\rho + \lambda)}{\sigma^2}} > 1 \\ \beta_2 \equiv \left[\dfrac{1}{2} - \dfrac{\rho - \delta}{\sigma^2}\right] - \sqrt{\left[\dfrac{\rho - \delta}{\sigma^2} - \dfrac{1}{2}\right]^2 + \dfrac{2(\rho + \lambda)}{\sigma^2}} < 0. \end{cases} \tag{10}$$

Now, consider the boundary conditions[25] that must be satisfied at a threshold value $\overline{\Pi}_f$ for entry to close the model. First, the value-matching condition can be written as

$$F_f(\overline{\Pi}_f) = v_f(\overline{\Pi}_f) - L_f I_f, \tag{11}$$

where $v_f(\overline{\Pi}_f) = \overline{\Pi}_f / \delta$ and I_f denotes the sunk costs[26] per unit of loan. Equation (11) states that at the threshold value $\overline{\Pi}_f$, the value of the option should equal the net value from exercising it.

24. Strictly speaking, one must write the solution as

$$F_f(\Pi_f) = A_1(\Pi_f)^{\beta_1} + A_2(\Pi_f)^{\beta_2} \quad (\beta_1 > 1 \text{ and } \beta_2 < 0)$$

instead of equation (8). The condition of $F_f(0) = 0$, however, enables one to omit the second term on the right-hand side of the equation.

25. Generally speaking, the value-matching and smooth-pasting conditions, together with the condition $F_f(0) = 0$, consist of so-called boundary conditions.

26. In this paper, I assume that monitoring costs account for nearly all of the sunk costs.

Second, the smooth-pasting condition is

$$F_f'(\overline{\Pi}_f) = v_f'(\overline{\Pi}_f), \tag{12}$$

which implies that $F_f(\Pi_f)$ and $v_f(\Pi_f) - L_f I_f$ should meet tangentially at the threshold value $\overline{\Pi}_f$. Specifically, each condition can be written as

$$\begin{cases} A(\overline{\Pi}_f)^{\beta_1} = \dfrac{\overline{\Pi}_f}{\delta} - L_f I_f \\ \beta_1 A(\overline{\Pi}_f)^{\beta_1 - 1} = \dfrac{1}{\delta}. \end{cases} \tag{13}$$

Solving $\overline{\Pi}_f$ from these conditions yields

$$\overline{\Pi}_f = \frac{\beta_1}{\beta_1 - 1} \delta L_f I_f. \tag{14}$$

Now let $\overline{Y}_f$ be a threshold value of Y at which the follower decides to enter the market. $\overline{Y}_f$ is found by

$$\overline{Y}_f = \frac{\beta_1}{\beta_1 - 1} \frac{\delta I_f}{(L_l + L_f)^{-\varepsilon}}. \tag{15}$$

Thus, the follower's value of the option to lend as a function of Y can be summarized as

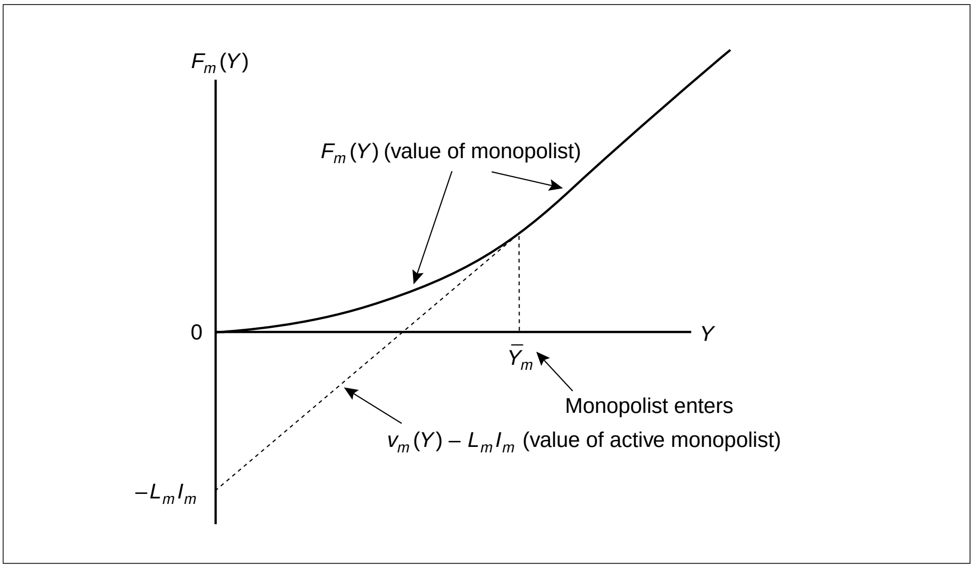
$$\begin{cases} \text{if } Y \geq \overline{Y}_f, \text{ then } F_f(Y) = \dfrac{1}{\delta} L_f Y (L_l + L_f)^{-\varepsilon} - L_f I_f. \\ \text{otherwise,} \qquad F_f(Y) = \left(\dfrac{Y}{\overline{Y}_f}\right)^{\beta_1} \left[\dfrac{1}{\delta} L_f \overline{Y}_f (L_l + L_f)^{-\varepsilon} - L_f I_f\right]. \end{cases} \tag{16}$$

Here, it should be noted that if the amount of the loan extended by the leader is zero ($L_l = 0$), the follower's value can be regarded as the monopolist's value denoted $F_m(Y)$. Figure 3 depicts the entry decision by this monopolist bank. At the threshold value of $\overline{Y}_m (= \overline{Y}_f)$, $F_m(Y)$, and $v_m(Y) - L_m I_m$ meet tangentially, which is required by boundary conditions (11) and (12).

Next, consider the lending decision by the leader. If $Y \geq \overline{Y}_f$, then the follower will lend immediately and the leader's cash flow will be $L_l Y (L_l + L_f)^{-\varepsilon}$. On the other hand, if $Y < \overline{Y}_f$, then the follower will prefer to wait until period T when $\overline{Y}_f$ is first hit. Hence, the leader will have cash flow equivalent to $L_l Y L_l^{-\varepsilon}$, implying that its expected (gross) value before netting out the sunk costs can be expressed as

$$E\left[\int_{s=0}^{T} e^{-\rho s} L_l Y L_l^{-\varepsilon} ds\right] + E[e^{-\rho T}] \frac{L_l \overline{Y}_f (L_l + L_f)^{-\varepsilon}}{\delta}. \tag{17}$$

Figure 3 Entry Decision by a Monopolist Bank



Thus, the leader's value of the option to lend as a function of Y can be summarized as

$$\begin{cases} \text{if } Y \geq \bar{Y}_f, \text{ then } F_l(Y) = \dfrac{1}{\delta} L_l Y (L_l + L_f)^{-\varepsilon} - L_l I_l. \\ \text{otherwise,} \qquad F_l(Y) = \dfrac{1}{\delta} L_l Y L_l^{-\varepsilon}\left[1 - \left(\dfrac{Y}{\bar{Y}_f}\right)^{\beta_1 - 1}\right] \\ \qquad\qquad\qquad + \left(\dfrac{Y}{\bar{Y}_f}\right)^{\beta_1}\left[\dfrac{1}{\delta} L_l \bar{Y}_f (L_l + L_f)^{-\varepsilon}\right] - L_l I_l. \end{cases} \tag{18}$$

And another threshold value of Y denoted $\bar{Y}_l$ at which the leader makes an entry decision must satisfy the condition

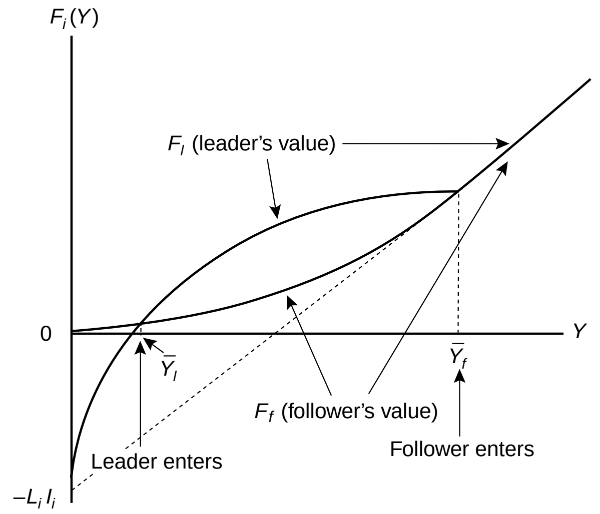
$$F_l(\bar{Y}_l) = F_f(\bar{Y}_l) > 0, \tag{19}$$

which shows that at $\bar{Y}_l$, both banks are indifferent about which role they assume, leader or follower.[27]

27. Note that in the region of Y, which satisfies $F_l(Y) < F_f(Y)$, the leader prefers to wait because in such a region the leader does not have an incentive to become a leader.

The decision-making rules by both banks are described in Figure 4. The figure shows the interaction between the leader and the follower banks when they are identical except for the interchangeable roles they assume. Two curves representing the leader's and follower's values cross each other at $\overline{Y}_l$ and meet tangentially at $\overline{Y}_f$.

Figure 4 The Case of Interchangeable Roles of Leader and Follower



Note: The figure is drawn under the assumption that both banks are identical except for the interchangeable (random) roles they assume.

C. The Case of Predetermined Roles of Leader and Follower

In this subsection, the strategic interaction is modified as follows. First, before actually paying the sunk (monitoring) costs, the predetermined leader declares the *ex ante* loan shares (amounts) of each bank. Then, both banks judge strategically whether they will enter the market based on current demand (return) conditions.

In this setting, the leader recognizes the values of two additional options.[28] First, there is an option to wait over a range of small values of Y, whose value should tangentially meet the leader's value of the option to lend $F_l(Y)$ at a threshold value $\overline{Y}_1$.[29] Its value denoted $G(Y)$ can be written as[30]

$$G(Y) = BY^{\beta_1}, \tag{20}$$

where B is a constant to be determined and β_1 is the positive root given by equation (10).

28. See Dixit and Pindyck (1994) for this idea.
29. Clearly, the condition $\overline{Y}_l < \overline{Y}_1 < \overline{Y}_f$ must hold.
30. Formally, this is written as $G(Y) = B_1Y^{\beta_1} + B_2Y^{\beta_2}$ ($\beta_1 > 1$ and $\beta_2 < 0$). Imposing the condition $G(0) = 0$ yields equation (20), however.

The boundary conditions are

$$\begin{cases} F_l(\overline{Y}_1) = G(\overline{Y}_1) \\ F_l'(\overline{Y}_1) = G'(\overline{Y}_1), \end{cases} \tag{21}$$

where $F_l(Y)$ corresponds to the case of $Y < \overline{Y}_f$ in equation (18). To be specific, the boundary conditions can be rewritten as

$$\begin{cases} \dfrac{1}{\delta} L_l \overline{Y}_1 L_l^{-\varepsilon}\left[1 - \left(\dfrac{\overline{Y}_1}{\overline{Y}_f}\right)^{\beta_1 - 1}\right] + \left(\dfrac{\overline{Y}_1}{\overline{Y}_f}\right)^{\beta_1}\left[\dfrac{1}{\delta}\overline{Y}_f L_l (L_l + L_f)^{-\varepsilon}\right] - L_l I_l = B\overline{Y}_1^{\beta_1} \\ \dfrac{1}{\delta} L_l L_l^{-\varepsilon} + \dfrac{1}{\delta}\beta_1\left(\dfrac{\overline{Y}_1}{\overline{Y}_f}\right)^{\beta_1 - 1}\{L_l[(L_l + L_f)^{-\varepsilon} - L_l^{-\varepsilon}]\} = B\beta_1 \overline{Y}_1^{\beta_1 - 1}. \end{cases} \tag{22}$$

This can be regarded as a system of two equations with the two unknown variables B and $\overline{Y}_1$. Thus, one can solve the system numerically.

Second, over a range of relatively large values of Y, there is also an option to wait, which should tangentially meet $F_l(Y)$ at two threshold values ($\overline{Y}_2$, $\overline{Y}_3$).[31] Its value denoted $H(Y)$ can be written as

$$H(Y) = C_1 Y^{\beta_1} + C_2 Y^{\beta_2}, \tag{23}$$

where C_1 and C_2 are constants to be determined and β_1 and β_2 are two roots given by equation (10).

The boundary conditions are

$$\begin{cases} \left.\begin{aligned} F_l(\overline{Y}_2) &= H(\overline{Y}_2) \\ F_l'(\overline{Y}_2) &= H'(\overline{Y}_2) \end{aligned}\right\} \quad \overline{Y}_2 < \overline{Y}_f \\ \left.\begin{aligned} F_l(\overline{Y}_3) &= H(\overline{Y}_3) \\ F_l'(\overline{Y}_3) &= H'(\overline{Y}_3) \end{aligned}\right\} \quad \overline{Y}_3 > \overline{Y}_f. \end{cases} \tag{24}$$

To be specific, these conditions can be rewritten as

$$\begin{cases} \dfrac{1}{\delta} L_l \overline{Y}_2 L_l^{-\varepsilon}\left[1 - \left(\dfrac{\overline{Y}_2}{\overline{Y}_f}\right)^{\beta_1 - 1}\right] + \left(\dfrac{\overline{Y}_2}{\overline{Y}_f}\right)^{\beta_1}\left[\dfrac{1}{\delta}\overline{Y}_f L_l (L_l + L_f)^{-\varepsilon}\right] - L_l I_l = C_1 \overline{Y}_2^{\beta_1} + C_2 \overline{Y}_2^{\beta_2} \\ \dfrac{1}{\delta} L_l L_l^{-\varepsilon} + \dfrac{1}{\delta}\beta_1\left(\dfrac{\overline{Y}_2}{\overline{Y}_f}\right)^{\beta_1 - 1}\{L_l[(L_l + L_f)^{-\varepsilon} - L_l^{-\varepsilon}]\} = C_1\beta_1 \overline{Y}_2^{\beta_1 - 1} + C_2\beta_2 \overline{Y}_2^{\beta_2 - 1} \\ \dfrac{1}{\delta} L_l \overline{Y}_3 (L_l + L_f)^{-\varepsilon} - L_l I_l = C_1 \overline{Y}_3^{\beta_1} + C_2 \overline{Y}_3^{\beta_2} \\ \dfrac{1}{\delta} L_l (L_l + L_f)^{-\varepsilon} = C_1 \beta_1 \overline{Y}_3^{\beta_1 - 1} + C_2 \beta_2 \overline{Y}_3^{\beta_2 - 1}. \end{cases} \tag{25}$$

31. By construction, they should satisfy $\overline{Y}_2 < \overline{Y}_f < \overline{Y}_3$.

This can be regarded as a system of four equations with the four unknown variables C_1, C_2, $\overline{Y}_2$, and $\overline{Y}_3$. Thus, one can solve the system numerically.

The decision-making rules by both banks can be summarized as follows:

$$\begin{cases} 0 < Y < \overline{Y}_1 & \text{both banks wait.} \\ \overline{Y}_1 \leq Y < \overline{Y}_2 & \text{the leader lends.} \\ \overline{Y}_2 \leq Y < \overline{Y}_3 & \text{both banks wait.} \\ \overline{Y}_3 \leq Y < \infty & \text{both banks lend.} \end{cases} \tag{26}$$

Figure 5 [1] shows the determination of each threshold value when the banks are identical except for the predetermined roles.[32] Over the very low demand level, both banks prefer to wait, similar to the case in which the roles are interchangeable. And once the demand level reaches $\overline{Y}_1$, the leader alone enters the loan market. But if the demand exceeds $\overline{Y}_2$, the leader prefers to wait rather than lend immediately. This is because the leader knows that the probability of the follower's entry is high over this region. Once $\overline{Y}_3$ is hit, however, both enter the market immediately because the profits made by entry are sufficiently high for both banks.

In comparison with the results when the roles are interchangeable, the following points are worthy of notice. If the roles of leader and follower are predetermined, as is the case with the Japanese main bank system, when the demand level is very low, the leader's incentive to enter the loan market alone becomes lower ($\overline{Y}_l < \overline{Y}_1$). Also, there is another inactive band for the leader ($\overline{Y}_2 \leq Y < \overline{Y}_3$), over which the leader waits, because the follower's threshold value of demand level for entry is drawing near.

Figure 5 [2] shows the case in which the *ex ante* loan share of the leader is larger than that of the follower ($L_l > L_f$), but the sunk costs per unit of loan are the same ($I_l = I_f$). In this case, at $\overline{Y}_f$, $F_l(Y)$ and $F_f(Y)$ do not meet, and they diverge as Y gets larger in the region of $Y \geq \overline{Y}_f$.[33]

Figure 5 [3] illustrates the case in which the sunk costs of the leader are lower than those of the follower ($I_l < I_f$), but the *ex ante* loan shares are the same ($L_l = L_f$). Also in this case, neither curve meets at $\overline{Y}_f$, although their slopes are the same from this point on.

D. Stochastic Version of Tobin's q

Here, note that equation (14) can be modified as

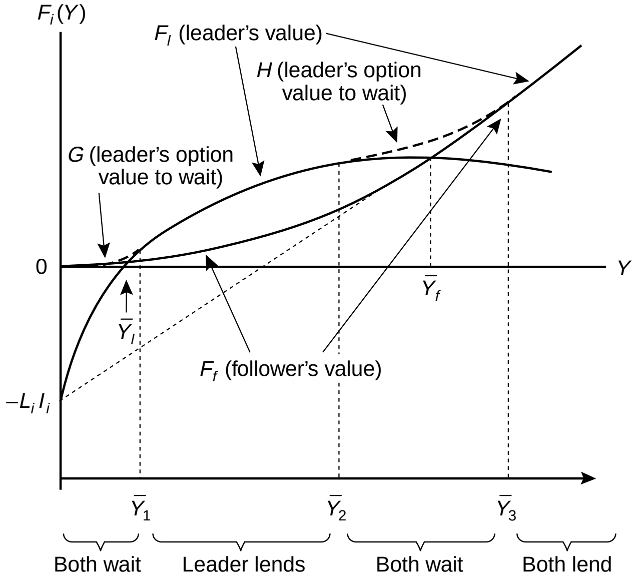
$$v_f(\overline{\Pi}_f) = \frac{\beta_1}{\beta_1 - 1} L_f I_f. \tag{27}$$

One can interpret equation (27) as saying there is a wedge between investment (sunk costs), denoted $L_f I_f$, and the present discounted value of the follower bank's cash flow from actual lending, which is denoted $v_f(\overline{\Pi}_f)$.

32. Two dashed curves show the values of the additional options to wait when the roles are predetermined.
33. See equations (16) and (18).

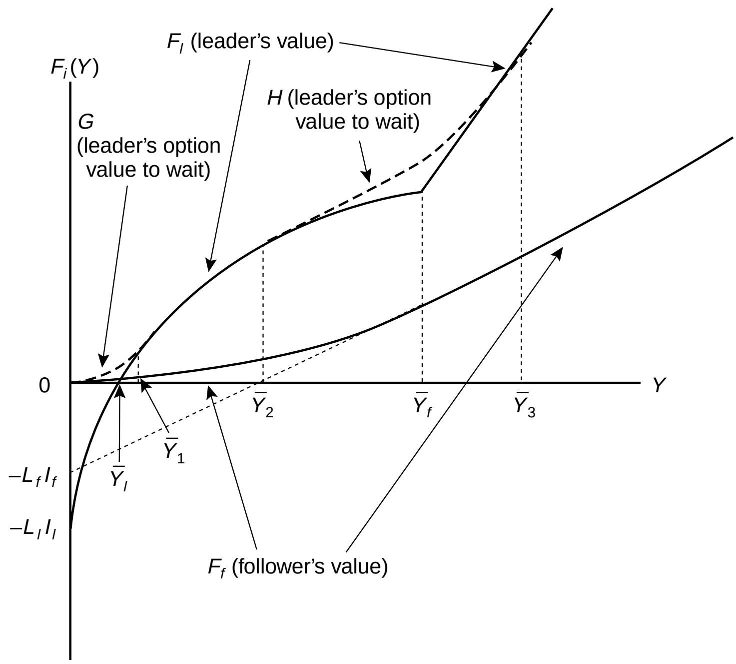
Figure 5 The Case of Predetermined Roles of Leader and Follower

[1] Identical Banks

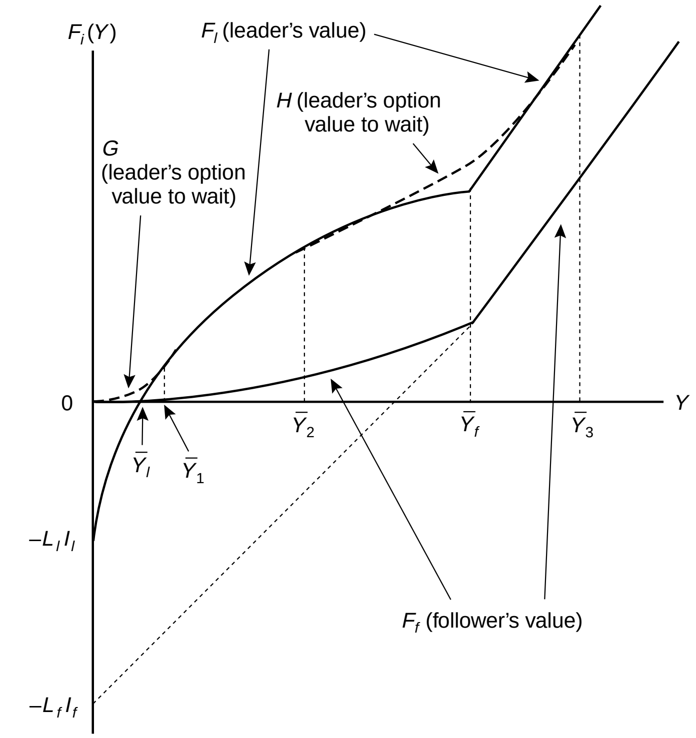


Note: The figure is drawn under the assumption that both banks are identical except for the predetermined roles they assume.

[2] Different *Ex Ante* Loan Shares ($L_l > L_f$) and the Same Sunk Costs per Unit of Loan ($I_l = I_f$)



[3] Different Sunk Costs per Unit of Loan ($I_l < I_f$) and the Same *Ex Ante* Loan Shares ($L_l = L_f$)



Thus, the wedge can be defined as

$$\overline{q} \equiv \frac{\beta_1}{\beta_1 - 1} > 1. \tag{28}$$

The index $\overline{q}$ captures a notion that is very similar and comparable to that introduced by Tobin (1969). It should be noted, however, that $\overline{q}$[34] defined as equation (28) depends on uncertainty[35] about future demand (and hence profit) conditions such as the expected growth rate α, the volatility σ, and the subjective probability of bankruptcy of the borrowing firm λ. Hence, there will be periods when $\overline{q}$ exceeds one without attracting investment.

34. Conceptually, this version of $\overline{q}$ is called the value of assets in place notion in contrast to the value of the firm notion. In the latter case, $\overline{q}$ should be defined as $[v(\Pi_f) - F(\Pi_f)]/L_f I_f$, which is net of the option value. For more details, see Dixit and Pindyck (1994).
35. This is why I call $\overline{q}$ the stochastic version of Tobin's q in this paper.

III. Numerical Analysis

In this section, I conduct numerical analysis by changing each parameter in turn, holding others fixed at plausible values. As a baseline case, I choose some parameter values such that $\varepsilon = 2$ and $(I_l, I_f) = (0.2, 0.2)$, unless otherwise stated. Also, I handle the cases in which $(L_l, L_f) = (10, 10)$ and $(15, 5)$ to analyze the effect of a change in the *ex ante* loan shares on entry decisions. Hence, if both banks enter the loan market, the current return from lending is 1.25 percent when $Y = 5$, for example. On the other hand, if only the leader enters, it is 5.00 percent when $L_l = 10$ and 2.22 percent when $L_l = 15$.[36]

The data state that the average contracted interest rate on new long-term loans and discounts extended by domestically licensed banks fell from 5.09 percent per annum in January 1993[37] to 2.38 percent in December 1999. Since, for example, the uncollateralized call rate was 3.88 percent in January 1993, and 0.05 percent in December 1999, the baseline parameter values of ε and (L_l, L_f) might not be so unrealistic.

Table 1 summarizes the main results of the analysis, and Tables 2–5 show the details. First, a rise in the volatility of the future demand condition σ raises each threshold value uniformly, holding other parameters constant. This result makes sense, since the coexistence of uncertainty and irreversibility yields the value of the option to wait.

Table 1 Summary of Numerical Analysis

Exogenous variables		Endogenous variables					
		Interchangeable roles		Predetermined roles			Stochastic Tobin's q
		Leader	Follower	Leader		Follower	
		$\bar{Y}_l$	$\bar{Y}_f$	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{q}$
Stochastic process: Volatility σ		+	+	+	+	+	+
Expected growth α	Case (i)	+	+	+	+	+	+
	Case (ii)	–	–	–	–	–	+
Probability of bankruptcy λ	Case (a)	–	–	–	–	–	–
	Case (b)	–	–	–	+	–	–
Ex ante loan share $L_l/(L_l+L_f)$		+*	0	+	+	–	0
Sunk costs: I_l & I_f		+	+	+	+	+	0
I_f		–	+	+	+	+	0
Inverse of interest rate elasticity of loan demand ε		+	+	+	+	+	0

Notes: 1. For definitions of $\bar{Y}_l$, $\bar{Y}_f$, $\bar{Y}_1$, $\bar{Y}_2$, $\bar{Y}_3$, and $\bar{q}$, see equations (19), (15), (22), (25), and (28).
2. + indicates that the endogenous variable rises when the exogenous variable rises. – indicates vice versa. 0 denotes no effect.
3. Case (i) denotes the case in which δ is held constant, while letting ρ adjust freely. Case (ii) denotes the case in which ρ is held constant while letting δ adjust freely.
4. Case (a) denotes the case in which α and ρ are fixed whatever the value of λ. Case (b) denotes the case in which α and ρ increase by the same amount as λ.
5. For comparative ease, total lending amount $L_l + L_f$ is always fixed at 20.

* If ε is small ($\varepsilon < 0.8$), this is negative.

36. Recall that the inverse loan demand function is specified as $r_L = Y(L_l + L_f)^{-\varepsilon}$.
37. The data can be obtained from January 1993. For details, see various issues of *Financial and Economic Statistics Monthly* (Bank of Japan).

Table 2 Dependence of Each Threshold Value on α and σ

[1] The Case of Fixed δ: δ = 0.03 (Adjustable ρ)

A. (L_l, L_f) = (10, 10), λ = 0, ε = 2, (I_l, I_f) = (0.2, 0.2)

Parameters		Interchangeable roles		Predetermined roles			Stochastic Tobin's q
		Leader	Follower	Leader		Follower	
α	σ	$\bar{Y}_l$	$\bar{Y}_f$	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{q}$
0.00	0.05	0.600	2.942	0.736	2.604	4.028	1.226
	0.10	0.615	3.600	0.900	2.908	5.694	1.500
	0.20	0.725	5.317	1.329	3.808	9.851	2.215
0.01	0.05	0.612	3.515	0.879	3.265	4.220	1.465
	0.10	0.642	4.149	1.037	3.512	5.885	1.729
	0.20	0.772	5.898	1.474	4.380	10.174	2.457
0.02	0.05	0.647	4.231	1.058	4.032	4.804	1.763
	0.10	0.686	4.800	1.200	4.211	6.348	2.000
	0.20	0.825	6.530	1.632	5.008	10.634	2.721
0.03	0.05	0.700	4.993	1.248	4.820	5.508	2.080
	0.10	0.740	5.509	1.377	4.956	6.954	2.295
	0.20	0.884	7.200	1.800	5.677	11.190	3.000

B. (L_l, L_f) = (15, 5), λ = 0, ε = 2, (I_l, I_f) = (0.2, 0.2)

Parameters		Interchangeable roles		Predetermined roles			Stochastic Tobin's q
		Leader	Follower	Leader		Follower	
α	σ	$\bar{Y}_l$	$\bar{Y}_f$	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{q}$
0.00	0.05	1.372	2.942	1.655	2.664	3.487	1.226
	0.10	1.473	3.600	2.025	3.091	4.496	1.500
	0.20	1.715	5.317	2.991	4.338	6.944	2.215
0.01	0.05	1.459	3.515	1.977	3.301	3.923	1.465
	0.10	1.561	4.149	2.334	3.660	4.982	1.729
	0.20	1.779	5.898	3.318	4.883	7.556	2.457
0.02	0.05	1.573	4.231	2.380	4.055	4.585	1.763
	0.10	1.652	4.800	2.700	4.322	5.602	2.000
	0.20	1.840	6.530	3.673	5.482	8.224	2.721
0.03	0.05	1.677	4.993	2.808	4.837	5.323	2.080
	0.10	1.737	5.509	3.099	5.057	6.297	2.295
	0.20	1.898	7.200	4.050	6.123	8.932	3.000

[2] The Case of Fixed ρ: ρ = 0.04 (Adjustable δ)
A. (L_l, L_f) = (10, 10), λ = 0, ε = 2, (I_l, I_f) = (0.2, 0.2)

Parameters		Interchangeable roles		Predetermined roles			Stochastic Tobin's q
		Leader	Follower	Leader		Follower	
α	σ	$\bar{Y}_l$	$\bar{Y}_f$	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{q}$
0.00	0.05	0.800	3.818	0.954	3.428	5.092	1.193
	0.10	0.811	4.549	1.137	3.756	6.968	1.422
	0.20	0.914	6.400	1.600	4.704	11.485	2.000
0.01	0.05	0.612	3.515	0.879	3.265	4.220	1.465
	0.10	0.642	4.149	1.037	3.512	5.885	1.729
	0.20	0.772	5.898	1.474	4.380	10.174	2.457
0.02	0.05	0.471	3.389	0.847	3.225	3.856	2.118
	0.10	0.510	3.881	0.970	3.379	5.177	2.425
	0.20	0.649	5.463	1.366	4.116	9.064	3.414
0.03	0.05	0.380	3.332	0.833	3.213	3.681	4.165
	0.10	0.415	3.710	0.927	3.310	4.728	4.637
	0.20	0.546	5.098	1.274	3.908	8.150	6.372

B. (L_l, L_f) = (15, 5), λ = 0, ε = 2, (I_l, I_f) = (0.2, 0.2)

Parameters		Interchangeable roles		Predetermined roles			Stochastic Tobin's q
		Leader	Follower	Leader		Follower	
α	σ	$\bar{Y}_l$	$\bar{Y}_f$	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{q}$
0.00	0.05	1.818	3.818	2.148	3.491	4.476	1.193
	0.10	1.923	4.549	2.559	3.951	5.621	1.422
	0.20	2.203	6.400	3.600	5.272	8.291	2.000
0.01	0.05	1.459	3.515	1.977	3.301	3.923	1.465
	0.10	1.561	4.149	2.334	3.660	4.982	1.729
	0.20	1.779	5.898	3.318	4.883	7.556	2.457
0.02	0.05	1.125	3.389	1.907	3.245	3.677	2.118
	0.10	1.181	3.881	2.183	3.486	4.546	2.425
	0.20	1.316	5.463	3.073	4.553	6.918	3.414
0.03	0.05	0.696	3.332	1.874	3.225	3.555	4.165
	0.10	0.717	3.710	2.087	3.387	4.259	4.637
	0.20	0.775	5.098	2.868	4.283	6.382	6.372

Table 3 Dependence of Each Threshold Value on λ

[1] The Case of Fixed α

Parameter	Interchangeable roles		Predetermined roles			Stochastic Tobin's q
	Leader	Follower	Leader		Follower	
λ	$\bar{Y}_l$	$\bar{Y}_f$	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{q}$
0.0	0.725	5.317	1.329	3.808	9.851	2.215
0.1	0.613	3.544	0.886	2.881	5.555	1.477
0.2	0.604	3.220	0.805	2.727	4.742	1.342
0.3	0.601	3.068	0.767	2.659	4.354	1.278
0.4	0.601	2.976	0.744	2.619	4.117	1.240
0.5	0.600	2.914	0.728	2.592	3.952	1.214
1.0	0.600	2.759	0.690	2.528	3.537	1.149

Note: Calculations are made under the assumption that the expected growth parameter α is fixed at zero. Other parameters are set as follows: δ = 0.03 (thus, ρ = 0.03), σ = 0.2, ε = 2, (L_l, L_f) = (10, 10), and (I_l, I_f) = (0.2, 0.2).

[2] The Case of Flexible α

Parameter	Interchangeable roles		Predetermined roles			Stochastic Tobin's q
	Leader	Follower	Leader		Follower	
λ	$\bar{Y}_l$	$\bar{Y}_f$	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{q}$
0.0	0.725	5.317	1.329	3.808	9.851	2.215
0.1	0.698	4.961	1.240	4.418	6.387	2.067
0.2	0.693	4.896	1.224	4.558	5.817	2.040
0.3	0.691	4.869	1.217	4.622	5.567	2.029
0.4	0.690	4.853	1.213	4.659	5.423	2.022
0.5	0.689	4.844	1.211	4.683	5.328	2.018
1.0	0.687	4.823	1.206	4.736	5.112	2.010

Note: Calculations are made under the assumption that the expected growth parameter α and ρ adjust by the same amount in the same direction as λ. α is set at zero when λ is zero. Other parameters are set as follows: δ = 0.03, σ = 0.2, ε = 2, (L_l, L_f) = (10, 10), and (I_l, I_f) = (0.2, 0.2).

Table 4 Dependence of Each Threshold Value on α and (I_l, I_f)

[1] The Case of Fixed δ: δ = 0.03 (Adjustable ρ)

A. (L_l, L_f) = (10, 10), λ = 0, ε = 2, σ = 0.1

Parameters			Interchangeable roles		Predetermined roles			Stochastic Tobin's q
			Leader	Follower	Leader		Follower	
α	I_l	I_f	$\bar{Y}_l$	$\bar{Y}_f$	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{q}$
0.00	0.1	0.1	0.307	1.800	0.450	1.454	2.847	1.500
		0.2	0.302	3.600	0.450	2.991	5.183	1.500
		0.3	0.301	5.400	0.450	4.521	7.586	1.500
	0.2	0.2	0.615	3.600	0.900	2.908	5.694	1.500
		0.3	0.606	5.400	0.900	4.448	7.994	1.500
0.01	0.1	0.1	0.321	2.074	0.519	1.756	2.942	1.729
		0.2	0.307	4.149	0.519	3.614	5.358	1.729
		0.3	0.304	6.223	0.519	5.462	7.863	1.729
	0.2	0.2	0.643	4.149	1.037	3.512	5.885	1.729
		0.3	0.623	6.223	1.037	5.376	8.248	1.729
0.02	0.1	0.1	0.343	2.400	0.600	2.106	3.174	2.000
		0.2	0.318	4.800	0.600	4.323	5.807	2.000
		0.3	0.312	7.200	0.600	6.527	8.549	2.000
	0.2	0.2	0.686	4.800	1.200	4.211	6.348	2.000
		0.3	0.652	7.200	1.200	6.437	8.914	2.000

B. (L_l, L_f) = (15, 5), λ = 0, ε = 2, σ = 0.1

Parameters			Interchangeable roles		Predetermined roles			Stochastic Tobin's q
			Leader	Follower	Leader		Follower	
α	I_l	I_f	$\bar{Y}_l$	$\bar{Y}_f$	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{q}$
0.00	0.1	0.1	0.737	1.800	1.012	1.546	2.248	1.500
		0.2	0.688	3.600	1.012	3.232	4.170	1.500
		0.3	0.680	5.400	1.012	4.894	6.158	1.500
	0.2	0.2	1.473	3.600	2.025	3.091	4.496	1.500
		0.3	1.397	5.400	2.025	4.792	6.378	1.500
0.01	0.1	0.1	0.780	2.074	1.167	1.830	2.491	1.729
		0.2	0.707	4.149	1.167	3.832	4.612	1.729
		0.3	0.693	6.223	1.167	5.797	6.824	1.729
	0.2	0.2	1.561	4.149	2.334	3.660	4.982	1.729
		0.3	1.452	6.223	2.334	5.686	7.043	1.729
0.02	0.1	0.1	0.826	2.400	1.350	2.166	2.801	2.000
		0.2	0.735	4.800	1.350	4.519	5.198	2.000
		0.3	0.712	7.200	1.350	6.828	7.709	2.000
	0.2	0.2	1.652	4.800	2.700	4.332	5.602	2.000
		0.3	1.521	7.200	2.700	6.715	7.923	2.000

Note: $\bar{Y}_1$ becomes larger slightly as I_f becomes larger, other things equal, although the figures in the tables are the same.

[2] The Case of Fixed ρ: ρ = 0.04 (Adjustable δ)

A. (L_l, L_f) = (10, 10), λ = 0, ε = 2, σ = 0.1

Parameters			Interchangeable roles		Predetermined roles			Stochastic Tobin's q
			Leader	Follower	Leader		Follower	
α	I_l	I_f	$\bar{Y}_l$	$\bar{Y}_f$	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{q}$
0.00	0.1	0.1	0.406	2.274	0.569	1.878	3.484	1.422
		0.2	0.401	4.549	0.569	3.857	6.332	1.422
		0.3	0.400	6.823	0.569	5.827	9.269	1.422
	0.2	0.2	0.811	4.549	1.137	3.756	6.968	1.422
		0.3	0.804	6.823	1.137	5.739	9.765	1.422
0.01	0.1	0.1	0.321	2.074	0.519	1.756	2.942	1.729
		0.2	0.307	4.149	0.519	3.614	5.358	1.729
		0.3	0.304	6.223	0.519	5.462	7.863	1.729
	0.2	0.2	0.643	4.149	1.037	3.512	5.885	1.729
		0.3	0.623	6.223	1.037	5.376	8.248	1.729
0.02	0.1	0.1	0.255	1.940	0.485	1.689	2.588	2.425
		0.2	0.228	3.881	0.485	3.474	4.733	2.425
		0.3	0.220	5.821	0.485	5.246	6.965	2.425
	0.2	0.2	0.510	3.881	0.970	3.379	5.177	2.425
		0.3	0.473	5.821	0.970	5.170	7.268	2.425

B. (L_l, L_f) = (15, 5), λ = 0, ε = 2, σ = 0.1

Parameters			Interchangeable roles		Predetermined roles			Stochastic Tobin's q
			Leader	Follower	Leader		Follower	
α	I_l	I_f	$\bar{Y}_l$	$\bar{Y}_f$	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{q}$
0.00	0.1	0.1	0.961	2.274	1.279	1.975	2.810	1.422
		0.2	0.910	4.549	1.279	4.128	5.201	1.422
		0.3	0.904	6.823	1.279	6.823	7.683	1.422
	0.2	0.2	1.923	4.549	2.559	3.951	5.621	1.422
		0.3	1.841	6.823	2.559	6.123	7.955	1.422
0.01	0.1	0.1	0.780	2.074	1.167	1.830	2.491	1.729
		0.2	0.707	4.149	1.167	3.832	4.612	1.729
		0.3	0.693	6.223	1.167	5.797	6.824	1.729
	0.2	0.2	1.561	4.149	2.334	3.660	4.982	1.729
		0.3	1.452	6.223	2.334	5.686	7.043	1.729
0.02	0.1	0.1	0.590	1.940	1.091	1.743	2.273	2.425
		0.2	0.519	3.881	1.091	3.642	4.217	2.425
		0.3	0.499	5.821	1.091	5.504	6.252	2.425
	0.2	0.2	1.181	3.881	2.183	3.486	4.546	2.425
		0.3	1.082	5.821	2.183	5.410	6.429	2.425

Note: $\bar{Y}_1$ becomes larger slightly as I_f becomes larger, other things equal, although the figures in the tables are the same.

Table 5 Dependence of Each Threshold Value on ε (δ = 0.03, ρ = 0.04 , σ = 0.2)

A. (L_l , L_f) = (10, 10), (I_l , I_f) = (0.2, 0.2), λ = 0

Parameters	Interchangeable roles		Predetermined roles			Stochastic Tobin's q
	Leader	Follower	Leader		Follower	
ε	$\bar{Y}_l$	$\bar{Y}_f$	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{q}$
0.2	0.021	0.027	0.023	0.025	0.029	2.457
0.4	0.030	0.049	0.037	0.044	0.056	2.457
0.6	0.043	0.089	0.059	0.076	0.107	2.457
0.8	0.063	0.162	0.093	0.135	0.206	2.457
1.0	0.094	0.295	0.147	0.239	0.395	2.457
1.2	0.142	0.537	0.234	0.425	0.758	2.457
1.4	0.215	0.977	0.370	0.758	1.451	2.457
1.6	0.328	1.779	0.587	1.358	2.778	2.457
1.8	0.502	3.240	0.930	2.436	5.317	2.457
2.0	0.772	5.898	1.474	4.380	10.174	2.457
2.2	1.191	10.737	2.337	7.886	19.468	2.457
2.4	1.844	19.548	3.704	14.219	37.254	2.457
2.6	2.862	35.689	5.870	25.665	71.303	2.457
2.8	4.454	64.791	9.303	46.369	136.510	2.457
3.0	6.944	117.957	14.745	83.847	261.434	2.457

B. (L_l , L_f) = (15, 5), (I_l , I_f) = (0.2, 0.2), λ = 0

Parameters	Interchangeable roles		Predetermined roles			Stochastic Tobin's q
	Leader	Follower	Leader		Follower	
ε	$\bar{Y}_l$	$\bar{Y}_f$	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{q}$
0.2	0.012	0.027	0.025	0.026	0.028	2.457
0.4	0.021	0.049	0.044	0.046	0.052	2.457
0.6	0.037	0.089	0.075	0.083	0.097	2.457
0.8	0.065	0.162	0.129	0.147	0.180	2.457
1.0	0.113	0.295	0.221	0.263	0.336	2.457
1.2	0.198	0.537	0.380	0.470	0.627	2.457
1.4	0.344	0.977	0.653	0.843	1.169	2.457
1.6	0.596	1.779	1.123	1.512	2.179	2.457
1.8	1.031	3.240	1.930	2.716	4.058	2.457
2.0	1.779	5.898	3.318	4.883	7.556	2.457
2.2	3.064	10.737	5.702	8.787	14.063	2.457
2.4	5.269	19.548	9.801	15.827	26.168	2.457
2.6	9.047	35.689	16.845	28.527	48.682	2.457
2.8	15.514	64.791	28.953	51.455	90.549	2.457
3.0	26.571	117.957	49.763	92.866	168.393	2.457

Second, in contrast, the direction of the effect of a rise in the expected growth parameter α on lending decisions depends on the presumption regarding which parameter is adjustable, the risk-adjusted discount rate ρ or the dividend rate δ when α changes.[38] When one assumes that ρ adjusts to accommodate the rise in α, it raises each threshold. On the other hand, if one assumes that δ adjusts to offset the rise in α, holding ρ constant, the rise in α lowers each threshold except for $\bar{q}$.

Third, look at the result of a rise in the probability of bankruptcy, λ. Generally speaking, the effects can be stated in the following ways.[39] First, it reduces the expected rate of capital gain on Π_l (= $r_L L_l$) and Π_f, which in turn decreases the value of the option to wait. Second, it increases the variance of changes in Π_l and Π_f, and thus raises the value of the option to wait.

The result shows that, in general, the net effect is to reduce the threshold values for both banks. But if one assumes that an increase in λ raises the value of α by the same amount, which means that an increase in λ is approximately equivalent to an increase in the risk-adjusted discount rate ρ, the direction of the effect is the same except for $\bar{Y}_2$, but the magnitude is reduced.

Fourth, a rise in the leader bank's share $L_l/(L_l + L_f)$ has no effect on $\bar{Y}_f$ and $\bar{q}$. But it raises $\bar{Y}_l$ (when $\varepsilon > 0.8$), $\bar{Y}_1$, and $\bar{Y}_2$, although it lowers $\bar{Y}_3$.

Fifth, a uniform rise in the sunk costs per unit of loan paid by both banks raises each threshold value. On the other hand, a relative rise in I_f lowers $\bar{Y}_l$, but raises other threshold values. However, $\bar{q}$ remains constant in each case.

Lastly, regarding the effect of a rise in the inverse of the interest elasticity of loan demand ε, $\bar{q}$ has nothing to do with the level of ε, while other threshold values change if one changes ε, holding other things constant. Each threshold value rises together with a rise in ε, since given the value of L_l and L_f, a larger value of ε implies a smaller value of interest rate r_L. It should be noted, however, that as the value of ε falls so that the loan market becomes more competitive, each threshold value tends to converge.

IV. Discussions

What implications can be derived from the analysis thus far regarding the recent slump in the Japanese loan market? In this section, I will explore the link between the actual bank lending situation in Japan and the insights derived from the theoretical model.

First and most importantly, the model shows that when the roles of leader and follower are predetermined as is the case with the Japanese main bank system, both leader and follower banks have a greater incentive to wait and see until the demand condition sufficiently improves than when the roles are interchangeable. And the degree of the tendency to wait becomes higher as the degree of uncertainty about the future demand condition becomes higher. It is generally acknowledged that the

38. Recall the relationship $\rho = \alpha + \delta$.

39. Note that in this paper, costs stemming from the real bankruptcy procedure such as the loss from liquidation are not taken into consideration. These considerations are likely to raise the value of the option to wait.

Japanese main bank system had succeeded in providing enough funds to the business sector by alleviating the informational problems. But the structure of the predetermined role of leader and follower in the loan market causes the banks to be more cautious in extending new loans.

Second, it is often said that, compared with the bubble period, the expected growth forecasts about future general demand conditions have bent downward. It is natural to think that the same tendency should occur in the bank loan market once one considers the significant presence of bank lending in fund-raising by Japan's business sector. This hypothesis can be roughly verified by looking at the data reported in various issues of the *Short-Term Economic Survey of Enterprises in Japan* issued by the Bank of Japan. These show that the Japanese business sector has decreased[40] its borrowing from financial institutions. This is because many corporations have revised their fixed investment plans downward,[41] reflecting their reduced forecasts about future demand conditions.

In this regard, the model says that if one assumes that the risk-adjusted discount rate ρ for private banks is constant regardless of the value of the expected growth parameter α,[42] a fall in α raises the threshold values, which implies that the banks' incentive to supply loans weakens given the current demand situation.

Third, various surveys state that the Japanese economy as a whole faces a much higher degree of uncertainty about future business conditions than before. That is likely to lead to a rise in uncertainty for banks about future corporate borrowing demand. In this regard, the model says that a rise in the uncertainty (volatility) parameter σ definitely dampens the incentive to enter the loan market.

Lastly, it is often pointed out that the interest rate elasticity of loan demand[43] has decreased recently. This kind of remark generally reflects the prolonged depressed condition of the loan market despite the significantly lowered lending interest rate. If this is the case, the model states that a fall in the interest rate elasticity of loan demand should cause the threshold values for both banks to rise, which implies a higher probability of not making an entry decision.

V. Concluding Remarks

This paper has explored banks' entry decisions into a duopolistic loan market to shed light on the recent slump in the Japanese loan market. The game-theoretic real options approach is employed to analyze the effects of uncertainty on lending decisions. The theoretical model shows that when the roles of leader and follower are predetermined as in the case of the Japanese main bank system, both banks have a greater incentive to wait than when the roles are interchangeable. This result might

40. The same source also reports that Japanese firms plan to restrain borrowing in the future.
41. Also, the continuing structural shift from indirect finance like bank borrowings to direct finance such as direct debt is thought to contribute to a fall in the expected growth rate of future loan demand, especially among large firms.
42. This assumption implicitly presumes that the risk-free interest rate, variance, risk price, and beta of the return are stable.
43. Note that in this paper ε denotes the inverse of interest rate elasticity.

explain why the Japanese loan market has been so stagnant since the bursting of the bubble economy in the early 1990s.

At least to my knowledge, the attempts to apply the real options approach to bank lending decisions particularly in the oligopolistic market structure have been quite rare thus far. I sincerely hope that this paper provides a useful starting point for future discussions in this field.

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