Model Risk and Its Control

Toshiyasu Kato and Toshinao Yoshiba

In this paper, we analyze model risks separately in pricing models and risk measurement models as follows. (1) In pricing models, model risk is defined as "the risk arising from the use of a model which cannot accurately evaluate market prices, or which is not a mainstream model in the market." (2) In risk measurement models, model risk is defined as "the risk of not accurately estimating the probability of future losses." Based on these definitions, we examine various specific cases and numerical examples to determine the sources of model risks and to discuss possible steps to control these risks.

Sources of model risk in pricing models include (1) use of wrong assumptions, (2) errors in estimations of parameters, (3) errors resulting from discretization, and (4) errors in market data. On the other hand, sources of model risk in risk measurement models include (1) the difference between assumed and actual distribution, and (2) errors in the logical framework of the model.

Practical steps to control model risks from a qualitative perspective include improvement of risk management systems (organization, authorization, human resources, etc.). From a quantitative perspective, in the case of pricing models, we can set up a reserve to allow for the difference in estimations using alternative models. In the case of risk measurement models, scenario analysis can be undertaken for various fluctuation patterns of risk factors, or position limits can be established based on information obtained from scenario analysis.

Key words: Model risk; Pricing model; Risk measurement model; Risk management systems; Reserves; Scenario analysis; Position limits

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I. Introduction

There has been an explosive growth in financial derivative products in recent years. In the case of Japan, a similar rush to develop new derivative products is expected with the implementation of the Financial Big Bang. Complex financial products require sophisticated financial engineering capabilities for proper risk control, including accurate valuation, hedging, and risk measurement. Parallel to the creation of more diverse financial products and the development of new markets for such products, both pricing models and risk measurement models used as risk management tools have also become increasingly complex. Recently, several major financial institutions have reported losses arising from the use of such complex models. This has drawn attention to the various types of risk which result from the use of such models.

Generally speaking, the use of models can carry various types of risks.[1] In this paper, however, we specify model risks as follows. In pricing models, model risk is defined as "the risk arising from the use of a model which cannot accurately evaluate market prices, or which is not a mainstream model in the market." In risk measurement models, model risk is defined as "the risk of not accurately estimating the probability of future losses." Hence, such types of risk as market price input errors, and bugs remaining in software in the model-building stage are excluded from our analysis.

Among possible sources of model risk, there has been a great deal of discussion regarding the volatility smile (hereinafter referred to as "smile") and the treatment of the distribution of underlying asset prices.[2] For instance, the Black-Scholes model[3] (hereinafter referred to as the "BS model"), a standard pricing model for options, assumes that underlying asset prices fluctuate according to a lognormal process, whereas actual market price fluctuations do not necessarily follow this process.

However, as pointed out by Kato and Yoshiba (1999), many market players continue to use the BS model as a pricing tool with full knowledge of its limitations. Moreover, models have become indispensable tools in the development of new financial products and the management of their risks. In view of this, market players no longer have the choice of terminating the use of models on the grounds of existing model risks.

The purpose of this paper is to determine the sources of model risks and to discuss possible steps to control their risks. The paper is organized as follows. Chapter II

1. For example, Derman (1996) refers to the following types of model risk.
   - Inapplicability of modelling
   - Incorrect model
   - Correct model, incorrect solution
   - Correct model, inappropriate use
   - Badly approximated solution
   - Software and hardware bugs
   - Unstable data
2. Simons (1997) states that analysis of underlying asset prices and evaluation of smile are vitally important in risk management. Jorion (1999) argues that one of the causal factors in the failure of Long-Term Capital Management (hereinafter referred to as LTCM) was an inappropriate assumption concerning the distribution of underlying asset price fluctuations.
3. See Appendix 1 for details of the Black-Scholes model.

presents some cases of model risks which have actually been realized. Numerical examples are examined in the following three chapters. Specifically, Chapter III treats long-term foreign exchange options; Chapter IV discusses barrier options on stock price; and Chapter V treats interest-rate strangle short strategies. Based on the implications gained from these analyses, Chapter VI examines available methods for managing model risks. Finally, Chapter VII briefly outlines some conclusions.

II. Cases of Realized Model Risks

When are model risks realized? One such case may occur when a financial institution revises its internal pricing model and registers a loss by changing its valuation of current prices. Such reports of the realization of model risks are occasionally heard from the market. While details of the causal factors in such cases are not made public, in this chapter, we present an outline of these cases based on media reports.[4]

A. Case of Index Swap

Index swaps are swap transactions in which floating interest rates are based on indices other than LIBOR. As such, Nikkei index-linked swaps fall under this category. In the case of index swaps, it is necessary to manage the position and the risks in line with the relevant index. This requires a full understanding of various types of indices, as well as the structure of index swap markets.

A certain financial institution accumulated a substantial position in a special type of index swap. At the time, the market participants were using several types of models for the valuation of this index swap. This financial institution began trading in this product using what was recognized at the time as the leading mainstream model. As the market for this index swap shrank, some participants left the market. Thereafter, another model, which was being used by some of the remaining participants, became the dominant model in the market.

While maintaining a very large position in this swap index, this financial institution fell behind in research of the most dominant pricing model for this product in the market. Consequently, it failed to recognize that a switch had been made in the dominant model until adjusting its position. As a result, it registered losses amounting to several billion yen when it finally adopted the new model and made the necessary adjustments in its current price valuations.

B. Case of Mark Cap

Caps[5] are a form of interest rate options and generally constitute an OTC product with relatively high liquidity. The broker screen displays the implied volatility for each strike price and time period as calculated for cap prices using the BS model. This volatility exhibits certain skew structures (hereinafter referred to as "skew") by

4. Cases cited in this section have been expressly selected for the purpose of presenting a more concrete image of model risks, and may include the authorís conjectures.
5. See Kato and Yoshiba (1999) for more on interest rate cap and skew structure.

strike price and by time period. To calculate the current price of any given cap, the volatility corresponding to the time period and strike price of the cap is first estimated (interpolated) on the basis of the skew which is normally observed in the market.

A certain financial institution was engaged in Deutsche mark cap transactions. At the time, the number of time periods and strike prices for which volatility could be confirmed on the screen was relatively small compared to yen caps. The estimation of volatility was particularly difficult for caps with significant differences between market interest rates and strike interest rates (hereinafter referred to as "far-out strikes").

The financial institution was using the BS model as its internal pricing model for caps. This institution uses the broker-screen volatility of the closest strike price as the volatility of far-out strikes. Some cap dealers attempted to capitalize on the inevitable difference between market prices and valuation prices by trading aggressively in far-out strikes. This strategy generated internal valuation profits.

The financial institution fell behind in improving its pricing model and failed to minimize the gap between market prices and valuation prices. Continued cap dealer transactions under an unimproved model resulted in the accumulation of substantial internal valuation profits. However, when the internal pricing model was finally revised, the financial institution reported several tens of billion yen in losses.

C. Case of Credit Spread Position Held by LTCM

LTCM had accumulated a highly leveraged credit spread position which combined emerging bonds, loans, and other instruments. The position was structured to generate profits as spreads narrowed. LTCM suffered huge losses as a result of the sudden increase in spreads following the Russian crisis in 1998.

Various reasons have been given for these huge losses. For instance, LTCM was unable to hedge or cancel its transactions because its liquidity had dried up in the market. On this point, it has been said that LTCM had not taken liquidity into account when building its model. Others have pointed to internal problems in LTCM's risk measurement model. Specifically, problems with wrong assumptions concerning the distribution of underlying asset prices and errors in data used in estimating the distribution of underlying asset prices have been pointed out. Both would lead to fatal errors in risk measurement (Jorion [1999]).

D. Summary

The critical points in the model risks described above can be outlined as follows:

- declining market liquidity and obsolescence of pricing models,
- trader transactions capitalizing on the difference between market prices and prices calculated by pricing models, and
- wrong assumptions concerning the distribution of underlying asset prices and errors in data used in estimation.

The above cases indicate that very large losses can result when improvement of internal models is neglected, or when organizational mechanisms for undertaking the necessary improvements fail.

III. Long-Term Foreign Exchange Options

Several cases of the realization of model risks were discussed in Chapter II. In Chapters III through V, model risks shall be more closely examined using numerical examples. This exercise shall start in this chapter with the analysis of long-term foreign exchange options.

Figure 1 presents the volatility of yen/dollar exchange rates and yen interest rates at end-of-month as announced by the Japanese Bankers Association.

Figure 1 Volatility in Foreign Exchange Options (Telerate Co. as of October 29, 1999)

05/11 2.44 GMT [JAPAN BANKER'S ASSOCIATION — (3)] 35302
[OCT 29, '99]
MONTHLY INFORMATION

* OPTION VOLATILITY * ATM FWD		*** JPY INTEREST RATE *** (360 DAYS BASE)			
[TERM]	[RATE]	[TERM]	[RATE]	[TERM]	[RATE]
1 MONTH	13.71	1 MONTH	0.04	2 YEAR	0.38
2 MONTH	13.96	2 MONTH	0.39	3 YEAR	0.63
3 MONTH	14.32	3 MONTH	0.33	4 YEAR	0.95
4 MONTH	14.48	4 MONTH	0.27	5 YEAR	1.28
5 MONTH	14.64	5 MONTH	0.26	7 YEAR	1.81
6 MONTH	14.81	6 MONTH	0.26	10 YEAR	2.28
9 MONTH	14.92	9 MONTH	0.26		
1 YEAR	15.05	1 YEAR	0.26		

[PROVIDED BY KYODO NEWS] [CONTINUED ON PAGE 35303]

"Option volatility" appearing in Figure 1 presents the option volatility for time periods ("Term") of less than one year, as calculated using the BS model. For the given day of October 29, 1999, it can be seen that the volatility structure is such that volatility increases as the period of the transaction is extended, going from 13.71% for a one-month transaction to 15.05% for a one-year transaction. "ATM FWD" appearing in the same figure refers to at-the-money (hereinafter referred to as "ATM") forward transactions. This indicates the volatility of options using the forward foreign exchange rate of the pertinent period as the strike price.[6]

As shown in Figure 1, volatility for periods exceeding one year are not given. This is because foreign exchange options rarely exceed one year, and the transaction amount is very small even when they do. This can be explained as follows. First of all, as compared to short-term options, long-term foreign exchange options constitute high-risk products because premiums become increasingly sensitive to volatility as the maturity is prolonged. Secondly, because the market for long-term foreign exchange options is not very liquid, in certain cases a deal cannot be quickly closed. Finally, as pointed out by Derman [1996] and as can be confirmed from Figure 1,

6. In actual OTC foreign exchange option transactions, the volatility of the smile is also taken into consideration.

the BS model's assumption of constant interest rates and volatility is not realistic, particularly when the term of the transaction is relatively long. As such, the valuation of long-term foreign exchange options presents difficult problems and is especially susceptible to model risks.

A. Amin-Jarrow Model[7]

Amin and Jarrow [1999] have developed a foreign exchange option model which treats two-country interest rates and foreign exchange rates as variables. Table 1 compares the premiums on long-term yen/dollar foreign exchange options (call options) exceeding one year (notional amount: 105.71 yen/dollar) as calculated using the Amin-Jarrow model (hereinafter referred to as the "AJ model") and the BS model as of October 15, 1999.[8] Note that the value of the correlation coefficient used in Table 1 for the yen/dollar exchange rate and the dollar interest rate (0.05) was obtained from historical data.

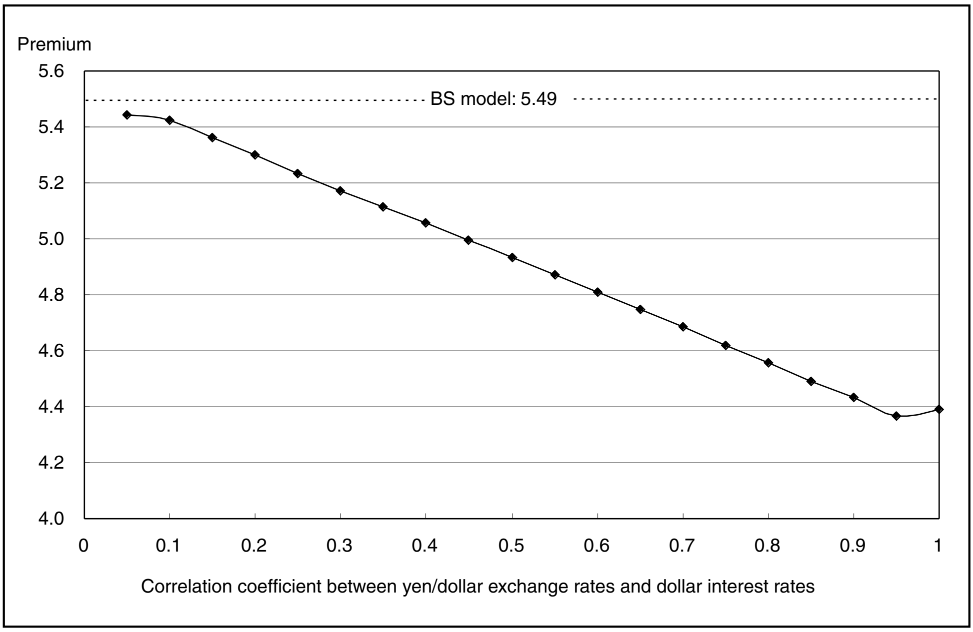
Strike prices are given in the left-hand column of Table 1, and corresponding premiums are indicated by option maturity. For example, for a strike price of 105.71 yen/dollar (spot rate for October 15, 1999), the five-year maturity premium was 5.48 yen/dollar and 5.49 yen/dollar for the AJ model and BS model, respectively. The largest observed difference in premiums for the two models was 0.08 yen/dollar (158.57 yen/dollar strike price with five-year maturity). This difference was equivalent to approximately 10% of the premiums (0.87 yen/dollar and 0.95 yen/dollar), or only 0.076% of the notional amount. At this level, the difference can be readily contained in the bid-offer spread of volatility. In other words, the calculated values of the two models are very close to each other.

Table 1 Comparison of Premiums (¥/$) in AJ model and BS model (Call Option for October 15, 1999)

Strike price	AJ model				Years	BS model				Years
	1	2	3	4	5	1	2	3	4	5
158.57	0.05	0.26	0.50	0.70	0.87	0.05	0.27	0.52	0.75	0.95
148.00	0.14	0.50	0.83	1.08	1.27	0.14	0.51	0.85	1.13	1.34
137.43	0.37	0.95	1.35	1.64	1.85	0.37	0.96	1.38	1.68	1.91
126.86	0.95	1.75	2.20	2.48	2.67	0.96	1.76	2.22	2.52	2.71
116.29	2.28	3.14	3.52	3.73	3.84	2.28	3.15	3.54	3.75	3.86
105.71	4.95	5.45	5.55	5.54	5.48	4.96	5.46	5.57	5.56	5.49
95.14	9.67	9.10	8.57	8.15	7.78	9.68	9.11	8.59	8.16	7.78
84.57	16.80	14.46	12.92	11.82	10.95	16.80	14.47	12.93	11.82	10.94
74.00	25.95	21.70	18.84	16.80	15.23	25.95	21.71	18.86	16.82	15.23
63.43	36.16	30.61	26.44	23.31	20.86	36.16	30.62	26.45	23.33	20.86
52.86	46.67	40.59	35.47	31.34	27.95	46.67	40.59	35.48	31.35	27.96

7. See Appendix 2 for details of the Amin-Jarrow model.
8. Data used in the analysis consists of: yen/dollar exchange rate, and term structures of dollar and yen interest rates of October 15, 1999; and yen/dollar exchange rate, and variance and covariance of dollar and yen interest rates from October 2, 1998 to October 15, 1999 (historical data for approximately one year). In this analysis, historical volatility was used. Although it is desirable to use implied volatility, historical volatility was used for the following reasons: because of low liquidity in long-term foreign exchange option markets, it is difficult to obtain implied volatility; and the present chapter focuses on differences in numerical results among various models.

Figure 2 Relation of Correlation Coefficient and Premiums in AJ Model (Strike price of 105.71 yen/dollar, call option with five-year maturity)



Given a strike price of 105.71 yen/dollar and a five-year maturity, Figure 2 depicts how premiums change in accordance with the correlation coefficient[9] for the yen/dollar exchange rate and the dollar interest rate.

In the case of the AJ model, the premium is diminished as the correlation coefficient approaches 1. Figure 2 indicates that the difference in premiums exceeds 20% when comparisons are made at correlation values of 0.05 and 1 (equivalent to a difference of slightly less than 1% in notional amount).

The differences in the calculated values of the AJ model and BS model were small in Table 1 because the correlation between the yen/dollar foreign exchange rate and the dollar interest rate was assumed to be fixed at 0.05. It should be noted that in the case of the BS model, the premium remains constant because the correlation coefficient is not incorporated into the model (Figure 2). Therefore, if the BS model is applied to long-term foreign exchange options, there is a risk of major mis-pricing at certain levels of correlation between exchange and interest rates.

B. Implications

The AJ model is a relatively detailed model which takes account of the term structures of interest rates and volatility, the correlation of interest rates and foreign exchange rates, and the fluctuation of these values over time. Thus, the AJ model is

9. Premiums were calculated based on estimations of the parameters of the AJ model for the given correlation coefficient.

based on more realistic assumptions than the BS model. Therefore, in the valuation of long-term foreign exchange options, it is more desirable to use the AJ model than the BS model.

However, in the case of long-term foreign exchange options, not only is it difficult to obtain the market price, but there is no guarantee that actual transactions can be carried out at the current price calculated using the model. Moreover, there is a high risk of mis-pricing caused by wrong estimation of the parameters.

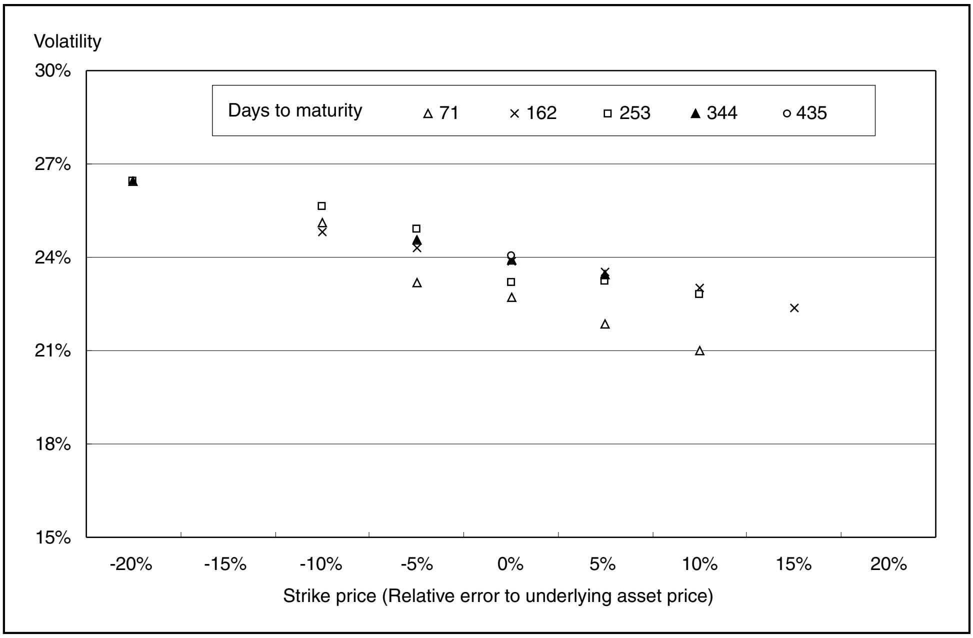
IV. Barrier Options on Stock Price

This chapter is given to the analysis of barrier options (barrier options on stock price), an exotic option with a complicated structure.

Stock prices are generally subject to higher levels of volatility than foreign exchange rates and bonds. In addition, stock prices tend to clearly exhibit volatility structures, such as skews and smiles. Figure 3 presents the volatility structure of OTC options of the Nikkei Stock Average index (five types of days to maturity).[10]

The relative error (in percent) between the underlying asset price on a given day and the strike price is measured on the horizontal axis of Figure 3, while the vertical axis measures the volatility corresponding to the plain call option or put option (hereinafter referred to as the "normal option") at the given strike price. The skew

Figure 3 Volatility Skew (As of September 30, 1999)



10. Premiums are used for indications of OTC stock index options. In Figure 3, implied volatility was calculated using the BS model based on the indications of the Traditional Co.

structure of stock options is generally such that volatility is higher for options with a lower strike price, and lower for options with a higher strike price. Similarly, the skew structure of options closer to maturity is more steeply sloped than options with longer maturity. Figure 3 confirms these features of the skew structure.

To properly value the current price of stock index options and to manage the risk, it is necessary to use models which conform to market data. However, as in the case of foreign exchange options, the BS model is widely used. This choice is based on the fact that almost all market transactions in options constitute normal option transactions.

In the case of such exotic options as barrier options, it is clearly dangerous to use the volatility of normal options for valuation of current prices and risk management. In the following section, barrier options are priced using the implied binomial tree model (a type of lattice method), and the results are examined.

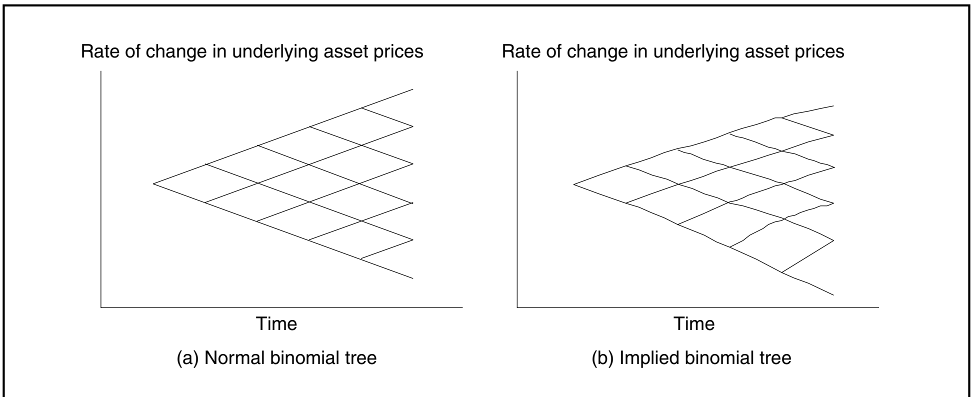
A. Implied Binomial Tree Model[11]

Implied binomial tree models (hereinafter referred to as the "IBT model") were designed to facilitate more accurate pricing of the types of volatility skews and term structures appearing in Figure 3. The IBT model used in the numerical examples of this chapter is from Derman and Kani (1994).[12] The lattice structures of the normal binomial lattice model and the IBT model are shown in Figure 4.

The lattice structure of IBT shown in Figure 4(b) is warped in comparison to the lattice structure in Figure 4(a) representing the binomial tree derived by Cox, Ross, and Rubinstein (1979). Although IBT assumes the same risk neutrality as the normal binomial tree, its lattice structure is warped to achieve a higher level of conformity with market data.

Using this IBT model, we analyze the "down and out put option" which is one

Figure 4 Comparison of Lattice Structures



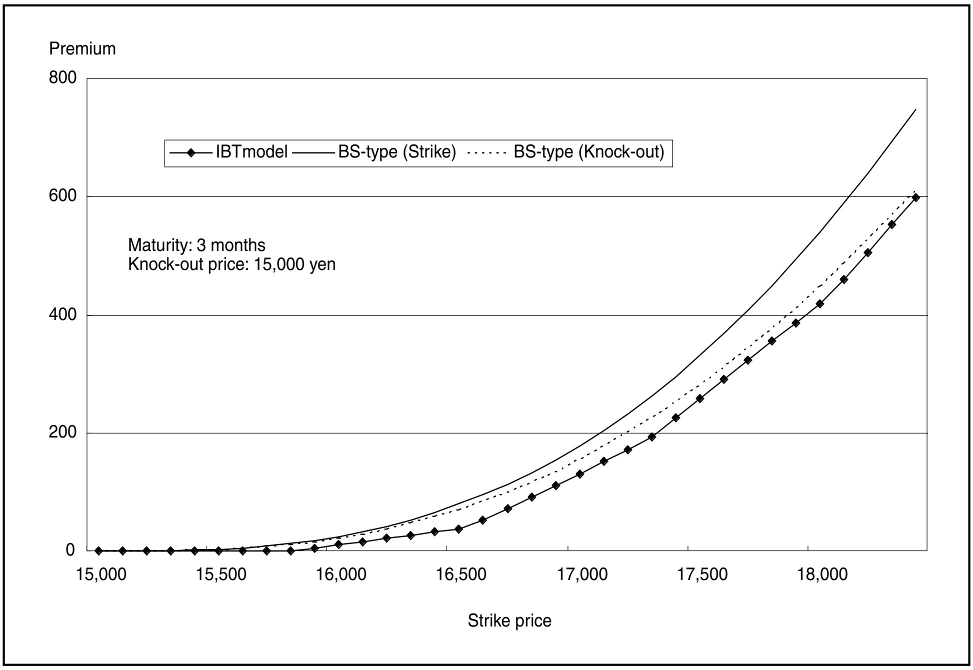
11. See Appendix 3 for a summary of the implied binomial tree model.
12. The Derman and Kani (1994) model is sometimes referred to as the implied volatility tree. See also Dupire (1994) for details of the implied volatility tree.

type of barrier option. The down and out put option is an exotic type of option wherein option rights are lost (the right to sell the underlying asset) if the barrier price (hereinafter referred to as the "knock-out price") is ever reached through maturity.[13] The down and out put option is structured so that the rate of change in the option price against the price of the underlying asset increases as the knock-out price is approached.

Figure 5 compares down and out put option premiums calculated using the results of a Black-Scholes type model (hereinafter referred to as the "BS-type")[14] and the IBT model. The underlying asset price is 17,605.46 yen, and the knock-out price is 15,000 yen. BS-type (strike) premiums shown in the figure were calculated by substituting volatility at strike price into the closed-form result of the BS-type, while BS-type (knock-out) premiums were similarly calculated using the volatility of the knock-out price.

Figure 5 indicates that IBT model premiums[15] are smaller than BS-type (knock-out) premiums, and that IBT model premiums approach the BS-type (knock-out)

Figure 5 Comparison of Down and Out Put Option Premiums (1) (As of September 30, 1999)



13. Judgment on whether the barrier price has been reached may be based on intra-day price fluctuations, closing prices, or end-of-week prices.
14. In this paper, "BS-type analytical results" refers to calculations carried out using an option model with the same framework as the BS model. See Appendix 1 for details.
15. All IBT model calculations in this chapter, excluding the calculations for Figure 6, are based on 30 lattice segments.

premiums as the strike price rises. This is an interesting observation as it means that, in comparison to the BS-type (knock-out), the IBT model assigns a greater weight to the possibility of the underlying asset price reaching the knock-out price.

B. Problems of the IBT Model

As mentioned above, the IBT model has been structured to conform to market data. On the other hand, various qualifications apply to the use of this model. Figure 6 depicts the relation between the number of lattice segments and premiums in the IBT model.

In the lattice method, as a rule, the calculated premiums are unstable when the number of lattice segments is small. Figure 6 shows that premiums become increasingly stable as the number of segments is increased. This observation applies to both normal options and down and out put options. Furthermore, a comparison of the down and out put option and normal option reveals that the former requires a larger number of segments for premiums to be stabilized.

As shown in Figure 7, premiums calculated using the lattice approach take the form of a discontinuous function in their relation to underlying asset prices. This discontinuity occurs for the following reason. Under the lattice method, judgment on knock-out of a down and out put option is made at a point located on the lattice. Therefore, if the knock-out price is not located on the lattice, the calculated results will be unstable. It can be seen that there is a tendency for the instability to be larger when the option is in-the-money.

Figure 6 Comparison of Number of Segments (As of September 30, 1999)

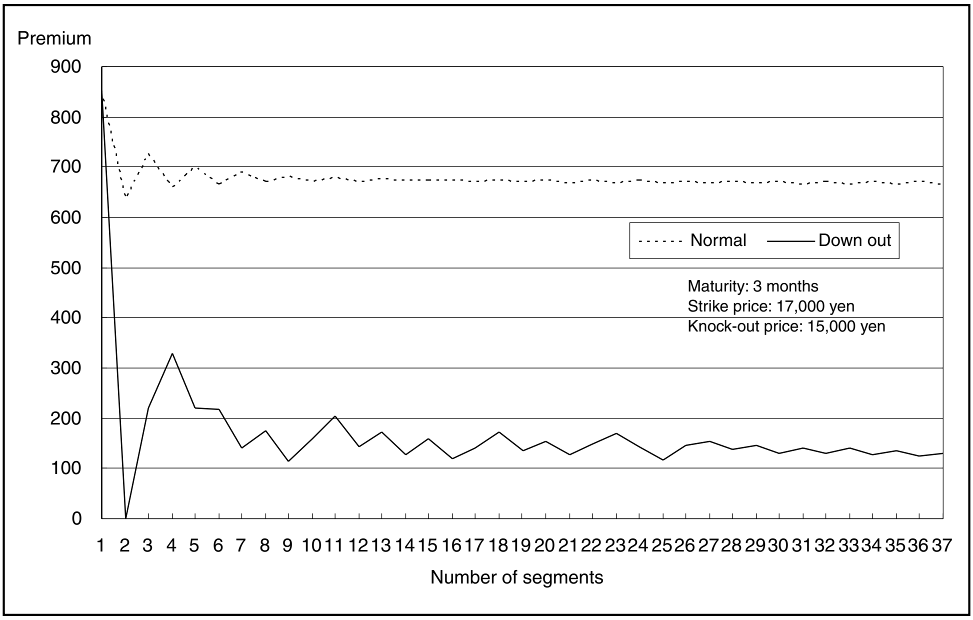


Figure 7 Comparison of Down and Out Put Option Premiums (2) (As of September 30, 1999)

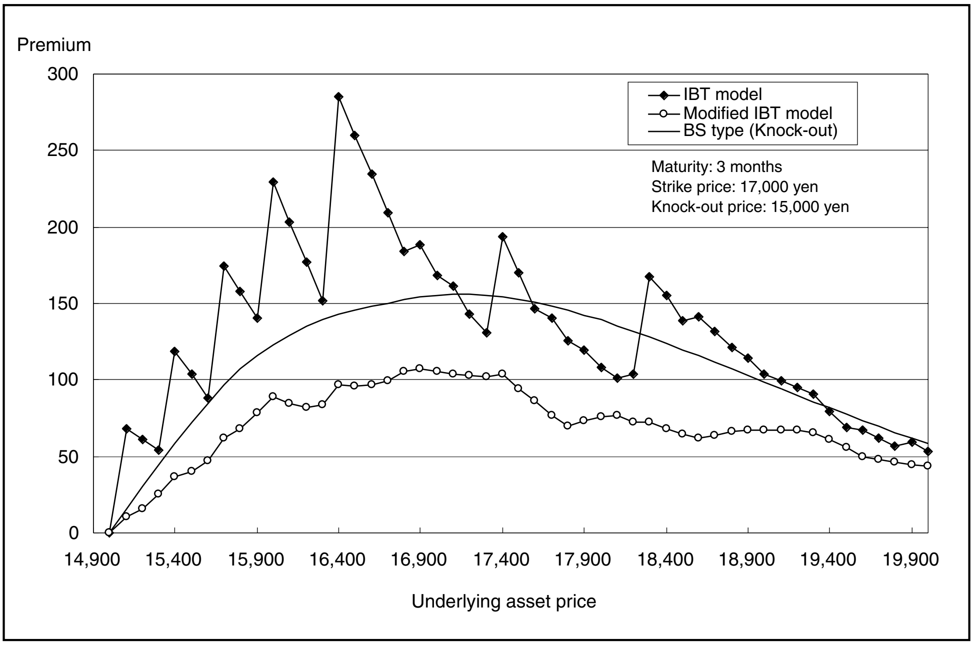


Figure 7 depicts the modified IBT model[16] which was designed to overcome this problem. The modified model approximates the premium in the following manner. Two points on the lattice lying on either side of a knock-out price are assumed to represent knock-out prices and are used in calculating two option prices located on the lattice. Then, the difference between the closest price on the lattice and the knock-out price is linearly apportioned to approximate the premium.[17] Figure 7 presents a comparison of down and out put option premiums calculated using the IBT model and the modified IBT model.

Figure 7 shows that the premiums calculated using the modified IBT model trace a relatively smooth curve, indicating that this is an easy-to-use model from the perspective of position management. However, compared to the BS-type (knock-out), premiums are consistently lower and move in the range of 1/2 - 2/3. This point differs significantly from the results observed in Figure 5.

Based on the foregoing analysis, the structures and computational processes of the IBT model and modified IBT model can be characterized as follows.

- Calculated results are more compatible to the market than the BS model.
- Because these models are based on a discretized approach, the results are affected

16. The modified IBT model is based on the procedures of Derman, Kani, Ergener, and Bardhan (1995).
17. The option price is adjusted as follows. First, the option price of a point on the lattice is calculated on the assumption that no barrier exits at the lattice-point prices located on either side of the knock-out price. Next, the distance (price difference) between the knock-out price and lattice-point price is linearly apportioned to the option prices of each of the lattice points. Following this process, the option prices on the two points on the lattice are added together and used as the option price of the lattice point located between the two points.

by the discretized segments (number of segments).
- An increase in the number of segments dramatically increases the computational burden.
- Reliability is low when market data is limited.
- The data handling burden is high, and discretionary factors cannot be removed.

C. Implications

The IBT model is designed for calculating current prices which conform to the market, based on the prices of heavily traded financial products. However, in reality, the IBT model depends on approximations such as discretization and adjustments in barrier prices. Therefore, the IBT model may not necessarily arrive at the theoretically correct current price.

The IBT model also carries the risk of calculating a price which may not conform with the current price at any given time. This risk arises from such considerations as the time lag in obtaining market data, the timing of the data display output, and the bid-offer spread. Furthermore, as can be confirmed in Figure 3, a wide range of market data cannot be obtained at all times, and interpolation processes are required in numerous instances. The IBT model may be particularly susceptible to the impact of data interpolation.

V. Strangle Short Strategy

One of the available option strategies is a strangle short strategy. A strangle constitutes a combination of calls and puts with different strike prices. (A combination of calls and puts with the same strike price is called a straddle.) The general objectives of making a strangle position can be described as follows. (1) Holders of options can use strangles as a hedge against sharp swings in underlying asset prices, and (2) writers of options can use strangles to earn premiums expecting that the underlying asset price may remain within the range of the window (the range of underlying asset price whereby the put-call strike-price difference does not result in payment on maturity). Figure 8 presents the profit-loss diagram of the strangle short position.

Traders frequently take short positions to accommodate the hedging needs of customers.[18] Moreover, some traders actively adopt a strangle short strategy featuring a wide window to earn premiums.

In this chapter, we shall measure the risks of strangle positions and examine some pertinent problems.

A. Interest Rate Distribution

Figure 9 depicts the 10-day fluctuation histogram of a five-year swap rate. The data covers a period of approximately six years extending between November 1993 and

18. It is more common for traders to take short positions of options in order to act as counterparties to the hedging needs of customers. In Japan, it appears that it is not unusual for customers to take short positions with the aim of earning premiums.

Figure 8 Strangle Short

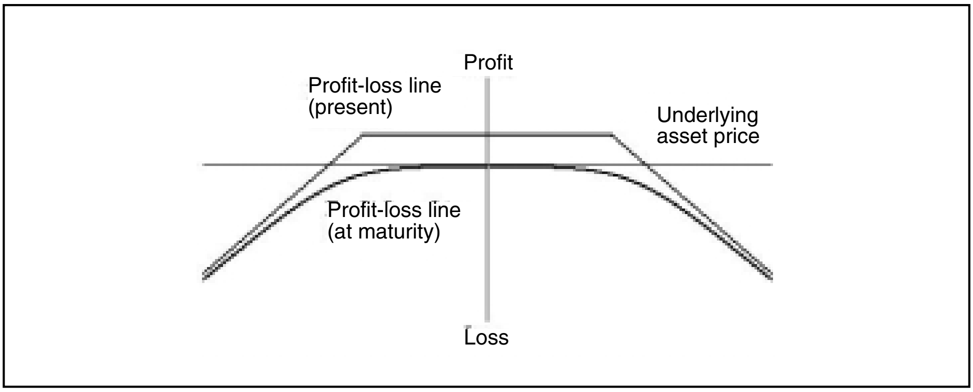
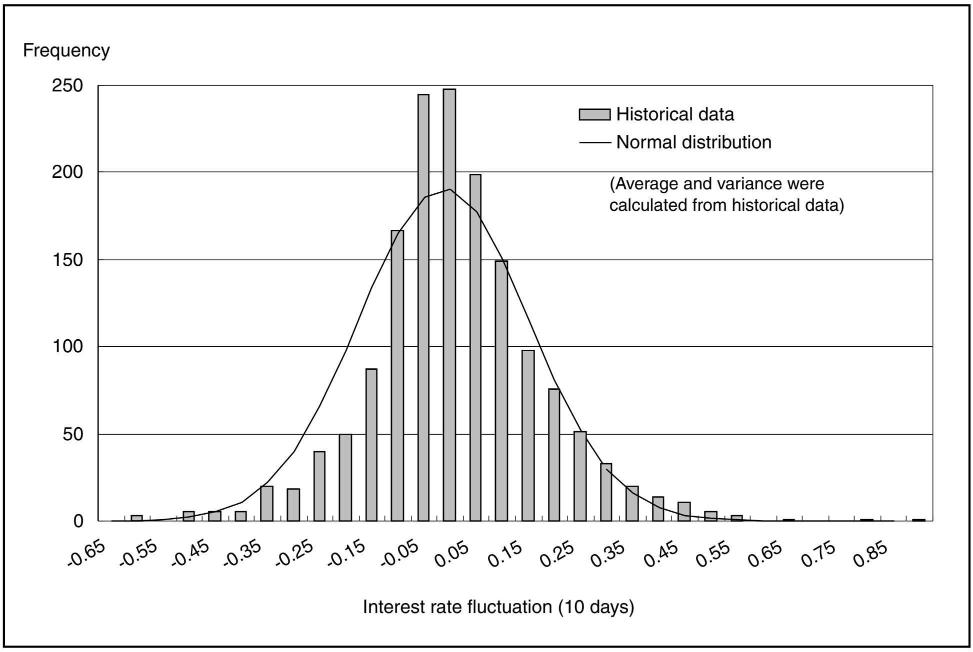


Figure 9 Distribution of Ten-Day Fluctuations of Five-Year Swap Rate (November 15, 1993 - November 25, 1999)



November 1999. It can be seen from Figure 9 that the distribution of interest rate fluctuations during this period differs from normal distribution which is generally used in the value-at-risk method (hereinafter referred to as "VaR"). The center of gravity of the actual distribution observed during this period lies to the right of normal distribution (skewness = 0.28). Moreover, the historical data exhibits a kurtosis of five compared to three in the case of normal distribution. Finally, the historical data has a fat tail with the 99% points located at -0.41 and 0.43 (compared to 99% points located at ±0.38 in the case of normal distribution).

B. Comparison of Risk Amount

A five-year swap rate is used as the underlying asset in the following example of a strangle short strategy. Let us assume that we have sold calls and puts with strike prices of ±0.30% from the current interest rate level. Let us further assume that the transactions were made with ten business days remaining to maturity, and that we hold the options through maturity. Figure 10 and Table 2 present a histogram of profits and losses registered if one unit of the same strangle short transaction had been contracted on every business day between the second half of 1996 and the first half of 1999.[19]

Under this strangle short strategy, per unit profits amounted to 0.00 - +0.01 on nearly 90% of all trading days, indicating a very high concentration of profit/loss distribution in an extremely narrow band. Cumulative profit/loss for the entire period amounted to a gain of +0.39.[20] The largest half-year gain was registered in the first half of 1999 with a profit of +0.39, while the largest half-year loss was registered in the second half of 1998 with a loss of -0.63. The largest fluctuation in profit/loss was observed during the second half of 1998. Almost all of the over ±0.05 fluctuations observed during the entire period were concentrated in this half-year period. Furthermore, the largest loss registered for a single transaction was also registered in the second half of 1998 at -0.23.

Next, the relationship between profit/loss under this strangle short strategy and VaR is shown in Figure 11.

After marking a brief climb in 1994, the five-year yen-swap rate generally continued to move downward until reversing itself in the second half of 1998. Thereafter, it has fluctuated in a relatively narrow band around 1%. Figure 11 shows that the profit/loss of the strangle short strategy adopted in this chapter fluctuated significantly when the trend in interest rates was reversed. Particularly large and continuous fluctuations were observed during the second half of 1998. On the other hand, fluctuations were small throughout the remainder of the period.

VaR shown in Figure 11 was calculated using the scenario method and the historical simulation method.[21] Normal distribution was assumed for VaR (one year) and VaR (three years), and calculations were based on the worst-case scenario of the 99% (2.33 σ) confidence level of the historical volatility for the latest one- and three-year periods, respectively.[22] On the other hand, the 99% point of the historical simulation method was calculated for VaR (HS: one year) and VaR (HS: three years) based on historical data for the latest one- and three-year periods. Because the volatility of the five-year swap rate continuously declined after interest rates began to climb in 1994, VaR (one year) and VaR (three years), which both assume normal distribution, traced

19. We assumed that we were able to sell both the call and put options based on a BS model with historical volatility of the ten immediately preceding days with zero drift. (In reality, we could probably have received a higher premium because of the existence of a smile in the market.) Note that in this chapter we are analyzing an option in which the five-year swap rate itself is the underlying asset.
20. Reinvestment of profits is not taken into account here.
21. Standard methods for calculation of VaR of options are (1) the delta + gamma + vega method, (2) the scenario method, and (3) the simulation method. Because (1) presents various problems for VaR measurement, (2) and (3) were used in this analysis.
22. Calculations are based on one year = 250 business days, and three years = 750 business days.

Figure 10 Profit and Loss of Interest Rate Strangle Short Strategy (Frequency distribution: October 1, 1996 - September 30, 1999)

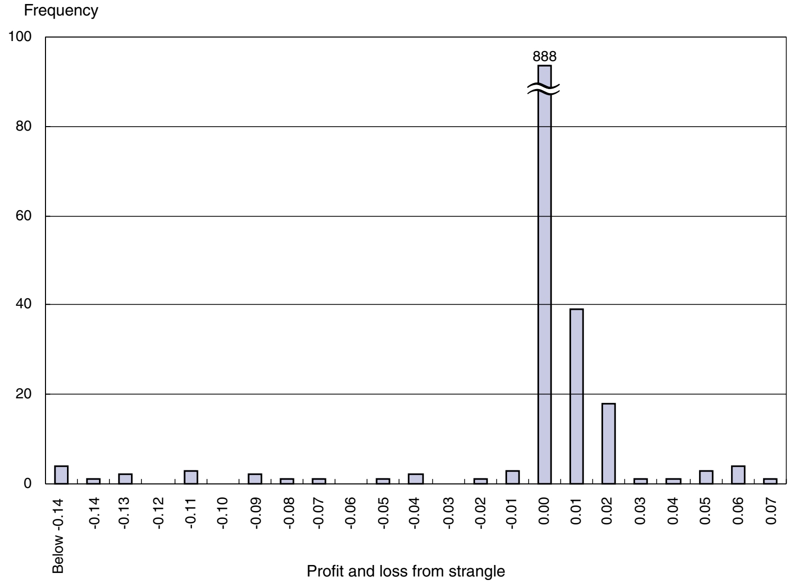
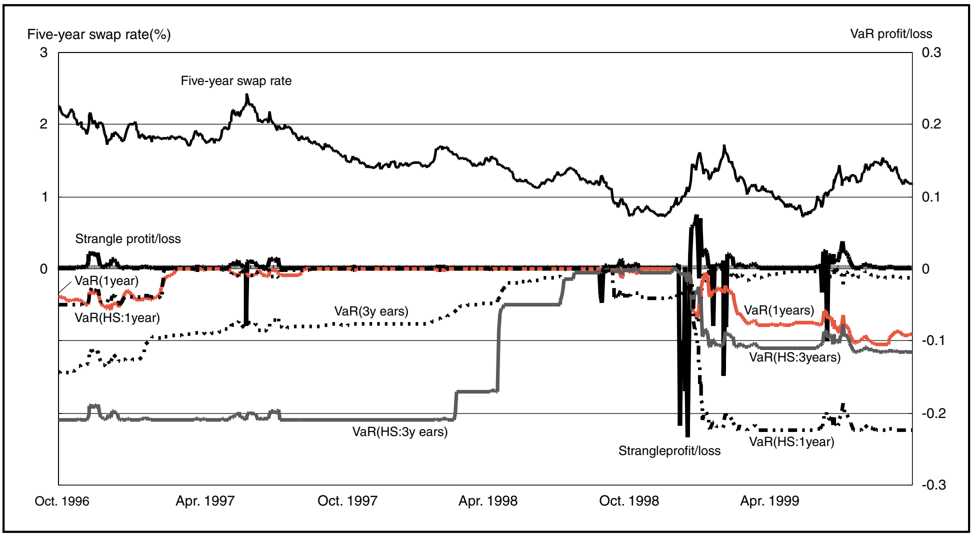


Table 2 Profit and Loss for Interest Rate Strangle Short Strategy (Per Period: October 1, 1996 - September 30, 1999)

Data Segments	2nd half of 1996	1st half of 1997	2nd half of 1997	1st half of 1998	2nd half of 1998	1st half of 1999	Total
Below -0.14	0	0	0	0	*4*	0	*4*
-0.14	0	0	0	0	1	0	1
-0.13	0	0	0	0	2	0	2
-0.12	0	0	0	0	0	0	0
-0.11	0	0	0	0	2	1	3
-0.10	0	0	0	0	0	0	0
-0.09	0	1	0	0	1	0	2
-0.08	0	0	0	0	1	0	1
-0.07	0	0	0	0	1	0	1
-0.06	0	0	0	0	0	0	0
-0.05	0	0	0	1	0	0	1
-0.04	0	0	0	1	0	1	2
-0.03	0	0	0	0	0	0	0
-0.02	0	0	0	0	1	0	1
-0.01	0	0	1	1	1	0	3
0.00	116	121	129	121	92	109	688
0.01	8	9	0	2	10	10	39
0.02	6	0	0	0	3	9	18
0.03	0	0	0	0	0	1	1
0.04	0	0	0	0	1	0	1
0.05	0	0	0	0	3	0	3
0.06	0	0	0	0	4	0	4
0.07	0	0	0	0	1	0	1
Number of Days	130	131	130	126	128	131	776
Profit/Loss	0.37	0.23	0.03	-0.01	*-0.63*	0.39	0.39

Figure 11 Risk Amount and Trends in Profit/Loss under Interest Rate Strangle Short Strategy (October 1, 1996 - September 30, 1999)



a steady downward trend. VaR (HS: three years) also declined sharply through the first half of 1998 because the extreme fluctuations of 1995 were removed from the data used for the historical simulation. Responding to market fluctuations, all VaRs began to climb after the second half of 1998. The most dramatic rise was observed in the case of VaR (HS: one year).

Following the reverse rise in interest rates in the second half of 1998, profit/loss of the strangle short strategy showed large fluctuations. Consequently, both the frequency and extent of excess loss over VaR were particularly high during this period (see Table 3).

Table 3 can be used to undertake the backtesting required by the market-risk regulations of the Basel Committee on Banking Supervision (hereinafter referred to as "market-risk regulations"). The mandated backtesting counts the number of times that daily trading profit/loss has exceeded VaR (at the 99% confidence level) over the past year. Four times or less is defined as constituting the "green zone," five - nine times as the "yellow zone," and ten or more times as the "red zone." Financial institutions whose loss frequently exceed VaR are required to add to their capital.[23] Table 3 indicates that the strangle short strategy of this chapter would have put us in the red zone during the second half of 1998.[24]

23. Market-risk regulations require backtesting using daily profit/loss and VaR values for the latest twelve-month period for each fiscal year. The backtesting results are used in determining the plus-factors to be added to the multiplication-factor of three. The plus-factor is zero for the green zone and one for the red zone. In the yellow zone, the plus-factors are 0.4, 0.5, 0.65, 0.75, and 0.85 for frequencies of excess of 5, 6, 7, 8, and 9, respectively. The market risk amount is obtained by multiplying the "multiplication-factor + plus-factor" (3 - 4) by the VaR value for ten business days of holding and 99% confidence level.
24. The analysis of this chapter compares profit/loss of ten business days and VaR values of ten business days of holding.

Table 3 Frequency of Excess Loss over VaR under Interest Rate Strangle Short Strategy (October 1, 1996 - September 30, 1999)

Risk measurement method		2nd half of 1996	1st half of 1997	2nd half of 1997	1st half of 1998	2nd half of 1998	1st half of 1999	Total
VaR(3 years)	Number of days exceeded	0	1	0	3	13	2	19
	maximum excess		-0.00		-0.05	-0.23	-0.09	
VaR(1 year)	Number of days exceeded	0	1	0	3	12	1	17
	maximum excess		-0.08		-0.05	-0.23	-0.03	
VaR(HS:3 years)	Number of days exceeded	0	0	0	3	9	1	13
	maximum excess				-0.05	-0.23	-0.01	
VaR(HS:1 year)	Number of days exceeded	0	1	0	3	8	0	12
	maximum excess		-0.08		-0.05	-0.20		

C. Implications

In this chapter, we have undertaken to empirically analyze risk measurement models and to identify pertinent problems using the strangle short strategy as a test case. Because the relation between option payoffs and underlying asset prices is non-linear, the measurement of risks is more complicated than in the case of linear payoff.

Risk measurement models are frequently based on the assumption of normal distribution or lognormal distribution. Even in the case of the historical simulation model, it is assumed that the distribution patterns of the historical data will be repeated in the future. Because risk measurement models must contain such internal assumptions, they are inevitably susceptible to the type of model risks whereby actual distributions differ from the assumed distribution.

VI. Practical Steps to Control Model Risks

In the foregoing chapters, we have examined model risks as separately defined for pricing models and risk measurement models. Our principal findings can be summarized as follows.

- Model Risks of Pricing Models

  Definition: The risk arising from the use of a model which cannot accurately express market prices, or which is not a mainstream model in the market.

Where market prices are obtainable, risks are realized when the valuations of internal pricing models are compared to actual market prices. In such cases, hedging discrepancies will lead to a gradual accumulation of real profits or losses. On the other hand, where market prices are unobtainable, risks are realized when the internal pricing model is switched to the market's dominant or mainstream model.

The sources of model risks in pricing models are generally as follows: (1) errors in the premises and assumptions of the model; (2) errors in estimations of parameters which cannot be directly observed such as default probability, correlation coefficients, and other factors; (3) errors arising from discretization and other approximations;

and (4) errors in market data. The cases investigated in Chapter III correspond to categories (1) and (2), while those of Chapter IV correspond to categories (3) and (4).

Probably the majority of cases of losses resulting from model risks belong to category (2), while the incidence of category (1) is thought to be quite low. Finally, model risks can be relatively easily controlled when market prices are obtainable at high frequencies.

- Model Risks of Risk Measurement Models

Definition: The risk of not accurately estimating the probability of future losses.

As seen in Section II. C and Section V, one type of model risk is attributable to wrong premises concerning the distribution of underlying asset prices and the failure to appropriately specify holding periods and confidence levels. In such cases, VaR cannot be accurately calculated and can lead to unexpectedly large losses. On the other hand, model risks can also take the form of over-reaction, such as unnecessarily large additions to the capital.

The VaR model, extensively used in the measurement of market risks, has the problem of not being able to measure loss amounts located beyond the confidence level. Furthermore, in risk measurement models, logical elements of the entire framework of the model, such as computational methods and the setting of risk factors for simplifying computational tasks, can also be a source of model risk.

The selection of practical steps to control model risks involves a trade-off between the level of the model risk and the necessary cost of controlling the risk. For instance, overly rigorous examinations of models can possibly undermine the initiative of the model development section in developing new products. On the other hand, if only very small positions are to be built up in a specific product, there is little justification in spending large amounts on verifying the pertinent pricing model. Furthermore, from the perspective of reducing computational burdens, a possible approach would be to combine the use of a relatively simple model for day-to-day risk control with a more refined model brought in for occasional position valuations and the verification of the model itself. Therefore, if a necessary minimum level of steps has been properly installed, it could be possible to adopt a more flexible approach to controlling model risks while taking account of such factors as type of product, transaction policy, market conditions, and size of positions.

In the following sections, we shall examine what constitutes a necessary minimum level of practical steps for the control of model risks.

A. Management System for Model Risks

In this section, we shall examine management systems for model risk, which may be considered to be necessary for financial institutions.

- Organization, Authorization, and Human Resources

The management of financial institutions must have a proper awareness of model risks. Moreover, this knowledge should not be treated merely as ancillary information, but must be actively utilized in management decisions. Financial institutions must establish independent "model control sections" which function independently of sections involved in the actual use of the models (such as the trading sections). Personnel in charge of model control must investigate the various models being used by the financial institution and should be authorized to issue directives for the revision and improvement of such models as needed. Finally, personnel assigned to model control must be able to understand the essence of the models and must be fully capable of analyzing and verifying the models.

The financial institution cited in the case of the mark cap appearing in Section II. B suffered large losses, although the mark cap is a very commonplace financial product. There is a possibility that this financial institution could have averted these losses if the personnel in its model control section had been capable of verifying the model and had been explicitly authorized to order the implementation of necessary changes.

- Examination of Models

When a new model is developed by the trading section (hereinafter referred to as the "front") or the model development section, it is vitally important that it be examined by an independent model examination section. The use of unexamined models developed by the front or other sections can be extremely dangerous. As such, an examination section which is completely independent of the front must thoroughly examine a new model, including such aspects as the theoretical justification (assumptions) of the model, the conformity of prices calculated by the model and actual market prices, and hedging effectiveness. Such examination systems and protocols should be explicitly established in the internal rules of the financial institution.
The case involving the wrong calculation of long-term foreign exchange options cited in Chapter III could have been avoided if the financial institution had instituted an appropriate model examination system. A properly manned examination section could have prevented the inadvertent use of the BS model in pricing this option.

- Regular Review of Models

The continued use of a previously approved model without regular review can be extremely dangerous. This is because models must regularly undergo minor adjustments to accurately reflect market changes, such as changes in the distribution pattern of underlying asset prices. Moreover, the various types of checks undertaken in the examination process (conformity of calculated theoretical prices and actual market prices, and hedging effectiveness) must also be regularly implemented.

The implementation of regular review could have averted the very large losses suffered in the index swap case cited in Section II. A by calling for the necessary modifications in the model at an early stage.

- Communicating with the Front

As models are built to reflect the structure of the markets, model control staff must have a full and up-to-date understanding of market developments. Likewise, they must be fully informed of the risk profile of the positions held by the front. It is the front that is the closest to the market and in a position to acquire the largest amount of information concerning the market. Therefore, personnel in charge of model control must maintain close and regular communications with the front, and endeavor to stay abreast of market developments and the risk profile of the current position.

For instance, as shown in Chapter V, positions under a strangle short strategy display non-linearity (optionality), and risk measurements must take this matter into account. Model control staff must have an up-to-date knowledge of the front's current positions and position management policies, in order to develop an awareness of possible types of future risks and to build and manage appropriate risk measurement models.

B. Other Steps to Control Model Risk

Further to the model risk management systems discussed above, in this section we shall consider other types of effective steps. The specific steps examined below are: reserves against loss, scenario analysis, and position limits.

- Reserves against Loss

Providing reserves against loss should constitute an effective step in controlling the model risks of pricing models.

Let us consider cases in which the lattice method or Monte Carlo method is used to calculate numerical values for pricing. When using these methods, it is not necessarily best to simply increase the number of segments or random number series.[25] An alternative approach would be to achieve a certain level of precision in pricing which satisfies the requirements of the financial institution, and to thereafter depend on reserves to cope with possible calculated discrepancies. For instance, in the normal Monte Carlo method, it is known that, given N random number series, calculated error[26] will be in the order of $N^{-1/2}$. Therefore, if pricing is based on 10,000 random number series, a viable step to control model risks would be to set up a reserve equivalent to a few percent of the valuation amount.[27]

The use of reserves is also a viable step in cases where market data on volatility and other factors is unobtainable. For instance, suppose that daily data on volatility for a particular financial product is unobtainable while monthly data is available. In this case, reserves can be used to cover pricing errors on all days, excluding the day on which volatility data is available (once a month).

Such responses would provide adequate provisions for the problems described in Chapter IV, such as instability in current price resulting from too few segments under the lattice method, or the unobtainability of volatility over a broad range.

25. In the computational procedures of the lattice method and the Monte Carlo method, calculated results will generally approach the theoretical values as the number of segments or random number series reaches infinity. The problem, however, is that merely increasing the number of segments or random number series will only add to the computational load and consume more time. Speedy pricing is particularly important in trading functions. As such, increasing the number of segments or random number series is not necessarily a viable solution.
26. See Tsuda (1995) pp. 91-113, and Kijima, Nagayama, and Omi (1996) pp. 143-155 for details of calculated discrepancies in the Monte Carlo method.
27. $10{,}000^{-1/2} = 1\%$

- Scenario Analysis

The front is always aware of current market conditions when using pricing models. Therefore, proper provisions must be made for dramatic market shifts which undermine the parametric foundations of the model. One approach is to use historical data (scenarios) from past incidents of dramatic market shifts to observe how the parameters of the model would have been affected in such cases, and to analyze the trends in the discrepancies between market prices and prices calculated by pricing model.

One of the problems of the standard VaR model is that the model does not take account of the costs of closing a position. Therefore, the question remains of how to account for risk when holding very large positions, or positions in low-liquidity products. In such cases, a viable approach would be to analyze various types of scenarios with different assumptions concerning holding period and confidence level, and different patterns of fluctuation in risk factors.

In Chapter V, we showed that the level of VaR in the historical simulation method can be significantly influenced by the data observation period. Scenario analysis provides an effective response to such problems which are endogenous to the VaR model.

- Position Limits

One of the available methods for reducing model risk is the setting of position limits. For instance, position limits could be set for a product whose valuation is highly complicated or which is subject to relatively high model risk. Such limits would be determined in light of the corresponding level of model risk.

An example of the application of this approach to pricing models is as follows. Assume a pricing model for an infrequently traded financial product. There exists the possibility of discrepancies arising between the internal model and other models used in the market. A viable response in this case would be the setting of position limits. This approach can be used to avoid losses arising from discrepancies between market and valuation prices, as described in Section II. A, II. B, and III. Next, in the case of risk measurement models, information from scenario analysis can be used in determining position limits. This approach can be used in reducing the risks which may result from errors in the model's assumed distribution of underlying asset prices, as described in Section II. C and Section V.

Position limits can be managed more easily and at less cost than the process of model review. Therefore, position limits can be used as a flexible and effective complement to the model review system.

VII. Conclusions

In this paper, we have presented various cases in which actual model risks have been realized and have used market data to empirically analyze the problems of the models. Based on the implications of our empirical analysis, we have examined various practical steps to control model risks.

Specifically, we have used examples of index swaps, mark cap, and the experiences of LTCM to develop a general image of model risks. From there, we have proceeded to analyze long-term foreign exchange options, barrier options on stock prices, and a strangle short strategy to identify some salient features of model risk. Finally, we have examined various steps and responses to model risks from a practical perspective.

In the area of qualitative steps to control model risks, we have investigated issues pertaining to the reinforcement of risk management systems. Our investigations have covered the following concrete issues: (1) organization, authorization, and human resources; (2) examination of models; (3) periodic review of models; and (4) maintenance of proper communications with the front.

In the area of quantitative steps to control model risks in pricing models, we have proposed setting up reserves to allow for the pricing difference among different models, and instituting position limits based on such differences. With regard to quantitative steps to control model risks in risk measurement models, we have proposed scenario analysis of various patterns in risk factor fluctuations, and instituting position limits based on the information gained from scenario analysis.

Financial institutions can, by no means, afford to ignore model risks. In the future, there will be a growing need to implement various types of steps to control model risks, including quantitative ones.

APPENDIX 1: BLACK-SCHOLES TYPES

Black and Scholes (1973) assumed a lognormal distribution of stock price fluctuations to derive the premiums of options on stocks as the underlying asset. (BS model) Various other models built on the same framework as the BS model are generally referred to as BS types.
The option valuation formula of the BS type used in this paper is as follows.[28]

(Normal Option)

$$\begin{aligned} &\text{Normal Call } = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2), \\ &\text{Normal Put } = Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1), \\ &\quad d_1 = \frac{\ln(S/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}, \\ &\quad d_2 = \frac{\ln(S/K) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}. \end{aligned} \tag{A.1}$$

(Down and Out Put Option)

$$\begin{aligned} &\text{Down and Out Put} = \text{Normal Put} - \text{Down and In Put}, \\ &\text{Down and In Put } = -Se^{-qT}N(-x_1) + Ke^{-rT}N(-x_1 + \sigma\sqrt{T}) + Se^{-qT}(H/S)^{2\lambda} \\ &\qquad [N(y) - N(y_1)] - Ke^{-rT}(H/S)^{2\lambda-2}[N(y - \sigma\sqrt{T}) - N(y_1 - \sigma\sqrt{T})], \\ &\quad \lambda = \frac{r - q + \sigma^2/2}{\sigma^2}, \quad y = \frac{\ln[H^2/(SK)]}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}, \\ &\quad x_1 = \frac{\ln(S/H)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}, \quad y_1 = \frac{\ln(H/S)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}. \end{aligned} \tag{A.2}$$

where:

- $N(\cdot)$: cumulative density function of standard normal distribution,
- S : underlying asset price,
- K : strike price,
- H : barrier price $(K > H)$,
- T : time to option maturity,
- r : risk-free domestic currency interest rate (for foreign exchange option),
- q : risk-free foreign currency interest rate (for foreign exchange option),
- σ : volatility of underlying asset price.

The price of the down and out put option can be derived by deducting the down and in put option price from the normal put option price. The option valuation formulas of BS-type models assume that parameters, such as interest rates and volatility of factors other than underlying asset prices, remain constant through maturity.

28. See Hull (1997) pp. 457-489 for details of valuation formulas for exotic options.

APPENDIX 2: AMIN-JARROW MODEL

The BS model assumes constant interest rates and volatility through maturity. Because interest rates are less volatile than foreign exchange rates, the above assumption does not pose a serious problem in the case of short-term options. However, interest-rate term structures and fluctuations cannot be ignored in the case of long-term options. As such, models for long-term options must take this factor into account. Amin and Jarrow (1991) developed a model which describes explicitly stochastic fluctuations in two-country interest rates and exchange rates. The AJ model assumes that the forward interest rates of the two countries can be expressed in HJM[29] type two-factor models, and that exchange rates can be expressed in four-factor models (two factors are the same as forward interest rates models) which assume that the rate of return follows a normal distribution. Finally, the AJ model expresses the variance and covariance of interest and exchange rates in terms of the relation between the functions of the volatility of each factor.

$$
\begin{aligned}
df_d(t,T) &= \alpha_d(t,T)\,dt + \sum_{i=1}^{2} \sigma_{di}(t,T,f_d(t,T))\,dW_i(t),\\
df_f(t,T) &= \alpha_f(t,T)\,dt + \sum_{i=2}^{3} \sigma_{fi}(t,T,f_f(t,T))\,dW_i(t),\\
dS_d(t) &= \mu_d(t)\,S_d(t)\,dt + \sum_{i=1}^{4} \delta_{di}(t)\,S_d(t)\,dW_i(t).
\end{aligned}
\qquad \text{(A.3)}
$$

Subscripts f and d are foreign and domestic interests rates for forward interest rates respectively, and:

$f_k(t,T)$: forward interest rate at time T observed at time t $(k : d, f)$,
$a_k(t,T)$: forward interest rate drift $(k : d, f)$,
$\sigma_{ki}(t,T,f_k(t,T))$: forward interest rate volatility $(k : d, f)$ (two-factor),
$S_d(t)$: foreign exchange rate,
$\mu_d(t)$: foreign exchange rate drift,
$\delta_{di}(t)$: foreign exchange rate volatility (four-factor)
$W_i(t)$: Wiener process[30],

where the following relation holds.

29. The Heath-Jarrow-Morton model is an interest-rate term structure model. See Heath, Jarrow, and Morton (1992) for details.
30. It is assumed that the four Wiener processes in (A.3) are mutually independent. Relations are established between the foreign exchange rate and interest rate fluctuations in the following manner. Of the four Wiener processes for the foreign exchange rate, two are assumed to be the same for foreign and domestic forward interest rates, and one of the Wiener processes for the foreign and domestic forward interest rates is assumed to be the same.

$$
\begin{aligned}
\mathrm{var}(df_d(t,T)) &= [\sigma_{d1}(t,T,f_d(t,T))^2 + \sigma_{d2}(t,T,f_d(t,T))^2]dt,\\
\mathrm{var}(df_f(t,T)) &= [\sigma_{f2}(t,T,f_f(t,T))^2 + \sigma_{f3}(t,T,f_f(t,T))^2]dt,\\
\mathrm{cov}(df_d(t,T),df_f(t,T)) &= [\sigma_{d2}(t,T,f_d(t,T))\sigma_{f2}(t,T,f_d(t,T))]dt,\\
\mathrm{var}(\frac{dS_d(t)}{S_d(t)}) &= \left[\sum_{i=1}^{4}\delta_{di}^2(t)\right]dt,\\
\mathrm{cov}(\frac{dS_d(t)}{S_d(t)}, df_d(\mathrm{t,T})) &= \left[\sum_{i=1}^{2}\sigma_{di}(t,T,f_d(t,T))\delta_{di}(t)\right]dt,
\end{aligned}
$$

Assuming that each volatility function is a deterministic function, the call option price can be expressed as follows.

$$
\begin{aligned}
&\mathrm{Call} = P_f(0,T)S_d(0)N(h) - KP_d(0,T)N(h-\zeta),\\
&h = \frac{\ln\left(\dfrac{P_f(0,T)S_d(0)}{KP_d(0,T)} + \dfrac{1}{2}\zeta^2\right)}{\zeta},\\
&\zeta^2 = \sum_{i=1}^{4}\int_0^T [a_{fi}(v,T) + \delta_{di}(v) - a_{di}(v,T)]^2\,dv,\\
&a_{ki}(t,T) = -\int_t^T \sigma_{ki}(t,u,f_k(t,u))\,du,\\
&P_k(t,T) = \exp\left[-\int_t^T f_k(t,u)\,du\right].
\end{aligned}
\tag{A.4}
$$

Unlike the BS model, the AJ model describes explicitly term structures and fluctuations in the term structures of interest rates and volatility. However, in this paper, we assumed that interest rate and foreign exchange rate volatility, variance and covariance remained constant through maturity, and derived our calculations from historical data. This choice is based on the fact that the market for long-term foreign exchange options is small and price data is difficult to obtain.

APPENDIX 3: IMPLIED BINOMIAL TREE MODEL

The behavior of underlying asset prices can be expressed as follows in the type of world assumed in the BS model.

$$dS = \mu_c S dt + \sigma_c S dW(t). \tag{A.5}$$

Here, S stands for underlying asset price, and it is assumed that drift μ_c and volatility σ_c remain constant through maturity. $W(t)$ is the Wiener process. Cox, Ross, and Rubinstein (1979) used a binomial tree to derive option prices, based on the assumption that underlying asset prices exhibit this type of behavior. While its basic structure is the same as that of the binomial tree proposed by Cox and others, the IBT model contains the following innovative feature. Time is divided into several smaller segments, and the behavior of underlying prices within each time segment is assumed to conform to the following geometric Brown movements.

$$dS = \mu(t) S dt + \sigma(S,t)\, S dW(t). \tag{A.6}$$

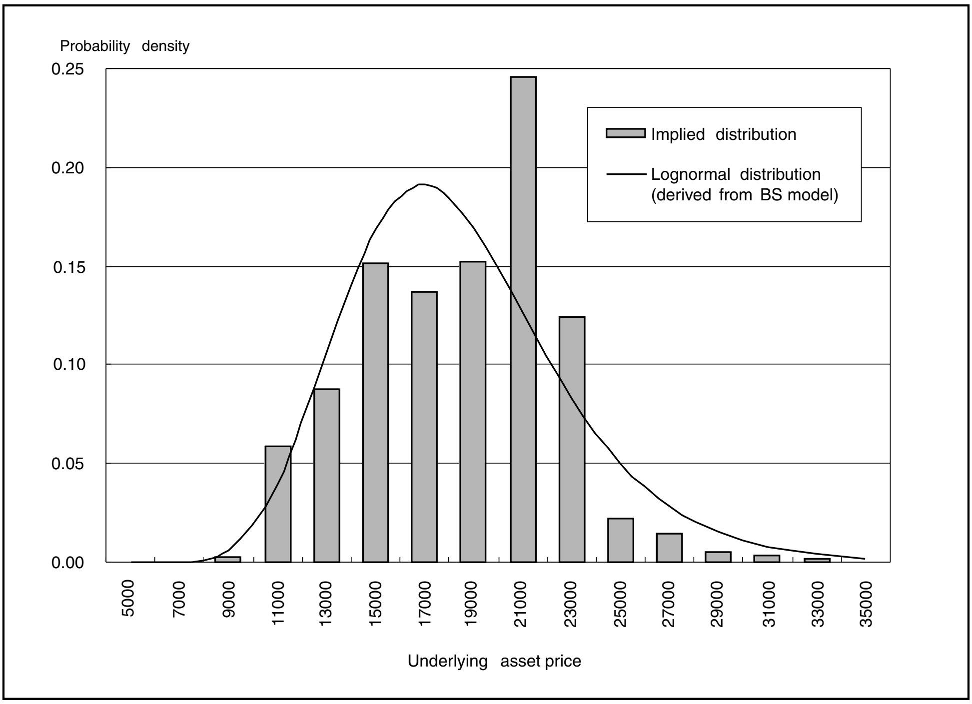
The IBT model was developed to cope with interest rate term structures and other market phenomena, such as smiles and skews. Therefore, the functions $\mu(t)$ and $\sigma(S,t)$ in (A.6) are determined to conform with option prices observed in the market. Furthermore, based on the following procedures, the normal binomial tree is designed to yield a regular lattice structure as shown in Figure 4(a): (1) the perstep rate of increase in underlying asset prices is given as $u = e^{\sigma_c \sqrt{\Delta t}}$, the perstep rate of decline in underlying prices is given as $d = e^{-\sigma_c \sqrt{\Delta t}}$, and the drift in underlying asset prices is given as $a=e^{\mu_c \Delta t}$; and (2) the upward probability is given as $p=(a-d)/(u-d)$, and the downward probability is given as $1-p$. On the other hand, the IBT model uses forward induction to develop the lattice structures. In other words, when moving from the starting-point asset price to the next step, the parameters $\mu(t)$ and $\sigma(S,t)$, which express the stochastic processes of the underlying asset price, are defined so that the asset price in the next step conforms to the option price observed in the market.[31] For this reason, the lattice structure differs from that of the standard binomial tree and is skewed as shown in Figure 4(b).

Appendix Figure shows the distribution of underlying asset prices as deduced by the IBT model using the market data as of September 30, 1999.

As can be seen from Appendix Figure, the implied distribution of underlying asset prices derived from the IBT model does not conform with a lognormal distribution.

31. See Derman and Kani (1994) and Rubinstein (1994) for details of structuring of lattices in the IBT model.

Appendix Figure Implied Distribution of Underlying Asset Prices in IBT model (As of September 30, 1999)



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