A Unit Root Test with Structural Change for Japanese Macroeconomic Variables

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The problem of a time trend in the unit root test is first discussed and then a unit root test for Japanese macroeconomic variables under the assumption that a structural change has occurred in the time trend is conducted. Many Japanese macroeconomic variables exhibit a structural change in the data generating process around 1970. Once such a structural change in the time trend is taken into account, many real economic variables (such as real GNP) are found to follow a trend stationary process.

Key Words : Unit Root; Deterministic Trend; Structural Break; Null Hypothesis.

I. Introduction

In recent years, there have been two important developments in the field of econometrics: an increasing recognition of the problem of "spurious" regressions and a new method of "cointegration" for a multi-variate time series model. These developments have led to the widespread use of a stationarity test as a pre-test for the analysis of macroeconomic time series data. In their well-known paper which conducted the first stationarity test for economic variables, Nelson and Plosser (1982) found that the nonstationarity hypothesis could not be rejected for many U.S. macroeconomic variables. Their study subsequently triggered developments in stationarity tests such as the Dickey-Fuller test as well as numerous Monte Carlo simulations aimed at comparing the required size and power of those tests. At the same time, many shortcomings of the tests have become apparent: for example, the results are sensitive to the choice of maintained hypothesis and null hypothesis. The purpose of this paper is therefore twofold: first to address the question of how to choose a time trend in maintained hypothesis and null hypothesis in the stationarity test, and second to conduct a stationarity test for main Japanese macroeconomic variables, taking into account the problem of choosing a time trend and null hypothesis.

In a growing economy, many variables exhibit an upward trend. The conventional approach has been to assume a linear deterministic trend to account for the upward trend in economic variables. But actual economic time series often exhibit changes in the trend when major economic events such as the Great Depression and the oil crises occur. Perron (1989) has shown that a time series will often be judged as non-stationary in a model which assume no change in the time trend even though, in the true model, it follows a stationary stochastic process around a trend which experiences a change. In fact, many Japanese macroeconomic variables exhibit a structural change in the trend around 1970. Therefore the non-stationarity hypothesis should state that macroeconomic variables follow a unit root process around a time trend which goes through a change. In this paper, I assume that a large shock has caused a permanent structural change in the trend of Japan's growth around 1970, and then test whether the rest of the economic shocks are permanent ones which generate a stochastic trend or temporary shocks which generate a stationary stochastic process around the trend.

Another important question is whether a unit root process should be assumed as a null hypothesis or as an alternative hypothesis. This cannot be decided *a priori* since economic theory provides little guidance on this question. However, in earlier tests, a time series was routinely judged as non-stationary if it failed to reject the null hypothesis of a unit root process. More recently, to resolve the problem of asymmetry in the testing, several tests have been developed which assume stationarity as a null hypothesis. In this paper, I examine whether these two testing methods will lead to a consistent result.

II. Problems with Unit Root Tests

A. The Problem of Structural Changes

Nelson and Plosser (1982) suggested the presence of a unit root in 14 U.S. macroeconomic time series (such as real GNP) by showing that the unit root hypothesis could not be rejected by these data. Their conclusion was based on long-term annual data including those before the war. The subsequent studies based on the post-war quarterly data also generally supported their conclusion.

However, Perron (1989) criticized those studies arguing that they only considered the case of a linear time trend. The long- term time series data cover the periods of major events such as the Great Depression and post-war oil crises, which might have affected the growth trend. Therefore, Perron (1989) introduced a structural change in the time trend, and performed a unit root test for the same 14 time series as those of Nelson and Plosser (1982). He rejected the unit root hypothesis for 11 of the time series. Later simulations have shown that there will be a bias in favor of a unit root if tests which do not consider structural changes are performed on data generated by a model with structural changes. These suggest that the results of the aforementioned studies depend critically on the implicit assumptions made about the time trend. These findings support the argument that permanent shocks are limited to a few rare incidents like the Great Depression and the oil crises, and weaken the real business cycle theory where permanent productivity shocks are the main source of business cycles.

While Perron (1989) has determined the timing of a structural change exogenously, Zivot and Andrews (1992) have devised a testing method that can determine the timing endogenously. Using their method, they have found that a unit root exists for 5 out of the 11 time series for which Perron earlier rejected the presence of a unit root.

B. Choice of a Deterministic Trend

When a time series model is considered for a unit root test, there are many possible forms that the time trend can take, which seem to be an important cause of the different results in unit root tests. The possible forms of the time trend may be summarized as follows: when a time trend is to be selected, one has to decide (1) whether it is going to be a linear or a higher polynomial trend and, (2) whether it is a trend with or without structural changes. Furthermore, if the trend goes through a structural change, then one has to decide next (2)-a whether it is a change in the slope of the trend or a shift in the trend with the same slope or a combination of the two; (2)-b whether the timing of the structural change is exogenously determined or endogenously determined by some method; and (2)-c whether it is a single structural change or multiple structural changes. These choices are shown in the following chart:

- (1) Choice of a trend
  - a linear trend
  - a higher polynomial trend
- (2) Change in trend
  - no
  - yes
    - a. type
      - slope
      - jump
      - both
    - b. time of structural change
      - exogenous
      - endogenous
    - c. number of changes
      - once
      - multiple

As shown above, there are many choices to be made with respect to a time trend. However, it is possible, to some extent, to determine appropriate choices by looking carefully at the plots of time series data, comparing the averages and variances of sub-period samples. It is important, therefore, to carefully analyze data and set up appropriate maintained hypothesis before conducting a unit root test.

C. Choice of Null Hypothesis

When economic theory suggests a hypothesis to be tested, the common practice is to set it up as an alternative hypothesis and then to accept it if the null hypothesis is rejected. In the case of a unit root test, however, economic theory does not normally provide justification for setting up either the existence or non-existence of a unit root as the null hypothesis. In fact, it is often the case that the validity of a theoretical proposition in question itself depends on whether a unit root exists or not. A test for the null hypothesis that assumes a unit root, such as the ADF test used by Nelson and Plosser (1982), has no choice but to accept the unit root hypothesis when the null hypothesis is not rejected. It lacks symmetry in the treatment of the stationarity and non-stationarity (unit root) hypotheses. Furthermore the power of the ADF test is known to decline when the true root of the characteristic equation is close to 1, which tends to create a bias in favor of the unit root hypothesis.

Kwiatkoswki, Phillips, Schmidt, and Shin (1992) have proposed a test that sets up the stationarity hypothesis as the null hypothesis. Using the same time series data as those used by Nelson and Plosser, they have found that the stationarity hypothesis cannot be rejected for most of the series. Their test, however, has the same weakness as the ADF test in that the power declines when the root is close to 1. Therefore, when an estimated root is a near unit root, it is desirable to judge on the basis of both tests. The problem arises of course when two test results turn out to be contradictory. To overcome this problem, Hatanaka and Koto (1993) have proposed a method that compares the *P*-values of the two tests.

D. Previous Studies in Japan

Iwamoto and Kobayashi (1992) tested for a unit root in real GNP and real GNP per capita, using quarterly data. They divided the sample period between a high growth period and a low growth period. And the break point between the two periods was set at 1971 : 2Q, which was determined by minimizing the standard errors of the 6 year rolling periods. They found that the unit root hypothesis could not be rejected for either period. Takeuchi (1991) used a Stepwise Chow test to narrow the range of the possible break points in real GNP series to the period between 1970 and 1973. He tested the unit root hypothesis for each break point in 1970-73, and found that the result depended on the choice of the break point. Ohara (1994) extended the method of Zivot and Andrews (1992) to include the case of 2 break points in the sample period and tested the unit root hypothesis for the full sample period. He found that the unit root hypothesis could be rejected for Japanese real GNP series for 1 and 2 break points. At the same time, he found the completely opposite results for U.S. real GNP series: the unit root hypothesis couldn't be rejected for 1 nor 2 break points.

III. Empirical Analysis

Two tests are attempted as follows. The first test adopts the model of Perron (1989) and examines the unit root hypothesis for Japanese macroeconomic variables. In the test, I make several reasonable assumptions. First, I assume the time trend to be linear and exclude those time series that apparently have a nonlinear time trend. Second, I use the combination of two kinds of trend changes, a change in the slope of the trend and a shift in the trend with the same slope, and determine the break point exogenously. Zivot and Andrews (1992), who determine the break point endogenously, test the alternative hypothesis of "a stationary process around the trend with a break point" against the null hypothesis of "a unit root process with no change in the drift term." Their test will be useful if there is a break point in a series that does not exhibit a visible change in the trend, as is the case with the post-war U.S. real GNP series. But, in the case of Japanese macroeconomic data, a break in the trend is clearly visible around the time of the so-called Nixon Shock and the first oil crisis. Therefore, for Japanese data, it is appropriate to use the method of Perron (1989) and test the null hypothesis of a non-stationary process with a structural change. However, since the choice of a break point may affect the test result, I perform a test for each of 32 possible break points in an 8 year span. I also assume a single break point because a permanent shock to the Japanese economy seems to have occurred only once (around 1970) in the post-war period.

The second test adopts the method of Kwiatkowski, Phillips, Schmidt, and Shin (1992), which, unlike the first test, sets the stationarity hypothesis as the null hypothesis. The sample period for this test is chosen to be 1975-93, during which it can be safely assumed that no break in the trend occurred.

A. A Time-Series Model with a Structural Change

(1) Test Hypothesis

The first test is the one that Perron (1989) has proposed as an application of the ADF test. In the test, an alternative hypothesis is set up such that the time series follow a stationary process around a trend, which experiences a structural change characterized by both a change in the slope and a shift in the trend at time T_B. It can be written as follows:

$$y_t = \alpha_1 + (\alpha_2 - \alpha_1) DU_t + b_1 t + (b_2 - b_1) DT_t + \varepsilon_t , \tag{1}$$

where

$$DU_t = 1 \ (t > T_B), \quad 0 \ (t \leqq T_B) ,$$

and

$$DT_t = t \ (t > T_B), \quad 0 \ (t \leqq T_B) .$$

Here DU_t and DT_t are dummy variables for a change in the data generating process. The equation may be rewritten for sub-periods before and after the structural change:

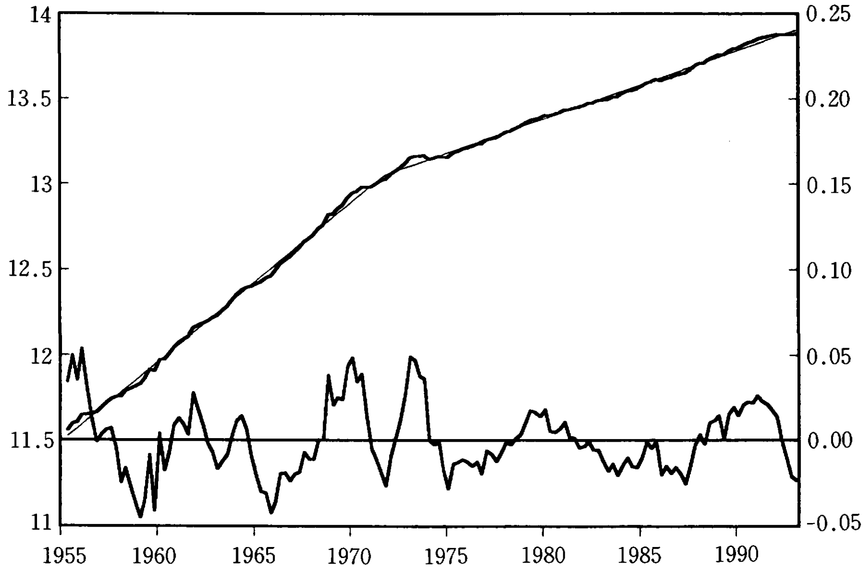
$$y_t = a_1 + b_1 t + \varepsilon_t \qquad (t \leqq T_B) ,$$

and

$$y_t = a_2 + b_2 t + \varepsilon_t \qquad (t > T_B) .$$

Figure 1 depicts real GNP and estimated equation (1) with the time trend change occurring at T_B = 1971 : 3Q. It also shows the residuals ε_t around the trend.[1] If instead a single regression were estimated without assuming any trend change, then the residuals

Figure 1
Real GNP and Residuals around the Time Trend
(Logarithm of seasonally - adjusted quarterly data, ⟨left scale⟩; residuals ⟨right scale⟩; 1971 : 3Q = a break point)



around the trend would have taken the form of a hill. It would be clearly inappropriate to discuss whether such residuals are stationary or non-stationary.

On the other hand, the null hypothesis is set up such that the series follow a unit root process as follows:

$$y_t = b_1 + (b_2 - b_1)\,DU_t + b_3 D\,(TB)_t + y_{t-1} + \varepsilon_t\,, \tag{2}$$

where

$$DU_t = 1\;(t > T_B),\quad 0\;(t \leqq T_B)\,,$$

and

$$D\,(TB)_t = 1\;(t = T_B + 1),\quad 0\;(t \neq T_B + 1).$$

[1]It may be more realistic to assume that a change in the trend occurs gradually over time. This can be done using the following trend change model:

$$a_1 + (a_2 - a_1)\,\Phi(L)\,DU_t + b_1 t + (b_2 - b_1)\,\Phi(L)\,DT_t\,,$$

where $\Phi(L)$ is a lag polynomial for a stationary process with the coefficients summing up to 1. Regression (1) can still be used with this trend model if ε_t is assumed to follow a stationary process defined by $\Phi(L)^{-1}$.

This can be rewritten for sub-periods as follows:

$$y_t = b_1 \qquad + y_{t-1} + \varepsilon_t \qquad (t < T_B)\ ,$$

and

$$y_t = b_1 + b_3 + y_{t-1} + \varepsilon_t \qquad (t = T_B)\ ,$$

and

$$y_t = b_2 \qquad + y_{t-1} + \varepsilon_t \qquad (t > T_B)\ ,$$

where the drift term differs according to the periods before, at, and after time T_B. The change in the growth rate from the high growth period to the low growth period is represented by the change in the drift term from b_1 to b_2, while the jump in the trend at the break point is represented by b_3.

As shown in Figure 1, residuals ε_t have a positive serial correlation. Therefore, in the actual test, I use the ADF test that assumes a high order AR process. The alternative hypothesis (1) and the null hypothesis (2) can be nested as in the following regression equation:

$$y_t = \alpha_1 + \alpha_2 DU_t + \alpha_3 D(TB)_t + \beta_1 t + \beta_2 DT_t + \theta y_{t-1} + \sum_{i=1}^{k} \theta_i \Delta y_{t-i} + \varepsilon_t\ . \tag{3}$$

In this equation, if a unit root exits, θ=1 and $\beta_1=\beta_2=0$ should hold. And if the drift term experiences a change, $\alpha_2 \neq 0$ and $\alpha_3 \neq 0$ should hold. If the process is stationary around the trend, θ<1, $\beta_1 \neq 0$ and $\beta_2 \neq 0$ should hold. If there is a change in the slope of the trend, $\alpha_2 \neq 0$ should hold; and if there is a jump in the trend, $\alpha_3 \neq 0$ should hold.

Just like the ADF test, the present test statistics are obtained from the results of Monte Carlo simulations. The distribution of the test statistic is characterized by a larger critical value in absolute terms than that of the Dickey and Fuller test on the left hand tail. It also depends on the position of the break point in the sample period. The closer the break point to the center of the sample period, the greater the value of the statistic. The test statistic becomes very close to the critical value of the Dickey and Fuller test as the break point moves to either end of the sample period.

(2) Test Results

I tested the unit root hypothesis for real GNP, real private consumption expenditures, real domestic gross capital formation, industrial production, M1, and M2+CDs, using quarterly data and the model specified by equation (3). All data are seasonally adjusted; the money supply figures are stock outstanding at the end of each quarter; and the industrial production figure is a period average.[2] The sample period starts with either

[2]The smoothing of the data by seasonal adjustment may affect the test result. Nevertheless, it is meaningful to test seasonally adjusted data as a pre-test because seasonally-adjusted quarterly data are often used in econometric analysis. Although this problem can be avoided by using annual data, I have not done so because the power and acuracy of the test fall with only 40 years of samples. For seasonal data, Hyllenberg, Engel, Granger and Yoo (1990) have proposed a seasonal unit root test.

1954 or 1955: and ends with 1993 or 1994 (see Table 1). As the nominal interest rate shows a steady downward trend without an apparent break, I do not assume a break point for the interest rate series. The considered break points range from 1969 : 1Q to 1976 : 4Q (32 break points in a span of 8 years), and I estimate the time trend for each. The number of lags is determined by the condition for mutually independent distribution of residuals: specifically, the number of lags is reduced one by one from the maximum of 12 periods if the autocorrelation and partial autocorrelation of residuals for 20 periods are all 0.[3]

Figure 2 shows the estimated θ and its t-value for each variable and for each break point. The line graphs represent the t-values, and the horizontal lines represent the associated significance levels calculated by Perron (1989). Table 1 shows the estimation results when it maximizes the t-value of each coefficient. According to these results, when the break point is set after 1974, real GNP has a low t-value and the estimated θ is close to 1; therefore, the unit root hypothesis cannot be rejected. But, when the break point is set between 1970 : 4Q and 1973 : 4Q, the unit root hypothesis is rejected at the 5% level of

Table 1
The Unit Root with Structual Change

Variable	Period	T_B	k	α_1	α_2	α_3	β_1	β_2	θ	$s.e$
Real GNP	55:2-93:1	71:3	10	4.824 (5.83)	0.392 (5.63)	-0.017 (-1.54)	0.0099 (5.79)	-0.0057 (-5.79)	0.578 (-5.80)***	0.0097
Real Private Consumption	55:2-93:1	71:3	3	3.691 (5.85)	0.288 (5.75)	-0.023 (-2.00)	0.0071 (5.82)	-0.0042 (-5.91)	0.669 (-5.82)***	0.0106
Real Gross Capital Formation	55:2-93:1	69:1	10	1.239 (3.80)	0.200 (3.19)	-0.003 (-0.09)	0.0045 (3.43)	-0.0033 (-3.29)	0.872 (-3.71)	0.0287
Industrial Production	54:2-94:1	73:1	4	0.215 (4.37)	0.184 (3.58)	-0.010 (-0.60)	0.0038 (3.86)	-0.0028 (-3.82)	0.878 (-3.95)*	0.0163
M1	55:1-93:4	70:4	12	0.793 (2.39)	0.135 (3.45)	-0.010 (-0.47)	0.0034 (2.52)	-0.0018 (-3.31)	0.919 (-2.78)	0.0180
M2 + CDs	55:1-93:4	70:3	12	0.273 (3.21)	0.042 (3.22)	-0.010 (-1.23)	0.0009 (2.47)	-0.0005 (-2.87)	0.976 (-2.92)	0.0075
Call Rate	61:1-94:1		12	2.214 (3.77)			-0.0072 (-2.869)		0.0750 (-3.96)*	0.6570
Interest-bearing Corporate Bonds	56:2-92:4		12	2.485 (3.79)			-0.0077 (-3.483)		0.772 (-3.81)	0.5297

Notes: T_B is a break point in the time trend; k is the order of lags in the estimated equation; t-values are in parentheses.
The critical values of $\theta = 1$ under the condition of $T_B/T = 0.4$ are -3.954, -4.22, and -4.81 for the 10, 5, and 1% level of significance, respectively. Figures with *, **, and *** represent those rejected at the 10, 5, and 1% level of significance, respectively.

[3]For the variables tested by this method, the number of lags determined turns out to be larger than that determined by the statistical significance of t-values.

Figure 2
Logarithm Line Graphs, t-Values, and Estimated Roots
(Shaded areas represent ranges of possible break points)

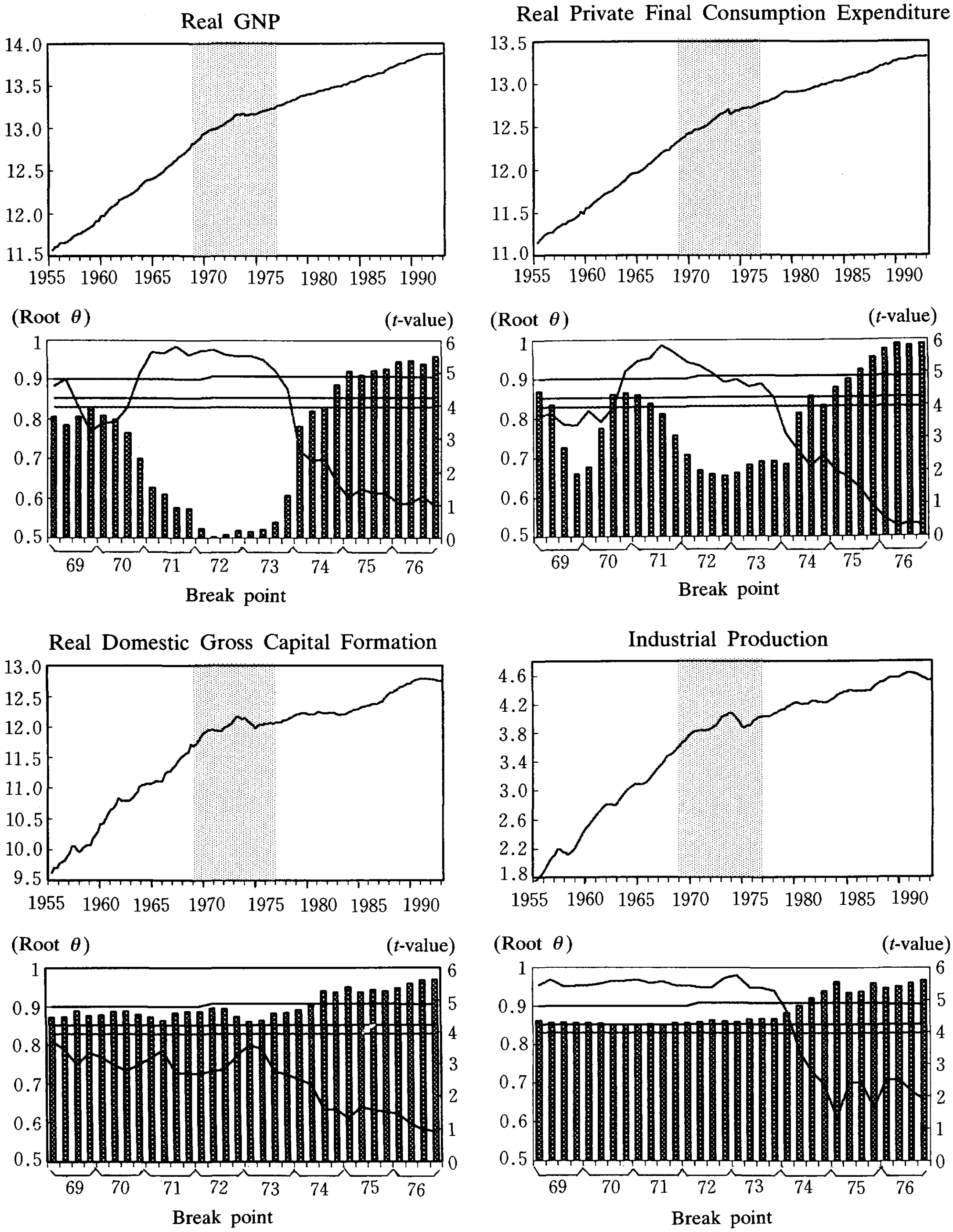
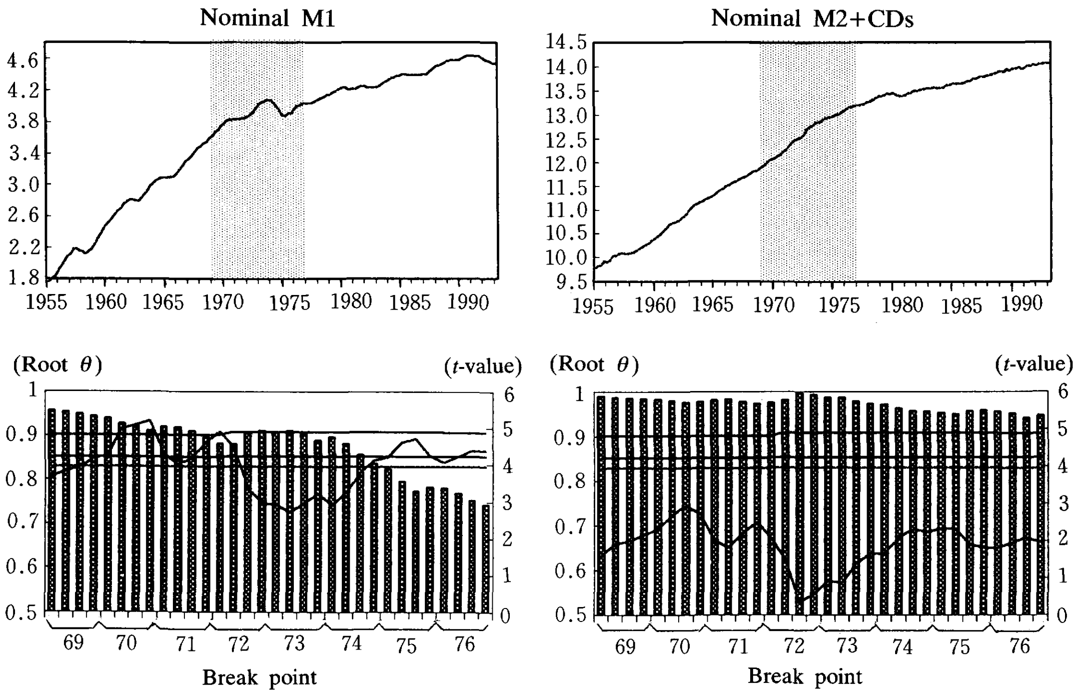


Figure 2 – continued



Note: Line graphs represent t-values in absolute terms (right scale); bar graphs represent estimated roots (left scale); horizontal lines represent the 1, 5, and 10% levels of significance according to Perron (1989).

significance with the estimated θ down to 0.5. It is important to recognize that if the break point is selected arbitrarily, a bias tends to appear in favor of accepting the unit root hypothesis for the residuals. This suggests a rule to test the unit root hypothesis by selecting the break point that provides the most favorable result for the stationarity hypothesis. Accordingly, real GNP appears to be a stationary process around the trend with a break point rather than a unit root process.

Similar results hold for real private final consumption expenditure and industrial production. Although industrial production has an estimated θ close to 1, the unit root hypothesis is rejected at the 1% level of significance. The associated t-value is stable for the break points between 1969 : 1Q and 1973 : 4Q. Real private final consumption expenditure exhibits a similar movement to that of real GNP with a slight lag. And the same rule concerning the selection of the break point gives rise to the same result as that of real GNP. In contrast, the unit root hypothesis cannot be rejected at the 10% level of significance for domestic gross capital formation regardless of the selection of the break point. This may be due to the fact that a single linear trend for the low growth period fails to capture its sharp increase after 1987, and that its movement is more volatile in the high growth period than the low growth period, which makes the standard errors of the residuals different between the two periods. They suggest the need for an improvement in the model and the estimation method. Therefore the above result should be treated with some caution. As for M1, the unit root hypothesis with a break point between 1970 : 2Q and 1972 : 2Q is rejected at the 10% level of significance, but it cannot be rejected if the break point is set outside of that period. Moreover, the estimated θ for M1 tends to be greater than 0.9, making it difficult to judge whether it is significantly different from 1 or not.

Figure 3 plots actual M1 and its trend as well as its residuals. It suggests that a trend change has occurred gradually over several years, rather than at a particular point in time. Therefore it is likely that the model is misspecified. The GNP deflator also exhibits a similar movement to that of M1. Figure 4 plots the movement of the logarithm of the GNP deflator, which traced a linear trend until the beginning of the 1970s but rose above this from 1973 through the rest of the decade, and then gradually returned to a new trend with a lower slope in the 1980s. Thus, an appropriate way to deal with this case is to set up an alternative hypothesis that a deterministic trend has a curvature against the null hypothesis that a stochastic trend exists. Similarly, M2+CDs has a very low t-value, which suggests a misspecification of the model just like the case of M1. A similar problem seems to appear for many nominal macroeconomic variables, presumably because they all reflect the characteristic movement of the GNP deflator.

Figure 3
M1 and Residuals around the Time Trend
(Logarithm of seasonally - adjusted quarterly data, ⟨left scale⟩; residuals ⟨right scale⟩; 1972 : 4Q = a break point)

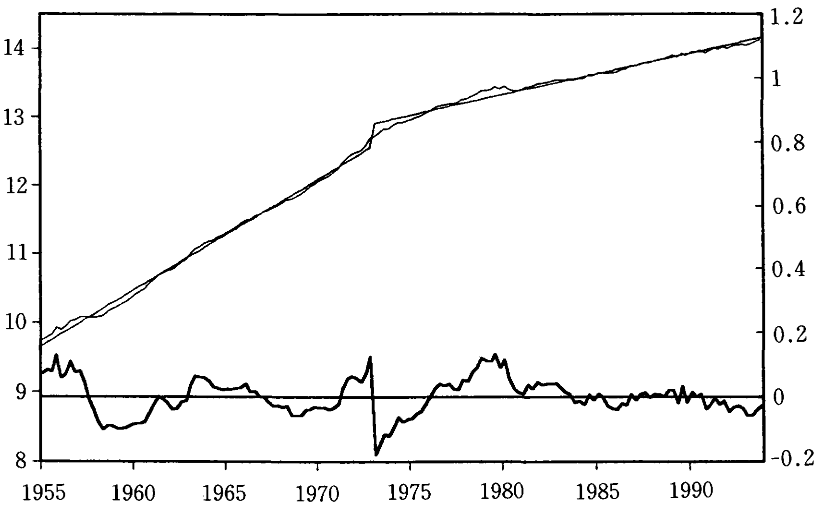
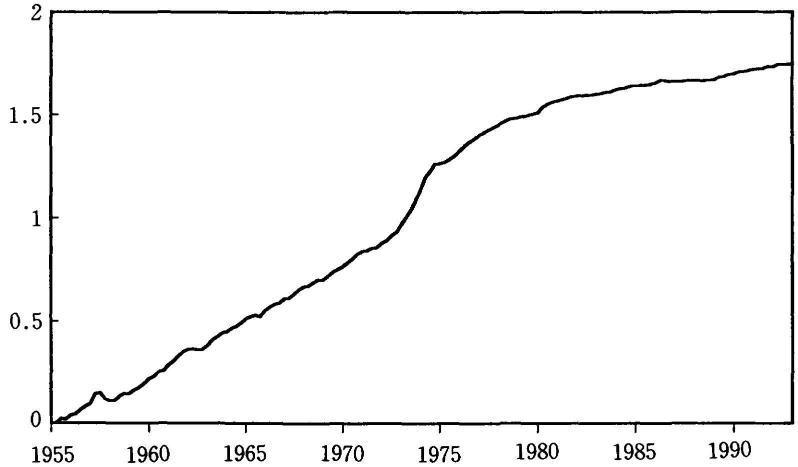


Figure 4
GNP Deflator
(Initial logarithm normalized at 0)



Finally, Table 1 shows that the unit root hypothesis for the call rate is rejected at the 10% level of significance. Although the hypothesis was not rejected at the same level for the yield of interest-bearing Telegraph and Telephone bonds, no definite conclusion can be drawn because the lag structure turns out to be very unstable.

B. A Test with the Stationarity Null Hypothesis

(1) Test Hypothesis

Next, I apply the ADF test and the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test to a sample period that appears to have no break point. Note that the KPSS test (just like the PP test) is based on an asymptotic theory which assumes a large sample size. To check the presence of a potential small sample bias, I also conduct the PP test which I can then compare with the ADF test.

The KPSS test assumes a data generating process which consists of a trend, random walk, and stationary error terms. Then, for the variation around the trend caused by the random walk and error terms, it checks a stationarity condition that the variance takes a finite value that is independent of time. If the variance is constant, then the random walk term should not exist. And, if it is not constant, then the stationarity hypothesis should be rejected.[4]

[4]When stationarity is to be rejected as a null hypothesis, it is sufficient to reject the necessary condition. However, when the necessary condition is not rejected, one cannot conclude from this result that the process is stationary.

and
$$\begin{aligned} y_t &= \xi t + r_t + \varepsilon_t & \varepsilon_t &\sim i.i.d\,(0, \sigma^2{}_{\varepsilon}) , \\ r_t &= r_{t-1} + \mu_t & \mu_t &\sim i.i.d\,(0, \sigma^2{}_{\mu}) , \end{aligned} \tag{4}$$

where ξt represents a trend term and r_t a random walk term. To obtain the variance of the residuals around the trend, this equation can be transformed as follows:

$$y_t - \xi t = r_0 + \sum_{i=1}^{t-1} \mu_{t-i} + \varepsilon_t ,$$

$$\text{var}\,(y_t - \xi t - r_0) = \sigma^2{}_{\mu} t + \sigma^2{}_{\varepsilon} .$$

As mentioned above, one of the necessary conditions for stationarity is that the variance of y_t is independent of time and is finite. Therefore, if the residual around the trend, $(y_t - \xi t - r_0)$, satisfies the stationarity condition, its variance $(\sigma_{\mu}^2 t + \sigma_{\varepsilon}^2)$ must be independent of t and take a constant value. Thus $\sigma_{\mu}^2 = 0$ becomes a necessary condition for the stationarity of residuals. The statistic LM that tests this condition is given as follows:[5]

$$LM = \sum_{t=1}^{T} S_t^2 / \hat{\sigma}^2{}_{\varepsilon}$$

where

$$S_t = \sum_{i=1}^{t} e_i \, (t = 1, 2, \ldots T) .$$

Here e_i is the residual of the regression of y_i on a trend and a constant. The variance of e_t is used as the estimate of σ_{ε}^2 because $\hat{\sigma}_{\varepsilon}^2 = \text{var}(e_t)$ holds under the null hypothesis. However, e_t does not satisfy the condition of ε_t that is assumed under equation (4) because it exhibits a strong positive autocorrelation. Therefore I use the PP test, which does not assume the strong *i.i.d.* condition for the distribution of error terms.

(2) Test Results

The data used for the test include quarterly data for real GNP, real private final consumption expenditure, real domestic gross capital formation, and monthly data for industrial production index, M1, M2+CDs, M3+CDs, the call rate, and the yield of 5-year interest-bearing bank debentures. All of them except interest rates are seasonally adjusted, and money stocks are end-of-period figures. The sample period for both quarterly and monthly data is chosen to be 1975-93 because the model can be assumed without break in the trend. The maximum number of lags used in the ADF test is 36 for monthly data and 12 for quarterly data. And the appropriate number of lags is determined by the same test on residuals as used in the previous section.

As shown in Table 2, the PP test and the ADF test reject the unit root hypothesis for M1 and the call rate, respectively. As the rest of the variables presumably have a unit root, the stationarity hypothesis for other variables should be rejected. In fact, the stationarity hypothesis for those variables, except real GNP and real private final

[5]The distribution of the statistic is obtained from Kwiatkowski, Phillips, Schmidt, and Shin (1992).

Table 2
The Unit Root Test: Sub Sample

Variable	ADF test	(lag)	PP test l = 4	l = 8	l = 12	KPSS test l = 4	l = 8	l = 12
(Quarterly Data)								
Real GNP	-1.85	(0)	-2.17	-2.29	-2.23	0.116	0.083	0.076
Real Private Consumption	-2.96	(3)	-2.18	-2.21	-1.97	0.085	0.065	0.066
Real Gross Capital Formation	-1.86	(2)	-1.42	-1.59	-1.66	0.288**	0.176*	0.139*
(Monthly data)								
Industrial Production	-2.13	(22)	-0.93	-1.24	-1.50	0.264**	0.161*	0.125*
M1	-2.35	(25)	-4.29**	-4.58**	-4.87**	0.310**	0.187*	0.141*
M2 + CDs	-2.18	(36)	-0.26	-0.53	-0.77	0.478**	0.287**	0.216**
M3 + CDs	-1.47	(36)	-2.25	-2.14	-2.06	0.832**	0.489**	0.358**
Call Rate	-3.54*	(36)	-2.82	-3.02	-3.11	0.149*	0.091	0.071
Interest-bearing Corporate Bonds	-2.82	(30)	-2.66	-2.54	-2.72	0.194*	0.117	0.087

Notes: The sample period is 1975 : 1Q – 1993 : 1Q for quarterly data and 1975 : 1M – 1993 : 12M for monthly data.
Figures with * and ** represent those rejected at the 10 and 5% level of significance, respectively.

consumption expenditure, is rejected. For these two variables, either the KPSS test or the ADF test fails to reject despite null hypothesis is wrong. This is probably because the power of the tests declines as the root is very close to 1. As this case shows, the problem of indetermination arises for those tests that accept or reject according to a given significance level if the two tests happen to produce conflicting results.[6]

IV. Concluding Remarks

I have reviewed the problem of a time trend in the unit root test, and experimented with an estimation method which specifies the null and alternative hypotheses that appropriately reflect actual time series movements. Many Japanese post-war macroeconomic variables exhibit visible changes in the data generating process around 1970. This suggests that the unit root hypothesis for those variables must assume that they follow two non-stationary processes with different drift terms. Therefore I have conducted a unit root test against an alternative hypothesis that macroeconomic variables follow a

[6]Hatanaka and Koto (1993) have used a modified method of Saikkonen and Luukkonen (1993) as a test with a stationarity null hypothesis. Saikkonen and Luukkonen estimate an ARMA model with an addition of MA process for ε_t while the KPSS test is a nonparametric test which applies the PP test method to the estimation of error terms.

stationary process around the trend that undergoes a structural change. Within the framework of the model and the sample data presented in this paper, the test has rejected the unit root hypothesis for the Japan's real GNP series.

Some caution may be in order, however. There are many *a priori* assumptions that have been made in the modeling of the time trend. For example, I have considered only a linear trend, which may have failed to adequately capture the gradual changes in the trend observed for monetary aggregates and which therefore has produced abnormal movement around the trend. Therefore, the estimation results may need to be interpreted with some caution. One way to address this problem would be to build a null hypothesis which assumes a linear function as the drift term during some period, in the stochastic drift term over time, together with an alternative hypothesis which assumes a quadratic function as the time trend.

Time series models used in cointegration tests do not take into consideration a possible change in the structure of a time trend. Regarding application of the cointegration method, many empirical studies have been undertaken with respect to the estimation of a money demand function. However, few studies have dealt with the possibility that a drift term in the stochastic trend may change during the sample period, and that the trend may involve a curvature during certain period as is the case for the monetary aggregates and the GNP deflator in the 70's. Therefore, the treatment of a time trend in monetary aggregates should be an important topic on the research agenda because it is essential to the estimation of a reliable money demand function.

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