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The Real Value of Postal Savings Certificates

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In Japan, there has been growing debate about the competitiveness between "postal savings certificates" (PSCs) offered by the Ministry of Posts and Communications and time deposits offered by private banks. Since PSCs have an "American put-option" characteristic, the "option premium" implied in PSCs should be taken into account explicitly. Since the interest rates have been regulated, we must devise a new option pricing theory based on imperfect arbitrage under such assumptions as depositor's risk neutrality and Markov property in the interest rates movements. Empirical analyses show that the advantage of PSCs over time deposits offered by private banks increases as depositor's holding period becomes longer and the current interest rates become higher.

I. Introduction

In recent years, there has been growing debate about the postal savings system in Japan, in particular "postal savings certificates" (hereafter "PSCs"). The main issues have been the economic rationale (profitability) of PSCs, their superiority over deposits provided by private financial institutions, and the macroeconomic effects of a shift of deposits to the postal savings system on money supply and real economic activity.

On the question of economic rationale and competitiveness of PSCs, the Ministry of Posts and Communications and private financial institutions are sharply opposed. The Ministry has argued that PSCs are a profitable financial instrument from a long-term perspective. On the other hand, private financial institutions have argued that if potential risks incurred by PSCs are taken into account, they are such a costly means of raising funds that they cannot be offered on a commercial basis. Despite growing interest in the debate, however, most arguments have tended to be abstract, and few quantitative cost-and-benefit analyses have been conducted, for example, to evaluate the profitability of PSCs or the special postal savings account of the government as a whole.

One reason for this lack of quantitative analysis is that PSCs have an "option" characteristic, which renders it difficult to evaluate their real value. PSCs yield a predetermined interest which depends on the period held after the minimum initial six months

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of commitment, and may be canceled (redeemed) any time after six months. This characteristic is the same as that of an "American put-option," which is the right to exercise an option to sell an equity or a bond any time at a predetermined price. In this respect, PSCs are equivalent to bonds with an American put-option. Therefore, in comparing the real value of PSCs with that of other financial merchandise, we must take into account the "option premium" implied in PSCs.

Private financial institutions are also providing deposits with an option characteristic, such as maturity-designated time deposits (hereafter "MTDs"). However, their maximum maturity period is limited to three years while PSCs have a maximum maturity period of ten years. Therefore, the option premium of PSCs that depositors can implicitly enjoy would be greater than that of MTDs.[1,2] It becomes important, therefore, to estimate explicitly the option premium of PSCs in order to evaluate their real value. Thus, the purpose of this paper is firstly to formulate a theoretical framework by which we can estimate the real value of PSCs, using some relevant results from option theory, and secondly to calculate their option premium using actual data.[3]

However, we must be careful when we apply the standard option theory to the evaluation of PSCs. The standard theory is based on the assumption that the market is frictionless and that therefore perfect arbitrage is guaranteed, which is not necessarily true in the case of PSCs, the interest rates of which are regulated.[4] To accommodate this difference, we will devise a new scheme to calculate the option premium of PSCs under the assumption that depositors are risk neutral.[5]

This paper is structured as follows: Section II discusses the option characteristic of PSCs. Section III sets up a three-period model and develops a scheme to calculate their real value and explains the assumptions we make. Section IV sets up a similar model of MTDs, which are the deposits with longest maturity period that commercial banks can offer. Section V calculates the option premium of PSCs with actual data, and evaluates the advantage of PSCs over MTDs. Appendixes present a general multi-period model for calculating the real values of PSCs and MTDs.

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[1]Logically speaking, so long as depositors purchase PSCs — bonds with an American put-option — from the special postal savings account of the government, it is the depositors that should pay for the "option premium" instead of taxpayers.

[2]According to the standard option pricing theory (*e.g.*, Cox and Rubinstein (1985)), an option premium increases with the length of the option period.

[3]The problem of PSCs should be analyzed from much broader perspectives, such as how their interest rates are determined and how they are taxed, in addition to the issues of public competition and profitability. Moreover, a broader question of the postal savings system should be addressed including the role of public financial institutions and the Fiscal Investment and Loan Program, and their relationships with private financial institutions. However, these questions are beyond the scope of this paper. Instead, this paper focuses on the problem of how to estimate the real value of PSCs.

[4]The interest rates of PSCs and commercial bank time deposits, including MTDs, were deregulated in June 1993.

[5]This assumption of risk neutrality will be discussed in detail in footnote 10 below.

The main conclusions of this paper may be summarized as follows:

(1) PSCs are equivalent to financial instruments with an American put-option that uses certificates of deposit as the underlying asset and may be canceled (redeemed) with a predetermined interest rate at any time before maturity.

(2) Although both publicly offered PSCs and privately offered MTDs have an option characteristic, one major difference is that the former have a maximum maturity period of ten years while the latter have a maximum maturity period of only three years.

(3) Therefore, in evaluating the real value of these deposits with an option characteristic, one must explicitly take into account the option premium implied in such deposits. This becomes particularly important for PSCs because they have a very long maturity period. For example, an estimated maximum option premium for PSCs is 0.69% if they are held for four years but 1.45% if held for ten years.

(4) In terms of real value, the advantage of PSCs over MTDs tends to increase as the current interest rates increase and as the depositor's holding period lengthens: the extra value could be as large as 0.57% for a four-year holding period and 1.17% for a ten-year holding period. Nevertheless, if interest rates are low and the holding period is short, the value of MTDs could be greater than that of PSCs: this would be the case if the holding period is less than two years.

(5) Finally, it is important to note that the special postal savings account of the government could make a substantial loss during a period of large interest rate fluctuations because there is no means of hedging the short position of a put-option with maximum maturity of ten years.[6]

II. The Option Characteristic of Postal Savings Certificates

A. Option Transactions and Characteristics of Postal Savings Certificates

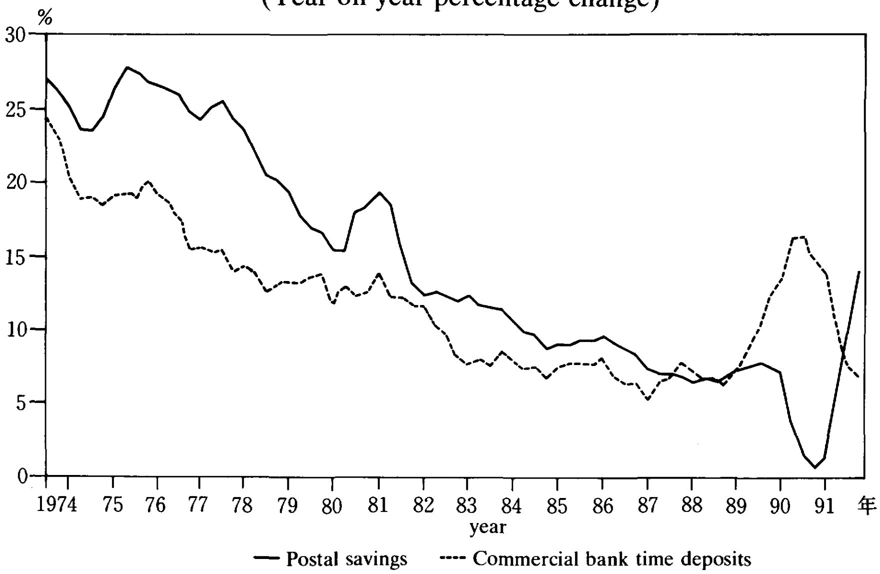
1. Recent developments in postal savings certificates

During the 1980s, PSCs grew consistently faster than private time deposits (Figure 1). From mid-1989 to 91, this trend was temporarily reversed because of the introduction of a new small savings instrument (Money Market Certificates, hereafter "MMC") by commercial banks. Since 1991, however, the growth rate of PSCs has recovered and again exceeded that of private time deposits.

Such faster growth of PSCs has occurred despite their lower interest rates (Figure 2).

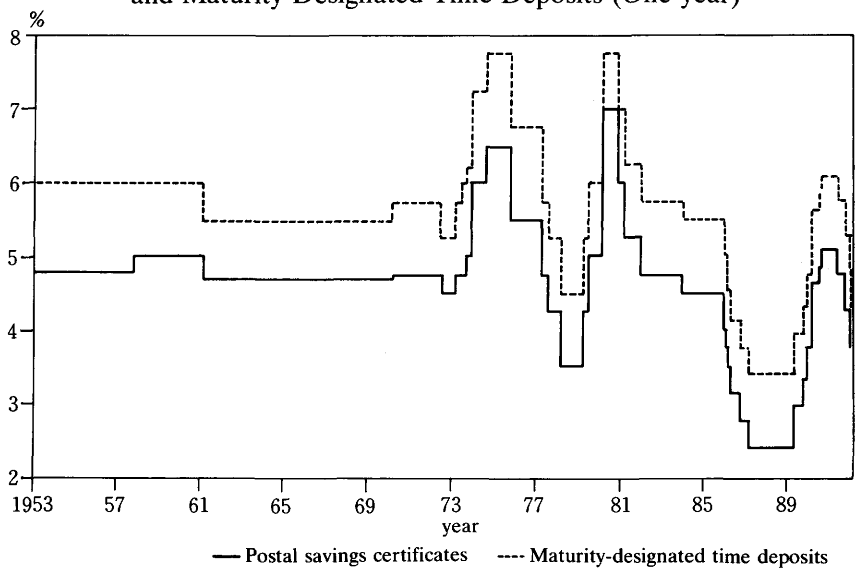
[6]Theoretically, an option seller can hedge against the risk of a potentially infinite loss, which depends on variations in the value of the underlying asset, by making offsetting transactions in the current market (*e.g.*, a delta hedge).

Figure 1
Balance of Personal Savings
(Year-on-year percentage change)



Note: Excludes corporate sector. Includes small MMCs and other deposits with deregulated interest rates in addition to regular time deposits with regulated interest rates (ceased from June 1992).

Figure 2
Interest Rates of Postal Savings Certificates
and Maturity-Designated Time Deposits (One year)



Although some advantages in calculating interest payments (compound interest rather than simple one) partially account for this phenomenon, the main reason must have been the option characteristic of PSCs. Thus, we first study the characteristics of PSCs as a financial instrument, and explain their option characteristic.

2. *A review of option transactions*

An option contract is a transaction of the right to buy (call-option) or sell (put-option) an equity or a bond (an underlying asset) at a predetermined date and price. Options may be classified according to type of right and exercise date of that right as follows:

[According to type of right]

A call option: the right to buy an underlying asset at a predetermined price from an option seller.

A put option: the right to sell an underlying asset at a predetermined price to an option buyer.

[According to exercise date]

An American option: an option that can be exercised at any time before maturity.

An European option: an option that can be exercised only at maturity.

Theoretically, an option premium is determined by the following five factors: (i) the price of an underlying asset; (ii) price volatility; (iii) the exercise price; (iv) return on riskless assets; and (v) the length of maturity. For example, a theoretical option premium will rise as volatility in the price of an underlying asset increases and as the length of maturity becomes longer.[7]

In comparison with futures and other related transactions called derivatives, a distinctive characteristic of an option is asymmetric risk sharing between buyer and seller. While the potential loss of an option buyer is limited, the potential loss of the seller could be infinite, depending on the variations in the price of the underlying asset.[8] It is important, therefore, to recognize that substantial risk is involved in offering a financial instrument with an option characteristic such as PSCs.

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[7]For determination of an option premium, *see* Cox and Rubinstein (1985).

[8]Suppose a call option has an exercise price of k yen and option premium of c yen. If the price of an underlying asset (x yen) is greater than the exercise price (*i.e.*, $x \geqq k$) at an exercise date, the option buyer will exercise the option and sell the underlying asset in the spot market and gain a cash flow of $x-k-ce^{rT}$ (where r is the interest rate to obtain the discounted present value of premium c). Even if $x < k$ at the exercise date, the buyer's loss will be limited to $-ce^{rT}$. In contrast, if the option is not exercised, the seller's cash outflow will be limited to ce^{rT}; but, if it is exercised, the seller's loss will be $k-x+ce^{rT}$, which could go to negative infinity depending on the value of x (Figure 3).

3. *Postal savings certificates and an option*

Now we examine the option characteristics of PSCs. First, we look at the basic characteristics of PSCs.

[PSCs]

Maximum maturity: ten years.

Minimum commitment: six months.

Interest rate: progressive interest rates on deposits with maturity of up to three years (with different rate applied every six months), and a constant interest rate on those with maturity of more than three years, both of which are predetermined when opening an account (Table 1).

Interest calculation method: semiannual compound interest.

Once a depositor receives a PSC, its predetermined yield curve (interest rate structure) is guaranteed as long as the certificate is held up to the maximum period of ten years. Thus, even if market interest rates decline in the future, the holder can still enjoy

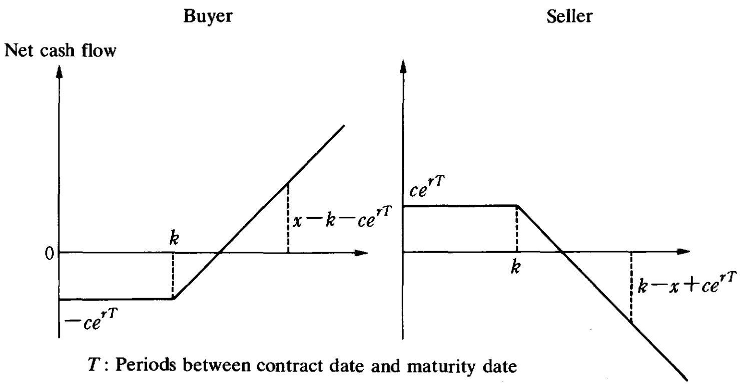
Table 1
Revisions in Interest Rate Structure
of Postal Savings Certificates

(%)

Revision date	Less than 1 year	1 year	1.5 years	2 years	2.5 years	3 years and over
7/29/91	4.25	4.75	5.5	5.85	5.9	6.0
11/25/91	3.75	4.25	5.0	5.35	5.4	5.5
1/20/92	3.25	3.75	4.5	4.85	4.9	5.0

Figure 3

Cash Flows of Call Options at Maturity



T: Periods between contract date and maturity date a higher interest rate. On the other hand, if interest rates go up and it turns out to be profitable to switch, the deposit can be canceled (redeemed) without penalty so long as it has been held for at least six months. Therefore, PSCs are nothing but a saving instrument which contains a ten-year American put-option with the savings certificate as an underlying asset and with a predetermined progressive yield curve.

B. Difference between Postal Savings Certificates and Maturity-Designated Time Deposits

PSCs are not the only savings instrument that have an option characteristic. For example, commercial banks offer maturity-designated time deposits (hereafter "MTDs"), which also have the same characteristic of being a saving instrument with an American put-option, except for a shorter maturity period. The basic characteristics of MTDs may be summarized as follows:

[MTDs]

Maximum maturity: three years.

Minimum commitment: one year.

Interest rate: the interest rate on one-year time deposits and two-year time deposits are applied respectively to deposits held for more than one year but less than two years and those held for more than two years. If a deposit is canceled before the minimum one year, the cancellation interest rate as that on an ordinary time deposit applies (Table 2).

Interest calculation method: annual compound interest.

The main difference between PSCs and MTDs is the length of maturity period: ten years for PSCs compared to three years for MTDs (Table 3). The difference plays an important role in calculating their real values because the longer maturity period of PSCs increases their option value.[9]

Table 2
Revisions in Interest Rate Structure
of Maturity-Designated Time Deposits

(%)

Revision date	Cancellation	1 year	2 years and over
7/29/91	4.25	5.75	6.0
11/25/91	3.75	5.25	5.5
1/20/92	3.25	4.75	5.0

[9]The option value depends not only on maturity length, but also the value of the underlying asset, the exercise price, and other factors. It is important to note, for example, that although PSCs have lower interest rates than MTDs, their exercise price is lower and thus their option value is greater.

Table 3
Option Characteristics of Postal Savings Certificates
and Maturity-Designated Time Deposits

	Postal savings certificates	Maturity-designated time deposits
Option type	American-put	American-put
Underlying asset	Certificate of savings	Certificate of deposit
Exercise price	Predetermined interest rate	Predetermined interest rate
Maximum maturity period	10 years	3 years

III. The Real Value of Postal Savings Certificates

A. Assumptions for Calculating Real Value

As mentioned earlier, we cannot directly apply the standard option theory, which assumes perfect interest rate arbitrage, to evaluate the real value of PSCs because their interest rates are regulated. Thus, we need to make some assumptions in order to evaluate real value. The main ones are:

(1) The same fixed structure of yield spreads is assumed for all presupposed interest rate structures in the simulations.
(2) The stochastic process of interest rate structures follows the first-order Markov process.
(3) Simulations are conducted by assuming different deposit holding periods.
(4) Depositors are risk neutral.[10]

Let us elaborate further on assumptions (1) – (3) that are essential for computing the real value of PSCs.

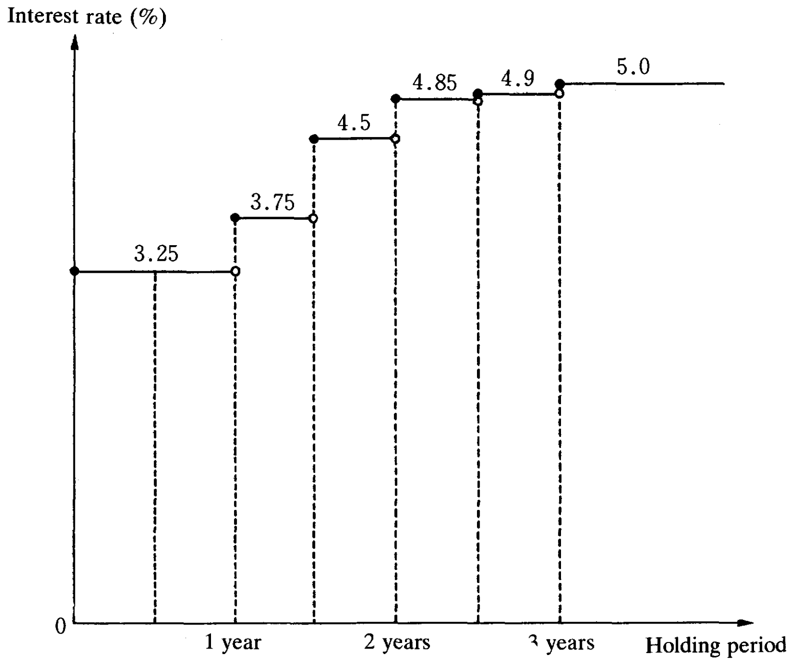
[10] An option is often used to hedge against the risk of high volatility in the price of its underlying asset. The risk premium of the option has the characteristic of being its insurance fee, and normally depends on the degree of price volatility and the investor's attitude toward risk. If an investor is risk averse, his expected return should be less than that of a risk neutral investor because in maximizing his utility he will consider not only the expected rate of return but also the risk (*i.e.*, the variance of returns). It is possible, in principle, to estimate the option premium for the case of a risk averse investor; but the calculation would become much more complicated than that presented below.

1. Interest rate structure assumption

As mentioned earlier, since PSCs are considered to be an underlying asset in the option theory, their interest rate structure corresponds to the exercise price of the option. Thus, the interest rate structure becomes an important factor in calculating the real value of PSCs. Figure 4 shows the interest rate structure of PSCs that was effective on January 20, 1992. According to this, the interest rate structure consists of six phases: progressive interest rates every six months up to three years, and a constant interest rate after three years.

We call an interest rate structure that holds at any moment of time "an interest rate state." In our analysis, we assume nine different "interest rate states" (interest rate state 1, interest rate state 2, ..., interest rate state 9) according to the nine levels of the interest rate for three-year and over PSCs, which take the following values: 4.0%, 4.5%, 5.0%, ..., 8.0%. The interest rate structure is assumed to be fixed so that interest rate differentials are the same between periods independent of the interest rate state. The structure of yield spreads assumed throughout the exercise is obtained by averaging historical data. Actual interest rate structures were in fact very rigid so that this assumption seems to be

Figure 4
Interest Rate Structure of Postal Savings Certificates
(January 20, 1992)



sufficiently realistic.[11] Table 4 shows the estimated interest rate structures for nine different interest rate states.

The interest rate structure of PSCs is predetermined at the point in time when the account was opened. Thus, if $r_{i,t}$ stands for the (annual) interest rate at time t for the savings certificate purchased at time 0 when the interest rate state was i (i=1, 2, ..., 9), then the return after time t will be given by $(1+r_{i,t}/2)^t$. For example, if the account was opened on January 20, 1992 and was held for two years and six months, the return would have been $(1+0.049/2)^5$ (Table 1).

2. *Assumption on the stochastic process of the interest rate state*

Another important factor necessary for calculating the real value of PSCs is the stochastic process of the interest rate state. Although, in principle, we should explicitly incorporate the mechanism of interest rate determination into the model, we will assume in this paper (for simplifying the computation) that the probability distribution of future interest rate states depends only on the current interest rate state. Such a stochastic process is called a first-order "Markov process."[12]

Table 4
Interest Rate States and Interest Rate Structure of Postal Savings Certificates

(%)

Interest rate state	Less than 1 year	1 year	1.5 years	2 years	2.5 years	3 years and over
1	2.35	2.75	3.50	3.82	3.90	4.00
2	2.85	3.25	4.00	4.32	4.40	4.50
3	3.35	3.75	4.50	4.82	4.90	5.00
4	3.85	4.25	5.00	5.32	5.40	5.50
5	4.35	4.75	5.50	5.82	5.90	6.00
6	4.85	5.25	6.00	6.32	6.40	6.50
7	5.35	5.75	6.50	6.82	6.90	7.00
8	5.85	6.25	7.00	7.32	7.40	7.50
9	6.35	6.75	7.50	7.82	7.90	8.00

[11]To be precise, there seems to be some tendency for interest rate differentials to shrink when interest rates are low. Our main conclusions, however, will not be affected even if we explicitly incorporate this tendency into our model.

[12]Consider a stochastic process $\{x_t\}$. In general, the probability distribution of x_{t+1} depends on "entire history of x_i" (x_i for all $i \leqq t$). In a special case when the probability distribution of x_{t+1} depends only on x_t, the stochastic process $\{x_t\}$ is called a first-order "Markov process."

We need to obtain the conditional probability distribution that an interest rate state j (j=1, ..., 9) will emerge in the next period when the current interest rate state is i (*i.e.*, $P(j \mid i)$). This conditional probability distribution is estimated for the case of a six-month period, using monthly data from July 1973 to January 1992 (Figure 5, Table 5).

Figure 5

Changes in the Three-Year Interest Rate of Postal Savings Certificates

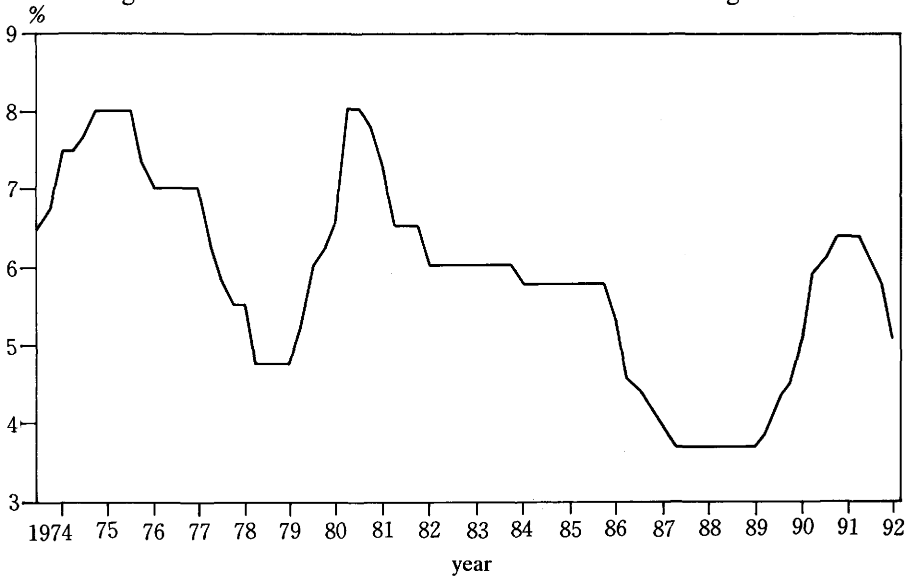


Table 5
Conditional Frequency Distributions
of Interest Rate States

State		Period $t+1$ 1	2	3	4	5	6	7	8	9
Period t	1	21	5	1	0	0	0	0	0	0
	2	6	4	4	0	1	0	0	0	0
	3	0	2	7	3	4	4	0	0	0
	4	0	1	6	1	0	3	0	0	0
	5	0	3	3	5	43	5	0	0	0
	6	0	0	0	2	10	8	0	4	5
	7	0	0	0	2	4	0	12	3	0
	8	0	0	0	0	0	4	0	2	7
	9	0	0	0	0	0	2	6	4	10

It is always possible that our estimated distribution is different from the true distribution because it is based on finite samples, and that even if it well approximates the true distribution, it may not be the one that depositors use to calculate the option premium of PSCs because they look at the future not the past. In fact, our estimated frequency distribution is not very smooth (Table 5) and therefore our methodology does not seem to have completely overcome these potential problems. In view of this, we have also calculated the option value for the case in which a depositor has *a priori* distribution that is smoothly mountain-shaped (Table 6).[13]

3. *Holding period assumption*

When market interest rates at the time of opening the account are high, a holder of PSCs could enjoy the high interest rate for ten years — more than three times longer than a holder of MTDs could. In contrast, a holder of MTDs will face an interest rate risk after three years: for example, if he holds the deposits for more than three years and if the interest rate falls after three years, he will end up with receiving a lower average interest rate.

Nonetheless, if the holding period is short, it is possible that a holder of MTDs enjoys a higher rate of return than a holder of PSCs because the former offer a higher

Table 6
Example of Mountain-Shaped Frequency Distribution
of Interest Rate States

State		Period $t+1$								
		1	2	3	4	5	6	7	8	9
	1	4	3	2	1	0	0	0	0	0
	2	3	4	3	2	1	0	0	0	0
	3	2	3	4	3	2	1	0	0	0
	4	1	2	3	4	3	2	1	0	0
Period t	5	0	1	2	3	4	3	2	1	0
	6	0	0	1	2	3	4	3	2	1
	7	0	0	0	1	2	3	4	3	2
	8	0	0	0	0	1	2	3	4	3
	9	0	0	0	0	0	1	2	3	4

[13]It is possible to make a more sophisticated formulation. For example, a depositor may think that the interest rate will rise above 8% in the future. In this case, it would be appropriate for us to consider the case of "interest rate state 10" in which the interest rate rises above 8%.

interest rate than the latter. Therefore, the holding period becomes an important factor in evaluating the advantages of PSCs vis-à-vis MTDs. We will present several simulations for different holding periods.[14]

B. Evaluation of the Real Value of Postal Savings Certificates

To evaluate the real value of PSCs, we have to take the following two characteristics into consideration:

(1) A PSC provides a depositor with an option to keep his deposit or to switch to a deposit with a new interest rate structure when the interest rate state changes in each period;
(2) A PSC offers a higher interest rate to depositors holding it for a longer period.

We proceed in two steps to explain the scheme of calculating the real value of PSCs. First, we focus on the option characteristic as stated in (1) above, assuming a flat interest rate structure (*i.e.*, ignoring the second characteristic of a progressive interest rate structure). Second, we consider the progressive interest rate structure as stated in (2) above, thus completing the scheme for calculating the real value of PSCs.

We assume a situation in which a depositor can choose between keeping his deposit or switching to a new deposit within the postal savings system. In reality, a depositor may find it more advantageous to switch to another financial instrument outside the postal savings system. We will ignore this possibility to simplify our analysis. If we allow such a possibility, a depositor may be able to enjoy a higher return so that our calculation may be regarded as representing the minimum of the real value of PSCs.

1. *The option characteristic and the real value of postal savings certificates*

We will first consider a simple three-period model with a flat interest rate structure (*i.e.*, a constant interest rate throughout all holding periods). We assume N possible interest rate states. Interest rate r_n (n=1, ..., N) applies in state n where $r_i > r_j$ if $i > j$.[15]

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[14]Although the holding period is a very important parameter, we do not have reliable information about the average holding period of depositors. This is a question that should be discussed in a broader context of a depositor's life-cycle. Because this is beyond the scope of our paper, we do not deal with this question in detail except that we present some preliminary estimations in footnote 18.

[15]Because interest rates on PSCs are expressed in terms of annual rate, we should, precisely speaking, write $r_i / 2$ instead of r_i for the interest rate per period (six months) in our model. To simplify our analysis, however, we simply write r_i to mean one-period (six months) interest rate. Similarly, we define the real value of a PSC in terms of a one-period rate. However, when we present our calculation results later on, we will express them in terms of an annual rate.

[Time Structure of the Model]

		First period		Second period		Third period	
Time:	0		1		2		3
State:	i		x		y		

a. At time 2

Consider a depositor at time 2. This depositor holds a PSC with interest rate r_j, and now faces interest rate state y at time 2. If he switches to a new PSC at time 2, the return in the third period will be $(1+r_y)$. If he does not switch, the return in the third period will be $(1+r_j)$. Therefore, if $r_y \geqq r_j$, the depositor should switch his deposit, and if $r_y < r_j$, he should keep it. Let $\phi_2(y \mid j)$ stand for "the maximum return that a depositor can obtain after time 2 (*i.e.*, in the third period) when he holds a savings certificate with interest rate r_j at time 1 and faces interest rate state y at time 2." Then, $\phi_2(y \mid j)$ can be written as follows:

$$\phi_2(y \mid j) = \begin{cases} 1+r_y & \text{for} \quad y \geqq j \\ 1+r_j & \text{for} \quad y < j \end{cases} \tag{1}$$

which takes a greater value for a higher y and j.

b. At time 1

Next consider a depositor at time 1. This depositor holds a PSC with interest rate r_i, and faces interest rate state x. If he switches to a new savings certificate at time 1, the return after time 2 (in the second and third periods) will be given by:

$$(1+r_x)\, \Sigma_y\, \phi_2(y \mid x)\, P(y \mid x) \tag{2}$$

where $\phi_2(y \mid x)$ represents "the maximum return that a depositor can obtain after time 2 (in the third period) when he holds a savings certificate with interest rate r_x at time 2 and faces interest rate state y at time 2." Notice that j is replaced by x in equation (1) because switching at time 1 means choosing interest rate r_x. $P(y \mid x)$ represents the probability that an interest rate state will be y at time 2 given interest rate state x at time 1. Therefore, $\Sigma_y\, \phi_2(y \mid x)\, P(y \mid x)$ represents the expected maximum value of the return after time 2 (in the third period). As $(1+r_x)$ is the return during time 1 and 2 (in the second period), equation (2) represents the return after time 1 (in the second and third periods) when the depositor switches his deposit at time 1.

If the depositor does not switch his deposit at time 1, the return will be given by:

$$(1+r_i)\, \Sigma_y\, \phi_2(y \mid i)\, P(y \mid x). \tag{3}$$

Notice that j is replaced by i in equation (1) when he keeps his deposit.

Now let us consider what choice the depositor should make in this situation. If $x \geqq i$, then

$$\begin{cases} 1+r_x \geqq 1+r_i \\ \phi_2(y \mid x) \geqq \phi_2(y \mid i). \end{cases}$$

In this case, it is better for the depositor to switch his deposit so that he can enjoy higher interest rate r_x during time 1 and 2 (in the second period) instead of r_i (*see* the top equation), and also a higher expected minimum return (shown in the bottom equation) after time 2 (in the third period). Therefore, from equations (2) and (3), we have:

return for switching (equation (2)) $\geqq$ return for keepig (equation (3)).

In short, if $x \geqq j$, switching the deposit is better, and if $x < j$, keeping it is better.

Let us define $\phi_1(x \mid i)$ to be "the maximum return that a depositor can obtain after time 1 (in the second and third periods) when he holds a savings certificate with interest rate r_i at time 1 and faces interest rate state x at time 1." Then, $\phi_1(x \mid i)$ can be written as follows:

$$\phi_1(x \mid i) = \begin{cases} (1+r_x)\, \Sigma_y\, \phi_2(y \mid x)\, P(y \mid x) \text{ for } x \geqq i \\ (1+r_i)\, \Sigma_y\, \phi_2(y \mid i)\, P(y \mid x) \text{ for } x < i \end{cases} \tag{4}$$

which takes a greater value for a higher x and i.

c. *At time 0*

Suppose the interest rate state was i at time 0. A depositor at time 0 holds a savings certificate with interest rate r_i. There is no uncertainty about the return in the first period, which is $(1+r_i)$, since the depositor has no choice to switch or keep the deposit. In contrast, the depositor is not sure at time 0 what interest rate state will emerge at time 1; he only knows the probability distribution of the state at that time. Thus, for him, $\phi_1(x \mid i)$ is a stochastic variable.

Using the same arguments of the previous section, we can derive the expected return in the subsequent three periods conditional upon the information available at time 0 as follows:

$$\phi_0(i) = (1+r_i)\, \Sigma_x\, \phi_1(x \mid i)\, P(x \mid i). \tag{5}$$

The third roots of this value represent the average return per period or the real value of PSCs in our three-period model.

2. *Treatment of the progressive interest rate structure*

Next we consider the progressive interest rate structure, and complete the scheme for calculating the real value of PSCs.

A PSC offers predetermined progressive interest rates according to the length of holding period: the longer the period, the higher the interest rate. Therefore, it is not necessarily better for a depositor to switch his deposit whenever market interest rates are higher because he may be able to enjoy a higher return by keeping it longer. In deciding whether to keep his deposit or switch it, a depositor must realize that his decision will affect not only the returns in future periods, but also returns in previous periods.

We continue to use the same three-period model, but now we assume a progressive interest rate structure as shown in Figure 4. Let $r_{i,s}$ stand for the interest rate that is offered for the holding period of s in interest rate structure i. Although it was always better to switch the deposit whenever a current rate was higher than the predetermined rate in the flat interest rate model of the previous section, it may so happen in the present model that even though a current rate is higher, it is better to keep the deposit. For example, suppose a depositor has held a deposit for two periods and the interest rate state turns out to be y at time 2. And let A stand for total return if the depositor switches the deposit, and B for the total return if the depositor keeps it. Then, we can express A and B as follows:

$$A = (1+r_{i,2})^2 (1+r_{y,1})$$
$$B = (1+r_{i,3})^3.$$

Thus, even if $(1+r_{y,1})$ is greater than $(1+r_{i,2})$, there exists a possibility that B is greater than A, depending on the relative sizes of $(1+r_{i,2})$ and $(1+r_{i,3})$.

To analyze more general cases, let us define the following three functions:

$\phi_t(x \mid i, h)$: the maximum return after time $t-h$ (*i.e.*, from period $t-h+1$ to period 3) for a depositor who has held a deposit for h periods in interest rate structure i and faces interest rate state x at time t.

$A_t(x \mid i, h)$: the maximum return after time $t-h$ (*i.e.*, from period $t-h+1$ to period 3) for a depositor who has held a deposit for h periods in interest rate structure i and switches at time t when the interest rate state is x.

$B_t(x \mid i, h)$: the maximum return after time $t-h$ (*i.e.*, from period $t-h+1$ to period 3) for a depositor who has held a deposit for h periods in interest rate structure i and does not switch at time t when the interest rate state is x.

Thus, the depositor should switch the deposit at time t if $A_t(x \mid i, h) \geqq B_t(x \mid i, h)$; in this case, he chooses interest rate structure x over i and $\phi_t(x \mid i, \mathrm{h}) = A_t(x \mid i, h)$ holds. The depositor should keep the deposit at time t if $A_t(x \mid i, h) < B_t(x \mid i, h)$; in this case, he chooses interest rate structure i over x and $\phi_t(x \mid i, h) = B_t(x \mid i, h)$ holds.

a. At time 2

Suppose a depositor has held the deposit up to time 2 in interest rate structure j, and faces interest rate state y at time 2. In this case, $A_t(\cdot)$ and $B_t(\cdot)$ can be written as follows:

$$A_2(y \mid j, k) = (1+r_{j,k})^k (1+r_{y,1})$$
$$B_2(y \mid j, k) = (1+r_{j,k+1})^{(k+1)} \tag{6}$$

where k=1 or 2. Let y^* stand for the minimum y such that $A_2(y \mid j, k) \geqq B_2(y \mid j, k)$ holds. Then, y^* becomes a function of j and k. The depositor will switch the deposit if $y \geqq y^*$ and keep it if $y < y^*$. Therefore, $\phi_2(y \mid j, k)$ can be written as follows:

$$\phi_2(y \mid j, k) = \begin{cases} A_2(y \mid j, k) \text{ for } y \geqq y^*(j, k) \\ B_2(y \mid j, k) \text{ for } y < y^*(j, k). \end{cases} \tag{7}$$

b. *At time 1*

Suppose a depositor has held the deposit up to time 1 in interest rate structure i, and faces interest rate state x at time 1. His holding period must be one period.

If the depositor switches the deposit at time 1 after one holding period, his return $A_1(x \mid i, 1)$ is given by the following:

$$A_1(x \mid i, 1) = (1+r_{i,1})\, \Sigma_y \phi_2(y \mid x, 1)\, P(y \mid x). \tag{8}$$

This corresponds to equation (2). It should be noted, however, that $(1+r_{i,1})$ of equation (8) represents a return during time 1 and 2 (the first period) while $(1+r_x)$ of equation (2) represents a return during time 2 and 3 (the second period), which is already included in $\phi_2(y \mid x, 1)$ of equation (8).

On the other hand, if the depositor keeps the deposit, his return $B_1(x \mid i, 1)$ is given by the following:

$$B_1(x \mid i, 1) = \Sigma_y\, \phi_2(y \mid i, 2)\, P(y \mid x). \tag{9}$$

Let x^* stand for the minimum x such that $A_1(x \mid i, 1) \geqq B_1(x \mid i, 1)$ holds. Then, x^* becomes a function of i. The depositor will switch the deposit if $x \geqq x^*$ and keep it if $x < x^*$. Therefore, $\phi_1(x \mid i, 1)$ can be written as follows:

$$\phi_1(x \mid i, 1) = \begin{cases} A_1(x \mid i, 1) \text{ for } x \geqq x^*(i) \\ B_1(x \mid i, 1) \text{ for } x < x^*(i). \end{cases} \tag{10}$$

c. *At time 0*

Suppose that the interest rate state is i at time 0. A depositor at time 0 holds the deposit with interest rate r_i. Function $\phi_1(x \mid i, 1)$ represents a return after time 0 (one period before time 1) or in the three periods, which is a stochastic variable governed by probability distribution $P(x \mid i)$. Return after time 0, $\phi_0(i)$, conditional upon information available at time 0 can be written as follows:

$$\phi_0(i) = \Sigma_x\, \phi_1(x \mid i, 1)\, P(x \mid i). \tag{11}$$

The third roots of this value represent the average return per period or the real value of PSCs in our three-period model.

IV. The Real Value of Maturity-Designated Time Deposits

We have assumed so far that a depositor can extend the holding period of a deposit if he thinks it advantageous. In other words, we have assumed that the maximum maturity period of a deposit is longer than his holding period (*i.e.*, three periods). However, if his holding period is longer than the maximum maturity period, the calculation scheme we have developed in Section III needs to be modified. For example, if the maximum maturity period is two periods and the depositor's holding period is three, then he has to switch his deposit after two periods even if he wants to keep it.

This point becomes important when we compare the real values of PSCs and MTDs. For example, suppose that a depositor's holding period is four years. In the case of PSCs, because their maximum maturity period is ten years, he could keep his deposit for the whole four-year holding period without switching if he wanted to. In the case of MTDs, however, he has to switch his deposit at least once during his four-year holding period. This difference will affect the decisions of depositors.

In this section we will develop a scheme to calculate the real value of a deposit whose maximum maturity period is less than his holding period, built on the arguments of Section III. For convenience, we call the scheme that calculates the real value of a deposit whose maximum maturity period is longer than a depositor's holding period, "postal scheme"; and the scheme that calculates the real value of a deposit whose maximum maturity period is shorter than a depositor's holding period, "maturity-designated scheme." Note that if a depositor's holding period is more than ten years, we need to use the "maturity-designated scheme" to calculate the real value of PSCs.

We will use the same three-period model that was presented in the previous section. A more general multi-period model is presented in the appendixes.

Let us consider what modifications are necessary to the "postal scheme" in order to develop a "maturity-designated scheme." Let us start from equation (7):

$$\phi_2(y \mid j, k) = \begin{cases} A_2(y \mid j, k) \text{ for } y \geqq y^*(j, k) \\ B_2(y \mid j, k) \text{ for } y < y^*(j, k). \end{cases} \tag{7}$$

This shows the optimal decision of a depositor who has kept his deposit up to time 2 for k periods in interest rate structure j and faces interest rate state y at time 2. For example, if $A_2(y \mid j, k)$ is less than $B_2(y \mid j, k)$, then it is better for the depositor to keep the deposit (*i.e.*, to choose interest rate structure j instead of y), and $\phi_2(y \mid j, k)=B_2(y \mid j, k)$ holds. In this case, there are no restrictions on k and the same formula applies for $k=1$ and $k=2$.

The situation will be different if a depositor cannot keep his deposit beyond the maximum maturity period. For example, suppose that the maximum maturity period is two and $k=2$, then the depositor has no choice but to switch in the next period to a deposit with interest rate structure y so that $\phi_2(y \mid j, 2)=A_2(y \mid j, 2)$ should hold. Thus, equation (7) needs to be modified as follows:

(1) When $k=1$ (the same equation as that in Section III):

$$\phi_2(y \mid j, 1) = \begin{cases} A_2(y \mid j, 1) \text{ for } y \geqq y^*(j, 1) \\ B_2(y \mid j, 1) \text{ for } y < y^*(j, 1). \end{cases} \tag{12}$$

(2) When $k=2$ (must switch):

$$\phi_2(y \mid j, 2) = A_2(y \mid j, 2) \text{ for any } j \text{ and } y.$$

V. Comparison of Postal Savings Certificates and Maturity-Designated Time Deposits

In this section, we will calculate and compare the real values of PSCs and MTDs using the methods outlined in the appendixes that generalize those explained in the previous sections.

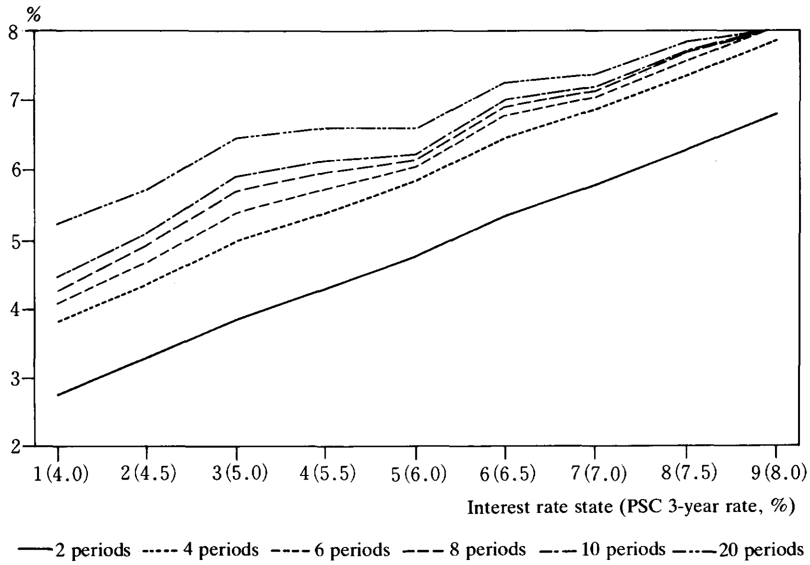
A. The Option Premium of Postal Savings Certificates

Figure 6 presents the calculated real values of PSCs and MTDs (the horizontal axis shows interest rate states, which are represented by the interest rate of three-year and over PSCs). We will consider the case of a depositor who plans to hold a deposit for four

Figure 6

Real Values of Postal Savings Certificates and Maturity-Designated Time Deposits by Holding Period

(1) Real Value of Postal Savings Certificates for Different Holding Periods



(2) Real Value of Maturity-Designated Time Deposits for Different Holding Periods

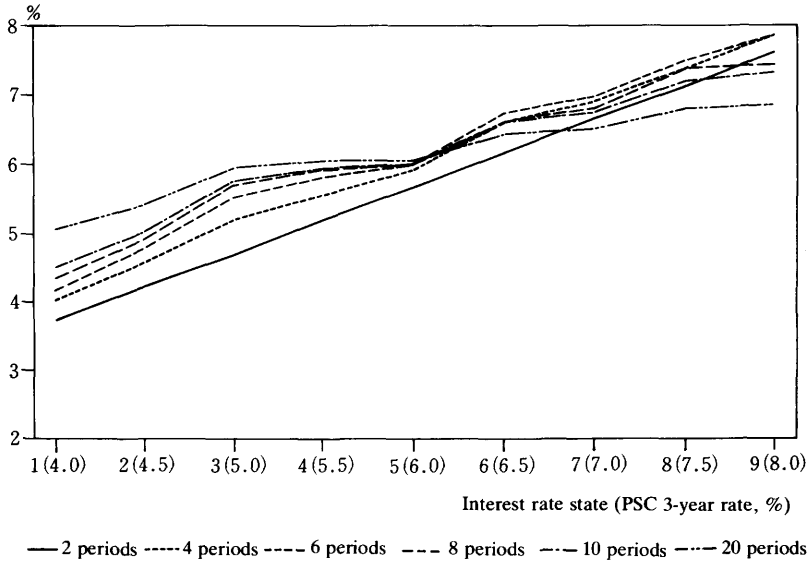
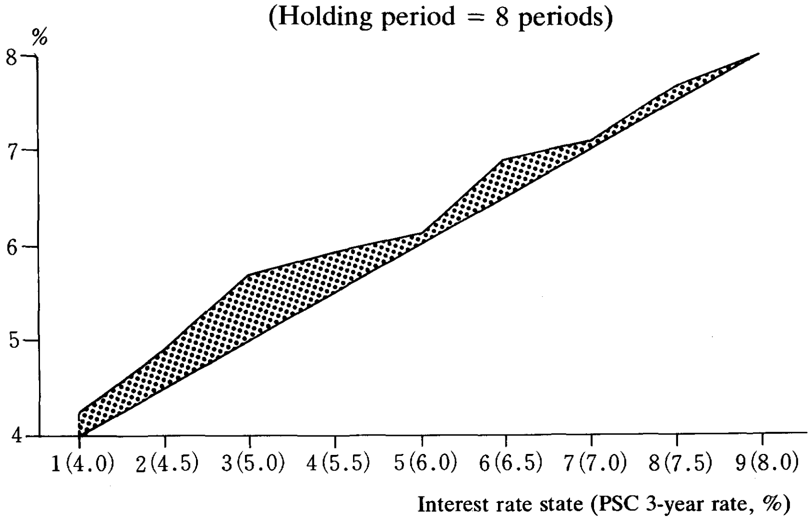


Figure 7
Derivation of the Option Premium of Postal Savings Certificates
(Holding period = 8 periods)



years, and calculate the option premium. As discussed in Section II, the option characteristic of PSCs comes from the fact that a depositor can cancel his deposit before maturity without any penalty, and therefore the option premium represents the price of such right to cancel his deposit.

Theoretically speaking, the option premium of PSCs in this case is the difference between its real value and the return on a four-year time deposit that has no such cancellation right. Figure 7 shows the real value of PSCs as a function of the interest rate state when a depositor wants to save for eight periods. The gap between this curve and a 45 degree line corresponds to the theoretical option premium of PSCs. Figure 8 on the next page shows such option premiums for various holding periods. It shows that the option premium can be as large as 0.69% (in interest rate state 3) for eight holding periods (four years).[16]

B. Difference in the Real Values of Postal Savings Certificates and Maturity-Designated Time Deposits

These option premiums that have been estimated in relation to a hypothetical four-year time deposit are not the interest rate differentials that depositors are actually interested in. They are interested in the interest rate differentials among competing financial instruments such as public-sector PSCs and private-sector MTDs.

Figure 10 compares the estimated real value of PSCs with that of MTDs for several different holding periods. For example, the difference in the real values of these two instruments for a depositor who wants to keep his savings for eight periods (four years) is shown as the gap between the solid and dotted lines in the chart for eight holding periods in Figure 10. Such gaps for various holding periods are plotted in Figure 11, which shows that, in the case of eight holding periods, PSCs offer up to 0.57% higher real value (in interest rate state 9) than MTDs.

The difference in the real values of these two instruments is attributable to the

[16]According to Figure 8 on the next page, the option premium of PSCs declines with ups and downs as the interest rate state increases. These ups and downs are due to a distorted probability distribution estimated from past data. If we use a smooth probability distribution (Table 6), the option premium declines monotonously (Figure 9).

Figure 9

Smooth Option Premiums of Postal Savings Certificates

Holding period = 20 periods (10 years)

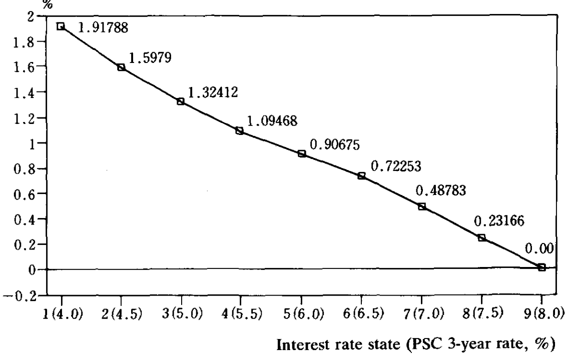
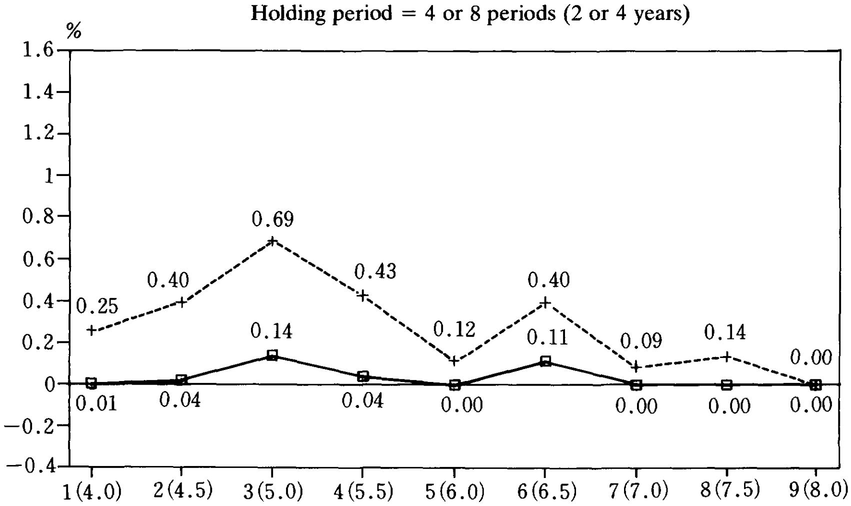


Figure 8
Option Premium of Postal Savings Certificates

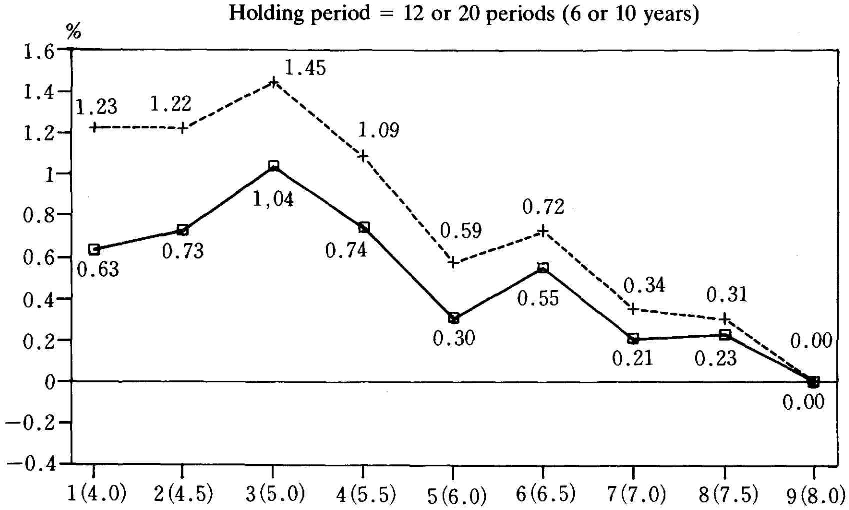
Holding period = 4 or 8 periods (2 or 4 years)



Interest rate state (PSC 3-year rate, %)

□ 4 periods (2 years) + 8 periods (4 years)

Holding period = 12 or 20 periods (6 or 10 years)



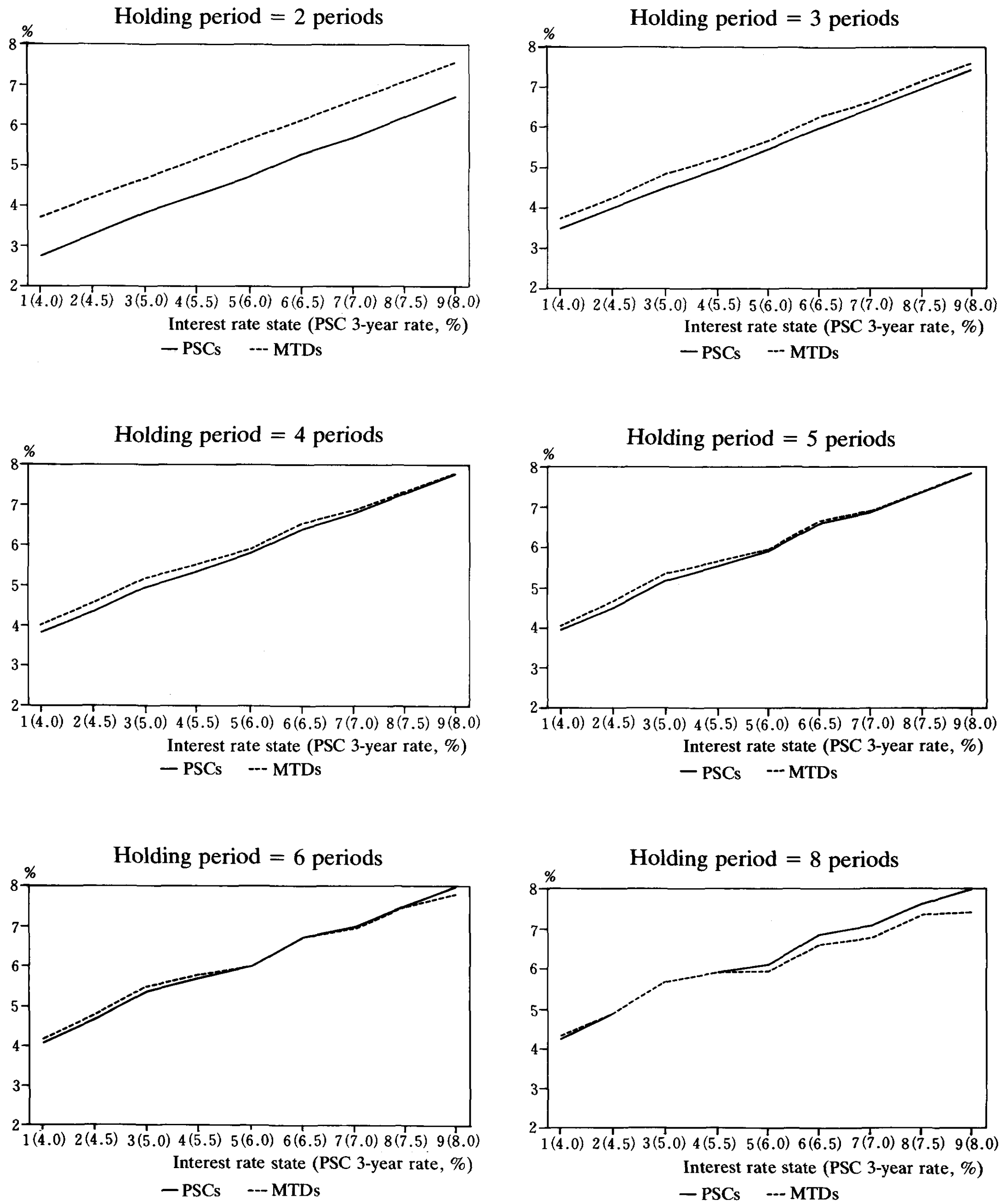
Interest rate state (PSC 3-year rate, %)

□ 12 periods (6 years) + 20 periods (10 years)

Figure 10

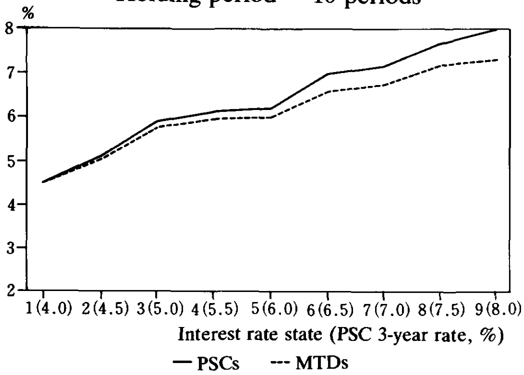
Real Values of Postal Savings Certificates (PSCs)

and Maturity-Designated Time Deposits (MTDs) by Holding Period

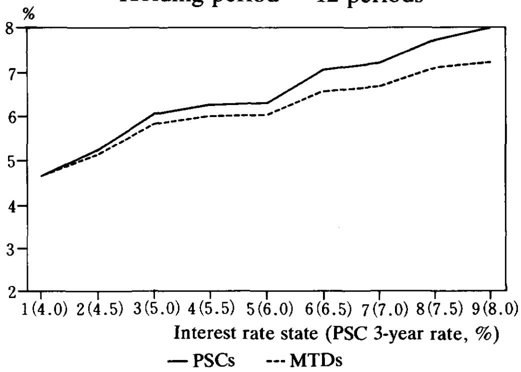


(*Continued on next page*)

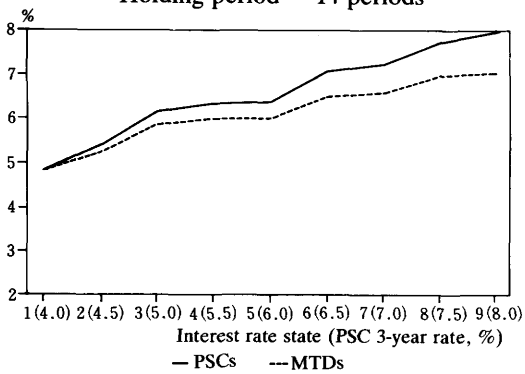
Holding period = 10 periods



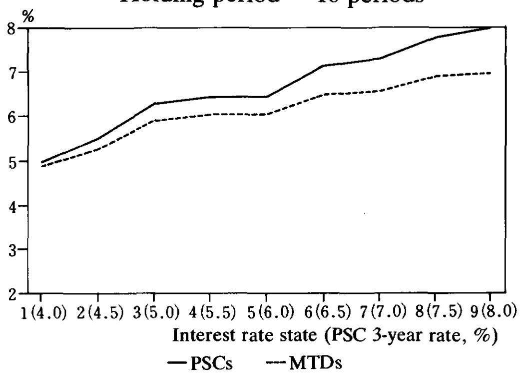
Holding period = 12 periods



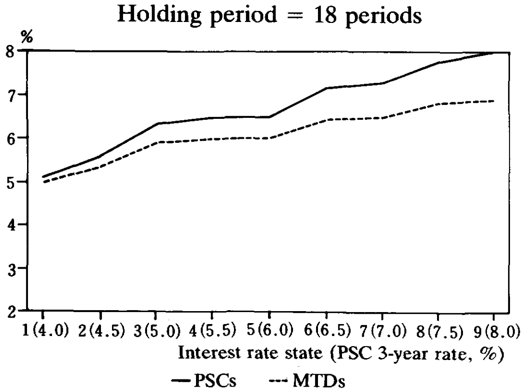
Holding period = 14 periods



Holding period = 16 periods



Holding period = 18 periods



Holding period = 20 periods

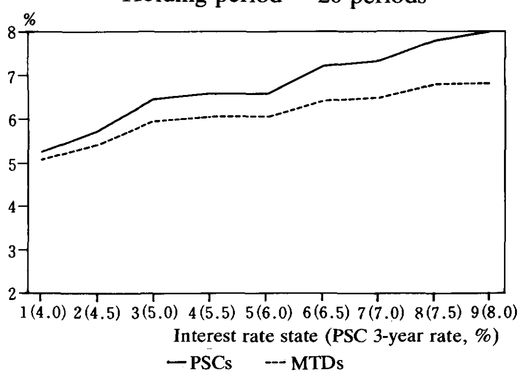
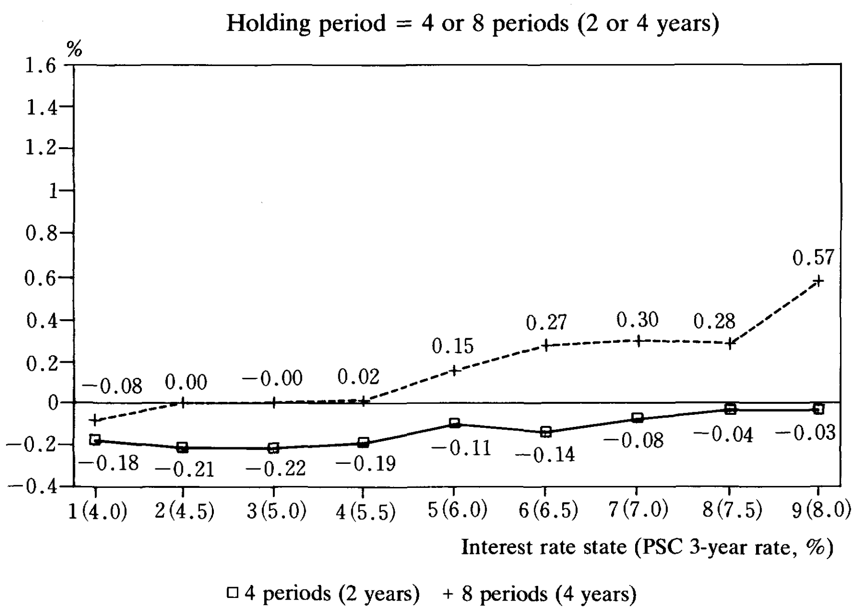
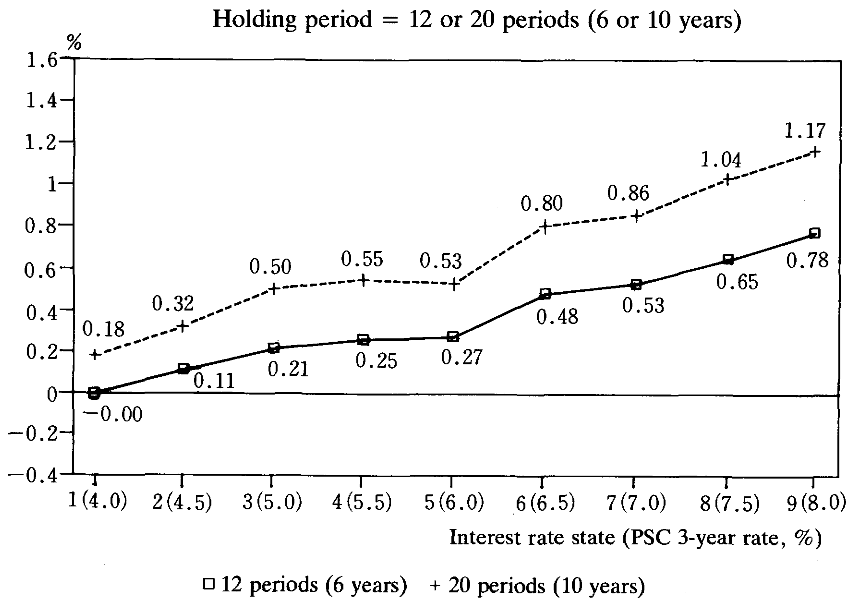


Figure 11
Extra Real Value of Postal Savings Certificates
Over Maturity-Designated Time Deposits





following three main factors: (i) the current interest rate (half-year compound interest basis)[17]; (ii) the option premium; and (iii) the expected interest rate after the maximum maturity period, which is important because if it is low, then the expected interest rate loss incurred by keeping MTDs after the maximum maturity of three years would increase. The nominal interest rate of PSCs is lower than that of MTDs, but the gap tends to narrow as the period gets longer (Tables 4 and 7). The option premium tends to decline for both instruments as the current interest rate rises because it reduces the possibility of switching the deposit in the future (shown in Figure 8 in the case of PSCs). Thus, when the holding period is longer and the current interest rate higher, the difference in the real values of the two instruments depends more on the expected decline in the interest rate of MTDs after its maximum maturity (or the extent to which a holder of PSCs can expect to enjoy a higher interest rate for a longer period than a holder of MTDs).

Next, let us examine the relationship between the difference in the real values of the two instruments and the holding period of a depositor. According to Figure 10, the real value of MTDs is higher than that of PSCs in every interest rate state up to four periods. Beyond five periods, however, which of the two instruments has a higher real value depends on interest rate state, and the boundary line in the interest rate state moves closer to the origin as the holding period becomes longer (Figure 12). Figure 13 shows the combination of an interest rate state and a holding period in which one of the two instruments has a higher real value. According to this, PSCs become more attractive to depositors as the current interest rate rises and the holding period becomes longer.

Table 7
Interest Rate Adjustments of Maturity-Designated Time Deposits to Semiannual Compound Interest Rates

(%)

Interest rate state	0.5 year	1 year	1.5 years	2 years	2.5 years	3 years
1	2.35	3.72	3.73	3.96	3.97	3.96
2	2.85	4.21	4.22	4.45	4.46	4.45
3	3.35	4.69	4.71	4.94	4.95	4.94
4	3.85	5.18	5.21	5.43	5.44	5.43
5	4.35	5.67	5.70	5.91	5.93	5.91
6	4.85	6.16	6.19	6.40	6.42	6.40
7	5.35	6.64	6.68	6.88	6.91	6.88
8	5.85	7.12	7.17	7.36	7.39	7.36
9	6.35	7.61	7.65	7.85	7.88	7.85

[17] *See* Appendix 2 for necessary adjustments from annual compound interest rates to semiannual compound interest rates.

Figure 12
Changes in Holding Period and Real Value of Postal Savings Certificates (PSCs) and Maturity-Designated Time Deposits (MTDs)

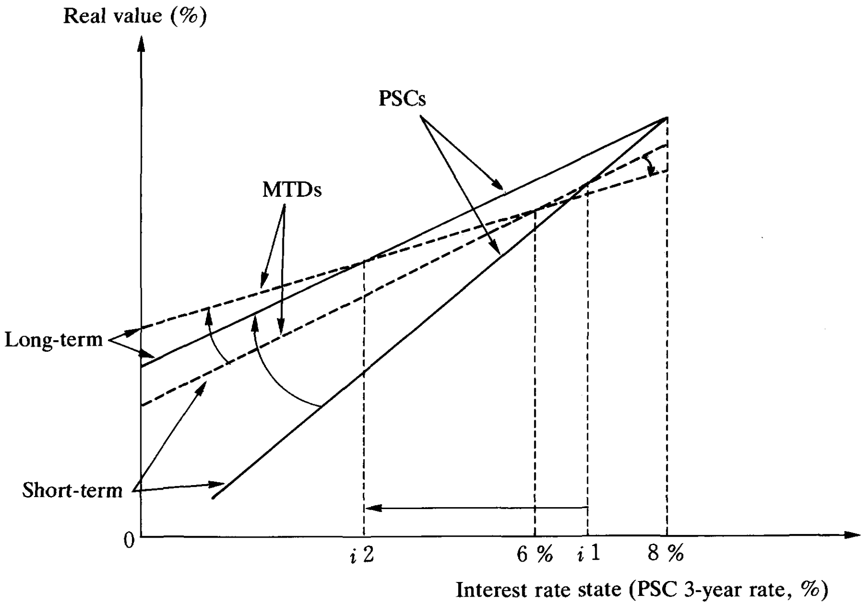
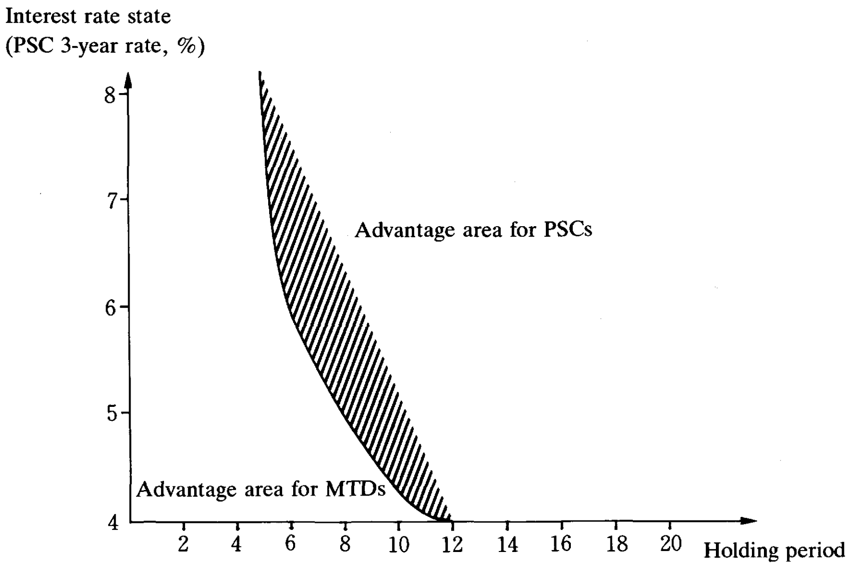


Figure 13
Comparative Advantages of Postal Savings Certificates (PSCs) and Maturity-Designated Time Deposits (MTDs)



VI. Conclusion

In this paper, we discussed the option characteristic of PSCs (an American put-option) and estimated their real value including option premium. Then, we examined the comparative advantages of PSCs and MTDs, comparing their real values (effective yields). Although the paper touched on many aspects of PSCs, the main conclusions may be summarized as follows:

(1) The option premium of PSCs tends to increase with the length of the holding period. For example, while the option premium is at most 0.69% for a four-year period, it is 1.45% for a ten-year period.
(2) The extra real value (including option premium) of PSCs over MTDs tends to increase as the current interest rate rises and the holding period lengthens. For example, while the extra value is at most 0.57% for a four-year period, it rises to 1.17% for a ten-year period.
(3) However, if the holding period is short, the real value of MTDs can be greater than that of PSCs.

These conclusions, however, depend on several assumptions we made in Section III and therefore our estimated figures for the size of the option premium and the real values of the two financial instruments should be treated with some reservation. Nevertheless, the analytical framework and the main conclusions that we presented in this paper should be useful for addressing the broader issues of the postal savings system.

In concluding, we would like to point out some of the remaining issues. First, postal savings are assigned to provide funds for the Fiscal Investment and Loan Program through the special postal savings account of the government. Therefore, in discussing issues concerning the postal savings system we must consider not only the question of how to raise the funds (*i.e.* cost-and-benefit analysis of postal savings), but also the question of how to use the funds (*i.e.* economic efficiency of the Fiscal Investment and Loan Program).

Second, we have treated a depositor's holding period as given, which is equivalent to assuming a zero interest rate elasticity of present consumption for future consumption (or an inelastic intertemporal substitution between present and future consumption). According to the life-cycle model, however, a higher interest rate induces households to save now and consume later; in other words, a higher interest rate means a longer holding period. In this case, the holding period becomes an increasing function of the real value of a financial instrument or the interest rate state: the real value declines in a low interest rate state because of a shorter holding period and increases in a high interest rate state because of a longer holding period. Incorporating this aspect into our scheme would require building a dynamic model and estimating the interest elasticity of the holding period.

Finally, we have stated that the real value of PSCs tends to increase as the interest rate rises and the holding period becomes longer. This implies, however, that postal savings are seeking deposits at an increasing marginal cost. Thus, to insure a stable flow of funds through the special postal savings account, we need to analyze postal savings system issues in terms of economic efficiency and stability.[18,19]

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[18]The average holding period of depositors is an important parameter for estimating the real financing cost of the postal savings system. Figure 14 plots quarterly percentage changes (year on year) in outstanding PSCs and MTDs, and Figure 15 changes in the interest rate of PSCs (three years and over). Figure 14 shows that the difference in their growth rates has widened since the interest rate was raised in 1990. The average interest rate during this period was 5%, at which PSCs become advantageous if the holding period is eight periods (four years) or over as seen in Figure 13. These developments suggest that the average holding period was four years. We hasten to add, however, that this figure is based on inference and should be treated with caution. In

Figure 14

Outstanding of Postal Savings Certificates (PSCs)
and Maturity-Designated Time Deposits (MTDs)
(Year-on-year percentage change)

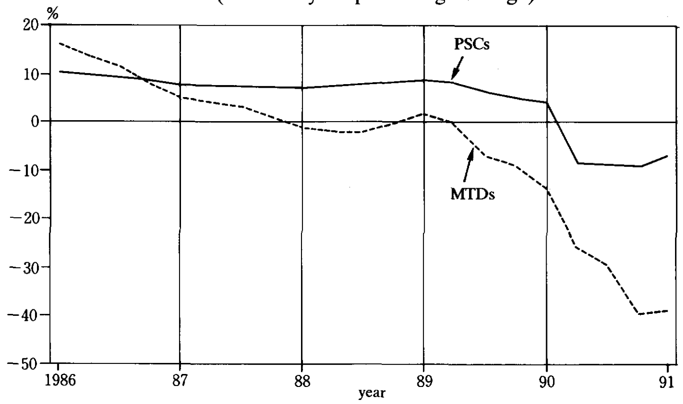
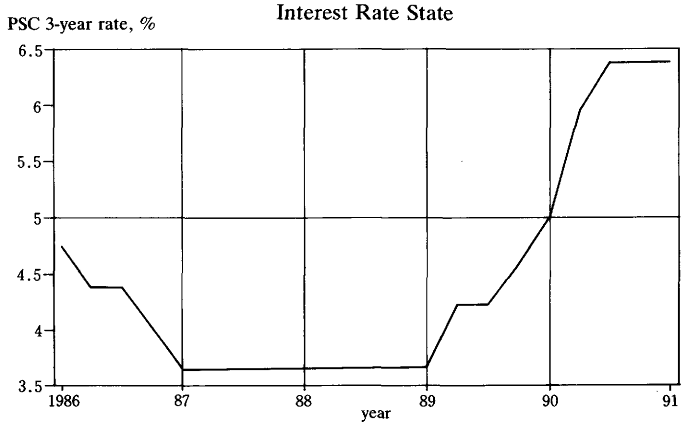


Figure 15

Interest Rate State



particular, there is a possibility that the growth of MTDs has been adversely affected by the introduction of new small MMCs during this period, which might bias the results of our estimation.

[19]The real value of PSCs that has been estimated in Section III represents their expected (average) return. However, in addition to expected return, the probability distribution of returns, which can be obtained from the maximization problem of depositors, is also important. For example, if we assume a holding period of eight periods (four years), the probability distribution of returns can be obtained as the nine cases (depending on the initial interest rate state) in Figure 16. From these distributions, we can estimate the risk of the postal savings system which sells the savings certificates with an American put-option.

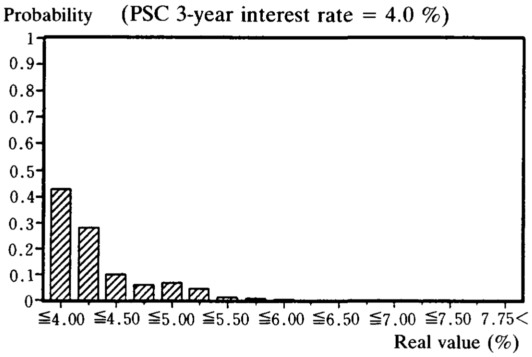
Figure 16

Probability Distribution of Returns on Postal Savings Certificates (PSCs)

(Holding period = 8 periods)

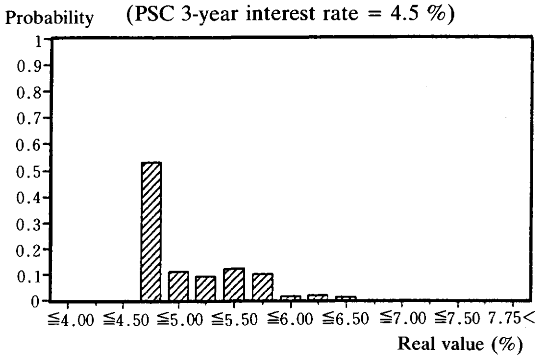
Initial interest rate state = 1

(PSC 3-year interest rate = 4.0 %)



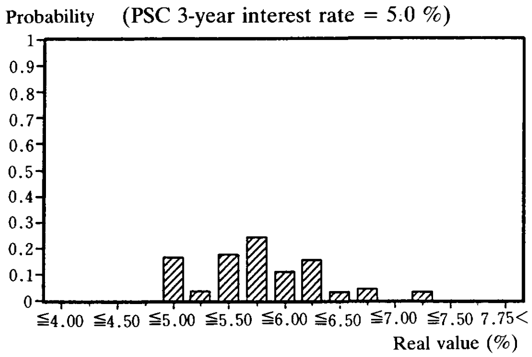
Initial interest rate state = 2

(PSC 3-year interest rate = 4.5 %)



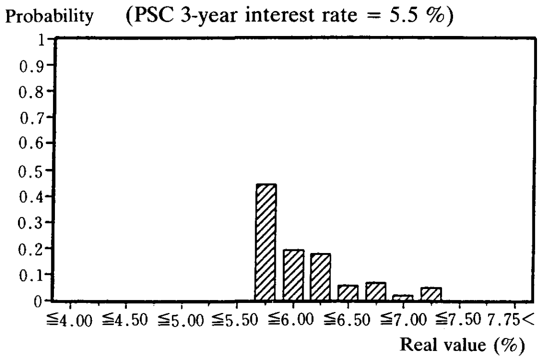
Initial interest rate state = 3

(PSC 3-year interest rate = 5.0 %)



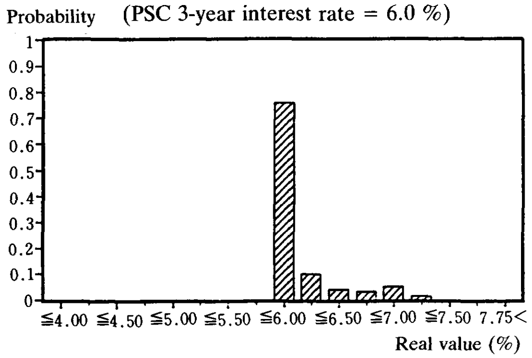
Initial interest rate state = 4

(PSC 3-year interest rate = 5.5 %)

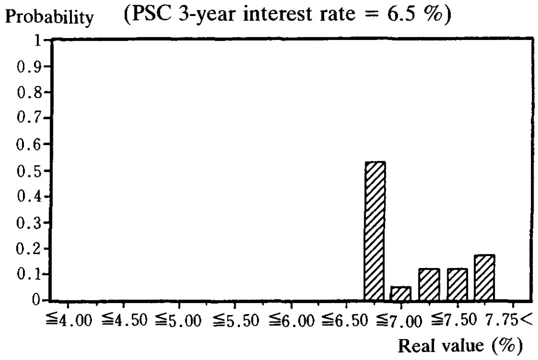


(Continued on next page)

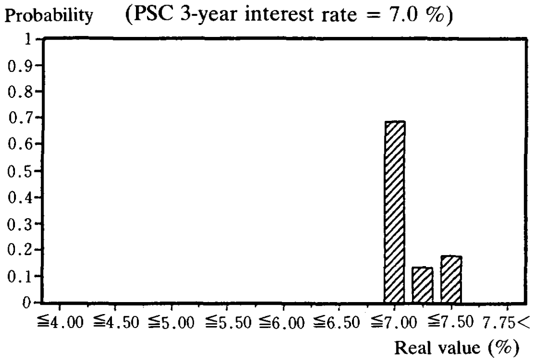
Initial interest rate state = 5
(PSC 3-year interest rate = 6.0 %)



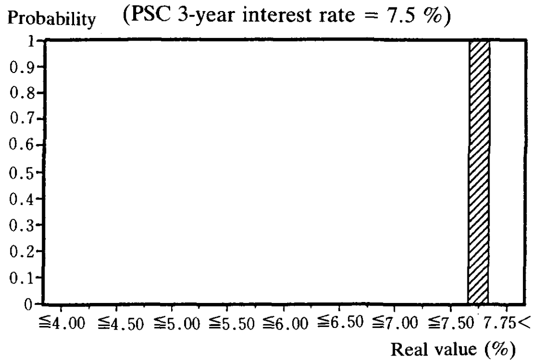
Initial interest rate state = 6
(PSC 3-year interest rate = 6.5 %)



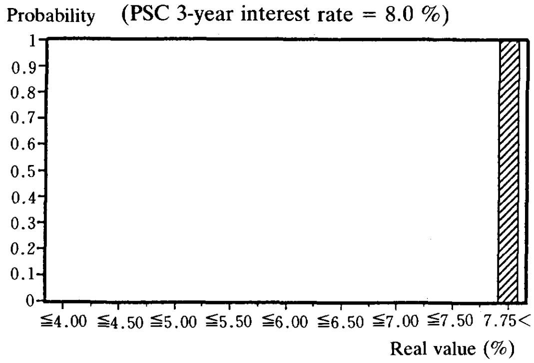
Initial interest rate state = 7
(PSC 3-year interest rate = 7.0 %)



Initial interest rate state = 8
(PSC 3-year interest rate = 7.5 %)



Initial interest rate state = 9
(PSC 3-year interest rate = 8.0 %)



Appendix A. Calculation of the Real Value of Postal Savings Certificates (A General Case)

The purpose of this appendix is to generalize the three-period model that has been developed in Section III to calculate the real value of PSCs. This appendix extends the model to T periods and applies it to developing a general calculation scheme for various holding periods.[20]

We first calculate the sum of principal and interest at the time of cancellation for a unit of initial deposit — we call the sum "the deposit return." Next, we take the Tth roots of the deposit return, and calculate the annual interest rate of a half-year compound type deposit — we call the interest rate "the real value of PSCs."

The initial interest rate state (at time 0) is given as an initial condition. A depositor starts with this initial condition: all he can do is to purchase a PSC in this given interest rate state. In this paper, we consider N different initial conditions in accordance with N interest rate states, and calculate the deposit returns for these N cases.

The key functions that play an important role in calculating real value are: (i) "deposit return functions" that show the deposit return for a depositor who cancels his initial deposit and switches to a new deposit and the deposit return for a depositor who keeps his initial deposit; (ii) a "response function" that states the best decision strategy between a switch and a keep, given the above deposit return functions.

A depositor does not have a choice at time 0 (initial period) and T (final period). However, a depositor has to decide whether he should switch his deposit (to a new interest rate structure) or keep it (in the same interest rate structure) at each time from 1 to $T-1$. As the decision problem of a depositor at time $T-1$ is slightly different from those at earlier times, we will consider the problem at time $T-1$ separately. Moreover, we call the minimum period beyond which the interest rate stays the same "period M," and treat the period beyond M (which is three years and over in the case of PSCs) separately.

1. Deposit Return Functions and Response Function

Deposit return functions

These refer to a group of three functions: $\phi_t(x \mid i, h)$, $A_t(x \mid i, h)$, and $B_t(x \mid i, h)$, discussed in Section III. Redefining these for the general case of T periods we have:

$\phi_t(x \mid i, h)$: the maximum return after time $t-h$ when the account is opened (*i.e.* from period $t-h+1$ to period T) for a depositor who held his deposit for h periods in interest rate structure i and faces interest rate state x at time t.

$A_t(x \mid i, h)$: the maximum return after time $t-h$ (*i.e.* from period $t-h+1$ to period T) for a depositor who held his deposit for h periods in interest rate structure i and switches at time t when the interest rate state is x.

[20] We will limit our analysis to the cases in which the holding period is 20 periods (ten years: the maximum maturity period for PSCs) or less.

$B_t(x \mid i, h)$: the maximum return after time $t-h$ (*i.e.* from period $t-h+1$ to period T) for a depositor who held his deposit for h periods in interest rate structure i and does not switch at time t when the interest rate state is x.

Response function

This refers to function $f_t(x \mid i, h)$ which states the optimal decision of a depositor who has held his deposit for h periods in interest rate structure i and faces interest rate state x at time t, and must decide whether to keep his deposit or to switch it:

$$A_t(x \mid i, h) \geqq B_t(x \mid i, h) \Longrightarrow \text{switch} \Longleftrightarrow f_t(x \mid i, h) = x$$

$$A_t(x \mid i, h) < B_t(x \mid i, h) \Longrightarrow \text{keep} \Longleftrightarrow f_t(x \mid i, h) = i.$$

The response function and the three deposit return functions have the following relationship:

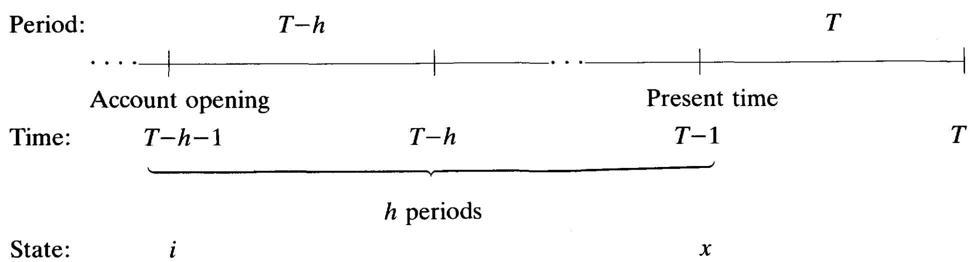
$$\phi_t(x \mid i, h) = \begin{cases} A_t(x \mid i, h) \text{ for } x \in \{x : f_t(x \mid i, h) = x\} \\ B_t(x \mid i, h) \text{ for } x \in \{x : f_t(x \mid i, h) = i\}. \end{cases}$$

It should be emphasized that, under the assumptions we made in Section III, a depositor only has to know three pieces of information: (i) the new interest rate state x at time t; (ii) the interest rate structure i in which he has held his deposit up to time t; and (iii) the holding period h. Moreover, the present decision affects not only future deposit return but also past deposit return because the interest rate of PSCs increases with the length of the deposit and applies to all the periods since account opening. This is why response function $f_t(x \mid i, h)$ depends on deposit return functions $A_t(x \mid i, h)$ and $B_t(x \mid i, h)$, which capture accumulated deposit returns since account opening. Before starting our analysis, let us introduce notation $r_{i,h}$, which represents a one-period (six months) interest rate for the deposit that has been held for h periods in interest rate structure i.

2. Decision at Time $T-1$

First, let us derive deposit return functions and the response function at time $T-1$. Suppose that a depositor has held his deposit for h periods in interest rate structure i and faces interest rate state x at time $T-1$.

[Time Structure of the Model]



Then, as we argued in Section III, we will have the following deposit return functions:

$$A_{T-1}(x \mid i, h) = (1+r_{i,h})^h (1+r_{x,1})$$
$$B_{T-1}(x \mid i, h) = (1+r_{i,h+1})^{(h+1)}.$$

Therefore, from the arguments in the previous section, we have the following response function:

$$A_{T-1}(x \mid i, h) \geqq B_{T-1}(x \mid i, h) \Longrightarrow \text{switch} \Longleftrightarrow f_{T-1}(x \mid i, h) = x$$

$$A_{T-1}(x \mid i, h) < B_{T-1}(x \mid i, h) \Longrightarrow \text{keep} \Longleftrightarrow f_{T-1}(x \mid i, h) = i.$$

Thus, we have the following functional relationship:

$$\phi_{T-1}(x \mid i, h) = \begin{cases} A_{T-1}(x \mid i, h) \text{ for } x \in \{x : f_{T-1}(x \mid i, h) = x\} \\ B_{T-1}(x \mid i, h) \text{ for } x \in \{x : f_{T-1}(x \mid i, h) = i\}. \end{cases}$$

3. Decision at Time t ($< T-1$)

Next let us consider the decision problem of a depositor who faces a new interest rate state at time t ($< T-1$).

[Time Structure of the Model]

Period:	$t-h+1$		$t+1$	
	Account opening		Present time	
Time:	$t-h$	$t-h+1$	t	$t+1$
	h periods			
State:	i		x	y

Figure A-1 describes the depositor's decision-making in the following situation:

{interest rate structure (i) of the deposit held until time t}

< {interest rate state (x) at time t}

< {expected interest rate state (y) at time $t+1$}.

Suppose the depositor faces interest rate state x at time t and decides to switch. Then, his deposit return function $A_t(x \mid i, h)$ can be written as follows:

$$\begin{aligned} A_t(x \mid i, h) &= (1+r_{i,h})^h \Sigma_y \phi_{t+1}(y \mid x, 1) P(y \mid x) \\ &= (1+r_{i,h})^h \{\Sigma_{S1} A_{t+1}(y \mid x, 1) P(y \mid x) + \Sigma_{S2} B_{t+1}(y \mid x, 1) P(y \mid x)\}, \end{aligned}$$
$$S1 = \{y : f_{t+1}(y \mid x, 1) = y\}, \quad S2 = \{y : f_{t+1}(y \mid x, 1) = x\}.$$

$(1 + r_{i,h})^h$ represents the return for h periods during which he holds his deposit in interest rate structure i (the return during time $t-h$ to t <period $t-h+1$ to t>). As this depositor switches his deposit at time t, he would have held his new deposit under interest rate state x for one period at time $t+1$. $S1$ is a set of the interest rate states at time $t+1$ that give him an incentive to switch again at time $t+1$. This corresponds to the path represented by $(G_{t,0} \rightarrow G_{t+1,1} \rightarrow G_{t+2,2})$ in Figure A-1. $A_{t+1}(y \mid x, 1)$ represents deposit return after time t for a depositor who has held his deposit in structure x for one period and switches in structure y at time $t+1$. $P(y \mid x)$ represents the conditional probability for state y given state x. Thus, the first term in $\{\cdot\}$ of the second equation of $A_t(x \mid i, h)$ represents deposit return after time t (period $t+1$ to T) if the depositor switches at time $t+1$.

$S2$ is a complement of $S1$: that is, the set of the interest rate states at time $t+1$ which does not give the depositor (who has switched his deposit at time t) an incentive to switch again at time $t+1$. This corresponds to the path represented by $(G_{t,0} \rightarrow G_{t+1,1} \rightarrow G_{t+2,1})$ in Figure A-1. $B_{t+1}(y \mid x, 1)$ represents deposit return after time t for a depositor who has held his deposit in structure x for one period and keeps it under state y at time $t+1$. $P(y \mid x)$ represents the conditional probability for state y given state x. Thus, the second term in $\{\cdot\}$ of the second equation of $A_t(x \mid i, h)$ represents the deposit return after time t (period $t+1$ to T) if the depositor keeps his deposit at time $t+1$.

In sum, $A_t(x \mid i, h)$ represents deposit return after time $t-h$ (period $t-h+1$ to T) for a depositor who has held his deposit in interest rate structure i for h periods and switches at time t when the interest rate state is x.

On the other hand, suppose that the depositor decides to keep his deposit at time t when the interest rate state is x. Then, his deposit return function $B_t(x \mid i, h)$ can be written as follows:

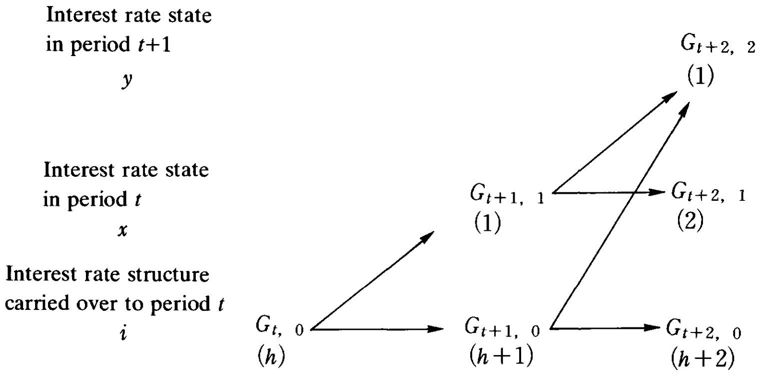
$$\begin{aligned} B_t(x \mid i, h) &= \Sigma_y\, \phi_{t+1}(y \mid i, h+1)\, P(y \mid x) \\ &= \Sigma_{S3} A_{t+1}(y \mid i, h+1)\, P(y \mid x) + \Sigma_{S4} B_{t+1}(y \mid i, h+1)\, P(y \mid x), \\ S3 &= \{y : f_{t+1}(y \mid i, h+1) = y\}, \quad S4 = \{y : f_{t+1}(y \mid i, h+1) = i\}. \end{aligned}$$

As this depositor keeps his deposit at time t, he would have held his deposit in interest rate structure i for $h+1$ periods at time $t+1$. $S3$ is a set of the interest rate states at time $t+1$ that give him an incentive to switch at time $t+1$. This corresponds to the path represented by $(G_{t,0} \rightarrow G_{t+1,0} \rightarrow G_{t+2,2})$ in Figure A-1. $A_{t+1}(y \mid i, h+1)$ represents the deposit return after time $t-h$ (period $t-h+1$ to T) for a depositor who has held his deposit in structure i for $h+1$ periods and switches under state y at time $t+1$. $P(y \mid x)$ represents the conditional probability for state y given state x. Thus, the first term in $\{\cdot\}$ of the second equation of $B_t(x \mid i, h)$ represents the deposit return after time $t-h$ (period $t-h+1$ to T) if the depositor switches at time $t+1$.

$S4$ is a complement of $S3$: that is, the set of the interest rate states at time $t+1$ that does not give the depositor who has kept his deposit at time t an incentive to switch at time $t+1$. This corresponds to the path represented by $(G_{t,0} \rightarrow G_{t+1,0} \rightarrow G_{t+2,0})$ in Figure A-1. $B_{t+1}(y \mid i, h+1)$ represents the deposit return after time $t-h$ (period $t-h+1$ to T) for a depositor who has held his deposit in structure i for $h+1$ periods and again keeps his deposit under state y at time $t+1$. $P(y \mid x)$ represents the conditional probability for state y given state x. Thus, the second term in $\{\cdot\}$ of the second equation of $B_t(x \mid i, h)$ represents the deposit return after time $t-h$ (period $t-h+1$ to T) if the depositor keeps his deposit at time $t+1$.

In sum, $B_t(x \mid i, h)$ represents deposit return after time $t-h$ (period $t-h+1$ to T) for a depositor who has held his deposit in interest rate structure i for h periods and keeps it at time t

Figure A-1
"Switch" / "Keep" Decision Tree



Note: Figures in parentheses are holding period.

when the interest rate state is x.

Finally, comparing $A_t(x \mid i, h)$ with $B_t(x \mid i, h)$, a depositor decides whether to switch his deposit or keep it at time t. In other words, he derives his response function as follows:

$$A_t(x \mid i, h) \geqq B_t(x \mid i, h) \Longrightarrow \text{switch} \Longleftrightarrow f_t(x \mid i, h) = x$$

$$A_t(x \mid i, h) < B_t(x \mid i, h) \Longrightarrow \text{keep} \quad \Longleftrightarrow f_t(x \mid i, h) = i.$$

Therefore, we have

$$\phi_t(x \mid i, h) = \begin{cases} A_t(x \mid i, h) \text{ for } x \in \{x : f_t(x \mid i, h) = x\} \\ B_t(x \mid i, h) \text{ for } x \in \{x : f_t(x \mid i, h) = i\}. \end{cases}$$

As A_t and B_t are both a function of ϕ_{t+1}, ϕ_t becomes a function of ϕ_{t+1}. Both A_t and B_t are a function of A_{t+1} and B_{t+1}, and likewise f_t is a function of f_{t+1}. Therefore, if we know the vector of deposit return functions $(\phi_{t+1}, A_{t+1}, B_{t+1})$ and the response function (f_{t+1}) at time $t+1$, we can derive the vector of deposit return functions (ϕ_t, A_t, B_t) and the response function (f_t) at time t.

4. Deposit Return during the Total Holding Period and Real Value of Postal Savings Certificates

The above arguments imply that we can obtain deposit return function $\phi_0(i)$ during the total holding period (from period 1 to T) by solving backward from time $T-1$.

As $\phi_0(i)$ represents the maximum return after time 0 for a depositor who has held a deposit in interest rate structure i for one period at time 1, we can write it as follows:

$$\phi_0(i) = \Sigma_y \phi_1(y \mid i, 1) P(y \mid i).$$

On the other hand, as $A_0(i \mid 1, 1)$ represents the maximum return after time -1 for a depositor who has held a deposit in interest rate structure 1 for one period and switches at time 0 when the interest rate state is i, we can write it as follows:

$$A_0(i \mid 1, 1) = (1+r_{1,1}) \Sigma_y \phi_1(y \mid i, 1) P(y \mid i).$$

Therefore, we obtain the following relationship:

$$\phi_0(i) = A_0(i \mid 1, 1) / (1+r_{1,1}).$$

From this, we can calculate the real value of PSCs as follows:

$$\text{The real value of PSC} = [\{\phi_0(i)\}^{1/T} - 1] \cdot 2.$$

5. The Case of the Holding Period Beyond M

In principle, the above arguments apply to all the cases irrespective of the length of the holding period. Nevertheless, if the holding period is longer than M, there is a special characteristic that helps make calculation easier. Let us discuss this useful characteristic.

The derivation of $A_t(x \mid i, h)$ is the same as above. On the other hand, the derivation of $B_t(x \mid i, h)$ becomes very simple because of the following characteristic: The length of the holding period is important in deciding whether to switch or to keep the deposit because the interest rate that applies to the deposit increases with the length of the period. However, if the holding period is longer than M, the relevant interest rate does not increase any more by definition. Therefore, in this case, we do not have to consider the length of the holding period in deriving the response function. In other words, we have the following response function:

$$f_t(x \mid i, M) = f_t(x \mid i, M+k) \quad k \geqq 0.$$

Moreover, even if the holding period becomes longer, the same interest rate applies. Thus, we have the following:

$$\begin{aligned} \phi_t(x \mid i, M+k) &= \phi_t(x \mid i, M)(1+r_{i,M})^k \quad k \geqq 0 \\ A_t(x \mid i, M+k) &= A_t(x \mid i, M)(1+r_{i,M})^k \quad k \geqq 0 \\ B_t(x \mid i, M+k) &= B_t(x \mid i, M)(1+r_{i,M})^k \quad k \geqq 0. \end{aligned}$$

In particular, if $k=1$, then we have the following:

$$\begin{aligned} f_t(x \mid i, M+1) &= f_t(x \mid i, M) \\ \phi_t(x \mid i, M+1) &= \phi_t(x \mid i, M)(1+r_{i,M}) \\ A_t(x \mid i, M+1) &= A_t(x \mid i, M)(1+r_{i,M}) \\ B_t(x \mid i, M+1) &= B_t(x \mid i, M)(1+r_{i,M}). \end{aligned}$$

Substituting these into the formula for $B_t(x \mid i, M)$, we obtain the following equation:

$$\begin{aligned} B_t(x \mid i, M) &= \Sigma_y \phi_{t+1}(y \mid i, M+1) P(y \mid x) \\ &= (1+r_{i,M}) \Sigma_y \phi_{t+1}(y \mid i, M) P(y \mid x) \\ &= (1+r_{i,M}) \{\Sigma_{S3} A_{t+1}(y \mid i, M) P(y \mid x) + \Sigma_{S4} B_{t+1}(y \mid i, M) P(y \mid x)\} \end{aligned}$$
$$S3 = \{y : f_{t+1}(y \mid i, M) = y\}, \quad S4 = \{y : f_{t+1}(y \mid i, M) = i\}.$$

This equation is very useful in our calculation. As discussed in Section III, to derive the deposit return functions and the response function for h holding periods, we need these functions for $h+1$ holding periods. To derive these functions for $h+1$ holding periods, we need these functions for $h+2$ holding periods, and so on. However, if we use the above relationship for M holding periods and longer, we only need to obtain at most M sets of the deposit return functions and the response functions.

Appendix B. Calculation of the Real Value of Maturity-Designated Time Deposits (A General Case)

Once the calculation scheme is constructed for the real value of PSCs, it is easy to derive the corresponding scheme for the real value of MTDs. A necessary modification is that a depositor has no choice but to switch to a new interest rate structure after the maturity date: that is,

$$f_t(x \mid i, M) = x \quad \text{for any } i \text{ and } x.$$

Note that the same modification will be necessary if we consider the case in which the holding period of PSCs is more than ten years.

In principle, this is the only modification we need for calculating the real value of MTDs. However, because they use an annual (instead of a semiannual) compound interest rate for MTDs, we need to make some adjustments. For example, if a deposit is held for one year and six months, the interest will be compounded only once. Therefore, to make it comparable to that of PSCs, we need to convert the original annual interest rate (r) into an equivalent semiannual compound interest rate (r^*) as follows:

$$(1+r^*/2)^3 = (1+r)(1+r/2)$$

Table 7 shows the adjusted interest rate structure (r^*) for the nine interest rate states.

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