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The Optimal Currency Composition of Government Debt

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Against the background of the increasing issues of government bonds in the United States and European countries, debt management policy has recently been receiving increasing attention. This paper questions the current practice of the United States, the largest debtor nation in the world, of continuing to issue government bonds denominated in its own currency. Based on the assumption that goods prices are sticky, we derive a formula for the optimal currency composition of government debt that maximizes the welfare of the U.S. private sector. The optimal currency composition computed under realistic parameter values suggests that one way to increase U.S. economic welfare is to continue issuing almost 100% of short-term bonds in the U.S. dollar and at the same time to reduce the share of the U.S. dollar in long-term bonds.

I. Introduction

From the viewpoint of the national economy, what is the optimal composition of government debt? Does varying maturities of government debt affect the economy differently? If so, what is the optimal maturity structure? Do nominal bonds (with repayment specified in money) and real bonds (with repayment specified in goods) have different effects on the economy? Is there any reason to prefer domestic-currency denominated bonds to foreign-currency denominated bonds, or vice versa?

These questions, which are related to the management of government bonds, were a focus of much attention in the 1960s, notably in the work of Tobin (1963). In recent years, the questions have been receiving increasing attention again, against the background of the increasing issues of government bonds in the United States and European countries. In particular, academic research in this area over the past several years has been active based on the broad perspective of both economics and political science.[1] The

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[1]Studies analyzing government debt from the viewpoint of political science include Alesina and Tabellini (1990), Cukierman and Meltzer (1989), Persson and Svensson (1989), and Aghion and Bolton (1990). Although our analysis is closely related to these studies, this paper will not deal with political elements such as voting or election behavior.

first purpose of this paper is to review the major theoretical issues of this literature in a unified theoretical model.

Bohn (1991) and others have questioned the current practice of the United States, the largest debtor nation in the world, of continuing to issue government bonds denominated in its own currency. They argue that dollar denominated bonds give the U.S. government an incentive to create surprise inflation, thereby deteriorating the welfare of the U.S. private sector. The second purpose of the paper is to theoretically examine this argument by computing the optimal currency composition of government debt that maximizes the welfare of the economy as a whole.

The paper is organized as follows. Section II introduces to the recent literature on the issues and management of government debt. Section III extends the existing models by making an additional assumption that prices are sticky. We derives a formula for the optimal currency composition of government debt that maximizes the welfare of the private sector. We then compute the optimal currency composition in the case of the United States under realistic parameter values. After a few concluding remarks, Section IV discusses issues for future research.

The major results of the paper can be summarized as follows:

(A) Under the assumption that economic welfare deteriorates with an increase in tax rates (*i.e.*, the presence of an excess burden of taxation), the optimal tax plan that minimizes the present discounted value of the excess tax burden can be characterized as a policy that avoids concentrating taxation in one particular period. For example, when a temporary increase in government expenditure is expected because of the outbreak of war, the optimal strategy of financing the military expenditures is issuing public bonds during the war and levying a uniform tax from the present into the future, corresponding to an increase in the present discounted value of government expenditure. In other words, minimizing the excess tax burden requires the government to spread the burden of financing the military expenditures over a lengthy period including peacetime. This type of behavior, which is called "tax smoothing," has empirically been found to constitute an important part of the government's behavioral principles;

(B) If we define the "government" as a consolidated entity of the fiscal and monetary authorities, the principle of tax smoothing applies to the choice of an inflation tax rate. This approach, which is called public finance approach to inflation, emphasizes the possibility that economic welfare is not maximized in equilibrium when the government conducts policy in a discretionary manner (*i.e.* the problem of time inconsistency) because of the following reason. The government with domestic currecy debt has an incentive to generate surprise inflation (inflation greater than the private sector had expected) to reduce the real value of debt. However, private agents who form rational expectations of government behavior know beforehand that the government has this incentive. Accordingly, they rationally expect a higher inflation rate, which then induces a higher nominal interest rate. As a result, the government fails to reduce the real value of debt. Reflecting a higher inflation rate, the resulting equilibrium becomes suboptimal.

(C) To avoid the deterioration of economic welfare, which is associated with issues of domestic currency bonds, it has been suggested by some researchers that the government should issue real bonds instead of domestic currency bonds including money. However, the first-best equilibrium can also be obtained by controlling the composition of government debt when it is made up of both domestic currency and foreign currency bonds. Under the realistic assumption that goods prices adjust only gradually and purchasing power parity does not hold in the short run, domestic currency bonds give a future government an incentive to create an unexpected inflation, while foreign currency bonds give an incentive to create an unexpected deflation. These two conflicting incentives can be cancelled out by controlling the currency composition of government debt. As a result, the equilibrium inflation rate is reduced and thus economic welfare improves. As the speed of price adjustment declines and the maturity of government debt increases, the deflationary incentive of foreign currency bonds is weakened, while the inflationary incentive of domestic currency bonds is strengthened. In this case, therefore, it is consequently desirable to reduce the share of domestic currency bonds. Examining the current U.S. situation using numerical examples based on realistic parameter values, we can conclude that one way to increase economic welfare is to continue issuing almost 100% of Treasury bills in the U.S. dollar and at the same time to reduce the share of the U.S. dollar in long-term bonds.

II. Literature on the Issues and Management of Government Debt

A. The Ricardian World

As a starting point, consider an economy in which the Ricardian equivalence theorem holds.[2] More specifically, we assume that (1) the private sector is represented by an infinitely-lived agent; (2) there is no uncertainty about future events;[3] (3) a lump-sum tax is available; and (4) the stream of government expenditures from the present to the future is exogenously given.

In this enviroment, a tax reduction accompanied by an issue of government debt increases the current disposable income of the private sector. However, the net wealth of the private sector, including the present discounted value of future disposable income, is unchanged because the repayment of that debt eventually leads to a tax increase. This means that the intertemporal budget constraint of the private sector is independent of the government's debt or tax policy. As a result, the private sector's consumption and

[2]See Barro (1974, 1989) for the Ricardian equivalence theorem and its underlying assumptions.

[3]Even if there is uncertainty about the future, the arguments of the paper will be unchanged as long as capital markets are complete.

investment decisions, as well as prices and interest rates which they determine, are also independent of government policies.

An important thing to note here is that there is no reason to prefer a particular level of government debt because government debt has no effect on the equilibrium of the economy. This implies that the optimal level of government debt is indeterminate. Moreover, the government's debt management policy does not have any effect on the equilibrium of the economy, either. For example, suppose the government replaces short-term bonds with long-term bonds. While this swap operation leads to a tax-reduction in the short run and a tax-increase in the longer run, the net wealth of the private sector is unchanged in terms of present discounted value, and the intertemporal budget constraint the private sector faces does not shift at all. Thus, a change in the maturity composition of government debt has no effect on the equilibrium of the economy. Likewise, replacing nominal bonds (bonds with repayment specified in money) with real bonds (bonds with repayment specified in goods), which may alter the real value of redemption in the future depending on the prevailing inflation rate, has no effect on the private sector's intertemporal budget constraint since the change in the real value of redemption is fully offset by a corresponding change in tax.

Although the framework of simple Ricardian world is a good place to start the analysis, these so-called irrelevance propositions[4] depend on various strong assumptions, some of which are too strong to deal with issues regarding government debt policy. In what follows, we will relax some of these assumptions to make the analysis more realistic.

B. Determining the Level of Government Debt

In this respect, Barro (1979) questions the availability of lump-sum tax. To trace his arguments, consider a case where taxation imposes an excess burden to the economy, while leaving intact other assumptions in the Ricardian world.[5]

Denote the excess burden measured in terms of goods by $u(x)$ corresponding to the tax amount x, and assume that $u(\cdot)$ is a convex function in x. Then, the amount of goods allocated to the private sector equals to outputs minus government expenditures minus $u(x)$, where outputs as well as government expenditures are given exogenously. The objective of the government is to maximize this value. That is, we assume a "benevolent government" in the sense of Fischer (1980), *i.e.*, in the sense of maximizing the private sector's economic welfare.

More specifically, the problem the government faces is to minimize the present discounted value of the excess burden:

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[4]Many studies deal with irrelevance propositions, including Wallance (1981) and Sargent and Wallance (1988).

[5]Of course, the other assumptions can be relaxed as well. For example, when we assume that the private economic agent has a finite life (as in an overlapping generations model), debt and tax may no longer be equivalent because a change in the timing of tax can generate an intergenerational redistribution of income.

$$\sum_{t=0}^{T} \beta^t u(x_t) \qquad \text{where } \beta \epsilon [0, 1] \text{ is a discount factor}$$

with respect to $\{x_0, \cdots, x_T\}$, subject to the intertemporal budget constraint:

$$\sum_{t=0}^{T} g_t (1+r)^{-t} = \sum_{t=0}^{T} x_t (1+r)^{-t}$$

where r is the real interest rate (exogenously given) which satisfy $\beta=(1+r)^{-1}$, and government expenditures $\{g_0, \cdots, g_T\}$ are assumed to be given exogenously and satisfy $g_0, \cdots, g_T>0$ (assumption (4) in the Ricardian world).

The solution to this problem may depend on the timing to solve it. In this respect, we assume for the moment that the government solves this problem at the beginning of period 0 and makes a precommitment to the private sector, agreeing that it will adhere to that optimal taxation plan in the future. Later we will discuss situations where a precommitment is impossible.

The first-order condition for minimization[6] is given by:

$$u'(x_0) = u'(x_1) = \cdots = u'(x_T).$$

Thus, the tax amount that minimizes the present discounted value of excess burden is given by:

$$x_0 = x_1 = \cdots = x_T.$$

This simply says that tax smoothing across time is desirable under the presence of excess burden of taxation. Since the same type of smoothing is also observed when a consumer makes a decision on a future consumption plan in order to maximize a concave utility function,[7] this result itself is not surprising. From the point of view of determinating the level of government debt, however, this has an important implication: it is possible to calculate the amount of debt issues and redemptions in each period by taking the difference between the amount of government expenditures and the amount of tax revenue. Therefore, under the assumption that there exists an excess tax burden, the level of government debt in each period is determined by specifying the level of government debt in some arbitrary period.

As an example, consider a situation where government expenditures are expected to increase temporalily because of war. According to the above argument, the best strategy for financing the war-related expenditures is to precommit itself to issue bonds during the war and to increase tax rates uniformly from the present into the future, corresponding to an increase in the present discounted value of government expenditures. In other words, the government should not increase tax revenue only during the war, but should spread

[6]Because $u(\cdot)$ is a convex function, the second order condition for minimization is also satisfied.

[7]Under the assumption that the utility function in each period is a concave function in current consumption, maximization of the present discounted value of all future utility leads to consumption smoothing over time.

tax burden over time, including peacetime. To apply the same principle, we can justify the issue of government debt to finance an increase in government expenditures during recession.[8]

To the extent that, regardless of time and place, we observe that governments increase the debt issues during war and recession, tax smoothing seems to be a principle governing the actual behavior of the fiscal authorities. In fact, Barro (1979, 1986, 1987a, 1987b) has broadly confirmed the tax smoothing behavior of the fiscal authorities on the basis of data from the United States and the United Kingdom.[9]

C. Time Inconsistency

Although Barro (1979) is reasonably successful in explaining the government behavior in determining the level of government debt, his analysis has the following problems. First, although introducing the excess burden of taxation into the model is useful in discussing the level of government debt, it does not help determine the composition of government debt. Second, if the government determines tax rates in a discretionary manner, there is no guarantee that the tax plan precomitted in time 0 will be implemented. In other words, the optimal tax plan of period 0 may be time-inconsistent. This possibility is not a serious problem if, as we assumed in Section II, B, the precommitment made in period 0 effectively binds the government from the first period on. However, it is hard to imagine that the actual process of policy-making reflects such a binding precommitment. To be realistic, therefore, it is more appropriate to assume that the government will revise its optimal tax plan in each period. In this case, the problem of time inconsistency can significantly affect the equilibrium and, consequently, the welfare of the economy.

These two problems are, in fact, closely related. Before we discuss this point, however, let us first apply the argument of Section II, B, to the inflation tax to illustrate the mechanism that generates time inconsistency in an optimal tax plan.[10]

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[8]This argument is contrary to the balanced budget principle and is closer to the Keynesian view of the 1960s, in the sense that the amount of government expenditures need not equal the amount of government revenue in each period. It should be noted, however, that in obtaining the government's intertemporal budget constraint, our analysis assumes that government debt has been fully repaid in the final period (T). In other words, we are assuming that the government expenditures equal government revenue in the long run, thus ruling out the possibility of budgetary failure which the balanced budget principle cautions us against. That is, we are simply advocating the usefulness of short-run budgetary imbalances as long as the long-term budgetary balance is assured. When a short-term budgetary imbalance is likely to lead to long-term budgetary failure, the argument for tax smoothing may be misleading.

[9]Fukuda, Teruyama, and Asako, *et al.* (1992) tests the hypothesis of tax smoothing in Japan and concludes that the hypothesis cannot be rejected either in the pre-war or in the post-war period.

[10]Beginning with Friedman (1971) and Phelps (1973), many studies have investigated seignorage. See Kanaya (1991) for the survey of the recent literatute.

1. Optimal inflation plan: the precommitment solution

Suppose inflation tax is the only feasible means of financing government expenditures,[11] with the government defined as the consolidated entity of the fiscal and monetary authorities.[12] Denote the balance of government debt at the end of period t by D_t, the inflation rate in period t by π_t, real money demand in period t by μ_{t-1}. Then, the government's flow budget constraint becomes,[13]

$$D_t = (1 + r)D_{t-1} + g_t - \pi_t \mu_{t-1} \tag{1}$$

where the third term on the right side, $(\pi_t \mu_{t-1})$, refers to the revenue from the inflation tax.

We assume that the private sector forms an expectation of the inflation rate during period t, π_t^e, based on information available in period $t-1$, and that it determines real money demand, μ_{t-1}, so that we can express the relationship between money demand and the expectation of the inflation rate as $\mu_{t-1} = \mu(\pi_t^e)$, $\mu'<0$. We further assume that government debt is all issued as real bonds and that the maturity is one year (we will consider issues of other types of bonds in Section III).

Under the assumption that government debt is fully repaid in the final period T,[14] we have the following intertemporal budget constraint:

$$\sum_{t=0}^{T} g_t (1+r)^{-t} = \sum_{t=0}^{T} \pi_t \mu(\pi_t^e)(1+r)^{-t}.$$

We assumed in Section II, B that taxation imposes an excess burden. Likewise, we assume here that the inflation tax reduces the private sector's economic welfare in two senses.[15] First, expected inflation creates a wedge between the actual amount of real money demand and the optimal money supply in the sense of Friedman ($\mu(-r)$ in our model),[16] thus reducing the welfare of the private sector. Second, whether or not it is

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[11]To simplify our analysis, we are ruling out other forms of taxation. When we discuss the currency composition of government debt in the next section, however, we will assume that the government can use inflation tax in addition to conventional tax.

[12]To justify this definition of the government, the fiscal and monetary authorities must coordinate their policies. If they do not, the arguments of this section may change significantly. Alesina and Tabellini (1987) discuss a situation where the relationship between the fiscal and monetary authorities is represented by a non-cooperative game.

[13]This flow budget constraint is an approximation which holds only when the length of each period is infinitesimal (so that we can think of time as passing continuously). To be more precise, the third term on the right hand side is $-[\pi_t \mu_{t-1} + (\mu_t - \mu_{t-1})]$, and the tax revenue from inflation is made up of two elements, namely, (1) the capital loss on the existing money balance arising from inflation (the first term in [ ]) and (2) the amount of new money absorbed by the private sector during period t (the second term in [ ]).

[14]That is, $D_t(1+r)^{-T}=0$ (*i.e.*, the transversality condition).

[15]See Driffill, Mizon, and Ulph (1990) for a detailed analysis of the cost of inflation.

[16]This is real money supply which equals marginal revenue from money creation (equal to the nominal interest rate) with its marginal cost (which is zero or negligible). Because, according to the Fisher equation, the nominal interest rate equals the real rate plus the inflation rate, the inflation rate which leads to a nominal interest rate of zero must be $-r$. Thus, the optimal supply of money is given by $\mu(-r)$.

anticipated, inflation reduces the efficiency of resource allocation in the economy. Let $v(\pi_t^e, \pi_t)$ denote the welfare loss caused by these two elements, measured in terms of goods.

Under this setting, consider a situation in which the government precommits itself to the optimal inflation plan $\{\pi_0, \cdots, \pi_T\}$ formed in period 0 by minimizing the present discounted value of the excess burdens:

$$\sum_{t=0}^{T} \beta^t v(\pi_t^e, \pi_t)$$

subject to the intertemporal budget constraint. Assuming that the expected inflation rate in the previous period, π_0^e, is given exogenously and that the inflation rate in each subsequent period is anticipated perfectly ($\pi_t^e = \pi_t$), we can obtain the following first order conditions for minimization with respect to $\pi_0, \cdots, \pi_T$:

$$v_2(\pi_0) = \lambda_0 \mu(\pi_0^e) \tag{2a}$$

$$v_1(\pi_t) + v_2(\pi_t) = \lambda_0 [\mu(\pi_t) + \pi_t \mu'(\pi_t)] \qquad t=1, \cdots, T \tag{2b}$$

where v_i (i=1, 2) is a partial derivative of $v(\cdot, \cdot)$ with respect to the i th element and λ_0 is a Lagrangean multiplier associated with the intertemporal budget constraint. The difference between equation (2a) and (2b) reflects the fact that money demand in period 0 is already determined by the expected inflation rate in the previous period. From equation (2b), we see that the inflation rate in each subsequent period is smoothed ($\pi_1 = \cdots = \pi_T$). By letting $\pi^P>0$ denote the smoothed inflation rate after the first period, what we call the precommitment solution refers to the inflation plan $\{\pi_0, \pi^P, \cdots, \pi^P\}$ which satisfies equations (2a) and (2b) as well as the intertemporal budget constraint.

2. *The problem of time inconsistency*

It is not necessarily realistic to assume that this type of precommitment to an inflation plan is feasible. That is, it may not be appropriate to think that the government once and for all forms an inflation plan in some period and simply implements that plan in all subsequent periods. Rather, actual monetary policy is conducted in a discretionary manner; *i.e.*, the government recalculates the optimal inflation rate in each period. In fact, Calvo and Leiderman (1992), comparing the theoretical inflation rate obtained under precommitment with the actual inflation rate in Argentina, Brazil and Israel, have found that the actual rate exceeded the theoretical rate in many instances.[17] They have interpreted this result as an indication of the importance of discretion in monetary policy.[18] Taking this fact into consideration, we will base the following discussion on the

[17]We will explain below why the actual inflation rate exceeds the precommitment rate under discretionary monetary policy.

[18]To empirically test the precommitment model, we need only to check whether or not the actual data support the following propositions: (1) the inflation rate tax (*i.e.*, the inflation rate) is positively correlated with

premise that the monetary policy is conducted in a discretionary manner.

An important issue in a discretionary policy is the lack of any guarantee that the optimal inflation plan formulated in period 0 will be executed in subsequent periods. To delve into this point further, let us see if the government has an incentive to maintain $\pi_s = \pi^P$ in period s ($s \geqq 1$).[19] When $\pi_s^e = \pi^P$ is the expected inflation rate of the private sector, the government's problem in period s is to minimize the present discounted value of inflation cost with respect to $\{\pi_s, \cdots, \pi_T\}$, subject to the following intertemporal budget constraint:

$$D_{s-1} + \sum_{t=s}^{T} g_t (1+r)^{-(t-s)} = \pi_s \mu(\pi^P) + \sum_{t=s+1}^{T} \pi_t \mu(\pi_t)(1+r)^{-(t-s)}.$$

We treat $\pi_s \mu_{s-1}$ ($= \pi_s \mu(\pi^P)$) on the right side of the budget constraint separately, because μ_{s-1} is already determined when the government determines π_s in period s. Then, the first order condition with respect to π_s becomes:

$$v_2(\pi_s) = \lambda_s \mu(\pi^P) \tag{3a}$$

and the first order conditions with respect to $\pi_{s+1}, \cdots, \pi_T$ are:

$$v_1(\pi_t) + v_2(\pi_t) = \lambda_s [\mu(\pi_t) + \pi_t \mu'(\pi_t)] \qquad t=s+1, \cdots, T \tag{3b}$$

where λ_s is a Lagrangean multiplier associated with the intertemporal budget constraint. By eliminating λ_s from both equations, we obtain the following necessary conditions:

$$[v_1(\pi_t)+v_2(\pi_t)]/[\mu(\pi_t)+\pi_t \mu'(\pi_t)] = v_2(\pi_s)/\mu(\pi^P) \qquad t=s+1, \cdots, T.$$

Substituting $\pi_t=\pi^P$ into the left side of this equation, we have $[v_1(\pi^P)+v_2(\pi^P)]/[\mu(\pi^P)+\pi^P \mu'(\pi^P)]$. On the other hand, if we substitute it into the right side of the equation, we have $v_2(\pi^P)/\mu(\pi^P)$. Consequently, it is clear that $\{\pi_s, \cdots, \pi_T\} = \{\pi^P, \cdots, \pi^P\}$ does not satisfy the necessary conditions except when $v_1 = \mu'(\cdot) = 0$.[20] Thus, the government has an incentive in period s to deviate from the inflation plan

the rate of other taxes (or they are cointegrated if they are non-stationary); and (2) the inflation rate follows a random walk. Mankiw (1987), which was the first empirical study of this type, tested both propositions using the U.S. data from 1952 to 1985 to find that the precommitment model is supported. Subsequent studies, however, have been less persuasive. For example, Poterba and Rotemberg (1990) tested prposition (1) against data from five countries (the United State, Germany, France, Japan and the United Kingdom) following the 1950s, and found that the proposition was rejected in Germany, France and the United Kingdom. Grilli (1988) tested proposition (2) against data from 10 European countries and found that the proposition was rejected in half the countries. Fukuda, Teruyama, and Asako, *et al.* (1992) tested the model against recent Japanese data and found evidence against it.

[19]To be more precise, it is necessary to make sure that $\pi_s=\pi_s^e=\pi^p$ is a Nash equilibrium between the government and the private sector. That is, we must examine (1) if the optimal strategy of the private sector is $\pi_s^e=\pi^p$ when the strategy of the government is givey by $\pi_s=\pi^p$; and (2) if the optimal strategy of the government is $\pi_s=\pi^p$ when the strategy of the private sector is given by $\pi_s=\pi^p$. Condition (1) is the rational expectations hypothesis itself and is assumed in this paper. In the text, we examine only condition (2).

[20]If we consider the intertemporal budget constraint in addition to the first order conditions, we can show that $\pi_s > \pi^p > \pi_{s+1} = \pi_{s+2} = \cdots = \pi_T$.

formed in period 0. In other words, the plan formed in period 0 is time inconsistent.[21, 22]

3. The process of generating time inconsistency

We must keep in mind the following points concerning the process of generating time inconsistency. First, the cost of the inflation tax is crucial to the creation of this problem. In the Ricardian world where taxation incurs no cost, all the inflation plans are equivalent as long as they satisfy the intertemporal budget constraint. Then, there is no reason for the government to deviate from the original plan. Second, we assume a "benevolent government" which seeks to maximize the private sector's economic welfare. Therefore, the problem of time inconsistency generated here is completely different from the phenomenon of a self-interested government trying to deceive the private sector. Third, time inconsistency is generated although we assume that the government has perfect information. Thus, inconsistency is unrelated to the uncertainty about the future or the associated forecast errors.

Why then does the government have an incentive to deviate from the original plan in period s? By comparing equation (2b) and equation (3a), we find two differences. First, the precommitment solution considers the effect of an increase in inflation to reduce real money demand (the second term of [ ] in equation (2b)) in calculating the marginal tax revenue, while equation (3a) ignores that effect. Second, the precommitment solution considers the cost associated with the effect of expected inflation to reduce real money demand (the first term of the left side in equation (2b)), while equation (3a) ignores it.

These differences can be understood as follows. From the viewpoint of the government in period 0, real money demand in period s is determined endogenously by the inflation rate in period s with perfect foresight. In contrast, from the viewpoint of the government in period s, the expected inflation rate in period s, π_s^e, has already been formed in period $s-1$. Thus, real money demand in period s, which is based on that expected inflation rate, is also predetermined. Therefore, the government pursuing discretionary policy ignores the change in real money demand associated with inflation during period s. The result is that, because the government has overestimated the marginal revenue from the inflation tax in period s, its chosen inflation rate exceeds the precommitted rate.

This result can also be interpreted in terms of the Ramsey principle, which focuses on determining the optimal tax rate when the government levies taxes on several commodities to secure a fixed amount of revenue. According to the Ramsey principle, the maximization of the sum of consumers' surpluses from different commodities requires

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[21] Auernheimer (1974) and Calvo (1978) have pointed out that this type of time inconsistency would arise with the inflation tax. Moreover, it is well known that a similar problem of inconsistency generally arises with taxation when the demand for goods depends on expected rate of return; capital levy is a typical example.

[22] The problem of time inconsistency was first pointed out by Kydland and Prescott (1977). Romer (1991) shows that this problem is also important empirically.

low tax rates on goods with high price elasticities and high rates on goods with low price elasticities. This follows from the fact that, because a tax-induced reduction in demand will be inconsequential for goods with low price elasticities, the resulting excess burden, as measured by the consumers' surplus, will also be insignificant. Thus, the overall excess tax burden can be minimized by levying a higher tax on these commodities. In the context of our model, if we consider each period's money as a separate product, we can apply the Ramsey principle to the inflation tax. It follows, then, that there should be higher inflation in those periods when the elasticity of money demand is low with respect to the inflation rate. From the viewpoint of the government in period s, the elasticity of money demand with respect to the inflation rate in period s is zero, because real money demand in period s has already been determined in the preceding period. From this, we can explain the desirability of levying a higher inflation tax in period s as compared with the subsequent periods.

As shown above, when monetary policy is conducted in a discretionary manner, the government in period s attempts to deviate from the inflation rate (π^P) which the private sector has anticipated. Thus, the expectation of inflation is not rational and does not satisfy the (Nash) equilibrium conditions. Therefore, when forming expectations in period $s-1$ about π_s, the private sector should incorporate the government's incentive to deviate in period s. Then, π_s must satisfy the following condition:

$$v_2(\pi_s) = \lambda_s \mu(\pi_s) \tag{3a'}$$

instead of equation (3a). When there is a series $\{\pi_0, \cdots, \pi_T\}$ which satisfies equation (3a′) for $s=0, \cdots, T$, it is called the (time-consistent) discretionary solution.[23]

D. The Effectiveness of Debt Management Policy

1. *Welfare comparison of precommitment and discretionary solutions*

From the point of view of the private sector's economic welfare, would precommitment or discretion yield a more desirable solution? The basic difference between the two is that, when minimizing the inflation cost, the solution under precommitment reflects the perfect foresight of future inflation, while the solution under discretion ignores the constraint of perfect foresight. Because perfect foresight has to hold eventually under either solution, however, it should, in general, be more desirable to optimize taking account of the effects of inflation on economic welfare through changes in the expected inflation rate. Thus, the solution under precommitment should be as good as or better than the solution under discretion.

[23]When computing time-consistent solutions, we need to note that the Lagrangean multipier in equation (3a′) depends on the inflation rates from period $s+1$ on. Thus, we must know $\pi_{s+1}, \cdots, \pi_T$ to calculate the inflation rate in period s. As a practical procedure, we first obtain the inflation rate in the final period (π_T), and move backward in time to obtain π_{T-1}, π_{T-2}, and so on (*i.e.*, backward induction method).

2. *The PPS model*

Then, the next question is if there is some means to achieve under discretionary policy the welfare level of precommitment solution. In this context, Persson, Persson and Svensson (1987, 1988), PPS hereafter, have shown that the precommitment solution can be achieved under discretionary policy through a debt management policy. An implication of the PPS's proposition is that the conceptual difficulties of Barro (1979) as discussed in Part C, namely, (1) time inconsistency and (2) the indeterminacy of debt composition, are not mutually independent, but solving one of the problems can lead to the solution of the remaining problem. In the following, we will explain how the precommitment solution can be achieved under discretionary policy by controlling the composition of government debt. We will do this in terms of the model developed in Part C.

Instead of assuming, as we did in Part C, that government debt is all real bonds, we assume that the government can issue either nominal bonds (with repayment specified in monetary units) or real bonds (with repayment specified in goods). Letting N_t represent the balance of nominal bonds, the balance of real bonds is represented by $D_t - N_t$ (where D_t is the total amount of debt as in the previous section). We assume that the maturity of both types is one year. Denoting the nominal interest rate by i_t, we can express the government's flow budget constraint as follows.

$$D_t = (1 + r)\,(D_{t-1} - N_{t-1}) + g_t - \pi_t\,\mu_{t-1} + (1 + i_{t-1} - \pi_t)\,N_{t-1} \tag{4}$$

This differs from equation (1), which is the budget constraint obtained when government debt is all real bonds, in that equation (4) contains a fourth term on the right hand side, which shows that the real value of nominal bond repayment could be reduced by inflation as long as the nominal interest rate is kept constant. Substituting the Fisher equation ($i_{t-1} = r + \pi_t^{\,e}$) into equation (4), we obtain:

$$D_t = (1 + r)D_{t-1} + g_t - \pi_t\,\mu_{t-1} - (\pi_t - \pi_t^{\,e})\,N_{t-1}. \tag{4'}$$

The last term shows that only unanticipated inflation reduces the real value of nominal bond repayment, reflecting the fact that anticipated inflation induces a rise in the nominal interest rate through the Fisher effect, while unanticipated inflation has no impacts on the nominal interest rate.

Following the argument in Part C, consider the problem the government faces in period s when $\pi_s^{\,e} = \pi^P$. Given equation (4′), the first order conditions for the minimization of the inflation cost are given by:

$$v_2\,(\pi_s) = \lambda_s\,[\mu\,(\pi^P) + N_{s-1}] \tag{5a}$$

$$v_1\,(\pi_t) + v_2\,(\pi_t) = \lambda_s\,[\mu\,(\pi_t) + \pi_t\,\mu'\,(\pi_t)] \qquad t = s+1, \cdots, T. \tag{5b}$$

By comparing equation (5a) with equation (3a), we find that although the marginal cost of inflation(*i.e.*, the left hand side) is the same, the marginal revenue of inflation (*i.e.*, the right hand side) is different; that is, there is an additional term of N_{s-1} in equation (5a).

This term indicates that, from the viewpoint of the government in period s, inflation reduces the real value of nominal bonds, because the expected rate of inflation in period s, π_s^e, and the resulting nominal interest rate (i_{s-1}) are both predetermined. In other words, the balance of nominal bonds constitutes a part of the taxation base of inflation tax for the government in period s. On the other hand, since nominal interest rates in periods after $s+1$ are determined endogenously reflecting expected inflation rates, it is impossible to generate unanticipated inflation. This means that the balance of nominal bonds does not constitute a part of the taxation base of inflation tax in periods after $s+1$, as indicated in equation (5b).

Suppose that the inflation plan $\{\pi_s, \cdots, \pi_T\}=\{\pi^P, \cdots, \pi^P\}$ satisfies equations (5a) and (5b). Then, by substituting this solution into equations (5a) and (5b) to eliminate λ_s, we obtain:

$$v_2(\pi^P)/[\mu(\pi^P)+N_{s-1}]=[v_1(\pi^P)+v_2(\pi^P)]/[\mu(\pi^P)+\pi^P\mu'(\pi^P)]. \tag{6}$$

Conversely, suppose N_{s-1} is determined to satisfy equation (6). Then, $\{\pi_s,\cdots, \pi_T\}=\{\pi^P, \cdots, \pi^P\}$ must satisfy equations (5a) and (5b). Thus, by showing that this solution satisfies the intertemporal budget constraint obtained from (4′):

$$D_{s-1}+\sum_{t=s}^{T} g_s(1+r)^{-(t-s)}=\pi_s\mu(\pi^P)+\sum_{t=s+1}^{T}\pi_t\mu(\pi_t)(1+r)^{-(t-s)}+(\pi_s-\pi^P)N_{s-1}$$

we can show that, in period s, the government has the incentive to carry out the inflation plan formulated in period 0. In what follows, we will show that the above intertemporal budget constraint is identical to the intertemporal budget constraint under the precommitment solution.

By substituting $\{\pi_s,\cdots, \pi_T\}=\{\pi^P, \cdots, \pi^P\}$ into the right hand side of the intertemporal budget constraint, we have:

$$\sum_{t=s}^{T}\pi^P\mu(\pi^P)(1+r)^{-(t-s)}$$

which equals the revenue from inflation tax in periods after s under the precommitment solution. As for the expenditure side (the left hand side of the intertemporal budget constraint), because we assume government expenditure is constant, all we have to show is that D_{s-1} is the same as the balance of debt at the end of period $s-1$ under the precommitment solution. First, it is clear that D_0 is equal to the balance of debt at the end of period 0 under the precommitment solution. From this, we can guarantee that $\pi_1=\pi^P$ holds by setting N_0 according to equation (6), with the result that D_1 is equal to the debt balance at the end of period 1 under the precommitment solution. Making use of this fact, we can achieve $\pi_2=\pi^P$ by setting N_1 again according to equation (6), with the result that D_2 is set equal to the outstanding balance of debt under the precommitment solution. By repeating the same procedure, it can be shown that the rate of inflation in each of subsequent periods is equal to π^P, so that $\{D_0, \cdots, D_T\}$ equals the series of debt balances under the precommitment solution. That is, the solution under discretion is identical to the solution under precommitment.

This result can intuitively be understood as follows. For the convenience of explanation, assume that inflation incurs cost only when it is anticipated ($v \equiv v(\pi^e)$).[24] Then, equation (6) reduces to:[25]

$$\mu(\pi^P) + N_{s-1} = 0 \tag{7}$$

As we saw in explaining equation (5a), the left hand side of equation (7) represents the taxation base of inflation tax from the viewpoint of the government in period s. Thus, equation (7) simply says that the taxation base of inflation tax should be equal to zero in order to achieve the precommitment solution. When this condition is met, the government in period s loses the incentive to create unanticipated inflation, thus preventing the occurrence of time inconsistency. Put differently, negative nominal bond gives an incentive for unanticipated *deflation* to the government in period s. Thus, by issuing negative nominal bond, the incentive for unanticipated *inflation* associated with the real money balance can be fully neutralized. This is the basic idea of the PPS model.[26, 27]

III. The Currency Composition of Government Debt

The purpose of this section is to discuss the optimal currency composition of government debt along the line of the discussion of PPS in the previous section. There has been increasing interest in the currency composition of government debt as a policy issue. For example, it has been argued since the late 1980s that the United States, as the world's largest debtor country, should change the way of money from overseas or that the U.S. government should borrow in yen rather than in dollars.[28]

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[24]The assumption that the cost of unanticipated inflation is zero not only is unrealistic but also contains a theoretically serious problem pointed out by Calvo and Obstfeld (1990). It should thus be emphasized that the argument of this subsection is only a convenient way to obtain an intuitive understanding of equation (6).

[25]When $v \equiv v(\pi^e)$, equation (5a) becomes $0=\lambda_s[\mu(\pi^P)+N_{s-1}]$. Because the Lagrangean multiplier λ_s associated with the cost of funds is positive, the bracketed term [ ] must be zero. We obtain equation (7) from this fact.

[26]At first sight, the condition that the government owes net assets to the private sector in the form of nominal bonds is unrealistic. One interpretation is that the government obtains nominal assets to the private sector when the private sector fails to pay taxes on time. See Calvo and Leiderman (1992) for details.

[27]The importance of debt composition was first pointed out by Lucas and Stokey (1983) as a way to prevent the decline of the private sector's economic welfare associated with time inconsistency. Unlike the PPS model discussed in this paper, the analysis of Lucas and Stokey deals with a barter economy, where they emphasize the importance of properly managing the term structure of government debt. Other studies along the lines of Lucas and Stokey as well as Persson, Persson and Svensson include Bohn (1990, 1991), Calvo and Guidotti (1989, 1990b), and Lucas (1986). Calvo and Guidotti (1990a) and Calvo, Guidotti and Leiderman (1991) confirm that the implications of the theoretical models are consistent with actual data.

[28]For example, see Takashi Hosomi, "Why International Cooperation Does Not Work?" Tokyo Business Today, April 1988.

However, there are the following problem in applying the PPS model to this issue. The PPS model emphasizes the contrast between the two types of bonds: nominal bonds offer the government in the future an incentive to create unanticipated inflation, while real bonds do not give such an incentive. If we follow this line of argument, we cannot distinguish between foreign currency bonds and real bonds because foreign currency bonds, whose real value (and real interest rate) is independent of domestic inflation rates under purchasing power parity, do not create an incentive for unanticipated inflation.[29] This means that there is a possibility that the PPS model might fail to capturte basic characteristics of foreign currency bonds.

One way to overcome this problem is to introduce a stochastic element into the PPS model. This was done in Bohn (1990) by assuming that the government expenditure is a random variable. Bohn has shown that these two types of bonds are distinguishable when they are correlated differently with the stochastic government expenditure.

In this paper we will adopt another approach to distinguish two types of bonds: we will relax the assumption of the PPS model about the flexibility of goods prices.[30] Let us use an example to illustrate the basic argument of this paper, by assuming that the U.S. government issues yen-denominated bonds. In the PPS world, where purchasing power parity holds, the real exchange rate between the yen and the dollar remains constant at a certain level and is beyond the control of the U.S. government. Consequently, issues of yen-denominated bonds will have no effect on the U.S. government's future monetary policy. In contrast, in a situation where U.S. goods' prices are not perfectly flexible and purchasing power parity does not always hold, monetary policy can control the real exchange rate in the short run, if not in the long run. Thus, the U.S. government has an incentive to decrease the real value of the yen by reducing the U.S. money supply, so that it can reduce the real value of yen-denominated debt measured in terms of U.S. goods. This fact shows that foreign currency bonds and real bonds have identical characteristics only in the special case where purchasing power parity always holds. In any other situation, foreign currency bonds differ from real bonds in their incentive effect on the government in the future.

In the following, we assume that the goods prices are sticky and that the government issues either domestic currency bonds or foreign currency bonds to finance fiscal expenditures which are exogenously given. We will analyze the effect of the currency composition of government debt on the future conduct of monetary policy, and derive the optimal

[29]See Calvo (1978) and Bohn (1990) for details.

[30]The relationship between the rigidity of goods prices and time inconsistency associated with the inflation tax is discussed, in a context somewhat different from ours, by Blanchard, Dornbusch, and Buiter (1986) and Calvo (1989). For example, Blanchard, Dornbusch, and Buiter point out that there is no problem of time inconsistency in the first place in the extreme situation where there is no discontinuity in the movement of goods prices and the maturity of nominal bonds is infinitesimal. This follows from the fact that, under those conditions, the real value of nominal bonds cannot be reduced by inflation.

currency composition of government debt which maximizes the private sector's economic welfare.

A. Model

1. *Issues and redemption of government debt*

To delineate the issues and redemption of government debt, we think of the following simple framework. First, the government spends $g(>0)$ in period 0 in real terms (as in Section II, we assume that government expenditures are determined exogenously). To finance this expenditure, the government issues at the end of period 0 two types of bonds—*i.e.*, domestic currency bonds and foreign currency bonds. We assume that the maturity of both types is the same and are given by $T \geqq 1$. One unit of domestic currency bond guarantees the payment of $(1+i_0)^T$ in domestic currency, while one unit of foreign currency bonds guarantees the payment of $(1+i_0^*)^T$ in foreign currency. The share of domestic currency bonds is given by $\theta \in [0, 1]$, while the share of foreign currency bonds is given by $(1-\theta)$. The choice of θ and T is made when the bonds are issued (*i.e.*, period 0), and the subsequent government behavior is determined given the values of these parameters.

To minimize the complications that would result from introducing stickiness of goods prices, we assume that all government bonds are redeemed at maturity (*i.e.*, at the end of period T). That is, our assumption rules out the possibility that bonds are redeemed before maturity or are refinanced by issues of new bonds. Ordinary tax (*e.g.* income tax) is levied on the repayment of government bonds. As in Section II, we assume that the ordinary tax incurs an excess burden, and that the excess burden corresponding to the tax rate of x is expressed by $u(x)$, which is measured in terms of goods.

2. *Devaluation of government debt by monetary policy*

Using these assumptions, let us consider government behavior from debt issue to redemption (*i.e.*, from period 1 until period T). The important point here is that, as far as the government during this period is concerned, (1) the nominal interest rate on domestic currency bonds i_0, (2) the nominal interest rate on foreign currency bonds i_0^*, (3) the currency composition of government bonds θ, and (4) the maturity of government bonds T are all given.

Then, by denoting the logarithm of the domestic goods price by p_t and the logarithm of the relative price of domestic goods to foreign goods (the price of foreign goods/the price of domestic goods) by q_t (which we will hereafter call the real exchange rate), we can write the tax amount (x_T) at maturity as:

$$x_T = [(1+i_0)^T/(1+p_T-p_0)]\theta_g + [(1+i_0^*)^T(1+q_T-q_0)]\,(1-\theta)g \tag{8}$$

where the bracketed expressions in the first and second terms on the right side indicate the real interest rates on domestic currency bonds and foreign currency bonds, respectively. In the context of Section II, equation (8) corresponds to equation (4). The difference between the two is that equation (8) ignores the inflation tax on holding money. This simplification has been made to focus our attention on the inflation tax aspect of nominal bonds.

As equation (8) demonstrates, there are two ways to reduce the excess tax burden in period T. One is to reduce the real value of domestic currency bonds by raising the price of domestic goods between period 0 and period T (the first term on the right side). This channel has already been analyzed in Section II. The other way is to reduce the value of foreign currency bonds measured in terms of domestic goods by lowering the real exchange rate between period 0 and period T (the second term on the right side).

We assume monetary policy is the only policy instrument. If the government seeks to lower the real interest rates of both domestic and foreign currency bonds, it needs to move p and q in opposite directions. However, as long as money supply control is the only policy instrument, it is impossible to control p and q in opposite directions. Thus, there is a trade-off between p and q. To specify relationship between p and q, we assume:

$$\pi_t = \gamma q_t, \quad \text{where} \quad \pi_t \equiv p_t - p_{t-1}, \quad \gamma \geqq 0. \tag{9}$$

Under the assumption that domestic goods and foreign goods are perfect substitutes, the relationship can be interpreted as follows. When $q>0$, domestic goods are relatively cheaper than foreign goods, so that the resulting excess demand for domestic goods increases their price. On the other hand, when $q<0$, domestic goods are relatively more expensive, so the resulting excess supply of domestic goods reduces their price. When $q=0$, purchasing power parity holds so that the prices of both types of goods are identical and the domestic goods market is in equilibrium. The parameter γ in equation (8) represents the speed of adjustment of the domestic goods price to the excess demand or supply of domestic goods.

We can interpret equations (8) and (9) as the constraints on the government when it seeks to reduce the real value of government bonds. Then, the government behavior from period 1 until period T can be specified as maximizing the present discounted value of the cost function defined as $w_0(\pi, q)$, subject to these constraints. There are two respects in which this cost function differs from the objective function $v(\cdot)$ defined in Section II. First, whereas in Section II we assumed that the cost of anticipated inflation differs from that of unanticipated inflation, we assume in this section that the cost of inflation is not affected by whether it is anticipated or not. This simplification will not affect the results of the analysis. Second, we assume that the cost depends on the real exchange rate. This is based on the assumption that there is a desirable level for the real exchange rate, and that economic welfare declines as the actual real exchange rate deviates from that level.

In the analysis below, we substitute equation (9) into $w_0(\cdot, \cdot)$ to express the cost as a function only of q, and use the $w(\cdot)$ function so defined:

$$w_0(\pi, q) = w_0(\gamma q, q) \equiv w(q).$$

As in Section II, we assume that $w(q)$ is a convex function in q and that, without loss of generality,[31]

$$w'(0) = 0. \tag{10}$$

3. The inefficiency of discretionary policy management

The discussion in Section II assumed that goods prices were perfectly flexible. In that case, we saw that the government control of the real value (or the real interest rate) of government debt would be perfectly anticipated and incorporated in the nominal interest rate when the bonds were issued, so that the *ex post* real interest rate would be independent of government policy. The same logic can be applied to the model in this section, where we have taken stickiness of goods prices into consideration.

Suppose the optimal policy $\{q_0, \cdots, q_T; \pi_0, \cdots, \pi_T\}$ is specified, which minimizes the present discounted current value of the cost $w(\cdot)$ from period 0 until period T under the constraints of equations (8) and (9). Because the private sector knows that the government will follow this policy in the future, it must seek to prevent an eventual fall in the real interest rate by adjusting the issuing conditions of government debt in period 0. With domestic currency bonds, this will be realized as an increase in the nominal interest rate corresponding to the expected inflation rate until maturity (the Fisher effect). To explain this in terms of equation (8), an increase in p_T lowers the real interest rate given by the bracketed expression on the right hand side. This, however, can be offset by increasing i_0, so that the bracketed expression remains constant (to use the same expression as Section II, it remains at $(1+r)^T$). On the other hand, to lower the real interest rate, the government with an outstanding balance of foreign currency bonds has an incentive to lower q_T at maturity. In this case, too, it is possible to equate the bracketed expression in the second term of the right hand side of equation (8) to $(1+r)^T$ by raising i_0^*. That is, if the government has an incentive to reduce the real value of the foreign currency at the time of maturity, and the private sector is aware of this incentive, there is a fall in the demand for foreign currency bonds in period 0, so the price of foreign currency bonds falls.

In this way, the *ex post* real interest rate will be independent of government behavior even in the framework of the present model. It is for this reason that many implications of the PPS model can be applied to this model. Of particular importance is that the level of

[31]A sufficient condition for this is that the partial derivatives of $w_0(\cdot, \cdot)$ with respect to π and q are zero when they are evaluated at $\pi=0$ and $q=0$. This condition means that the government's cost is minimized when the inflation rate is zero and purchasing power parity holds.

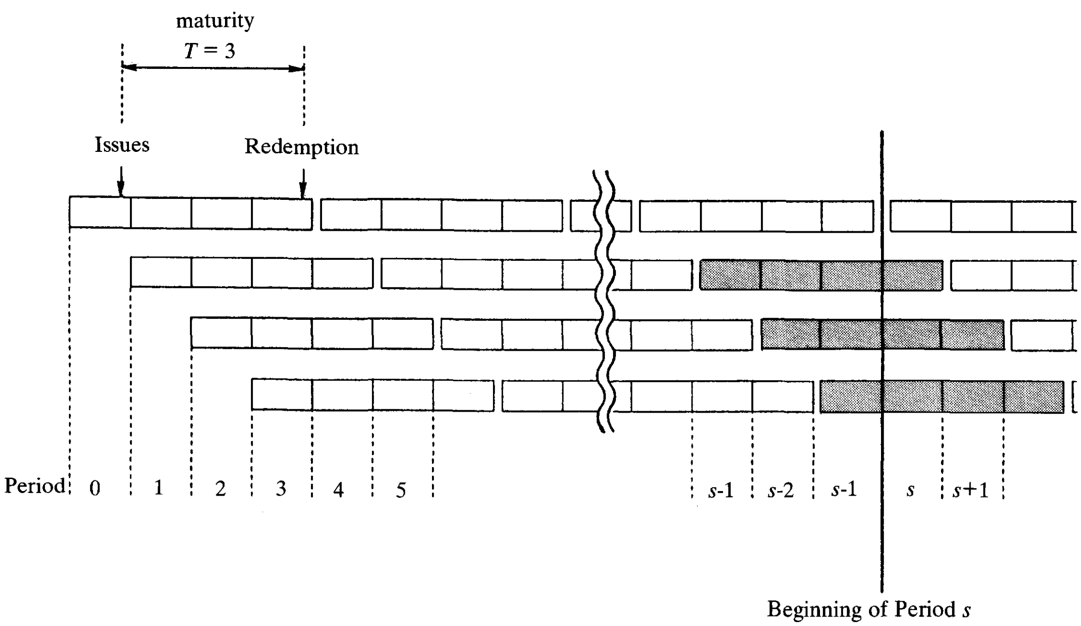
economic welfare under discretionary policy is lower than that of economic welfare under the precommitment solution. In the following, we will specify the precommitment solution as the benchmark, and then derive the condition under which the precommitment solution coincides with the solution under discretionary policy.

4. *Stratified issuance and redemption*

To specify the solutions under precommitment and discretion, we need to extend the pattern of issuing and redeeming government debt described in Section III, A, *1*. We assume here that government spends the amount denoted by g in every period after period 0, and that a corresponding amount of government bonds with the maturity of T is issued in each period.

Figure 1 depicts the extended pattern of issuance and redemption when T=3. Let us use this figure to see the composition of outstanding government bonds by date of issue at the beginning of some period (period s) after sufficient time has elapsed since period 0. As Figure 1 indicates, at the beginning of period s, there are three types of bonds issued in periods $s-3$, $s-2$, and $s-1$ with the remaining maturities of one, two and three periods. It is important to note that the composition of government debt measured by remaining maturity is the same at each point, which reflects the recursive nature of the model. This will turn out to be useful in simplifying the specification of the solutions

Figure 1
Issuing and Redemption Patterns of Government Debt



Note: The shaded areas indicate the outstanding balance of debt at the beginning of period s.

under precommitment and discretion, as explained below.

B. Precommitment Solution

Using these assumptions, consider a case where the government in period 0 sets $\{\pi_0, \cdots, \pi_\infty; q_0, \cdots, q_\infty\}$ and makes a precommitment to it. The objective of the government is to minimize:

$$\sum_{t=0}^{\infty} \beta^t [u(x_t) + w(q_t)]$$

where x_t is equal to zero from period 0 to period $T-1$ ($x_0 = \cdots = x_{T-1} = 0$) because there is no bond redemption.

As the discussion in the previous section noted, the key to the precommitment solution is that the government minimizes cost by taking into consideration the fact that the private sector has perfect foresight of government behavior. As we have seen before, when the private sector has perfect foresight of the government behavior, the real interest rates on both domestic currency and foreign currency bonds are independent of π and q, which are under the control of the government. Thus, the tax amount $\{x_T, \cdots, x_\infty\}$ is also independent of the series of π and q. Then, we can write:

$$x_t = x^p \equiv g(1+r)^T, \qquad t=T, \cdots, +\infty.$$

The problem the government faces, then, is to minimize:

$$\sum_{t=0}^{T-1} \beta^t u(0) + \sum_{t=T}^{\infty} \beta^t u(x^p) + \sum_{t=0}^{\infty} \beta^t w(q_t)$$

with respect to $q_0, \cdots, q_\infty$. It is important to note that θ does not appear in this exercise at all. That is, as in Section II, the precommitment solution is independent of the currency composition of government bonds.

Because the first and second terms in the above expression are constants, the first-order conditions for cost minimization are given by:

$$w'(q_t) = 0, \qquad t=0, \cdots, +\infty.$$

The second-order condition is satisfied by the assumption of convexity. Under the assumption of equation (10), moreover, the first-order conditions mean:

$$q_t = 0 = \pi_t, \qquad t=0, \cdots, +\infty. \tag{11}$$

That is, purchasing power parity always holds under the precommitment solution, and inflation is set at zero. Therefore, the nominal interest rate under the precommitment solution becomes:

$$i_t = r = i_t^*$$

C. Solution under Discretionary Policy

1. *The problem the government faces*

Suppose that the precommitment solutions have been realized from period 0 through period $s-1$. The problem the government pursuing discretionary policy management faces in period s is to minimize:

$$\sum_{t=s}^{\infty} \beta^{t-s}[u(x_t) + w(q_t)] \tag{12}$$

with respect to $q_s, \cdots, q_\infty$, subject to:[32]

$$x_t = x^p-(p_t-p_{s-1})\,\theta x^p + (q_t-q_{s-1})(1-\theta)x^p, \qquad t=s, \cdots, s+T-1 \tag{13a}$$

$$x_t = x^p, \qquad t=s+T, \cdots, +\infty \tag{13b}$$

$$\pi_t \equiv p_t-p_{t-1} = \gamma q_t, \qquad t=s, \cdots, +\infty. \tag{13c}$$

Equations (13a) and (13b) indicate the characteristics of the solution under discretion in the following respects. Since, by assumption, the nominal interest rate under the precommitment solution is applied to the government debt outstanding at the beginning of period s (equation (13a)), $x=x^p$ is realized as long as the government implements $\pi=0=q$ according to its precommitment from period s on. In practice, however, because nominal interest rates for those bonds outstanding at the beginning of period s are given subsequent to period s, the amount of tax can be reduced by raising p and lowering q. In contrast, it is recognized by the government that the nominal interest rates of bonds issued after period s are determined endogenously reflecting the expectations of the private sector. Thus, the governemnt regards it as a constraint that each element of $\{x_{s+T}, \cdots, x_\infty\}$ equals x^p.

The problem expressed by equations (12) and (13) is a dynamic optimization problem with q as a control variable and p as a state variable. The solution to this problem must satisfy the following Bellman equation:[33]

$$V(p_{t-1}) = \underset{\{q_t\}}{\text{Min}}\,[u(x_t)+w(q_t)+\beta V(p_t)], \qquad \text{s.t. (13a) – (13c)} \tag{14}$$

where $V(\cdot)$ is a value function, which can be interpreted as the sum of the present discounted value of $u(\cdot)+w(\cdot)$ in each period, when the time paths of p and q are given as a solution to the dynamic optimization problem.

By obtaining the first-order conditions for the minimization of q from equation (14) and following the procedure explained in the appendix, we obtain the following necessary condition for q_s:[34]

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[32]Equation (13a) is a budget constraint similar to equation (8). We obtain equation (13a) by substituting the nominal interest rate under the precemmitment solution into equation (8).

[33]See Stokey and Lucas (1989) for details on dynamic optimization, Bellman equation, and value function.

[34]For the procedure to obtain equation (15), see Appendix.

$$\theta[\gamma \sum_{j=0}^{T-1} \beta^j u'(x_{s+j})] - (1-\theta)u'(x_s) = w'(q_s)/x^P. \tag{15}$$

Equation (15) shows the marginal benefit and cost when q_s is increased by one unit in period s. In particular, the right hand side of equation (15) shows the cost associated with the deviation of the real exchange rate from purchasing power parity and the corresponding increase in the inflation rate when q_s increases. The second term on the left hand side shows the increased excess burden of tax in period s, when the rise of q increases the real value of foreign currency bonds maturing in period s. Finally, the first term on the left hand side shows that an increase in q_s eventually reduces the tax in each period after period s. This follows from the fact that an increase in q_s raises π_s and thus uniformly increases the goods prices after period s, thereby reducing the real value of all domestic currency bonds maturing after period s.

2. Necessary conditions for the precommitment solution

What conditions are necessary for the government pursuing discretionary policy management to choose the precommitment solution (*i.e.*, $q_s=0$)? A necessary condition is that equation (15) holds when $q_s=0$, $x_s=\cdots=x_{s+T}=x^P$, namely:

$$\theta[\gamma \sum_{j=0}^{T-1} \beta^j u'(x_p)] - (1-\theta)u'(x^P) = w'(0)/x^P \tag{16}$$

To put the argument another way, when θ is given to satisfy equation (16), the government chooses the precommitment solution in period s as a result of cost minimization.[35]

Let us summarize the preceding discussion of the solution under discretion. As stated at beginning of Section III, A, *1*, we examined whether or not the government in period s had an incentive to deviate from the precommitment solution, under the assumption that "the precommitment solution had been realized from period 0 through period $s-1$." Our analysis has indicated that the government will implement the precommitment solution in period s if θ is given to satisfy equation (16). Consequently, when equation (16) is satisfied, "the precommitment solution will be realized from period 0 through period s." This means that the government in period $s+1$ faces the same problem as the government in period s. Thus, as long as θ is given to satisfy equation (16), the precommitment solution is always realized. In other words, the solution under discretion coincides with the solution under precommitment.

[35]Strictly speaking, this conjecture may be incorrect. The criticism of Calvo and Obstfeld (1990) about the PPS model—that its condition is not a sufficient condition for a government pursuing discretionary policy to choose the precommitment solution—also applies to the derivation of the optimal currency composition analyzed in this paper. The condition used to derive the optimal currency composition is only one of several necessary conditions, and we have not proved that it is also sufficient.

D. The Optimal Currency Composition

1. *The formula for the optimal currency composition*

By substituting $w'(0)=0$ into equation (16) and solving for θ, we obtain the following formula for the optimal currency composition:

$$\theta^* = [1 + \gamma \sum_{j=0}^{T-1} \beta^j]^{-1}.$$

In a continuous time model, this corresponds to:

$$\theta^* = [1 + \gamma \int_0^T \exp(-\delta t)dt]^{-1} \tag{17}$$

where $\delta > 0$ is the discount rate. Equation (17) has the following implications.

2. *The relationship with the speed of adjustment of goods prices*

To understand the relationship between θ^* and γ, consider the two extreme cases where goods prices are perfectly flexible (*i.e.*, $\gamma \to +\infty$) and where the prices are perfectly rigid (*i.e.*, $\gamma=0$). First, as discussed in Lucas and Stokey as well as in PPS, when goods prices are perfectly flexible, we have, from quation (17):

$$\theta^* = 0.$$

Recall that, in Section II, to equate the solution under discretion with the solution under precommitment, the total value of government debt, including the balance of money supply, had to be equal to zero (see equation (7)). Because this section assumes that the balance of money is zero, the above result obtained under the assumption of perfectly flexible goods prices is the same as that obtained in Section II.

To understand this result, let us see how equation (9) changes as γ approaches infinity. By dividing both sides of equation (9) by γ and setting their limits as $\gamma \to +\infty$, we find that q also equals zero. This means that purchasing power parity always holds, so that the third term on the right hand side of equation (13a) is always zero. That is, the government cannot manipulate the real value of foreign currency bonds when goods prices are perfectly flexible. In this sense, foreign currency bonds are indistinguishable from real bonds. On the other hand, there is an incentive to reduce the real value of domestic currency bonds through inflation, because the inflation rate is under government control. Consequently, it is preferable to issue all debt as foreign currency bonds to achieve the precommitment solution.

Next, when goods prices are perfectly rigid (*i.e.*, $\gamma=0$), we obtain the following optimal currency composition from equation (17):

$$\theta^* = 1.$$

This simply says that, when goods prices are perfectly rigid, it is preferable to issue all debt as domestic currency bonds.

To interpret this result, go back again to equation (9). We find that π identically equals zero when $\gamma=0$, and that q is freely controllable, a sharp contrast to the case where $\gamma\rightarrow+\infty$. That is, while there is an incentive to reduce the real value of foreign currency bonds through deflationary monetary policy, there is no incentive to reduce the real value of domestic currency bonds through inflation. Consequently, it is appropriate to issue all debt as domestic currency bonds in this case.

Because we have $\theta^*=0$ when $\gamma\rightarrow+\infty$ and $\theta^*=1$ when $\gamma=0$, it is easy to imagine that, in an intermediate case between these two extremes, γ^* takes a value between 0 and 1, and that θ^* increases as the value of γ falls.

From equation (17), we can also immediately confirm that $\partial\theta^*/\partial\gamma<0$. To develop an intuitive understanding of this, we can take the government perspective in period s, as follows. First, suppose that goods prices are perfectly flexible (*i.e.*, $\gamma\rightarrow+\infty$). As we saw before, the optimal currency composition in this case is $\theta^*=0$ (*i.e.*, all debt is issued as foreign currency bonds). Next, suppose that γ approaches 0 in small increments. In this case, the real exchange rate can be moved away from purchasing power parity, so that the government in period s will probably want to reduce q.[36] To achieve the precommitment solution in period s, this incentive must be eliminated, which requires that we increase the composition of domestic currency bonds by an increment. This follows from the fact that, while a fall in the amount of foreign currency bonds weakens the incentive to reduce q, an increase in the amount of domestic currency bonds strengthens the incentive to increase π. Thus, it is possible to eliminate completely the incentive change associated with the fall in γ by appropriately adjusting the composition of domestic and foreign currency bonds incrementally. When γ approaches 0 further, a similar method can be employed to find the value of θ which completely eliminates the incentive to deviate from the precommitment solution. The value approaches unity monotonically as the value of γ falls.

3. *The relationship with maturity length*

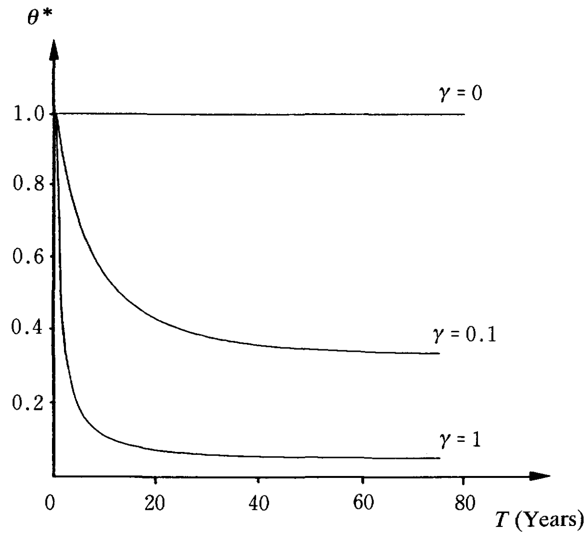
As is clear from equation (17), θ^* monotonically decreases when T increases (*i.e.*, $\partial\theta^*/\partial T<0$). This relationship is depicted in Figure 2. This relationship basically results from the convexity of $w(\cdot)$ with respect to π. Intuitively, this can be understood as follows.

Let us examine the difference in cost between the case where it takes one year for price to increase by 10% and the case where it takes two years to do so.[37] Where price

[36]However, because γ still takes a large value, the government must be willing to accept a substantial inflation (or deflation) rate if it wants to move the real exchange rate away from purchasing power parity. Thus, the government in period s has only a small incentive to reduce q.

[37]In the following discussion, we assume for simplicity that the discount rate is zero (*i.e.*, $\beta=1$). This, however, does not change the substance of the argument.

Figure 2
Optimal Currency Composition of Government Debt



Notes: The value of θ^* derived from equation (17). δ is set at 0.05.

increases by 10% in one year, the cost is $w\,(0.1)$ in the first period and $w\,(0)$ in the second period. On the other hand, where price increases by 5% per year for two years, the cost is $w\,(0.05)$ in both years. Thus, the problem is reduced to the comparison of $w\,(0.1)+w\,(0)$ and $2w\,(0.05)$. If we apply the previous section's discussion on tax smoothing (Section II, A), however, it is easy to see that inflation cost is minimized when inflation tax is levied uniformly over two years instead of only during the first year (*i.e.*, $w\,(0.1)+w\,(0)>2w\,(0.05)$).

The characteristic of a convex function shows that cost decreases with the length of time required for price to increase by a certain amount.[38] This means that, as the maturity of domestic currency bonds increases, the government in period s has a greater incentive to create inflation. In other words, as the maturity of government bonds increases, there is a greater risk that the real value of domestic currency bonds will be reduced by inflation.

In contrast, there is no advantege in tax smoothing with foreign currency bonds, because $w\,(\cdot)$ is a function not of the fluctuation of q but of its level.[39,40] Consequently,

[38]A similar result obtains when the optimal investment is calculated in situations where the cost of adjustment is expressed as a convex function. That is, cost increases as time for completion of a given project is shortened.

[39]To compare the cost of reducing q from 0 to -0.1 in two years, (i) the cost of completing the entire reduction in the first year is given by $w(-0.1)+w(-0.1)$; (ii) the cost of making the entire reduction in the second year is given by $w(0)+w(-0.1)$; and (iii) the cost of increasing q by -0.05 per year for two years is given by $w(-0.05)+w(-0.1)$. It is easy to see that (i) > (iii) > (ii), and that there is no advantage in tax smoothing.

[40]To be more realistic, it may be more reasonable to assume that there is a cost associated not only with the

the risk that the real value of foreign currency bonds will be reduced by manipulating the real exchange rate is independent of maturity length.

As T increases, there is a greater risk that the real value of domestic currency bonds will be reduced, while there is no change in the risk that the real value of foreign currency bonds will be reduced. Thus, to realize the precommitment solution,[41,42] foreign currency bonds must be increased at the expense of domestic currency bonds.

E. Numerical Example

To examine some of the specific values of the optimal currency composition given by equation (17), we will compute the value of θ^* by assuming realistic parameter values.

Of the parameters in equation (17), the most difficult parameter to estimate is the speed of adjustment of goods prices (γ). To give a reasonable value to this parameter, it may be useful to think of a situation where there is an unexpected change in the par value of a currency under the fixed exchange rate regime. At the instant when the par value changes, the real exchange rate surges away from zero. At the same time, a gradual price adjustment begins to reestablish purchasing power parity. If time is measured in terms of years, the value of γ shows the number of years it takes for this price adjustment.[43] For example, the amount of time necesssary for price to complete 70% of the required adjustment is one year when $\gamma=1.2$ and three years when $\gamma=0.4$.[44] However, the speed of adjustment of goods prices might change significantly according to circumstances, so it is not easy to choose a particular value. Thus, we will assume that the value takes the range of one to three years (*i.e.*, 0.4 and 1.2 in terms of γ) in calculating the numerical example below.

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level of the real exchange rate but also with its change. When we specify the model in this way, the quantitative characteristics of the model may change. As long as there is no cost associated with the price level, however, an asymmetry remains between the price level and the real exchange rate. Thus, the relationship that θ^* falls as the maturity length increases is unchanged.

[41]The relationship between T and θ^* can be interpreted from a different angle with equation (15). According to equation (15), the effect of an increase in q_s on the real value of debt applies only to foreign currency bonds maturing in periods, but to all domestic currency bonds maturing from period s to period $s+t$. Consequently, the incentive to reduce the real value of debt is independent of maturity length in the case of foreign currency bonds, but depends on maturity length with domestic currency bonds.

[42]This paper's analysis adds a theoretical explanation to the relationship between the incentive to reduce the real value of debt through inflation and maturity length, which was simply assumed by Missale and Blanchard (1991).

[43]If we let q_0 denote the level of the real exchange rate immediately after the jump, we can express the subsequent movement of q as $dq_t/dt=-\gamma q_t$, from equation (9) as well as the definition of q. By solving this equation, we have $q_t=q_0 \exp(-\gamma t)$, so we know that the value of γ indicates the speed with which q_t converges to zero.

[44]We need to solve $\exp(-\gamma t)=1-0.7$ to calculate the time required for price to complete 70% of the required adjustment. By making the approximtation of $\gamma t \fallingdotseq 1.2$, we obtain $t=1$ year when $\gamma=1.2$ and $t=3$ years when $\gamma=0.4$.

Table 1 shows the values of θ^* obtained from changing the values of γ and T.[45] From this, we find the following. First, when the maturity of bonds is greater than 10 years, it is necessary to increase the share of foreign currency bonds to over 80%. Second, in the case of short-term bonds with the maturities of less than three months, it is desirable to keep the share of domestic currency bonds at a level higher than 80%. This follows from the fact that, during a short period of this magnitude, the risk that the real value of foreign currency bonds will be reduced by depreciation of the foreign currency is greater than the risk that the real value of domestic currency bonds will be reduced by inflation.

Because this result is based on the assumption of identical maturities for domestic currency bonds and foreign currency bonds, caution is required when this argument is applied to cases where there are diverse maturities as in the United States. However, if we divide the bond market into two segments, so that long-term bonds are used to control a particular government incentive in the long-term bond market and short-term bonds are used to control a different government incentive in the short-term bond market, we can draw the following conlusions about today's currency composition of the U.S. government bonds.[46] First, it is preferable to issue long-term bonds in currencies other than the U.S. dollar, which is consistent with the assertion of Bohn (1991) and others. Second, the current situation, almost 100% of Treasury bills denominated in the U.S. dollar, is reasonable, as it is nearly consistent with the optimal currency composition

Table 1
Numerical Example of Optimal Currency Composition

the speed of adjustment of goods prices	Maturity T (in years)				
	0.25	1	5	10	30
γ = 0.2	0.95	0.84	0.53	0.39	0.24
0.4	0.91	0.72	0.36	0.24	0.14
0.6	0.87	0.63	0.27	0.17	0.10
0.8	0.83	0.56	0.22	0.14	0.07
1.0	0.80	0.51	0.18	0.11	0.06
1.2	0.77	0.46	0.16	0.10	0.05

Notes: The value of θ^* derived from equation (17). δ is set at 0.05.

[45] Table 3 assumes that the discount rate (δ) is 5% per year (*i.e.*, δ=0.05). We did not find a significant change in the value of θ^* by using a discount rate of up to 10% per year.

[46] Of course, there is no absolute need to divide up the government bond in this manner. It is possible to eliminate the government's future incentives to create inflation by, for example, allowing the composition to exceed the optimal value in the long-term bond and allowing it to fall short of the optimal value in the short-term bond, or vice versa.

obtained from the theoretical model of this paper.

IV. Conclusion

This paper reviewed the literature on the level and composition of government debt. To build upon this, it then analyzed the currency composition of government debt by relaxing the assumption about the speed of adjustment of goods prices. Because this paper's major results have already been summarized in Section I, here we will make only a few remarks about remaining issues and the direction of future research in this area.

As mentioned at the outset, the direct motivation for the current analysis of the currency composition of government debt came from the question of how to understand the existing situation where U.S. government debt is all denominated in the U.S. dollar. The analysis, as well as the numerical examples of Section III, E, offers one answer to this question. However, it should be noted that we derived these results keepig the deviation from the Recardian world as small as possible. In other words, this analysis consistently accepted three of the four assumptions of the Ricardian world noted at the beginning of Section II, namely: (1) there is a representative agent; (2) there is no uncertainty; (3) a lump-sum tax is available; and (4) government expenditures are given exogenously. Only assumption (3) was relaxed by introducing an excess tax burden. If we relax the other assumptions, we will be able to extend the analytical results in the following manner.

First, the paper has assumed that the private sector is represented by a single agent (Ricardian assumption (1)), and that this representative agent possesses all government debt and assumes all tax obligations associated with debt repayment. In reality, however, there are many private economic agents who have diverse patterns of debt holding and tax obligations. For example, in a situation where the private sector consists of the "old" (who hold all government debt) and the "young" (who pay all taxes associated with repayment),[47] creating surprise inflation not only reduces excess burden of taxation (as emphasized in this paper) but also transfers income from the old to the young. As a result, if the government finds income transfer of this kind desirable, it has a greater incentive for inflation than is the case where a representative agent is assumed. In another example, we can also consider a situation where the private sector consists of "citizens" (who pay all taxes associated with repayment) and "foreigners" (who hold all debt). In this case, the real devaluation of domestic currency bonds through inflation transfers income from "foreigners" to "citizens." If the government's first priority is the economic welfare of its own citizens, there should be a greater incentive for inflation.[48] In these examples, the relaxation of the representative agent assumption strengthens the

[47]This kind of situation is often assumed in an overlapping generations model.

[48]Bohn (1991) notes the important fact that a portion of the U.S. government debt is held by foreigners, and

government incentive for inflation, so there is a greater problem of time inconsistency. Consequently, it would be an interesting extension of this paper to relax the assumption of a representative agent.

Second, this analysis focused on the distinction between foreign currency bonds and domestic currency bonds. To be more realistic, however, there is the additional problem of choosing among bonds denominated in different foreign currencies. To consider this problem, it may be necessary to relax the assumption that there is no uncertainty about the future (Ricardian assumption (2)). For example, suppose that the income tax revenue of the United States varies stochastically with business fluctuations and that there is a tendency for the U.S. current account balance to improve and for the mark to depreciate in real terms when a recession reduces U.S. income tax revenue. Then, the U.S. government can reduce the risk of fluctuations of income tax revenue by issuing mark-denominated bonds. Thus, one can not only control the future government behavior, as emphasized in this paper, but also hedge the future risk by issuing nominal bonds denominated in domestic and foreign currencies. Incorporating this element into our analysis will make it possible to discuss more intelligently the question of whether the U.S. government should issue long-term bonds in the Deutsche mark or the Japanese yen.[49]

Third, the discussion in this paper focused on how to finance the given stream of government expenditures (Ricardian assumption (4)). It would be of interest, however, to extend the model to allow the government to determine both the financing method and financing amount endogenously. Practitioners suggest that the practice of issuing government bonds denominated only in U.S. dollars is a serious hindrance to cutting down government expenditures by reassuring the U.S. government and Congress that the real value of debt can easily be reduced by inflation.[50] In this context, it is important to strengthen our understanding of the relationship between government expenditures and the composition of debt from both theoretical and policy perspectives.[51]

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extends the analysis of the PPS model to the case where there is external debt. See Obstfeld (1989) for this argument as well as the problems of the Bohn's model.

[49]Bohn (1988, 1990) discusses the composition of government debt from the point of view of risk hedging. Calvo and Guidotti (1990b) introduce uncertainty into their analysis along the lines of Lucas and Stokey as well as Persson, Persson, and Svensson. They discuss the relationship between its incentive effect on future governments and the risk hedging effect.

[50]See, for example, Hosomi's article (reference in footnote 28).

[51]Obstfeld (1990) extends the framework of the PPS model in an attempt to endogenize government expenditures.

Appendix. Derivation of Equation (15)

From equation (14), we obtain the following first order conditions for the minimization of q_t:

$$[(1-\theta)-\theta\gamma]x^p u'(x_t) + w'(q_t) + \beta\theta V'(p_t)=0, \qquad t=s,\cdots,s+T-1 \tag{A-1a}$$

$$w'(q_t) = 0, \qquad t=s+T,\cdots,+\infty. \tag{A-1b}$$

As the interpretation of equation (A-1b) is obvious, we will look only at equation (A-1a) term by term. The left side of equation (A-1a) shows the marginal benefit and cost when q is incrementally increased by one unit in period t. The first term shows how the amount of tax in period t changes as q_t increases, as well as the resulting changes in excess burden of tax $u(\cdot)$. As is clear from equation (13a), an increase in q_t raises the real value of foreign currency bonds maturing in period t. At the same time, given p_{t-1}, an increase in q_t has the opposite effect of reducing the real value of domestic currency bonds maturing in period t. The former effect is reflected in the first element of the bracketed expression in the first term, while the latter effect is reflected in the second term. Whether the amount of tax increases or decreases depends on the sign of the bracketed expression. Next, the second term on the left side of equation (A-1a) shows that the cost $w(\cdot)$ rises with the increase in q_t, reflecting the fact that the real exchange rate deviates from purchasing power parity and the inflation rate in period t increases. Finally, the third term on the left hand side shows that the value function changes as the increase in q_t raises the value of the state variable p_t.

The following method obtains the expression in which $V'(p_t)$ is eliminated from the third term of the left hand side of equation (A-1a). First, by applying the envelop theorem to equation (14), we obtain:

$$V'(p_{t-1}) = \beta V'(p_t) - \theta x^p u'(x_t), \qquad t=s+1,\cdots,s+T-1. \tag{A-2}$$

This is a first-order differential equation in $V'(p)$. If we solve this equation, subject to the following initial condition:

$$V'(p_{s+T-1}) = 0$$

we obtain the following:

$$V'(p_t)=-\theta x^p \sum_{j=1}^{s+T-1-t} \beta^{j-1}u'(x_{t+j}), \qquad t=s,\cdots,s+T-1. \tag{A-3}$$

By substituting equation (A-3) into equation (A-1a) and rearranging, we obtain:

$$\theta[\gamma \sum_{j=0}^{s+T-1-t} \beta^j u'(x_{t+j})]-(1-\theta)u'(x_t) = w'(q_t)/x^p, \qquad t=s,\cdots,s+T-1. \tag{A-4}$$

By substituting $t=s$ into equation (A-4), we obtain equation (15) in the text.

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