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Dynamic Equilibrium Price Index: Asset Price and Inflation

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The purpose of this paper is to develop the theory of dynamic equilibrium price index (DEPI), and to discuss its theoretical as well as policy implications. DEPI is derived from intertemporal optimization and arbitrage equilibrium condition as a weighted geometric mean of product price inflation and asset price inflation. DEPI measures a change in ex ante intertemporal cost of living, which is a more fundamental indicator of inflation and the dynamic state of macroeconomy than conventional price indexes. Thus, DEPI functions as an important information variable for monetary policy.

I. Introduction

In the late 1980s, the Japanese economy experienced sharp increases in asset prices (stocks and real estate) while the prices of goods and services generally remained stable. In other words, a divergence in the movement of asset prices and product prices emerged. There are two contrasting views regarding this development. One view holds that this situation posed no danger of inflation because both CPI and the GDP deflator remained stable, and that asset price inflation of this period represented a relative price change between stock and flow prices, which should be distinguished from inflation defined as "a continuous rise in the general price level of goods and services."

Another view, however, regards asset price inflation as posing a threat to the monetary policy objective of price stability, and raises questions about the adequacy of CPI and other existing price indexes as the main information variables for conducting monetary policy. Is CPI a reliable indicator of a change in the inflationary trend? Is asset price relevant to the question of inflation? Should the monetary authorities pay attention to asset price inflation? These questions are becoming increasingly important because a

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successful monetary policy requires the correct understanding of the relationship between asset price and the phenomenon of inflation.

This paper addresses these questions and develops the theory of dynamic equilibrium price index (DEPI), which extends the idea of "intertemporal cost of living" proposed by Alchian and Klein (1973) and Pollak (1975).[1] The paper theoretically derives DEPI from intertemporal optimization and arbitrage equilibrium condition as a weighted geometric mean of product price inflation and asset price inflation, and discusses its theoretical as well as policy implications. Moreover, the paper actually calculates DEPI using Japanese data, and interprets its movement.

The main message of this paper is as follows: Existing price indexes such as CPI are insufficient information variables for conducting monetary policy because they only capture the current change in the price level while the monetary authorities are concerned with changing inflationary trend over future periods. DEPI is designed to capture a change in the inflationary trend in terms of *ex ante* intertemporal cost of living that is implicit in current product and asset markets. Such DEPI can be derived as a weighted geometric mean of product price and asset price changes. Our calculation shows that DEPI rose much faster than CPI and the GDP deflator in 1972-73, 1979-80, and 1986-89, which suggests that the inflationary trend shifted upward in these periods.

The rest of the paper is organized as follows: Section II defines intertemporal cost of living in terms of expenditure function, and then derives the intertemporal price index (IPI) as the ratio of expenditure functions for two intertemporal price regimes. Section III formulates and calculates DEPI, combining IPI with the implicit futures prices obtained from arbitrage equilibrium condition in the asset market. Section IV discusses the theoretical and policy implications of DEPI. Section V concludes this paper.

II. Intertemporal Cost of Living and Price Index

This section formalizes the intertemporal cost of living that is required to achieve a given level of intertemporal utility. This is achieved by the expenditure function whose argument is an intertemporal price vector, which gives rise to the intertemporal price index (IPI), that measures a change in intertemporal cost of living.

A. Intertemporal Cost of Living and Expenditure Function

Throughout this paper we assume a multi-period economy with one product. Let $P = (p_0, p_1, \ldots, p_n)$ stand for the intertemporal price vector, $X = (x_0, x_1, \ldots, x_n)$ for the corresponding consumption vector, and $R = (i_0, i_1, \ldots, i_{n-1})$ for the interest rate vector. Then, the present value of the intertemporal cost of living during these periods ($t = 0, 1, \ldots, n$ where $0 \leq n \leq \infty$) is given by

[1]See also Carlson (1989) and Shigehara (1990).

$$E = p_0 x_0 + \sum_{t=1}^{n} p_t x_t \prod_{s=0}^{t-1} (1+i_s)^{-1} \tag{1}$$

which may be rewritten as

$$E = \sum_{t=0}^{n} \hat{p}_t x_t \tag{2}$$

where

$$\hat{p}_0 = p_0 \quad \text{and} \quad \hat{p}_t = p_t \prod_{s=0}^{t-1} (1+i_s)^{-1} \quad (t \geq 1). \tag{3}$$

They represent the *futures prices* which we define as the present values of the future product prices.

Consumption vector $X = (x_0, x_1, \ldots, x_n)$, however, normally depends on present-value intertemporal price vector $\hat{P} = (\hat{p}_0, \hat{p}_1, \ldots, \hat{p}_n)$ because a consumer makes his intertemporal consumption plans given price vector $\hat{P}$. To capture this, consider the following intertemporal expenditure minimization problem:

$$E(\hat{P}, U) \equiv \min_{\{x_t\}} \sum_{t=0}^{n} \hat{p}_t x_t \quad \text{subject to } U(x_0, \ldots, x_n) = U \tag{4}$$

where $U(x_0, x_1, \ldots, x_n)$ represents the intertemporal utility function. Expenditure function $E(\hat{P}, U)$ represents the minimum cost of living that is required to achieve utility level U.[2]

The intertemporal utility function $U(\cdot)$ typically takes the following form:

$$U(x_0, x_1, \ldots, x_n) = \sum_{t=0}^{n} \left(\frac{1}{1+\rho}\right)^t u(x_t) \tag{5}$$

where ρ = the rate of time preference, $u(\cdot)$ = one-period utility function. Let $u(x_t) = \log x_t$ and exponentially transform equation (5). Then, we obtain the following Cobb-Douglas intertemporal utility function (we retain the same symbol U for convenience):

$$U(x_0, x_1, \ldots, x_n) = \prod_{t=0}^{n} x_t^{\alpha_t} \tag{6}$$

where

$$\alpha_t = (1+\rho)^{-t} / \{ \sum_{s=0}^{n} (1+\rho)^{-s} \} \tag{7}$$

These are the normalized factors of time preference, which add up to one.

Then, expenditure function (4) will take the following explicit form:

$$E(\hat{P}, U) = \left(\prod_{t=0}^{n} \alpha_t^{\alpha_t} \right)^{-1} \left(\prod_{t=0}^{n} \hat{p}_t^{\alpha_t} \right) \cdot U \tag{8}$$

which represents the minimum cost of living that is required to achieve utility level U under price vector $\hat{P}$. For a given U, therefore, the intertemporal cost of living may be regarded as a function of price vector $\hat{P}$.

[2]See Varian (1984) for the expenditure function.

B. Intertemporal Price Index

The intertemporal price index (IPI) is obtained from the above expenditure function, and is meant to capture a change in the intertemporal cost of living over future periods. Let $\hat{P}^T = (\hat{p}_0^T, \hat{p}_1^T, \ldots, \hat{p}_n^T)$ be the intertemporal price vector seen from year T, and $\hat{P}^S = (\hat{p}_0^S, \hat{p}_1^S, \ldots, \hat{p}_n^S)$ the intertemporal price vector seen from year S. Then, IPI is defined as the ratio of the two expenditure functions, which correspond to intertemporal price vectors $\hat{P}^T$ and $\hat{P}^S$:

$$\mathrm{IPI} = \frac{E(\hat{P}^T, U)}{E(\hat{P}^S, U)} \tag{9}$$

where U is a given intertemporal utility level. One shortcoming of this formulation is that IPI depends on the choice of utility level U. In other words, IPI cannot be uniquely determined: there are conceivably as many IPI as the number of possible U.

This problem will be avoided if we assume a homothetic (such as Cobb-Douglas) intertemporal utility function. This is because if the intertemporal utility function $U(\cdot)$ is homothetic, then the corresponding expenditure function can be written as follows:

$$E(\hat{P}, U) = e(\hat{P}) \cdot U \tag{10}$$

As a result, IPI becomes independent of utility level U and, therefore, becomes a function of intertemporal price vectors $\hat{P}^T$ and $\hat{P}^S$ alone:[3]

$$\mathrm{IPI} = \frac{E(\hat{P}^T, U)}{E(\hat{P}^S, U)} = \frac{e(\hat{P}^T) \cdot U}{e(\hat{P}^S) \cdot U} = \frac{e(\hat{P}^T)}{e(\hat{P}^S)} \tag{11}$$

This IPI measures a change in intertemporal cost of living as a result of the price regime change from S to T.[4]

With Cobb-Douglas intertemporal utility function (6), IPI can be written explicitly as:

$$\mathrm{IPI} = \prod_{t=0}^{n} \left(\frac{\hat{p}_t^T}{\hat{p}_t^S}\right)^{\alpha_t} \tag{12}$$

which is a function of intertemporal price vectors alone with parameter α_t given by equation (7). It shows that a change in intertemporal cost of living under a constant utility

[3]Malmquist (1953), Pollak (1971), and Samuelson and Swamy (1974) show that the homogeneity of the utility function is the necessary and sufficient condition for IPI to be independent of utility level U. See also Diewert (1976, 1988).

[4]Furthermore, it follows that the indirect utility function becomes $V(\hat{P}, I) = \{e(\hat{P})\}^{-1} I$. Thus, IPI can be written as $\mathrm{IPI} = E(\hat{P}^T, U)/E(\hat{P}^S, U) = V(\hat{P}^S, I)/V(\hat{P}^T, I)$. This shows that IPI may also be interpreted as measuring the decline in the intertemporal utility level under a given income I as a result of the price regime change from S to T.

level can be expressed as a weighted geometric mean of current and futures price changes $(\hat{p}_t^T/\hat{p}_t^S)$.[5]

III. Dynamic Equilibrium Price Index

The previous section has shown that a change in intertemporal cost of living can be measured by the intertemporal price index (IPI). IPI, however, is not operational because futures prices are not directly observable in general. This section resolves this problem as follows: First, from an arbitrage equilibrium condition, we derive futures prices that are implicit in the asset market. Then, combining these implicit futures prices with IPI, we obtain the dynamic equilibrium price index (DEPI), which measures a change in *ex ante* intertemporal cost of living.

A. Asset Price and Futures Prices

According to economic theory, asset price represents the present value of asset returns over future periods. Thus, if we assume a constant marginal product of the aggregate asset of the economy, profit maximization implies that the rate of return on asset (s_t) must be equal to product price (p_t) times the marginal product of asset (MP_K). Thus, asset price (q_0) is given by

$$\begin{aligned} q_0 &= \sum_{t=1}^{\infty} (1-\delta)^{t-1} s_t \prod_{s=0}^{t-1} (1+i_s)^{-1} \\ &= \prod_{t=1}^{\infty} (1-\delta)^{t-1} p_t \cdot MP_K \prod_{s=0}^{t-1} (1+i_s)^{-1} \end{aligned} \tag{13}$$

where δ represents rate of depreciation. It is assumed that production takes one period.

The above equation can be rewritten using futures prices ($\hat{p}_t$) as follows:

$$q_0 = MP_K \cdot \sum_{t=1}^{\infty} (1-\delta)^{t-1} \cdot \hat{p}_t \tag{14}$$

which shows the relationship between asset price and futures prices of the product.

Next, let us define the mean futures price of the product ($\hat{p}_f$) by

$$\sum_{t=1}^{\infty} (1-\delta)^{t-1} \cdot \hat{p}_f = \sum_{t=1}^{\infty} (1-\delta)^{t-1} \cdot \hat{p}_t \tag{15}$$

Then, equation (14) can be written as follows:

$$q_0 = \hat{p}_f \cdot MP_K/\delta \tag{16}$$

Thus, the arbitrage equilibrium condition of the asset market implies that the mean futures price of the product can be written as a function of the asset price as follows:

$$\hat{p}_f = \delta \cdot q_0/MP_K \tag{17}$$

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[5]The conventional price indexes such as CPI and the GDP deflator correspond to a special case of IPI with $n = 0$. In other words, they belong to "one-period price indexes."

B. DEPI and *Ex Ante* Intertemporal Cost of Living

The dynamic equilibrium price index (DEPI) is obtained from combining the intertemporal price index (IPI) and the implicit mean futures price of the product ($\hat{p}_f$). Substituting $\hat{p}_f$ for $\hat{p}_t$ ($t \geq 1$) in equation (12), we can formulate DEPI as follows:

$$\begin{aligned} \text{DEPI} &= \left(\frac{p_0^T}{p_0^S}\right)^{\alpha_0} \cdot \prod_{t=1}^{n} \left(\frac{\delta \cdot q_0^T / MP_K}{\delta \cdot q_0^S / MP_K}\right)^{\alpha_t} \\ &= \left(\frac{p_0^T}{p_0^S}\right)^{\alpha_0} \cdot \left(\frac{q_0^T}{q_0^S}\right)^{1-\alpha_0} \end{aligned} \tag{18}$$

where $\alpha_0 = 1 / \sum_{s=0}^{n} (1+\rho)^{-s}$. Note that we have assumed MP_K to be constant.

DEPI compares *ex ante* intertemporal cost of living under price regime T with that under price regime S; it measures a change in *ex ante* intertemporal cost of living that is necessary to achieve the same utility level under the two price regimes. As a higher intertemporal cost of living means the lower intertemporal purchasing power of money, DEPI may be regarded as a new index of inflation which measures a change in the "dynamic value of money" as opposed to the "static value of money," which one-period price indexes measure.

Conventional price indexes such as CPI and the GDP deflator measure a change in the current cost of living, and therefore do not necessarily measure a change in the true intertemporal cost of living. DEPI is designed to remedy this shortcoming of one-period price indexes. DEPI is derived from optimization and market equilibrium as a geometric mean of product price and asset price changes. That is to say, a change in *ex ante* intertemporal cost of living or a change in the dynamic value of money can be measured in terms of the weighted geometric mean of product price inflation and asset price inflation.

The reason for the importance of asset price in the measurement of intertemporal cost of living may be understood intuitively as follows: Assume, for simplicity, that one unit of labor and asset produce one unit of product in each period, and that the asset does not depreciate. Then, there are two ways to obtain the future product: one is to purchase it in each period and the other is to purchase the asset at the outset. Both methods should cost the same in equilibrium through arbitrage.

C. DEPI and Changing Inflationary Trend

Let us calculate DEPI and compare its movement with CPI and GDP deflator. To calculate DEPI, we use the GDP deflator for the current product price (p_0) and create an asset price series (q_0) using data in *Annual Report on National Accounts* (Economic Planning Agency). We also estimate the parameter value $\rho = 0.03$ and set $n = \infty$ (Appendix B).

Our calculation shows that a divergence in the movement of DEPI and conventional

Figure 1
DEPI and GDP Deflator
(year-to-year percent change)

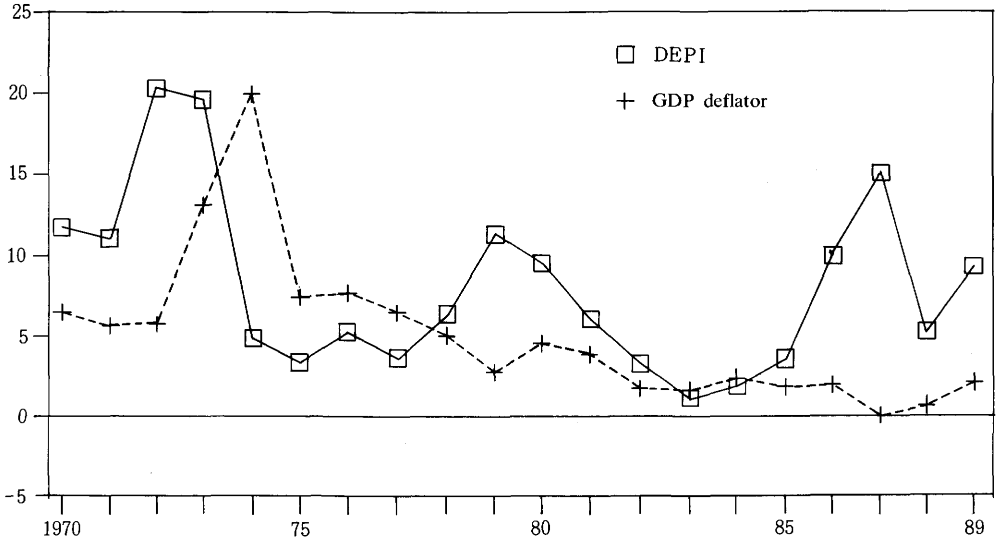
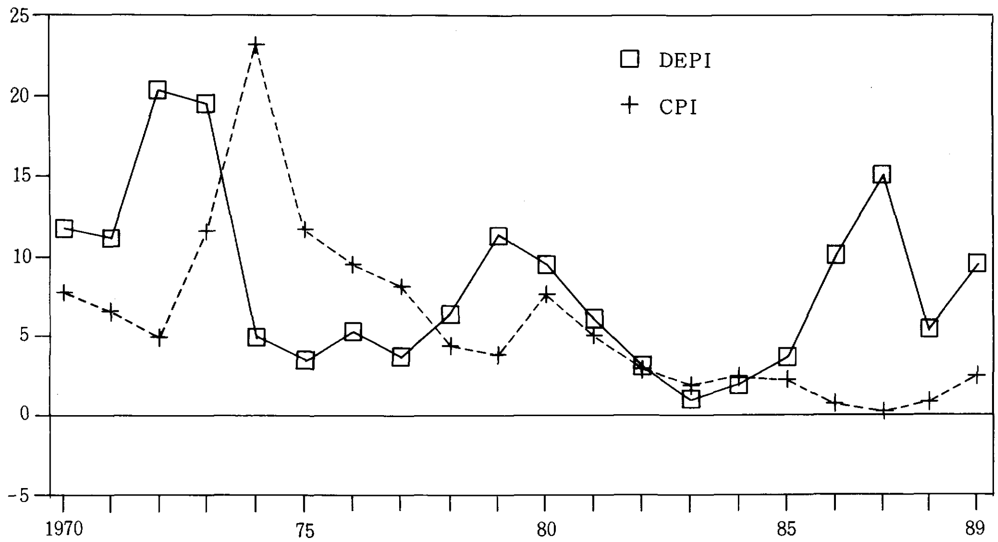


Figure 2
DEPI and CPI
(year-to-year percent change)



price indexes (CPI and the GDP deflator) occurred three times during 1970-89 (Figures 1 and 2).[6] The first time was in the early 1970s before the first oil crisis and galloping product price inflation. In particular, DEPI rose sharply in 1972-73, indicating an upward shift in the inflationary trend. Neither CPI nor the GDP deflator, however, showed any sign of a worsening inflationary trend until 1973. In fact, movement in CPI gave a false impression that inflation was declining during 1970-72. Moreover, DEPI showed a downward shift in the inflationary trend in 1974 following the restrictive monetary policy that started in the previous year, while CPI and the GDP deflator continued to rise.

The second time was in 1979-80 when the second oil crisis came. Neither CPI nor the GDP deflator rose until 1980. The stance of monetary policy, however, changed swiftly in 1979 partly because the monetary authorities had learned from the experience of the first oil crisis and galloping product price inflation and partly because WPI started to rise in 1979. As a result, inflation remained moderate throughout this period, and the adverse impact of the oil crisis on real economic activity was limited. More importantly, the stable monetary policy maintained throughout the second half of the 1970s helped create a noninflationary economic environment.

The third time was in 1986-89, during which asset prices rose sharply while the prices of goods and services remained relatively stable. Speculation, excess liquidity, and the distortional nature of the property tax system and land policy have been cited as factors that might have contributed to the asset price inflation of this period. DEPI indicated that *ex ante* intertemporal cost of living rose, suggesting the need for restrictive monetary policy during this period. CPI and the GDP deflator, on the other hand, indicated little need for a change in the stance of monetary policy. In response to increasing concern about asset price inflation and tightening labor market conditions, however, Bank of Japan raised its official discount rate in 1989, signaling a change in the stance of monetary policy.

IV. Theoretical and Policy Implications

This section further explores the theoretical and policy implications of DEPI. In particular, it relates DEPI to the Wicksellian view of inflation as a phenomenon of disequilibrium dynamics, which illustrates the signaling role of DEPI with respect to the deviation of the real interest rate from the equilibrium rate. The deviation of the real interest rate will cause disequilibrium dynamics and change the inflationary trend. Such a development will be reflected in the movement of DEPI before product price starts to rise.

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[6]See Bank of Japan (1986), Komiya (1988), and Fujii and Kawahara (1990) for a history of the Japanese economy and monetary policy during this period.

According to Wicksell, inflation is a phenomenon of disequilibrium dynamics caused by the deviation of the real interest rate from the "natural rate of interest."[7] The deviation of the real interest rate will directly bring about the intertemporal misallocation of economic resources and the distortion of income distribution, and subsequently product price inflation. That is to say, economic dislocation will occur when the real interest rate is not properly adjusted, and a rising product price in subsequent periods will signal the existence of interest rate maladjustment.[8] This maladjustment, however, will be captured more directly by DEPI.

Consider the following Wicksellian process: Imagine an ideal macroeconomy on a balanced growth path. We call the real interest rate that will support this economy the natural rate of interest. Now, suppose the real interest rate has fallen below the natural rate for some reason.[9] Then, the asset price and Tobin's q will rise along with investment, despite the fact that the marginal product of the economy has not changed.[10] The resulting economic boom, however, will lead to a rising product price because the economy temporarily grows faster than the potential growth rate which is determined by the marginal product of the economy.

The presence of such disequilibrium and changing inflationary trend will be captured by DEPI when conventional price indexes may reveal no such risks. To see this, rewrite equation (18) as follows:

$$\mathrm{DEPI} = \left(\frac{p_0^T}{p_0^S}\right) \cdot \left(\frac{q_0^T / p_0^T}{q_0^S / p_0^S}\right)^{1-\alpha_0} \tag{19}$$

where (q_0 / p_0) is the ratio of the present value of returns on asset to its replacement cost, which is nothing but Tobin's q. According to the q theory, investment is an increasing function of Tobin's q.[11] Asset accumulates if Tobin's q is greater than one (the balanced growth path value of Tobin's q in the simple model) and it decumulates if it is less than one. Therefore, if a deviation of Tobin's q from one is caused by the deviation of the real interest rate from the natural rate, DEPI will signal the risk of dynamic disequilibrium and inflation faster than one-period price indexes.

It should be noted that the informational value of DEPI depends on the assumption that the long-term marginal product of the economy is relatively stable. If this assumption should not hold, DEPI movements *per se* would not necessarily imply the maladjustment

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[7]See Wicksell (1989, 1906), Leijonhufvud (1981), and Humphrey (1990).

[8]In addition to its signaling role, product price inflation will have its own distortional effects on the price system as it affects intratemporal relative prices such as real wages, effective tax rates, etc.

[9]For example, when expected inflaion is not completely reflected in the nominal interest rate or monetary policy becomes expansionary, the real interest rate will decline at least for a certain period of time.

[10]Substituting $p_t = p_0(1+\pi)^t$ and $i_t = i$ into equation (13) and using the definition of the real interest rate $(1+r) = (1+i) / (1+\pi)$, we obtain the following relationship: $(q_0 / p_0) = MP_K / (r+\delta)$. This implies that a decline in the real interest rate (r) will raise Tobin's q ($= q_0 / p_0$) and investment without any change in MP_K.

of the interest rate. This is because DEPI could also rise if the marginal product of the economy should increase due to productivity shocks. This problem would be resolved if we could explicitly introduce the change in the marginal product $(MP_K{}^T / MP_K{}^S)$ in equation (18) in place of the assumption that MP_K remains constant. But, this would require the precise knowledge of the long-term productivity of the economy, which we do not normally possess.

It is possible, however, to tell whether a rise in (q_0/p_0) is due to increased productivity of the economy or due to the maladjustment of the interest rate because the real interest rate of the economy should respond differently. Consider the case in which an increase in (q_0/p_0) was caused by increased productivity of the economy (MP_K). What would happen to the real interest rate? The real interest rate (r_t) should go up because both consumption and investment would increase as a result of the productivity shock and the increases in consumption and investment would require $r_t > \rho$ and $(q_t/p_t) > 1$ during the transition to the new balanced growth path.[12]

Therefore, a rising ratio of asset price to product price (q_t/p_t) and a rising real interest rate (r_t) should be associated with a positive productivity shock, while a rising (q_t/p_t) and a falling r_t should be associated with the maladjustment of the interest rate. In the former case, the real interest rate is adjusting properly; in the latter case, the real interest rate should be raised. The important point is that, to steer the economy to a balanced growth path, the real interest rate must be guided, either by market forces or by economic policy, to move in the same direction as the ratio of asset price to product price.

V. Conclusion

The purpose of this paper has been to develop the theory of dynamic equilibrium price index (DEPI), and to discuss its theoretical as well as policy implications. DEPI is derived from intertemporal optimization and arbitrage equilibrium condition as a weighted geometric mean of product price inflation and asset price inflation. DEPI measures a change in *ex ante* intertemporal cost of living, which is a more precise measure of a change in the true cost of living than one-period price indexes (CPI or the GDP deflator). DEPI is, moreover, compatible with the Wicksellian view of inflation as a phenomenon of disequilibrium dynamics, which will be reflected in the movement of DEPI before product price actually starts to rise. Our calculation shows that DEPI rose much faster than CPI and the GDP deflator in 1972–73, 1979–80, and 1986-89, which suggests that the inflationary trend shifted upward in these periods.

[11] See Tobin (1969), Tobin and Brainard (1977), and Hayashi (1982).

[12] In a balanced growth path (with no population growth and no technological progress), the following must hold: $r_t = \rho$ and $(q_t/p_t) = 1$, and therefore $MP_K = \rho + \delta$. See, for example, Blanchard and Fischer (1989).

There are, however, several practical and theoretical issues that remain as the agenda for future research. First, the accurate calculation of DEPI requires reliable and promptly available data on a comprehensive range of asset prices. Second, the effects of long-term productivity changes need to be adjusted in order to improve the informational value of DEPI. Third, further analysis of DEPI is warranted within the framework of an explicit dynamic model that incorporates the process of disequilibrium dynamics and the phenomenon of inflation.

Appendix A.

This appendix presents a numerical example of DEPI in a simple two-period economy ($t = 0$, 1 and $\delta = 1$). It is intended to provide the reader with an overview of the DEPI theory. For simplicity, we assume in this example that $\rho = 0.03$ and $MP_K = 1$.

(1) Suppose that the price structure of the product and asset markets have changed from regime S to regime T (Table A-1).

Table A-1
Changing Price Structure

		S	$\rightarrow$	T
Product Market:	Product price (p_0)	100		105
Asset Market:	Market interest rate (i)	0.05		0.04
	Asset price (q_0)	100		115

Now, the arbitrage equilibrium condition of the asset market is given by:

$$q_0 = p_1 \cdot MP_K/(1+i)$$

Note that, in this example, the asset is assumed to depreciate completely after one period ($\delta = 1$).

Therefore, future price (p_1) and futures price ($\hat{p}_1$), which is the present value of the future price, must satisfy the following relationship:

$$p_1 = (1+i)q_0/MP_K \quad \text{and} \quad \hat{p}_1 = q_0/MP_K$$

Using this relationship, future price (p_1) and futures price ($\hat{p}_1$) of the product are obtained as in Table A-2.

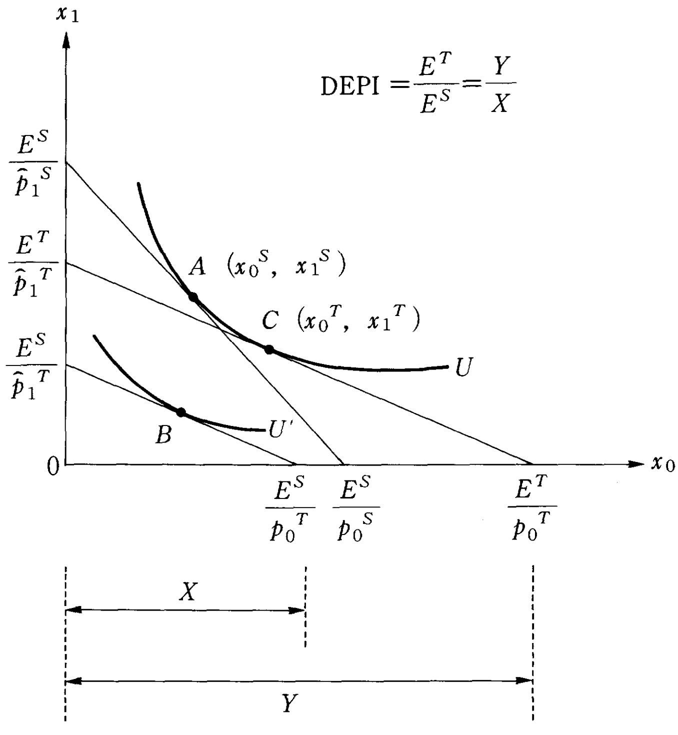
Table A-2
Changing Future and Futures Product Prices

	S	$\rightarrow$	T
Future price (p_1)	105		120
Futures price ($\hat{p}_1$)	100		115

(2) Next, consider how intertemporal cost of living will be affected as a result of the regime change from S to T. Suppose that intertemporal consumption vector $A = (x_0^S, x_1^S)$ is chosen under price regime S (Figure A). Let U and E^S be the corresponding utility level and the intertemporal cost of living, respectively. Now, if price regime changes from S to T, the intertemporal consumption vector will move from point A to B while utility level declines from U to U'. Therefore, to maintain the original level of utility, intertemporal cost of living must be raised from E^S *to* E^T. Then, the consumer can choose consumption vector $C = (x_0^T, x_1^T)$ which gives utility U.

In short, intertemporal cost of living increases from E^S to E^T as price regime changes from S to T. Such a change in intertemporal cost of living can be measured by using the

Figure A
Dynamic Equilibrium Price Index



following expenditure function:

$$E(\hat{P}, U) \equiv \min_{\{x_t\}} \sum_{t=0}^{1} \hat{p}_t x_t \quad \text{subject to } U(x_0, x_1) = U$$

where $\hat{P} = (\hat{p}_0, \hat{p}_1)$ and $\hat{p}_t = p_t/(1+i)^t$. If we assume a Cobb-Douglas (more generally, homothetic) intertemporal utility function for $U(x_0, x_1)$, the corresponding expenditure function can be written as $E(\hat{P}, U) = e(\hat{P}) \cdot U$.

(3) DEPI measures a change in the intertemporal cost of living from price regime S to price regime T that is necessary to achieve the same level of utility. If we take S as the base year, DEPI can be expressed as DEPI $= E(\hat{P}^T, U)/E(\hat{P}^S, U) = e(\hat{P}^T)/e(\hat{P}^S)$. As has been explained in the text, it can be expressed in the following final form:

$$\text{DEPI} = \left(\frac{p_0^T}{p_0^S}\right)^{\alpha_0} \cdot \left(\frac{\hat{p}_1^T}{\hat{p}_1^S}\right)^{1-\alpha_0}$$

$$= \left(\frac{p_0^T}{p_0^S}\right)^{\alpha_0} \cdot \left(\frac{q_0^T}{q_0^S}\right)^{1-\alpha_0}$$

where the Cobb-Douglas intertemporal utility function $U(x_0, x_1) = x_0^{\alpha_0} \cdot x_1^{1-\alpha_0}$ is assumed.

Substituting the numbers and parameter values used in the present example, DEPI can be calculated as follows:

Table A-3
DEPI and Inflation

	S	$\rightarrow$	T
One-period price index (p_0)	100		105
Dynamic equilibrium price index (DEPI)	100		110

That is to say, while the rate of inflation is only 5% according to the conventional one-period price index which measures a change in the current cost of living , it is 10% according to DEPI which measures a change in the intertemporal cost of living.

Appendix B.

This appendix states the calculation method of DEPI and the data sources that have been used in this paper.

(1) We take 1980 as the base year (S), and 1970-89 as the sample period.

(2) We use the GDP deflator for the current product price (p_0).

(3) We obtain the asset price (q_0) from stock data in *Annual Report on National Accounts* (Economic Planning Agency). The rate of increase in the asset price is calculated as the ratio of the valuational adjustment in national net assets (national wealth) to the value of the asset at the end of the previous year. The asset price index is thus derived.

(4) The rate of time preference (ρ) is estimated from the modified golden rule: $\rho = f - \delta - z - g$ where f = real return on asset, ρ = rate of depreciation, z = growth rate of labor, g = rate of technical progress.

f is estimated to be 0.13 as the average ratio of private sector operational profits to private sector net assets; δ is estimated to be 0.06 as the average ratio of private sector asset depreciation to private sector net asset. z is estimated to be 0.01 from labor data; g is estimated to be 0.03 as the difference between the growth rate of real GDP (0.04) and the growth rate of labor ($z = 0.01$). Thus, we obtain $\rho = 0.03$.

(5) Finally, we calculate DEPI using equation (18) and the above data and parameter values.

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